General Instructions:

Read the following instructions very carefully and strictly follow them :

(i) This Question paper contains 38 questions. All questions are compulsory.
(ii) Question paper is divided into FIVE Sections - Section A, B, C, D and E.
(iii) In **Section A -** Question Number 1 to 18 are Multiple Choice Questions (MCQs) and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.
(iv) In **Section B -** Question Number 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
(v) In **Section C -** Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In **Section D -** Question Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.
(vii) In **Section E-** Question Number 36 to 38 are case study based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section −D and 2 questions in Section - E.
(ix) Use of calculator is NOT allowed.

SECTION-A

1. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is

(a) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. If vector $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}$ and vector $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, then which of the following is correct?

(a) $\vec{a} || \vec{b}$ (b) $\vec{a} \perp \vec{b}$
(c) $|\vec{b}| > |\vec{a}|$ (d) $|\vec{a}| = |\vec{b}|$

3. $\int_{-1}^{1} \frac{|x|}{x}\, dx, x \neq 0$ is equal to

(a) -1 (b) 0
(c) 1 (d) 2

4. Which of the following is not a homogeneous function of x and y ?

(a) $y^2 - xy$ (b) $x - 3y$
(c) $\sin^2 \frac{y}{x} + \frac{y}{x}$ (d) $\tan x - \sec y$

5. If $f(x) = |x| + |x - 1|$, then which of the following is correct ?

(a) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$.
(b) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$.
(c) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.
(d) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.

6. If A is a square matrix of order 2 such that $\det(A) = 4$, then $\det(4 \operatorname{adj} A)$ is equal to:

(a) 16 (b) 64
(c) 256 (d) 512

7. If E and F are two independent events such that $P(E) = \frac{2}{3}$, $P(F) = \frac{3}{7}$, then $P(E/\overline{F})$ is equal to:

(a) $\frac{1}{6}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{7}{9}$

8. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is:

(a) 0 (b) 2
(c) 4 (d) 5

9. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}, C = \begin{bmatrix} 9 & 8 & 7 \end{bmatrix}$, which of the following is defined?

(a) Only AB (b) Only AC

(c) Only BA (d) All AB, AC and BA

10. If $\int \frac{2^{\frac{1}{x}}}{x^2}\, dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to

(a) $\frac{-1}{\log 2}$ (b) $-\log 2$

(c) -1 (d) $\frac{1}{2}$

11. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}, |\vec{a}| = \sqrt{37}, |\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

12. The integrating factor of differential equation $(x + 2y^3)\frac{dy}{dx} = 2y$ is

(a) $e^{\frac{y^2}{2}}$ (b) $\frac{1}{\sqrt{y}}$

(c) $\frac{1}{y^2}$ (d) $e^{-\frac{1}{y^2}}$

13. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to

(a) 0 (b) 1

(c) 7 (d) ± 7

14. The corner points of the feasible region in graphical representation of a L.P.P. are $(2, 72), (15, 20)$ and $(40, 15)$. If $Z = 18x + 9y$ be the objective function, then

(a) Z is maximum at (2,72), minimum at (15,20)

(b) Z is maximum at (15,20) minimum at (40,15)

(c) Z is maximum at (40,15), minimum at (15,20)

(d) Z is maximum at (40,15), minimum at (2,72)

15. If A and B are invertible matrices, then which of the following is not correct ?

(a) $(A + B)^{-1} = B^{-1} + A^{-1}$

(b) $(AB)^{-1} = B^{-1} A^{-1}$

(c) $\text{adj}(A) = |A|A^{-1}$

(d) $|A|^{-1} = |A^{-1}|$

16. If the feasible region of a linear programming problem with objective function $Z = ax + by$, is bounded, then which of the following is correct ?

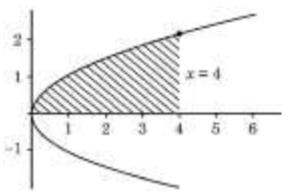
(a) It will only have a maximum value.

(b) It will only have a minimum value.

(c) It will have both maximum and minimum values.

(d) It will have neither maximum nor minimum value.

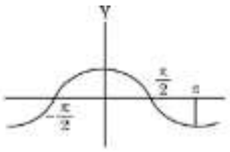
17. The area of the shaded region bounded by the curves $y^2 = x, x = 4$ and the x-axis is given by

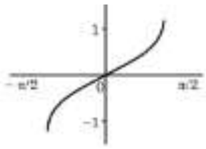


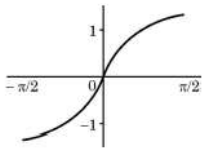
(a) $\int_0^4 x\, dx$ (b) $\int_0^2 y^2 dy$

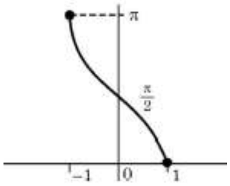
(c) $2\int_0^4 \sqrt{x}\, dx$ (d) $\int_0^4 \sqrt{x}\, dx$

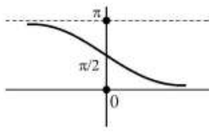
18. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse?



(a) 

(b) 

(c) 

(d) 

Assertion-Reason Based Questions

Direction: Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A): Let Z be the set of integers. A function $f: Z \to Z$ defined as $f(x) = 3x - 5, \forall x \in Z$ is a bijective.

Reason (R): A function is a bijective if it is both surjective and injective.

20. Assertion (A) : $f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason (R) : For a function f to be continuous at $x = a$,

$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$.

SECTION – B

21. (a) Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$.

OR

(b) If $\tan^{-1}(x^2 + y^2) = a^2$, then find $\frac{dy}{dx}$.

22. Evaluate: $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$

23. The diagonals of a parallelogram are given by $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$. Find the area of the parallelogram.

24. Find the intervals in which function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is (i) increasing (ii) decreasing.

25. (a) Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings.

OR

(b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points $A(2, 1, 3)$ and $B(8, -1, 0)$ respectively.

SECTION - C

26. The side of an equilateral triangle is increasing at the rate of 3 cm/s. At what rate its area increasing when the side of the triangle is 15 cm ?

27. Solve the following linear programming problem graphically :

Maximise $Z = x + 2y$
Subject to the constraints:
$x - y \geq 0$
$x - 2y \geq -2$
$x \geq 0, y \geq 0$

28. (a) Find: $\int \frac{x+\sin x}{1+\cos x}\,dx$

OR

(b) Evaluate: $\int_0^{\frac{\pi}{4}} \frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$

29. (a) Verify that lines given by $\vec{r} = (1-\lambda)\hat{\imath} + (\lambda-2)\hat{\jmath} + (3-2\lambda)\hat{k}$ and $\vec{r} = (\mu+1)\hat{\imath} + (2\mu-1)\hat{\jmath} - (2\mu+1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines.

OR

(b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{\imath} + 8\hat{\jmath}$, $\vec{W} = 6\hat{\imath} + 12\hat{\jmath}$ and $\vec{F} = 12\hat{\imath} + 18\hat{\jmath}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder.

30. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows:

X	0	2	4	5
P(X)	p	2 p	3 p	p

Where X is the number of students absent.
(i) Calculate p.
(ii) Calculate the mean of the number of absent students on Saturday.

OR

(b) For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35 . Find the probability that the candidate chosen at random will have a distinction in the written est.

31. Sketch the graph of $y = |x+3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration.

SECTION - D

32. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

33. Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$.

34. (a) Find the image A' of the point $A(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'.

OR

(b) Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point $Q(2, 4, -1)$ is 7 units. Also, find the equation of line joining P and Q .

35. A school wants to allocate students into three clubs: Sports, Music and Drama, under following conditions :
• The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
• The number of students in Music club should be 20 more than half the number of students in Sports club.
• The total number of students to be allocated in all three clubs are 180.
Find the number of students allocated to different clubs, using matrix method.

SECTION-E

36.

A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.
Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres.
Based on this information, answer the following questions :
(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.
(ii) Write the area of the solar panel as a function of x.
(iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

OR

(iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

37. **A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$:**

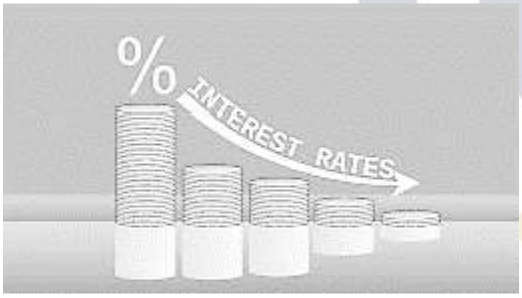
$R_1 = \{(2,3),(3,2)\}$
$R_2 = \{(1,2),(1,3),(3,2)\}$
$R_3 = \{(1,2),(2,1),(1,1)\}$
$R_4 = \{(1,1),(1,2),(3,3),(2,2)\}$
$R_5 = \{(1,1),(1,2),(3,3),(2,2),(2,1),(2,3),(3,2)\}$

The students are asked to answer the following questions about the above relations :
(i) Identify the relation which is reflexive, transitive but not symmetric.
(ii) Identify the relation which is reflexive and symmetric but not transitive.
(iii) (a) Identify the relations which are symmetric but neither reflexive nor transitive.

