SECTION - A

1. **(d)**
2. **(b)**
3. **(b)**
4. **(d)**
5. **(c)**
6. **(b)**
7. **(c)**
8. **(c)**
9. **(a)**
10. **(a)**
11. **(c)**
12. **(b)**
13. **(b)**
14. **(c)**
15. **(a)**
16. **(c)**
17. **(d)**
18. **(c)**
19. **(d)** Assertion (A) is false, but Reason (R) is true.
20. **(d)** Assertion (A) is false, but Reason (R) is true.

SECTION B

21. (a) Let $u = 2^{\cos^2 x} \Rightarrow \frac{du}{dx} = 2^{\cos^2 x}(-2\cos x \sin x)\log 2$

Let $v = \cos^2 x \Rightarrow \frac{dv}{dx} = -2\cos x \sin x$

Now $\frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)} = 2^{\cos^2 x}\log 2$

OR

(b) $\tan^{-1}(x^2 + y^2) = a^2 \Rightarrow x^2 + y^2 = \tan a^2$

Differentiate both sides wrt x,

$$2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

22. $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

$= \tan^{-1}\left[2\sin\left(2 \times \frac{\pi}{6}\right)\right] = \tan^{-1}\left[2\sin\frac{\pi}{3}\right]$

$= \tan^{-1}\left[2 \times \frac{\sqrt{3}}{2}\right] = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

23. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -2\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$

Area of parallelogram $= \frac{1}{2}|\vec{a} \times \vec{b}|$

$= \frac{1}{2}\sqrt{(-2)^2 + 3^2 + 7^2} = \frac{\sqrt{62}}{2}$

24. $f(x) = 5x^{3/2} - 3x^{5/2} \Rightarrow f'(x) = \frac{15}{2}\sqrt{x}(1 - x)$

For increasing / decreasing, put $f'(x) = 0$

$\Rightarrow x = 0,1$

(i) When $x \in [0,1], f'(x) \geq 0$.So, f is increasing when $x \in [0,1]$
(The intervals $(0,1), [0,1)$ or $(0,1]$ canalso beconsidered.)
(ii) When $x \in [1, \infty), f'(x) \leq 0$.So, f is decreasing when $x \in [1, \infty)$
(Theinterval $(1, \infty)$ can also beconsidered.)

25. **(a)** Let the required angle between the kite strings be θ.

Then, $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$

$\Rightarrow \cos\theta == \dfrac{(3\hat{\imath}+\hat{\jmath}+2\hat{k})(2\hat{\imath}-2\hat{\jmath}+4\hat{k})}{\sqrt{9+1+4}\sqrt{4+4+16}} = \dfrac{12}{\sqrt{336}} = \dfrac{3}{\sqrt{21}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{12}{\sqrt{336}}\right)\text{or}\cos^{-1}\left(\dfrac{3}{\sqrt{21}}\right)$

OR

(b)$\overrightarrow{BA} = -6\hat{\imath}+2\hat{\jmath}+3\hat{k}$
Required unit vector of magnitude 21

$= 21 \times \left(\dfrac{-6\hat{\imath}+2\hat{\jmath}+3\hat{k}}{\sqrt{36+4+9}}\right)$

$= 3(-6\hat{\imath}+2\hat{\jmath}+3\hat{k})$ or $-18\hat{\imath}+6\hat{\jmath}+9\hat{k}$

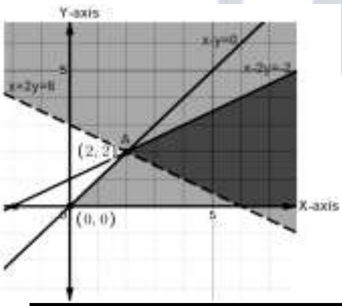
SECTION C

26. Let' a ' be the side of the triangle, $50\frac{da}{dt} = 3$ cm/s Now area of an equilateral triangle, $A = \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \dfrac{dA}{dt} = \dfrac{\sqrt{3}a}{2} \times \dfrac{da}{dt}$

$\therefore \left.\dfrac{dA}{dt}\right]_{a=15\text{ cm}} = \dfrac{\sqrt{3}\times 15}{2} \times 3 = \dfrac{45\sqrt{3}}{2}\ \text{cm}^2/\text{s}$

27.



Corner Point	Value of $Z = x + 2y$
$O(0,0)$	0
$A(2,2)$	6

Since feasible region is unbounded. Plot $x + 2y > 6$ which has common region with feasible region, thus Z has no maximum value.

28. **(a)** $\int \frac{x+\sin x}{1+\cos x}dx$

$= \int \dfrac{x + 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx$

$= \int x\left(\dfrac{1}{2}\sec^2\dfrac{x}{2}\right)dx + \int \tan\dfrac{x}{2}dx$

$= x\tan\dfrac{x}{2} - \int \tan\dfrac{x}{2}dx + \int \tan\dfrac{x}{2}dx$

$= x\tan\dfrac{x}{2} + C$

OR

(b) $\int_0^{\pi/4} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

$= \frac{1}{2}\int_0^{\pi/4} \frac{dx}{\cos^4 x\sqrt{\tan x}}$

$= \frac{1}{2}\int_0^{\pi/4} \frac{(1+\tan^2 x)\sec^2 x}{\sqrt{\tan x}} dx$

Put $\tan x = t \Rightarrow \sec^2 x dx = dt$

$I = \frac{1}{2}\int_0^1 \frac{1+t^2}{\sqrt{t}} dt$

$= \frac{1}{2}\int_0^1 \left(\frac{1}{\sqrt{t}} + t^{3/2}\right) dt$

$= \frac{1}{2}\left[2\sqrt{t} + \frac{2}{5}t^{5/2}\right]_0^1$

$= \frac{6}{5}$

29. **(a)** Rewriting the lines, we get

$\vec{r} = (\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) + \lambda(-\hat{\imath} + \hat{\jmath} - 2\hat{k})$ and $\vec{r} = (\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$

Let $\vec{a}_1 = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}, \vec{a}_2 = \hat{\imath} - \hat{\jmath} - \hat{k}, \vec{b}_1 = -\hat{\imath} + \hat{\jmath} - 2\hat{k}, \vec{b}_2 = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

Note that the dr's of given lines are not proportional so, they are not parallel lines. The lines will be skew if they do not intersect each other also.

Here$\vec{a}_2 - \vec{a}_1 = \hat{\jmath} - 4\hat{k}, \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = 2\hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

Consider $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$

$= (\hat{\jmath} - 4\hat{k}) \cdot (2\hat{\imath} - 4\hat{\jmath} - 3\hat{k}) = 8 \neq 0$

Hence lines will not intersect. So the lines are skew.

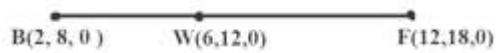
Shortest Distance $= \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$

$= \frac{8}{\sqrt{4+16+9}} = \frac{8}{\sqrt{29}}$

OR

(b) Let the wicket keeper divides the line segment in ratio $k:1$

$\vec{W} = \frac{k\vec{F} + 1.\vec{B}}{k+1}$



$\Rightarrow 6\hat{\imath} + 12\hat{\jmath} = \left(\frac{12k+2}{k+1}\right)\hat{\imath} + \left(\frac{18k+8}{k+1}\right)\hat{\jmath}$

$\Rightarrow k = \frac{2}{3}$

Hence, the required ratio is $2:3$

30. **(a)** **(i)**

Since $\sum P(X) = 1 \Rightarrow p + 2p + 3p + p = 1$

$\Rightarrow p = \frac{1}{7}$

(ii) Mean $= \sum X.P(X) = 0(p) + 2(2p) + 4(3p) + 5(p)$

$= 21p = 21\left(\frac{1}{7}\right) = 3$

OR

(b) Let E_1 : The applicant is a male

E_2 : The applicant is a female

A : The candidate chosen will have distinction in the written test.

$P(E_1)=\frac{1}{3}, P(E_2)=\frac{2}{3}, P(A \mid E_1)=0.4, P(A \mid E_2)=0.35$

$P(A)=P(E_1)P(A \mid E_1)+P(E_2)P(A \mid E_2)$

$=\frac{1}{3}\times 0.4+\frac{2}{3}\times 0.35$

$=\frac{11}{30}$

31. Required Area

$=\int_{-6}^{0} y dx$

$=2\int_{-3}^{0}(x+3)dx$

$=2\left[\frac{(x+3)^2}{2}\right]_{-3}^{0}$

$=9$

SECTION D

32. **(a)** Let $x=\sin A, y=\sin B \Rightarrow A=\sin^{-1}x, B=\sin^{-1}y$

$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

$\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$

$\Rightarrow 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)=2a\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

$\Rightarrow \cot\left(\frac{A-B}{2}\right)=a \Rightarrow A-B=2\cot^{-1}a$

$\Rightarrow \sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$

differentiate both sides wrt x,

$$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

OR

(b) $x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right)$

$\Rightarrow \frac{dx}{d\theta}=a\left(-\sin\theta+\frac{1}{\tan\frac{\theta}{2}}\times\sec^2\frac{\theta}{2}\times\frac{1}{2}\right)$

$=a\left(-\sin\theta+\frac{1}{\sin\theta}\right)=a\left(\frac{1-\sin^2\theta}{\sin\theta}\right)$

$\frac{dx}{d\theta}=a\cot\theta\cos\theta$

Also, $y=\sin\theta \Rightarrow \frac{dy}{d\theta}=\cos\theta$

$\therefore \frac{dy}{dx}=\frac{\tan\theta}{a}$

Differetiating wrt x,

$\frac{d^2y}{dx^2}=\frac{\sec^2\theta}{a}\times\frac{d\theta}{dx}$

$$= \frac{\sec^3 \theta \tan \theta}{a^2}$$

$$\left.\frac{d^2y}{dx^2}\right]_{\text{at}\theta=\frac{\pi}{4}} = \frac{2\sqrt{2}}{a^2}$$

33. $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow f'(x) = 6(x^2 - 5x + 6) = 6(x-2)(x-3)$

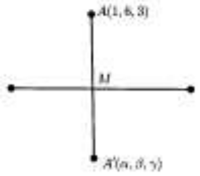
$f'(x) = 0 \Rightarrow x = 2,3 \in [1,5]$

Now $f(1) = 24, f(2) = 29, f(3) = 28, f(5) = 56$

Hence, the absolute maximum value is 56 and the absolute minimum value is 24 .

34. (a) The equation of given line is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$

Any arbitrary point on the line is $M(\lambda, 2\lambda + 1, 3\lambda + 2)$



dr's of AM are $< \lambda - 1, 2\lambda - 5, 3\lambda - 1 >$

Here$1(\lambda - 1) + 2(2\lambda - 5) + 3(3\lambda - 1) = 0$

$\Rightarrow \lambda = 1$

$M(1,3,5)$ is the foot perpendicular of the point **A** to the given line.

Let image of point **A** in the line be $\boldsymbol{A'}(\alpha, \beta, \gamma)$

Since M is the mid-point of AA', so $M\left(\frac{1+\alpha}{2}, \frac{6+\beta}{2}, \frac{3+\gamma}{2}\right) = M(1,3,5)$

$\Rightarrow A'(1,0,7)$ is the image of A .

Also, Equation of AA' is $\frac{x-1}{0} = \frac{y-6}{-3} = \frac{z-3}{2}$

OR

(b) The given line is $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9} = \lambda$ and $Q(2,4,-1)$

Any random point on the line will be given by $P(\lambda - 5, 4\lambda - 3, -9\lambda + 6)$

Since $PQ = 7 \Rightarrow \sqrt{(\lambda - 7)^2 + (4\lambda - 7)^2 + (-9\lambda + 7)^2} = 7$

$\Rightarrow 98(\lambda^2 - 2\lambda + 1) = 0 \Rightarrow \lambda = 1$

Hence, the required point is $P(-4,1,-3)$

The equation of line PQ is $\frac{x+4}{6} = \frac{y-1}{3} = \frac{z+3}{2}$ or $\frac{x-2}{6} = \frac{y-4}{3} = \frac{z+1}{2}$

35. Let x, y and z be the no. of students allocated to Sports, Music and Drama clubs respectively.

Here, $x = y + z, y = \frac{x}{2} + 20, x + y + z = 180$

$\Rightarrow x - y - z = 0, x - 2y = -40, x + y + z = 180$

Given equations can be written as $AX = B$

where, $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -2 & 0 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$|A| = -4 \neq 0 \Rightarrow A^{-1}$ exists.

$$\text{adj}A = \begin{bmatrix} -2 & 0 & -2 \\ -1 & 2 & -1 \\ 3 & -2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{adj}A = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}$$

$X = A^{-1}B$

$$= \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 1 & -2 & 1 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -40 \\ 180 \end{bmatrix} = \begin{bmatrix} 90 \\ 65 \\ 25 \end{bmatrix}$$

$x = 90, y = 65, z = 25$

Number of students allocated in sports, music and drama are 90,65 and 25 respectively.

SECTION E

36. **(i)** $2x+3y=300$

(ii) $A=xy=\frac{x}{3}(300-2x)$

(iii) (a) $A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$

$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$

For critical points, put $\frac{dA}{dx}=0 \Rightarrow x=75$

Also, $\frac{d^2A}{dx^2}=-\frac{4}{3}<0$. So, A is maximum at $x=75$

Also, maximum area is $A=\frac{75}{3}(300-150)=3750\ \mathrm{m}^2$

OR

(iii) (**b**) $A=\frac{x}{3}(300-2x)=\frac{1}{3}(300x-2x^2)$

$\Rightarrow \frac{dA}{dx}=\frac{1}{3}(300-4x)$

For critical points, put $\frac{dA}{dx}=0 \Rightarrow x=75$

As $\frac{dA}{dx}$ changes its sign from positive to negative as x passes through $x=75$ from left to right, which means $x=75$ is the point of maximum.

Also, maximum area is $A=\frac{75}{3}(300-150)=3750\ \mathrm{m}^2$

Note : Full credit to be given if the student takes equation as

$2x+2y=300$ or $2x+4y=300$ or $4x+4y=300$ or $4x+3y=300$

The solutions of sub-parts will differ and marks may be given accordingly.

