1. **(c)** The principal value range of $\cos^{-1}x$ is $[0,\pi]$. Let $\cos^{-1}x=\theta$, so $\theta\in[0,\pi]$.
Then, $y=2\theta$, giving $y\in[0,2\pi]$.
2. **(a)** A row matrix has exactly one row and any number of columns, so its order is always $1\times n$.
3. **(d)** $(A+B)'=A'+B'$ is always true. Also, the transpose of scalar multiplication and product gives $(kAB)'=k(AB)'=kB'A'$, matching option (iv). However, $(A-B)'=A'-B'$, not $B'-A'$, and $(AB)'=B'A'$, not $A'B'$.
4. **(b)** Δ_2 is obtained by interchanging the first two rows of Δ_1, which changes the sign, and also multiplying one row by 2 , which multiplies the determinant by 2 .
Hence, overall $\Delta_2=-2\Delta_1$.
5. **(b)** Determinant $=\cos x\times\sin x-(-\cos x\times\sin x)=2\cos x\sin x=\sin 2x$.
Given $\sin 2x=1$, so $2x=\frac{\pi}{4}$
6. **(d)** For skew-symmetric matrices, $A'=-A$ and $B'=-B$. Then $(A-B)'=A'-B'=-A+B=-(A-B)$.
Hence, $A-B$ is skew-symmetric, while the other given expressions do not satisfy this property.
7. **(a)** Given $f(x)=x^3-12x$ on $[0,3]$.
Differentiate: $f'(x)=3x^2-12=0\Rightarrow x=2$
Check values: $f(0)=0, f(2)=-16, f(3)=-9$
Thus, the least value in the interval is -16 at $x=2$.
8. **(d)** Let $I=\int\frac{3ax}{b^2+c^2x^2}dx$
Put $u=b^2+c^2x^2$, So $du=2c^2xdx$
Then,
$$I=\frac{3a}{2c^2}\int\frac{du}{u}$$
$$I=\frac{3a}{2c^2}\log|b^2+c^2x^2|+K$$
Hence, $A=\frac{3a}{2c^2}$.
9. **(a)** The denominator $x^2+2|x|+1$ is an even function, since it contains x^2 and $|x|$. The numerator x^3 is odd.
Hence, the integrand is odd.
The integral of an odd function over $[-1,1]$ is 0 .
10. **(c)** Area bounded by the curve and x -axis is $\int_{-1}^{1}|y|dx$
Since $y=x|x|, |y|=x^2$
Therefore,
$$\text{Area }=\int_{-1}^{1}x^2dx$$
$$=2\int_0^1 x^2dx$$
$$=2\left[\frac{x^3}{3}\right]_0^1$$
$$\therefore\text{ Area }=\frac{2}{3}$$
11. **(c)** Given $R\frac{dx}{dy}+Px=Q$,
Divide by R: $\frac{dx}{dy}+\frac{P}{R}x=\frac{Q}{R}$
A linear equation in x . The integrating factor is $e^{\int\frac{P}{R}dy}$, obtained from standard form $\frac{dx}{dy}+Pyx=Qy$.
12. **(b)** Given $\frac{d}{dx}(e^y)=0$
Differentiating, $e^y\frac{dy}{dx}=0$,
Highest derivative present is $\frac{dy}{dx}$,

So the order is 1 . Power of the derivative is 1 , hence the degree is 1 after expressing the equation in polynomial form.

13. **(c)** For perpendicular vectors, the dot product is zero:
$1 \times 2 + 2(-p) + 3 \times 1 = 0$
$\Rightarrow 2 - 2p + 3 = 0$
Thus, $5 - 2p = 0$
$\Rightarrow p = \frac{5}{2}$
Hence, vectors are perpendicular for this value.

14. **(a)** Let points be $A(-1,-1,2), B(2, m, 5), C(3,11,6)$.
For collinearity,
$\overrightarrow{AB} = \lambda\overrightarrow{AC}$
$\overrightarrow{AB} = (3, m + 1,3)$
$\overrightarrow{AC} = (4,12,4)$
Comparing ratios:
$\frac{3}{4} = \frac{m+1}{12} = \frac{3}{4}$
Thus, $m + 1 = 9$
$\therefore m = 8$

15. **(c)** Using the formula $|\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}||\sin\theta|$
We get $\theta = \pm\frac{\pi}{6}$.
Therefore, $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}|\cos\theta$
$= 8 \times 3 \times \frac{\sqrt{3}}{2}$
$= 12\sqrt{3}$

16. **(c)** Line is $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$, so it passes through the origin with direction vector $(1,0,0)$.
Distance from the point $(2,5,7)$ to the line is $\sqrt{5^2 + 7^2} = \sqrt{74}$

17. **(d)** For $Z = 5x + 7y$, evaluate at the corner points of the feasible region: $(0,0), (0,2), (3,4), (7,0)$.
Values are $0,14,43,35$. The maximum value is 43 , the minimum value is 0 .
Hence, maximum - minimum $= 43$

18. **(b)** In a linear programming problem, the objective function is always linear in variables, meaning each variable has degree one and no higher powers occur. Hence, the overall degree of the objective function is 1 by definition of linearity.

19. **(c)** In a die throw, the odd numbers are $\{1,3,5\}$ and odd primes are $\{3,5\}$, making $P(A \mid B) = \frac{2}{3}$, However, the formula for $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$, not $\frac{P(A \cup B)}{P(B)}$.

20. **(d)** Direction ratios are $(p, 1, r)$ and $(p' + 1 + r')$.
For perpendicular lines,
$pp' + 1 + rr' = 0$
$pp' + rr' = -1$, not 1
Hence, the assertion is false and the reason $\vec{b}_1 \cdot \vec{b}_2 = 0$ is correct, so Reason is true.

21. **(A)** For $x < 3, |x - 3| = 3 - x$
$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} \frac{3-x}{2(x-3)} = -\frac{1}{2}$$
For $x \geq 3$,
$$f(3) = \frac{3-6}{6}$$
$$f(3) = -\frac{1}{2}$$
Also,

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{x-6}{6} = -\frac{1}{2}$

Since LHL = RHL = f(3), f(x) is continuous at x = 3.

OR

(B) Given: $\sqrt{3}(x^2 + y^2) = 4xy$

Differentiate implicitly:

$$\sqrt{3}\left(2x + 2y\frac{dy}{dx}\right) = 4\left(y + x\frac{dy}{dx}\right)$$

Simplify:

$$2\sqrt{3}x + 2\sqrt{3}y\frac{dy}{dx} = 4y + 4x\frac{dy}{dx}$$

Rearrange:

$$(2\sqrt{3}y - 4x)\frac{dy}{dx} = 4y - 2\sqrt{3}x$$

$$2\sqrt{3}y\frac{dy}{dx} - 4x\frac{dy}{dx} = 4y - 2\sqrt{3}x$$

$$\frac{dy}{dx}(2\sqrt{3}y - 4x) = (4y - 2\sqrt{3}x)$$

$$\frac{dy}{dx} = \frac{4y - 2\sqrt{3}x}{2\sqrt{3}y - 4x}$$

Substitute $x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}$

Numerator:

$$= 4 \cdot \frac{\sqrt{3}}{2} - 2\sqrt{3} \cdot \frac{1}{2}$$

$$= 2\sqrt{3} - \sqrt{3}$$

$$= \sqrt{3}$$

Denominator:

$$= 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} - 4 \cdot \frac{1}{2}$$

$$= 3 - 2$$

$$= 1$$

$$\therefore \frac{dy}{dx} = \sqrt{3}$$

22. For cone, semi-vertical angle $= \frac{\pi}{6}$

$$\tan\left(\frac{\pi}{6}\right) = \frac{r}{h}$$

$$\frac{1}{\sqrt{3}} = \frac{r}{h}$$

$$\therefore r = \frac{h}{\sqrt{3}}$$

Volume of perfume:

$$V = \frac{1}{3}\pi r^2 h$$

Substitute $r = \frac{h}{\sqrt{3}}$:

$$V = \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h$$

$$V = \frac{1}{3}\pi \cdot \frac{h^2}{3} \cdot h$$

$$V = \frac{\pi h^3}{9}$$

Differentiate with respect to t :

$$\frac{dV}{dt} = \frac{\pi}{9} \cdot 3h^2 \frac{dh}{dt}$$

$\frac{dV}{dt}=\frac{\pi h^2}{3}\frac{dh}{dt}$

Given:

$\frac{dV}{dt}=-1\ \text{mm}^3/\text{min},\ h=10\ \text{mm}$

$-1=\frac{\pi(10)^2}{3}\frac{dh}{dt}$

$-1=\frac{100\pi}{3}\frac{dh}{dt}$

$\frac{dh}{dt}=-\frac{3}{100\pi}$

Hence, the rate at which the level of perfume is dropping:

$\frac{3}{100\pi}\ \text{mm/min}$

23. Points: $P(1,3,2), Q(-1,0,8)$

$\overrightarrow{QP}=P-Q$

$=(1-(-1),3-0,2-8)$

$=(2,3,-6)$

Magnitude:

$|\overrightarrow{QP}|=\sqrt{2^2+3^2+(-6)^2}$

$=\sqrt{4+9+36}$

$\therefore |\overrightarrow{QP}|=7$

Unit vector:

$\frac{\overrightarrow{QP}}{|\overrightarrow{QP}|}=\frac{2\hat{\imath}+3\hat{\jmath}-6\hat{k}}{7}$

Required vector of magnitude 14:

$14\left(\frac{2\hat{\imath}+3\hat{\jmath}-6\hat{k}}{7}\right)$

$=2(2\hat{\imath}+3\hat{\jmath}-6\hat{k})$

$=(4\hat{\imath}+6\hat{\jmath}-12\hat{k})$

Hence, the vector of magnitude 14 is $(4\hat{\imath}+6\hat{\jmath}-12\hat{k})$.

24. Given:

$\vec{a}=3\hat{\imath}-2\hat{\jmath}+2\hat{k}$ and $\vec{b}=\hat{\imath}+2\hat{k}$

Diagonals of a parallelogram:

$\Rightarrow \dot{d}_1=\dot{a}+\vec{b}$

$=(3+1)\hat{\imath}+(-2)\hat{\jmath}+(2+2)\hat{k}$

$\therefore \dot{d}_1=4\hat{\imath}-2\hat{\jmath}+4\hat{k}$

$\Rightarrow \dot{d}_2=\dot{a}-\hat{b}$

$=(3-1)\hat{\imath}-2\hat{\jmath}+(2-2)\hat{k}$

$\therefore \dot{d}_2=2\hat{\imath}-2\hat{\jmath}$

Lengths:

$\Rightarrow |\vec{d}_1|=\sqrt{4^2+(-2)^2+4^2}$

$=\sqrt{16+4+16}$

$=\sqrt{36}$

$\therefore |\vec{d}_1|=6$

$\Rightarrow |\vec{d}_2|=\sqrt{2^2+(-2)^2+0}$

$=\sqrt{4+4}$

$=\sqrt{8}$

$=2\sqrt{2}$

$\therefore |\vec{d}_2|=2\sqrt{2}$

Hence, the vectors representing its diagonals and their lengths are:

$\dot{d}_1 = 4\hat{i} - 2\hat{j} + 4\hat{k}, |\dot{d}_1| = 6$

$\dot{d}_2 = 2\hat{i} - 2\hat{j}, |\dot{d}_2| = 2\sqrt{2}$

25. **(A)** $\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$

Divide numerator and denominator by cos 2x :

$$\tan^{-1}\left(\frac{1 - \tan 2x}{1 + \tan 2x}\right)$$

Using,

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{1 - \tan 2x}{1 + \tan 2x}$$

$$= \tan\left(\frac{\pi}{4} - 2x\right)$$

Since, $0 < x < \frac{\pi}{4}$,

$$\therefore \tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right) = \frac{\pi}{4} - 2x$$

OR

(B) $\tan\left(\sin^{-1}1 - \cos^{-1}\left(-\frac{1}{2}\right)\right)$

$$\sin^{-1}1 = \frac{\pi}{2}$$

$$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$$

So,

$$\tan\left(\frac{\pi}{2} - \frac{2\pi}{3}\right) = \tan\left(-\frac{\pi}{6}\right)$$

$$\tan\left(-\frac{\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

26. Let $I = \int_0^1 x\tan^{-1}x \cdot dx$

$$= \int_0^1 (\tan^{-1}x)(x) \cdot dx$$

$$= \left[(\tan^{-1}x)\int x \cdot dx\right]_0^1 - \int_0^1 \left[\frac{d}{dx}(\tan^{-1}x) \cdot \int x \cdot dx\right] \cdot dx$$

$$= \left[\frac{x^2 \tan^{-1}x}{2}\right]_0^1 - \int_0^1 \frac{1}{1 + x^2} \cdot \frac{x^2}{2} \cdot dx$$

$$= \left(\frac{1^2 \tan^{-1}1}{2} - 0\right) - \frac{1}{2}\int_0^1 \frac{1 + x^2 - 1}{1 + x^2} \cdot dx$$

$$= \frac{\frac{\pi}{4}}{2} - \frac{1}{2}\int_0^1 \left(1 - \frac{1}{1 + x^2}\right) \cdot dx$$

$$= \frac{\pi}{8} - \frac{1}{2}[x - \tan^{-1}(x)]_0^1$$

$$= \frac{\pi}{8} - \frac{1}{2}[(1 - \tan^{-1}1) - 0]$$

$$= \frac{\pi}{8} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right)$$

$$= \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$$

$$= \frac{\pi}{4} - \frac{1}{2}$$

27. **(A)** $\int \sqrt{\frac{x+2}{x-2}}\,dx$

$$\sqrt{\frac{x + 2}{x - 2}} = \frac{x + 2}{\sqrt{x^2 - 4}}$$

So,

$$\int \sqrt{\frac{x+2}{x-2}} = \int \frac{x+2}{\sqrt{x^2-4}}$$
$$= \int \frac{x}{\sqrt{x^2-4}}dx + 2\int \frac{1}{\sqrt{x^2-4}}dx$$
$$= \sqrt{x^2-4} + 2\log\left|x+\sqrt{x^2-4}\right| + C$$
$$\therefore \int \sqrt{\frac{x+2}{x-2}}dx = \sqrt{x^2-4} + 2\log\left|x+\sqrt{x^2-4}\right| + C$$

OR

(B) $\int \frac{x^2}{(x^2+9)(x^2+16)}dx$

$$\frac{x^2}{(x^2+9)(x^2+16)} = \frac{-9}{7(x^2+9)} + \frac{16}{7(x^2+16)}$$
$$= -\frac{9}{7}\int \frac{dx}{x^2+9} + \frac{16}{7}\int \frac{dx}{x^2+16}$$
$$= -\frac{3}{7}\tan^{-1}\frac{x}{3} + \frac{4}{7}\tan^{-1}\frac{x}{4} + C$$
$$\therefore \int \frac{x^2}{(x^2+9)(x^2+16)}dx = \frac{4}{7}\tan^{-1}\frac{x}{4} - \frac{3}{7}\tan^{-1}\frac{x}{3} + C$$

28. We have

$$I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}, I_2 = \int_{-1/2}^{1/2} |x|dx$$

We shall evaluate both integrals.

Step 1: Evaluate $\boldsymbol{I_1}$

Use the identity

$1 + \cos 2x = 2\cos^2 x$

Hence,

$$I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{2\cos^2 x} = \frac{1}{2}\int_{-\pi/4}^{\pi/4} \sec^2 x dx.$$

Since

$$\int \sec^2 x dx = \tan x$$

we get

$$I_1 = \frac{1}{2}[\tan x]_{-\pi/4}^{\pi/4} = \frac{1}{2}(1-(-1)) = 1$$

Step 2: Evaluate I_2

Since $|x|$ is an even function,

$$I_2 = 2\int_0^{1/2} x dx = 2\left[\frac{x^2}{2}\right]_0^{1/2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

Step 3: Verify the result

$$I_1 - 4I_2 = 1 - 4\left(\frac{1}{4}\right) = 1 - 1 = 0.$$

Hence,

$I_1 - 4I_2 = 0.$

29. **(A)** $x^2\frac{dy}{dx} = x^2 + xy + y^2$

Divide by x^2 :

$$\frac{dy}{dx} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$$

Put $y = vx$. Then,

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$v + x\frac{dv}{dx} = 1 + v + v^2$

$x\frac{dv}{dx} = 1 + v^2$

$\frac{dv}{1 + v^2} = \frac{dx}{x}$

Integrate:

$\tan^{-1}v = \log|x| + c$

Substitute $v = \frac{y}{x}$

$\therefore \tan^{-1}\left(\frac{y}{x}\right) = \log|x| + C$

OR

(B) $xy\frac{dy}{dx} = (x + 2)(y + 2)$

$\frac{y}{y + 2}dy = \frac{x + 2}{x}dx$

$\int \frac{y}{y + 2}dy = \int \left(1 + \frac{2}{x}\right)dx$

$\int \left(1 - \frac{2}{y + 2}\right)dy = x + 2\log|x| + C$

$y - 2\log|y + 2| = x + 2\log|x| + C$

Given $y(1) = -1$:

$-1 - 2\log 1 = 1 + 2\log 1 + C$

$C = -2$

$\therefore y - 2\log|y + 2| = x + 2\log|x| - 2$

30. $x + y \leq 7$,

$2x - 3y + 6 \geq 0, \Rightarrow y \leq \frac{2x + 6}{3}$

$x \geq 0, y \geq 0$

The corner points of the shaded feasible region are:

(1) Origin: $(0,0)$

(2) x-intercept of $x + y = 7$: $(7,0)$

(3) Intersection of $x + y = 7$ and $2x - 3y = -6$:

(i) From the first: $x = 7 - y$

(ii) Substitute:

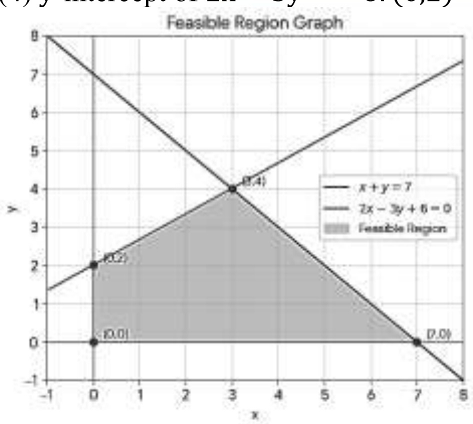
$2(7 - y) - 3y = -6$

$14 - 5y = -6$

$5y = 20$

$y = 4, x = 3$

(4) y-intercept of $2x - 3y = -6$: $(0,2)$



Evaluate $Z = 13x - 15y$:

(I) $Z(0,0) = 0$

(II) $Z(0,2) = 13(0) - 15(2)$

$= -30$

(III) $Z(3,4) = 39 - 60$

$= -21$

(IV) $Z(7,0) = 91$

So, the minimum value is -30 at $(x, y) = (0,2)$.

31. **(A)** Die shows number less than 3: $\{1,2\}$

$P(B_1) = \frac{2}{6} = \frac{1}{3}, P(B_2) = \frac{4}{6} = \frac{2}{3}$

$P(R \mid B_1) = \frac{3}{7}, P(R \mid B_2) = \frac{8}{14} = \frac{4}{7}$

Required probability:

$P(R) = P(B_1)P(R \mid B_1) + P(B_2)P(R \mid B_2)$

$= \frac{1}{3} \cdot \frac{3}{7} + \frac{2}{3} \cdot \frac{4}{7}$

$= \frac{1}{7} + \frac{8}{21}$

$= \frac{3+8}{21}$

$= \frac{11}{21}$

Hence, the probability that the ball drawn from one of the bags is a red ball is $\frac{11}{21}$.

OR

(B) $P(X \cup Y) = a$

Exactly one event occurs:

$P(X \cap Y') + P(X' \cap Y) = b$

$P(X') + P(Y') = 2 - [P(X) + P(Y)]$

$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$b = P(X) + P(Y) - 2P(X \cap Y)$

$P(X) + P(Y) = 2a - b$

$P(X') + P(Y') = 2 - (2a - b)$

$P(X') + P(Y') = 2 - (2a + b)$

Hence proved.

32. **(A)** Given:

$R = \{(x, y): |x - y|$ is divisible by a prime $p\}$, this means $x - y$ is divisible by p .

⇒ Reflexive:

$|x - x| = 0$, and 0 is divisible by p . So, R is reflexive.

⇒ Symmetric:

If $|x - y|$ is divisible by p , then $|y - x|$ is also divisible by p . So, R is symmetric.

⇒ Transitive:

If $p \mid (x - y)$ and $p \mid (y - z)$, then

$p|[(x - y) + (y - z)]|$

$p \mid (x - z)$

So, R is transitive.

Hence, R is an equivalence relation.

OR

(B) $f(x) = \frac{3x+2}{5x-3}$

One-one:

Let $f(x_1) = f(x_2)$,

$\frac{3x_1 + 2}{5x_2 - 3} = \frac{3x_2 + 2}{5x_2 - 3}$

Cross-multiply:

$(3x_1 + 2)(5x_2 - 3) = (3x_2 + 2)(5x_1 - 3)$

$x_1 = x_2$

So, f is one-one.
Onto: Let,

$$y = \frac{3x+2}{5x-3}$$

$y(5x-3) = 3x+2$

$5xy - 3y = 3x+2$

$5xy - 3x = 3y+2$

$x(5y-3) = 3y+2$

$$x = \frac{3y+2}{5y-3}$$

Since $y \neq \frac{3}{5}$, x exists in $R - \left\{\frac{3}{5}\right\}$

Therefore, f is onto,
Hence, f is one-one and onto.

33. (A) $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$

$|A| = 0(-1-2) - 2(0-(-2)) + 1(2-(-1))$

$|A| = 0 - 4 + 3 = -1$

$\Rightarrow$ Cofactor matrix:

$$C = \begin{bmatrix} -2 & -2 & 3 \\ -2 & -1 & 2 \\ -3 & -2 & 4 \end{bmatrix}$$

$$\text{adj}A = C^T = \begin{bmatrix} -2 & -2 & 3 \\ -2 & -1 & 2 \\ -3 & -2 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}\text{adj}A$$

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} -2 & -2 & 3 \\ -2 & -1 & 2 \\ -3 & -2 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 2 & 1 & 3 \\ 2 & 1 & 2 \\ -3 & -2 & -4 \end{bmatrix}$$

$\Rightarrow$ Given equations:

$-2y + z = 7$

$2x - y - z = 8$

$x - 2y = 10$

$\therefore$ The coefficient matrix is $\begin{bmatrix} 0 & -2 & 1 \\ 2 & -1 & -1 \\ 1 & -2 & 0 \end{bmatrix} = A^T$

So, $A^T X = B$

$X = \left(A^T\right)^{-1} B$

$X = (A^{-1})^T B$

$$(A^{-1})^T = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 2 & -3 \\ 1 & 1 & -2 \\ 3 & 2 & -4 \end{bmatrix}\begin{bmatrix} 7 \\ 8 \\ 10 \end{bmatrix}$$

$$X = \begin{bmatrix} 14+16-30 \\ 7+8-20 \\ 21+16-40 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} 0 \\ -5 \\ -5 \end{bmatrix}$$

Hence,

$x = 0, y = -5, z = -3$

OR

(B) ⇒ Given matrix is singular:

$$\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix} = 0$$

⇒ Expand along the first row:

$$= 3\begin{vmatrix} 4 & \cos 2x \\ 7 & 2 \end{vmatrix} - (-1)\begin{vmatrix} -7 & \cos 2x \\ -11 & 2 \end{vmatrix} + \sin 3x\begin{vmatrix} -7 & 4 \\ -11 & 7 \end{vmatrix}$$

$= 3(8 - 7\cos 2x) + 1(-14 + 11\cos 2x) + \sin 3x(-49 + 44)$

$= 24 - 21\cos 2x - 14 + 11\cos 2x - 5\sin 3x$

$= 10 - 10\cos 2x - 5\sin 3x$

⇒ Since the matrix is singular,

$10 - 10\cos 2x - 5\sin 3x = 0$

Divide by 5 :

$2 - 2\cos 2x - \sin 3x = 0$

$2(1 - \cos 2x) = \sin 3x$

⇒ Using identity:

$1 - \cos 2x = 2\sin^2 x$

$4\sin^2 x = \sin 3x$

$4\sin^2 x = 3\sin x - 4\sin^3 x$

$4\sin^3 x + 4\sin^2 x - 3\sin x = 0$

$\sin x(4\sin^2 x + 4\sin x - 3) = 0$

$4\sin^2 x + 4\sin x - 3 = 0$

⇒ Let $\sin x = t$:

$4t^2 + 4t - 3 = 0$

$4t^2 + 6t - 2t - 3 = 0$

$2t(2t + 3) - 1(2t + 3) = 0$

$(2t + 3)(2t - 1) = 0$

$t = -\frac{3}{2}$ or $t = \frac{1}{2}$

⇒ Since $x \in \left[0, \frac{\pi}{2}\right]$

$\sin x = 0 \Rightarrow x = 0$

$\sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}$

Therefore, $x = 0, \frac{\pi}{6}$

34. ⇒ Given: $x = \cos t, y = \cos mt$,

And we know that,

$\frac{dx}{dt} = -\sin t$

$\frac{dy}{dt} = -m\sin mt$

⇒ So, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$

$\frac{dy}{dx} = \frac{-m\sin mt}{-\sin t}$

$\frac{dy}{dx} = \frac{m\sin mt}{\sin t}$

⇒ Now, $(1 - x^2) = 1 - \cos^2 t = \sin^2 t$

$\therefore (1 - x^2)\frac{dy}{dx} = \sin^2 t \cdot \frac{m\sin mt}{\sin t}$

$(1 - x^2)\frac{dy}{dx} = m\sin t\sin mt$

⇒ Differentiate with respect to x :

$\frac{d}{dx}\left[(1-x^2)\frac{dy}{dx}\right] = \frac{d}{dx}(msintsinmt)$

$\Rightarrow$ Using: $\frac{d}{dx} = \frac{1}{\frac{dx}{dt}} \cdot \frac{d}{dt} = -\frac{1}{sint} \cdot \frac{d}{dt}$

$\frac{d}{dx}(msintsinmt) = -\frac{1}{sint} \cdot d(dt)(msintsinmt)$

$= -\frac{m}{\sin t}(costsinmt + msintcosmt)$

$= -m\frac{costsinmt}{sint} - m^2cosmt$

$= -x\frac{dy}{dx} - m^2y$

$\therefore \frac{d}{dx}\left[(1-x^2)\frac{dy}{dx}\right] = -x\frac{dy}{dx} - m^2y$

$\Rightarrow$ But, $\frac{d}{dx}\left[(1-x^2)\frac{dy}{dx}\right] = (1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx}$

$\Rightarrow$So ,

$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} = -x\frac{dy}{dx} - m^2y$

$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$

Hence proved.

35. Given lines:

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$

For the first line, direction ratios are (2,3,4),

For the second line:

$\frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1}$, direction ratios are (5,2,1),

Since $\frac{2}{5} \neq \frac{3}{2} \neq \frac{4}{1}$, the lines are not parallel.

Now let the first line be:

$x = 1 + 2\lambda, y = 2 + 3\lambda, z = 3 + 4\lambda$

Second line:

$x = 4 + 5\mu, y = 1 + 2\mu, z = \mu$

Let's equate them:

$1 + 2\lambda = 4 + 5\mu$ …(i)

$2 + 3\lambda = 1 + 2\mu$ ….(ii)

$3 + 4\lambda = \mu$ ….(iii)

From third equation: $\mu = 3 + 4\lambda$ ….(iv)

Substitute equation (iv) in the second:

$2 + 3\lambda = 1 + 2(3 + 4\lambda)$

$2 + 3\lambda = 7 + 8\lambda$

$-5 = 5\lambda$

$\therefore \lambda = -1$

Let's put $\lambda = -1$ in equation (iv):

$\mu = 3 + 4(-1)$

$\mu = 3 + (-4)$

$\therefore \mu = -1$

Now substitute $\lambda = -1$ and $\mu = -1$ in equation (i):

$1 + 2(-1) = 4 + 5(-1)$

$-1 = -1$

So lines intersect.

Point of intersection:

- $x = 1 + 2(-1) = -1$
- $y = 2 + 3(-1) = -1$

• $z = 3 + 4(-1) = -1$

Hence, the two lines are not parallel, and they intersect; their point of intersection is $(-1, -1, -1)$.

36. **(i)** For every ₹ 1 increase, 10 subscribers discontinue.

For ₹ x increase:

10x subscribers will discontinue.