OR

(iii) (b) What pairs should be added to the relation R_2 to make it an equivalence relation?

38.



A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities 10%, 20% and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is 5%, 3% and 1% respectively.
Based on the above information, answer the following:
(i) What is the probability that a customer after availing the loan will default on the loan repayment?
(ii) A customer after availing the loan, defaults on loan repayment. What is the probability that he availed the loan at a variable rate of interest?

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(vi) In **Section D -** Question Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.
(vii) In **Section E-** Question Number 36 to 38 are case study based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section −D and 2 questions in Section - E.
(ix) Use of calculator is NOT allowed.

SECTION-A

39. The projection vector of vector $\vec{a}$ on vector $\vec{b}$ is

(a) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\right)\vec{b}$ (b) $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

(c) $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$ (d) $\left(\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|^2}\right)\vec{b}$

40. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval

(a) $(0,2)$ (b) $(-\infty, 2]$

(c) $[1,2]$ (d) $[2, \infty)$

41. If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)dx$ is

(a) $\int_0^{2a} f\left(\frac{x}{2}\right) dx$ (b) $\int_0^{a} f(x)dx$

(c) $2\int_a^0 f(x)dx$ (d) $2\int_0^a f(x)dx$

42. If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x + y)$ is

(a) -8 (b) 0

(c) 6 (d) 8

43. If $y = \sin^{-1} x, -1 \leq x \leq 0$, then the range of y is

(a) $\left(\frac{-\pi}{2}, 0\right)$ (b) $\left[\frac{-\pi}{2}, 0\right]$

(c) $\left[\frac{-\pi}{2}, 0\right)$ (d) $\left(\frac{-\pi}{2}, 0\right]$

44. If a line makes angles of $\frac{3\pi}{4}, \frac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is

(a) $\frac{-\pi}{3}$ only (b) $\frac{\pi}{3}$ only

(c) $\frac{\pi}{6}$ (d) $\pm\frac{\pi}{3}$

45. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\bar{E}/\bar{F})$ is

(a) $\frac{P(\bar{E})}{P(\bar{F})}$ (b) $1 - P(\bar{E}/F)$

(c) $1 - P(E/F)$ (d) $\frac{1-P(E\cup F)}{P(\bar{F})}$

46. Which of the following can be both a symmetric and skew-symmetric matrix?

(a) Unit Matrix (b) Diagonal Matrix

(c) Null Matrix (d) Row Matrix

47. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is :

(a) $x = 4t + 3, y = -3t + 1, z = 7t + 2$

(b) $x = 3t + 4, y = t + 3, z = 2t + 7$

(c) $x = 3t + 4, y = t - 3, z = 2t + 7$

(d) $x = 3t + 4, y = -t + 3, z = 2t + 7$

48. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4AB + 3(AB + BA) - 4BA$, where A and B are both matrices of order 2×2.

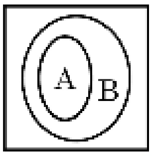
It is known that $A \neq B \neq I$ and $A^{-1} \neq B$.
Their answers are given as :
Abhay : 6 AB
Bina: 7AB − BA
Chhaya: 8 AB
Devesh : 7BA − AB
Who answered it correctly?
(a) Abhay (b) Bina
(c) Chhaya (d) Devesh

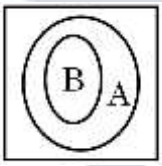
49. A cylindrical tank of radius 10 cm is being filled with sugar at the rate of $100\pi\ \text{cm}^3/\text{s}$. The rate, at which the height of the sugar inside the tank is increasing, is:
(a) 0.1 cm/s (b) 0.5 cm/s
(c) 1 cm/s (d) 1.1 cm/s

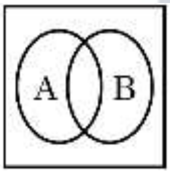
50. Let $\vec{p}$ and $\vec{q}$ be two unit vectors and α be the angle between them. Then $(\vec{p}+\vec{q})$ will be a unit vector for what value of α ?
(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$
(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

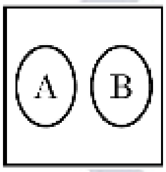
51. The line $x = 1 + 5\mu, y = -5 + \mu, z = -6 - 3\mu$ passes through which of the following point?
(a) $(1, -5, 6)$ (b) $(1, 5, 6)$
(c) $(1, -5, -6)$ (d) $(-1, -5, 6)$

52. If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set *A* and *B* ?

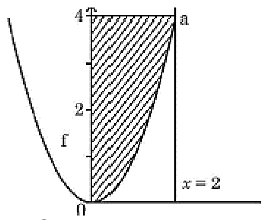
(a) 

(b) 

(c) 

(d) 

53. The area of the shaded region (figure) represented by the curves $y = x^2, 0 \leq x \leq 2$ and y -axis is given by



(a) $\int_0^2 x^2\,dx$ (b) $\int_0^2 \sqrt{y}\,dy$
(c) $\int_0^4 x^2\,dx$ (d) $\int_0^4 \sqrt{y}\,dy$

54. A factory produces two products X and Y . The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct ?
(a) The objective function maximizes the difference of the profit earned from products X and Y .
(b) The objective function measures the total production of products X and Y .
(c) The objective function maximizes the combined profit earned from selling *X* and *Y*.
(d) The objective function ensures the company produces more of product X than product Y .

55. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct ?

(a) $A = B$ (b) $AB = BA$

(c) $A = 0$ or $B = 0$ (d) $A = I$ or $B = I$

56. If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then $(p - q)$ is

(a) 0 (b) 1

(c) 2 (d) 3

ASSERTION - REASON BASED QUESTIONS

Direction: Question number 19 and 20 are Assertion (A) and Reason (R) based questions. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below :

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

57. Assertion (A): $A = \text{diag}[3 \quad 5 \quad 2]$ is a scalar matrix of order 3×3.

Reason (R): If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

58. Assertion (A): Every point of the feasible region of a Linear Programming Problem is an optimal solution.

Reason (R): The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.

SECTION - B

(This section comprises of 5 Very Short Answer (VSA) type questions.)

59. (a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$.

OR

(b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2}QP$.

60. Evaluate: $\int_0^{\frac{\pi}{4}} \sqrt{1 + \sin 2x}\, dx$

61. Find the values of ' a ' for which $f(x) = \sin x - ax + b$ is increasing on R .

62. If $\vec{a}$ and $\vec{b}$ are two non-collinear vectors, then find x, such that $\vec{\alpha} = (x - 2)\vec{a} + \vec{b}$ and $\vec{\beta} = (3 + 2x)\vec{a} - 2\vec{b}$ are collinear.

63. (a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x-y}{x\log x}$.

OR

(b) If $f(x) = \begin{cases} 2x - 3, & -3 \le x \le -2 \\ x + 1, & -2 < x \le 0 \end{cases}$

Check the differentiability of $f(x)$ at $x = -2$.

SECTION - C

(This section comprises of 6 Short Answer (SA) type questions.)

64. (a) Solve the differential equation $2(y + 3) - xy\frac{dy}{dx} = 0$; given $y(1) = -2$.

OR

(b) Solve the following differential equation:

$$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2.$$

65. Let R be a relation defined over N, where N is set of natural numbers, defined as " mRn if and only if m is a multiple of $n, m, n \in N$." Find whether R is reflexive, symmetric and transitive or not.

66. Solve the following linear programming problem graphically:

Minimise $Z = x - 5y$ subject to the constraints :

$x - y \ge 0$

$-x+2y \geq 2$
$x \geq 3, y \leq 4, y \geq 0$

67. (a) If $y=\log\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2 y_2+(x+1)^2 y_1=2$.

OR

(b) If $x\sqrt{1+y}+y\sqrt{1+x}=0, -1<x<1, x \neq y$, then prove that $\frac{dy}{dx}=\frac{-1}{(1+x)^2}$.

68. (a) A die with number 1 to 6 is biased such that $P(2)=\frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B : $A=\{(x,y): x+y=9\}, B=\{(x,y): x \neq 3\}$, where (x,y) denote a point in the sample space.
Check if events A and B are independent or mutually exclusive.