37. **(i)** R_4

(ii) R_5

(iii) (**a**) R_1 and R_3

OR

(iii)(b) Required pairs to be added to make the relation R_2 as an equivalence relation are: $(1,1),(2,2),(3,3),(2,1),(3,1)$ and $(2,3)$

38. E_1 : customer avails loan on fixed rate

E_2 : customer avails loan on floating rate

E_3 : customer avails loan on variable rate

A:the person defaults on the loan

$\mathrm{P}(E_1)=\frac{1}{10}, \mathrm{P}(E_2)=\frac{2}{10}, \mathrm{P}(E_3)=\frac{7}{10}$

$\mathrm{P}(A \mid E_1)=\frac{5}{100}, \mathrm{P}(A \mid E_2)=\frac{3}{100}, \mathrm{P}(A \mid E_3)=\frac{1}{100}$

(i) $\mathrm{P(A)}=\mathrm{P}(E_1)\cdot \mathrm{P}(A \mid E_1)+\mathrm{P}(E_2)\cdot \mathrm{P}(A \mid E_2)+\mathrm{P}(E_3)\cdot \mathrm{P}(A \mid E_3)$

$=\frac{1}{10}\times\frac{5}{100}+\frac{2}{10}\times\frac{3}{100}+\frac{7}{10}\times\frac{1}{100}$

$=\frac{18}{1000}$ or $\frac{9}{500}$

(ii) $\mathrm{P(E_3 \mid A)}$

$=\frac{\mathrm{P}(E_3)\cdot \mathrm{P}(A \mid E_3)}{\mathrm{P}(E_1)\cdot \mathrm{P}(A \mid E_1)+\mathrm{P}(E_2)\cdot \mathrm{P}(A \mid E_2)+\mathrm{P}(E_3)\cdot \mathrm{P}(A \mid E_3)}$

$=\frac{\frac{7}{10}\times\frac{1}{100}}{\frac{18}{1000}}$

$=\frac{7}{18}$

SECTION-A

39.(a)

40. **(d)**

41. (d)
42. (d)
43. (b)
44. (d) Step 1: Recall the direction cosine relation

If a line makes angles α, β, γ with the x, y, z-axes, then:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Step 2: Substitute given values

$\cos\alpha = \cos\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} \Rightarrow \cos^2\alpha = \frac{1}{2}$

$\cos\beta = \cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \Rightarrow \cos^2\beta = \frac{1}{4}$

So: $\cos^2\alpha + \cos^2 \downarrow = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

Step 3: Solve for $\cos^2\theta$

$\cos^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$

So:$\cos\theta = \pm\frac{1}{2}$

Step 4: Find θ

If $\cos\theta = \frac{1}{2}$, then $\theta = \frac{\pi}{3}$.

If $\cos\theta = -\frac{1}{2}$, then $\theta = \frac{2\pi}{3}$ or $-\frac{\pi}{3}$.

But since options are in terms of $\pm$, we write:

$\theta = \pm\frac{\pi}{3}$

45. (d)
46. (c)
47. (c)
48. (b)
49. (c)
50.(d)
51. (c)
52. (b)
53. (d)
54. (c)
55. (b)
56. (b)
57. (d) Assertion (A) is false and Reason (R) is true.
58. (d) Assertion (A) is false and Reason (R) is true.

SECTION-B

59. (a) Let α be the angle which the vector $\vec{a}$ makes with all the three axes.

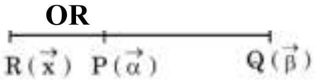
Then $3\cos^2\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$

The unit vector along the vector $\vec{a} = \frac{1}{\sqrt{3}}(\hat{\imath} + \hat{\jmath} + \hat{k})$

$\vec{a} = 5(\hat{\imath} + \hat{\jmath} + \hat{k})$

OR



(b)

$\frac{QR}{QP} = \frac{3}{2}$

Hence, R divides PQ, externally, in the ratio $1:3$.

The Position vector of R $= \vec{x} = \frac{\vec{\beta} - 3\vec{\alpha}}{1-3} = \frac{3\vec{\alpha} - \vec{\beta}}{2}$

60. Given definite integral $= \int_0^{\frac{\pi}{4}} \sqrt{(\sin x + \cos x)^2}dx$

$= \int_0^{\frac{\pi}{4}} (\sin x + \cos x)dx$

$= [-\cos x + \sin x]_0^{\frac{\pi}{4}}$

$= 1$

61. $f'(x) = \cos x - a$

For f(x) to be increasing, $f'(x) \geq 0$

i.e., $\cos x \geq a$

Since, $-1 \leq \cos x \leq 1$

$\Rightarrow a \leq -1$

Hence, $a \in (-\infty, -1]$. (Also, accept $a \in (-\infty, -1)$)

62. $\vec{\alpha}$ and $\vec{\beta}$ are collinear

$\Rightarrow \dfrac{x-2}{3+2x} = \dfrac{1}{-2}$

$\Rightarrow x = \dfrac{1}{4}$

63. **(a)** $x = e^{\frac{x}{y}}$

$\Rightarrow \log x = \dfrac{x}{y}$

$\Rightarrow y\log x = x$

Differentiating both sides w.r.to x , we get

$\dfrac{y}{x} + \log x \dfrac{dy}{dx} = 1$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{x-y}{x\log x}$

OR

(b) $Lf'(-2) = \lim_{h\to 0} \frac{f(-2-h)-f(-2)}{-h}$ $(h > 0)$

$= \lim_{h\to 0} \dfrac{2(-2-h) - 3 - (-7)}{-h}$

$= \lim_{h\to 0} 2 = 2$

$Rf'(-2) = \lim_{h\to 0} \dfrac{f(-2+h) - f(-2)}{h}$ $(h > 0)$

$= \lim_{h\to 0} \dfrac{-2+h+1-(-7)}{h}$

$= \lim_{h\to 0} \dfrac{6+h}{h}$, which does not exist, i.e., RHD does not exist.

Therefore, the function is not differentiable at -2 .

Note: (1) If a student finds only RHD and concludes the result, full marks may be awarded.

(2) If a student proves that the function is discontinuous at -2 and hence not differentiable at -2 , full marks may be awarded.

SECTION-C

64. **(a)** Given differential equation can be written as

$\dfrac{y}{y+3}dy = \dfrac{2}{x}dx$

$\Rightarrow \int \left(1 - \dfrac{3}{y+3}\right) dy = 2\int \dfrac{1}{x}dx$

$\Rightarrow y - 3\log|y+3| = 2\log|x| + C$

$y = -2$, when $x = 1 \Rightarrow C = -2$

Hence, the required particular solution is

$\Rightarrow y - 3\log|y+3| = 2\log|x| - 2$

OR

(b) Given differential equation can be written as $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$, which is linear in y .

I.F. $= e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1 + x^2$

The solution is given by

$y(1+x^2) = \int 4x^2 dx$

$\Rightarrow y(1+x^2) = \frac{4}{3}x^3 + C$

or $y = \frac{4x^3}{3(1+x^2)} + C\frac{1}{(1+x^2)}$, which is the required general solution

65. Let $x \in N$. Then we know that x is a multiple of itself.

$\Rightarrow xRx$

Hence, R is reflexive.

We have $2,8 \in N$ such that 8 is a multiple of 2

$\Rightarrow 8R2$

But, 2 is not a multiple of 8 . Hence, 2 is not R-related to 8 .

Therefore, R is not symmetric.

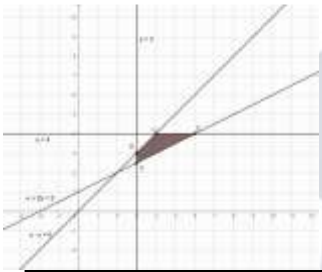
Let $x, y, z \in N$ such that xRy, yRz

Then $x = my, y = nz$ for some $m, n \in N$

$\Rightarrow x = mnz \Rightarrow x = pz$, where $p = mn \in N$. Hence, xRz

Therefore, R is transitive.

66.



Corner point	Value of Z = x − 5y
A(3,2.5)	-9.5
B(3,3)	-12
C(4,4)	-16
D(6,4)	-14

The minimum value of Z is -16 , which is attained at $x = 4, y = 4$.

67. (a) The given function can be written as

$y = 2\log(x+1) - \log x$

$\Rightarrow y_1 = \frac{2}{x+1} - \frac{1}{x} = \frac{x-1}{x(x+1)}$

$\Rightarrow (x+1)y_1 = \frac{x-1}{x} = 1 - \frac{1}{x}$

$\Rightarrow (x+1)y_2 + y_1 = \frac{1}{x^2}$

$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + \frac{1}{x}$

$\Rightarrow x(x+1)^2 y_2 + x(x+1)y_1 = 1 + 1 - (x+1)y_1$

$\Rightarrow x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$

OR

(b) $x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow (x-y)(x+y) + xy(x-y) = 0$

$\Rightarrow (x-y)(x+y+xy)=0$
$x \neq y \Rightarrow x+y+xy=0$
$\Rightarrow y=\dfrac{-x}{1+x}$
$\Rightarrow \dfrac{dy}{dx}=\dfrac{-1}{(1+x)^2}$

68. **(a)** $P(2)=\frac{3}{10}, P(\text{ any other number })=1-\frac{3}{10}=\frac{7}{10}$

Let X represent the Random Variable "the number of 2's".
Then X = 0,1,2
The probability distribution is

X	P(X)	XP(X)
0	$\frac{7}{10}\times\frac{7}{10}=\frac{49}{100}$	0
1	$\frac{3}{10}\times\frac{7}{10}\times 2=\frac{42}{100}$	$\frac{42}{100}$
2	$\frac{3}{10}\times\frac{3}{10}=\frac{9}{100}$	$\frac{18}{100}$

Mean $=\sum \text{XP(X)}=\dfrac{60}{100}=0.6$

OR

(b) $A=\{(3,6),(4,5),(5,4),(6,3)\}$

$P(A)=\dfrac{4}{36}=\dfrac{1}{9}, P(B)=\dfrac{30}{36}=\dfrac{5}{6}$

$P(A\cap B)=\dfrac{3}{36}=\dfrac{1}{12}$

$P(A)\times P(B)=\dfrac{5}{54}\neq P(A\cap B)$

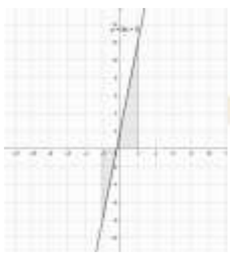
Therefore, A and B are not independent.
A and B are not mutually exclusive as $A\cap B\neq \emptyset$

69. $\mathrm{I}=\int \frac{1}{x}\frac{x+a}{\sqrt{x^2-a^2}}dx=\int \frac{1}{\sqrt{x^2-a^2}}dx+a\int \frac{1}{x\sqrt{x^2-a^2}}dx$

$=\log\left|x+\sqrt{x^2-a^2}\right|+\sec^{-1}\left(\frac{x}{a}\right)+C$

SECTION-D

70.



The required area

$$=\left|\int_{-2}^{-\frac{2}{5}}(5x+2)dx\right|+\int_{-\frac{2}{5}}^{2}(5x+2)dx$$

$$=\left|\left[\frac{(5x+2)^2}{10}\right]_{-2}^{-\frac{2}{5}}\right|+\left[\frac{(5x+2)^2}{10}\right]_{-\frac{2}{5}}^{2}$$

$$=\frac{64}{10}+\frac{144}{10}=\frac{104}{5}$$

71. $\frac{x^2+x+1}{(x+2)(x^2+1)}=\frac{A}{(x+2)}+\frac{Bx+c}{x^2+1}$

Getting $A=\frac{3}{5}, B=\frac{2}{5}, C=\frac{1}{5}$

Given integral $=\frac{3}{5}\int \frac{1}{x+2}dx+\frac{1}{5}\int \frac{2x}{x^2+1}dx+\frac{1}{5}\int \frac{1}{x^2+1}dx$

$=\frac{3}{5}\log|x+2|+\frac{1}{5}\log(x^2+1)+\frac{1}{5}\tan^{-1}x+C$

72. (a) The vector equations of the lines are

$\vec{r}=-\hat{\imath}+\hat{\jmath}+9\hat{k}+\lambda(2\hat{\imath}+\hat{\jmath}-3\hat{k})$

$\vec{r}=3\hat{\imath}-15\hat{\jmath}+9\hat{k}+\mu(2\hat{\imath}-7\hat{\jmath}+5\hat{k})$

$\overrightarrow{a_1}=-\hat{\imath}+\hat{\jmath}+9\hat{k}, \overrightarrow{a_2}=3\hat{\imath}-15\hat{\jmath}+9\hat{k}$

$\overrightarrow{b_1}=2\hat{\imath}+\hat{\jmath}-3\hat{k}, \overrightarrow{b_2}=2\hat{\imath}-7\hat{\jmath}+5\hat{k}$

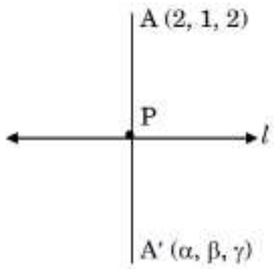
$\overrightarrow{a_2}-\overrightarrow{a_1}=4\hat{\imath}-16\hat{\jmath}$

$$\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 1 & -3\\ 2 & -7 & 5\end{vmatrix}=-16\hat{\imath}-16\hat{\jmath}-16\hat{k}$$

$$\text{S.D.}=\frac{|\,\overrightarrow{a_2}-\overrightarrow{a_1})\cdot(\overrightarrow{b_1}\times\overrightarrow{b_2})\,|}{|\overrightarrow{b_1}\times\overrightarrow{b_2}|}=\frac{12}{\sqrt{3}}=4\sqrt{3}$$

OR

(b)



Let the image of A in the line be $A'(\alpha,\beta,\gamma)$

The point P , which is the point of intersection of the lines l and AA', will have coordinates $(\lambda+4,-\lambda+2,-\lambda+2)$ for some λ.