(ii) New subscription fee: $300 + x$

Remaining subscribers: $5000 - 10x$

So revenue will be,

$R(x) = (300 + x)(5000 - 10x)$
$= 300(5000 - 10x) + x(5000 - 10x)$
$= 1500000 - 3000x + 5000x - 10x^2$
$= 1500000 + 2000x - 10x^2$
$\therefore R(x) = -10x^2 + 2000x + 1500000$

(iii) (A)

$R(x) = -10x^2 + 2000x + 1500000$

$\Rightarrow$ Differentiate:

$R'(x) = -20x + 2000$

$\Rightarrow$ For maximum:

$R'(x) = 0$
$-20x + 2000 = 0$
$\therefore x = 100$

So, revenue is at its maximum when the annual fee is increased by ₹ 100 .

OR

(B) $R'(x) = -20x + 2000$

$\Rightarrow$ For increasing:

$R'(x) > 0$
$-20x + 2000 > 0$
$x < 100$

$\Rightarrow$ For decreasing:

$\mathbb{R}'(x) < 0$
$x > 100$

$\Rightarrow$ Hence, in (0,5000):

$R(x)$ is increasing on $(0,100)$ and $R(x)$ is decreasing on $(100,5000)$.

37. **(i)** A person who buys a ticket can win any one of the designated prizes. The possible amounts are:

• ₹ 3,00,000 (First Prize)
• ₹ 2,00,000 (Second Prize)
• ₹ 50,000 (Third Prize)

(Note: They can also "win" ₹ 0 if they do not hold a winning ticket.)

(ii) (A)

Total number of tickets sold $(n) = 1{,}00{,}000$

Winning "at least ₹2,00,000" means winning either the first prize or one of the second prizes.

• Number of lst prize tickets $= 1$
• Number of 2nd prize tickets $= 2$
• Total favourable tickets $= 1 + 2 = 3$

$\therefore P(\text{ wins atleast } 2{,}00{,}000) = \frac{3}{1{,}00{,}000}$, or $P = 0.00003$

OR

(B) To find the probability of not winning, we first find the total number of winning tickets.

• Total winning tickets $= 1(1st) + 2(2nd) + 3(3rd) = 6$
• Total non-winning tickets $= 1{,}00{,}000 - 6 = 99{,}994$

$\therefore P(\text{ no win }) = \frac{99{,}994}{1{,}00{,}000}$ or $P = 0.99994$

(iii) In this scenario, Rohan has two tickets for two different jackpots.

• Event A: Wins Jackpot 1 (1st prize). $P(A) = \frac{1}{1{,}00{,}000}$

• Event B: Wins Jackpot 2. $P(B) = \frac{1}{1{,}00{,}000}$

To win on exactly one ticket, he must win the first and lose the second, OR lose the first and win the second:

$P(\text{ Exactly one }) = P(A) \times P(B') + P(A') \times P(B)$

$P(\text{ Exactly one }) = \left(\frac{1}{10^5} \times \frac{99{,}999}{10^5}\right) + \left(\frac{99{,}999}{10^5} \times \frac{1}{10^5}\right)$

$P(\text{ Exactly one }) = \frac{99{,}999+99{,}999}{10{,}00{,}00{,}00{,}000}$

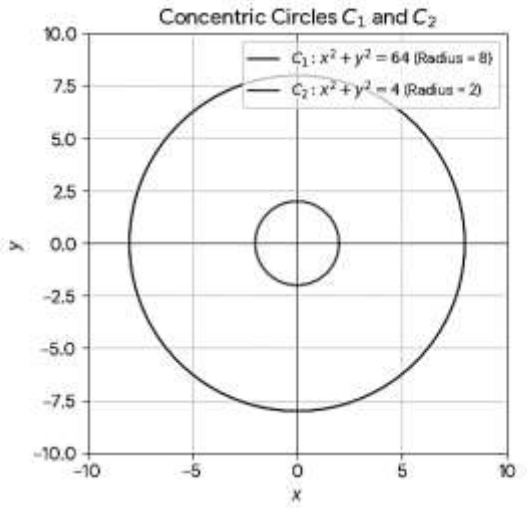
$P(\text{ Exactly one }) = \frac{1{,}99{,}998}{10^{10}}$

$\therefore P(\text{ Exactly one }) \approx 0.0000199998$

38. **(i)** Diagram of C_1 and C_2

• $C_1 : x^2 + y^2 = 64$ represents a circle centered at the origin $(0,0)$ with radius $R_1 = \sqrt{64} = 8$.

• $C_2 : x^2 + y^2 = 4$ represents a smaller concentric circle centered at the origin $(0,0)$ with radius $R_2 = \sqrt{4} = 2$.



(ii) Express y as a function of x

To express $y = f(x)$, rearrange each equation to solve for y :

• For C_1 :

$x^2 + y^2 = 64 \Longrightarrow y^2 = 64 - x^2 \Longrightarrow y = \pm\sqrt{64 - x^2}$

• For C_2 :

$x^2 + y^2 = 4 \Longrightarrow y^2 = 4 - x^2 \Longrightarrow y = \pm\sqrt{4 - x^2}$

(Note: For integration in the first quadrant, we use the positive branch $y = \sqrt{a^2 - x^2}$

(iii) (A) Area of the region covered by the roundabout (C_1)

The total area is 4 times the area in the first quadrant (x goes from 0 to 8):

$$\text{Area } = 4\int_0^8 y\,dx = 4\int_0^8 \sqrt{8^2 - x^2}\,dx$$

Using the standard integral formula $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

$$\text{Area } = 4\left[\frac{x}{2}\sqrt{64 - x^2} + \frac{64}{2}\sin^{-1}\left(\frac{x}{8}\right)\right]_0^8$$

$$\text{Area } = 4[(0 + 32\sin^{-1}(1)) - (0 + 32\sin^{-1}(0))]$$

$$\text{Area } = 4\left[32 \cdot \frac{\pi}{2} - 0\right] = 4 \times 16\pi = 64\pi \text{ sq. units}$$

OR

(B) Area of the region covered by the circular pond $(\mathbf{C}_2)$

The total area is 4 times the area in the first quadrant (x goes from 0 to 2):

$$\text{Area } = 4\int_0^2 y\,dx = 4\int_0^2 \sqrt{2^2 - x^2}\,dx$$

Using the same integration formula:

$$\text{Area } = 4\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2$$

$$\text{Area } = 4[(0 + 2\sin^{-1}(1)) - (0 + 2\sin^{-1}(0))]$$

Area $= 4\left[2 \cdot \frac{\pi}{2} - 0\right] = 4 \times \pi = 4\pi$ sq. units

39. (d) • Reflexive check: For R to be reflexive, $(a, a) \in R$ must hold for all $a \in A$. Here, $(3, 3) \notin R$. It is not reflexive.

• Symmetric check: $(1,2) \in R$ but $(2,1) \notin R$. It is not symmetric.

• Transitive check: If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$. The pairs $(1,1)$ and $(1,2)$ give $(1,2) \in R$, and $(1,2)$ and $(2,2)$ give $(1,2) \in R$. It is transitive.

40. (c) • Statement (I): $(A + B)(A - B) = A^2 - AB + BA - B^2$. Since matrix multiplication is generally not commutative $(AB \neq BA)$, this does not equal $A^2 - B^2$. (False)

• Statement (II): $AB = BA$. Matrix multiplication is not commutative in general. (False)

• Statement (III): $(A + B)^2 = (A + B)(A + B) = A^2 + AB + BA + B^2$. This correctly applies the distributive property. (True)

• Statement (IV): $AB = 0 \Longrightarrow A = 0$ or $B = 0$. The product of two non-zero matrices can be a zero matrix. (False)

41. (a) Problem: If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $3a + b + c$ is:

• For a symmetric matrix, $A = A^T$, which means $a_{ij} = a_{ji}$.

• Comparing elements:

$a = -1$

$b = 0$

$c = 5$

• Substitute values: $3a + b + c = 3(-1) + 0 + 5 = -3 + 5 = 2$.

42. (c) • Find the transpose A' :

$A' = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$

• Add A and A' :

$A + A' = \begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

• Compare corresponding entries:

$2\cos x = 1 \Longrightarrow \cos x = \frac{1}{2}$

• Since $x \in \left[0, \frac{\pi}{2}\right], x = \frac{\pi}{3}$.

43. (c) • Use the scalar property of matrix inverses: $(kA)^{-1} = \frac{1}{k}A^{-1}$ (where k is a non-zero scalar).

• Substituting $k = 3$:

$(3A)^{-1} = \frac{1}{3}A^{-1}$

44. (c) Expand the determinant along the first column:

$-1 \cdot \begin{vmatrix} a & -1 \\ 4 & 2a \end{vmatrix} - (-2) \cdot \begin{vmatrix} -2 & 5 \\ 4 & 2a \end{vmatrix} + 0 = -86$

$-1 \cdot (2a^2 + 4) + 2 \cdot (-4a - 20) = -86$

$-2a^2 - 4 - 8a - 40 = -86$

$-2a^2 - 8a - 44 = -86$

$-2a^2 - 8a + 42 = 0$

Divide the entire equation by -2 :

$a^2 + 4a - 21 = 0$

For a quadratic equation $Ax^2 + Bx + C = 0$, the sum of the roots is $-\frac{B}{A}$:

Sum of values of $a = -\frac{4}{1} = -4$

45. (d) Differentiate both sides with respect to x :

$\frac{d}{dx}(e^{-x}) + \frac{d}{dx}(e^{-y}) = \frac{d}{dx}$

$-e^{-x} - e^{-y}\frac{dy}{dx} = 0$

$$-e^{-y}\frac{dy}{dx}=e^{-x}$$
$$\frac{dy}{dx}=-\frac{e^{-x}}{e^{-y}}=-e^{-x-(-y)}=-e^{y-x}$$

46. (c) Find the first derivative:
$$f'(x)=1-\frac{1}{x^2}$$
Set $f'(x)=0$ to find critical points:
$$1-\frac{1}{x^2}=0\Longrightarrow x^2=1\Longrightarrow x=1\text{ or }x=-1$$
Find the second derivative:
$$f''(x)=\frac{2}{x^3}$$
Evaluate at critical points:
- At $x=1$: $f''(1)=2>0\Longrightarrow$ local minimum at $x=1$.

Local minimum value $=f(1)=1+\frac{1}{1}=2$
- At $x=-1$: $f''(-1)=-2<0\Longrightarrow$ local maximum at $x=-1$.

Local maximum value $=f(-1)=-1+\dfrac{1}{-1}=-2$

Therefore, the local maximum value is -2 .

47. (a) Rewrite the integrand:
$$\int_0^{2a}\frac{1}{1+(2x)^2}dx=\frac{\pi}{6}$$
Let $u=2x$, then $dx=\frac{du}{2}$.
When $x=0, u=0$. When $x=2a, u=4a$.
$$\frac{1}{2}\int_0^{4a}\frac{1}{1+u^2}du=\frac{\pi}{6}$$
$$\frac{1}{2}[\tan^{-1}(u)]_0^{4a}=\frac{\pi}{6}$$
$$\tan^{-1}(4a)-\tan^{-1}(0)=\frac{\pi}{3}$$
$$\tan^{-1}(4a)=\frac{\pi}{3}$$
$$4a=\tan\left(\frac{\pi}{3}\right)=\sqrt{3}$$
$$a=\frac{\sqrt{3}}{4}$$

48. (d) The intersection points of $y=x^2$ and $y=16$ are $x=\pm 4$.
- Integrating with respect to x :

Area $=\displaystyle\int_{-4}^{4}(16-x^2)dx=2\int_0^4(16-x^2)dx$

(None of the given options match this form).
- Integrating with respect to y :

The curve is symmetric about the y -axis, and x = $\sqrt{y}$ for the right half.

Area $=\displaystyle\int_0^{16}2\sqrt{y}dy=2\int_0^{16}\sqrt{y}dy$

49. (c) Separate variables:
$$\frac{1}{\sqrt{y}}dy=\frac{1}{\sqrt{x}}dx$$
$$y^{-1/2}dy=x^{-1/2}dx$$
Integrate both sides:
$$\int y^{-1/2}dy=\int x^{-1/2}dx$$

$2\sqrt{y}=2\sqrt{x}+C_1$
Divide by 2:
$\sqrt{y}=\sqrt{x}+C \Longrightarrow \sqrt{y}-\sqrt{x}=C$

50. (b) Write the linear differential equation in standard form $\frac{dy}{dx}+P(x)y=Q(x)$:

$\frac{dy}{dx}-\frac{1}{2x}y=\frac{3}{2x}$

Here, $P(x)=-\frac{1}{2x}$.

Find the Integrating Factor (I.F.):

L.F. $=e^{\int P(x)dx}=e^{\int -\frac{1}{2x}dx}$

L.F. $=e^{-\frac{1}{2}\ln|x|}=e^{\ln(x^{-1/2})}=x^{-1/2}=\frac{1}{\sqrt{x}}$

51. (c) $-2\leq\lambda\leq 1$

$|\lambda\vec{a}|=|\lambda||\vec{a}|$
$=5|\lambda|$
Greatest value $=5(2)=10$
Smallest value $=5(0)=0$
$=10+0$
$=10$

52. (c) $\vec{v}=\sqrt{a}^2+a^2+0^2=3$

$\sqrt{2}a^2=3$
$a\sqrt{2}=3$
$\Rightarrow \mathrm{a}=\pm\frac{3}{\sqrt{2}}$
$\vec{v}=\pm\frac{3}{\sqrt{2}}\hat{\imath}\pm\frac{3}{\sqrt{2}}\hat{\jmath}$
$\vec{v}=\pm\frac{3}{\sqrt{2}}(\hat{\imath}+\hat{\jmath})$

53. (b) $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

$\frac{2x-1}{4}=\frac{1-y}{3}=\frac{-z}{6}$

$\frac{x-\frac{1}{2}}{2}=\frac{y-1}{-3}=\frac{z-0}{-6}$

Direction ratio $=(a,b,c)=(2,-3,-6)$
$\sqrt{a^2+b^2+c^2}=\sqrt{2^2+(-3)^2+(-6)^2}$
$=\sqrt{4+9+36}$
$=\sqrt{49}$
$=7$
Direction cosines $=\frac{2}{7},\frac{-3}{7},\frac{-6}{7}$

54. (b) In a linear programming problem, the linear function which has to be maximized or minimized is called the objective function. This function represents the goal of the optimization, such as maximizing profit or minimizing cost, subject to a set of constraints.

55. (c) The two sloping lines are $x+y=5$ (intercepts $(0,5),(5,0)$) and $x+3y=9$ (intercepts $(0,3),(9,0)$); they meet at (3,2). The shaded strip is the region above $x+y=5$ and below $x+3y=9$ (together with $x,y\geq 0$), so the non-trivial constraints are $x+y\geq 5$ and $x+3y\leq 9$.

56. (d) $\mathrm{P}(\mathrm{A}'/\mathrm{B}')=\frac{P(A'\cap B')}{P(B')}$

$\frac{P(A\cup B)'}{P(B')}=\frac{1-P(A\cup B)}{P(B')}$

57. (a) $|\vec{a}\times\vec{b}|=|\vec{a}||\vec{b}|\sin\theta$

$|\vec{a}\times\vec{b}|^2=|\vec{a}|^2|\vec{b}|^2\sin^2\theta$
$|\vec{a}\cdot\vec{b}|=|\vec{a}||\vec{b}|\cos\theta$
$|\vec{a}\times\vec{b}|^2=|\vec{a}|^2|\vec{b}|^2\cos^2\theta$
$|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2\sin^2\theta+|\vec{a}|^2|\vec{b}|^2\cos^2\theta$
$=|\vec{a}|^2|\vec{b}|^2\sin^2+\cos^2\theta$
Since $\sin^2\theta+\cos^2\theta=1$, we get:
$|\vec{a}\times\vec{b}|^2+(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2$
$\frac{|\vec{a}\times\vec{b}|}{\vec{a}\cdot\vec{b}}=\frac{|\vec{a}||\vec{b}|\sin\theta}{|\vec{a}||\vec{b}|\cos\theta}$
$=\tan\theta$
$|\vec{a}\times\vec{b}|=(\vec{a}\cdot\vec{b})\tan\theta$
So, Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

58. (d) • Assertion (A) is False: Direction cosines $1,m,n$ must satisfy the condition $1^2+m^2+n^2=1$. For the values (1,1,1), the sum of squares is $1^2+1^2+1^2=3\neq 1$. Therefore, (1,1,1) cannot be the direction cosines of any line.
• Reason (R) is True: The statement $\cos\theta=1$ is true when $\theta=0$, which is a valid trigonometric property.

59. (A) $y=f(x)$
$y=\frac{x-2}{x-3}$
$y(x-3)=(x-2)$
$xy-3y=x-2$
$xy-x=3y-2$
$x(y-1)=3y-2$
$x=\frac{3y-2}{y-1}$
$y\neq 1$
Range $R-\{1\}$

OR

(B) Let $f(x_1,y_1)=f(x_2,y_2)$
$(2y_1,3x_1)=(2y_2,3x_2)$
$2y_1=2y_2$
$y_1=y_2$
$3x_1=3x_2$
$x_1=x_2$
∴ The function is injective.

60. $x=a\sin^3 t$
$\frac{dx}{dt}=a\cdot 3\sin^2 t\cdot\cos t$
$y=b\cos^3 t$
$\frac{dy}{dt}=b\cdot 3\cos^2 t(-\sin t)$
$\frac{dy}{dx}=\frac{3b\cos^2 t(-\sin t)}{3a\sin^2 t\cdot\cos t}$
$=\frac{-b}{a}\cos t$
$=\frac{-b}{a}\cos\left(\frac{\pi}{4}\right)$
$=\frac{-b}{a}(1)$
$=\frac{-b}{a}$

61. (A) $f(x)=\cos x+\sin^2 x$
$f(x)=-\sin x+2\sin x\cdot\cos x$

$f(x) = 0$
$-\sin x + 2\sin x \cdot \cos x = 0$
$\sin x(2\cos x - 1) = 0$
$\sin x = 0$ or $2\cos x = +1$
$x = 0$ or $\cos x = \dfrac{1}{2}$
$x = \pi$ or $x = \dfrac{\pi}{3}$
$f(x) = \cos x + \sin^2 x$
$f(0) = \cos 0 + \sin^2 0$
$= 1 + 0$
$= 1$
$f\left(\dfrac{\pi}{3}\right) = \dfrac{\cos\pi}{3} + \dfrac{\sin^2\pi}{3}$
$= \dfrac{1}{2} + \left(\dfrac{\sqrt{3}}{2}\right)^2$
$= \dfrac{1}{2} + \dfrac{3}{4}$
$= \dfrac{5}{4}$
$f(\pi) = \cos\pi + \sin^2\pi$
$= -1 + (0)^2$
$= 1$
$\therefore$ The absolute maximum value is $\frac{5}{4}$.

OR

(B) $V = \frac{2}{3}\pi r^3$
$S = 3\pi r^2$
$\dfrac{dV}{dt} = k$
$\dfrac{dV}{dt} = \dfrac{d}{dt}\left(\dfrac{2}{3}\pi r^3\right)$
$k = \dfrac{2}{3}\pi\left(3r^2\dfrac{dr}{dt}\right)$
$k = 2\pi r^2\dfrac{dr}{dt}$
$\dfrac{dr}{dt} = \dfrac{k}{2\pi r^2}$
Differentiate Surface Area with Respect to Time (t):
$\dfrac{dS}{dt} = \dfrac{d}{dt}(3\pi r^2)$
$\dfrac{dS}{dt} = 6\pi r\dfrac{dr}{dt}$
Substitute $\frac{dr}{dt}$ into the surface area equation:
Substitute $\frac{dr}{dt}\frac{k}{2\pi r^2}$ into $\frac{dS}{dt} = 6\pi r\left(\frac{k}{2\pi r^2}\right)$
$\dfrac{dS}{dt} = \dfrac{3k}{r}$
$\dfrac{dS}{dt} \propto \dfrac{1}{r}$

62. $\vec{a} = 0$
$\vec{b} = \overrightarrow{AB}$
$\vec{c} = \overrightarrow{AC}$
$\vec{d} = \dfrac{\vec{b} + \vec{c}}{2}$

$= \frac{\hat{j}+\hat{k}+3\hat{\imath}-\hat{j}+4\hat{k}}{2}$

$= \frac{3\hat{\imath}}{2}+\frac{5\hat{k}}{2}$

$\vec{A}D = \vec{d}-\vec{a}$

$\frac{3}{2}\hat{\imath}+\frac{5}{2}\hat{k}$

$|\vec{A}D| = \sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{5}{2}\right)^2}$

$= \sqrt{\frac{9}{4}+\frac{25}{4}}$

$= \sqrt{\frac{34}{4}}$

$= \frac{\sqrt{34}}{2}$

63. $\vec{r} = -\vec{j}+3\vec{k}+\lambda(2\hat{\imath}-2\hat{j}+\hat{k})$

$\vec{r} = (2\lambda)\hat{\imath}+(-1-2\lambda)\hat{j}+(3+\lambda)\hat{k}$

$x = 2\lambda, y = -1, z = 3+\lambda$

$x+y+z = 3$

$(2\lambda)+(-1+-2\lambda)+(3+\lambda) = 3$

$(2\lambda-2\lambda+\lambda)+(-1+3) = 3$

$\lambda+2 = 3$

$\lambda = 1$

$x = 2(1) = 2$

$y = -1-2(1) = -3$

$z = 3+(1) = 4$

∴ The point is $(2,-3,4)$.

64. Let $I = \int \frac{x+2}{\sqrt{9x-x^2}}dx$

$I = \int \frac{x+2}{\sqrt{-x^2-9x}}dx$

$= \int \frac{x+2}{\sqrt{-\left[x^2-9x+\left(\frac{9}{2}\right)^2-\left(\frac{9}{2}\right)^2\right]}}dx$

$= \int \frac{x+2}{\sqrt{-\left[\left(x-\frac{9}{2}\right)^2-\frac{81}{4}\right]}}dx$

$= \int \frac{x+2}{\sqrt{\frac{81}{4}-\left(x-\frac{9}{2}\right)^2}}dx$

Let $t = x-\frac{9}{2}$

$x = t+\frac{9}{2}$

Add 2 to both sides,

$x+2 = t+\frac{9}{2}+2$

$\Rightarrow x+2 = t+\frac{13}{2}$

Diff. both sides w.r. ' x '

$\Rightarrow$ dx = dt

$$I = \int \frac{\left(t + \frac{13}{2}\right)}{\sqrt{\frac{81}{4} - t^2}} dt$$

$$= \int \frac{t}{\sqrt{\frac{81}{4} - t^2}} dt + \frac{13}{2} \int \frac{dt}{\sqrt{81}} /4 - t^2$$

$$I_1 = \int \frac{t}{\sqrt{\frac{81}{4} - t^2}} dt$$

Let $\frac{81}{4} - t^2 = u$

-2tdt = du

$\Rightarrow$ tdt $= \frac{-du}{2}$

$$= \frac{-1}{2} \int \frac{du}{\sqrt{4}}$$

$$= \frac{-1}{2} \int 4^{\frac{-1}{2}} du$$

$$= \frac{-1}{2} \frac{4^{\frac{-1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= -\sqrt{4} + C$$

$$= -\sqrt{\frac{81}{4} - t^2 + C}$$

Substitute t $= x - \frac{9}{2}$

$$= -\sqrt{\frac{81}{4} - \left(x - \frac{9}{2}\right)^2 + C_1}$$

$$= -\sqrt{\frac{81}{4} - x^2 - \frac{81}{4} + 2 \cdot x \cdot \frac{9}{2} + C_1}$$

$$= -\sqrt{-x^2 + 9x} + C_1$$

$I_1 = -\sqrt{9x - x^2} + C_1$ ….(i)

$$I_2 = \frac{13}{2} \int \frac{dt}{\sqrt{\frac{81}{4} - t^2}}$$

$$= \frac{13}{2} \int \frac{dt}{\sqrt{\left(\frac{9}{2}\right)^2 - t^2}}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$$

$$= \frac{13}{2} \sin^{-1}\frac{t}{\frac{9}{2}} + C_2$$

Substitute, t $= x - \frac{9}{2}$

$$= \frac{13}{2} \sin^{-1}\frac{\left(x - \frac{9}{2}\right)}{\frac{9}{2}} + C_2$$

$$\frac{13}{2}\sin^{-1}\frac{\left(\frac{2x-9}{2}\right)}{\frac{9}{2}}+C_2$$

$\mathrm{I}_2=\frac{13}{2}\sin^{-1}\left(\frac{2x-9}{9}\right)+C_2$ ....(ii)

Now by adding (i) and (ii)

$\mathrm{I}=\mathrm{I}_1+\mathrm{I}_2$

$=-\sqrt{9x-x^2}+\frac{13}{2}\sin^{-1}\left(\frac{2x-9}{9}\right)+C\ ...[\because \mathrm{C}=\mathrm{C}_1+\mathrm{C}_2]$

65. **(A)** Step 1: Simplify the integrand

Express $\cot x$ in terms of $\sin x$ and $\cos x$:

$I=\int_{\pi/12}^{5\pi/12}\frac{dx}{1+\frac{\sqrt{\cos x}}{\sqrt{\sin x}}}=\int_{\pi/12}^{5\pi/12}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ...(i)

Step 2: Apply the integration property

Use the property $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$.

Here, $a+b=\frac{x}{12}+\frac{5\pi}{12}=\frac{6\pi}{12}=\frac{\pi}{2}$.