69. Find: $\int \frac{1}{x}\sqrt{\frac{x+a}{x-a}}\,dx$.

SECTION - D

(This section comprises of 4 Long Answer (LA) type questions.)

70. Using integration, find the area of the region bounded by the line $y=5x+2$, the x-axis and the ordinates $x=-2$ and $x=2$.

71. Find: $\int \frac{x^2+x+1}{(x+2)(x^2+1)}\,dx$.

72. (a) Find the shortest distance between the lines:
$\frac{x+1}{2}=\frac{y-1}{1}=\frac{z-9}{-3}$ and
$\frac{x-3}{2}=\frac{y+15}{-7}=\frac{z-9}{5}$

OR

(b) Find the image A' of the point $A(2,1,2)$ in the line $l: \vec{r}=4\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-\hat{j}-\hat{k})$. Also, find the equation of line joining AA'. Find the foot of perpendicular from point A on the line l.

73. (a) Given $A=\begin{bmatrix}-4 & 4 & 4\\ -7 & 1 & 3\\ 5 & -3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1 & 1\\ 1 & -2 & -2\\ 2 & 1 & 3\end{bmatrix}$, find AB .
Hence, solve the system of linear equations:
$x-y+z=4$
$x-2y-2z=9$
$2x+y+3z=1$

OR

(b) If $A=\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 1\end{bmatrix}$, then find A^{-1}.
Hence, solve the system of linear equations:
$x-2y=10$
$2x-y-z=8$
$-2y+z=7$

SECTION-E

(This section comprises of 3 case study based questions of 4 marks each.)

74. A school is organizing a debate competition with participants as speakers $S=\{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J=\{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R=\{(x,y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.

Based on the above, answer the following :

(i) How many relations can be there from S to J ?

(ii) A student identifies a function from S to J as $f = \{(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)\}$ Check if it is bijective.

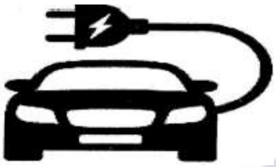
(iii) (a) How many one-one functions can be there from set S to set J ?

OR

(iii) (b) Another student considers a relation $R_1 = \{(S_1, S_2), \{S_2, S_4)\}$ in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric.

75. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is $60\%, 30\%$ and 10% respectively. Of their respective production capacities, $20\%, 10\%$ and 5% cars respectively are electric (or battery operated).

Based on the above, answer the following:



(i) (a) What is the probability that a randomly selected car is an electric car?

OR

(i) (b) What is the probability that a randomly selected car is a petrol car?

(ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet?

(iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi ?

76.

A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following :

(i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$.

(ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion.

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) This question paper is divided into five Sections - A, B, C, D and E.

(iii) In **Section A,** Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In **Section B,** Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

(v) In **Section C,** Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

(vi) In **Section D,** Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

(vii) In **Section E,** Questions no. 36 to 38 are case study based questions carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is not allowed.

SECTION A

This section comprises multiple choice questions .

77. The principal value of $\sin^{-1}\left(\sin\left(-\frac{10\pi}{3}\right)\right)$ is :

(a) $-\frac{2\pi}{3}$ (b) $-\frac{\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

78. If A and B are square matrices of same order such that $AB = A$ and $BA = B$, then $A^2 + B^2$ is equal to :

(a) $A + B$ (b) BA

(c) $2(A + B)$ (d) 2 BA

79. For real x, let $f(x) = x^3 + 5x + 1$. Then :

(a) f is one-one but not onto on R

(b) f is onto on R but not one-one

(c) f is one-one and onto on R

(d) f is neither one-one nor onto on R

80. If $y = \sin^{-1} x$, then $(1 - x^2)\frac{d^2y}{dx^2}$ is equal to :

(a) $x\frac{dy}{dx}$ (b) $-x\frac{dy}{dx}$

(c) $x^2\frac{dy}{dx}$ (d) $-x^2\frac{dy}{dx}$

81. The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are :

(a) $1 < \lambda < \sqrt{2}$ (b) $\lambda \geq 1$

(c) $\lambda \geq \sqrt{2}$ (d) $\lambda < 1$

82. If P is a point on the line segment joining (3,6,-1) and (6,2,-2) and y-coordinate of P is 4 , then its z-coordinate is :

(a) $-\frac{3}{2}$ (b) 0

(c) 1 (d) $\frac{3}{2}$

83. If M and N are square matrices of order 3 such that $\det(M) = m$ and $MN = mI$, then $\det(N)$ is equal to :

(a) -1 (b) 1

(c) $-m^2$ (d) m^2

84. If $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to:

(a) -4 (b) $-\frac{7}{2}$

(c) -2 (d) -1

85. If $f: N \to W$ is defined as $f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases}$ then f is:

(a) injective only

(b) surjective only

(c) a bijection

(d) neither surjective nor injective

86. The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ is a :

(a) diagonal matrix

(b) symmetric matrix

(c) skew symmetric matrix

(d) scalar matrix

87. If the sides AB and AC of $\triangle ABC$ are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is :

(a) $2\sqrt{2}$ units (b) $\sqrt{18}$ units

(c) $\frac{\sqrt{34}}{2}$ units (d) $\frac{\sqrt{48}}{2}$ units

88. The function f defined by $f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$ is not continuous at:

(a) $x = 0$ (b) $x = 1$

(c) $x = 2$ (d) $x = 5$

89. If $f(x) = 2x + \cos x$, then $f(x)$:

(a) has a maxima at $x = \pi$ (b) has a minima at $x = \pi$

(c) is an increasing function (d) is a decreasing function

90. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ is equal to :

(a) $2(\sin x + x\cos \alpha) + C$

(b) $2(\sin x - x\cos \alpha) + C$

(c) $2(\sin x + 2x\cos \alpha) + C$

(d) $2(\sin x + \sin \alpha) + C$

91. The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is :

(a) $-\frac{\pi}{4}$ (b) $\frac{\pi}{4}$

(c) $\tan^{-1} e - \frac{\pi}{4}$ (d) $\tan^{-1} e$

92. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ are :

(a) order 2, degree 2

(b) order 2, degree 1

(c) order 2, degree not defined

(d) order 1, degree not defined

93. The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is :

(a) $\frac{16}{9}$ sq. units (b) $\frac{32}{9}$ sq. units

(c) $\frac{16}{3}$ sq. units (d) $\frac{32}{3}$ sq. units

94. The corner points of the feasible region of a Linear Programming Problem are $(0, 2), (3, 0), (6, 0), (6, 8)$ and $(0, 5)$. If $Z = ax + by$; $(a, b > 0)$ be the objective function, and maximum value of Z is obtained at $(0, 2)$ and $(3, 0)$, then the relation between a and b is :

(a) $a = b$ (b) $a = 3b$

(c) $b = 6a$ (d) $3a = 2b$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

95. Assertion (A) : If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events.

Reason (R) : Two events are independent if the occurrence of one does not effect the occurrence of the other.

96. Assertion (A): In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution.

Reason (R): A feasible region is defined as the region that satisfies all the constraints.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions.

97. Let A and B be two square matrices of order 3 such that $\det(A) = 3$ and $\det(B) = -4$. Find the value of $\det(-6AB)$.

98. (a) Find the least value of ' a ' so that $f(x)=2x^2-ax+3$ is an increasing function on [2,4].

OR

(b) If $f(x)=x+\frac{1}{x}, x\geq 1$, show that f is an increasing function.

99. (a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$.

OR

(b) Find domain of $\sin^{-1}\sqrt{x-1}$.

100. Calculate the area of the region bounded by the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the x-axis using integration.

101. For the curve $y=5x-2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x=2$?

SECTION C

This section comprises 6 Short Answer (SA) type questions.

102. (a) If $f: R^+ \to R$ is defined as $f(x)=\log_a x (a>0$ and $a\neq 1)$, prove that f is a bijection. ($R^+$is a set of all positive real numbers.)

OR

(b) Let $A=\{1,2,3\}$ and $B=\{4,5,6\}$. A relation R from A to B is defined as $R=\{(x,y): x+y=6, x\in A, y\in B\}$.

(i) Write all elements of R.

(ii) Is R a function? Justify.

(iii) Determine domain and range of R.

103. (a) Find k so that $f(x)=\begin{cases}\frac{x^2-2x-3}{x+1}, & x\neq -1\\ k, & x=-1\end{cases}$ is continuous at $x=-1$.

OR

(b) Check the differentiability of function $f(x)=x|x|$ at $x=0$.

104. Evaluate: $\int_{\pi/2}^{\pi} e^x\left(\frac{1-\sin x}{1-\cos x}\right)dx$

105. (a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

OR

(b) A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.