Drs of AP are $\langle\lambda+2,-\lambda+1,-\lambda\rangle$

$AP\perp l$

$(\lambda+2)-(-\lambda+1)-(-\lambda)=0$

$\Rightarrow \lambda=-\frac{1}{3}$

Therefore, the coordinates of P are $\left(\frac{11}{3},\frac{7}{3},\frac{7}{3}\right)$

P is the mid-point of AA'

$\Rightarrow \frac{2+\alpha}{2}=\frac{11}{3}, \frac{1+\beta}{2}=\frac{7}{3}, \frac{2+\gamma}{2}=\frac{7}{3}$

$\Rightarrow \alpha=\frac{16}{3}, \beta=\frac{11}{3}, \gamma=\frac{8}{3}$

The coordinates of the image are $\left(\frac{16}{3},\frac{11}{3},\frac{8}{3}\right)$

The equation of AA' is

$$\frac{x-2}{\frac{10}{3}}=\frac{y-1}{\frac{8}{3}}=\frac{z-2}{\frac{2}{3}}$$

or,

$$\frac{3(x-2)}{5}=\frac{3(y-1)}{4}=\frac{3(z-2)}{1}$$

73. (a) $AB=\begin{bmatrix}8 & 0 & 0\\ 0 & 8 & 0\\ 0 & 0 & 8\end{bmatrix}=8I$

The system of equations is equivalent to the matrix equation:

$BX = C$, where $C = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$\Rightarrow X = B^{-1}C$

$AB = 8I$

$\Rightarrow B^{-1} = \frac{1}{8}A$

$X = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$

$\therefore x = 3, y = -2, z = -1$

(b) $|A| = 1 \neq 0 \Rightarrow A^{-1}$ exists.

$\text{adj}A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj}A = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$

The given system of equations is equivalent to the matrix equation

$A^T X = B$, where $\text{B} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}, \text{X} = \begin{bmatrix} \text{x} \\ \text{y} \\ \text{z} \end{bmatrix}$

$\Rightarrow X = \left(A^{\text{T}}\right)^{-1}B$

$\Rightarrow X = (A^{-1})^T B$

$\Rightarrow X = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}\begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$

$\text{x} = 0, \text{y} = -5, \text{z} = -3$

SECTION-E

74. (i) The number of relations = 24×3=212

(ii) Since, S_2 and S_3 have been assigned the same judge J_2, the function is not one-one. Hence, it is not bijective.

(iii) (a) There cannot exist any one-one function from S to J as $n(S) > n(J)$. Hence, the number of one-one functions from S to J is 0.

OR

(iii) (b) To make R_1 reflexive and not symmetric we need to add the following ordered pairs:
$(S_1, S_1), (S_2, S_2), (S_3, S_3), (S_4, S_4)$

75. (a)(i) Let $A =$ Amber manufactures the car

B = Bonzi manufactures the car

C = Comet manufactures the car

E = The selected car is electric

$P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$

$P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P\left(\frac{E}{C}\right)$

$= \frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}$

$= \frac{155}{1000}$ or $\frac{31}{200}$

OR

(b)(i) Let A = Amber manufactures the car

B = Bonzi manufactures the car

C = Comet manufactures the car

E = The selected car is a petrol car

$P(A) = \frac{60}{100}, P(B) = \frac{30}{100}, P(C) = \frac{10}{100}$

$P(E) = P(A) \times P\left(\frac{E}{A}\right) + P(B) \times P\left(\frac{E}{B}\right) + P(C) \times P\left(\frac{E}{C}\right)$

$= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{90}{100} + \frac{10}{100} \times \frac{95}{100}$

$= \frac{845}{1000}$ or $\frac{169}{200}$

(ii) $P\left(\frac{C}{E}\right) = \frac{P(C)\times P\left(\frac{E}{C}\right)}{P(E)}$

$= \frac{\frac{10}{100} \times \frac{5}{100}}{\frac{60}{100} \times \frac{20}{100} + \frac{30}{100} \times \frac{10}{100} + \frac{10}{100} \times \frac{5}{100}}$

$= \frac{\frac{50}{10000}}{\frac{1550}{10000}} = \frac{1}{31}$

(iii)

$P\left(\frac{A \text{ or } B}{E}\right) = 1 - P\left(\frac{C}{E}\right) = 1 - \frac{1}{31} = \frac{30}{31}$

76. (i) $f'(x) = e^x(\cos x + \sin x)$

For critical points, $f'(x) = 0$

$\Rightarrow \cos x + \sin x = 0$

$\Rightarrow \cos x = -\sin x$

For x to be a critical point $x \in (0, \pi)$, hence, $x = \frac{3\pi}{4}$

For all $x \in \left[0, \frac{3\pi}{4}\right], f'(x) \geq 0$

Hence, f is increasing in $\left[0, \frac{3\pi}{4}\right]$

Note: If a student concludes the answer in any of the following intervals, full marks may be awarded:

$\left(0, \frac{3\pi}{4}\right)$ or $\left[0, \frac{3\pi}{4}\right)$ or $\left(0, \frac{3\pi}{4}\right]$

For all $x \in \left[\frac{3\pi}{4}, \pi\right], f'(x) \leq 0$

Hence, f is decreasing in $\left[\frac{3\pi}{4}, \pi\right]$

Note: If a student concludes the answer in any of the following intervals, full marks may be awarded:

$\left(\frac{3\pi}{4}, \pi\right)$ or $\left(\frac{3\pi}{4}, \pi\right]$ or $\left[\frac{3\pi}{4}, \pi\right)$

(ii)

$x = \frac{3\pi}{4}$ is a critical point

$f''(x) = e^x(\cos x - \sin x) + e^x(\cos x + \sin x)$

$= 2e^x \cos x$

$f''\left(\frac{3\pi}{4}\right) = -ve$

Hence, $\frac{3\pi}{4}$ is a point of local maximum.

SECTION A

77. (c)

78. (a)

79. (c)

80. (a)

81. (c)

82. (a)

83. (d)

84. (d)

85. (b)

86. (c)

87. (c)

88. **(b)**
89. **(c)**
90. **(a)**
91. **(c)**
92. **(c)**
93. **(c)**
94. **(d)**
95. **(d)** Assertion (A) is false, but Reason (R) is true.
96. **(b)** Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

SECTION B

97. $|-6AB| = (-6)^3|A||B|$
$= -216 \times 3 \times (-4) = 2592$

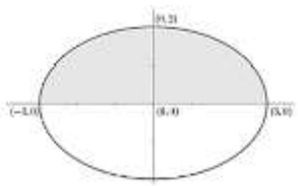
98. **(a)** $f(x) = 2x^2 - ax + 3 \Rightarrow f'(x) = 4x - a$
Now $2 \leq x \leq 4 \Rightarrow 8 - a \leq 4x - a \leq 16 - a$
For f to bean increasing function, $f'(x) \geq 0$
$\Rightarrow 8 - a \geq 0 \Rightarrow a \leq 8$
Least value of a does not exist.

OR

(b)$f(x) = x + \frac{1}{x} \Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$
Now $\frac{x^2-1}{x^2} \geq 0$ for all $x \geq 1$
$\Rightarrow f'(x) \geq 0 \Rightarrow f$ is an increasing function.

99. **(a)** Put $x = \tan\theta \Rightarrow \theta = \tan^{-1} x$
Now $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$
$= \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right) = \sin^{-1}(\sin\theta)$
$= \theta = \tan^{-1} x$
(b) Here $-1 \leq \sqrt{x-1} \leq 1$
$\Rightarrow 0 \leq x - 1 \leq 1 \Rightarrow 1 \leq x \leq 2$
Hence, domainis $x \in [1,2]$

100.



$A = 2 \times \frac{2}{3}\int_0^3 \sqrt{9-x^2}dx$
$= \frac{4}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$
$= \frac{4}{3}\left[\left(0 + \frac{9}{2}\sin^{-1} 1\right) - 0\right]$
$= 3\pi$

101. $y = 5x - 2x^3$
Given $\frac{dx}{dt} = 2$ units /s
slope of the curve $= \frac{dy}{dx} = 5 - 6x^2 = m$
$\frac{dm}{dt} = -12x\frac{dx}{dt} = -12x(2) = -24x$

at $x=2, \frac{dm}{dt} = -24(2) = -48$

Hence, slope of curve is decreasing at the rate of 48

SECTION C

102. **(a)** $f(x) = \log_a x \ (a > 0, a \neq 1)$

Let $x_1, x_2 \in R^+$ such that $f(x_1) = f(x_2)$

$\Rightarrow \log_a x_1 = \log_a x_2$

$\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.

Let $f(x) = y \Rightarrow \log_a x = y \Rightarrow a^y = x$

for every $y \in R$, there exists $x \in R^+$

f is onto.

f is a bijection.

OR

(b) (i) $R = \{(1,5),(2,4)\}$

(ii) R is not a function as $3 \in A$ do not have an image in co-domain.

(iii) Domain of $R = \{1,2\}$, Range of $R = \{4,5\}$

103. **(a)** $\lim_{x\to -1} \frac{x^2-2x-3}{x+1} = \lim_{x\to -1} \frac{(x-3)(x+1)}{x+1} = \lim_{x\to -1}(x-3) = -4$

Also, $f(-1) = k$

as f is continuous, $k = -4$

OR

(b) $f(x) = x|x| = \begin{cases} -x^2 & , x \leq 0 \\ x^2 & , x > 0 \end{cases}$

$$\text{LHD} = \lim_{h\to 0} \frac{f(0-h)-f(0)}{-h} = \lim_{h\to 0} \frac{-h^2-0}{-h} = 0$$

$$\text{RHD} = \lim_{h\to 0} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0} \frac{h^2-0}{h} = 0$$

Since LHD $=$ RHD, f is differentiableat $x = 0$

104. $\mathrm{I} = \int_{\pi/2}^{\pi} e^x \left(\frac{1-\sin x}{1-\cos x}\right) dx$

$$= \int_{\pi/2}^{\pi} e^x \left(\frac{1 - 2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}}\right) dx$$

$$= \int_{\pi/2}^{\pi} e^x \left(\frac{1}{2}\text{cosece}^2\frac{x}{2} - \cot\frac{x}{2}\right) dx$$

$$\mathrm{I} = -\left[e^x \cot\frac{x}{2}\right]_{\pi/2}^{\pi}$$

$$= -\left[e^{\pi}\cot\frac{\pi}{2} - e^{\frac{\pi}{2}}\cot\frac{\pi}{4}\right]$$

$$= e^{\frac{\pi}{2}}$$

105. **(a)** Let X denote the random variable which counts the number of boys.

$X = 0,1,2,3$

$P(\text{ Boy }) = P(\text{ Girl }) = \frac{1}{2}$

Required Probability Distribution

X	$P(X)$
0	$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$
1	$3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{3}{8}$
2	$3\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{3}{8}$
3	$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

OR

(b) Possible values of X are $-2,0,2$

X	-2	0	2
$P(X)$	$\frac{1}{4}$	$\frac{2}{4}=\frac{1}{2}$	$\frac{1}{4}$

Mean $=\sum XP(X)=-2\left(\frac{1}{4}\right)+0\left(\frac{1}{2}\right)+2\left(\frac{1}{4}\right)=0$

106. $l_1:\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$

Any point on l_1 is $(2\lambda+1,3\lambda+2,4\lambda+3)$

$l_2:\frac{x-4}{5}=\frac{y-1}{2}=\frac{z-0}{1}=\mu$

Any point on l_2 is $(5\mu+4,2\mu+1,\mu)$

For point of intersection,

$2\lambda+1=5\mu+4,3\lambda+2=2\mu+1$

Solving, $\lambda=\mu=-1$

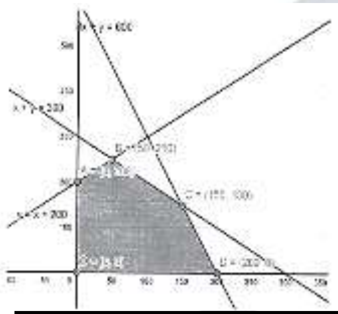
Since, $\lambda=\mu=-1$ satisfy $4\lambda+3=\mu$

$\therefore$ Point of intersection is $(-1,-1,-1)$

Now distance of $(-1,-5,-10)$ from $(-1,-1,-1)$ is:

$\sqrt{(-1+1)^2+(-1+5)^2+(-1+10)^2}=\sqrt{97}$ units

107.



Corner Point	**Value of $Z=100x+50y$**
$O(0,0)$	0
$A(0,200)$	10000
$B(50,250)$	17500
$C(150,150)$	22500
$D(200,0)$	20000

$Z_{\max}=22500$ when $x=150,y=150$

SECTION D

108. Consider $(kA)\left(\frac{1}{k}A^{-1}\right)=k\cdot\frac{1}{k}(A\cdot A^{-1})=I$

$\Rightarrow kA$ and $\frac{1}{k}A^{-1}$ are inverse of each other.