Therefore, replace x with $\left(\frac{\pi}{2}-x\right)$:

$$I=\int_{x/12}^{5x/12}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx$$

Since $\sin\left(\frac{\pi}{2}-x\right)=\cos x$ and $\cos\left(\frac{\pi}{2}-x\right)=\sin x$:

$I=\int_{x/12}^{5x/12}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$ ....(ii)

Step 3: Add Equation 1 and Equation 2

$$2I=\int_{\pi/12}^{5\pi/12}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$2I=\int_{\pi/12}^{5\pi/12}1\cdot dx=[x]_{\pi/12}^{5\pi/12}$$

$$2I=\frac{5\pi}{12}-\frac{\pi}{12}=\frac{4\pi}{12}=\frac{\pi}{3}$$

$$I=\frac{\pi}{6}$$

OR

(B) Step 1: Split the integral based on absolute value definition

Since $|x|=-x$ for $x<0$ and $|x|=x$ for $x\geq 0$:

$$I=\int_{-\pi/6}^{0}(\sin(-x)+\cos(-x))dx+\int_0^{\pi}(\sin x+\cos x)dx$$

Since $\sin(-x)=-\sin x$ and $\cos(-x)=\cos x$:

$$I=\int_{-\pi/6}^{0}(-\sin x+\cos x)dx+\int_0^{\pi}(\sin x+\cos x)dx$$

Step 2: Integrate each part

• First Integral:

$$\int_{-\pi/6}^{0}(-\sin x+\cos x)dx=[\cos x+\sin x]_{-\pi/6}^{0}$$

$$=(\cos 0+\sin 0)-\left(\cos\left(-\frac{\pi}{6}\right)+\sin\left(-\frac{\pi}{6}\right)\right)$$

$$=(1+0)-\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)=1-\frac{\sqrt{3}}{2}+\frac{1}{2}=\frac{3-\sqrt{3}}{2}$$

• Second Integral:

$$\int_0^{\pi} (\sin x + \cos x)dx = [-\cos x + \sin x]_0^{\pi}$$
$= (-\cos\pi + \sin\pi) - (-\cos 0 + \sin 0)$
$= (-(-1) + 0) - (-1 + 0) = 1 + 1 = 2$
Step 3: Combine results
$$I = \frac{3-\sqrt{3}}{2} + 2 = \frac{3-\sqrt{3}+4}{2} = \frac{7-\sqrt{3}}{2}$$

66. Step 1: Integrate to find $F(x)$
$$F(x) = \int \frac{1}{e^x + 1} dx$$
Multiply the numerator and denominator by e^{-x} :
$$F(x) = \int \frac{e^{-x}}{1 + e^{-x}} dx$$
Step 2: Use substitution
Let $u = 1 + e^{-x} \Rightarrow du = -e^{-x}dx \Rightarrow e^{-x}dx = -du$:
$$F(x) = \int \frac{-du}{u} = -\log|u| + C = -\log(1 + e^{-x}) + C$$
Step 3: Use the initial condition to find C
Given $F(0) = \log\frac{1}{2} = -\log 2$:
$-\log(1 + e^0) + C = -\log 2$
$-\log(1 + 1) + C = -\log 2 \Rightarrow -\log 2 + C = -\log 2 \Rightarrow C = 0$
Step 4: Write the final function
$$F(x) = -\log(1 + e^{-x}) = \log\left(\frac{1}{1+e^{-x}}\right) = \log\left(\frac{e^x}{e^x + 1}\right)$$

67. **(A)** Step 1: Substitute $y = vx$
This is a homogeneous differential equation. Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$.
Substitute these into the equation:
$$x\left(v + x\frac{dv}{dx}\right) = vx - x\sin^2(v)$$
Divide the entire equation by x :
$$v + x\frac{dv}{dx} = v - \sin^2(v)$$
$$x\frac{dv}{dx} = -\sin^2(v)$$
Step 2: Separate variables and integrate
$$\frac{dv}{\sin^2(v)} = -\frac{dx}{x} \Rightarrow \csc^2(v)dv = -\frac{dx}{x}$$
$$\int \csc^2(v)dv = -\int \frac{dx}{x}$$
$$-\cot(v) = -\log|x| + C \Rightarrow \cot\left(\frac{y}{x}\right) = \log|x| + C'$$
Step 3: Apply initial condition
Given $y = \frac{\pi}{6}$ when $x = 1$:
$$\cot\left(\frac{\pi/6}{1}\right) = \log(1) + C' \Rightarrow \sqrt{3} = 0 + C' \Rightarrow C' = \sqrt{3}$$
Step 4: Final particular solution
$$\cot\left(\frac{y}{x}\right) = \log|x| + \sqrt{3}$$

OR

(B) Step 1: Put it in standard linear form
Divide the entire equation by ylogy:
$$\frac{dx}{dy} + \frac{1}{y\log y}x = \frac{2}{y^2\log y}$$

This is a linear differential equation of the form $\frac{dx}{dy}+Px=Q$, where:

$$P=\frac{1}{y\log y}, Q=\frac{2}{y^2\log y}$$

Step 2: Find the Integrating Factor (I.F.)

$$\text{I.F.} = e^{\int P dy} = e^{\int \frac{1}{y\log y} dy}$$

Let $u=\log y \Longrightarrow du=\frac{1}{y}dy$.

$$\int \frac{1}{y\log y}dy=\int \frac{1}{u}du=\log|u|=\log(\log y)$$

$$\text{I.F.} = e^{\log(\log y)}=\log y$$

Step 3: Solve the equation

$$x\cdot(\text{I.F.})=\int Q\cdot(\text{I.F.})dy+C$$

$$x\log y=\int\left(\frac{2}{y^2\log y}\right)\cdot\log y dy+C$$

$$x\log y=\int\frac{2}{y^2}dy+C=2\int y^{-2}dy+C$$

$$x\log y=-\frac{2}{y}+C$$

68. Maximize $Z=10500x+9000y$

Subject to: $x+y\leq 50, 2x+y\leq 80, x, y\geq 0$

Step 1: Find boundary line intersection points

(1) Line 1: $x+y=50$

- If $x=0 \Longrightarrow y=50 \rightarrow (0,50)$
- If $y=0 \Longrightarrow x=50 \rightarrow (50,0)$

(2) Line 2: $2x+y=80$

- If $x=0 \Longrightarrow y=80 \rightarrow (0,80)$
- If $y=0 \Longrightarrow x=40 \rightarrow (40,0)$
- Intersection point of both lines:

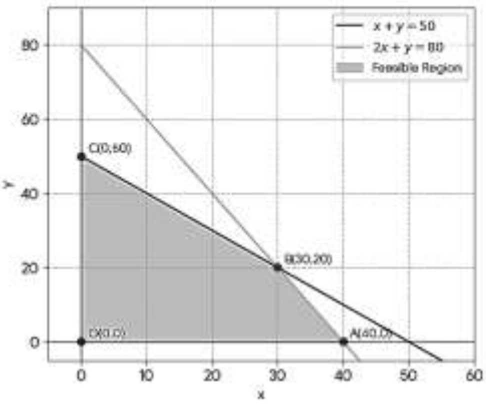
Subtract Line 1 from Line 2: $(2x+y)-(x+y)=80-50 \Longrightarrow x=30$

Substitute $x=30$ into Line 1: $30+y=50 \Longrightarrow y=20 \rightarrow (30,20)$

Step 2: Identify the Feasible Region

Since the inequalities are $\leq$, the shading goes towards the origin (0,0). The corner points of the bounded feasible region are:

- $O(0,0)$
- $A(40,0)$
- $B(30,20)$
- $C(0,50)$



Step 3: Evaluate objective function $Z=10500x+9000y$ at corner points

Corner Point	Value of $Z=10500x+9000y$

$O(0,0)$	$Z=0$
$A(40,0)$	$Z = 10500(40) + 0 = 420,000$
B(30,20)	$Z = 10500(30) + 9000(20) = 315,000 + 180,000 = 495,000$
$\alpha(0,50)$	$Z = 0 + 9000(50) = 450,000$

69. (A) (i) Let P(H) be the probability of hitting the target.

P(M) be the probability of missing the target.

$\mathrm{P(H)} + \mathrm{P(M)} = 1$
$\mathrm{P(H)} = 3\mathrm{P(M)}$
$\mathrm{P(H)} = 3(1 - \mathrm{P(H)})$
$\mathrm{P(H)} = 3 - 3\mathrm{P(H)}$
$4\mathrm{P(H)} = 3$
$\mathrm{P(H)} = \frac{3}{4}$

(ii) If both shot hit $= \mathrm{P(H)} \cdot \mathrm{P(H)}$

$= \frac{3}{4} \cdot \frac{3}{4}$
$= \frac{9}{16}$

$P($ at least one shot missing out $) = 1 - \frac{9}{16}$

$= \frac{7}{16}$

OR

(B) Let m, f and s denote respectively the mother, father and the son, then sample space S is S = {mfs, msf, fms, fsm, smf, sfm}.

E: son on one end

F: father in the middle

i.e., E = {mfs, fms, sfm, smf } and F = {sfm, mfs)

⇒ E ∩ F = {mfs, sfm)

$P(E) = \frac{4}{6} = \frac{2}{3}$

$P(F) = \frac{2}{6} = \frac{1}{3}$

$P(E \cap F) = \frac{2}{6} = \frac{1}{3}$

$\therefore P(E \mid F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{\frac{1}{3}}{\frac{1}{3}}$

$= 1$

70. (A) QP $= \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -1 \\ -4 & 2 & -1 \\ 2 & -1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$= 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= 61$

By the system of equations:

$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & 2 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$AX = B$

$X = A^{-1} B$

$QA = 6I$

Post multipication of A^{-1} both sides,

$QAA^{-1} = 6IA^{-1}$ ... $[\because AA^{-1} = I]$

$Q = 6A^{-1}$

$A^{-1} = \frac{1}{6}Q$

$X = A^{-1}B$

$= \frac{1}{6}QB$

$A^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -1 \\ -4 & 2 & -1 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$

$X = \frac{1}{6}\begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-3+35 \end{bmatrix}$

$X = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

Hence, $x = 2, y = -1, z = 4$

OR

(B) $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$

By multiply and divide in R_1 by x, R_2 by y and R_3 by z, we get

$\Delta xyz = \begin{vmatrix} \frac{1}{x}+1 & \frac{1}{x} & \frac{1}{x} \\ \frac{1}{y} & \frac{1}{y}+1 & \frac{1}{y} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}+1 \end{vmatrix}$

$R_1 \rightarrow R_1 + R_2 + R_3$

$\Delta = xyz\begin{vmatrix} 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} & 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{x} & 1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z} \\ \frac{1}{y} & \frac{1}{y}+1 & \frac{1}{y} \\ \frac{1}{2} & \frac{1}{z} & \frac{1}{z}+1 \end{vmatrix}$

From R_1 take $\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$,

$$\Delta = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{y} & \frac{1}{y}+1 & \frac{1}{y} \\ \frac{1}{z} & \frac{1}{z} & \frac{1}{z}+1 \end{vmatrix}$$

$C_2 \to C_2 - C_1; C_3 \to C_3 - C_1$

$$\Delta = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{y} & 1 & 0 \\ \frac{1}{z} & 0 & 1 \end{vmatrix}$$

$\Delta =$ Expanding along R_1

$$\Delta = xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$$

Now, $\Delta = 0$

$$\Rightarrow xyz\left(1+\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right) = 0$$

$$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z} = -1$$

$$\Rightarrow x^{-1}+y^{-1}+z^{-1} = -1$$

71. **(A)** Given,

$f(x) = \cot^{-1}(\sin x + \cos x)$

Differential both sides w.r.t ' x ', we get

$$f'(x) = \frac{-1}{1+(\sin x+\cos x)^2}\cdot(\cos x - \sin x)$$

$$f'(x) = \frac{\sin x - \cos x}{1+(\sin x+\cos x)^2}$$

$$f'(x) = \frac{\sin x - \cos x}{2+\sin 2x}$$

If $f(x)$ is increasing, $f(x) => 0$

$\sin x - \cos x > 0$

$\Rightarrow \sin x > \cos x$

If f(x) is decreasing, $f(x) < 0$

$\sin x - \cos x < 0$

$\Rightarrow \sin x < \cos x$

If $f'(x) = 0, \sin x = \cos x; x = \frac{\pi}{4}$

In interval $\left(0,\frac{\pi}{4}\right)$; $\cos x > \sin x$;

Hence, $f(x)$ is decreasing in $\left(0,\frac{\pi}{4}\right)$.

In interval $\left(\frac{\pi}{4},\pi\right)$; $\sin x > \cos x$;

Hence, $f(x)$ is increasing in $\left(\frac{\pi}{4},\pi\right)$.

OR

(B) Let 1, w and p be the length and width and perimeter of the rectangle, respectively. and V be the volume of the cylinder.

Now, $P = 2(1+w) = 36$

$\Rightarrow 1 + w = 18$

Volume of the cylinder $= V = \pi r^2 h$ ___ $\{h = w; r = 1\}$

$V = \pi r^2 w$

$\Rightarrow V = \pi r^2(18-1)$

$\Rightarrow V = 18\pi r^2 - \pi r^3$

For maximum volume differential volume w.r.t ' 1 ' both sides,

$$\frac{dv}{dl} = \frac{d}{dl}(18\pi r^2 - \pi r^3)$$

$\frac{dv}{dl} = 36\pi l - 3\pi l^2$
$= 3\pi 1(12 - 1)$
For maximum volume; $\frac{dv}{dl} = 0$
$\Rightarrow 3\pi l(12 - 1) = 0$
$l = 0,12$
again differential both sides w.r.t ' 1 '
$\frac{d^2v}{dl^2} = 36\pi - 6\pi l$
At $1 = 12$,
$\frac{d^2v}{dl^2} = 36\pi - 72\pi$
$= -36\pi < 0$
Hence, volume of cylinder is max. at $1 = 12$ cm and width $\mathrm{w} = 18 - 12 = 6$ cm

72. Given,
$g(x) = \cos^{-1}(x^2 - 1)$
$\cos^{-1}: [-11] \rightarrow [0, \pi]$
$-1 \leq x^2 - 1 \leq 1$
adding ' 1 ' both sides,
$0 \leq \mathrm{x}^2 \leq 2$
Hence, $x^2 \geq 2 \forall x \in R$
The domain $-\sqrt{2} \leq x \leq \sqrt{2}$
$\{x \in R \mid -\sqrt{2} \leq x \leq \sqrt{2}\}$
Now, $\mathrm{g(x)} = \frac{\pi}{3}$
$\Rightarrow \cos^1(x^2 - 1) = \frac{\pi}{3}$
$\Rightarrow (x^2 - 1) = \frac{\cos\pi}{3}$
$\Rightarrow (x^2 - 1) = \frac{1}{2}$
$\Rightarrow x^2 = 1 + \frac{1}{2}$
$\Rightarrow x^2 = \frac{3}{2}$
$\Rightarrow \mathrm{x} = \pm\sqrt{\frac{3}{2}}$
$\Rightarrow \mathrm{x} = \pm\frac{\sqrt{6}}{2}$
$\sqrt{2} \approx 1.414; \frac{\sqrt{6}}{2} \approx \frac{2.449}{2}$
$\Rightarrow \approx 1.225$
Range of $\cos^{-1}x$ other than principal value branch
$\{[n\pi, (n + 1)\pi]; n \in 2\}$

73. Equation of a line passing through two points A and B .
$\vec{r}_1 = \vec{a} + t(\vec{b} - \vec{a})$
$= \vec{\imath} + 2\vec{\jmath} + 3\vec{k} + t[(5\hat{\imath} + 8\hat{\jmath} + 11\hat{k}) - (\hat{l} + 2\hat{\jmath} + 3\hat{k})]$
$\vec{r}_1 = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} + t(4\hat{\imath} + 6\hat{\jmath} + 8\hat{k})$
$\vec{r}_2 = 4\hat{\imath} + \hat{\jmath} + \lambda(5\hat{\imath} + 2\hat{\jmath} + \hat{k})$ ... (Given)
At a point of intersection co-ordinates of both the lines are equal.
$1 + 4t = 4 + 5\lambda$
$\Rightarrow 4t - 5\lambda = 3 \qquad \ldots(\mathrm{i})$
$2 + 6t = 1 + 2\lambda$
$\Rightarrow 6t - 2\lambda = -1 \qquad \ldots(\mathrm{ii})$

$3 + 8t = \lambda$ ....(iii)

Put (iii) in equation (ii)

$\Rightarrow 6t - 2(3 + 8t) = -1$

$\Rightarrow 6t - 6 - 16t = -1$

$\Rightarrow -10t = 5$

$\Rightarrow t = \frac{-1}{2}$

Put $t = \frac{-1}{2}$ in equation (iii)

$\Rightarrow 3 + 3\left(\frac{-1}{2}\right) = \lambda$

$\Rightarrow \lambda = -1$

Coordinates of the point of intersection:

$\vec{r}_1 = \hat{\imath} + 2\hat{\jmath} + 3\hat{k} - \frac{1}{2}(4\hat{\imath} + 6\hat{\jmath} + 8\hat{k})$

$= \hat{\imath} + 2\hat{\jmath} + 3\hat{k} - 2\hat{\imath} - 3\hat{\jmath} - 4\hat{k}$

$\vec{r}_1 = -\hat{\imath} - \hat{\jmath} - \hat{k}$

Coordinates are $(-1, -1, -1)$

Direction of given lines are;

$\vec{d}_1 = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$

$\vec{d}_2 = 5\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & 6 & 8 \\ 5 & 2 & 1 \end{vmatrix}$

$= (6 - 16)\hat{\imath} - (4 - 40)\hat{\jmath} + (8 - 30)\hat{k}$

$= -10\hat{\imath} + 36\hat{\jmath} - 22\hat{k}$

Hence, equation of line passing through point of intersect

$\vec{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(-10\hat{\imath} + 36\hat{\jmath} - 22\hat{k})$

$\vec{r} = \vec{a} + \mu\vec{b}$

μ in any scalar.

74. **(i)** Given, Total no. of people be 50

no. of males $= 30$

no. of females $= 20$

$P(F) = \frac{\text{No. of favourable outcome}}{\text{No. of toal outcome}}$

$\frac{20}{50}$

$= 0.4$

(ii) Let C be the event that person suffers from lung complications.

C^1 be the event that person not suffers from lung complications.

$\mathrm{P(c/m)} = \frac{170}{1000} = 0.17$

$P\left(\frac{C^1}{m}\right) = 1 - P\left(\frac{C}{m}\right)$

$= 1 - 0.17$

$= 0.83$

(iii) (A) $\mathrm{P(F)} = 0.4; \mathrm{P(m)} = \frac{30}{50} = 0.6$

$P\left(\frac{C}{F}\right) = \frac{120}{1000} = 0.120$

$P\left(\frac{C^1}{F}\right) = 1 - 0.120$

$= 0.88$

By using total probability,

$P(C) = P(M) \cdot P\left(\frac{C}{M}\right) + P(F) \cdot P\left(\frac{C}{F}\right)$
$= 0.6 \times 0.17 + 0.4 \times 0.120$
$= 0.102 + 0.048$
$= 0.15$

$P\left(\frac{F}{C}\right) = P(F) \cdot \frac{P\left(\frac{C}{F}\right)}{P(C)}$
$= \frac{0.4 \times 0.120}{0.15}$
$= \frac{0.048}{0.15}$
$= 0.32$

OR

(B) $P(C^1) = P(M) \cdot P\left(\frac{M}{C^1}\right) + P(F) \cdot P\left(\frac{F}{C^1}\right)$ ... { By using total probability }
$= 0.6 \times 0.83 + 0.4$
$= 0.83$
$P\left(\frac{M}{C^1}\right) = \frac{0.6 \times 0.83}{0.85}$
$= \frac{498}{850}$
$= 0.585$

75. **(i)** Given, $9x^2 + 16y^2 = 144$
Divide both sides by ' 144 '
$\frac{x^2}{16} + \frac{y^2}{9} = 1$
$\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$
$\Rightarrow \mathrm{y} = \pm\frac{3}{4}\sqrt{16 - x^2}$

(ii) $\int\ ydx = \frac{3}{4}\int\ \sqrt{16} - x^2 dx$
$\int\ \sqrt{a^2 - x^2}dx$
$= \frac{x}{2}\sqrt{a}^2 - x^2 + \frac{a^2}{2}\frac{\sin^{-1}x}{a} + C$
$= \frac{3}{4}\left[\frac{x}{2}\sqrt{16 - x^2} + 8\sin^{-1}\frac{x}{4}\right] + C$
$\int\ ydx = \frac{3x}{8}\sqrt{16} - x^2 + 6\sin^{-1}\frac{x}{4} + C$

(iii) **(A)** $\mathrm{A} = 4\int_0^4\ y \cdot dx = 4\frac{\int_0^{43}}{}\sqrt{16 - x^2}dx$
$= 3\left[\frac{x}{2}\sqrt{16 - x^2} + 8\sin^{-1}\frac{x}{4}\right]_0^4$
$= 3\left[\left(8 \cdot \frac{\pi}{2}\right) - (0)\right]$
$= 3 \times 4\pi$
$= 12$ sq.units

OR

(B) Coordinates of P and Q are (7,0) and (0,6) respectively.
$\frac{x}{7} + \frac{y}{6} = 1$
$\Rightarrow \frac{y}{6} = 1 - \frac{x}{7}$
$\Rightarrow \mathrm{y} = 6 - \frac{6x}{7}$

Area $= \int_0^7 \left(6 - \frac{6x}{7}\right) dx$

$A = \left[6x - \frac{6x^2}{14}\right]_0^7$

$= \left[6x - 3\frac{x^2}{7}\right]_0^7$

$= [(42 - 21) - (0)]$

$= 21$ sq.units

76. Given,

$f(x) = \begin{cases} (x^4 - 4x^2 + 4), & 0 \le x < 3 \\ x^2 + 40, & x \ge 3 \end{cases}$

(i) $f(x) = x^4 - 4x^2 + 4$ for $0 \le x < 3$

Differential both sides w.r.t. ' x '

$f(x) = 4x^3 - 8x$

(ii) $f(x) = x^2 + 40, x \ge 3$

Differential both sides w.r.t. ' x '

$f(x) = 2x$

$f(x) = 2 \times 4$

$f(x) = 8$

(iii) (A) LHL =RHL = f(3)

$f(3) = 32 + 40 = 49$

LHL $= \lim_{x \to 3} f(x)$

$= \lim_{h \to 0} f(3 - h)$

$= \lim_{h \to 0} (3 - h)^4 - 4(3 - h)^2 + 4$

$= \lim_{h \to 0} (3 - h)^2(3 - h)^2 - 4(3 - h)^2 + 4$

$= 81 - 36 + 4$

$= 49$

RHL $= \lim_{x \to 3^+} f(x)$

$= \lim_{h \to 0} f(3 + h)$

$= \lim_{h \to 0} (3 + h)^2 + 40$

$= 9 + 40$

$= 49$

Hence, LHL = RHL = f(3) = 49

So, f(x) is continuous at x = 3

OR

(B) $f(x) = \begin{cases} (x^4 - 4x^2 + 4), & 0 \le x < 3 \\ x^2 + 40, & x \ge 3 \end{cases}$

$f'(x) = \begin{cases} (4x^3 - 8x), & 0 \le x < 3 \\ 2x, & x > 3 \end{cases}$

For probability,

$x < 3; \lim_{x \to 3} f'(x) = \lim_{x \to 3} 4(3) - 8(3)$

$= 108 - 24$

$= 84$

$x > 3; \lim_{x \to 3^+} f'(x) = \lim_{x \to 3^+}$

$= 2 \times 3$

$= 6$

So, Lf(x) = Rf(x)

Hence, f(x) is not diff. at x = 3

77. **(d)** • Domain of $f(x) = \cos^{-1}(2x - 5)$

• Condition: The domain of $\cos^{-1}(\theta)$ is $-1 \le \theta \le 1$.

• Inequality: $-1 \leq 2x - 5 \leq 1$
Step 1: Add 5 to all parts: $4 \leq 2x \leq 6$
Step 2: Divide by 2: $2 \leq x \leq 3$
Result: Domain is [2,3].

78. (a) • Value of $(x + y)$ if $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$
• Equation: $A^2 = 4A + 3I$
Step 1: Multiply both sides by A^{-1}.
$A = 4I + 3A^{-1}$
Step 2: Rearrange to isolate A^{-1} :
$3A^{-1} = A - 4I \Rightarrow A^{-1} = \frac{1}{3}A - \frac{4}{3}I$
Step 3: Compare coefficients: $x = \frac{1}{3}$ and $y = -\frac{4}{3}$
• Calculation: $x + y = \frac{1}{3} - \frac{4}{3} = -1$

79. (a) Nature of $AB' + BA'$ if A and B are skew-symmetric
• Given: $A' = -A$ and $B' = -B$
Let: $C = AB' + BA'$
Transpose: Take the transpose of C :
$C' = (AB' + BA')' = (AB')' + (BA')'$
Property: Use $(XY' = Y'X'$:
$C' = (B')'A' + (A')'B' = B(-A) + A(-B) = -BA - AB$
Substitute $B' = -B$ and $A' = -A$ directly:
$C = A(-B) + B(-A) = -AB - BA$
$C' = (AB' + BA')' = (B')'A' + (A')'B' = BA' + AB' = AB' + BA' = C$
• Result: Since $C' = C$, it is a symmetric matrix.

80. (b) • Order of Matrix A
• Given Equation: $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$
• Dimensions:
Left matrix dimension: 3×1
Result matrix dimension: 3×3
Multiplication Rule: $(3 \times 1) \times (1 \times 3) = (3 \times 3)$
• Result: Order of A must be 1×3.

81. (d) • Value of x if $A^2 = A$ and $(I - A)^3 = xA + I$
• Expansion: $(I - A)^3 = I^3 - 3I^2A + 3IA^2 - A^3$
Simplify powers: Since $A^2 = A$, then $A^3 = A^2 \cdot A = A \cdot A = A$.
Substitute:
$(I - A)^3 = I - 3A + 3A - A = I - A$
• Compare: $I - A = xA + I \Rightarrow x = -1$

82. (c) • Value of $\det(B^{-1})$
• Property: $B(\text{adj}B) = |B|I$
• Given:
$$B(\text{adj}B) = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix} = \frac{1}{3}I$$
• Determinant: $|B| = \frac{1}{3}$
• Inverse Determinant: $\det(B^{-1}) = \frac{1}{|B|} = \frac{1}{1/3} = 3$

83. (d) Find the value of k for which f(x) is continuous at $x = 0$.
(1) For continuity, $\lim_{x\to 0} f(x) = f(0)$.
(2) Calculate the limit:
$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$

(Since $\sin(I/x)$ oscillates between $-I$ and I, and $x^2 \to 0$)
(3) Find f(0) :
$f(0) = k(0+1) = k$
(4) Equating both: $k = 0$.

84. **(c)** If $\sin^{-1}x = y$, then find $\frac{dy}{dx}$.
(1) Differentiate $y = \sin^{-1}x$ directly with respect to x :
$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

85. **(c)** Rate of change of volume (V) of a sphere with respect to its diameter (D), when radius $r = 5$ cm.
(1) Express volume in terms of diameter D (where $r = \frac{D}{2}$):
$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 = \frac{\pi}{6}D^3$$
(2) Differentiate with respect to D :
$$\frac{dV}{dD} = \frac{\pi}{6} \cdot 3D^2 = \frac{\pi}{2}D^2$$
(3) Substitute $r = 5$ cm $\Longrightarrow D = 10$ cm :
$$\frac{dV}{dD} = \frac{\pi}{2}(10)^2 = 50\pi \text{ cm}^3/\text{cm}$$

86. **(c)** Evaluate $\int \frac{dx}{2^x+2^{-x}}$.
(1) Rewrite the integrand:
$$\int \frac{dx}{2^x + \frac{1}{2^x}} = \int \frac{2^x}{(2^x)^2+1}dx$$
(2) Use substitution $u = 2^x \Longrightarrow du = 2^x \log 2 dx$:
$$\int \frac{1}{u^2+1} \cdot \frac{du}{\log 2} = \frac{1}{\log 2}\tan^{-1}(u) + C$$
(3) Substitute back $u = 2^x$:
$$\frac{\tan^{-1}(2^x)}{\log 2} + C$$

87. **(b)** Evaluate $\int_{-1}^{1}(1-|x|)dx$.
(1) The function $f(x) = 1-|x|$ is an even function ($f(-x) = f(x)$).
(2) By property of even functions:
$$\int_{-1}^{1} f(x)dx = 2\int_{-1}^{0} f(x)dx$$
(3) In the interval $[-1,0]$, $|x| = -x$, so $1-|x| = 1+x$:
$$2\int_{-1}^{0}(1+x)dx$$

88. **(b)** Find the area of the shaded region.
(1) The region is bounded vertically from $y = 1$ to the top of the circle $y = 3$.
(2) Express the circle $x^2 + y^2 = 9$ in terms of y.
$$x = \pm\sqrt{9-y^2}$$
(3) Due to symmetry about the y-axis, integrate from $y = 1$ to $y = 3$ and multiply by 2 :
$$\text{Area} = 2\int_{1}^{3}\sqrt{9-y^2}dy$$

89. **(d)** A differential equation $\frac{dy}{dx} = F(x,y)$ is homogeneous if $F(\lambda x, \lambda y) = F(x,y)$ for any non-zero constant λ. This means the total degree of every term must be identical, or the variables must only appear in the form of a ratio like $\frac{y}{x}$ or $\frac{x}{y}$.
(I) $F(x,y) = 3x+2y \Longrightarrow F(\lambda x, \lambda y) = \lambda(3x+2y) \neq F(x,y)$ (Homogeneous of degree 1 , but $\frac{dy}{dx} = F(x,y)$ requires F to be degree 0 to make the ODE homogeneous).

(II) $F(x, y) = \sin\frac{y}{x} + \log y - \log x = \sin\frac{y}{x} + \log\left(\frac{y}{x}\right)$. Replacing x, y with λx, λy leaves the ratios unchanged. (Homogeneous)

(III) $F(x, y) = e^{y/x} + 1$. The variable appears as a ratio $\frac{y}{x}$, which remains unchanged. (Homogeneous)

(IV) $F(x, y) = \sqrt{x^2 + y^2} - y \Rightarrow F(\lambda x, \lambda y) = \lambda\sqrt{x^2 + y^2} - \lambda y = \lambda F(x, y) \neq F(x, y)$. (Not degree 0).

Therefore, only functions (II) and (III) satisfy the condition.

90. (a) According to the Cauchy-Schwarz Inequality, for any two vectors:

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\theta$

Since $\cos\theta \leq 1$, it is always true that $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$.

91. (b) Expand the given dot product:

$(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$

$|\vec{a}|^2 - |\vec{b}|^2 = 198$

Substitute the given relation $|\vec{a}| = 10|\vec{b}|$:

$(10|\vec{b}|)^2 - |\vec{b}|^2 = 198$

$100|\vec{b}|^2 - |\vec{b}|^2 = 198$

$99|\vec{b}|^2 = 198$

$|\vec{b}|^2 = 2 \Rightarrow |\vec{b}| = \sqrt{2}$

92. (d) The angle θ between two lines with direction cosines (I_1, m_1, n_1) and (I_2, m_2, n_2) is given by the formula:

$\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$

Since θ is explicitly stated to be an acute angle, cos θ must be positive. Therefore, the absolute value is used:

$\cos\theta = |l_1l_2 + m_1m_2 + n_1n_2|$

93. (c) Two lines are parallel if their direction ratios are proportional:

$$\frac{12}{4} = \frac{-3}{q} = \frac{9}{-p}$$

Simplify the first ratio:

$$3 = \frac{-3}{q} \Rightarrow q = -1$$

$$3 = \frac{9}{-p} \Rightarrow -3p = 9 \Rightarrow p = -3$$

Thus, the values of p and q are -3 and -1 respectively.

94. (c) • Probability of independent events

Given: $P(E) = \frac{3}{10}, P(E \cup F) = \frac{1}{2}$, and E, F are independent.