106. Find the distance of the point $(-1,-5,-10)$ from the point of intersection of the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$.

107. Solve the following Linear Programming Problem using graphical method : Maximise $Z=100x+50y$ subject to the constraints

$3x+y\leq 600$
$x+y\leq 300$
$y\leq x+200$
$x\geq 0, y\geq 0$

SECTION D

This section comprises 4 Long Answer (LA) type questions.

108. If A is a 3×3 invertible matrix, show that for any scalar $k\neq 0$, $(kA)^{-1}=\frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$

109. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y=4x-\frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight.
(ii) In how many days will the plant attain its maximum height ? What is the maximum height?

110. (a) Find: $\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)} dx$

OR

(b) Evaluate: $\int_0^{\pi} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$

111. (a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}|\vec{a}\times\vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{\imath}-\hat{\jmath}+\hat{k}$ and $\hat{\imath}+3\hat{\jmath}-\hat{k}$.

OR

(b) Find the equation of a line in vector and cartesian form which passes through the point $(1, 2, -4)$ and is perpendicular to the lines $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$, and $\vec{r}=15\hat{\imath}+29\hat{\jmath}+5\hat{k}+\mu(3\hat{\imath}+8\hat{\jmath}-5\hat{k})$

SECTION E

This section comprises 3 case study based questions.

Case Study - 1

112. Some students are having a misconception while comparing decimals. For example, a student may mention that $78.56 > 78.9$ as $7856 > 789$. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question: In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table:

Name of student	Distance of javelin (in meters)
Ajay	47.7
Bijoy	47.07
Kartik	43.09
Dinesh	43.9
Devesh	45.2

The students were asked to identify who has thrown the javelin the farthest.
Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.
On the basis of the above information, answer the following questions :
(i) What is the probability of a student not having misconception but still answers Bijoy in the test?
(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ?
(iii) (a) What is the probability that a student who answered as Bijoy is having misconception?

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ?

Case Study-2

113. An engineer is designing a new metro rail network in a city.

Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1: \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$, while the track for Line B is represented by $l_2: \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$.

Based on the above information, answer the following questions:

(i) Find whether the two metro tracks are parallel.

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$.

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point ($3, 2, 1$). Determine the equation of the pedestrian walkway.

OR

(iii) (b) Find the shortest distance between Line A and Line B .

Case Study-3

114. During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature ($25^\circ C$). Initially the processor's temperature is $85^\circ C$. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$, where $T(t)$ represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions :

(i) Find the expression for temperature of processor, $T(t)$ given that $T(0) = 85^\circ C$.

(ii) How long will it take for the processor's temperature to reach $40^\circ C$? Given that $k = 0 \cdot 03, \log_e 4 = 1 \cdot 3863$.

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(vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is not allowed.

SECTION A

This section comprises multiple choice questions ($MCQs$) of .

115. If $A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^3 is :

(a) $3\begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 125 & 0 & 0 \\ 0 & 125 & 0 \\ 0 & 0 & 125 \end{bmatrix}$

(c) $\begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$ (d) $\begin{bmatrix} 5^3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

116. If $P(A \cup B) = 0.9$ and $P(A \cap B) = 0.4$, then $P(\bar{A}) + P(\bar{B})$ is :

(a) 0.3 (b) 1
(c) $1 \cdot 3$ (d) 0.7

117. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 3 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 3 \\ -1 & 2 \\ 0 & 5 \end{bmatrix}$, then the correct statement is :

(a) Only AB is defined.
(b) Only BA is defined.
(c) AB and BA , both are defined.
(d) AB and BA , both are not defined.

118. If $\begin{vmatrix} 2x & 5 \\ 12 & x \end{vmatrix} = \begin{vmatrix} 6 & -5 \\ 4 & 3 \end{vmatrix}$, then the value of x is :

(a) 3 (b) 7
(c) ± 7 (d) ± 3

119. If $f(x) = \begin{cases} \frac{\sin^2 ax}{x^2}, & x \neq 0 \\ 1, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of a is :

(a) 1 (c) -1
(b) ± 1 (d) 0

120. If $A = [a_{ij}]$ is a 3×3 diagonal matrix such that $a_{11} = 1, a_{22} = 5$ and $a_{33} = -2$, then $|A|$ is :

(a) 0 (b) -10
(c) 10 (d) 1

121. The principal value of $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ is :

(a) $-\frac{\pi}{3}$ (b) $-\frac{2\pi}{3}$
(c) $\frac{\pi}{3}$ (d) $\frac{2\pi}{3}$

122. If $\begin{bmatrix} 4+x & x-1 \\ -2 & 3 \end{bmatrix}$ is a singular matrix, then the value of x is :

(a) 0 (b) 1
(c) -2 (d) -4

123. If $f(x) = \{[x], x \in R\}$ is the greatest integer function, then the correct statement is :

(a)f is continuous but not differentiable at $x = 2$.
(b) f is neither continuous nor differentiable at $x = 2$.
(c) f is continuous as well as differentiable at $x = 2$.
(d)f is not continuous but differentiable at $x = 2$.

124. The slope of the curve $y = -x^3 + 3x^2 + 8x - 20$ is maximum at :

(a) $(1, -10)$ (b)$(1,10)$
(c)$(10,1)$ (d)$(-10,1)$

125. $\int \sqrt{1 + \sin x}\, dx$ is equal to:

(a)$2\left(-\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$

(b)$2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + C$

(c) $-2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$

(d)$2\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) + C$

126. $\int_0^{\pi/2} \cos x \cdot e^{\sin x} dx$ is equal to :

(a)0 (b)$1 - e$
(c)$e - 1$ (d)e

127. **The area of the region enclosed between the curve $y = x|x|$, x-axis, $x = -2$ and $x = 2$ is :**

(a) $\frac{8}{3}$ (b) $\frac{16}{3}$

(c) 0 (d) 8

128. **The integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ is :**

(a) $e^{-1/\sqrt{x}}$ (b) $e^{2/\sqrt{x}}$

(c) $e^{2\sqrt{x}}$ (d) $e^{-2\sqrt{x}}$

129. **The sum of the order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ is :**

(a) 2 (b) $\frac{5}{2}$

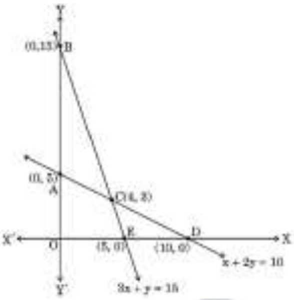
(c) 3 (d) 4

130. **For a Linear Programming Problem (LPP), the given objective function $Z = 3x + 2y$ is subject to constraints:**

$\boldsymbol{x + 2y \leq 10}$

$\boldsymbol{3x + y \leq 15}$

$\boldsymbol{x, y \geq 0}$



The correct feasible region is:

(a) AB (b) AOEC

(c) CED (d) Open unbounded region BCD

131. **Let $\vec{a}$ be a position vector whose tip is the point $(2, -3)$. If $\overrightarrow{AB} = \vec{a}$, where coordinates of A are $(-4, 5)$, then the coordinates of B are :**

(a) $(-2, -2)$ (b) $(2, -2)$

(c) $(-2, 2)$ (d) $(2, 2)$

132. **The respective values of $|\vec{a}|$ and $|\vec{b}|$, if given $(\vec{a} - \vec{b}) \cdot (\vec{a} + \vec{b}) = 512$ and $|\vec{a}| = 3|\vec{b}|$, are :**

(a) 48 and 16 (b) 3 and 1

(c) 24 and 8 (d) 6 and 2

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

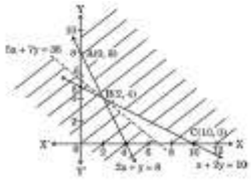
(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

133. **Assertion (A):** The shaded portion of the graph represents the feasible region for the given Linear Programming Problem (LPP).



$\text{Min} Z = 50x + 70y$

subject to constraints

$2x + y \geq 8, x + 2y \geq 10, x, y \geq 0$

$Z = 50x + 70y$ has a minimum value $= 380$ at $B(2,4)$.

Reason (R): The region representing $50x + 70y < 380$ does not have any point common with the feasible region.

134. **Assertion (A):** Let $A = \{x \in R: -1 \leq x \leq 1\}$. If $f: A \to A$ be defined as $f(x) = x^2$, then f is not an onto function.

Reason (R): If $y = -1 \in A$, then $x = \pm\sqrt{-1} \notin A$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions.