$(kA)^{-1}=\frac{1}{k}A^{-1}$

$(3A)^{-1}=\frac{1}{3}A^{-1}$

Here, $|A|=4\neq 0 \therefore A^{-1}$ exists.

$$\operatorname{adj}A=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}\cdot \operatorname{adj} A=\frac{1}{4}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

$$(3A)^{-1}=\frac{1}{12}\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$$

109. **(i)** $y=4x-\frac{1}{2}x^2 \Rightarrow \frac{dy}{dx}=(4-x)\text{cm/ day}$

(ii) For maximum height, $\frac{dy}{dx}=0 \Rightarrow x=4$ days

as $\frac{d^2y}{dx^2}<0$, number of days $=4$

Now, Maximum height $=y(4)=16-\frac{1}{2}(16)=8$ cm

110. **(a)** $\mathrm{I}=\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)} dx$

$$=\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)} dx$$

$$=\int \frac{\cos x}{(4+\sin^2 x)(1+4\sin^2 x)} dx$$

$\sin x=t$ gives

$$\mathrm{I}=\int \frac{dt}{(4+t^2)(1+4t^2)}$$

$$=-\frac{1}{15}\int \frac{dt}{4+t^2}+\frac{4}{15}\int \frac{dt}{1+4t^2} \quad (\because \text{ using Partial Fraction })$$

$$=-\frac{1}{30}\tan^{-1}\left(\frac{t}{2}\right)+\frac{2}{15}\tan^{-1}(2t)+C$$

$$=-\frac{1}{30}\tan^{-1}\left(\frac{\sin x}{2}\right)+\frac{2}{15}\tan^{-1}(2\sin x)+C$$

OR

(b) $I=\int_0^\pi \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$

$$=2\int_0^{\pi/2} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$$

$$=2\int_0^{\pi/2} \frac{\sec^2 x}{a^2+b^2\tan^2 x} dx$$

$\tan x=t$ gives

$$\mathrm{I}=2\int_0^\infty \frac{dt}{a^2+b^2t^2}$$

$$=\frac{2}{b^2}\cdot\frac{b}{a}\tan^{-1}\left(\frac{bt}{a}\right)\Big]_0^\infty$$

$$=\frac{\pi}{ab}$$

111. **(a)** Let $ABCD$ be the parallelogram with diagonals $\overrightarrow{AB}=\vec{a}$ and $\overrightarrow{BD}=\vec{b}$

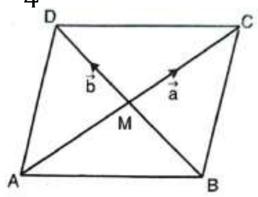
$\overrightarrow{AB}=\frac{1}{2}(\vec{a}-\vec{b})$ and $\overrightarrow{AD}=\frac{1}{2}(\vec{a}+\vec{b})$

Area of $ABCD$

$=|\overrightarrow{AB}\times\overrightarrow{AD}|$

$$=\left|\frac{1}{2}(\vec{a}-\vec{b})\times\frac{1}{2}(\vec{a}+\vec{b})\right|$$

$$=\frac{1}{4}|\vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}|$$



$= \frac{1}{4}|\vec{a} \times \vec{b} + \vec{a} \times \vec{b}|\ (\because \vec{a} \times \vec{a} = \vec{0})$

$= \frac{1}{4}|2(\vec{a} \times \vec{b})|$

$= \frac{1}{2}|\vec{a} \times \vec{b}|$

Given $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$

Area of parallelogram $= \frac{1}{2}|\vec{a} \times \vec{b}|$

Now $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 1 & 3 & -1 \end{vmatrix} = -2\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$

$|\vec{a} \times \vec{b}| = \sqrt{62}$

Area of parallelogram $= \frac{1}{2}\sqrt{62}$

OR

(b) Given lines are $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$

and $\vec{r} = (15\hat{\imath} + 29\hat{\jmath} + 5\hat{k}) + \mu(3\hat{\imath} + 8\hat{\jmath} - 5\hat{k})$

The first line in vector formis $\vec{r} = (8\hat{\imath} - 19\hat{\jmath} + 10\hat{k}) + \lambda(3\hat{\imath} - 16\hat{\jmath} + 7\hat{k})$

$\vec{a}_1 = 8\hat{\imath} - 19\hat{\jmath} + 10\hat{k}, \vec{a}_2 = 15\hat{\imath} + 29\hat{\jmath} + 5\hat{k}$

$\vec{b}_1 = 3\hat{\imath} - 16\hat{\jmath} + 7\hat{k}, \vec{b}_2 = 3\hat{\imath} + 8\hat{\jmath} - 5\hat{k}$

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix} = 24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}$

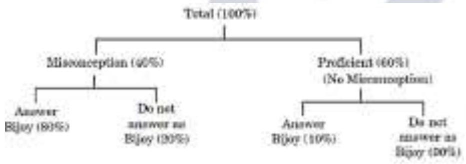
Equation of line passing through $(1,2,-4)$ and parallel to $\vec{b}$ is

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + t(24\hat{\imath} + 36\hat{\jmath} + 72\hat{k})$ or $\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + t'(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$

Cartesian formof line is $\frac{x-1}{24} = \frac{y-2}{36} = \frac{z+4}{72}$ or $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$

SECTION E

112.



Let E_1 : Student has a misconception

E_2 : Student does not have misconception

A: Student answered Bijoy as correct

$P(E_1) = \frac{40}{100}, P(E_2) = \frac{60}{100}$

$P(A \mid E_1) = \frac{80}{100}, P(A \mid E_2) = \frac{10}{100}$

$P(\bar{A} \mid E_1) = \frac{20}{100}, P(\bar{A} \mid E_2) = \frac{90}{100}$

(i) $P(A \mid E_2) = \frac{10}{100}$ or $\frac{1}{10}$

(ii) $P(A) = P(E_1)P(A \mid E_1) + P(E_2)P(A \mid E_2)$

$= \frac{40}{100} \times \frac{80}{100} + \frac{60}{100} \times \frac{10}{100} = \frac{40}{100} \times \frac{80}{100} + \frac{60}{100} \times \frac{10}{100}$

$= \frac{38}{100}$ or $\frac{19}{50}$

(iii)(a)$P(E_1 \mid A) = \frac{P(E_1)P(A \mid E_1)}{P(A)}$

$= \frac{\frac{40}{100} \times \frac{80}{100}}{\frac{38}{100}} = \frac{16}{19}$

OR

(iii)(a)$P(E_1 \mid A) = \dfrac{P(E_2)P(A \mid E_2)}{P(A)}$

$= \dfrac{\frac{60}{100} \times \frac{80}{100}}{\frac{38}{100}} = \dfrac{3}{19}$

113. $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}; l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$

(i) Drs of l_1 are $\langle 3, -2, 4 \rangle$, drs of l_2 are $\langle 2, 1, -3 \rangle$ as drs are not proportional, hence l_1 is not parallel to l_2.

(ii) Equations of line parallel to l_1 and passing through $(1, -2, -3)$ is

$\dfrac{x-1}{3} = \dfrac{y+2}{-2} = \dfrac{z+3}{4}$ or $\vec{r} = (\hat{\imath} - 2\hat{\jmath} - 3\hat{k}) + \lambda(3\hat{\imath} - 2\hat{\jmath} + 4\hat{k})$

(iii) (a) The pathway is perpendicular to l_1 and l_2 ... It is parallel to $\vec{b}_1 \times \vec{b}_2$

$$\vec{b} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = 2\hat{\imath} + 17\hat{\jmath} + 7\hat{k}$$

$\therefore$ Equation of pathwayis $\vec{r} = (3\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(2\hat{\imath} + 17\hat{\jmath} + 7\hat{k})$

$(\boldsymbol{iii})(\boldsymbol{b})\vec{a}_1 = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}, \vec{a}_2 = \hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$\vec{b}_1 = 3\hat{\imath} - 2\hat{\jmath} + 4\hat{k}, \vec{b}_2 = 2\hat{\imath} + \hat{\jmath} - 3\hat{k}$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \left| \frac{(-\hat{\imath} + 4\hat{\jmath} - 5\hat{k}) \cdot (2\hat{\imath} + 17\hat{\jmath} + 7\hat{k})}{\sqrt{4 + 289 + 49}} \right|$$

$$= \frac{31}{\sqrt{342}}$$

114. **(i)** $\frac{dT}{dt} = -k(T - 25)$

$\Rightarrow \dfrac{dT}{T - 25} = -kdt$

$\Rightarrow \displaystyle\int \frac{dT}{T - 25} = -k \int dt$

$\Rightarrow \log|T - 25| = -kt + C \;\; \ldots (a)$

When $t = 0, T = 85$

$\Rightarrow \log 60 = C$

Using in equation (a), $\log|T - 25| = -kt + \log 60$ .... (b)

(ii) When $k = 0.03, \log|T - 25| = -0.03t + \log 60$

$\Rightarrow \log\left|\dfrac{T - 25}{60}\right| = -0.03t$

$\Rightarrow T - 25 = 60. e^{-0.03t}$

When $T = 40, t = t_1$

$\Rightarrow \dfrac{15}{60} = e^{-0.03t_1}$

$\Rightarrow e^{-0.03t_1} = \dfrac{1}{4} \Rightarrow -0.03t_1 = -\log 4$

$\Rightarrow t_1 = \dfrac{\log 4}{0.03} = \dfrac{1.3863}{0.03} = 46.21$ m

SECTION-A

115. **(b)**

116. **(d)**

117. **(c)**

118. **(c)**

119. **(c)**

120. **(b)**

121. **(d)**
122. **(c)**
123. **(b)**
124. **(a)**
125. **(b)**
126. **(c)**
127. **(b)**
128. **(c)**
129. **(c)**
130. **(b)**
131. **(c)**
132. **(c)**
133. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
134. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

SECTION-B

135. Domain of $\cos^{-1} x$ is $[-1,1]$

$\Rightarrow - \quad 1 \leq x^2 - 4 \leq 1 \Rightarrow 3 \leq x^2 \leq 5$

$\Rightarrow x \quad x[-\sqrt{5}, -\sqrt{3}] \cup [\sqrt{3}, \sqrt{5}]$

136. $\frac{dS}{dt} = 5\ \mathrm{mm}^2/\mathrm{s}, \left(\frac{dV}{dt}\right)_{r=8} = ?$

$$S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{5}{8\pi r}$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt} \Rightarrow \frac{dV}{dt} = \frac{5}{2} r$$

$$\Rightarrow \left(\frac{dV}{dt}\right)_{r=8} = 20\ \mathrm{mm}^3/\mathrm{s}$$

137. **(a)** Let $y = \frac{\sin x}{\sqrt{\cos x}}$

$$\frac{dy}{dx} = \frac{\sqrt{\cos x} \cdot \cos x - \sin x \cdot \left(\frac{-\sin x}{2\sqrt{\cos x}}\right)}{\cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2\cos^2 x + \sin^2 x}{2(\cos x)^{3/2}} \text{ or } \frac{1 + \cos^2 x}{2(\cos x)^{3/2}}$$

OR

(b) $y = 5\cos x - 3\sin x$, then $\frac{dy}{dx} = -5 \cdot \sin x - 3 \cdot \cos x$

$$\Rightarrow \frac{d^2y}{dx^2} = -5 \cdot \cos x + 3 \cdot \sin x = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

138. **(a)** Let $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}, \vec{b} = 4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = \hat{\imath} + 10\hat{\jmath} + 17\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{1^2 + 10^2 + 17^2} = \sqrt{390}$$

$$\text{Unit vector } \hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|} = \frac{1}{\sqrt{390}}(\hat{\imath} + 10\hat{\jmath} + 17\hat{k})$$

$$\text{Required vector } = \frac{5}{\sqrt{390}}(\hat{\imath} + 10\hat{\jmath} + 17\hat{k})$$

OR

(b) $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$

$\Rightarrow$ either $\vec{b} = \vec{c}$ or $\vec{a} \perp (\vec{b} - \vec{c})$, since $\vec{a} \neq 0$

Also, $\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = 0$

$\Rightarrow$ either $\vec{b} = \vec{c}$ or $\vec{a} || (\vec{b} - \vec{c})$, since $\vec{a} \neq 0$

Since vectors $\vec{a}$ and $(\vec{b} - \vec{c})$ cannot be $||$ and $\perp$ simultaneously

Hence $\vec{b} = \vec{c}$

139.

Let P and Q trisect the wire AB.

P divides AB in the ratio 1: 2 then, coordinate of point $P = \left(\frac{14}{3}, \frac{4}{3}, -\frac{7}{3}\right)$

Q divides AB in the ratio 2: 1 then, coordinate of point $Q = \left(\frac{16}{3}, \frac{5}{3}, -\frac{8}{3}\right)$

SECTION-C

140. Since f(x) is a decreasing function $\Rightarrow f'(x) \leq 0$

$\Rightarrow \sqrt{3} \cdot \cos x + \sin x - 2a \leq 0$

$\Rightarrow 2\left(\frac{\sqrt{3}}{2} \cdot \cos x + \frac{1}{2} \sin x\right) - 2a \leq 0$

$\Rightarrow \cos\left(x - \frac{\pi}{6}\right) \leq a$

Since, $-1 \leq \cos\left(x - \frac{\pi}{6}\right) \leq 1 \Rightarrow a \geq 1$ i.e. $a \in [1, \infty)$ or $(1, \infty)$

141. **(a)** Let $I = \int \frac{2x}{(x^2+3)(x^2-5)} dx$

Put $x^2 = t \Rightarrow 2x.dx = dt$

$\Rightarrow I = \int \frac{dt}{(t+3)(t-5)}$

$= \int \left(-\frac{1}{8(t+3)} + \frac{1}{8(t-5)}\right) dt$

$= \frac{1}{8}[\log|t-5| - \log|t+3|] + c$

$= \frac{1}{8} \log\left|\frac{x^2-5}{x^2+3}\right| + c$

OR

(b) $\int_1^4 (|x-2| + |x-4|)dx$

$= \int_1^2 (2-x)dx + \int_2^4 (x-2)dx - \int_1^4 (x-4)dx$

$= \left[\frac{(2-x)^2}{-2}\right]_1^2 + \left[\frac{(x-2)^2}{2}\right]_2^4 - \left[\frac{(x-4)^2}{2}\right]_1^4$

$= \frac{1}{2} + 2 + \frac{9}{2} = 7$

142. $\left[x.\sin^2 \frac{y}{x} - y\right].dx + x.dy = 0$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sin^2 \frac{y}{x}$

Put $y = vx \Rightarrow \frac{dy}{dx} = x\frac{dv}{dx} + v$

$x\frac{dv}{dx} + v = v - \sin^2 v$

$\Rightarrow -\int \text{cosec}^2 v dv = \int \frac{dx}{x}$

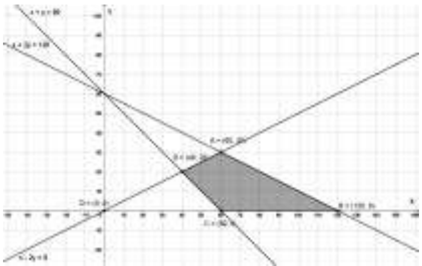
$\Rightarrow \cot v = \log|x| + c$

$\Rightarrow \cot \frac{y}{x} = \log|x| + c$

$x=1, y=\frac{\pi}{4} \Rightarrow c=1$

$\Rightarrow \cot\frac{y}{x}=\log|x|+1$

143.



Corner Points	Value of Z
A(60,30)	600
B(120,0)	600
C(60,0)	300
D(40,20)	400

Since Z is maximum on points A and B

Hence all points lying on segment AB give maximum Z.