• For independent events, $P(E \cup F) = P(E) + P(F) - P(E)P(F)$.

$$\frac{1}{2} = \frac{3}{10} + P(F) - \frac{3}{10}P(F)$$

$$\frac{1}{2} - \frac{3}{10} = P(F)\left(1 - \frac{3}{10}\right)$$

$$\frac{5-3}{10} = P(F)\left(\frac{7}{10}\right) \Rightarrow \frac{2}{10} = P(F)\left(\frac{7}{10}\right) \Rightarrow P(F) = \frac{2}{7}.$$

Since they are independent, $P(E \mid F) = P(E) = \frac{3}{10}$ and $P(F \mid E) = P(F) = \frac{2}{7}$.

$$P(E \mid F) - P(F \mid E) = \frac{3}{10} - \frac{2}{7} = \frac{21-20}{70} = \frac{1}{70}.$$

95. (a) • Assertion (A): The equation $\frac{dy}{dx} = e^{x+y} = e^x \cdot e^y$. Separating variables: $\int e^{-y}dy = \int e^x dx \Rightarrow -e^{-y} = e^x + C \Rightarrow e^x + e^{-y} = -C$. This is a valid family of solutions.

Assertion is True.

• Reason(R) : The general solution derived above is $e^x + e^{-y} =$ constant. Reason is True and explains the Assertion.

96. (c) • Assertion (A): Two vectors a and b are collinear if a = kb. Here $a = -\frac{1}{2}(-2a)$, so they are collinear.

Assertion is True.

• Reason (R): For collinear vectors, the dot product $a \cdot b = |a||b|\cos\theta$. For collinear vectors, $\theta = 0°$ or $180°$. The dot product is zero only if they are orthogonal $\theta = 90°$). Reason is False.

97. Given:

• $x = t + \frac{1}{t}$

• $y = t - \frac{1}{t}$

Step 1: Differentiate **x** with respect to **t**

$$\frac{dx}{dt} = \frac{d}{dt}(t + t^{-1}) = 1 - \frac{1}{t^2}$$

Step 2: Differentiate y with respect to t

$$\frac{dy}{dt} = \frac{d}{dt}(t - t^{-1}) = 1 + \frac{1}{t^2}$$

Step 3: Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 + \frac{1}{t^2}}{1 - \frac{1}{t^2}} = \frac{\frac{t^2+1}{t^2}}{\frac{t^2-1}{t^2}} = \frac{t^2+1}{t^2-1}$$

Step 4: Substitute $\mathbf{t = 2}$

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{2^2+1}{2^2-1} = \frac{4+1}{4-1} = \frac{5}{3}$$

98. Given:

• $f(x) = \tan x - 4x$ for $x \in \left(0, \frac{\pi}{2}\right)$

Step 1: Find the first derivative $f'(x)$

$f'(x) = \sec^2 x - 4$

Step 2: Set condition for an increasing function $(f'(x) > 0)$

$\sec^2 x - 4 > 0 \Longrightarrow \sec^2 x > 4$

Step 3: Solve for **x** in the given interval

Since $x \in \left(0, \frac{\pi}{2}\right)$, $\sec x$ is strictly positive:

$$\sec x > 2 \Longrightarrow \frac{1}{\cos x} > 2 \Longrightarrow \cos x < \frac{1}{2}$$

In the interval $\left(0, \frac{\pi}{2}\right)$, $\cos x = \frac{1}{2}$ at $x = \frac{\pi}{3}$. Since cosine decreases as x increases in this quadrant:

$x \in \left(\frac{\pi}{3}, \frac{\pi}{2}\right)$

99. **(A)** Expression:

$$\sin\left[\cot^{-1}\left(\sqrt{2}\cos(\tan^{-1}1)\right)\right]$$

Step 1: Simplify the innermost term

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Step 2: Evaluate the cosine term

$$\cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

Step 3: Substitute back into the expression

$$\sin\left[\cot^{-1}\left(\sqrt{2} \cdot \frac{1}{\sqrt{2}}\right)\right] = \sin[\cot^{-1}(1)]$$

Step 4: Evaluate the remaining terms

$$\cot^{-1}(1) = \frac{\pi}{4}$$

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

OR

(B) Given:

• $A = \{1,2,3\}$

• $R = \{(1,1), (3,3), (1,2)\}$

Part 1: Is R symmetric?

• Answer: No

• Justification: For a relation to be symmetric, if $(a, b) \in R$, then (b, a) must also belong to R. Here, $(1,2) \in R$ but $(2,1) \notin R$.
Part 2: Find the smallest relation set R_1 to make $R \cup R_1$ an equivalence relation To be an equivalence relation, the combined set must be:
(1) Reflexive: Needs (1, 1), (2,2), (3,3). It lacks (2,2).
(2) Symmetric: Needs (2,1) to counter (1,2).
(3) Transitive: Adding {(2,2), (2,1)} keeps the relation transitive automatically.
Therefore:
$R_1 = \{(2,2), (2,1)\}$

100. Given:

• Unit vectors $\vec{a}$ and $\vec{b} \Rightarrow |\vec{a}| = 1, |\vec{b}| = 1$
• $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$
Step 1: Square both sides
$|\vec{a} + 2\vec{b}|^2 = |2\vec{a} - \vec{b}|^2$
Step 2: Expand using dot products
$|\vec{a}|^2 + 4(\vec{a} \cdot \vec{b}) + 4|\vec{b}|^2 = 4|\vec{a}|^2 - 4(\vec{a} \cdot \vec{b}) + |\vec{b}|^2$
Step 3: Substitute magnitude values (1)
$1 + 4(\vec{a} \cdot \vec{b}) + 4 = 4 - 4(\vec{a} \cdot \vec{b}) + 1$
$5 + 4(\vec{a} \cdot \vec{b}) = 5 - 4(\vec{a} \cdot \vec{b})$
Step 4: Solve for the angle (θ)
$8(\vec{a} \cdot \vec{b}) = 0 \Rightarrow \vec{a} \cdot \vec{b} = 0$
$|\vec{a}||\vec{b}|\cos\theta = 0 \Rightarrow \cos\theta = 0$
$\theta = \frac{\pi}{2}$ or $90°$

101. **(A)** (1) Convert the equations into standard form: $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$

• First line:
$$\frac{x-3}{1} = \frac{-(y-1)}{1} = \frac{z+2}{p} \Rightarrow \frac{x-3}{1} = \frac{y-1}{-1} = \frac{z+2}{p}$$
Direction ratios $\vec{d_1} = (1, -1, p)$
• Second line:
$$\frac{-(x-2)}{3} = \frac{y+1}{5} = \frac{z+56}{2p} \Rightarrow \frac{x-2}{-3} = \frac{y+1}{5} = \frac{z+56}{2p}$$
Direction ratios $\vec{d_2} = (-3,5,2p)$
(2) Apply the condition for perpendicular lines: $a_1a_2 + b_1b_2 + c_1c_2 = 0$
$(1)(-3) + (-1)(5) + (p)(2p) = 0$
$-3 - 5 + 2p^2 = 0$
$2p^2 = 8$
$p^2 = 4 \Rightarrow p = \pm 2$

OR

(B) (1) Identify the direction vectors of the given lines:
• $\vec{b_1} = 3\hat{i} + 4\hat{j} + 2\hat{k}$
• $\vec{b_2} = \hat{i} - \hat{j} + \hat{k}$
(2) Find the direction vector ($\vec{b}$) of the required line:
Since the line is perpendicular to both, its direction is given by the cross product $\vec{b_1} \times \vec{b_2}$:
$$\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 1 & -1 & 1 \end{vmatrix}$$
$\vec{b} = \hat{i}(4 - (-2)) - \hat{j}(3 - 2) + \hat{k}(-3 - 4)$
$\vec{b} = 6\hat{i} - \hat{j} - 7\hat{k}$
(3) Write the equation of the line:
The line passes through the origin (0,0,0), so its position vector is $\vec{a} = 0\hat{i} + 0\hat{j} + 0\hat{k}$
$\vec{r} = \vec{a} + t\vec{b} \Rightarrow \vec{r} = t(6\hat{i} - \hat{j} - 7\hat{k})$

102. **(A)** (1) Substitute to clear the fractional exponents:

Let $x = t^6$ (since the LCM of 2 and 3 is 6). Then $dx = 6t^5 dt$.

• $x^{1/2} = (t^6)^{1/2} = t^3$

• $x^{1/3} = (t^6)^{1/3} = t^2$

(2) Rewrite the integral:

$$\int \frac{6t^5}{t^3 + t^2} dt = 6\int \frac{t^5}{t^2(t+1)} dt = 6\int \frac{t^3}{t+1} dt$$

(3) Perform algebraic division or add/subtract terms:

$t^3 = (t^3 + 1) - 1 = (t+1)(t^2 - t + 1) - 1$

$$\frac{t^3}{t+1} = t^2 - t + 1 - \frac{1}{t+1}$$

(4) Integrate term by term:

$$6\int \left(t^2 - t + 1 - \frac{1}{t+1}\right) dt = 6\left[\frac{t^3}{3} - \frac{t^2}{2} + t - \ln|t+1|\right] + C$$

$= 2t^3 - 3t^2 + 6t - 6\ln|t+1| + C$

(5) Substitute back $t = x^{1/6}$;

$= 2x^{1/2} - 3x^{1/3} + 6x^{1/6} - 6\ln\left|x^{1/6} + 1\right| + C$

OR

(B) (1) Simplify the integrand using trigonometry:

Let $x = \tan\theta \implies \theta = \tan^{-1}x$.

$$\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \theta\right)\right) = \frac{\pi}{4} - \theta = \frac{\pi}{4} - \tan^{-1}x$$

(2) Rewrite the integral:

$$\int \left(\frac{\pi}{4} - \tan^{-1}x\right) dx = \frac{\pi}{4}x - \int \tan^{-1}x dx$$

(3) Evaluate $\int \tan^{-1} x dx$ using Integration by Parts ($\int udv = uv - \int vdu$) :

• Let $u = \tan^{-1}x \implies du = \frac{1}{1+x^2} dx$

• Let $dv = dx \implies v = x$

$$\int \tan^{-1}x dx = x\tan^{-1}x - \int \frac{x}{1+x^2} dx = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2)$$

(4) Combine the results:

$$\frac{\pi}{4}x - \left(x\tan^{-1}x - \frac{1}{2}\ln(1+x^2)\right) + C = \frac{\pi}{4}x - x\tan^{-1}x + \frac{1}{2}\ln(1+x^2) + C$$

103. (1) Let the integral be I :

$I = \int_0^{\pi} \frac{\sin^{2026}x}{\sin^{20126}x + \cos^{2026}x} dx \quad \ldots(i)$

(2) Apply the definite integral property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:

$$I = \int_0^{\pi} \frac{\sin^{2026}(\pi - x)}{\sin^{20126}(\pi - x) + \cos^{20026}(\pi - x)} dx$$

Since $\sin(\pi - x) = \sin x$ and $\cos(\pi - x) = -\cos x$(and $(-\cos x)^{2026} = \cos^{2026}x$ due to the even power):

$I = \int_0^{\pi} \frac{\sin^{2026}x}{\sin^{2026}x + \cos^{2026}x} dx \quad \ldots(ii)$

Note: This yields the same equation, so we apply the property $\int_0^{2a} f(x)dx = 2\int_0^{at} f(x)dx$ if $f(2a - x) = f(x)$.

(3) Reduce the upper limit:

Since the integrand remains unchanged when replacing x with $\pi - x$:

$I = 2\int_0^{\pi/2} \frac{\sin^{2026}x}{\sin^{2026}x + \cos^{2026}x} dx \quad \ldots(iii)$

(4) Apply King's Property ($\int_0^a f(x)dx = \int_0^a f(a-x)dx$) on Equation 3:

$$I = 2\int_0^{\pi/2} \frac{\sin^{2026}\left(\frac{\pi}{2} - x\right)}{\sin^{2026}\left(\frac{\pi}{2} - x\right) + \cos^{2026}\left(\frac{\pi}{2} - x\right)} dx$$

$I = 2\int_0^{\pi/2} \frac{\cos^{2026}x}{\cos^{2026}x + \sin^{2026}x} dx \quad \ldots(iv)$

(5) Add Equation 3 and Equation 4:

$$2I = 2\int_0^{\pi/2} \frac{\sin^{2026}x + \cos^{2026}x}{\sin^{2026}x + \cos^{2026}x} dx$$

$$2I = 2\int_0^{\pi/2} 1 \cdot dx$$

$$I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

104. **(A)** Let $I = \int \frac{\cos x}{(2+\sin x)(4+\sin x)} dx$.

(1) Substitution:

Let $t = \sin x \Longrightarrow dt = \cos x dx$.

$$I = \int \frac{dt}{(2+t)(4+t)}$$

(2) Partial Fraction Decomposition:

$$\frac{1}{(2+t)(4+t)} = \frac{A}{2+t} + \frac{B}{4+t}$$

$I = A(4+t) + B(2+t)$

• Setting $t = -2 \Longrightarrow I = 2A \Longrightarrow A = \frac{1}{2}$

• Setting $t = -4 \Longrightarrow I = -2B \Longrightarrow B = -\frac{1}{2}$

(3) Integration:

$$I = \frac{1}{2}\int \left(\frac{1}{2+t} - \frac{1}{4+t}\right) dt$$

$$I = \frac{1}{2}(\ln|2+t| - \ln|4+t|) + C = \frac{1}{2}\ln\left|\frac{2+t}{4+t}\right| + C$$

(4) Substitute back $t = \sin x$:

$$I = \frac{1}{2}\ln\left|\frac{2+\sin x}{4+\sin x}\right| + C$$

OR

(B) Let $I = \int \frac{x+3}{x^2+4x+5} dx$.

(1) Express the numerator in terms of the derivative of the denominator:

The derivative of $(x^2 + 4x + 5)$ is $(2x + 4)$.

$$x + 3 = \frac{1}{2}(2x + 4) + 1$$

(2) Split into two integrals:

$$I = \frac{1}{2}\int \frac{2x+4}{x^2+4x+5} dx + \int \frac{1}{x^2+4x+5} dx$$

(3) Evaluate both parts:

• First integral: Use substitution $u = x^2 + 4x + 5$.

$$\frac{1}{2}\ln|x^2 + 4x + 5|$$

• Second integral: Complete the square in the denominator: $x^2 + 4x + 5 = (x+2)^2 + 1$.

$$\int \frac{1}{(x+2)^2 + 1^2} dx = \tan^{-1}(x+2)$$

(4) Combine results:

$$I = \frac{1}{2}\ln|x^2 + 4x + 5| + \tan^{-1}(x+2) + C$$

105. **(A)** Rearranging gives a homogeneous differential equation:

$$\frac{dy}{dx} = \frac{y^2 + 2xy}{2x^2}$$

(1) Substitution:

Let $y = vx \Longrightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$.

$$v + x\frac{dv}{dx} = \frac{(vx)^2 + 2x(vx)}{2x^2} = \frac{v^2 + 2v}{2}$$

$$x\frac{dv}{dx}=\frac{v^2+2v}{2}-v=\frac{v^2}{2}$$

(2) Separation of Variables:

$$\frac{2}{v^2}dv=\frac{1}{x}dx\Longrightarrow 2v^{-2}dv=\frac{1}{x}dx$$

(3) Integration:

$$-\frac{2}{v}=\ln|x|+C$$

(4) Substitute back $v=\frac{y}{x}$:

$$-\frac{2x}{y}=\ln|x|+C\Longrightarrow y=\frac{-2x}{\ln|x|+C}$$

OR

(B) (1) Separation of Variables:

$$\frac{dy}{2e^{-y}-1}=\frac{dx}{x+1}\Longrightarrow\frac{e^y dy}{2-e^y}=\frac{dx}{x+1}$$

(2) Integration:

Let $u=2-e^y\Longrightarrow du=-e^y dy.$

$$\int\frac{-du}{u}=\int\frac{dx}{x+1}$$

$$-\ln|2-e^y|=\ln|x+1|+\ln C\Longrightarrow\ln\left|\frac{1}{2-e^y}\right|=\ln|C(x+1)|$$

$$\frac{1}{2-e^y}=C(x+1)$$

(3) Apply Initial Condition $(x=0,y=0)$:

$$\frac{1}{2-e^0}=C(0+1)\Longrightarrow\frac{1}{1}=C\Longrightarrow C=1$$

(4) Find Particular Solution:

$$\frac{1}{2-e^y}=x+1\Longrightarrow 2-e^y=\frac{1}{x+1}$$

$$e^y=2-\frac{1}{x+1}=\frac{2x+1}{x+1}\Longrightarrow y=\ln\left|\frac{2x+1}{x+1}\right|$$

106. (1) Take the natural logarithm on both sides:

$$y\ln(\sin x)=\cos x\ln y$$

(2) Differentiate implicitly with respect to x using the product rule:

$$\frac{dy}{dx}\ln(\sin x)+y\cdot\frac{1}{\sin x}\cdot\cos x=(-\sin x)\ln y+\cos x\cdot\frac{1}{y}\frac{dy}{dx}$$

$$\frac{dy}{dx}\ln(\sin x)+y\cot x=-\sin x\ln y+\frac{\cos x}{y}\frac{dy}{dx}$$

(3) Rearrange to isolate $\frac{dy}{dx}$:

$$\frac{dy}{dx}\left[\ln(\sin x)-\frac{\cos x}{y}\right]=-\sin x\ln y-y\cot x$$

$$\frac{dy}{dx}\left[\frac{y\ln(\sin x)-\cos x}{y}\right]=-(\sin x\ln y+y\cot x)$$

$$\frac{dy}{dx}=\frac{y(y\cot x+\sin x\ln y)}{\cos x-y\ln(\sin x)}$$

107. Given:

- Success probability $(p)=90\%=0.9$
- Failure probability $(q)=1-0.9=0.1$
- Number of trials $(n)=3$

Using Binomial distribution formula: $P(X=k)=\binom{n}{k}p^kq^{n-k}$

(i) Exactly one surgery is successful $(X=1)$:

$$P(X=1)=\binom{3}{1}(0.9)^1(0.1)^2=3\times0.9\times0.01=0.027$$

(ii) At most two surgeries are successful $(X\le 2)$:

$P(X \le 2) = 1 - P(X = 3)$

$P(X = 3) = \binom{3}{3}(0.9)^3(0.1)^0 = 1 \times 0.729 \times 1 = 0.729$

$P(x \le 2) = 1 - 0.729 = 0.271$

108. (1) Prove f(x) is One-One (Injective)

Let $x_1, x_2 \in R$ such that $f(x_1) = f(x_2)$:

$$\frac{x_1}{\sqrt{1+x_1^2}} = \frac{x_2}{\sqrt{1+x_2^2}}$$

Squaring both sides:

$$\frac{x_1^2}{1+x_1^2} = \frac{x_2^2}{1+x_2^2}$$

$$x_1^2(1+x_2^2) = x_2^2(1+x_1^2)$$

$$x_1^2 + x_1^2x_2^2 = x_2^2 + x_1^2x_2^2$$

$$x_1^2 = x_2^2 \Longrightarrow x_1 = \pm x_2$$

From the original definition, $f(x) = \frac{x}{\sqrt{1+x^2}}$, the denominator $\sqrt{1+x^2}$ is always positive. Therefore, the sign of f(x) depends entirely on the sign of x.

• If x_1 and x_2 have opposite signs ($x_1 = -x_2$), then $f(x_1)$ and $f(x_2)$ would have opposite signs, which contradicts $f(x_1) = f(x_2)$.

Thus, $x_1 = -x_2$ is impossible (unless $x_1 = x_2 = 0$). Hence, $x_1 = x_2$.

• Conclusion: f(x) is one-one.

(2) Prove f(x) is Not Onto (Surjective)

Let $y = \frac{x}{\sqrt{1+x^2}}$. Since $I + x^2 > x^2$ for all $x \in R$, taking square roots gives:

$$\sqrt{1+x^2} > |x|$$

Therefore:

$$|y| = \frac{|x|}{\sqrt{1+x^2}} < 1 \Longrightarrow -1 < y < 1$$

The Range of f(x) is $(-1,1)$. Since the Co-domain is given as R, and Range ≠ Co-domain, there are elements in the co-domain (like $y = 2$) that have no preimage in the domain.

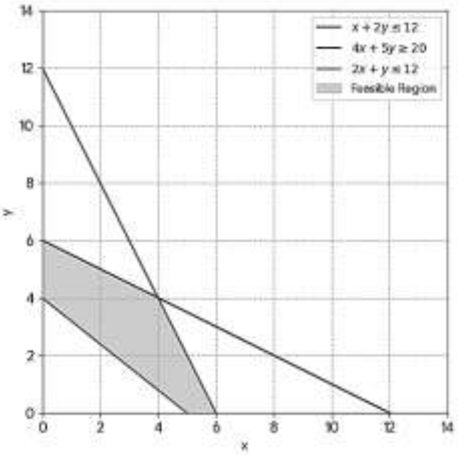
• Conclusion: f(x) is not onto.

109. To maximize $Z = 600x + 400y$, we plot the boundary lines for the given linear constraints:

(1) $x + 2y = 12 \Longrightarrow$ passes through (12,0) and (0,6)

(2) $4x + 5y = 20 \Longrightarrow$ passes through (5,0) and (0,4)

(3) $2x + y = 12 \Longrightarrow$ passes through (6,0) and (0,12)



Evaluating Corner Points:

Corner Point (x, y)	Value of Z = 600x + 400y

A(4,4)	$600(4)+400(4) = 2400 + 1600 = 4000$
B(6,0)	$600(6)+400(0) = 3600$
C(5,0)	$600(5)+400(0) = 3000$
D(0,4)	$600(0)+400(4) = 1600$
E(0,6)	$600(0)+400(6) = 2400$

The maximum value of Z is 4000 , which occurs at the point (4,4).

110. **(A)** Let x be the number of ₹1,000 vouchers and y be the number of ₹500 vouchers.

(1) System of Equations:

$x+y=60$

$x+3y=100$

(2) Matrix Method Formulation (AX = B) :

$\begin{bmatrix}1 & 1\\1 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}60\\100\end{bmatrix}$

Find A^{-1} :

$|A|=(1)(3)-(1)(1)=2$

$A^{-1}=\frac{1}{2}\begin{bmatrix}3 & -1\\-1 & 1\end{bmatrix}$

Solve for $X=A^{-1}B$:

$\begin{bmatrix}x\\y\end{bmatrix}=\frac{1}{2}\begin{bmatrix}3 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}60\\100\end{bmatrix}=\frac{1}{2}\begin{bmatrix}180-100\\-60+100\end{bmatrix}=\frac{1}{2}\begin{bmatrix}80\\40\end{bmatrix}=\begin{bmatrix}40\\20\end{bmatrix}$

• Final Answer: There are 40 vouchers of ₹1,000 and 20 vouchers of ₹500.

OR

(B) Given $PQ-RS=O \Rightarrow RS=PQ$.

Step 1: Compute PQ

$PQ=\begin{bmatrix}2 & -1\\3 & 4\end{bmatrix}\begin{bmatrix}5 & 2\\7 & 4\end{bmatrix}=\begin{bmatrix}2(5)+(-1)(7) & 2(2)+(-1)(4)\\3(5)+4(7) & 3(2)+4(4)\end{bmatrix}=\begin{bmatrix}3 & 0\\43 & 22\end{bmatrix}$

Step 2: Solve for S

Let matrix $S=\begin{bmatrix}a & b\\c & d\end{bmatrix}$.

$RS=\begin{bmatrix}2 & 5\\3 & 8\end{bmatrix}\begin{bmatrix}a & b\\c & d\end{bmatrix}=\begin{bmatrix}2a+5c & 2b+5d\\3a+8c & 3b+8d\end{bmatrix}$

Equating elements with PQ :

(1) $2a+5c=3$

(2) $3a+8c=43$

(3) $2b+5d=0$

(4) $3b+8d=22$

• Solving (1) and (2):

$3(2a+5c)-2(3a+8c)=3(3)-2(43) \Rightarrow -c=-77 \Rightarrow c=77$

$2a+5(77)=3 \Rightarrow 2a=-382 \Rightarrow a=-191$

• Solving (3) and (4):

$3(2b+5d)-2(3b+8d)=3(0)-2(22) \Rightarrow -d=-44 \Rightarrow d=44$

$2b+5(44)=0 \Rightarrow 2b=-220 \Rightarrow b=-110$

$S=\begin{bmatrix}-191 & -110\\77 & 44\end{bmatrix}$

111. **(A)** (1) Vector Form of the Lines

A line given in Cartesian form $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$ has the vector form $\vec{r}=(x_1\hat{i}+y_1\hat{j}+z_1\hat{k})+\lambda(a\hat{i}+b\hat{j}+c\hat{k})$.

• Line l_1 :

$$\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$$

• Passing point: $(-3,1,5) \Longrightarrow \vec{a}_1=-3\hat{i}+\hat{j}+5\hat{k}$

• Direction vector: $\vec{b}_1=-3\hat{i}+\hat{j}+5\hat{k}$

• Vector Form: $\vec{r}=(-3\hat{i}+\hat{j}+5\hat{k})+\lambda(-3\hat{i}+\hat{j}+5\hat{k})$

• Line I_2 :

First, rewrite the y-term to standard form: $\frac{2-y}{-2}=\frac{y-2}{2}$.

$$\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$$

• Passing point: $(-1,2,5) \Longrightarrow \vec{a}_2=-\hat{i}+2\hat{j}+5\hat{k}$

• Direction vector: $\vec{b}_2=-\hat{i}+2\hat{j}+5\hat{k}$

• Vector Form: $\vec{r}=(-\hat{i}+2\hat{j}+5\hat{k})+\mu(-\hat{i}+2\hat{j}+5\hat{k})$

(2) Check for Intersection

Any point on I_1 is:

$x=-3-3\lambda, y=1+\lambda, z=5+5\lambda$

Any point on l_2 is:

$x=-1-\mu, y=2+2\mu, z=5+5\mu$

Equating the z -coordinates:

$5+5\lambda=5+5\mu \Longrightarrow \lambda=\mu$

Equating the y-coordinates and substituting $\mu=\lambda$:

$1+\lambda=2+2\lambda \Longrightarrow \lambda=-1 \Longrightarrow \mu=-1$

Now, substitute $\lambda=-1$ and $\mu=-1$ into the x-coordinates to check compatibility:

• For $I_1: x=-3-3(-1)=0$

• For $I_2: x=-1-(-1)=0$

Since the values are consistent $(0=0)$, the lines intersect at the point $(0,0,0)$.

OR

(B) (1) The given lines for the opposite sides are parallel because they share the same direction vector direction ratios $\langle 2,3,6\rangle$.

Since it is a square, the adjacent sides must be perpendicular to these lines.

• Direction ratios of given lines: $\vec{b}_1=\langle 2,3,6\rangle$

• Direction ratios of other sides: $\vec{b}_2=\langle -3,6,p\rangle$

Since they are perpendicular, their dot product is zero:

$(2)(-3)+(3)(6)+(6)(p)=0$

$-6+18+6p=0 \Longrightarrow 12+6p=0 \Longrightarrow p=-2$

(2) Area of the Square

The distance between the parallel lines is equal to the side length (s) of the square.

• Point on line 1: $\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k}$

• Point on line 2: $\vec{a}_2=3\hat{i}+3\hat{j}-5\hat{k}$

• Vector between points: $\vec{a}_2-\vec{a}_1=2\hat{i}+\hat{j}-\hat{k}$

• Direction vector of lines: $\vec{b}=2\hat{i}+3\hat{j}+6\hat{k}$

The perpendicular distance formula is:

$$s=\frac{\left|(\vec{a}_2-\vec{a}_1)\times\vec{b}\right|}{|\vec{b}|}$$

Calculate the cross product:

$$(\vec{a}_2-\vec{a}_1)\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 2 & 3 & 6\end{vmatrix}=\hat{i}(6+3)-\hat{j}(12+2)+\hat{k}(6-2)=9\hat{i}-14\hat{j}+4\hat{k}$$

Magnitude of cross product:

$\sqrt{9^2+(-14)^2+4^2}=\sqrt{81+196+16}=\sqrt{293}$

Magnitude of $\vec{b}$:

$|\vec{b}|=\sqrt{2^2+3^2+6^2}=\sqrt{4+9+36}=\sqrt{49}=7$

Side length (s):

$s = \dfrac{\sqrt{293}}{7}$

Area $= s^2 = \frac{293}{49} \approx 5.98$ sq. units

112. From the diagram:

- Smaller pole height CD = 16 m
- Taller pole height AB = 22 m
- Distance RD = x, so BR = 20 − x

Using the Pythagorean theorem in right triangles $\triangle$ RDC and $\triangle$ RBA :

- $I_1 = \sqrt{AB^2 + BR^2} = \sqrt{22^2 + (20-x)^2} = \sqrt{484 + (20-x)^2}$
- $I_2 = \sqrt{CD^2 + RD^2} = \sqrt{16^2 + x^2} = \sqrt{256 + x^2}$

(i) Express $p(x) = l_1 + l_2$ in terms of x

$p(x) = \sqrt{484 + (20-x)^2} + \sqrt{256 + x^2}$

(ii) Find $p'(x)$

Differentiating with respect to x :

$$p'(x) = \frac{1}{2\sqrt{484 + (20-x)^2}} \cdot 2(20-x)(-1) + \frac{1}{2\sqrt{256 + x^2}} \cdot 2x$$

$$p'(x) = \frac{-(20-x)}{\sqrt{484 + (20-x)^2}} + \frac{x}{\sqrt{256 + x^2}}$$

(iii) (A) Find the value of x for which $l_1^2 + l_2^2$ is minimum Let $S(x) = l_1^2 + l_2^2$.