135. **Find the domain of the function $f(x) = \cos^{-1}(x^2 - 4)$.**

136. **Surface area of a balloon (spherical), when air is blown into it, increases at a rate of 5 mm^2/s. When the radius of the balloon is 8 mm , find the rate at which the volume of the balloon is increasing.**

137. **(a) Differentiate $\frac{\sin x}{\sqrt{\cos x}}$ with respect to x .**

OR

(b) If $y = 5\cos x - 3\sin x$, prove that $\frac{d^2y}{dx^2} + y = 0$.

138. **(a) Find a vector of magnitude 5 which is perpendicular to both the vectors $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $4\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$.**

OR

(b) Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} \neq 0$. Show that $\vec{b} = \vec{c}$.

139. **A man needs to hang two lanterns on a straight wire whose end points have coordinates A$(4, 1, -2)$ and B$(6, 2, -3)$. Find the coordinates of the points where he hangs the lanterns such that these points trisect the wire AB.**

SECTION C

This section comprises 6 Short Answer (SA) type questions.

140. **Find the value of ' a ' for which $f(x) = \sqrt{3}\sin x - \cos x - 2ax + 6$ is decreasing in R .**

141. **(a) Find: $\int \frac{2x}{(x^2+3)(x^2-5)} dx$**

OR

(b) Evaluate: $\int_1^4 (|x - 2| + |x - 4|) dx$

142. **Find the particular solution of the differential equation $\left[x\sin^2\left(\frac{y}{x}\right) - y\right] dx + x dy = 0$ given that $y = \frac{\pi}{4}$, when $x = 1$.**

143. **In the Linear Programming Problem (LPP), find the point/points giving maximum value for $Z = 5x + 10y$**

subject to constraints

$x + 2y \leq 120$

$x + y \geq 60$

$x - 2y \geq 0$

$x, y \geq 0$

144. **(a) If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ such that $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, then find the angle between $\vec{a}$ and $\vec{b}$.**

OR

(b) If $\vec{a}$ and $\vec{b}$ are unit vectors inclined with each other at an angle θ, then prove that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$.

145. (a) The probability that a student buys a colouring book is 0.7 and that she buys a box of colours is 0.2 . The probability that she buys a colouring book, given that she buys a box of colours, is 0.3 . Find the probability that the student:

(i) Buys both the colouring book and the box of colours.

(ii) Buys a box of colours given that she buys the colouring book.

OR

(b) A person has a fruit box that contains 6 apples and 4 oranges. He picks out a fruit three times, one after the other, after replacing the previous one in the box. Find:

(i) The probability distribution of the number of oranges he draws.

(ii) The expectation of the random variable (number of oranges).

SECTION D

This section comprises 4 Long Answer (LA) type questions.

146. Sketch a graph of $y = x^2$. Using integration, find the area of the region bounded by $y = 9, x = 0$ and $y = x^2$.

147. A furniture workshop produces three types of furniture - chairs, tables and beds each day. On a particular day the total number of furniture pieces produced is 45 . It was also found that production of beds exceeds that of chairs by 8 , while the total production of beds and chairs together is twice the production of tables. Determine the units produced of each type of furniture, using matrix method.

148. (a) For a positive constant ' a ', differentiate $a^{t+\frac{1}{t}}$ with respect to $\left(t+\frac{1}{t}\right)^a$, where t is a non-zero real number.

OR

(b) Find $\frac{dy}{dx}$ if $y^x + x^y + x^x = a^b$, where a and b are constants.

149. (a) Find the foot of the perpendicular drawn from the point $(1,1,4)$ on the line $\frac{x+2}{5} = \frac{y+1}{2} = \frac{-z+4}{-3}$.

OR

(b) Find the point on the line $\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-4}{3}$ at a distance of $2\sqrt{2}$ units from the point $(-1,-1,2)$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study - 1

150. A carpenter needs to make a wooden cuboidal box, closed from all sides, which has a square base and fixed volume. Since he is short of the paint required to paint the box on completion, he wants the surface area to be minimum.

On the basis of the above information, answer the following questions :

(i) Taking length = breadth = xm and height = ym, express the surface area (S) of the box in terms of x and its volume (V), which is constant.

(ii) Find $\frac{dS}{dx}$.

(iii) (a) Find a relation between x and y such that the surface area (S) is minimum.

OR

(iii) (b) If surface area (S) is constant, the volume (V) $= \frac{1}{4}\left(Sx - 2x^3\right)$, x being the edge of base. Show that volume (V) is maximum for $x = \sqrt{\frac{S}{6}}$.

Case Study - 2

151. Let A be the set of 30 students of class XII in a school. Let $f: A \rightarrow N, N$ is a set of natural numbers such that function $f(x) =$ Roll Number of student x .

On the basis of the given information, answer the following :

(i) Is f a bijective function ?

(ii) Give reasons to support your answer to (i).

(iii) (a) Let R be a relation defined by the teacher to plan the seating arrangement of students in pairs, where

$R = \{(x,y): x, y$ are Roll Numbers of students such that $y = 3x\}$.

List the elements of R . Is the relation R reflexive, symmetric and transitive ? Justify your answer.

OR

(iii) (b) Let R be a relation defined by
$R = \{(x, y): x, y$ are Roll Numbers of students such that $y = x^3\}$.
List the elements of R. Is R a function? Justify your answer.

Case Study - 3

152. **A gardener wanted to plant vegetables in his garden. Hence he bought 10 seeds of brinjal plant, 12 seeds of cabbage plant and 8 seeds of radish plant. The shopkeeper assured him of germination probabilities of brinjal, cabbage and radish to be 25%, 35% and 40% respectively. But before he could plant the seeds, they got mixed up in the bag and he had to sow them randomly.**

Radish Cabbage Brinjal

Based upon the above information, answer the following questions:
(i) Calculate the probability of a randomly chosen seed to germinate.
(ii) What is the probability that it is a cabbage seed, given that the chosen seed germinates?

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) This question paper contains 38 questions. All questions are compulsory.
(ii) This question paper is divided into five Sections - A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculator is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

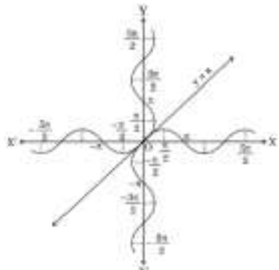
153. **Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is:**

(a) $n \times n$ (b) $n \times m$
(c) $m \times m$ (d) $m \times n$

154. **If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A is a/an :**

(a) scalar matrix
(b) identity matrix
(c) symmetric matrix
(d) skew-symmetric matrix

155. **The following graph is a combination of :**



(a) $y = \sin^{-1} x$ and $y = \cos^{-1} x$

(b) $y=\cos^{-1}x$ and $y=\cos x$
(c) $y=\sin^{-1}x$ and $y=\sin x$
(d) $y=\cos^{-1}x$ and $y=\sin x$

156. Sum of two skew-symmetric matrices of same order is always a/an :

(a) skew-symmetric matrix
(b) symmetric matrix
(c) null matrix
(d) identity matrix

157. $\left[\sec^{-1}(-\sqrt{2})-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$ is equal to :

(a) $\frac{11\pi}{12}$ (b) $\frac{5\pi}{12}$
(c) $-\frac{5\pi}{12}$ (d) $\frac{7\pi}{12}$

158. If $f(x)=\begin{cases}\frac{\log(1+ax)+\log(1-bx)}{x}, & \text{for } x\neq 0\\ k & \text{, for } x=0\end{cases}$ is continuous at $x=0$, then the value of k is :

(a) a (b) a + b
(c) a − b (d) b

159. If $\tan^{-1}(x^2-y^2)=a$, where ' a ' is a constant, then $\frac{dy}{dx}$ is :

(a) $\frac{x}{y}$ (b) $-\frac{x}{y}$
(c) $\frac{a}{x}$ (d) $\frac{a}{y}$

160. If $y=a\cos(\log x)+b\sin(\log x)$, then $x^2y_2+xy_1$ is :

(a) $\cot(\log x)$ (b) y
(c) -y (d) $\tan(\log x)$

161. Let $f(x)=|x|, x\in R$. Then, which of the following statements is incorrect?

(a) f has a minimum value at $x=0$.
(b) f has no maximum value in R.
(c) f is continuous at $x=0$.
(d) f is differentiable at $x=0$.