144. **(a)** Given $\vec{a}+\vec{b}+\vec{c}=\vec{0} \Rightarrow |\vec{a}+\vec{b}|=|-\vec{c}|$

$\Rightarrow |\vec{a}+\vec{b}|^2=|\vec{c}|^2 \Rightarrow |\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}=|\vec{c}|^2$

$\Rightarrow 9+25+2\vec{a}\cdot\vec{b}=49$

$\Rightarrow 2|\vec{a}||\vec{b}|\cos\theta=15$

$\Rightarrow \cos\theta=\frac{1}{2} \therefore \theta=\frac{\pi}{3}$

OR

(b) $|\vec{a}|=|\vec{b}|=1$

$|\vec{a}-\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}$

$=1+1-2|\vec{a}||\vec{b}|\cos\theta$

$=2-2\cos\theta$

$=2\left(2\sin^2\frac{\theta}{2}\right)=4\sin^2\frac{\theta}{2}$

$\Rightarrow \sin\frac{\theta}{2}=\frac{1}{2}|\vec{a}-\vec{b}|$

145. **(a)** Let A be the event of buying colouring book and B be the event of buying coloured box.

$P(A)=0.7, P(B)=0.2, P(A/B)=0.3$

(i) $P\left(\frac{A}{B}\right)=\frac{P(A\cap B)}{P(B)} \Rightarrow 0.3=\frac{P(A\cap B)}{0.2}$

$\Rightarrow P(A\cap B)=0.06$ or $\frac{3}{50}$

(ii) $P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$

$=\frac{0.06}{0.7}=\frac{3}{35}$ or 0.086

OR

(b) Let **X** be random variable for number of oranges.

$X=0,1,2,3$

Let A be the event that orange is drawn.

$P(A)=\frac{4}{10}=\frac{2}{5}, P(\bar{A})=1-\frac{2}{5}=\frac{3}{5}$

(i)

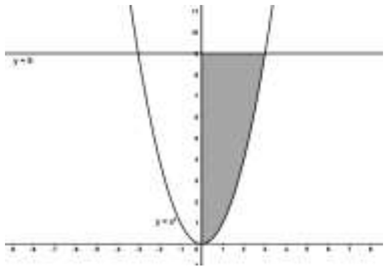
X	0	1	2	3
P(X)	$\frac{27}{125}$	$\frac{54}{125}$	$\frac{36}{125}$	$\frac{8}{125}$

(ii) $E(X) = \sum p_i X_i = 0 \times \frac{27}{125} + 1 \times \frac{54}{125} + 2 \times \frac{36}{125} + 3 \times \frac{8}{125}$

$= \frac{150}{125}$ or $\frac{6}{5}$

SECTION-D

146.



Required area $= \int_0^9 \sqrt{y}\,dy$

$= \frac{2}{3}\left[y^{3/2}\right]_0^9$

$= 18$

147. Let the numbers of chairs, tables and beds produced be x, y and z respectively.

$x + y + z = 45;\ -x + 0.y + z = 8;\ x - 2y + z = 0$

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix}$

$|A| = 1(0+2) - 1(-1-1) + 1(2-0) = 6 \neq 0$

A^{-1} exists

$AX = B \Rightarrow X = A^{-1}B$

$\operatorname{adj}(A) = \begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -2 \\ 2 & 3 & 1 \end{bmatrix}\begin{bmatrix} 45 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 15 \\ 19 \end{bmatrix}$

So, $x = 11, y = 15, z = 19$

Hence the numbers of chairs, tables and beds produced are 11, 15 and 19 respectively.

148. **(a)** Let $u = a^{t+\frac{1}{t}} \Rightarrow \frac{du}{dt} = a^{t+\frac{1}{t}} \cdot \log a \cdot \left(1 - \frac{1}{t^2}\right)$

$v = \left(t + \frac{1}{t}\right)^a \Rightarrow \frac{dv}{dt} = a\left(t + \frac{1}{t}\right)^{a-1} \cdot \left(1 - \frac{1}{t^2}\right)$

$\frac{du}{dv} = \frac{du/dt}{dv/dt} = \frac{a^{t+\frac{1}{t}} \cdot \log a}{a\left(t + \frac{1}{t}\right)^{a-1}}$

OR

(b) Let $u = y^x, v = x^y$ and $w = x^x$

$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0 \ldots\ldots\ldots\ldots (i)$

$u = y^x \Rightarrow \log u = x \cdot \log y \Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = \frac{x}{y} \cdot \frac{dy}{dx} + \log y$

$$\Rightarrow \frac{du}{dx} = y^x\left(\frac{x}{y}\cdot\frac{dy}{dx} + \log y\right) = xy^{x-1}\frac{dy}{dx} + y^x \log y$$

$$v = x^y \Rightarrow \log v = y\cdot\log x \Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = \frac{y}{x} + \log x\cdot\frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = x^y\left(\frac{y}{x} + \log x\cdot\frac{dy}{dx}\right) = yx^{y-1} + x^y \log x\frac{dy}{dx}$$

$$w = x^x \Rightarrow \log w = x\cdot\log x \Rightarrow \frac{1}{w}\cdot\frac{dw}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dw}{dx} = x^x\cdot(1+\log x)$$

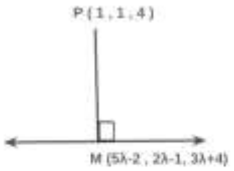
$\therefore$ From (i), we get

$$xy^{x-1}\cdot\frac{dy}{dx} + y^x\cdot\log y + yx^{y-1} + x^y\cdot\log x\cdot\frac{dy}{dx} + x^x\cdot(1+\log x) = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^x\cdot(1+\log x) + y^x\cdot\log y + yx^{y-1}}{x\cdot y^{x-1} + x^y\cdot\log x}$$

149. **(a)** Let $\frac{x+2}{5} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$

Coordinate of general point on the given line are $M(5\lambda-2, 2\lambda-1, 3\lambda+4)$



Direction Ratios of PM vector are $\langle 5\lambda-3, 2\lambda-2, 3\lambda\rangle$

Since, PM $\perp$ ***l***

$$\Rightarrow 5(5\lambda-3) + 2(2\lambda-2) + 3(3\lambda) = 0$$

$$\Rightarrow \lambda = \frac{1}{2}$$

Hence, coordinates of M are $\left(\frac{1}{2}, 0, \frac{11}{2}\right)$

OR

(b) Equation of given line be $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3} = \lambda$ (say)

Coordinate of any general point on the line are $P(3\lambda+1, 2\lambda-1, 3\lambda+4)$.

Let distance of point P from $(-1,-1,2)$ is $2\sqrt{2}$.

$$\Rightarrow \sqrt{(3\lambda+2)^2 + (2\lambda)^2 + (3\lambda+2)^2} = 2\sqrt{2}$$

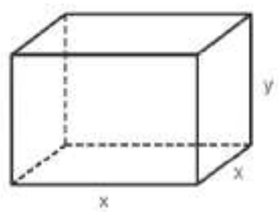
$$\Rightarrow 22\lambda^2 + 24\lambda = 0$$

$$\Rightarrow \lambda = 0 \text{ or } \lambda = -\frac{12}{11}$$

Hence, coordinates of point P are $(1,-1,4)$ or $\left(-\frac{25}{11}, -\frac{35}{11}, \frac{8}{11}\right)$

SECTION-E

150.



(i) $V = x^2y \Rightarrow y = \frac{V}{x^2}$ ... ...(i)

Hence, $S = 2x^2 + 4xy = 2x^2 + \frac{4V}{x}$

(ii) $\frac{dS}{dx} = 4\left(x - \frac{V}{x^2}\right)$

(iii) (a) $\frac{dS}{dx} = 0 \Rightarrow V = x^3 \Rightarrow x^2y = x^3 \Rightarrow y = x$

$\frac{d^2S}{dx^2}=4\left(1+\frac{2V}{x^3}\right)>0 \Rightarrow S$ is minimum if $y=x$.

OR

(iii) (b) $V=\frac{1}{4}(Sx-2x^3) \Rightarrow \frac{dV}{dx}=\frac{1}{4}(S-6x^2)$

Put $\frac{dV}{dx}=0 \Rightarrow x=\sqrt{\frac{S}{6}}$

$\left(\frac{d^2v}{dx^2}\right)_{x=\sqrt{\frac{S}{6}}}=-3\sqrt{\frac{S}{6}}<0 \Rightarrow$ Volume is maximum for $x=\sqrt{\frac{S}{6}}$.

151. **(i)** No, f is not bijective function

(ii) Range $=\{1,2,3,4,\ldots\ldots\ldots\ldots,30\}$ and codomain $=N$

Since, Range $\neq$ codomain $\Rightarrow$ **f** is not onto and hence **f** is not bijective.

(iii) (a) $R=\{(1,3),(2,6),(3,9),(4,12),(5,15),(6,18),(7,21),(8,24),(9,27),(10,30)\}$

Since $(1,1)\notin R \Longrightarrow R$ is not reflexive.

$(1,3)\in R$ but $(3,1)\notin R \Longrightarrow R$ is not symmetric $(1,3)\in R,(3,9)\in R$ but $(1,9)\notin R \Longrightarrow R$ is not transitive.

OR

(iii) (b) $R=\{(\mathbf{1},\mathbf{1}),(\mathbf{2},\mathbf{8}),(\mathbf{3},\mathbf{27})\}$

elements 4,5,6 ... 30 do not have an image. Hence the above relation is not a function.

152. Let A: Event that chosen seed germinates.

B: Event that Brinjal seed is chosen.

C: Event that Cabbage seed is chosen.

R: Event that Radish seed is chosen.

$P(B)=\frac{10}{30}; P(C)=\frac{12}{30}; P(R)=\frac{8}{30};$

$P\left(\frac{A}{B}\right)=\frac{25}{100}; P\left(\frac{A}{C}\right)=\frac{35}{100}; P\left(\frac{A}{R}\right)=\frac{40}{100}$

(i) $P(A)=P(B)\cdot P\left(\frac{A}{B}\right)+P(C)\cdot P\left(\frac{A}{C}\right)+P(R)\cdot P\left(\frac{A}{R}\right)$

$=\frac{10}{30}\times\frac{25}{100}+\frac{12}{30}\times\frac{35}{100}+\frac{8}{30}\times\frac{40}{100}$

$=\frac{990}{3000}$ or $\frac{33}{100}$

(ii) $P\left(\frac{C}{A}\right)=\frac{P(C)\cdot P\left(\frac{A}{C}\right)}{P(B)\cdot P\left(\frac{A}{B}\right)+P(C)\cdot P\left(\frac{A}{C}\right)+P(R)\cdot P\left(\frac{A}{R}\right)}$

$=\frac{\frac{12}{30}\times\frac{35}{100}}{\frac{990}{3000}}$

$=\frac{42}{99}$ or $\frac{14}{33}$

SECTION-A

153. **(b)**
154. **(c)**
155. **(c)**
156. **(a)**
157. **(d)**
158. **(c)**
159. **(a)**
160. **(c)**
161. **(d)**
162. **(b)**
163. **(c)**
164. **(b)**
165. **(d)**

166. **(a)**

167. **(c)**

168. 1 mark for any attempt as correct answer is not given in any option

169. **(d)**

170. **(c)**

171. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

172. **(d)** Assertion (A) is false, but Reason (R) is true.

SECTION-B

173. **(a)** $-1 \leq -x^2 \leq 1 \Rightarrow -1 \leq -x^2 \leq 0$

$\Rightarrow 0 \leq x^2 \leq 1 \Rightarrow -1 \leq x \leq 1$

174. **(a)** Let $u = \sqrt{e^{\sqrt{2x}}}$ and $v = e^{\sqrt{2x}}$

Derivative of $\sqrt{v}$ w.r.t. $v = \frac{1}{2\sqrt{v}}$.

Required derivative $= \dfrac{1}{2\sqrt{e^{\sqrt{2x}}}}$

OR

Taking log on both sides, we get $y\log x = x\log y$

Differentiating both sides w.r.t. x, we get

$$\frac{y}{x} + \log x\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(x\log y - y)}{x(y\log x - x)}$$

175. $f'(x) = \frac{x+1-x+4}{(x+1)^2} = \frac{5}{(x+1)^2} > 0$

Hence f is increasing in its domain.

176. **(a)** C divides BA in the ratio 3: 2 externally

Required vector $= \vec{c} = \dfrac{3\vec{a} - 2\vec{b}}{3-2} = 3\vec{a} - 2\vec{b}$

OR

(b) Unit vector equally inclined along coordinate axes is $\frac{\hat{\imath}}{\sqrt{3}} + \frac{\hat{\jmath}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$

$$\vec{r} = 5\sqrt{3}\left(\frac{\hat{\imath}}{\sqrt{3}} + \frac{\hat{\jmath}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}\right) = 5\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$$

(or $-5\hat{\imath} - 5\hat{\jmath} - 5\hat{k}$)

177. $\vec{b}_1 = 3\hat{\imath} - \hat{\jmath}, \vec{b}_2 = 2\hat{\imath} + 3\hat{k}, \vec{a}_2 = 4\hat{\imath} - \hat{k}, \vec{a}_1 = \hat{\imath} + \hat{\jmath} - \hat{k}$

$(\vec{a}_2 - \vec{a}_1) = 3\hat{\imath} - \hat{\jmath}$

$\vec{b}_1 \times \vec{b}_2 = -3\hat{\imath} - 9\hat{\jmath} + 2\hat{k}$

$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (3\hat{\imath} - \hat{\jmath}) \cdot (-3\hat{\imath} - 9\hat{\jmath} + 2\hat{k}) = 0,$

hence lines intersect

SECTION-C

178. **(a)** Let $B = [x \quad y \quad z]$

$$AB = C \Rightarrow \begin{bmatrix} x & y & z \\ 4x & 4y & 4z \\ -2x & -2y & -2z \end{bmatrix} = \begin{bmatrix} 3 & 4 & 2 \\ 12 & 16 & 8 \\ -6 & -8 & -4 \end{bmatrix} \text{ which}$$

gives $x = 3, y = 4$ and $z = 2$

$B = [3 \quad 4 \quad 2]$

179. **(a)** $x = \sin t$ gives $y = \sin^{-1}(\sin 3t) = 3t = 3\sin^{-1} x$

$$\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$$

Aliter: $\frac{dy}{dx} = \frac{3-12x^2}{\sqrt{1-(3x-4x^3)^2}}$

OR

(b) $x = \tan t$ gives $y = \cos^{-1}(\cos 2t) = 2t = 2\tan^{-1} x$

$$\frac{dy}{dx} = \frac{2}{1+x^2}$$

Aliter: $\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-4x}{(1+x^2)^2}$.