$S(x) = [484 + (20-x)^2] + [256 + x^2]$

$S(x) = 484 + 400 - 40x + x^2 + 256 + x^2 = 2x^2 - 40x + 1140$

To minimize, find $S'(x) = 0$.

$S'(x) = 4x - 40 = 0 \Longrightarrow 4x = 40 \Longrightarrow \mathbf{x} = 10$ m

(Since $S''(x) = 4 > 0$, it is a minimum value).

OR

(B) If both poles are 16 m long:

- $AB = 16$ m, $CD = 16$ m
- $S(x) = [16^2 + (20-x)^2] + [16^2 + x^2]$
- $S(x) = 256 + 400 - 40x + x^2 + 256 + x^2 = 2x^2 - 40x + 912$

Differentiating to find the minimum:

$S'(x) = 4x - 40 = 0 \Longrightarrow x = 10$m

Thus, the ladders should be placed exactly in the middle, at a distance of 10 m from either pole.

113. Given Data Summary

Let the relevant events be defined as:

- D: Student is a dropout
- R: Student is a regular student
- Q : Student qualifies the examination
- Q′ : Student does not qualify the examination

From the question statement:

- Percentage of dropouts: $P(D) = 40\% = 0.4$
- Percentage of regular students: $P(R) = 100\% - 40\% = 60\% = 0.6$
- Probability a dropout qualifies: $P(Q \mid D) = 5\% = 0.05$
- Probability a regular student qualifies: $P(Q \mid R) = 10\% = 0.10$

(i) The probability is simply the proportion of regular students in the total population:

$P(R) = 60\% = 0.6$ or $\dfrac{3}{5}$

(ii) Since the student is chosen specifically from the dropout group, we need the conditional probability that a dropout student does not qualify (Q') :

$P(Q' \mid D) = 1 - P(Q \mid D)$

$P(Q' \mid D) = 1 - 0.05 = 0.95$ or $\dfrac{19}{20}$

(iii) (A) "Not a dropout" means the student is a regular student (R). We use Bayes' Theorem to find $P(R \mid Q)$:

$$P(R \mid Q) = \frac{P(R) \cdot P(Q \mid R)}{P(R) \cdot P(Q \mid R) + P(D) \cdot P(Q \mid D)}$$

Substitute the values:

$$P(R \mid Q) = \frac{0.6 \times 0.10}{(0.6 \times 0.10) + (0.4 \times 0.05)}$$

$$P(R \mid Q) = \frac{0.06}{0.06 + 0.02}$$

$$P(R \mid Q) = \frac{0.06}{0.08} = \frac{6}{8} = 0.75 \text{ or } \frac{3}{4}$$

OR

(B) We need to find $P(R \mid Q')$ using Bayes' Theorem.

First, calculate the conditional non-qualification probabilities:

- $P(Q' \mid R) = 1 - 0.10 = 0.90$
- $P(Q' \mid D) = 1 - 0.05 = 0.95$

Now, apply Bayes' Theorem:

$$P(R \mid Q') = \frac{P(R) \cdot P(Q' \mid R)}{P(R) \cdot P(Q' \mid R) + P(D) \cdot P(Q' \mid D)}$$

$$P(R \mid Q') = \frac{0.6 \times 0.90}{(0.6 \times 0.90) + (0.4 \times 0.95)}$$

$$P(R \mid Q') = \frac{0.54}{0.54 + 0.38}$$

$$P(R \mid Q') = \frac{0.54}{0.92} = \frac{27}{46} \approx 0.587$$

114. **(i)** Equation of the boundary line AB

The line passes through the points $A(0,4)$ and $B(-2,0)$.

(1) Find the slope (m):

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{-2-0} = \frac{-4}{-2} = 2$$

(2) Write the equation using the slope-intercept form ($y = mx + c$, where the y intercept $c = 4$):

$y = 2x + 4$

$\Rightarrow 2x - y + 4 = 0$

(ii) Equation of the boundary line AC

The line passes through the points $A(0,4)$ and $C(3,0)$.

(1) Find the slope (m):

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-4}{3-0} = -\frac{4}{3}$$

(2) Write the equation using the slope-intercept form ($c = 4$):

$$y = -\frac{4}{3}x + 4$$

$\Rightarrow 3y = -4x + 12$

$\Rightarrow 4x + 3y - 12 = 0$

(iii) (A) Area of region OAC using integration

The region OAC lies in the first quadrant and is bounded by $x = 0, x = 3$, the x-axis ($y = 0$), and the line AC $\left(y = -\frac{4}{3}x + 4\right)$.

$$\text{Area } = \int_0^3 y_{AC} dx$$

$$\text{Area } = \int_0^3 \left(-\frac{4}{3}x + 4\right) dx$$

$$\text{Area } = \left[-\frac{4}{3}\left(\frac{x^2}{2}\right) + 4x\right]_0^3 = \left[-\frac{2}{3}x^2 + 4x\right]_0^3$$

$$\text{Area } = \left(-\frac{2}{3}(3)^2 + 4(3)\right) - (0)$$

Area $= -6 + 12 = 6$ sq. units

OR

(B) Area of region AOB using integration

The region AOB lies in the second quadrant and is bounded by $x = -2, x = 0$, the x axis $(y = 0)$, and the line $AB(y = 2x + 4)$.

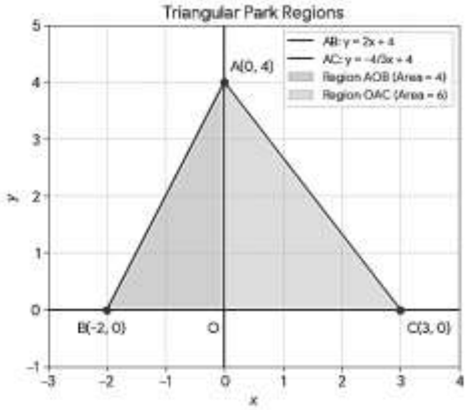
$$\text{Area } = \int_{-2}^{0} y_{AB}dx$$

$$\text{Area } = \int_{-2}^{0} (2x + 4)dx$$

$$\text{Area } = [x^2 + 4x]_{-2}^{0}$$

$$\text{Area } = (0) - ((-2)^2 + 4(-2))$$

$$\text{Area } = -(4 - 8) = 4 \text{ sq. units}$$



115. **(d)** The following graph represents:

The graph has:

• Domain: $(-\infty, -1] \cup [1, \infty)$

• Range: $[-\pi/2, \pi/2] \setminus \{0\}$

• Passes through $(1, \pi/2)$ and $(-1, -\pi/2)$

This is the graph of $y = \operatorname{cosec}^{-1}x$.

116. **(b)** • If $a_{ij} = \frac{|i-3j|}{2}$, then

$$a_{11} = \frac{|1-3|}{2} = 1, a_{12} = \frac{|1-6|}{2} = \frac{5}{2}$$

$$a_{21} = \frac{|2-3|}{2} = \frac{1}{2}, a_{22} = \frac{|2-6|}{2} = 2$$

$$A = \begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$$

117. **(b)** • Principal value of

$$\sec^{-1}(\sqrt{2}) + 2\operatorname{cosec}^{-1}(-\sqrt{2})$$

$$\sec^{-1}(\sqrt{2}) = \frac{\pi}{4}$$

$$\operatorname{cosec}^{-1}(-\sqrt{2}) = -\frac{\pi}{4}$$

Therefore,

$$\frac{\pi}{4} + 2\left(-\frac{\pi}{4}\right) = -\frac{\pi}{4}$$

118. **(c)** • If points $(2,3), (0,4)$ and $(p, 2)$ are collinear

Slope between $(2,3)$ and $(0,4)$:

$$m = \frac{4-3}{0-2} = -\frac{1}{2}$$

For collinearity:

$$\frac{2-3}{p-2} = -\frac{1}{2}$$

$$\frac{-1}{p-2} = -\frac{1}{2}$$
$$p - 2 = 2$$
$$p = 4$$

119. **(c)** $\frac{d}{dx}\left(e^{e^x}\right)$

(not e').

Using the chain rule:

Let

$y = e^{c^x}$

Then

$$\frac{dy}{dx} = e^{e^x} \cdot \frac{d}{dx}(e^x)$$
$$= e^{e^{e^x}} \cdot e^{e^x}$$
$$\frac{d}{dx}\left(e^{e^x}\right) = e^x e^{e^x}$$

120. **(b)** • Surface area of a sphere when its volume changes at the same rate as its radius

$$V = \frac{4}{3}\pi r^3$$
$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Given:

$$\frac{dV}{dt} = \frac{dr}{dt}$$
$$4\pi r^2 = 1$$

Hence surface area

$$S = 4\pi r^2 = 1$$

121. **(d)**• If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$

is continuous at $x = 0$, then

$$k = \lim_{x\to 0}\left(\frac{\sin x}{x} + \cos x\right)$$
$$= 1 + 1 = 2$$

122. **(b)** • Greatest Integer Function $f(x) = [x], 0 < x < 3$

The GIF is not differentiable at integers.

In (0,3), integers are:

1,2

So it is not differentiable at 2 points.

123. **(c)** • Evaluate

$$\int \frac{dx}{\sqrt{25 - 16x^2}}$$

Using

$$\int \frac{dx}{\sqrt{a^2 - b^2x^2}} = \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right) + C$$

with $a = 5, b = 4$.

$$= \frac{1}{4}\sin^{-1}\left(\frac{4x}{5}\right) + C$$

124. **(d)** If $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1}e + k$

Let $t = e^x, dx = \frac{dt}{t}$.

$$I = \int_1^e \frac{dt}{t^2 + 1}$$
$$= |\tan^{-1}t|_1^e$$

$= \tan^{-1}\varepsilon - \frac{\pi}{4}$
Comparing with
$\tan^{-1}e + k$
gives
$k = -\frac{\pi}{4}$

125. **(a)** Area bounded by $y = x$, x-axis, $x = 0$ and $x = 2$

$A = \int_0^2 x dx$

$= \left[\frac{x^2}{2}\right]_0^2$

$= \frac{4}{2} = 2$

126. **(b)** Given the differential equation:

$1 + \left(\frac{dy}{dx}\right)^3 = \lambda\left(\frac{d^3y}{dx^3}\right)^2$

Step 1: Find the Order
The highest order derivative present is
$\frac{d^3y}{dx^3}$
So,
Order = 3
Step 2: Find the Degree
The equation is already polynomial in derivatives, and the highest order derivative $\frac{d^by}{dz^3}$ is raised to the power 2.
Therefore,
Degree = 2
Step 3: Product of Order and Degree
$3 \times 2 = 6$

127. **(c)** (a) $(1 + x^2)\frac{dy}{dx} + 2xy = \cot x$

Linear in y.
(b) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

$y + x\frac{dy}{dx} + y = x(\sin x + \log x)$

$x\frac{dy}{dx} + 2y = x(\sin x + \log x)$
Linear.
(c) $x(1 + y^2)dx - y(1 + x^2)dy = 0$

$\frac{dy}{dx} = \frac{x(1 + y^2)}{y(1 + x^2)}$
Contains y^2, hence not linear.
(d) $ydx - (x + 3y^2)dy = 0$

$\frac{dy}{dx} = \frac{y}{x + 3y^2}$

128. **(a)** Region represented by
$3x + y \geq 3, 2x - y \geq -5, x, y \geq 0$
Rewrite:
$y \geq 3 - 3x$
$y \leq 2x + 5$
Since $x, y \geq 0$, the region lies in the 1st quadrant.
The region extends indefinitely to the right, so it is unbounded.

129. **(c)** • If all points on segment AB give maximum value of $Z = px + qy$ Line AB passes through:

$A(0,5), B(3,4)$
Slope of AB:
$m = \frac{4-5}{3-0} = -\frac{1}{3}$
For infinitely many optimal solutions, the objective line
$px + qy = k$
must be parallel to AB .
Slope of objective line:
$-\frac{p}{q}$
Hence,
$\frac{p}{q} = -\frac{1}{3}$
$\frac{p}{q} = \frac{1}{3}$
$q = 3p$

130. **(b)** If $(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}) \times (4\hat{\imath} + p\hat{\jmath} + q\hat{k}) = 0$
then the vectors are parallel.
So, $(4, p, q) = \lambda(3, -2, 5)$
Using first component:
$4 = 3\lambda$
$\lambda = \frac{4}{3}$
Therefore,
$p = -2 \cdot \frac{4}{3} = -\frac{8}{3}$
$q = 5 \cdot \frac{4}{3} = \frac{20}{3}$

131. **(a)** Area of Triangle ABC
Given:
$A(0,1,1), B(2,0,-1), C(1,0,3)$
$\overrightarrow{AB} = (2, -1, -2)$
$\overrightarrow{AC} = (1, -1, 2)$
$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & -2 \\ 1 & -1 & 2 \end{vmatrix} = (-4)\hat{\imath} - 6\hat{\jmath} - 1\hat{k}$
Magnitude:
$|\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{16 + 36 + 1} = \sqrt{53}$
Area of triangle:
$\text{Area} = \frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{\sqrt{53}}{2}$

132. **(d)** Probability that at least one marble is red
Total marbles:
$4 + 5 + 1 = 10$
Probability of not red in one draw:
$\frac{6}{10} = \frac{3}{5}$
Since replacement is made, draws are independent.
Probability of no red in 3 draws:
$\left(\frac{3}{5}\right)^3 = \frac{27}{125}$
Therefore,
$P(\text{ at least one red }) = 1 - \frac{27}{125} = \frac{98}{125}$

133. **(b)** • Assertion (A): For two square matrices A and B, AB is not necessarily equal to BA. This is True. Matrix multiplication is generally non-commutative.

• Reason (R): Product of two diagonal matrices of the same order is commutative. This is also True. (If D_1 and D_2 are diagonal, $D_1D_2 = D_2D_1$).

• Verdict: While both statements are true, the fact that diagonal matrices commute does not explain why matrix multiplication is not generally commutative.

134. **(a)** • Assertion (A): $f(x) = x^3 + 2$ is one-one because $x_1^3 + 2 = x_2^3 + 2 \Longrightarrow x_1 = x_2$. It is not onto because for $y = 3$ (in codomain N), $x^3 + 2 = 3 \Longrightarrow x^3 = 1 \Longrightarrow x = 1$ (which is in N). However, for $y = 1, x^3 = -1 \Longrightarrow x = -1 \notin N$. Since not all $y \in \mathbb{N}$ have a pre-image in N, it is not onto. Assertion is True.

• Reason (R): The stated reason correctly explains that there is no natural number x for certain y values, making the function not onto. Reason is True.

135. Evaluate

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$$

Term-wise:

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

$$\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{3}$$

$$\tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) = \tan^{-1}(-1) = -\frac{\pi}{4}$$

Principal value of $\tan^{-1}$lies in $(-\pi/2, \pi/2)$.

$$\tan^{-1}\left(\tan\frac{2\pi}{3}\right) = -\frac{\pi}{3}$$

Adding.

$$-\frac{\pi}{6} + \frac{\pi}{3} - \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{6} - \frac{\pi}{4} = -\frac{5\pi}{12}$$

136. **(A)** Show continuity at $x = \frac{\pi}{2}$

$$f(x) = \begin{cases} \dfrac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \dfrac{\pi}{2} \\ 1, & x = \dfrac{\pi}{2} \end{cases}$$

Check limit:

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos x}{-\left(x - \frac{\pi}{2}\right)}$$

Using L'Hospital's Rule,

$$= \lim_{x \to \frac{\pi}{2}} \frac{-\sin x}{-1} = \sin\frac{\pi}{2} = 1$$

Since

$$\lim_{x \to \frac{\pi}{2}} f(x) = 1 = f\left(\frac{\pi}{2}\right)$$

f is continuous at $x = \frac{\pi}{2}$

OR

(B) Differentiability at $x = 2$

$$f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$$

Continuity:

$LHL = 2 - 1 = 1$

$RHL = 2(2) - 3 = 1$

$f(2) = 1$

Continuous.

LHD:

$\frac{d}{dx}(x-1)=1$

RHD:

$\frac{d}{dx}(2x-3)=2$

Since

LHD $\neq$ RHD

Not differentiable at $x=2$

137. Find values of x for which $f(x)=x^x$ is increasing Let

$y=x^2$

$\ln y = x\ln x$

Differentiating.

$\frac{1}{y}\frac{dy}{dx}=1+\ln x$

$\frac{dy}{dx}=x^x(1+\ln x)$

Since $x^z>0$,

$\frac{dy}{dx}>0 \Leftrightarrow 1+\ln x>0$

$\ln x>-1$

$x>e^{-1}$

$x>\frac{1}{e}$

138. Show that triangle is isosceles

Points:

$A=(3,1,0), B=(5,6,-3), C=(0,4,0)$

$AB^2=(5-3)^2+(6-1)^2+(-3)^2=4+25+9=38$

$AC^2=(0-3)^2+(4-1)^2=9+9=18$

$BC^2=(0-5)^2+(4-6)^2+(0+3)^2=25+4+9=38$

$AB=BC=\sqrt{38}$

Hence,

$\triangle$ ABC is isosceles

139. **(A)** Vectors representing rods:

$\vec{a}=4\hat{i}-\hat{j}+3\hat{k}$

$\vec{b}=-2\hat{i}+\hat{j}-2\hat{k}$

Required vector is perpendicular to both:

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & -1 & 3\\ -2 & 1 & -2\end{vmatrix}=(-1)\hat{i}+2\hat{j}+2\hat{k}$$

Magnitude:

$\sqrt{1+4+4}=3$

Unit vector:

$\frac{\hat{i}+2\hat{j}+2\hat{k}}{3}$

For height 5 m ,

$\vec{v}=\frac{5}{3}(-\hat{i}+2\hat{j}+2\hat{k})$

OR

(B) Given:

$\alpha=\frac{\pi}{4}, \beta=\frac{\pi}{3}$

Direction cosines:

$l=\cos\alpha=\frac{1}{\sqrt{2}}$

$m = \cos\beta = \frac{1}{2}$

Using

$l^2 + m^2 + n^2 = 1$

$\frac{1}{2} + \frac{1}{4} + n^2 = 1$

$n^2 = \frac{1}{4}$

Since angle with z-axis is acute,

$n = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}$

Components of $\vec{a}$:

$\vec{a} = \frac{1}{\sqrt{2}}\hat{\imath} + \frac{1}{2}\hat{\jmath} + \frac{1}{2}\hat{k}$

140. **(A)** $A = R - \{3\}$, $B = R - \{1\}$

$f(x) = \frac{x-2}{x-3}$

One-One

Let

$f(x_1) = f(x_2)$

$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

Cross-multiplying:

$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$x_1 = x_2$

Hence f is one-one.

Onto

Let

$y = \frac{x-2}{x-3}$

$yx - 3y = x - 2$

$x(y-1) = 3y - 2$

$x = \frac{3y-2}{y-1}$

Since $y \neq 1$ (codomain $B = \mathbb{R} - \{1\}$), x exists and $x \neq 3$.

Hence f is onto.

f is one-one and onto (bijective)

OR

(B) $R = \{(x, y): (x - y) \text{ is divisible by } n\}$

Reflexive

$x - x = 0$

and 0 is divisible by n.

Reflexive

Symmetric

If $n \mid (x - y)$.

$n \mid -(x - y) = (y - x)$

Symmetric

Transitive

If

$n|(x - y), n|(y - z)$

Then

$n \mid [(x - y) + (y - z)]$

$n \mid (x-z)$

Transitive

Therefore,

R is an equivalence relation

141. $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

First compute A^2 :

$A^2 = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix}$

Now

$A^2 - 7A = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -10 & 0 \\ 0 & 0 & -10 \end{bmatrix}$

Adding 10I,

$A^2 - 7A + 10I = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A^2 - 7A + 10I = O$

142. **(A)** Given

$xy = e^{x-y}$

Taking logarithm:

$\ln x + \ln y = x - y$

Differentiate:

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$

Collecting terms:

$\frac{dy}{dx}\left(1 + \frac{1}{y}\right) = 1 - \frac{1}{x}$

$\frac{dy}{dx} = \frac{y(x-1)}{x(y+1)}$

OR

(B) Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$

Let $x = \sin\theta$. Then:

$\sqrt{1-x^2} = \cos\theta, \sqrt{1+x^2} = \sqrt{1+\sin^2\theta}$

So the expression simplifies to a standard identity form:

$y = \frac{\theta}{2} = \frac{1}{2}\sin^{-1} x$

Now we need:

$\frac{dy}{d(\cos^{-1} x^2)}$

Step 1:

$y = \frac{1}{2}\sin^{-1} x \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}}$

Step 2:

$\frac{d}{dx}(\cos^{-1} x^2) = \frac{-2x}{\sqrt{1-x^4}}$

Step 3:

$$\frac{dy}{d(\cos^{-1} x^2)} = \frac{\frac{1}{2\sqrt{1-x^2}}}{\frac{-2x}{\sqrt{1-x^4}}}$$

Since $\sqrt{1-x^4} = \sqrt{(1-x^2)(1+x^2)}$, it simplifies to a constant:

$-\frac{1}{2}$

143. Maximize $Z = 200x + 120y$ subject to:

- $x + y \leq 300$
- $3x + y \leq 600$
- $x - y \geq -100 \Longrightarrow y - x \leq 100$
- $x, y \geq 0$

Corner Points of Feasible Region

By finding the intersection points of the boundary lines, we get the following corner vertices:

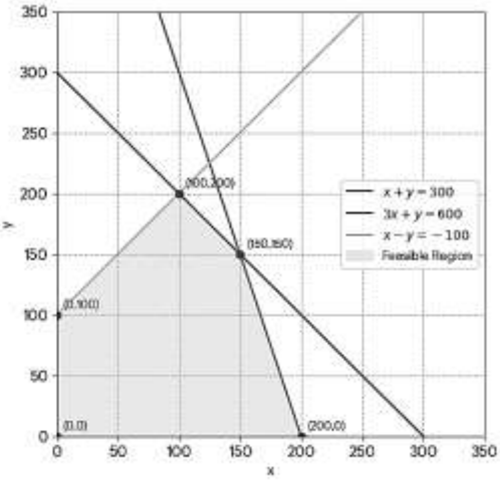
(1) $(0,0)$

(2) $(200,0)$-Intersection of $3x + y = 600$ and $y = 0$

(3) $(150, 150)$ - Intersection of $3x + y = 600$ and $x + y = 300$

(4) $(100,200)$ - Intersection of $x + y = 300$ and $y - x = 100$

(5) $(0,100)$ - Intersection of $y - x = 100$ and $x = 0$



Objective Function Values

Corner Point (x, y)	$Z = 200x + 120y$
(0.0)	$Z = 0$
(200,0)	$Z = 200(200) + 0 = 40{,}000$
(150, 150)	$Z = 200(150) + 120(150) = 48{,}000 \leftarrow$ Maximum
(100,200)	$Z = 200(100) + 120(200) = 44{,}000$
(0,100)	$Z = 200(0) + 120(100) = 12{,}000$

The maximum value of Z is 48,000 at the point $(150,150)$.

144. **(A)** Given line:

$$\frac{x-2}{3} = \frac{1-y}{2} = \frac{z-3}{2} = t$$

So,

$x = 2 + 3t, y = 1 - 2t, z = 3 + 2t$

Point on line:

$P(2 + 3t, 1 - 2t, 3 + 2t)$

Distance from (1,2,3) is $\sqrt{2}$:
$(1+3t)^2+(-1-2t)^2+(2t)^2=2$
$1+6t+9t^2+1+4t+4t^2+4t^2=2$
$17t^2+10t=0$
$t(17t+10)=0$
$t=0 \text{ or } t=-\frac{10}{17}$
Hence the required points are
(2,1,3)
and
$\left(\frac{4}{17},\frac{37}{17},\frac{31}{17}\right)$

OR

(B) Lines:
$\vec{r}=(4+\lambda)\hat{i}+(2\lambda-1)\hat{j}-3\lambda\hat{k}$
$\vec{r}=\hat{i}+4\mu\hat{j}+(2-5\mu)\hat{k}+2\mu\hat{i}-\hat{j}$
i.e.
$\vec{r}=(1+2\mu)\hat{i}+(4\mu-1)\hat{j}+(2-5\mu)\hat{k}$
Direction vectors:
$\vec{b}_1=(1,2,-3)$
$\vec{b}_2=(2,4,-5)$
Points on lines:
$A(4,-1,0), B(1,-1,2)$
$\overrightarrow{AB}=(-3,0,2)$
$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}=(2,-1,0)$
Shortest distance:
$d=\frac{|\overrightarrow{AB}\cdot(\vec{b}_1\times\vec{b}_2)|}{|\vec{b}_1\times\vec{b}_2|}$
$=\frac{|(-3,0,2)\cdot(2,-1,0)|}{\sqrt{4+1}}=\frac{6}{\sqrt{5}}$

145. Probability of debate competition:
$P(D)=\frac{1}{3}$
Probability of quiz competition:
$P(Q)=\frac{2}{3}$
Case 1: Debate
Students selected from Team A.
Team A: 4 girls, 6 boys.
$P(GB \mid D)=\frac{\binom{4}{1}\binom{6}{1}}{\binom{10}{2}}=\frac{24}{45}=\frac{8}{15}$
Contribution:
$\frac{1}{3}\times\frac{8}{15}=\frac{8}{45}$
Case 2: Quiz
Students selected from Team B.
Team B: 7 girls, 3 boys.
$P(GB \mid Q)=\frac{\binom{7}{1}\binom{3}{1}}{\binom{10}{2}}=\frac{21}{45}=\frac{7}{15}$
Contribution:

$\frac{2}{3}\times\frac{7}{15}=\frac{14}{45}$

Total probability:

$\frac{8}{45}+\frac{14}{45}=\frac{22}{45}$

146. **(A)** $I=\int \frac{x}{(x-1)(x^2+4)}dx$

Partial fractions:

$$\frac{x}{(x-1)(x^2+4)}=\frac{1}{5(x-1)}-\frac{x-4}{5(x^2+4)}$$

Hence

$$I=\frac{1}{5}\int\frac{dx}{x-1}-\frac{1}{5}\int\frac{xdx}{x^2+4}+\frac{4}{5}\int\frac{dx}{x^2+4}$$

$$I=\frac{1}{5}\ln|x-1|-\frac{1}{10}\ln(x^2+4)+\frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right)+C$$

$$I=\frac{1}{5}\ln|x-1|-\frac{1}{10}\ln(x^2+4)+\frac{2}{5}\tan^{-1}\left(\frac{x}{2}\right)+C$$

OR

(B) $I=\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}dx$

Integrate by parts:

$u=\tan^{-1}x,\ dv=\frac{xdx}{(1+x^2)^{3/2}}$

$du=\frac{dx}{1+x^2},\ v=-\frac{1}{\sqrt{1+x^2}}$

Therefore

$$I=\left[-\frac{\tan^{-1}x}{\sqrt{1+x^2}}\right]_0^1+\int_0^1\frac{dx}{(1+x^2)^{3/2}}$$

$$=-\frac{\pi}{4\sqrt{2}}+\left[\frac{x}{\sqrt{1+x^2}}\right]_0^1$$

$$=-\frac{\pi}{4\sqrt{2}}+\frac{1}{\sqrt{2}}$$

$$I=\frac{1-\frac{\pi}{4}}{\sqrt{2}}$$

147. (1) Define the limits and functions:

The given curve is $y=|x-6|$. The critical point where the definition changes is $x=6$

- For $4\le x\le 6: y=-(x-6)=6-x$
- For $6\le x\le 8: y=x-6$

(2) Set up the definite integrals:

$$\text{Area }=\int_4^8|x-6|dx=\int_4^6(6-x)dx+\int_6^8(x-6)dx$$

(3) Evaluate the integrals:

- First Part:

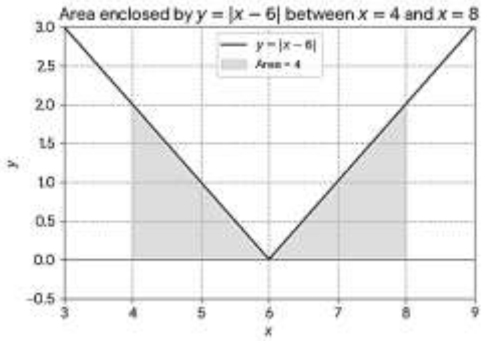
$$\int_4^6(6-x)dx=\left[6x-\frac{x^2}{2}\right]_4^6=(36-18)-(24-8)=18-16=2$$

- Second Part:

$$\int_6^8(x-6)dx=\left[\frac{x^2}{2}-6x\right]_6^8=(32-48)-(18-36)=-16-(-18)=2$$

(4) Total Area:

Area $=2+2=4$ square units



148. **(A)** (1) Rearrange into standard linear form:

Given: $ye^y dx = (y^3 + 2xe^y)dy$

$$\frac{dx}{dy} = \frac{y^3 + 2xe^y}{ye^y} = \frac{y^2}{e^y} + \frac{2x}{y}$$

$$\frac{dx}{dy} - \frac{2}{y}x = y^2e^{-y}$$

(2) Find the Integrating Factor (I.F.):

$$P(y) = -\frac{2}{y} \Rightarrow I \cdot F \cdot = e^{\int -\frac{2}{y}dy} = e^{-2\ln y} = \frac{1}{y^2}$$

(3) Solve the differential equation:

$$x \cdot \left(\frac{1}{y^2}\right) = \int \left(y^2e^{-y} \cdot \frac{1}{y^2}\right) dy + C$$

$$\frac{x}{y^2} = \int e^{-y}dy + C \Rightarrow \frac{x}{y^2} = -e^{-y} + C$$

$$x = -y^2e^{-y} + Cy^2$$

(4) Apply the initial condition $y(0) = 1$:

$$0 = -1^2e^{-1} + C(1)^2 \Rightarrow C = \frac{1}{e}$$

(5) Final Particular Solution:

$$x = y^2\left(\frac{1}{e} - e^{-y}\right)$$

OR

(B) (1) Express as a homogeneous differential equation:

$$(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy \Rightarrow \frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$$

(2) Substitute $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$:

$$v + x\frac{dv}{dx} = \frac{x^3 - 3x(vx)^2}{(vx)^3 - 3x^2(vx)} = \frac{1 - 3v^2}{v^3 - 3v}$$

$$x\frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} - v = \frac{1 - v^4}{v^3 - 3v}$$

(3) Separate variables and integrate:

$$\int \frac{v^3 - 3v}{1 - v^4}dv = \int \frac{dx}{x}$$

Split the integral:

$$\int \frac{v^3}{1 - v^4}dv - \int \frac{3v}{1 - v^4}dv = \ln|x| + C_1$$

$$-\frac{1}{4}\ln|1 - v^4| - \frac{3}{4}\ln\left|\frac{1 + v^2}{1 - v^2}\right| = \ln|x| + C_1$$

Multiplying by -4 and simplifying using $1 - v^4 = (1 - v^2)(1 + v^2)$ simplifies to:

$$\ln\left[\frac{(1 + v^2)^2}{1 - v^2}\right] = \ln|x^{-2}| + \ln|K|$$

$$\frac{(1 + v^2)^2}{1 - v^2} = \frac{K}{x^2}$$

(4) Substitute back $v = \frac{y}{x}$:

$$\frac{\left(1+\frac{y^2}{x^2}\right)^2}{1-\frac{y^2}{x^2}} = \frac{K}{x^2} \Rightarrow \frac{(x^2+y^2)^2}{x^2(x^2-y^2)} = \frac{K}{x^2}$$

$(x^2+y^2)^2 = K(x^2-y^2)$

149. (1) Find the point of intersection:

Let the first line be: $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda \Rightarrow (2\lambda+1, 3\lambda+2, 4\lambda+3)$

Let the second line be: $\frac{x-4}{5} = \frac{y-1}{2} = z = \mu \Rightarrow (5\mu+4, 2\mu+1, \mu)$

Equating the z-coordinates: $\mu = 4\lambda + 3$

Equating the y-coordinates: $2(4\lambda+3)+1 = 3\lambda+2 \Rightarrow 8\lambda+7 = 3\lambda+2 \Rightarrow 5\lambda = -5 \Rightarrow \lambda = -1$

Substitute $\lambda = -1$:

• Point coordinates: $x = -1, y = -1, z = -1$

• Intersection point: $P(-1,-1,-1)$

(2) Equation of the required line:

The line passes through $P(-1,-1,-1)$ and is parallel to $\vec{b} = 3\hat{i} + 2\hat{j} - 8\hat{k}$.