162. Let $f'(x)=3(x^2+2x)-\frac{4}{x^3}+5, f(1)=0$. Then, $f(x)$ is :

(a) $x^3+3x^2+\frac{2}{x^2}+5x+11$
(b) $x^3+3x^2+\frac{2}{x^2}+5x-11$
(c) $x^3+3x^2-\frac{2}{x^2}+5x-11$
(d) $x^3-3x^2-\frac{2}{x^2}+5x-11$

163. $\int \frac{x+5}{(x+6)^2}e^x dx$ is equal to:

(a) $\log(x+6)+C$
(b) e^x+C
(c) $\frac{e^x}{x+6}+C$
(d) $\frac{-1}{(x+6)^2}+C$

164. The order and degree of the following differential equation are, respectively:

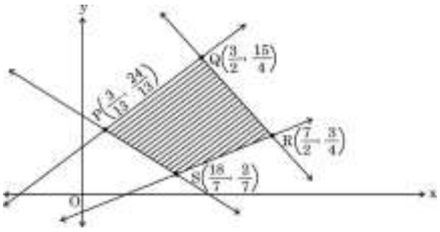
$$-\frac{d^4y}{dx^4}+2e^{dy/dx}+y^2=0$$

(a) −4,1 (b) 4, not defined
(c) 1,1 (d) 4,1

165. The solution for the differential equation $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ is :

(a) $3e^{4y} + 4e^{-3x} + C = 0$
(b) $e^{3x+4y} + C = 0$
(c) $3e^{-3y} + 4e^{4x} + 12C = 0$
(d) $3e^{-4y} + 4e^{3x} + 12C = 0$

166. For a Linear Programming Problem (LPP), the given objective function is Z = x + 2y. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.

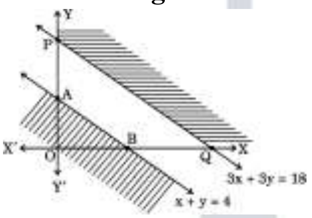


$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$

Which of the following statements is correct ?

(a) Z is minimum at S $\left(\frac{18}{7}, \frac{2}{7}\right)$
(b) Z is maximum at R $\left(\frac{7}{2}, \frac{3}{4}\right)$
(c) (Value of Z at P) > (Value of Z at Q)
(d) (Value of Z at Q) < (Value of Z at R)

167. In a Linear Programming Problem (LPP), the objective function Z = 2x + 5y is to be maximised under the following constraints: $x + y \leq 4, 3x + 3y \geq 18, x, y \geq 0$ Study the graph and select the correct option.



The solution of the given LPP :

(a) lies in the shaded unbounded region.
(b) lies in △ AOB.
(c) does not exist.
(d) lies in the combined region of △ AOB and unbounded shaded region.

168. Let $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $|\lambda\vec{a}|$ is :

(a) [5,10] (b) [−2,5]
(c) [−2,1] (d) [−10,5]

169. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(a) $\frac{3}{2}$ sq units (b) $\frac{2}{3}$ sq units
(c) 3 sq units (d) $\frac{4}{3}$ sq units

170. A box has 4 green, 8 blue and 3 red pens. A student picks up a pen at random, checks its colour and replaces it in the box. He repeats this process 3 times. The probability that at least one pen picked was red is:

(a) $\frac{124}{125}$ (b) $\frac{1}{125}$
(c) $\frac{61}{125}$ (d) $\frac{64}{125}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

171. Assertion (A): If $|\vec{a}\times\vec{b}|^2+|\vec{a}\cdot\vec{b}|^2=256$ and $|\vec{b}|=8$, then $|\vec{a}|=2$.

Reason (R): $\sin^2\theta+\cos^2\theta=1$ and $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$ and $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|\cos\theta$.

172. **Assertion (A):** Let $f(x)=e^x$ and $g(x)=\log x$. Then $(f+g)x=e^x+\log x$ where domain of $(f+g)$ is R.

Reason (R): $\text{Dom}(f+g)=\text{Dom}(f)\cap\text{Dom}(g)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

173. Find the domain of $f(x)=\sin^{-1}(-x^2)$.

174. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x>0$.

OR

(b) If $(x)^y=(y)^x$, then find $\frac{dy}{dx}$.

175. Determine the values of x for which $f(x)=\frac{x-4}{x+1}, x\neq-1$ is an increasing or a decreasing function.

176. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

177. Determine if the lines $\vec{r}=(\hat{i}+\hat{j}-\hat{k})+\lambda(3\hat{i}-\hat{j})$ and $\vec{r}=(4\hat{i}-\hat{k})+\mu(2\hat{i}+3\hat{k})$ intersect with each other.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

178. Let $A=\begin{bmatrix}1\\4\\-2\end{bmatrix}$ and $C=\begin{bmatrix}3&4&2\\12&16&8\\-6&-8&-4\end{bmatrix}$ be two matrices. Then, find the matrix B if $AB=C$.

179. (a) Differentiate $y=\sin^{-1}(3x-4x^3)$ w.r.t. x, if $x\in\left[-\frac{1}{2},\frac{1}{2}\right]$.

OR

(b) Differentiate $y=\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x\in(0,1)$.

180. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x+y=41, x,y\in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f: N\to N$, where N is a set of natural numbers, given by $f(n)=\begin{cases}n-1, \text{ if } n \text{ is even}\\ n+1, \text{ if } n \text{ is odd}\end{cases}$ is a bijection.

181. Consider the Linear Programming Problem, where the objective function $Z=(x+4y)$ needs to be minimized subject to constraints

$2x+y\geq1000$
$x+2y\geq800$
$x,y\geq0$

Draw a neat graph of the feasible region and find the minimum value of Z.

182. (a) Find the distance of the point $P(2,4,-1)$ from the line

$$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$$

OR

(b) Let the position vectors of the points A, B and C be $3\hat{\imath} - \hat{\jmath} - 2\hat{k}$, $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\hat{\imath} + 5\hat{\jmath} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

183. A person is Head of two independent selection committees I and II. If the probability of making a wrong selection in committee I is 0.03 and that in committee II is 0.01 , then find the probability that the person makes the correct decision of selection :

(i) in both committees

(ii) in only one committee

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

184. (a) Find:

$$\int \frac{x^2+1}{(x-1)^2(x+3)}dx$$

OR

(b) Evaluate: $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}dx$

185. Draw a rough sketch for the curve $y = 2 + |x+1|$. Using integration, find the area of the region bounded by the curve $y = 2 + |x+1|$, $x = -4$, $x = 3$ and $y = 0$.

186. (a) Solve the differential equation : $x^2y dx - (x^3 + y^3)dy = 0$.

OR

(b) Solve the differential equation $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

187. Let the polished side of the mirror be along the line $\frac{x}{1} = \frac{1-y}{-2} = \frac{2z-4}{6}$. A point $P(1, 6, 3)$, some distance away from the mirror, has its image formed behind the mirror. Find the coordinates of the image point and the distance between the point P and its image.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study - 1

188. Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions :

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B.

(ii) Find |A| and confirm if it is possible to find A^{-1}.

(iii) (a) Find A^{-1}, if possible, and write the formula to find X .

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix.

Case Study – 2

189.

A ladder of fixed length ' h ' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions:

(i) Express the distance (y) between the wall and foot of the ladder in terms of ' h ' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer.

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point.

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point.

OR

(iii) (b) If the foot of the ladder whose length is 5 m , is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ?

Case Study - 3

190. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective.

(ii) What is the probability that this defective smartphone was manufactured by company B?

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) This question paper is divided into five Sections - A, B, C, D and E.

(iii) In **Section A,** Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In **Section B,** Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.

(v) In **Section C,** Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.

(vi) In **Section D,** Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.

(vii) In **Section E,** Questions no. 36 to 38 are case study based questions carrying 4 marks each.