180. **(a)** $\mathrm{R} = \{(1,39), (2,37), \_\_\_, (20,1)\}$

Domain $= \{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$

Range $= \{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39\}$

$(1,1)$ does not belong to R hence not reflexive

$(1,39)$ belongs to R but $(39,1)$ does not belong to R hence not symmetric

$(11,19)$ and $(19,3)$ belong to R but $(11,3)$ does not belong to R hence not transitive. Hence R is not an equivalence relation.

OR

(a) Let $f(x) = f(y)$

Let x and y are both odd or both even

Then either $x + 1 = y + 1$ or $x - 1 = y - 1$ gives

$x = y$

x odd and y even is rejected as

$x + 1 = y - 1$ gives $x - y = -2$ not possible as odd number and even number cannot differ by 2

Hence f is one-one

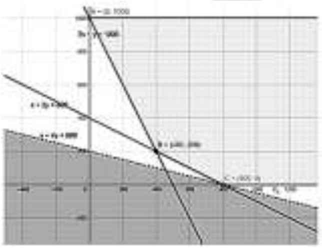
For onto: Let $\mathrm{f(x)} = \mathrm{y}$ gives $\mathrm{x} = \mathrm{y} + 1$ or $\mathrm{x} = \mathrm{y} - 1$

If y is odd, x is even and if y is even, x is odd.

Range $= \mathrm{N} =$ co-domain, hence onto

As f is both one-one and onto hence bijective

181.



Correct Graph and shading:

Corner points	Value of Z
(800,0)	800
(400,200)	1200
(0,1000)	4000

$x + 4y < 800$ has no region common with feasible region, hence 800 is minimum

182. **(a)** Let $\vec{a_2} = 2\hat{\imath} + 4\hat{\jmath} - \hat{k}, \vec{a_1} = -5\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ and $\vec{b} = \hat{\imath} + 4\hat{\jmath} - 9\hat{k}$

Distance between point and line is given by $\mathrm{d} = \frac{|(\vec{a_2}-\vec{a_1})\times\vec{b}|}{|\vec{b}|}$

Here$(\vec{a_2} - \vec{a_1}) = 7\hat{\imath} + 7\hat{\jmath} - 7\hat{k}$

$(\vec{a_2} - \vec{a_1}) \times \vec{b} = -35\hat{\imath} + 56\hat{\jmath} + 21\hat{k}$

$$d = \frac{49\sqrt{2}}{7\sqrt{2}} = 7$$

OR

(b) Vector parallel to given line $= 3\hat{j} + 4\hat{k}$

Vector equation is $\vec{r} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k} + \mu(3\hat{\jmath} + 4\hat{k})$

Cartesian equation is $\frac{x-3}{0} = \frac{y+1}{3} = \frac{z+2}{4}$

183. **(i)** P(correct decision in both committees) $= (1 - 0.03) \cdot (1 - 0.01) = 0.9603$

(ii) P(correct decision in one committee) $= 0.03 \cdot (1 - 0.01) + (1 - 0.03) \cdot 0.01$

$= 0.0394$

SECTION-D

184. **(a)** $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3} = \frac{3/8}{x-1} + \frac{1/2}{(x-1)^2} + \frac{5/8}{x+3}$

$$I = \frac{3}{8}\log|x-1| - \frac{1}{2(x-1)} + \frac{5}{8}\log|x+3| + C$$

OR

$$\text{Let } I = \int_0^{\frac{\pi}{2}} \frac{x}{\sin x + \cos x} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2} - x}{\cos x + \sin x} dx \text{ using property}$$

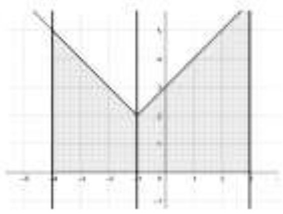
$$2I = \int_0^{\frac{\pi}{2}} \frac{\frac{\pi}{2}}{\sin x + \cos x} dx$$

$$= \frac{\pi}{2}\frac{1}{\sqrt{2}}\int_0^{\frac{\pi}{2}} \frac{1}{\sin\left(\frac{\pi}{4} + x\right)} dx$$

$$2I = \frac{\pi}{2\sqrt{2}} \log\left|\operatorname{cosec}\left(\frac{\pi}{4} + x\right) - \cot\left(\frac{\pi}{4} + x\right)\right|_0^{\frac{\pi}{2}}$$

$$I = \frac{\pi}{4\sqrt{2}} \log\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

185.



Correct Graph:

$$\text{Required area} = \int_{-4}^{-1} (2 + |x+1|)dx + \int_{-1}^{3} (2 + |x+1|)dx$$

$$= \int_{-4}^{-1} (1-x)dx + \int_{-1}^{3} (3+x)dx$$

$$= -\frac{(1-x)^2}{2}\Bigg|_{-4}^{-1} + \frac{(3+x)^2}{2}\Bigg|_{-1}^{3}$$

$$= \frac{21}{2} + 16 = \frac{53}{2}$$

186. **(a)** Given differential equation can be written as $\frac{dy}{dx} = \frac{yx^2}{x^3+y^3}$

Put $y = vx$, so $\frac{dv}{dx} = v + x\frac{dv}{dx}$

Therefore,

$$e, v + x\frac{dv}{dx} = \frac{vx^3}{x^3 + v^3x^3} = \frac{v}{1+v^3}$$

$x\frac{dv}{dx} = \frac{-v^4}{1+v^3}$

$\left(\frac{1}{v^4}+\frac{1}{v}\right)dv = \frac{-dx}{x}$

Integrating we get

$\frac{-1}{3v^3} + \log|v| = -\log|x| + C$

$\frac{-x^3}{3y^3} + \log|y| = C$

OR

(b) Given D.E. is $\frac{dy}{dx}+\frac{2x}{1+x^2}y = \frac{4x^2}{1+x^2}$

Integrating factor is $e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = (1+x^2)$

Solution is $y(1+x^2) = \int 4x^2 dx + C$

$y(1+x^2) = \frac{4x^3}{3} + C$

$y(0) = 0$ gives $C = 0$, hence solution is $y(1+x^2) = \frac{4x^3}{3}$

187. Equation of given line is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$

Let coordinates of point on the line be $(\lambda, 2\lambda+1, 3\lambda+2)$ for some λ

Drs of line perpendicular to line along mirror are $\langle \lambda-1, 2\lambda-5, 3\lambda-1\rangle$

$(\lambda-1)\cdot 1 + (2\lambda-5)\cdot 2 + (3\lambda-1)\cdot 3 = 0$ gives $\lambda = 1$

Coordinates of foot of perpendicular are (1,3,5)

For image

$\frac{x+1}{2} = 1, \frac{y+6}{2} = 3, \frac{z+3}{2} = 5$ gives image as (1,0,7)

Required distance $= \sqrt{0+36+16} = 2\sqrt{13}$

SECTION-E

188. **(i)**Let the price of each pen, notepad, eraser be ₹x, ₹y and ₹z respectively Given system in the form AX = B is

$$\begin{pmatrix}4&3&2\\2&4&6\\6&2&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}60\\90\\70\end{pmatrix}$$

(ii) $|A| = 50 \neq 0$, hence A^{-1} exists

(iii) (a) $A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{50}\begin{pmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{pmatrix}$

$X = A^{-1}B$

OR

(iii)(b) $A^2 = \begin{pmatrix}4&3&2\\2&4&6\\6&2&3\end{pmatrix}\begin{pmatrix}4&3&2\\2&4&6\\6&2&3\end{pmatrix} = \begin{pmatrix}34&28&32\\52&34&46\\46&32&33\end{pmatrix}$

$A^2 - 8I = \begin{pmatrix}26&28&32\\52&26&46\\46&32&25\end{pmatrix}$

189. **(i)** $y^2 = h^2 - x^2$

$A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{h^2-x^2}$

(ii) $\frac{dA}{dx} = \frac{1}{2}\sqrt{h^2-x^2} + \frac{1}{2}x\frac{-x}{\sqrt{h^2-x^2}}$

$\frac{dA}{dx} = 0$ gives $x = \frac{h}{\sqrt{2}}$

(iii)(a) $A'' = \frac{1}{2}\frac{-4x\cdot\sqrt{h^2-x^2} - (h^2-2x^2)\frac{-x}{\sqrt{h^2-x^2}}}{h^2-x^2}$ is < 0 at $x = \frac{h}{\sqrt{2}}$

Hence A is maximum at critical point

OR

(iii)(b) $y^2 = 25 - x^2$ hence $y = 3$ gives $x = 4$

$2y\frac{dy}{dt} = -2x\frac{dx}{dt}$

$\frac{dx}{dt} = 1.5\ m/s$

190. **(i)** P(defective smartphone) $= 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$

$= 0.0345$

(ii) P(B/ Defective) $= \frac{0.35\times0.04}{0.25\times0.05+0.35\times0.04+0.40\times0.02}$

$= \frac{140}{345}$ or $\frac{28}{69}$

SECTION-A

191. **(a)**
192. **(c)**
193. **(b)**
194. **(d)**
195. **(b)**
196. **(c)**
197. **(b)**
198. **(a)**
199. **(b)**
200. **(c)**
201. **(b)**
202. **(c)**
203. **(a)**
204. **(d)**
205. **(d)**
206. **(a)**
207. **(d)**
208. **(b)**
209. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
210. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

SECTION-B

211. **Let $x_1, x_2 \in A$ such that $f(x_1) = f(x_2) \Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \Rightarrow x_1 = x_2, \therefore$ ' f ' is one-one. For each $y \in B$, there exists $x = \frac{3y-2}{y-1} \in R - \{3\}$, such that $f(x) = y, \therefore$ ' f ' is onto $\Rightarrow$ ' f ' is one-one & onto, or ' f ' is a bijective function.**

212. $A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$.

L.H.S. $= A^2 - 4A + 7I = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O =$ R.H.S.

213. **(a)** Let,

$y = \frac{5^x}{x^5} = 5^x \cdot x^{-5} \Rightarrow \frac{dy}{dx} = (5^x)' \cdot x^{-5} + 5^x \cdot (x^{-5})'$

$= \frac{5^x}{x^5}\log 5 - \frac{5^{x+1}}{x^6}$

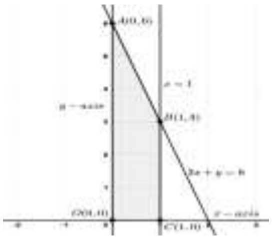
OR

(b) Differentiating $-2x^2 - 5xy + y^3 = 76$, with respect to ' x '

$$-4x-5y-5x\frac{dy}{dx}+3y^2\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{4x+5y}{3y^2-5x}$$

214.



Correct Plotting of the two lines

Correct shading of the feasible region and marking the corner points

215. **(a)** Probability distribution table is:

X	0	1	2	3
P(X)	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{1}{10}$

Mean $=E(X)=\sum p_i x_i=0\cdot\frac{2}{10}+1\cdot\frac{3}{10}+2\cdot\frac{4}{10}+3\cdot\frac{1}{10}=\frac{14}{10}=\frac{7}{5}$ (or 1.4)

OR

(b) $A=A$ randomly chosen person is a woman

$B=A$ randomly chosen person works outside village.

$$\mathrm{P}(A)=\frac{4000}{8000}=\frac{1}{2},\mathrm{P}(B)=\frac{3000}{8000}=\frac{3}{8},\mathrm{P}(A\cap B)=\frac{1200}{8000}=\frac{3}{20}$$

Required probability $=P(A\cup B)=P(A)+P(B)-P(A\cap B)=\frac{1}{2}+\frac{3}{8}-\frac{3}{20}=\frac{29}{40}$

216. **(a)** One-One: Let $x_1,x_2\in R$ such that $f(x_1)=f(x_2)\Rightarrow 4x_1{}^3-5=4x_2{}^3-5\Rightarrow x_1{}^3=x_2{}^3\Rightarrow x_1=x_2,\therefore f'$ is one-one Onto: $x\in R(D_f)\Rightarrow x^3\in R\Rightarrow 4x^3-5\in R\Rightarrow f(x)\in R,\therefore R_f=$ Co-domain (f)

' f ' is an onto function

$\Rightarrow$ ' f ' is one-one & onto both OR

(b) Reflexive: For any $x\in N, x\cdot x=x^2$, which is square of the natural number ' x '.

$\Rightarrow (x,x)\in R$

' R ' is a Reflexive relation.

Symmetric: Let $(x,y)\in R\Rightarrow xy$ is a square of a natural number

$\Rightarrow$ yx is a square of a natural number, $\because xy=yx$.

$\Rightarrow (y,x)\in R$

' R ' is a Symmetric relation.

Transitive: Let $(x,y),(y,z)\in R\Rightarrow xy=a^2, yz=b^2$ for some $a,b\in N$,

$$\frac{a^2}{y}=x,\frac{b^2}{y}=z\in N$$

$$\Rightarrow xz=\frac{a^2}{y}\cdot\frac{b^2}{y}=\left(\frac{ab}{y}\right)^2,\frac{ab}{y}\in N$$

$\Rightarrow (x,z)\in R$

' R ' is a Transitive relation.

Hence, R is an Equivalence relation

217. **(a)** The system of equations in matrices is:

$AX=B$, where $A=\begin{bmatrix}2&5\\3&2\end{bmatrix}, X=\begin{bmatrix}x\\y\end{bmatrix}, B=\begin{bmatrix}1\\7\end{bmatrix}$

The solution is given by $X=A^{-1}B\Rightarrow\begin{bmatrix}x\\y\end{bmatrix}=\frac{-1}{11}\begin{bmatrix}2&-5\\-3&2\end{bmatrix}\begin{bmatrix}1\\7\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$

Point common to paths of the ants is $(3,-1)$.