• Vector Form:

$\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \kappa(3\hat{i} + 2\hat{j} - 8\hat{k})$

• Cartesian Form:

$$\frac{x+1}{3} = \frac{y+1}{2} = \frac{z+1}{-8}$$

150. **(i)** Relation between Height (h) and Radius (r)

From the geometry of the right circular cone:

$$\tan\alpha = \frac{\text{Radius}}{\text{Height}}$$

For the entire cup:

$$\tan\alpha = \frac{R}{H} = \frac{5}{15} = \frac{1}{3}$$

For the juice level inside the cup:

$$\tan\alpha = \frac{r}{h}$$

Equating the two expressions:

$$\frac{r}{h} = \frac{1}{3} \Rightarrow r = \frac{h}{3} \quad \text{or} \quad h = 3r$$

(ii) Rate at which the Juice Level is Rising $\left(\frac{dh}{dt}\right)$

The volume V of a cone is:

$$V = \frac{1}{3}\pi r^2 h$$

Substitute $r = \frac{h}{3}$ into the volume formula:

$$V = \frac{1}{3}\pi\left(\frac{h}{3}\right)^2 h = \frac{\pi h^3}{27}$$

Differentiate both sides with respect to time t :

$$\frac{dV}{dt} = \frac{\pi}{27} \cdot 3h^2 \frac{dh}{dt} = \frac{\pi h^2}{9}\frac{dh}{dt}$$

Given that $\frac{dV}{df} = 0.1\ \text{cm}^3/\text{s}$, substitute $h = 6$ cm :

$$0.1 = \frac{\pi(6)^2}{9}\frac{dh}{dt}$$

$$0.1 = 4\pi\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{40\pi}\ \text{cm/s}$$

(iii) (A) Rate of Increase of Upper Surface Area $\left(\frac{dA}{dt}\right)$
The upper surface area A is a circle:
$A = \pi r^2$
Substitute $r = \frac{h}{3}$:
$$A = \pi\left(\frac{h}{3}\right)^2 = \frac{\pi h^2}{9}$$
Differentiate with respect to time t :
$$\frac{dA}{dt} = \frac{2\pi h}{9}\frac{dh}{dt}$$
Substitute $h = 6$ cm and $\frac{dh}{dt} = \frac{1}{40\pi}$ cm/s :
$$\frac{dA}{dt} = \frac{2\pi(6)}{9} \cdot \frac{1}{40\pi} = \frac{12\pi}{360\pi} = \frac{1}{30}\ \text{cm}^2/\text{s}$$
OR

(B) Rate of Increase of Wetted Surface Area $\left(\frac{dS}{dt}\right)$
The curved (wetted) surface area S is:
$S = \pi r l$
Where the slant height $l = \sqrt{r^2 + h^2} = \sqrt{\left(\frac{h}{3}\right)^2 + h^2} = \frac{\sqrt{10}}{3}h$.
Substitute r and I into the area formula:
$$S = \pi\left(\frac{h}{3}\right)\left(\frac{\sqrt{10}}{3}h\right) = \frac{\sqrt{10}\pi h^2}{9}$$
Differentiate with respect to time t :
$$\frac{dS}{dt} = \frac{2\sqrt{10}\pi h}{9}\frac{dh}{dt}$$
Substitute $h = 6$ cm and $\frac{dh}{df} = \frac{1}{40\pi}$ cm/s :
$$\frac{dS}{dt} = \frac{2\sqrt{10}\pi(6)}{9} \cdot \frac{1}{40\pi} = \frac{12\sqrt{10}\pi}{360\pi} = \frac{\sqrt{10}}{30}\ \text{cm}^2/\text{s}$$

151. **(i)** Formulating the System of Equations and Matrix Form
Let the length, breadth, and height of the cuboid box be x, y and z respectively (in cm).
(1) Equation 1: Sum of length and breadth is 3 cm more than height.
$x + y = z + 3 \Longrightarrow x + y - z = 3$
(2) Equation 2: Twice the length, thrice the breadth, and height add up to 10 cm .
$2x + 3y + z = 10$
(3) Equation 3: Breadth added to 7 times height is 1 cm less than 3 times length.
$y + 7z = 3x - 1 \Longrightarrow -3x + y + 7z = -1$
Expressing these linear equations as a matrix equation $AX = B$:
$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 10 \\ -1 \end{bmatrix}$$
(ii) Existence of A^{-1}
A matrix inverse A^{-1} exists if and only if its determinant is non-zero ($\det(A) \neq 0$).
$$\det(A) = 1\begin{vmatrix} 3 & 1 \\ 1 & 7 \end{vmatrix} - 1\begin{vmatrix} 2 & 1 \\ -3 & 7 \end{vmatrix} + (-1)\begin{vmatrix} 2 & 3 \\ -3 & 1 \end{vmatrix}$$
$\det(A) = 1(21 - 1) - 1(14 - (-3)) - 1(2 - (-9))$
$\det(A) = 20 - 17 - 11 = -8$
Justification: Since $\det(A) = -8 \neq 0$, the matrix A is non-singular. Therefore, A^{-1} exists.
(iii) (A) Finding A^{-1}
The formula for the inverse is $A^{-1} = \frac{1}{\det(A)}\text{adj}(A)$. Let's find the cofactors C_{ij} of matrix A :
- $C_{11} = +(21 - 1) = 20$
- $C_{12} = -(14 + 3) = -17$
- $C_{13} = +(2 + 9) = 11$

- $C_{21} = -(7+1) = -8$
- $C_{22} = +(7-3) = 4$
- $C_{23} = -(1+3) = -4$
- $C_{31} = +(1+3) = 4$
- $C_{32} = -(1+2) = -3$
- $C_{33} = +(3-2) = 1$

The adjugate matrix adj(A) is the transpose of the cofactor matrix:

$$\operatorname{adj}(A) = \begin{bmatrix} 20 & -8 & 4 \\ -17 & 4 & -3 \\ 11 & -4 & 1 \end{bmatrix}$$

Thus, the inverse matrix is:

$$A^{-1} = -\frac{1}{8}\begin{bmatrix} 20 & -8 & 4 \\ -17 & 4 & -3 \\ 11 & -4 & 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -20 & 8 & -4 \\ 17 & -4 & 3 \\ -11 & 4 & -1 \end{bmatrix}$$

OR

(B) Finding $A^2 + 7I$

First, calculate A^2 :

$$A^2 = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ -3 & 1 & 7 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1(1)+1(2)+(-1)(-3) & 1(1)+1(3)+(-1)(1) & 1(-1)+1(1)+(-1)(7) \\ 2(1)+3(2)+1(-3) & 2(1)+3(3)+1(1) & 2(-1)+3(1)+1(7) \\ -3(1)+1(2)+7(-3) & -3(1)+1(3)+7(1) & -3(-1)+1(1)+7(7) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 6 & 3 & -7 \\ 5 & 12 & 8 \\ -22 & 7 & 53 \end{bmatrix}$$

Now, add 7I (where I is the 3×3 identity matrix):

$$A^2 + 7I = \begin{bmatrix} 6 & 3 & -7 \\ 5 & 12 & 8 \\ -22 & 7 & 53 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix} = \begin{bmatrix} 13 & 3 & -7 \\ 5 & 19 & 8 \\ -22 & 7 & 60 \end{bmatrix}$$

152. We have three boxes:

Box	Red caps	Green caps	Total caps
I	30	0	30
II	20	10	30
III	0	30	30

The probability of selecting Box i is $P(B_i) = \frac{i}{6}$:

$$P(B_1) = \frac{1}{6}, P(B_2) = \frac{2}{6} = \frac{1}{3}, P(B_3) = \frac{3}{6} = \frac{1}{2}$$

(i) Probability of selecting a red cap

Use law of total probability:

$$P(\text{ Red }) = \sum_{i=1}^{3} P(B_i) \cdot P(\text{ Red } \mid B_i)$$

- Box I: $P(\text{Red} \mid B_1) = \frac{30}{30} = 1$
- Box ||: $P(\text{Red} \mid B_2) = \frac{20}{30} = \frac{2}{3}$
- Box III: $P(\text{Red} \mid B_3) = 0$

$$P(\text{ Red }) = \frac{1}{6} \cdot 1 + \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{2} \cdot 0 = \frac{1}{6} + \frac{2}{9} = \frac{3}{18} + \frac{4}{18} = \frac{7}{18}$$

(ii) Probability that cap came from Box II given it is green

Use Bayes' theorem:

$$P(B_2 \mid \text{ Green }) = \frac{P(B_2) \cdot P(\text{ Green } \mid B_2)}{\sum_{i=1}^{3} P(B_i) \cdot P(\text{ Green } \mid B_i)}$$

- P (Green | B_1) = 0
- P (Green | B_2) $= \frac{10}{30} = \frac{1}{3}$
- P (Green | B_3) $= \frac{30}{30} = 1$

Numerator:

$$P(B_2) \cdot P(\text{ Green } \mid B_2) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$$

Denominator:

$$0 + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{2} \cdot 1 = \frac{1}{9} + \frac{1}{2} = \frac{1}{9} + \frac{9}{18} = \frac{2}{18} + \frac{9}{18} = \frac{11}{18}$$

$$P(B_2 \text{ Green }) = \frac{\frac{1}{9}}{\frac{11}{18}} = \frac{2}{11}$$

153. **(b)** Compute A^2 :

$$A^2 = \begin{bmatrix} -p & q \\ r & p \end{bmatrix}\begin{bmatrix} -p & q \\ r & p \end{bmatrix} = \begin{bmatrix} (-p)(-p) + qr & (-p)(q) + qp \\ r(-p) + pr & rq + p^2 \end{bmatrix} = \begin{bmatrix} p^2 + qr & 0 \\ 0 & p^2 + qr \end{bmatrix}$$

Given $A^2 = I$:

$$\begin{bmatrix} p^2 + qr & 0 \\ 0 & p^2 + qr \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Comparing the corresponding elements:

$p^2 + qr = 1 \Longrightarrow 1 - p^2 - qr = 0$

154. **(b)** • Since A and identity matrix I commute (AI = IA = A), expand $(A - I)^3$ binomially:

$(A - I)^3 = A^3 - 3A^2I + 3AI^2 - I^3 = A^3 - 3A^2 + 3A - I$

Since $A^2 = A$, then $A^3 = A \cdot A^2 = A \cdot A = A^2 = A$. Substitute these values back into the expression:

$(A - I)^3 = A - 3A + 3A - I = A - I$

Now, calculate the complete expression:

$(A - I)^3 - A = (A - I) - A = -I$

155. **(d)** The standard principal value branch (range) for $\operatorname{cosec}^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$. Option (d) states the branch as $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ without excluding 0 (where cosecant is undefined). Therefore, it is incorrect.

156. **(c)** • Rearranging the equation gives: $C = -(A + B)$ First, calculate $A + B$:

$$A + B = \begin{bmatrix} 0 + (-3) & -3 + 0 & 4 + 1 \\ 1 + 2 & 0 + 4 & 2 + 0 \end{bmatrix} = \begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$$

Now, find C :

$$C = -\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$$

157. **(a)** For a non-singular matrix, determinant $|A| \neq 0$.

The determinant of the adjoint matrix is given by $|\operatorname{adj} A| = |A|^{n-1}$.

Since $|A| \neq 0$, it follows that $|\operatorname{adj} A| \neq 0$, meaning $\operatorname{adj} A$ must also be nonsingular. Therefore, statement (a) is false.

158. **(d)** • For continuity at $x = -1$, the limit must equal the value of the function:

$$\lim_{x \to -1} f(x) = f(-1)$$

Calculate the limit by factoring the numerator:

$$\lim_{x \to -1} \frac{x^2 - 4x - 5}{x + 1} = \lim_{x \to -1} \frac{(x - 5)(x + 1)}{x + 1} = \lim_{x \to -1} (x - 5) = -1 - 5 = -6$$

Since $f(-1) = k$, we get:

$k = -6$

159. **(b)** Area of $\triangle ABC = 5$ sq. units

Vertices:

$A(3,1), B(-2,1), C(0,k)$

Using area formula:

$$\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 5$$

$\frac{1}{2}|3(1-k)+(-2)(k-1)|=5$

$\frac{1}{2}|5-5k|=5$

$|1-k|=2$

$k=3$ or $k=-1$

160. **(a)** Differentiate

$y=\cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$

Since

$\frac{\sin x+\cos x}{\sqrt{2}}=\sin\left(x+\frac{\pi}{4}\right)$

Given

$-\frac{\pi}{4}<x<\frac{\pi}{4}$

so

$x+\frac{\pi}{4}\in\left(0,\frac{\pi}{2}\right)$

Hence

$\cos^{-1}\left(\sin\left(x+\frac{\pi}{4}\right)\right)=\frac{\pi}{2}-\left(x+\frac{\pi}{4}\right)=\frac{\pi}{4}-x$

Differentiating.

$\frac{dy}{dx}=-1$

161. **(c)** Absolute minimum value of

$f(x)=(x-2)^2+5, x\in[-3,2]$

Since $(x-2)^2\geq 0$,

minimum occurs at $x=2$.

$f(2)=5$

162. **(b)** Evaluate

$\int\frac{1}{\sqrt{1+\cos 2x}}dx$

Using

$1+\cos 2x=2\cos^2 x$

$\int\frac{dx}{\sqrt{2}\cos x}=\frac{1}{\sqrt{2}}\int\sec x dx$

$=\frac{1}{\sqrt{2}}\log|\sec x+\tan x|+C$

163. **(a)** Evaluate

$\int_{-5}^{-1}\frac{1}{x}dx$

Since $1/x$ is continuous on $[-5,-1]$.

$=[\ln|x|]_{-5}^{-1}$

$=\ln 1-\ln 5=-\ln 5$

164. **(b)** • Area bounded by

$y=2x-4, x=0, x=1, y=0$

Between $x=0$ and $x=1$, the line lies below the x-axis.

Area

$=\int_0^1(0-(2x-4))dx=\int_0^1(4-2x)dx$

$=[4x-x^2]_0^1=4-1=3$

165. **(a)** The order and degree of the differential equation $1+\left(\frac{d^3y}{dx^3}\right)^3=\lambda\frac{d^2y}{dx^2}$ is:

• Order: The highest order derivative present in the equation is $\frac{d^3y}{dx^3}$, which is of order 3.

• Degree: The power of the highest order derivative when the differential equation is a polynomial in its derivatives is 3.

166. **(a)** The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is:

Using the method of separation of variables:

$$\frac{dy}{dx} = e^{3x} \cdot e^{-y}$$

Rearranging the terms:

$$e^y dy = e^{3x} dx$$

Integrating both sides:

$$\int e^y dy = \int e^{3x} dx$$

$$e^y = \frac{e^{3x}}{3} + C'$$

Multiplying by 3:

$$3e^y = e^{3x} + 3C'$$

$3e^y = e^{3x} + C$ (where $C = 3C'$)

167. **(b)** The corner points of the feasible region are $(0,0), (0,40), (20,40), (60,20)$ and $(60,0)$. If the objective function is $Z = 4x + 3y$, find the maximum value.

Evaluate Z at each corner point:

• At $(0,0): Z = 4(0) + 3(0) = 0$
• At $(0,40): Z = 4(0) + 3(40) = 120$
• At $(20,40): Z = 4(20) + 3(40) = 80 + 120 = 200$
•At $(60,20): Z = 4(60) + 3(20) = 240 + 60 = 300$
• At $(60,0): Z = 4(60) + 3(0) = 240$

The maximum value is 300 at point $(60,20)$.

168. **(d)** If the position vector $\vec{p}$ of a point ($24, n$) is such that $|\vec{p}| = 25$, find the value of n.

The vector is given by $\vec{p} = 24\hat{\imath} + n\hat{\jmath}$.

$$|\vec{p}| = \sqrt{24^2 + n^2} = 25$$

Squaring both sides:

$$576 + n^2 = 625$$

$$n^2 = 625 - 576 = 49$$

$$n = \pm 7$$

169. **(c)** If vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + \lambda\hat{k}$ and $\vec{b} = 2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ represent the two strips of the Red Cross sign, find the value of λ.

Since the strips of a Red Cross sign are perpendicular to each other, the dot product of their vectors must equal zero ($\vec{a} \cdot \vec{b} = 0$):

$$(3)(2) + (2)(-4) + (\lambda)(5) = 0$$

$$6 - 8 + 5\lambda = 0$$

$$-2 + 5\lambda = 0$$

$$5\lambda = 2 \Rightarrow \lambda = \frac{2}{5}$$

170. **(d)** If $3P(A) = P(B) = \frac{3}{5}$ and $P(A \mid B) = \frac{2}{3}$, find $P(A \cup B)$.

From the given values:

• $P(B) = \frac{3}{5}$
• $3P(A) = \frac{3}{5} \Rightarrow P(A) = \frac{1}{5}$

Using the conditional probability formula:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{2}{3} = \frac{P(A \cap B)}{3/5} \Rightarrow P(A \cap B) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$$

Now use the addition theorem of probability:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cup B) = \frac{1}{5} + \frac{3}{5} - \frac{2}{5} = \frac{2}{5}$

171. **(a)** • Assertion (A): The relation $R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$ on the set $\{1,2,3\}$.

Reflexive: Since $(1,1), (2,2), (3,3) \in R$, it is reflexive.
Symmetric: Since $(1,2) \in R \Longrightarrow (2,1) \in R$, it is symmetric.
Transitive: Since $(1,2) \in R$ and $(2,1) \in R \Longrightarrow (1,1) \in R$, it is transitive.
Therefore, R is an equivalence relation. Assertion (A) is True.

• Reason (R): A relation that is reflexive, symmetric, and transitive is an equivalence relation. This is the correct definition. Reason (R) is True.

• Conclusion: So, Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

172. **(a)** • Assertion (A): For minimizing $Z = x + 2y$ subject to $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0$:

Testing the corner points $(0,3)$ and $(6,0)$:
At $(0,3)$: $Z = 0 + 2(3) = 6$
At $(6,0)$: $Z = 6 + 2(0) = 6$
Since both points give the same minimum value, any point on the line segment joining them also minimizes Z, resulting in infinitely many solutions. Assertion (A) is True.

• Reason (R): If two corner points produce the same minimum value, every point on the line segment joining them gives the same minimum value. Reason (R) is True and explains Assertion (A).

• Conclusion: So, Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

173. Evaluate $\sin\left[\tan^{-1}\left(\tan\left(\frac{3x}{4}\right)\right)\right]$

(1) Simplify the inner tangent:
$\tan\left(\frac{3\pi}{4}\right) = \tan\left(\pi - \frac{\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1$
(2) Evaluate the principal value of $\tan^{-1}(-1)$:
$\tan^{-1}(-1) = -\frac{\pi}{4}$ since $-\frac{\pi}{2} < -\frac{\pi}{4} < \frac{\pi}{2}$
(3) Substitute back into the sine function:
$\sin\left(-\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

174. **(A)** Differentiate x^x with respect to $x\log x$

Let $u = x^x$ and $v = x\log x$. We need to find $\frac{du}{dv}$.
(1) Take the natural logarithm of u :
$\log u = \log(x^x) = x\log x = v$
(2) Rewrite u in terms of v :
$u = e^v$
(3) Differentiate u with respect to v :
$\frac{du}{dv} = e^v = x^x$

OR

(B) If $y = P\cos(ux) + Q\sin(ux)$, show that $\frac{d^2y}{dx^2} + u^2y = 0$
(1) Differentiate once with respect to x :
$\frac{dy}{dx} = -Pu\sin(ux) + Qu\cos(ux)$
(2) Differentiate a second time:
$\frac{d^2y}{dx^2} = -Pu^2\cos(ux) - Qu^2\sin(ux)$
(3) Factor out $-u^2$:
$\frac{d^2y}{dx^2} = -u^2(P\cos(ux) + Q\sin(ux)) = -u^2y$
$\Longrightarrow \frac{d^2y}{dx^2} + u^2y = 0$ (Proved)

175. Determine the values of x for which $f(x)=\frac{x-3}{x+1}, x \neq -1$ is increasing

(1) Find the derivative $f'(x)$ using the quotient rule:

$$f'(x)=\frac{(x+1)\cdot(1)-(x-3)\cdot(1)}{(x+1)^2}=\frac{x+1-x+3}{(x+1)^2}=\frac{4}{(x+1)^2}$$

(2) For an increasing function, $f'(x)>0$:

$$\frac{4}{(x+1)^2}>0$$

(3) Since the numerator is positive and the denominator $(x+1)^2$ is always positive for any real value except $x=-1$, the function is strictly increasing everywhere except at $x=-1$.

176. **(A)** Find λ such that $\vec{a}+\lambda\vec{b}$ is perpendicular to $\vec{c}$

Given: $\vec{a}=2\hat{\imath}-3\hat{\jmath}+\hat{k}, \vec{b}=4\hat{\jmath}-2\hat{k}$, and $\vec{c}=3\hat{\imath}+2\hat{k}$.

(1) Compute $\vec{a}+\lambda\vec{b}$:

$$\vec{a}+\lambda\vec{b}=(2\hat{\imath}-3\hat{\jmath}+\hat{k})+\lambda(4\hat{\jmath}-2\hat{k})=2\hat{\imath}+(4\lambda-3)\hat{\jmath}+(1-2\lambda)\hat{k}$$

(2) Set the dot product with $\vec{\boldsymbol{c}}$ equal to $\mathbf{0}$ for perpendicularity:

$$(\vec{a}+\lambda\vec{b})\cdot\vec{c}=0$$
$$2(3)+(4\lambda-3)(0)+(1-2\lambda)(2)=0$$
$$6+2-4\lambda=0 \Longrightarrow 4\lambda=8 \Longrightarrow \lambda=2$$

OR

(B) Find the area of $\triangle ABC$ if $\overrightarrow{AB}=\hat{\imath}+2\hat{\jmath}-\hat{k}$ and $\overrightarrow{AC}=2\hat{\imath}-3\hat{\jmath}$

(1) Compute the cross product $\overrightarrow{AB}\times\overrightarrow{AC}$:

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 2 & -1\\ 2 & -3 & 0\end{vmatrix}$$

$$=\hat{\imath}(0-3)-\hat{\jmath}(0-(-2))+\hat{k}(-3-4)=-3\hat{\imath}-2\hat{\jmath}-7\hat{k}$$

(2) Find the magnitude of the cross product:

$$|\overrightarrow{AB}\times\overrightarrow{AC}|=\sqrt{(-3)^2+(-2)^2+(-7)^2}=\sqrt{9+4+49}=\sqrt{62}$$

(3) Calculate the area:

$$\text{Area }=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|=\frac{\sqrt{62}}{2}\text{ square units}$$

177. Find the angle between the pair of lines

(1) Rewrite the given equations in standard symmetrical form:

- Line 1: $\frac{x-2}{3}=\frac{y+5}{2}=\frac{z-1}{6} \Longrightarrow \overrightarrow{b_1}=3\hat{\imath}+2\hat{\jmath}+6\hat{k}$
- Line 2: $\frac{x-7}{1}=\frac{y}{2}=\frac{z-6}{2} \Longrightarrow \overrightarrow{b_2}=\hat{\imath}+2\hat{\jmath}+2\hat{k}$

(2) Use the angle formula $\cos\theta=\frac{|\vec{b}_1-\vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}$:

- $\overrightarrow{b_1}\cdot\overrightarrow{b_2}=(3)(1)+(2)(2)+(6)(2)=3+4+12=19$
- $|\vec{b}_1|=\sqrt{3^2+2^2+6^2}=\sqrt{49}=7$
- $|\vec{b}_2|=\sqrt{1^2+2^2+2^2}=\sqrt{9}=3$

(3) Compute θ.

$$\cos\theta=\frac{19}{7\times 3}=\frac{19}{21} \Longrightarrow \theta=\cos^{-1}\left(\frac{19}{21}\right)$$

178. Spherical Balloon

Given that the rate of decrease of volume is proportional to surface area:

$$\frac{dV}{dt}=-kS$$

where k is a positive constant.

For a sphere,

$$V=\frac{4}{3}\pi r^3$$

and

$$S=4\pi r^2$$

Differentiating volume w.r.t. time:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Substituting into the given relation:

$$4\pi r^2 \frac{dr}{dt}$$

$$= -k(4\pi r^2)$$

$$\frac{dr}{dt} = -k$$

which is a constant.

Hence,

$$\frac{dr}{dt} = \text{ constant}$$

Therefore, the radius decreases at a constant rate.