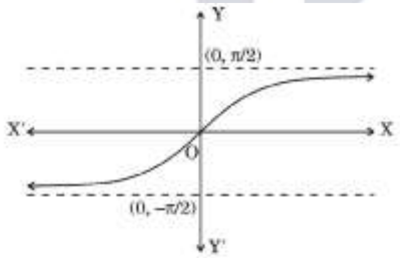
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

191. The given graph illustrates:



(a) $y = \tan^{-1} x$ (b) $y = \operatorname{cosec}^{-1} x$

(c) $y = \cot^{-1} x$ (d) $y = \sec^{-1} x$

192. Domain of $f(x) = \cos^{-1} x + \sin x$ is :

(a) R (b) $(-1,1)$

(c) $[-1,1]$ (d) ϕ

193. What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?

(a) 9 (b) 512

(c) 615 (d) 64

194. The matrix A = $\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ is a/an :

(a) scalar matrix

(b) identity matrix

(c) null matrix

(d) symmetric matrix

195. **If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is:**

(a) 30 (b) 120
(c) 15 (d) 225

196. **Let A be a square matrix of order 3. If $|A| = 5$, then $|\text{adj}A|$ is:**

(a) 5 (b) 125
(c) 25 (d) -5

197. **If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$, then the value of $(x-y)$ is :**

(a) 2 or 10 (b) - 2 or 10
(c) 2 or -10 (d) -2 or -10

198. **If $f(x) = \begin{cases} 1, & \text{if } x \le 3 \\ ax+b, & \text{if } 3 < x < 5 \\ 7, & \text{if } 5 \le x \end{cases}$ is continuous in R , then the values of a and b are :**

(a) $a = 3, b = -8$ (b) $a = 3, b = 8$
(c) $a = -3, b = -8$ (d) $a = -3, b = 8$

199. **If $f(x) = -2x^8$, then the correct statement is :**

(a) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (b) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$
(c) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (d) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

200. **A spherical ball has a variable diameter $\frac{5}{2}(3x+1)$. The rate of change of its volume w.r.t. x, when $x = 1$, is:**

(a) 225π (b) 300π
(c) 375π (d) 125π

201. **If $f: R \to R$ is defined as $f(x) = 2x - \sin x$, then f is :**

(a) a decreasing function
(b) an increasing function
(c) maximum at $x = \frac{\pi}{2}$
(d) maximum at $x = 0$

202. **$\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}} dx$ is equal to:**

(a) $x + C$ (b) $\frac{x^2}{2} + C$
(c) $\frac{x^4}{4} + C$ (d) $\frac{x^3}{3} + C$

203. **For a function $f(x)$, which of the following holds true ?**

(a) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$
(b) $\int_{-a}^a f(x)dx = 0$, if f is an even function
(c) $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if f is an odd function
(d) $\int_0^{2a} f(x)dx = \int_0^a f(x)dx - \int_0^a f(2a+x)dx$

204. **$\int \frac{e^x}{\sqrt{4-e^{2x}}} dx$ is equal to:**

(a) $\frac{1}{2}\cos^{-1}(e^x) + C$ (b) $\frac{1}{2}\sin^{-1}(e^x) + C$
(c) $\frac{e^x}{2} + C$ (d) $\sin^{-1}\left(\frac{e^x}{2}\right) + C$

205. **A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is :**

(a) 6 (b) 1
(c) $\frac{1}{4}$ (d) 4

206. If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are:

(a) orthogonal vectors
(b) parallel to each other
(c) unit vectors
(d) collinear vectors

207. If $P(A) = \frac{1}{7}, P(B) = \frac{5}{7}$ and $P(A \cap B) = \frac{4}{7}$, then $P(\overline{A} \mid B)$ is :

(a) $\frac{6}{7}$ (b) $\frac{3}{4}$
(c) $\frac{4}{5}$ (d) $\frac{1}{5}$

208. A coin is tossed, and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is:

(a) $\frac{2}{13}$ (b) $\frac{3}{26}$
(c) $\frac{19}{26}$ (d) $\frac{3}{13}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

209. Assertion (A): $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$ is continuous at $x = 0$.

Reason (R): When $x \to 0, \sin\frac{1}{x}$ is a finite value between -1 and 1.

210. Assertion (A): Set of values of $\sec^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R): $\sec^{-1} x$ is defined for $x \in R - (-1,1)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

211. Let $f: A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = R - \{3\}$ and $B = R - \{1\}$. Discuss the bijectivity of the function.

212. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then show that $A^2 - 4A + 7I = 0$.

213. (a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x.

OR

(b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.

214. In a Linear Programming Problem, the objective function $Z = 5x + 4y$ needs to be maximised under constraints $3x + y \leq 6, x \leq 1, x, y \geq 0$. Express the LPP on the graph and shade the feasible region and mark the corner points.

215. (a) 10 identical blocks are marked with ' 0 ' on two of them, ' 1 ' on three of them, ' 2 ' on four of them and ' 3 ' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

OR

(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village?

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

216. (a) Show that the function $f: R \rightarrow R$ defined by $f(x) = 4x^3 - 5, \forall x \in R$ is one-one and onto.

OR

(b) Let R be a relation defined on a set N of natural numbers such that R = {(x, y) : xy is a square of a natural number, x, y ∈ N}. Determine if the relation R is an equivalence relation.

217. (a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

OR

(b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000 , what profit did he earn after the sale of two days?

218. Differentiate $y = \sqrt{\log\left\{\sin\left(\frac{x^3}{3} - 1\right)\right\}}$ with respect to x.

219. Amongst all pairs of positive integers with product as 289, find which of the two numbers add up to the least.

220. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints
$4x + y \geq 20$
$2x + 3y \geq 30$
$x, y \geq 0$
find the minimum value of Z.

221. (a) The scalar product of the vector $\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{c} = \lambda\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ is equal to 1 . Find the value of λ.

OR

(b) Find the shortest distance between the lines:
$\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$
$\vec{r} = (\hat{\imath} + 4\hat{k}) + \mu(3\hat{\imath} - 6\hat{\jmath} + 9\hat{k})$

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

222. (a) Find: $\int \frac{x^2+1}{(x^2+2)(2x^2+1)} dx$

OR

(b) Evaluate: $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x} dx$

223. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

224. Solve the differential equation $\frac{dy}{dx} = \cos x - 2y$.

225. (a) Find the point Q on the line $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point $P(1, 2, 3)$.

OR

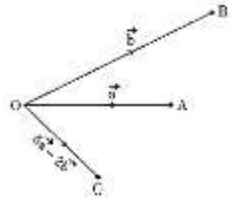
(b) Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study - 1

226. Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based upon the above information, answer the following questions :

(i) Complete the given figure to explain their entire movement plan along the respective vectors.

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$.

(iii) (a) If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km , then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a} \times \vec{b}|$.

OR

(iii) (b) If $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\jmath} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$.

Case Study-2

227. Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.

(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions :

(i) Write the order and degree of the given differential equation.

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that $r(0) = 5$ mm.

(iii) (a) If it is given that $r = 3$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm.

OR

(iii) (b) If it is given that $r = 1$ mm when $t = 1$ hour, find the value of k. Hence, find t for $r = 0$ mm.

Case Study - 3

228. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health,

and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively.

Based upon the above information, answer the following questions :

(i) A person was tested randomly. What is the probability that he/she has contracted the disease?

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ?

General Instructions:

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE Sections - Section A, B, C, D and E.
(iii) In **Section A-** Question Number 1 to 18 are Multiple Choice Questions (MCQs) type and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.
(iv) In **Section B -** Question Number 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
(v) In **Section C -** Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In **Section D-** Question Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.
(vii) In **Section E -** Question Number 36 to 38 are case study based questions, carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section - B, 3 questions in Section - C, 2 questions in Section - D and 2 questions in Section-E.
(ix) Use of calculator is NOT allowed.

SECTION - A

This section consists of 20 multiple choice questions, each of 1 mark.

229. Which of the following functions from Z to Z is both one-one and onto ?

(a) $f(x) = 2x - 1$ (b) $f(x) = 3x^2 + 5$
(c) $f(x) = x + 5$ (d) $f(x) = 5x^3$

230. Value of $4\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ is

(a) 3 (b) -3
(c) 1 (d) -1

231. If $A = \begin{bmatrix} x & 0 & m \\ y & z & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$, where I is a unit matrix, then $x + y + z + m$ is equal to

(a) 18 (b) 12
(c) 6 (d) 2

232. If $B = [23 \quad 41 \quad 57]\begin{bmatrix} 31 & 42 \\ 53 & 64 \\ 75 & 86 \end{bmatrix}$, then the order of B is :

(a) 3×2 (b) 2×2
(c) 1×3 (d) 1×2

233. If A and B are square matrices of the same order, then $(A - B)^2 = ?$

(a) $A^2 - 2AB + B^2$ (b) $A^2 - AB - BA + B^2$
(c) $A^2 - 2BA + B^2$ (d) $A^2 - AB + BA + B^2$

234. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is

(a) 0 (b) 9
(c) -6 (d) 6

235. If $A^{-1} = \begin{bmatrix} 7 & 2 \\ 8 & 2 \end{bmatrix}$, then matrix A is

(a) $\begin{bmatrix} 2 & -2 \\ -8 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} -7 & 8 \\ 2 & -2 \end{bmatrix}$
(c) $\begin{bmatrix} -1 & 1 \\ 4 & -\frac{7}{2} \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1 \\ -4 & \frac{7}{2} \end{bmatrix}$

236. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\frac{dy}{dx}$ is

(a) $\frac{-\sqrt{x}}{\sqrt{y}}$ (b) $-\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}$
(c) $-\frac{\sqrt{y}}{\sqrt{x}}$ (d) $\frac{-2\sqrt{y}}{\sqrt{x}}$