OR

(b) Let $A = \begin{bmatrix} 50 & 60 & 35 \\ 40 & 45 & 50 \end{bmatrix} \begin{matrix} \text{Day I} \\ \text{Day II} \end{matrix}$, $B = \begin{bmatrix} 150 \\ 175 \\ 180 \end{bmatrix}$ be the day wise sale and the selling price per subject, matrices respectively.

Total sales day wise $= \begin{bmatrix} 50 & 60 & 35 \\ 40 & 45 & 50 \end{bmatrix} \begin{bmatrix} 150 \\ 175 \\ 180 \end{bmatrix} = \begin{bmatrix} 24,300 \\ 22,875 \end{bmatrix} \begin{matrix} \text{Day I} \\ \text{Day II} \end{matrix}$

Total sales in two days = ₹ 24,300 + ₹22,875 = ₹47,175
Profit = ₹47,175 − ₹35,000 = ₹12,175.

218. $\frac{dy}{dx} = \frac{1}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}} \cdot \frac{1}{\sin\left(\frac{x^3}{3}-1\right)} \cdot \cos\left(\frac{x^3}{3}-1\right) \cdot \frac{3x^2}{3}$

$$= \frac{x^2 \cot\left(\frac{x^3}{3}-1\right)}{2\sqrt{\log\left\{\sin\left(\frac{x^3}{3}-1\right)\right\}}}$$

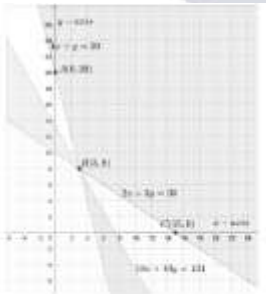
219. Let numbers be 'x' and 'y' such that $xy = 289 \Rightarrow y = \frac{289}{x}$, '$S$' be their sum, then

$S = x + y = x + \frac{289}{x}$

$\frac{dS}{dx} = 1 - \frac{289}{x^2}, \frac{dS}{dx} = 0 \Rightarrow x = 17$, a positive integer

$\left.\frac{d^2S}{dx^2}\right]_{x=17} = 289\left(\frac{2}{x^3}\right)\Big]_{x=17} > 0, \therefore$ S is minimum when $x = 17, y = 17$

220.



Correct Fig.

Corner points	**Value of $Z = 18x + 10y$**
$A(0,20)$	200
$B(3,8)$	134
$C(15,0)$	270

Also, $Z < 134$, does not have any common point with the feasible region,
$\text{Min}(Z) = 134$ at $B(3,8)$

221. **(a)** Let $\vec{d} = \vec{b} + \vec{c} = (2+\lambda)\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$

$$\hat{d} = \frac{(2+\lambda)\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}}$$

$$\vec{a} \cdot \hat{d} = (\hat{\imath} - \hat{\jmath} + 2\hat{k}) \cdot \frac{(2+\lambda)\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{\sqrt{(2+\lambda)^2 + 40}} = 1$$

$\Rightarrow (2+\lambda) + 6 + 4 = \sqrt{(2+\lambda)^2 + 40} \Rightarrow \lambda = -5$

OR

(b) The two given lines are parallel with,
$\vec{a}_1 = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}, \vec{a}_2 = \hat{\imath} + 4\hat{k}$

Then $\vec{a}_2 - \vec{a}_1 = -\hat{\imath} + \hat{\jmath} + \hat{k}$ and the parallel vector is $\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

$$\vec{b} \times (\vec{a}_2 - \vec{a}_1) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -2 & 3 \\ -1 & 1 & 1 \end{vmatrix} = -5\hat{\imath} - 4\hat{\jmath} - \hat{k}$$

Shortest Distance $= \dfrac{|\vec{b} \times (\vec{a}_2 - \vec{a}_1)|}{|\vec{b}|} = \dfrac{\sqrt{42}}{\sqrt{14}} = \sqrt{3}$

SECTION-D

222. **(a)** $\int \frac{x^2+1}{(x^2+2)(2x^2+1)} dx = \frac{1}{3}\int \frac{1}{x^2+2} dx + \frac{1}{3}\int \frac{1}{2x^2+1} dx$ (Using Partial Fractions)

$$= \frac{1}{3} \cdot \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \frac{1}{3} \cdot \frac{1}{\sqrt{2}} \tan^{-1}(\sqrt{2}x) + C$$

or $= \dfrac{1}{3\sqrt{2}}\left(\tan^{-1}\dfrac{x}{\sqrt{2}} + \tan^{-1}\sqrt{2}x\right) + C$

OR

(b) Let $I = \int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ .... (i)

$$\Rightarrow I = \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x + \tan x} dx \text{ .... (ii)}$$

Adding (i) & (ii), we get

$$2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x + \tan x} dx \Rightarrow 2I = \pi \int_0^{\pi} \frac{\tan x(\sec x - \tan x)}{\sec^2 x - \tan^2 x} dx$$

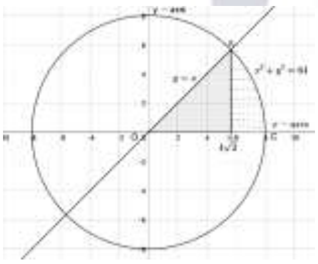
$$= \pi \int_0^{\pi} (\sec x \tan x - \sec^2 x + 1) dx$$

$$= \pi(\sec x - \tan x + x)]_0^{\pi}$$

$$= \pi(-1 + \pi - 1) = \pi(\pi - 2)$$

$$I = \frac{\pi}{2}(\pi - 2) \text{ or } \pi\left(\frac{\pi}{2} - 1\right)$$

223.



Correct graph

Equation of the circular tabletop:

$x^2 + y^2 = 64$

Equation of line (scratch): $x = y$

The line and circle intersect at $x = 4\sqrt{2}$

Area of the shaded region

$$= \int_0^{4\sqrt{2}} x dx + \int_{4\sqrt{2}}^{8} \sqrt{64 - x^2} dx$$

$$= \left.\frac{x^2}{2}\right]_0^{4\sqrt{2}} + \left[\frac{x}{2}\sqrt{64 - x^2} + 32\sin^{-1}\frac{x}{8}\right]_{4\sqrt{2}}^{8}$$

$$= \frac{32}{2} + 32\sin^{-1} 1 - 2\sqrt{2} \cdot 4\sqrt{2} - 32\sin^{-1}\frac{1}{\sqrt{2}}$$

$$= 16 + 16\pi - 16 - 8\pi = 8\pi \text{ cm}^2$$

224. The given differential equation can be written as:

$\dfrac{dy}{dx} + 2y = \cos x$, Taking $P = 2, Q = \cos x$

Integrating factor is given by, $I = e^{\int 2dx} = e^{2x}$
The solution is, $y \cdot e^{2x} = \int \; e^{2x} \cos x dx$

Let, $\mathrm{I}_1 = \int \; \cos x \cdot e^{2x} dx = \cos x \frac{e^{2x}}{2} - \int \; (-\sin x) \frac{e^{2x}}{2} dx$

$= \frac{e^{2x} \cos x}{2} + \frac{1}{2}\left[\sin x \cdot \frac{e^{2x}}{2} - \int \; \cos x \cdot \frac{e^{2x}}{2} dx\right]$

$\Rightarrow \mathrm{I}_1 = \frac{e^{2x} \cos x}{2} + \frac{e^{2x} \sin x}{4} - \frac{1}{4}\mathrm{I}_1 \Rightarrow \mathrm{I}_1 = \frac{e^{2x}}{5}(2\cos x + \sin x)$

The solution of the differential equation is

$y \cdot e^{2x} = \frac{e^{2x}}{5}(2\cos x + \sin x) + C \Rightarrow y = \frac{1}{5}(2\cos x + \sin x) + Ce^{-2x}$

225. **(a)** The general point on the line $(3\lambda - 2, 2\lambda - 1, 2\lambda + 3)$ is Q, from some $\lambda \in R$

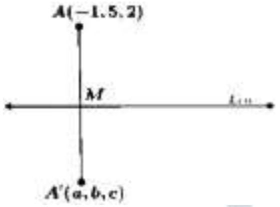
$\mathrm{PQ} = 3\sqrt{2} \Rightarrow (\mathrm{PQ})^2 = 18 \Rightarrow (3\lambda - 3)^2 + (2\lambda - 3)^2 + (2\lambda)^2 = 18$

$17\lambda^2 - 30\lambda = 0 \Rightarrow \lambda = 0$ or $\lambda = \frac{30}{17}$

Thus, the point is $\mathrm{Q}(-2, -1, 3)$ or $\mathrm{Q}\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$

OR

(b) Let $\mathrm{A}'(a, b, c)$ be the image of the point $\mathrm{A}(-1,5,2)$ in the given line, also assume ' M ' as the point of intersection of AA' with the given line, then ' M ' is the mid-point of the line segment AA'



The Line in the standard form is: $\frac{x-2}{1} = \frac{y}{2} = \frac{z-2}{-3}$, then M is the point $(\lambda + 2, 2\lambda, -3\lambda + 2)$, for some $\lambda \in R$

Direction Ratios of AM are $\lambda + 3, 2\lambda - 5, -3\lambda$ AM $\perp$ Line, $\therefore 1(\lambda + 3) + 2(2\lambda - 5) - 3(-3\lambda) = 0 \Rightarrow \lambda = \frac{1}{2}$

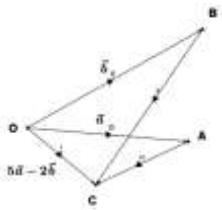
$M\left(\frac{5}{2}, 1, \frac{1}{2}\right) = M\left(\frac{a-1}{2}, \frac{b+5}{2}, \frac{c+2}{2}\right) \Rightarrow a = 6, b = -3, c = -1$

The Image of A in the line is $A'(6, -3, -1)$

And, $AA' = \sqrt{49 + 64 + 9} = \sqrt{122}$

SECTION-E

226. **(i)** The Complete figure of their entire movement plan is:



(ii) $\overrightarrow{\mathrm{AC}} = \overrightarrow{\mathrm{OC}} - \overrightarrow{\mathrm{OA}} = 4\vec{a} - 2\vec{b}, \overrightarrow{\mathrm{BC}} = \overrightarrow{\mathrm{OC}} - \overrightarrow{\mathrm{OB}} = 5\vec{a} - 3\vec{b}$

(iii) (a) we are given: $|\vec{a}| = 1, |\vec{b}| = 2$, assuming ' θ ' as the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

$$\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right) = \cos^{-1}\frac{1}{1 \times 2} = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}$$

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta = 1(2)\frac{\sqrt{3}}{2} = \sqrt{3}$$

OR

(iii) (b) $\vec{a} + \vec{b} = 2\hat{\imath} + 3\hat{k}, \vec{a} - \vec{b} = 2\hat{\imath} - 2\hat{\jmath} + 5\hat{k}$, let $\vec{c}$ be $\perp$ to both $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$

Then, $\vec{c} = (\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 0 & 3 \\ 2 & -2 & 5 \end{vmatrix} = 6\hat{\imath} - 4\hat{\jmath} - 4k$ and $|\vec{c}| = \sqrt{68}$

The required unit vector is, $\hat{c} = \frac{1}{2\sqrt{17}}(6\hat{\imath} - 4\hat{\jmath} - 4\hat{k}) = \frac{1}{\sqrt{17}}(3\hat{\imath} - 2\hat{\jmath} - 2\hat{k})$

Case Study – 2

227. **(i)** Order = 1, Degree = 1

(ii) Separating the variable and integrating, $\int dr = \frac{2k}{3}\int dt \Rightarrow r = \frac{2}{3}kt + C$

Putting $t = 0, r = 5$, we get $C = 5$

$r = \frac{2}{3}kt + 5$

(iii) (a) Putting $\mathrm{r} = 3, \mathrm{t} = 1, 3 = \frac{2}{3}\mathrm{k}(1) + 5 \Rightarrow \mathrm{k} = -3$

$r = -2t + 5$, For $r = 0, t = \frac{5}{2}$ hours or 2.5 hours

OR

(iii) (b) Putting $\mathrm{r} = 1, \mathrm{t} = 1, 1 = \frac{2}{3}\mathrm{k} + 5 \Rightarrow \mathrm{k} = -6$

$r = -4t + 5$, For $r = 0, t = \frac{5}{4}$ hours or 1.25 hours

Case Study – 3

228. **(i)** Let A: Person contracted the disease

$= \mathrm{P}(A_1) \cdot \mathrm{P}(A \mid A_1) + \mathrm{P}(A_2) \cdot \mathrm{P}(A \mid A_2) + \mathrm{P}(A_3) \cdot \mathrm{P}(A \mid A_3)$

P(A)

$= \frac{7}{10}\left(\frac{25}{100}\right) + \frac{2}{10}\left(\frac{35}{100}\right) + \frac{1}{10}\left(\frac{50}{100}\right)$

$= \frac{295}{1000} = 0.295$ or $\left(\frac{59}{200}\right)$

(ii) $\mathrm{P}(A_2 \mid \overline{A}) = \frac{\mathrm{P}(A_2)\cdot\mathrm{P}(\overline{A}/A_2)}{\mathrm{P}(A_1)\cdot\mathrm{P}(\overline{A}/A_1) + \mathrm{P}(A_2)\cdot\mathrm{P}(\overline{A}/A_2)}$

$= \frac{\frac{2}{10}\times\frac{65}{100}}{\frac{7}{10}\times\frac{75}{100} + \frac{2}{10}\times\frac{65}{100} + \frac{1}{10}\times\frac{50}{100}}$

$= \frac{2\times 13}{7\times 15 + 2\times 13 + 1\times 10} = \frac{26}{141}$

SECTION-A

229. **(c)**
230. **(a)**
231. **(b)**
232. **(d)**
233. **(b)**
234. **(c)**
235. **(c)**
236. **(c)**
237. **(b)**
238. **(d)**
239. **(a)**
240. **(b)**
241. **(a)**
242. **(b)**
243. **(b)**
244. **(c)**
245. **(d)**

246. **(c)**

247. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

248. **(a)** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

SECTION-B

249. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \frac{\pi}{3} - \left[\pi - \frac{\pi}{3}\right]$