179. **(A)** Evaluate

$$I = \int \frac{x - \sin x}{1 - \cos x} dx$$

$$I = \int \frac{x}{1 - \cos x} dx - \int \frac{\sin x}{1 - \cos x} dx$$

Using

$$1 - \cos x = 2\sin^2 \frac{x}{2}$$

and

$$\frac{\sin x}{1 - \cos x} = \cot \frac{x}{2}$$

Therefore,

$$I = \int x\mathrm{cosec}^2 \frac{x}{2} \frac{dx}{2} - \int \cot \frac{x}{2} dx$$

Using integration by parts:

$$\int x\mathrm{cosec}^2 \frac{x}{2} dx = -2x\cot \frac{x}{2} + 4\ln \left|\sin \frac{x}{2}\right|$$

Hence,

$$\int \frac{x - \sin x}{1 - \cos x} dx = -x\cot \frac{x}{2} + \ln(1 - \cos x) + C$$

(B) Evaluate

$$I = \int_0^2 \frac{dx}{\sqrt{x^2 + 2x + 3}}$$

$$x^2 + 2x + 3 = (x + 1)^2 + 2$$

Let

$$u = x + 1$$

Then

$$I = \int_1^3 \frac{du}{\sqrt{u^2 + 2}}$$

Using

$$\int \frac{du}{\sqrt{u^2 + a^2}} = \ln \left|u + \sqrt{u^2 + a^2}\right|$$

$$I = \left[\ln \left(u + \sqrt{u^2 + 2}\right)\right]_1^3$$

$$= \ln(3 + \sqrt{11}) - \ln(1 + \sqrt{3})$$

$$I = \ln \left(\frac{3 + \sqrt{11}}{1 + \sqrt{3}}\right)$$

180. Solve $(x + 2y^3)dy = ydx$

$$\frac{dx}{dy} = \frac{x + 2y^3}{y}$$

$\frac{dx}{dy}-\frac{x}{y}=2y^2$

Linear differential equation in x.

Integrating factor:

$IF=e^{\int(-1/y)dy}=\frac{1}{y}$

Multiplying by $1/y$:

$\frac{d}{dy}\left(\frac{x}{y}\right)=2y$

Integrating:

$\frac{x}{y}=y^2+C$

$x=y^3+Cy$

181. Step 1: Plot the boundary lines

(I) Line 1: $2x+y=1000$

(1) If $x=0, y=1000 \Longrightarrow (0,1000)$

(2) If $y=0,2x=1000 \Longrightarrow x=500 \Longrightarrow (500,0)$

(II) Line 2: $x+y=800$

(1) If $x=0, y=800 \Longrightarrow (0,800)$

(2) If $y=0, x=800 \Longrightarrow (800,0)$

Step 2: Find the Intersection Point

Subtract Line 2 from Line 1 :

$(2x+y)-(x+y)=1000-800 \Longrightarrow x=200$

Substitute $x=200$ into Line 2:

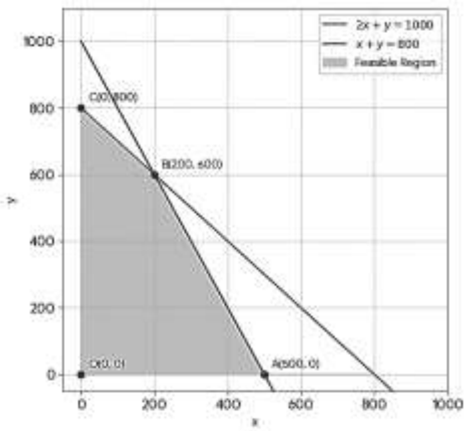
$200+y=800 \Longrightarrow y=600$

Intersection point is $B(200,600)$.

Step 3: Identify the Feasible Region

Since both inequalities are $\leq$ and include $x, y \geq 0$, the feasible region is bounded in the first quadrant with the corner points:

- $O(0,0)$
- $A(500,0)$
- $B(200,600)$
- $C(0,800)$



Step 4: Evaluate Z at the corner points

$Z=\frac{4x+3y}{10}$

- At $O(0,0): Z=0$
- At $A(500,0): Z=\frac{4(500)+3(0)}{10}=200$
- At $B(200,600): Z=\frac{4(200)+3(600)}{10}=\frac{800+1800}{10}=260$
- At $C(0,800): Z=\frac{4(0)+3(800)}{10}=240$

Conclusion:
The maximum value of Z is 260 at the point $(200,600)$.

182. **(A)** Given position vectors:
$A=(55,-2), B=(5,8), C=(a,-52)$
Since A, B and C are collinear,

$$\frac{y_2-y_1}{x_2-x_1}=\frac{y_3-y_2}{x_3-x_2}$$

$$\frac{8-(-2)}{5-55}=\frac{-52-8}{a-5}$$

$$\frac{10}{-50}=\frac{-60}{a-5}$$

$$\frac{1}{5}=\frac{-60}{a-5}$$

$a-5=300$
$a=305$

OR

(B) Given $\vec{a},\vec{b},\vec{c}$ are unit vectors.
$|\vec{a}-\vec{b}|^2=(\vec{a}-\vec{b})\cdot(\vec{a}-\vec{b})$
$=|\vec{a}|^2+|\vec{b}|^2-2\vec{a}\cdot\vec{b}$
$=2-2\vec{a}\cdot\vec{b}$
Similarly,
$\vec{b}-\vec{c}|^2=2-2\vec{b}\cdot\vec{c}$
$|\vec{c}-\vec{a}|^2=2-2\vec{c}\cdot\vec{a}$
Adding.
$S=6-2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a})$
Since for unit vectors,

$$\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}\geq-\frac{3}{2}$$

Therefore,

$$S\leq 6-2\left(-\frac{3}{2}\right)=9$$

Hence,
$|\vec{a}-\vec{b}|^2+\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2\leq 9$
Proved.

183. **(A)** $S=\{1,2,3,4,5,6\}$
$A=\{1,2,3\}$
$n(A)=3$
$B=\{3,5\}$
$n(B)=2$
$C=\{2,3,4,5\}$
$n(C)=4$
(i) $A\cap C=\{2,5\}$
$n(A\cap C)=2$

$$P(A\mid C)=\frac{n(A\cap C)}{n(C)}$$

$$=\frac{2}{4}$$

$$=\frac{1}{2}$$

$$P(C\mid A)=\frac{n(A\cap C)}{n(A)}$$

$$=\frac{2}{3}$$

(ii) $P(A\cap B\mid C)$

$A \cap B = \{5\}$
$(A \cap B) \cap C = \{5\}$
$n = ((A \cap B) \cap C) = 1$
$P(A \cap B \mid C) = \dfrac{((A \cap B) \cap C)}{n(C)}$
$= \dfrac{1}{4}$
$P(A \cup B \mid C)$
$A \cup B = \{1,2,3,5\}$
$(A \cup B) \cap C = \{2,3,5\}$
$n((A \cup B) \cap C) = 3$
$P(A \cup B \mid C) = \dfrac{n((A \cup B) \cap C)}{n(C)}$
$= \dfrac{3}{4}$

OR

(B) Since two cards are picked with replacement from 6 cards, the total number of outcomes is:
$n(S) = 6 \times 6$
$= 36$
The possible outcomes for a sum of 10 are:
$A = \{(4,6) \cdot (5,5), (6,4)\}$
$n(A) = 3$
$p(A) = \dfrac{3}{36}$
$= \dfrac{1}{12}$
First card can be 1, 2, 3, 5, or 6
$n(B) = 5 \times 6 = 30$
$P(B) = \dfrac{30}{36}$
$= \dfrac{5}{6}$
$P(A \cap B)$
$A \cap B$ refers to the outcomes in A where the first card is not 4 .
Looking at event A:
(4, 6) - First card is 4 (Exclude)
(5, 5) - First card is not 4 (Include)
(6, 4) - First card is not 4 (Include)
so,
$A \cap B = \{(5,5), (6,4)\}$
$n(A \cap B) = 2$
$P(A \cap B) = \dfrac{2}{36}$
$= \dfrac{1}{18}$
Two events are independent if $P(A \cap B) = P(A) \cdot P(B)$
$P(A) \cdot P(B) = \dfrac{1}{12} \times \dfrac{5}{6}$
$= \dfrac{5}{72}$
$P(A \cap B) = \dfrac{1}{18}$
$= \dfrac{4}{72}$
Since, $\frac{4}{72} \neq \frac{5}{72}$, the events A and B are not independent.

184. Let the cost of one apple be x , one orange be x , and one banana be z .

Write the following system of linear equations:

$4x+3y+2z=60$
$2x+4y+6z=90$
$6x+2y+3z=70$

This can be written in the matrix form AX = B :

$$\begin{bmatrix}4&3&2\\2&4&6\\6&2&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}60\\90\\70\end{bmatrix}$$

Find the determinant of matrix A :

$|A|=4(12-12)-3(6-36)+2(4-24)$
$=4(0)-3(-30)+2(-20)$
$=0+90-40$
$=50$

Since $|A|\neq 0$, the system has a unique solution given by $X=A^{-1}B$.

The cofactors C_{ij} are:

$C_{11}=+(12-12)=0, C_{12}=-(6-36), C_{13}=+(4-24)=-20$
$C_{21}=-(9-4)=-5, C_{22}=+(12-12)=0, C_{23}=-(8-18)=10$
$C_{31}=+(18-8)=10, C_{32}=-(24-4)=-20, C_{33}=+(16-6)=10$

$$\text{adj}A=\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}$$

Using $X=\frac{1}{|A|}(\text{adj}A)B$:

$$\begin{bmatrix}x\\y\\z\end{bmatrix}=\frac{1}{50}\begin{bmatrix}0&-5&10\\30&0&-20\\-20&10&10\end{bmatrix}\begin{bmatrix}60\\90\\70\end{bmatrix}$$

$$=\frac{1}{50}\begin{bmatrix}(0\times 60)+(-5\times 90)+(10\times 70)\\(30\times 60)+(0\times 90)+(-20\times 70)\\(-20\times 60)+(10\times 90)+(10\times 70)\end{bmatrix}$$

$$=\frac{1}{50}\begin{bmatrix}-450+700\\1800-1400\\-1200+900+700\end{bmatrix}$$

$$=\frac{1}{50}\begin{bmatrix}250\\400\\400\end{bmatrix}$$

$$=\begin{bmatrix}5\\8\\8\end{bmatrix}$$

The cost per fruit is ₹ 5 per apple, ₹ 8 per orange and ₹ 8 per banana.

185. **(A)** Given:

$$y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$$

Differentiate the Left-Hand Side:

Using the product rule $(uv)'=u'v+uv'$:

Let u = y and v = $\sqrt{x^2+1}$

$$\frac{d}{dx}\left(y\sqrt{x^2+1}\right)=\frac{dy}{dx}\cdot\sqrt{x^2+1}+y\cdot\frac{d}{dx}\left(\sqrt{x^2+1}\right)$$

$$=\sqrt{x^2+1}\frac{dy}{dx}+y\cdot\left(\frac{1}{2\sqrt{x^2+1}}\cdot 2x\right)$$

$$=\sqrt{x^2+1}\frac{dy}{dx}+\frac{xy}{\sqrt{x^2+1}}\qquad\text{....(i)}$$

Differentiate the Right-Hand Side:

Using the chain rule for log(u) :

$$\frac{d}{dx}\left[\log\left(\sqrt{x^2+1}-x\right)\right]=\frac{1}{\sqrt{x^2+1}-x}\cdot\frac{d}{dx}\left(\sqrt{x^2+1}-x\right)$$

$$= \frac{1}{\sqrt{x^2+1}-x} \cdot \left(\frac{x}{\sqrt{x^2+1}} - 1\right)$$

Take the LCM in the bracket:

$$= \frac{1}{\sqrt{x^2+1}-x} \cdot \left(\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right)$$

$$= \frac{1}{\sqrt{x^2+1}-x} \cdot \left(\frac{-\sqrt{x^2+1}-x}{\sqrt{x^2+1}}\right)$$

$= -\frac{1}{\sqrt{x^2+1}}$ ….(ii)

Equate LHS and RHS

$$\sqrt{x^2+1}\frac{dy}{dx} + \frac{xy}{\sqrt{x^2+1}} = -\frac{1}{\sqrt{x^2+1}}$$

Multiply the entire equation by $\sqrt{x^2+1}$ to clear the denominators:

$$\left(\sqrt{x^2+1} \cdot \sqrt{x^2+1}\right)\frac{dy}{dx} + xy = -1$$

$$(x^2+1)\frac{dy}{dx} + xy = -1$$

$$(x^2+1)\frac{dy}{dx} + xy + 1 = 0$$

Hence proved

OR

(B) $x^{\cot x} + \frac{2x^2-3}{2x^2-x+2}$

Split the expression into two parts: u and v.

Let $y = u + v$,

$u = x^{\cot x}$

$$v = \frac{2x^2-3}{2x^2-x+2}$$

Then,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{du}{dx}$$

Differentiate $u = x^{\cot x}$

We use logarithmic differentiation:

$\log u = \log(x^{\cot x})$

$\log u = \cot x \cdot \log x$

Differentiating both sides with respect to x :

$$\frac{1}{u}\frac{du}{dx} = \frac{d}{dx}(\cot x) \cdot \log x + \cot x \cdot \frac{d}{dx}(\log x)$$

$$\frac{1}{u}\frac{du}{dx} = (-\csc^2 x)\log x + \cot x\left(\frac{1}{x}\right)$$

$$\frac{du}{dx} = u\left[\frac{\cot x}{x} - \csc^2 x \log x\right]$$

$$\frac{du}{dx} = x^{\cot x}\left[\frac{\cot x}{x} - \csc^2 x \log x\right]$$

Differentiate $v = \frac{2x^2-3}{2x^2-x+2}$

We use quotient rule: $\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g-fg'}{g^2}$

Let $f = 2x^2 - 3 \Rightarrow f' = 4x$

Let $g = 2x^2 - x + 2 \Rightarrow g' = 4x - 1$

$$\frac{dv}{dx} = \frac{(4x)(2x^2-x+2)-(2x^2-3)(4x-1)}{(2x^2-x+2)^2}$$

$$\frac{dv}{dx} = \frac{(8x^3-4x^2+8x)-(8x^3-2x^2-12x+3)}{(2x^2-x+2)^2}$$

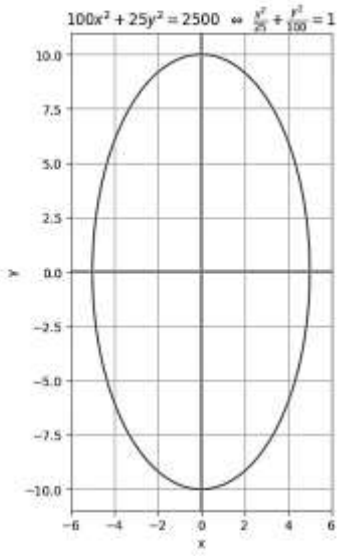
$$\frac{dv}{dx} = \frac{8x^3-4x^2+8x-8x^3+2x^2+12x-3}{(2x^2-x+2)^2}$$

$$\frac{dv}{dx} = \frac{-2x^2 + 20x - 3}{(2x^2 - x + 2)^2}$$

Combining the two results:

$$\frac{dy}{dx} = x^{\cot x}\left[\frac{\cot x}{x} - \csc^2 x \log x\right] + \frac{-2x^2 + 20x - 3}{(2x^2 - x + 2)^2}$$

186.



Curve Sketching and Area

(1) Standard Form and Sketching

Divide the equation $100x^2 + 25y^2 = 2500$ by 2500:

$$\frac{x^2}{25} + \frac{y^2}{100} = 1$$

This represents an ellipse centered at (0,0) with:

• Semi-minor axis (a) = 5 (along the x-axis)

• Semi-major axis (b) = 10 (along the y-axis)

(2) Area Using Integration

The total area A is 4 times the area enclosed in the first quadrant:

$$y = 10\sqrt{1 - \frac{x^2}{25}} = 2\sqrt{25 - x^2}$$

$$A = 4\int_0^5 2\sqrt{25 - x^2}dx = 8\int_0^5 \sqrt{25 - x^2}dx$$

Using the standard integral formula $\int \sqrt{a^2 - x^2}dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$:

$$A = 8\left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)\right]_0^5$$

$$A = 8\left[\left(0 + \frac{25}{2}\sin^{-1}(1)\right) - 0\right] = 8\left(\frac{25}{2}\cdot\frac{\pi}{2}\right) = 50\pi \text{ sq. units}$$

187. **(A)** (1) Standardize the Equation of the Line

Rewrite $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ in symmetric form:

$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$$

Any general point P on this line is given by:

$P = (5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

(2) Find the Foot of the Perpendicular

Let $Q(0,2,3)$ be the given external point. The direction ratios of QP are:

DRs $= (5\lambda - 3 - 0, 2\lambda + 1 - 2, 3\lambda - 4 - 3) = (5\lambda - 3, 2\lambda - 1, 3\lambda - 7)$

Since QP is perpendicular to the line (which has direction ratios 5,2,3):

$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$25\lambda - 15 + 4\lambda - 2 + 9\lambda - 21 = 0$
$38\lambda - 38 = 0 \Longrightarrow \lambda = 1$
Substitute $\lambda = 1$ into P.
Foot of the perpendicular $P = (5(1) - 3, 2(1) + 1, 3(1) - 4) = (2, 3, -1)$
(3) Length of the Perpendicular
Use the distance formula between $Q(0,2,3)$ and $P(2,3,-1)$:
Length $= \sqrt{(2-0)^2 + (3-2)^2 + (-1-3)^2} = \sqrt{4+1+16} = \sqrt{21}$ units

OR

(B) (1) Identify Line Vector Components
The lines are:
• $\vec{r}_1 = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k}) \Longrightarrow \vec{a}_1 = \hat{\imath} + 2\hat{\jmath} + \hat{k}, \vec{b}_1 = \hat{\imath} - \hat{\jmath} + \hat{k}$
• $\vec{r}_2 = (p\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k}) \Longrightarrow \vec{a}_2 = p\hat{\imath} - \hat{\jmath} - \hat{k}, \vec{b}_2 = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$
(2) Compute Cross Product and Difference Vector
$\vec{a}_2 - \vec{a}_1 = (p-1)\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{\imath}(-2-1) - \hat{\jmath}(2-2) + \hat{k}(1+2) = -3\hat{\imath} + 3\hat{k}$$

Magnitude $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 0^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$
(3) Set Up Shortest Distance Formula

$$\text{Shortest Distance (SD)} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (p-1)(-3) + (-3)(0) + (-2)(3) = -3p + 3 - 6 = -3p - 3$
Given SD $= \frac{3}{\sqrt{2}}$:

$$\frac{|-3p-3|}{3\sqrt{2}} = \frac{3}{\sqrt{2}}$$

$|-3p-3| = 9 \Longrightarrow |p+1| = 3$
(4) Solve for p
• $p + 1 = 3 \Longrightarrow p = 2$
• $p + 1 = -3 \Longrightarrow p = -4$
Values of p: 2 or -4

188. **(i)** A relation from Set A to B is any subset of the Cartesian product $A \times B$.
Number of elements in Set A, $n(A) = 4$
Number of elements in Set B, $n(B) = 3$
Number of elements in $A \times B = 4 \times 3 = 12$
The total number of possible relations is given by $2^{n(A) \times n(B)}$
Total relations $= 2^{12} = 4096$
(ii) Let $R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$
To be considered a function, each element in the domain (Set A) must map to exactly one element in the codomain (Set B).
• Not a function: Notice that G_1 is related to both B_1 and B_2. R is not a function because it has two different outputs for a single input.
• Not injective: Even if we ignore the first point, G_2 and G_3 map to B_2. An injective (one-to-one) function requires unique inputs and outputs.
Conclusion: No R is not an injective function (it is not a function at all).
(iii) (A)
$R = \{(x,y), x, y \in A$, x and y are from same colony $\}$ to be an equivalence relation, it must satisfy three conditions:
• Reflexive: Every student x lives in the same colony as themselves. So, $(x,x) \in R$ for all $\in A$.
• Symmetric: If student x lives in the same colony as student y, then student y necessarily lives in the same colony as student x. So, if $(x,y) \in R$, then $(y,x) \in R$.
• Transitive: If students x and y are in the same colony, and y and z are in the same colony, then x and z must be as well. so, if $(x,y) \in R$ and $(y,z) \in R$, then $(x,z) \in R$.

Conclusion: Since it is reflexive, symmetric, and transitive, R is an equivalence relation.

OR

(B) A function is bijective if it is both injective (one-to-one) and surjective (onto).
$n(B) = 3$ (Domain)
$n(A) = 4$ (Codomain)
For a function to be surjective, the number of elements in the domain must be greater than or equal to the number of elements in the codomain ($n(B) \geq n(A)$). Here, $3 < 4$. This means that at least one element in Set A will always be missing a corresponding element from Set B. Therefore, the function cannot be onto.
Conclusion: No, no function from $B \rightarrow A$ can be bijective because the sets have different cardinalities.

189. Define the events:

A_1, A_2, A_3 : Person was given vaccine A_1, A_2, A_3 respectively.
$P(A_1) = 0.25, P(A_2) = 0.35, P(A_3) = 0.40$
E: Person is protected
$\mathrm{P}(\mathrm{E} \mid \mathrm{A}_1) = 0.60, \mathrm{P}(\mathrm{E} \mid \mathrm{A}_2) = 0.55, \mathrm{P}(\mathrm{E} \mid \mathrm{A}_3) = 0.50$
E' : Person is infected (Not protected).
$\mathrm{P}(\mathrm{E}' \mid \mathrm{A}_1) = 0.40, \mathrm{P}(\mathrm{E}' \mid \mathrm{A}_2) = 0.45, \mathrm{P}(\mathrm{E}' \mid \mathrm{A}_3) = 0.50$
(i) This is the conditional probability $\mathrm{P}(\mathrm{E}' \mid \mathrm{A}_2)$.
Since 55% are protected, the remaining are infected:
$\mathrm{P}(\mathrm{E}' \mid \mathrm{A}_2) = 1 - 0.55$
$= 0.45$
(ii) We use the Law of Total Probability:
$\mathrm{P}(\mathrm{E}) = \mathrm{P}(\mathrm{A}_1)\mathrm{P}(\mathrm{E} \mid \mathrm{A}_1) + \mathrm{P}(\mathrm{A}_2)\mathrm{P}(\mathrm{E} \mid \mathrm{A}_2) + \mathrm{P}(\mathrm{A}_3)\mathrm{P}(\mathrm{E} \mid \mathrm{A}_3)$
$= (0.25 \times 0.60) + (0.35 \times 0.55) + (0.40 \times 0.50)$
$= 0.15 + 0.1925 + 0.20$
$= 0.5425$
(iii) (A) Total probability of being infected:
$\mathrm{P}(\mathrm{E}') = 1 - \mathrm{P}(\mathrm{E})$
$= 1 - 0.5425$
$= 0.4575$
Now apply Bayes' Theorem:
$$\mathrm{P}(\mathrm{A1} \mid \mathrm{E}') = \frac{P(A_1)P(E' \mid A_1)}{P(E')}$$
$$= \frac{0.25 \times 0.40}{0.4575}$$
$$= \frac{0.10}{0.4575}$$
$= 0.2186$

OR

(B) Apply Bayes' Theorem using the total probability of protection from (ii):
$$\mathrm{P}(\mathrm{A}_3 \mid \mathrm{E}) = \frac{P(A_3)P(E \mid A_3)}{P(E)}$$
$$= \frac{0.40 \times 0.50}{0.5425}$$
$$= \frac{20}{0.5425}$$
$= 0.3687$

190. Let the fixed surface area be S .

Surface area (S): $S = \pi r^2 + 2\pi \mathrm{rh}$ (Area of base + curved surface)
Volume (V): $\mathrm{V} = \pi \mathrm{r}^2\ \mathrm{h}$
From the surface area equation, we can express height h in terms of r and S :
$2\pi r\ \mathrm{h} = \mathrm{S} - \pi r^2$
$$\mathrm{h} = \frac{S - \pi r^2}{2\pi r}$$
Now, substitute h into the Volume formula:

$V = \pi r^2 \left(\frac{S - \pi r^2}{2\pi r}\right)$
$= \frac{r(S - \pi r^2)}{2}$
$= \frac{Sr - \pi r^3}{2}$

(i) Now we differentiate V with respect to r(S constant):
$\frac{dv}{dr} = \frac{d}{dr}\left(\frac{Sr}{2} - \frac{\pi r^3}{2}\right)$
$\frac{dv}{dr} = \frac{S}{2} - \frac{3\pi r^2}{2}$

(ii) To find the maximum volume, we set the first derivative to zero:
$\frac{dv}{dr} = 0$
$\frac{S}{2} - \frac{3\pi r^2}{2} = 0$
$S = 3\pi r^2$

Now, substitute the value of S back into the original surface area equation to find the relation between h and r.
$3\pi r^2 = \pi r^2 + 2\pi rh$
$3\pi r^2 - \pi r^2 = 2\pi rh$
$2\pi r^2 = 2\pi rh$
$h = r$

For maximum volume, the height of the tumbler must be equal to the radius of its base.

191. **(a)** Let $f: R \to R$ be defined as $f(x) = x^3$, then f is:

• One-one (Injective):
Let $f(x_1) = f(x_2) \Longrightarrow x_1^3 = x_2^3 \Longrightarrow x_1 = x_2$. Since every real number has a unique cube, the function is one-one.
• Onto (Surjective):
For every $y \in R$ (codomain), there exists a pre-image $x = \sqrt[3]{y} \in R$ such that $f(x) = (\sqrt[3]{y})^3 = y$. Thus, the range equals the codomain, making it onto.

192. **(c)**• A matrix of order 2×3 contains $2 \times 3 = 6$ element positions.
• Each position can be filled in 2 ways (either with 1 or 2).
• Total number of possible matrices $= 2^6 = 64$.

193. **(b)** Find the transpose A′ :
$A' = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$
• Add A and A′ :
$A + A' = \begin{bmatrix} 2\cos 2\alpha & 0 \\ 0 & 2\cos 2\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
• Equate corresponding elements:
$2\cos 2\alpha = 1 \Longrightarrow \cos 2\alpha = \frac{1}{2}$
$2\alpha = \frac{\pi}{3} \Longrightarrow \alpha = \frac{\pi}{6}$

194. **(d)** • Using the determinant formula for the area of a triangle:
$\pm 15 = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$
$\pm 30 = [3(3 - 3) + 6(3 - (-7)) + k(-7 - 3)]$
$\pm 30 = [0 + 60 - 10k]$
• Case 1: $30 = 60 - 10k \Longrightarrow 10k = 30 \Longrightarrow k = 3$
• Case 2: $-30 = 60 - 10k \Longrightarrow 10k = 90 \Longrightarrow k = 9$

195. **(d)** • Differentiate with respect to x :
$\frac{dy}{dx} = 3x^2 \cdot e^{-3x} + x^3 \cdot (-3e^{-3x})$
$\frac{dy}{dx} = 3x^2 e^{-3x}(1 - x)$

• For an increasing function, $\frac{dy}{dx} \geq 0$:
Since $3x^2 \geq 0$ and $e^{-3x} > 0$ for all x, the sign depends completely on $(1-x)$.
$1 - x \geq 0 \Rightarrow x \leq 1 \Rightarrow x \in (-\infty, 1]$
• Among the given options, (0,1) is a proper subset of $(-\infty, 1]$ where the function strictly increases.

196. **(b)** • For continuity at $x = 0, \lim\limits_{x\to 0} f(x) = f(0)$:

$$\lim_{x\to 0} \frac{\sin 7x}{5x} = k$$
$$\frac{7}{5} \cdot \lim_{x\to 0} \left(\frac{\sin 7x}{7x}\right) = k$$
• Since $\lim\limits_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$:
$$\frac{7}{5} \cdot (1) = k \Rightarrow k = \frac{7}{5}$$

197. **(c)** $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

Use:
$$\frac{1-\cos x}{\sin x} = \tan\frac{x}{2}$$
So,
$$y = \tan^{-1}\left(\tan\frac{x}{2}\right) = \frac{x}{2}$$
$$\frac{dy}{dx} = \frac{1}{2}$$

198. **(d)** $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$

Use
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\frac{\cos 2x}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x} = \csc^2 x - \sec^2 x$$
Therefore
$$\int (\csc^2 x - \sec^2 x)dx = -\cot x - \tan x + C$$

199. **(b)** $\int \frac{dx}{x^2+6x+10}$

$$x^2 + 6x + 10 = (x+3)^2 + 1$$
$$\int \frac{dx}{(x+3)^2+1} = \tan^{-1}(x+3) + C$$

200. **(c)** $\frac{dy}{dx} = e^{y-x}$

$$e^{-y}dy = e^{-x}dx$$
Integrating,
$$-e^{-y} = -e^{-x} + C$$
$$e^{-y} - e^{-x} = C$$

201. **(c)** $(1+y^2)dx = (\tan^{-1}y - x)dy$

Write as
$$(1+y^2)dx + (x - \tan^{-1}y)dy = 0$$
Here
$$M = 1 + y^2, N = x - \tan^{-1}y$$
$$\frac{\partial M}{\partial y} = 2y, \frac{\partial N}{\partial x} = 1$$
$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = \frac{1-2y}{1+y^2}$$
Function of y only, so
$$IF = e^{\int \frac{1-2y}{1+y^2}dy}$$
$$= e^{\tan^{-1}y - \ln(1+y^2)}$$

$$= \frac{e^{\tan^{-1}y}}{1+y^2}$$

Ignoring the removable factor $\frac{1}{1+y^2}$ present in all terms, the matching option is

$e^{\tan^{-1}y}$

202. **(d)** $\vec{a}, \vec{b}$ are unit vectors.