237. If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is

(a) 1 (b) $\frac{1}{2}$
(c) $-\frac{1}{2}$ (d) -1

238. When x is positive, the minimum value of x^x is

(a) e^e (b) $\frac{1}{e}$

(c) $e^{\frac{1}{e}}$ (d) $e^{\frac{-1}{e}}$

239. $\int \frac{2x^3}{4+x^8} dx$ is equal to

(a) $\frac{1}{4}\tan^{-1}\frac{x^4}{2} + C$ (b) $\frac{1}{2}\tan^{-1}\frac{x^4}{2} + C$

(c) $\frac{1}{4}\tan^{-1}\frac{x^4}{4} + C$ (d) $\frac{1}{4}\tan^{-1} x^4 + C$

240. $\int e^x \cdot \frac{x}{(1+x)^2} dx$ is equal to

(a) $e^x \cdot \frac{x}{1+x} + C$ (b) $e^x \cdot \frac{1}{1+x} + C$

(c) $e^x \cdot \frac{1}{x} + C$ (d) $e^x \cdot \frac{1}{(1+x)^2} + C$

241. The area of the region bounded by the lines $y = x + 1, x = 1, x = 3$ and x-axis is

(a) 6 sq units (b) 8 sq units

(c) 7.5 sq units (d) 2 sq units

242. The integrating factor for solving the differential equation $x \cdot \frac{dy}{dx} - y = 2x^2$ is

(a) x (b) $\frac{1}{x}$

(c) e^{-x} (d) $-\log x$

243. The number of vector(s) of unit length perpendicular to the vectors $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$ is (are) :

(a) one (b) two

(c) three (d) infinite

244. Of all the points of the feasible region, for maximum or minimum values of the objective function, the point lies:

(a) inside the feasible region

(b) at the boundary line of the feasible region

(c) at the corner points of the feasible region

(d) at the coordinate axes

245. The common region for the inequalities $x \geq 0, x + y \leq 1$ and $y \geq 0$, lies in

(a) IV Quadrant (b) II Quadrant

(c) III Quadrant (d) I Quadrant

246. A and B appeared for an interview for two vacancies. The probability of A's selection is $\frac{1}{5}$ and that of B's selection is $\frac{1}{3}$. The probability that none of them is selected is:

(a) $\frac{11}{15}$ (b) $\frac{7}{15}$

(c) $\frac{8}{15}$ (d) $\frac{1}{5}$

Assertion - Reason Based Questions

Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true and Reason (R) is false.

(d) Assertion (A) is false and Reason (R) is true.

247. Assertion (A) : The vectors $\vec{a} = 4\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = -2\hat{\imath} + 3\hat{\jmath} - 5\hat{k}$ are mutually perpendicular vectors.

Reason (R) : Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, if $\vec{a} \cdot \vec{b} = 0$.

248. **Assertion (A) :** $x^2 \mathrm{d}y = (2xy + y^2)\mathrm{d}x$ is a homogeneous differential equation.

Reason (R) : A differential equation of the form $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{F}\left(\frac{y}{x}\right)$ is a homogeneous differential equation.

SECTION - B

This section consists of 5 very short answer type questions, each of 2 marks.

249. Evaluate: $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

250. (a) Show that the function $\mathrm{f}(x) = (x-1)^{\frac{1}{3}}$ is not differentiable at $x = 1$.

OR

(b) Differentiate $\mathrm{y} = \log\left(x + \sqrt{x^2 + \mathrm{a}^2}\right)$ w.r.t. x.

251. If $\mathrm{y} = 7x - x^3$ and x increases at the rate of 2 units per second, then how fast is the slope of the curve changing, when $x = 5$?

252. (a) If $|\vec{a} + \vec{b}| = 60, |\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then find $|\vec{a}|$.

OR

(b) Using vectors, find the value of K such that the points $(\mathrm{K}, -11, 2), (0, -2, 2)$ and $(2, 4, 2)$ are collinear.

253. Find the angle between the two lines whose equations are $2x = 3y = -z$ and $6x = -y = -4z$.

SECTION - C

In this section there are 6 short answer type questions, each of 3 marks.

254. Find the intervals in which the function $\mathrm{f}(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(a) strictly increasing

(b) strictly decreasing

255. (a) Find: $\int \frac{x^2 - x + 1}{(x-1)(x^2+1)} \mathrm{d}x$

OR

(b) Evaluate: $\int_1^4 (|x| + |3 - x|)\mathrm{d}x$

256. (a) Find the particular solution of the differential equation, $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + x^2 + \mathrm{y}^2 + x^2\mathrm{y}^2$, given that $\mathrm{y} = 1$ when $x = 0$.

OR

(b) Solve the differential equation: $2x\mathrm{y}\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 + 3\mathrm{y}^2$.

257. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} + \hat{k}, \vec{b} = 2\hat{\imath} + \hat{\jmath}$ and $\vec{c} = 3\hat{\imath} - 4\hat{\jmath} - 5\hat{k}$, then find a unit vector perpendicular to both the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.

258. The corner points of the feasible region determined by some system of linear inequations, are $(0, 0), (5, 0), (3, 4)$ and $(0, 5)$. Let $\mathrm{Z} = \mathrm{a}x + \mathrm{by}$, where $\mathrm{a, b} > 0$. Find the condition on a and b so that the maximum of Z occurs at both points $(3, 4)$ and $(0, 5)$.

259. (a) Find the probability distribution of the number of doublets in three throws of a pair of dice.

OR

(b) If E and F are two independent events with $\mathrm{P(E)} = \mathrm{p}, \mathrm{P(F)} = 2\mathrm{p}$ and P (exactly one of $\mathrm{E, F}) = \frac{5}{9}$, then find the value of p.

SECTION - D

This section consists of 4 long answer type questions, each of 5 marks.

260. If $\mathrm{A} = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the system of equations :

$2x - 3y + 5z = 11$
$3x + 2y - 4z = -5$
$x + y - 2z = -3$

261. (a) Differentiate $x^{\sin x} + (\sin x)^x$ w.r.t. x.

OR

(b) If $\mathrm{y} = x + \tan x$, then prove that

$\cos^2 x \dfrac{d^2y}{dx^2} - 2y + 2x = 0$

262. The region enclosed between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x =$ a. Find the value of a.

263. (a) Find the shortest distance between the lines given by
$\vec{r} = (4\hat{\imath} - \hat{\jmath} + 2\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} - 3\hat{k})$ and $\vec{r} = (2\hat{\imath} + \hat{\jmath} - \hat{k}) + \mu(3\hat{\imath} + 2\hat{\jmath} - 4\hat{k})$

OR

(b) Find the coordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point $P(5, 4, 2)$ to the line $\vec{r} = -\hat{\imath} + 3\hat{\jmath} + \hat{k} + \lambda(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$.

SECTION - E

In this section there are 3 case-study based questions of 4 marks each.

264. An architect designs a building for a Company. The design of window on the ground floor is proposed to be different than at the other floors. The window is in the shape of a rectangle whose top length is surmounted by a semi-circular opening. This window has a perimeter of 10 m .
Based on the above information, answer the following :
(i) If $2x$ and $2y$ represent the length and breadth of the rectangular portion of the window, then establish a relation between x and y .
(ii) Find the total area of the window in terms of x.
(iii) (a) Find the values of x and y for the maximum area of the window.

OR

(iii) (b) If x and y represent the length and breadth of the rectangle, then establish the expression for the area of the window in terms of x only.

265. There are three categories of students in a class of 60 students : A : Very hardworking students, B : Regular but not so hard working, C : Careless and irregular students. It is known that 6 students in category A, 26 in category B and the rest in category C. It is also found that probability of students of category A, unable to get good marks in the final year examination, is 0.002 , of category B it is 0.02 and of category C, this probability is 0.20 .
Based on the above information, answer the following :
(i) Find the probability that a student selected at random is unable to get good marks in the final examination.
(ii) A student selected at random was found to be one who could not get good marks in the final examination. Find the probability, that this student is NOT of category A.

266. Rajesh, a student of Class-XII, visited an exhibition with his family. There he saw a huge swing and found that it traced the path of a parabola $y = x^2$. The following questions came to his mind. Answer the questions :
(i) Let $f: R \to R$ be a function defined as $f(x) = x^2$. Find whether f is one-one function.
(ii) Let $f: R \to R$ be defined as $f(x) = x^2$. Find whether f is an onto function.
(iii) (a) Let $f: N \to N$ be defined as $f(x) = x^2$. Find whether f is one-one function. Also, find if it is an onto function.

OR

(iii) (b) Let $f: N \to \{1, 4, 9, 16, \dots\dots\}$ defined as $f(x) = x^2$, find where f is one-one function. Also, find if it is an onto function.