$= \frac{-\pi}{3}$

250. **(a)** $f'(x) = \frac{1}{3}(x-1)^{\frac{-2}{3}} f'(1)$ is not defined

OR

$y = \log\left(x + \sqrt{x^2 + a^2}\right)$

$\frac{dy}{dx} = \frac{1 + \frac{x}{\sqrt{x^2+a^2}}}{x + \sqrt{x^2+a^2}}$

$= \frac{1}{\sqrt{x^2+a^2}}$

251. $y = 7x - x^3$

$m = 7 - 3x^2$

$\frac{dm}{dt} = -6x\frac{dx}{dt}$

$\left[\frac{dm}{dt}\right]_{x=5} = -30(2) = -60$

252. **(a)** $|\vec{a}+\vec{b}|^2 + |\vec{a}-\vec{b}|^2 = 2\left(|\vec{a}|^2 + |\vec{b}|^2\right)$

$(60)^2 + (40)^2 = 2(|\vec{a}|^2 + (46)^2)$

$\frac{(3600+1600)}{2} = (|\vec{a}|^2 + (46)^2)$

$2600 - 2116 = |\vec{a}|^2$

$484 = |\vec{a}|^2$

$|\vec{a}| = 22$

OR

(b) Let A, B, C be the points $(k,-11,2)$, $(0,-2,2)$ and $(2,4,2)$ respectively

$\overrightarrow{AB} = -k\hat{\imath} + 9\hat{\jmath}$

$\overrightarrow{BC} = 2\hat{\imath} + 6\hat{\jmath}$

Since $\overrightarrow{AB}$ is parallel to $\overrightarrow{BC}$

$\frac{-k}{2} = \frac{9}{6}$

$\Rightarrow k = -3$

253. $2x = 3y = -z \Rightarrow \frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$

$6x = -y = -4z \Rightarrow \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$\cos\theta = \frac{3\times2 + 2\times(-12) - 6\times(-3)}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}}$

$3\times2 + 2\times(-12) - 6\times(-3) = 0 \therefore \theta = \frac{\pi}{2}$

SECTION-C

254. $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$

$f'(x) = 12x^3 - 12x^2 - 24x$

$f'(x) = 12x(x-2)(x+1)$
$f'(x) = 0 \Rightarrow x = -1,0,2$

Sign of f/(x)

f is strictly increasing in $(-1,0)$ as well as $(2,\infty)$ or $[-1,0]$ as well as $[2,\infty)$
and strictly decreasing in $(-\infty,-1)$ as well as $(0,2)$ or $(-\infty,-1]$ as well as $[0,2]$

255. **(a)** $\int \frac{x^2-x+1}{(x-1)(x^2+1)}dx$

Let $\frac{x^2-x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$

Getting $A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{-1}{2}$

$$\frac{1}{2}\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{x}{x^2+1}dx - \frac{1}{2}\int \frac{1}{x^2+1}dx$$

$$= \frac{1}{2}\log|x-1| + \frac{1}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + C$$

OR

(b) $\int_1^4 (|x| + |3-x|)dx$

$$\int_1^4 |x|dx + \int_1^4 |3-x|dx$$

$$\int_1^4 xdx + \int_1^3 (3-x)dx - \int_3^4 (3-x)dx$$

$$= \frac{1}{2}[x^2]_1^4 + \left[3x - \frac{x^2}{2}\right]_1^3 - \left[3x - \frac{x^2}{2}\right]_3^4$$

$$= \frac{15}{2} + 2 + \frac{1}{2} = 10$$

256. **(a)** $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$

$$\frac{dy}{dx} = (1+x^2)(1+y^2)$$

$$\int \frac{dy}{1+y^2} = \int (1+x^2)dx$$

$$=> \tan^{-1}y = x + \frac{x^3}{3} + C$$

$y = 1$ when $x = 0 => C = \frac{\pi}{4}$

Particular solution is $\tan^{-1}y = x + \frac{x^3}{3} + \frac{\pi}{4}$

(b) The given differential equation can be written as $\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$

putting y = vx, we get

$$v + x\frac{dv}{dx} = \frac{x^2+3v^2x^2}{2xvx}$$

$$\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$

$\Rightarrow \log(1+v^2) = \log|x| + \log C$

$=> 1 + v^2 = Cx$

$$=> 1 + \frac{y^2}{x^2} = Cx \text{ or } x^2 + y^2 = Cx^3$$

257. let $\vec{p} = \vec{a} - \vec{b} = -\hat{\imath} + \hat{\jmath} + \hat{k}$

$\vec{q} = \vec{c} - \vec{b} = \hat{\imath} - 5\hat{\jmath} - 5\hat{k}$

Vector perpendicular to $\vec{p}$ and $\vec{q} = \vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix} = -4\hat{j} + 4\hat{k}$

Unit Vector perpendicular to $\vec{p}$ and $\vec{q} = \frac{-4\hat{j}+4\hat{k}}{\sqrt{4^2+4^2}}$

$= \dfrac{-\hat{j}+\hat{k}}{\sqrt{2}}$ or $\dfrac{-1}{\sqrt{2}}\hat{j} + \dfrac{1}{\sqrt{2}}\hat{k}$

258. $Z = ax + by$

At $(3,4), Z = 3a + 4b$

At $(0,5), Z = 5b$

$3a + 4b = 0 + 5b$

$3a = b$

259. **(a)** Let X denote the number of doublets

$X = 0,1,2,3$

$P(\text{ doublet }) = \dfrac{1}{6}, P(\text{ not a doublet }) = \dfrac{5}{6}$

X	P(X)
0	$\left(\frac{5}{6}\right)^3 = \frac{125}{216}$
1	$3 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^2 = \frac{75}{216}$
2	$3 \times \left(\frac{1}{6}\right)^2 \times = \frac{15}{216}$
3	$\left(\frac{1}{6}\right)^3 = \frac{1}{216}$

OR

(b) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= p + 2p - p \times 2p$

$P(E \cup F) = P(\text{ exactly one of } E, F) + P(E \cap F)$

$= \dfrac{5}{9} + p \times 2p$

$\Rightarrow 3p - 2p^2 = \dfrac{5}{9} + 2p^2$

$\Rightarrow 36p^2 - 27p + 5 = 0$

$\Rightarrow (3p - 1)(12p - 5) = 0$

$\Rightarrow p = \dfrac{1}{3}$ or $\dfrac{5}{12}$

SECTION-D

260. $\text{adj}A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$

$|A| = 2 \times 0 - 3 \times 2 + 5 \times 1 = -6 + 5 = -1 \neq 0 \Rightarrow A^{-1}$ exists

$A^{-1} = \dfrac{1}{|A|}(\text{adj}A) = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 - 5 + 6 \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$\Rightarrow x = 1, y = 2, z = 3$

261. **(a)**Let $y = x^{\sin x} + (\sin x)^x = \text{u} + \text{v} \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$u = x^{\sin x}$

$\Rightarrow \log u = \sin x \log x$

$\Rightarrow \frac{du}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \log x\right]$

$v = (\sin x)^x$

$\Rightarrow \log v = x\log(\sin x)$

$\Rightarrow \frac{dv}{dx} = (\sin x)^x[x\cot x + \log(\sin x)]$

$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x}{x} + \cos x \log x\right] + (\sin x)^x[x\cot x + \log(\sin x)]$

OR

(b) $y = x + \tan x$

$\Rightarrow \frac{dy}{dx} = 1 + \sec^2 x$

$\Rightarrow \frac{d^2y}{dx^2} = 2\sec^2 x\tan x$

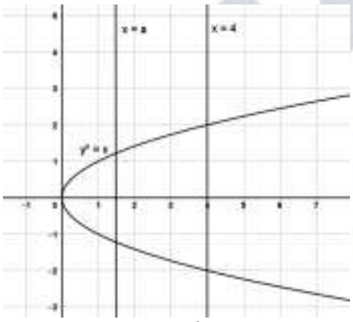
$\Rightarrow \cos^2 x\frac{d^2y}{dx^2} = 2\tan x$

$\Rightarrow \cos^2 x\frac{d^2y}{dx^2} = 2x + 2\tan x - 2x$

$\Rightarrow \cos^2 x\frac{d^2y}{dx^2} = 2y - 2x$

$\Rightarrow \cos^2 x\frac{d^2y}{dx^2} - 2y + 2x = 0$

262.



$2\int_0^a \sqrt{x}dx = \int_0^4 \sqrt{x}dx$

$\Rightarrow 2 \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^a = \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^4$

$\Rightarrow \frac{4}{3}\left[a^{\frac{3}{2}}\right] = \frac{2}{3}[8]$

$\Rightarrow a^{\frac{3}{2}} = 4$

$\Rightarrow a = 4^{\frac{2}{3}}$

263. **(a)** $\overrightarrow{a_1} = 4\hat{\imath} - \hat{\jmath} + 2\hat{k}\ \overrightarrow{b_1} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

$\overrightarrow{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}\ \overrightarrow{b_2} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

$\overrightarrow{a_2} - \overrightarrow{a_1} = -2\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$

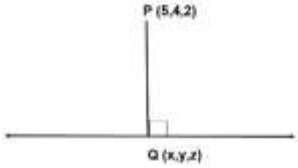
$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -3 \\ 3 & 2 & -4 \end{vmatrix} = -2\hat{\imath} - 5\hat{\jmath} - 4\hat{k}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{4 + 25 + 16} = \sqrt{45} = 3\sqrt{5}$

$(\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2}) = 4 - 10 + 12 = 6$

$$SD = \left|\frac{|\overrightarrow{a_2} - \overrightarrow{a_1}) \cdot (\overrightarrow{b_1} \times \overrightarrow{b_2})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right| = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}}$$

OR



(b)

Let coordinates of Q be $(2\lambda - 1, 3\lambda + 3, 1 - \lambda)$

D. Ratios of PQ $\langle 2\lambda - 6, 3\lambda - 1, -1 - \lambda\rangle$

PQ $\perp$ line

$\Rightarrow 2(2\lambda - 6) + 3(3\lambda - 1) - (-1 - \lambda) = 0$

$\Rightarrow \lambda = 1$

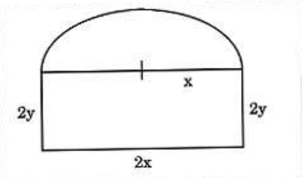
Foot of perpendicular is $Q(1,6,0)$

Length of perpendicular $= \sqrt{(4)^2 + (-2)^2 + (2)^2}$

$= \sqrt{16 + 4 + 4} = \sqrt{24} = 2\sqrt{6}$

SECTION-E

264. **(i)** $2x + 4y + \pi x = 10$



(ii) $A = (2x)(2y) + \frac{1}{2}\pi x^2$

$y = \dfrac{10 - 2x - \pi x}{4}$

$A = 4x\dfrac{(10 - 2x - \pi x)}{4} + \dfrac{1}{2}\pi x^2$

$A = 10x - 2x^2 - \dfrac{1}{2}\pi x^2$

(iii) (a) $\frac{dA}{dx} = 10 - 4x - \pi x = 0$

$\Rightarrow 10 = (\pi + 4)x$

$\Rightarrow x = \dfrac{10}{\pi + 4}$

$\dfrac{d^2y}{dx^2} = -4 - \pi < 0$

$\Rightarrow y = \dfrac{1}{4}\left[10 - 2\left(\dfrac{10}{\pi + 4}\right) - \pi\left(\dfrac{10}{\pi + 4}\right)\right]$

$\Rightarrow y = \dfrac{1}{4}[10\pi + 40 - 20 - 10\pi]$

$y = \dfrac{1}{4}\left[\dfrac{20}{\pi + 4}\right] = \dfrac{5}{\pi + 4}$

(iii)(b) $A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$

$x + 2y + \pi\left(\dfrac{x}{2}\right) = 10$

$\Rightarrow y = \dfrac{20 - 2x - \pi x}{4}$

$A = x\left(\dfrac{20 - 2x - \pi x}{4}\right) + \dfrac{1}{8}\pi x^2$

$A = 5x - \dfrac{1}{2}x^2 - \dfrac{\pi}{8}x^2$

265. Let G be an event that student is not able to get good marks We have

$P(A)=\frac{6}{60}\ P(G \mid A)=0.002$

$P(B)=\frac{26}{60}\quad P(G \mid B)=0.02$

$P(C)=\frac{28}{60}\quad P(G \mid C)=0.20$

(i) $P(G)=P(A)P(G \mid A)+P(B)P(G \mid B)+P(C)P(G \mid C)$

$=\frac{6}{60}\times 0.002+\frac{26}{60}\times 0.02+\frac{28}{60}\times 0.2$

$=\frac{511}{5000}$

(ii) $1-P(A \mid G)$

$=1-\frac{P(A)P(G \mid A)}{P(A)P(G \mid A)+P(B)P(G \mid B)+P(C)P(G \mid C)}$

$=1-\frac{\frac{1}{10}\times 0.002}{\frac{511}{5000}}$

$=1-\frac{1}{511}$

$=\frac{510}{511}$

266. **(i)** $f: R \rightarrow R$

$f(x)=x^2$

For $x_1=-1$ and $x_2=1$ we have $f(x_1)=f(x_2)=1$

Hence f is not a one -one function

(ii) $f: R \rightarrow R$

$f(x)=x^2$

for, $y=-4 \in R$ (Codomain) there does not exist any $x \in R$ such that f(x) $=-4$ So f is not onto

(iii) (a) f: N → N

$f(x)=x^2$

Let, $f(x_1)=f(x_2)$ for some $x_1, x_2 \in N$

$\Rightarrow x_1^2=x_2^2$

$\Rightarrow x_1=x_2$

Hence, f is one- one

For, $y=5 \in N$ (codomain)

there does not exist any $\boldsymbol{x}$ (domain) such that f(x) = 5

So f is not onto

OR

(iii) (b) $f: N \rightarrow \{1,4,9,16 \ldots\}$

$f(x)=x^2$

Let$f(x_1)=f(x_2)$

$\Rightarrow {x_1}^2={x_2}^2$

$\Rightarrow x_1=x_2$

Hence f is one- one

$\forall y \in\{1,4,9,16, \ldots\}$ there exist $x \in N$ such that f(x) = y

f is onto