$|\vec{a} + 2\vec{b}| = 1$

Squaring,

$1 = |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a}\cdot\vec{b}$

$1 = 1 + 4 + 4\cos\theta$

$4\cos\theta = -4$

$\cos\theta = -1$

$\theta = \pi$

203. **(b)** Area of parallelogram

Diagonals:

$\vec{d}_1 = \hat{\imath} + \hat{\jmath} - 2\hat{k} = (1,1,-2)$

$\vec{d}_2 = \hat{\imath} - 3\hat{\jmath} + 4\hat{k} = (1,-3,4)$

Area of parallelogram:

$$A = \frac{1}{2}|\vec{d}_1 \times \vec{d}_2|$$

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix} = (-2,-6,-4)$$

$$|\vec{d}_1 \times \vec{d}_2| = \sqrt{4+36+16} = \sqrt{56} = 2\sqrt{14}$$

$$A = \frac{1}{2}(2\sqrt{14}) = \sqrt{14}$$

204. **(a)** Maximum Z - Minimum Z

$Z = 4x + 6y$

At corner points:

Point	Z
(0,2)	12
(6,0)	24
(6,8)	72
(0,5)	30

$Z_{max} = 72, Z_{min} = 12$

$Z_{max} - Z_{min} = 72 - 12 = 60$

205. **(c)** Same minimum value at two corner points

If an objective function has the same optimum value at two corner points, then every point on the line segment joining them gives the same value.

Hence infinitely many solutions.

206. **(d)** If $P(A \mid B) = P(B \mid A)$

$$\frac{P(A\cap B)}{P(B)} = \frac{P(A\cap B)}{P(A)}$$

Assuming $P(A\cap B) \neq 0$.

$P(A) = P(B)$

207. **(b)** Odd prime number on each die

Odd prime numbers on a die:

3,5

Probability on one die:

$\frac{2}{6}=\frac{1}{3}$

For two dice:

$\left(\frac{1}{3}\right)^2=\frac{1}{9}$

208. **(d)** • A: "Eldest child is a boy"

• B: "Family has at least one boy"

For 3 children, the equally likely outcomes are:

BBB, BBG, BGB, BGG, GBB, GBG, GGB, GGG

Since the family has at least one boy, exclude GGG.

So the sample space becomes:

{BBB, BBG, BGB, BGG, GBB, GBG, GGB}

Total outcomes = 7.

Outcomes where the eldest child is a boy:

BBB, BBG, BGB, BGG

Number of favorable outcomes = 4.

Therefore,

$P(A \mid B)=\frac{4}{7}$

209. **(a)** Analysis of Assertion (A):

• Given function: $f(x)=e^{-x}$ for $x \in R$

• Differentiating with respect to x :

$f'(x)=-e^{-x}$

• Since the exponential function e^{-x} is always positive for any real value of **x**, $-e^{-x}$ will always be negative ($f'(x)<0$).

• Since the derivative is negative, $f(x)=e^{-x}$ is a decreasing function.

• Thus, Assertion (A) is True.

Analysis of Reason (R):

• By the derivative test for monotonicity, if $f'(x)<0$ on an interval, then the function $f(x)$ is decreasing on that interval.

• Thus, Reason (R) is True.

Conclusion:

Since we used the condition $f'(x)<0$ from Reason (R) to prove that Assertion (A) is a decreasing function, Reason (R) is the correct explanation for Assertion (A).

So, Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

210. **(a)** Analysis of Assertion (A):

• By definition, conditional probability is given by:

$P(A \mid B)=\frac{P(A \cap B)}{P(B)}$

• For independent events, the occurrence of event B does not affect the probability of event A, meaning $P(A \mid B)=P(A)$.

• Thus, Assertion (A) is True.

Analysis of Reason (R):

• The fundamental multiplication rule for two independent events states that:

$P(A \cap B)=P(A) \cdot P(B)$

• Thus, Reason (R) is True.

Conclusion:

To verify the assertion, we substitute the reason's formula into the conditional probability formula:

$P(A \mid B)=\frac{P(A) \cdot P(B)}{P(B)}=P(A)$

This shows that Reason (R) is the direct mathematical explanation for why Assertion (A) holds true.

So, Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

211. **(A)** For $A=\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, find a, b such that

$A^2+aA+bI=0$

First find A^2 :

$A^2 = \begin{bmatrix}3 & 1\\2 & 1\end{bmatrix}\begin{bmatrix}3 & 1\\2 & 1\end{bmatrix} = \begin{bmatrix}11 & 4\\8 & 3\end{bmatrix}$

Using Cayley-Hamilton theorem,

$|A-\lambda I| = \begin{vmatrix}3-\lambda & 1\\2 & 1-\lambda\end{vmatrix}$

$= (3-\lambda)(1-\lambda) - 2 = \lambda^2 - 4\lambda + 1$

Hence,

$A^2 - 4A + I = 0$

Comparing with $A^2 + aA + bI = 0$,

$a = -4, b = 1$

OR

(B) Equation of line through $(-1,-1)$ and $(1,3)$

Using determinant form,

$\begin{vmatrix}x & y & 1\\-1 & -1 & 1\\1 & 3 & 1\end{vmatrix} = 0$

Expanding.

$4x - 2y + 2 = 0$

$2x - y + 1 = 0$

212. **(A)** Given,

$\frac{dV}{dt} = 20\ \text{cm}^3/\text{s}$

Volume of sphere:

$V = \frac{4}{3}\pi r^3$

Differentiate:

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$20 = 4\pi(4)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{5}{16\pi}$

Surface area:

$S = 4\pi r^2$

$\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

At $r = 4$,

$\frac{dS}{dt} = 8\pi(4)\left(\frac{5}{16\pi}\right) = 10$

$\frac{dS}{dt} = 10\ \text{cm}^2/\text{s}$

OR

(B) $f(x) = x^3 + 2x^2 - 1$

$f'(x) = 3x^2 + 4x = x(3x+4)$

For increasing function,

$f'(x) > 0$

$x(3x+4) > 0$

Critical points:

$x = 0, x = -\frac{4}{3}$

Sign analysis gives

$\left(-\infty, -\frac{4}{3}\right)(+), \left(-\frac{4}{3}, 0\right)(-), (0,\infty)(+)$

Therefore,

$\left(-\infty, -\frac{4}{3}\right) \cup (0,\infty)$

213. $I = \int \frac{(x-4)e^x}{(x-2)^3} dx$

Observe that

$$\frac{d}{dx}\left(\frac{e^x}{(x-2)^2}\right) = e^x\left[\frac{1}{(x-2)^2} - \frac{2}{(x-2)^3}\right] = \frac{(x-4)e^x}{(x-2)^3}$$

Hence,

$$\int \frac{(x-4)e^x}{(x-2)^3} dx = \frac{e^x}{(x-2)^2} + C$$

214. Given

$A(3,-1,2), B(1,-1,-3), C(4,-3,1)$

$\overrightarrow{AB} = B - A = (-2,0,-5)$

$\overrightarrow{BC} = C - B = (3,-2,4)$

A vector perpendicular to both is

$$\overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0 & -5 \\ 3 & -2 & 4 \end{vmatrix}$$

$= (-10)\hat{i} - 7\hat{j} + 4\hat{k}$

Magnitude:

$\sqrt{100 + 49 + 16} = \sqrt{165}$

Therefore unit vectors are

$$\pm\frac{-10\hat{i} - 7\hat{j} + 4\hat{k}}{\sqrt{165}}$$

215. Line:

$\vec{r} = (11,-2,-8) + \lambda(10,-4,-11)$

Point:

$P = (2,-1,5)$

Take point on line

$A = (11,-2,-8), d = (10,-4,-11)$

Image of P in the line:

First find foot F of perpendicular.

$$\lambda = \frac{(P-A)\cdot d}{d \cdot d}$$

$P - A = (-9,1,13)$

$(P-A)\cdot d = -90 - 4 - 143 = -237$

$d \cdot d = 100 + 16 + 121 = 237$

$\lambda = -1$

$F = A - d = (1,2,3)$

Reflection point

$P' = 2F - P$

$= (2,4,6) - (2,-1,5)$

$= (0,5,1)$

216. **(A)** $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

Since principal value of $\sin^{-1}x$ lies in $[-\pi/2, \pi/2]$.

$$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

Also.

$$\tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$$

Since principal value of $\tan^{-1}x$ lies in $(-\pi/2, \pi/2)$.

$$\tan^{-1}\left(\tan\frac{5\pi}{6}\right) = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\frac{\pi}{6}$$

Further,

$\cos\frac{13\pi}{6} = \cos\left(2\pi + \frac{\pi}{6}\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

Since principal value of $\cos^{-1}x$ lies in $[0, \pi]$.

$\cos^{-1}\left(\cos\frac{13\pi}{6}\right) = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

Hence,

$\frac{\pi}{3} - \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{3}$

$\frac{\pi}{3}$

OR

(B) Let

$f(a) = f(b)$

Then

$a^2 + a + 1 = b^2 + b + 1$

$a^2 - b^2 + a - b = 0$

$(a - b)(a + b + 1) = 0$

Since $a + b + 1 > 0$,

$a - b = 0$

$a = b$

Therefore f is one-one (injective).

Not Onto

For $n \in \mathbb{N}$.

$f(n) = n^2 + n + 1$

Values are

$3, 7, 13, 21, \ldots$

Clearly $2, 4, 5, 6, \ldots$ are not images of any natural number.

Hence every element of $\mathbb{N}$ is not attained.

Therefore f is not onto (surjective).

f is one-one but not onto

217. **(A)** $I = \int \frac{x}{\sqrt{7-6x-x^2}}dx$

Solution

$7 - 6x - x^2 = 16 - (x + 3)^2$

Put

$t = x + 3$

Then

$x = t - 3, dx = dt$

$I = \int \frac{t-3}{\sqrt{16-t^2}}dt$

$= \int \frac{t}{\sqrt{16-t^2}}dt - 3\int \frac{dt}{\sqrt{16-t^2}}$

Now

$\int \frac{t}{\sqrt{16-t^2}}dt = -\sqrt{16-t^2}$

and

$\int \frac{dt}{\sqrt{16-t^2}} = \sin^{-1}\left(\frac{t}{4}\right)$

Therefore

$I = -\sqrt{16-t^2} - 3\sin^{-1}\left(\frac{t}{4}\right) + C$

Substituting $t = x + 3$.

$I = -\sqrt{7-6x-x^2} - 3\sin^{-1}\left(\frac{x+3}{4}\right) + C$

OR

(B) $I = \int \frac{2}{(1-x)(1+x^2)} dx$

Partial Fractions

Let

$$\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx + C}{1+x^2}$$

Multiplying by $(1-x)(1+x^2)$.

$2 = A(1+x^2) + (Bx + C)(1-x)$

Comparing coefficients,

$A = 1, B = 1, C = 1$

Thus

$$\frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x+1}{1+x^2}$$

Hence

$$I = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$= -\ln|1-x| + \frac{1}{2}\ln(1+x^2) + \tan^{-1}x + C$$

$$I = -\ln|1-x| + \frac{1}{2}\ln(1+x^2) + \tan^{-1}x + C$$

218. **(A)** $x\frac{dy}{dx} = y - x\tan\frac{y}{x}$

$$\frac{dy}{dx} = \frac{y}{x} - \tan\frac{y}{x}$$

Put

$$v = \frac{y}{x}$$

Then

$y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Substituting.

$$v + x\frac{dv}{dx} = v - \tan v$$

$$x\frac{dv}{dx} = -\tan v$$

$$\frac{dv}{\tan v} = -\frac{dx}{x}$$

$$\int \cot v dv = -\int \frac{dx}{x}$$

$\ln|\sin v| = -\ln|x| + C$

$\ln|x\sin v| = C$

$x\sin v = C_1$

Since $v = \frac{y}{x}$,

$$x\sin\left(\frac{y}{x}\right) = C$$

OR

(B) $\frac{dy}{dx} = e^{x+y} + e^{x-y}$

$= e^x(e^y + e^{-y})$

Separate variables:

$$\frac{dy}{e^y + e^{-y}} = e^x dx$$

Multiply numerator and denominator by e^y :

$$\frac{e^y dy}{e^{2y} + 1} = e^2 dx$$

Let

$t = \epsilon^y$
Then
$dt = e^y dy$
So
$$\int \frac{dt}{1+t^2} = \int e^x dx$$
$\tan^{-1} t = e^x + C$
Substituting $t = \epsilon^y$.
$\tan^{-1}(e^y) = e^x + C$
or
$e^y = \tan(e^z + C)$
which is the required solution.

219. Let
$$I = \int_0^\pi \frac{x}{1+\sin x}dx$$
Using the property
$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$
we get
$$I = \int_0^\pi \frac{\pi - x}{1+\sin(\pi - x)}dx$$
Since $\sin(\pi - x) = \sin x$,
$$I = \int_0^\pi \frac{\pi - x}{1+\sin x}dx$$
Adding the two expressions,
$$2I = \int_0^\pi \frac{\pi}{1+\sin x}dx$$
$$I = \frac{\pi}{2}\int_0^\pi \frac{dx}{1+\sin x}$$
Now,
$$\frac{1}{1+\sin x} = \frac{1-\sin x}{1-\sin^2 x} = \frac{1-\sin x}{\cos^2 x}$$
$= \sec^2 x - \sec x \tan x$
Hence
$$\int_0^\pi \frac{dx}{1+\sin x} = \int_0^\pi (\sec^2 x - \sec x \tan x)dx$$
$= [\tan x - \sec x_0^\pi$
$= (0-(-1)) - (0-1)$
$= 2$
Therefore
$$I = \frac{\pi}{2} \times 2$$
$I = \pi$

220. Let
$\vec{r} = \vec{a} + \vec{b} + \vec{c}$
Since $\vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular and have equal magnitudes,
$|\vec{a}| = |\vec{b}| = |\vec{c}| = m$
Now,
$\vec{r} \cdot \vec{a} = (\vec{a} + \vec{b} + \vec{c}) \cdot \vec{a}$
$= \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} + \vec{c} \cdot \vec{a}$
$= m^2$
Similarly,
$\vec{r} \cdot \vec{b} = m^2, \vec{r} \cdot \vec{c} = m^2$

Let α, β, γ be the angles made by $\vec{r}$ with $\vec{a}, \vec{b}, \vec{c}$ respectively.
Then

$$\cos\alpha = \frac{\vec{r} \cdot \vec{a}}{|\vec{r}||\vec{a}|} = \frac{m^2}{|\vec{r}|m} = \frac{m}{|\vec{r}|}$$

Similarly,

$$\cos\beta = \frac{m}{|\vec{r}|}, \cos\gamma = \frac{m}{|\vec{r}|}$$

Hence

$$\cos\alpha = \cos\beta = \cos\gamma$$

Therefore

$$\alpha = \beta = \gamma$$

Thus $\vec{a} + \vec{b} + \vec{c}$ is equally inclined to $\vec{a}, \vec{b}, \vec{c}$.

221. LPP

Corner points:
$(0,10), (5,5), (15,15), (0,20)$
Objective function:
$Z = px + qy, p, q > 0$
For maximum Z to occur at both (15,15) and (0,20).
their objective values must be equal.
At (15, 15).
$Z_1 = 15p + 15q$
At (0, 20).
$Z_2 = 20q$
Equating.
$15p + 15q = 20q$
$15p = 5q$
$q = 3p$
or
$p : q = 1 : 3$
Checking:
$Z(0,10) = 10q = 30p$
$Z(5,5) = 5p + 5q = 20p$
$Z(15,15) = 60p$
$Z(0,20) = 60p$
Thus the maximum value occurs at both (15,15) and (0,20).
$p : q = 1 : 3$

222. **(A)** Given

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

Step 1: Find |A|

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}$$

$$= (1 + 3) + (2 + 3) + (2 - 1)$$

$$= 10$$

Step 2: Adjoint

$$\text{adj}(A) = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Hence

$$A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{10} & \frac{1}{5} & \frac{3}{10} \end{bmatrix}$$

Solve the system

$x - y + z = 4$

$2x + y - 3z = 0$

$x + y + z = 2$

Matrix form:

$$AX = B, B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$X = A^{-1}B$

$$= \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Therefore

$x = 2, y = -1, z = 1$

OR

(B) Given

$$A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$$

To prove:

$(AB)^{-1} = B^{-1}A^{-1}$

We know

$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = I$

Similarly,

$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB = I$

Since $B^{-1}A^{-1}$ is both left and right inverse of AB.

$(AB)^{-1} = B^{-1}A^{-1}$

Hence proved.

223. **(A)** A closed cylindrical can has volume

$V = \pi r^2 h = 200$

$$h = \frac{200}{\pi r^2}$$

Surface area

$S = 2\pi rh + 2\pi r^2$

Substitute h :

$$S(r) = \frac{400}{r} + 2\pi r^2$$

Differentiate:

$$\frac{dS}{dr} = -\frac{400}{r^2} + 4\pi r$$

For minimum area:

$$-\frac{400}{r^2} + 4\pi r = 0$$

$4\pi r^3 = 400$

$$r^3 = \frac{100}{\pi}$$

$$r = \left(\frac{100}{\pi}\right)^{1/3}$$

Now

$h = \frac{200}{\pi r^2} = 2r$
Hence
$h = 2\left(\frac{100}{\pi}\right)^{1/3}$
Thus the cylinder of minimum surface area satisfies
$h = 2r$

OR

(B) For a cone of height h inscribed in a sphere of radius r,
Volume:
$V = \frac{1}{3}\pi R^2 h$
Using sphere geometry,
$R^2 = r^2 - (r-h)^2 = 2rh - h^2$
Therefore
$V = \frac{1}{3}\pi h(2rh - h^2) = \frac{1}{3}\pi(2rh^2 - h^3)$
Differentiate:
$\frac{dV}{dh} = \frac{1}{3}\pi(4rh - 3h^2)$
For maximum volume:
$4rh - 3h^2 = 0$
$h(4r - 3h) = 0$
$h = \frac{4r}{3}$
Hence
$h = \frac{4r}{3}$

224. $y = 3x + 2, y = 0, x = -1, x = 1$
Area
$A = \int_{-1}^{1} (3x + 2)dx$
$= \left[\frac{3}{2}x^2 + 2x\right]_{-1}^{1}$
$= \left(\frac{3}{2} + 2\right) - \left(\frac{3}{2} - 2\right)$
$= \frac{7}{2} + \frac{1}{2}$
$= 4$

225. Lines:
$\vec{r} = (1,2,3) + \lambda(1,-3,2)$
$\vec{r} = (4,5,6) + \mu(2,3,1)$
Let
$\vec{a} = (1,2,3), \vec{b} = (4,5,6)$
Direction vectors
$\vec{d}_1 = (1,-3,2), \vec{d}_2 = (2,3,1)$
Shortest distance between skew lines:
$$D = \frac{|(\vec{b} - \vec{a}) \cdot (\vec{d}_1 \times \vec{d}_2)|}{|\vec{d}_1 \times \vec{d}_2|}$$
First,
$\vec{b} - \vec{a} = (3,3,3)$
$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$= (-9)\hat{i} + 3\hat{j} + 9\hat{k}$
$= (-3,1,3) \times 3$
Magnitude:
$|\vec{d}_1 \times \vec{d}_2| = \sqrt{81+9+81} = \sqrt{171} = 3\sqrt{19}$
Numerator:
$|(3,3,3)\cdot(-9,3,9)| = |-27+9+27| = 9$
Thus
$D = \dfrac{9}{3\sqrt{19}}$
$\dfrac{3}{\sqrt{19}}$
or
$\dfrac{3\sqrt{19}}{19}$

226. **(i)** The relation is defined as $R = \{(x,y) : y \text{ is divisible by } x\}$.
By testing each element $x \in B$ to see which $y \in B$ it divides:
- $x = 1$: divides $1,2,3,4,5,6 \rightarrow (1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$
- $x = 2$: divides $2,4,6 \rightarrow (2,2),(2,4),(2,6)$
- $x = 3$: divides $3,6 \rightarrow (3,3),(3,6)$
- $x = 4$: divides $4 \rightarrow (4,4)$
- $x = 5$: divides $5 \rightarrow (5,5)$
- $x = 6$: divides $6 \rightarrow (6,6)$

$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6)$
$,(3,3),(3,6),(4,4),(5,5),(6,6)\}$
(ii) Definition: A relation R on set B is reflexive if $(x,x) \in R$ for all $x \in B$.
- Check: Every number divides itself, so $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)$ are all present in R.

Yes, R is reflexive.
(iii) (A) Check whether R in B is symmetric or transitive
- Symmetric:
- A relation is symmetric if $(x,y) \in R \Rightarrow (y,x) \in R$.
- Here, $(1,2) \in R$ (since 2 is divisible by 1), but $(2,1) \notin R$ (since 1 is not divisible by 2).
- Therefore, R is not symmetric.
- Transitive:
- A relation is transitive if $(x,y) \in R$ and $(y,z) \in R \Rightarrow (x,z) \in R$.
- If y is divisible by $x(y = k_1x)$ and z is divisible by $y(z = k_2y)$, then $z = k_2(k_1x) = (k_1k_2)x$. Since k_1k_2 is an integer, z is divisible by x.
- Therefore, R is transitive.

OR

(B) Let $R_1 : B \rightarrow B$ be defined as $R_1 = \{(x,y): x = 2y, x,y \in B\}$
First, let's list the elements of R_1 :
- If $y = 1 \Rightarrow x = 2(1) = 2 \rightarrow (2,1)$
- If $y = 2 \Rightarrow x = 2(2) = 4 \rightarrow (4,2)$
- If $y = 3 \Rightarrow x = 2(3) = 6 \rightarrow (6,3)$

$R_1 = \{(2,1),(4,2),(6,3)\}$
- Reflexive: $(1,1) \notin R_1$ since $1 \neq 2(1)$. Thus, it is not reflexive.
- Symmetric: $(2,1) \in R_1$ but $(1,2) \notin R_1$. Thus, it is not symmetric.
- Transitive: We have $(4,2) \in R_1$ and $(2,1) \in R_1$, but $(4,1) \notin R_1$ since $4 \neq 2(1)$. Thus, it is not transitive.

227. **(i)** Given parametric equations:
- $x = at^3$
- $y = at^5$

Step 1: Differentiate x and y with respect to t
- $\frac{dx}{dt} = 3at^2$
- $\frac{dy}{dt} = 5at^4$

Step 2: Find $\frac{dy}{dx}$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{5at^4}{3at^2}=\frac{5}{3}t^2$$

(ii) Given parametric equations:

- $x=t+\frac{1}{t}$
- $y=t-\frac{1}{t}$

Step 1: Differentiate x and y with respect to t

- $\frac{dx}{dt}=1-\frac{1}{t^2}=\frac{t^2-1}{t^2}$
- $\frac{dy}{dt}=1+\frac{1}{t^2}=\frac{t^2+1}{t^2}$

Step 2: Find $\frac{dy}{dx}$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{\frac{t^2+1}{t^2}}{\frac{t^2-1}{t^2}}=\frac{t^2+1}{t^2-1}$$

(iii) (A)

Given parametric equations:

- $x=2\sin t+\sin 2t$
- $y=2\cos t+\cos 2t$

Step 1: Differentiate x and y with respect to t

- $\frac{dx}{dt}=2\cos t+2\cos 2t$
- $\frac{dy}{dt}=-2\sin t-2\sin 2t$

Step 2: Find the derivative expression $\frac{dy}{dx}$

$$\frac{dy}{dx}=\frac{-2(\sin t+\sin 2t)}{2(\cos t+\cos 2t)}=-\frac{\sin t+\sin 2t}{\cos t+\cos 2t}$$

Step 3: Evaluate at $t=\frac{\pi}{6}$

- $\sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$
- $\sin\left(2\cdot\frac{\pi}{6}\right)=\sin\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$
- $\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$
- $\cos\left(2\cdot\frac{\pi}{6}\right)=\cos\left(\frac{\pi}{3}\right)=\frac{1}{2}$

Substitute these values into the expression:

$$\frac{dy}{dx}=-\frac{\frac{1}{2}+\frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}+\frac{1}{2}}=-1$$

OR

(B) Given parametric equations:

- $x=a\left(\cos t+\log\tan\frac{t}{2}\right)$
- $y=a\sin t$

Step 1: Differentiate y with respect to t

- $\frac{dy}{dt}=a\cos t$

Step 2: Differentiate x with respect to t

$$\frac{dx}{dt}=a\left(-\sin t+\frac{1}{\tan\frac{t}{2}}\cdot\sec^2\frac{t}{2}\cdot\frac{1}{2}\right)$$

Simplify the trigonometric inner part:

$$\frac{1}{\frac{\sin(t/2)}{\cos(t/2)}}\cdot\frac{1}{\cos^2(t/2)}\cdot\frac{1}{2}=\frac{1}{2\sin(t/2)\cos(t/2)}=\frac{1}{\sin t}$$

Substitute it back into $\frac{dx}{dt}$:

$$\frac{dx}{dt}=a\left(-\sin t+\frac{1}{\sin t}\right)=a\left(\frac{1-\sin^2 t}{\sin t}\right)=a\frac{\cos^2 t}{\sin t}$$

Step 3: Find $\frac{dy}{dx}$

$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{a\cos t}{a\frac{\cos^2 t}{\sin t}}=\frac{\sin t}{\cos t}=\tan t$$

228. Initial Data

- Bag I: 5 Red (R) + 6 Black (B) = 11 total balls
- Bag II: 3 Red (R) + 4 Black (B) = 7 total balls

(i) Probability of getting both red balls

One ball is drawn from each bag independently.

- P(Red from Bag I) $=\frac{5}{11}$
- P(Red from Bag II) $=\frac{3}{7}$

Probability $=\frac{5}{11}\times\frac{3}{7}=\frac{15}{77}$

(ii) Probability of getting one red and one black ball

There are two possible cases:

(1) Case 1: Red from Bag I and Black from Bag II

P(Case 1) $=\frac{5}{11}\times\frac{4}{7}=\frac{20}{77}$

(2) Case 2: Black from Bag I and Red from Bag II

P(Case 2) $=\frac{6}{11}\times\frac{3}{7}=\frac{18}{77}$

Total Probability $=\frac{20}{77}+\frac{18}{77}=\frac{38}{77}$

(iii) (A) Probability of getting a red ball after transfer

Two mutually exclusive cases depending on what is transferred:

- Case 1: Red ball is transferred from Bag I to Bag II
- Probability of transferring Red: $\frac{5}{11}$
- New composition of Bag II: 4 Red +4 Black = 8 total balls
- Probability of drawing Red from Bag II: $\frac{4}{8}$
- P(Case 1) $=\frac{5}{11}\times\frac{4}{8}=\frac{20}{88}$
- Case 2: Black ball is transferred from Bag I to Bag II
- Probability of transferring Black: $\frac{6}{11}$
- New composition of Bag II: 3 Red +5 Black = 8 total balls
- Probability of drawing Red from Bag II: $\frac{3}{8}$
- P(Case 2) $=\frac{6}{11}\times\frac{3}{8}=\frac{18}{88}$

Total Probability $=\frac{20}{88}+\frac{18}{88}=\frac{38}{88}=\frac{19}{44}$

OR

(B) Probability of getting a black ball after transfer

Two mutually exclusive cases depending on what is transferred:

- Case 1: Red ball is transferred from Bag I to Bag II
- Probability of transferring Red: $\frac{5}{11}$
- New composition of Bag II: 4 Red +4 Black = 8 total balls
- Probability of drawing Black from Bag II: $\frac{4}{8}$

- P(Case 1) $= \frac{5}{11} \times \frac{4}{8} = \frac{20}{88}$
- Case 2: Black ball is transferred from Bag 1 to Bag II
- Probability of transferring Black: $\frac{6}{11}$
- New composition of Bag II: 3 Red +5 Black $=8$ total balls
- Probability of drawing Black from Bag II: $\frac{5}{8}$
- P(Case 2) $= \frac{6}{11} \times \frac{5}{8} = \frac{30}{88}$

Total Probability $= \frac{20}{88} + \frac{30}{88} = \frac{50}{88} = \frac{25}{44}$