General Instructions :

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions. All questions are compulsory.
(ii) This question paper is divided into five Sections $-$A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculator is not allowed.

SECTION – A

1. If $2\cos^{-1}x = y$, then

(a) $0 \le y \le \pi$ (b) $-\pi \le y \le \pi$
(c) $0 \le y \le 2\pi$ (d) $-\pi \le y \le 0$

2. Which of the following cannot be the order of a row-matrix ?

(a) 2×1 (b) 1×2
(c) 1×1 (d) $1 \times n$

3. Which of the following properties is/are true for two matrices of suitable orders?

(I) $(A + B)' = A' + B'$
(II) $(A - B)' = B' - A'$
(III) $(AB)' = A'B'$
(IV) $(kAB)' = kB'A'$ (k is a scalar)

(a) (I) only (b) (I), (II) and (III)
(c) (I) and (II) (d) (I) and (IV)

4. If $\Delta_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{vmatrix}$ and $\Delta_2 = \begin{vmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{vmatrix}$, then

(a) $\Delta_1 = 2\Delta_2$ (b) $\Delta_2 = -2\Delta_1$
(c) $\Delta_1 = \Delta_2$ (d) $\Delta_2 = -\Delta_1$

5. One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is

(a) 0 (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

6. If A and B are skew symmetric matrices of same order, then which of the following matrices is also skew symmetric?

(a) AB (b) $AB + BA$
(c) $(A + B)^2$ (d) $A - B$

7. The least value of $f(x) = x^3 - 12x, x \in [0, 3]$ is

(a) -16 (b) -9
(c) 0 (d) 16

8. If $\int \frac{3ax}{b^2+c^2x^2} dx = A\log|b^2 + c^2x^2| + K$, then the value of A is

(a) $3a$ (b) $\frac{3a}{2b^2}$
(c) $\frac{3a}{b^2c^2}$ (d) $\frac{3a}{2c^2}$

9. The value of $\int_{-1}^{1} \frac{x^3}{x^2+2|x|+1}\,dx$ is

(a) 0 (b) log2

(c) 2log2 (d) $\frac{1}{2}$log2

10. The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

(a) 0 (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{4}{3}$

11. The integrating factor of differential equation $R\frac{dx}{dy} + Px = Q$ where P, Q, R are functions of y is

(a) $e^{\int \frac{P}{Q}dy}$ (b) $e^{\int Pdy}$

(c) $e^{\int \frac{P}{R}dy}$ (d) $e^{\int \frac{P}{R}dx}$

12. The order and degree of the differential equation $\frac{d}{dx}(e^y) = 0$ respectively are

(a) 0,1 (b) 1,1

(c) 2,1 (d) 1, not defined

13. The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is

(a) 0 (b) 1

(c) $\frac{5}{2}$ (d) $-\frac{5}{2}$

14. The value of m for which the points with position vectors $-\hat{i} - \hat{j} + 2\hat{k}$, $2\hat{i} + m\hat{j} + 5\hat{k}$ and $3\hat{i} + 11\hat{j} + 6\hat{k}$ are collinear, is

(a) 8 (b) -8

(c) 2 (d) $\frac{5}{2}$

15. If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a} \times \vec{b}| = 12$, then the value of $|\vec{a} \cdot \vec{b}|$

(a) $6\sqrt{3}$ (b) $8\sqrt{3}$

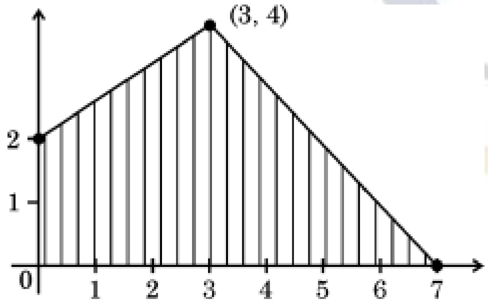
(c) $12\sqrt{3}$ (d) $3\sqrt{12}$

16. The length of perpendicular drawn from point $(2, 5, 7)$ on line $\frac{x}{1} = \frac{y}{0} = \frac{z}{0}$ is

(a) 2 (b) 5

(c) $\sqrt{74}$ (d) $\sqrt{78}$

17. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below :



The maximum value of Z - minimum value of Z is

(a) 8 (b) 29

(c) 35 (d) 43

18. The degree of an objective function of a linear programming problem is

(a) 0 (b) 1

(c) 2 (d) Any natural number

Direction : Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R).

Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion

(A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

19. **Assertion (A) :** In an experiment of throwing an unbiased die, the probability of getting a prime number given that number appearing on the die being odd is $\frac{2}{3}$.

Reason (R) : For any two events A and B, $P(A \mid B) = \frac{P(A \cup B)}{P(B)}$

20. **Assertion (A) :** Lines given by $x = py + q, z = ry + s$ and $x = p'y + q', z = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$.

Reason (R) : Two lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.

SECTION – B

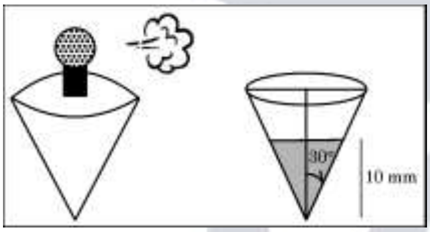
21. **(A)** Check whether function f(x) defined as

$$f(x) = \begin{cases} \frac{|x-3|}{2(x-3)}, & x < 3 \\ \frac{x-6}{6}, & x \geq 3 \end{cases}$$ is continuous at $x = 3$ or not?

OR

(B) If $\sqrt{3}(x^2 + y^2) = 4xy$, then find $\frac{dy}{dx}$ at $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$.

22. A room freshner bottle in the shape of an inverted cone sprays the perfume at regular intervals such that volume of the perfume in the bottle decreases at the steady rate of $1\ mm^3/min$. Find the rate at which level of perfume is dropping at an instant when level of perfume in the bottle is 10 mm , if the semi-vertical angle of conical bottle is $\frac{\pi}{6}$.



23. Find the vector of magnitude 14 in the direction of $\overrightarrow{QP}$, where P and Q are the points (1,3,2) and (−1,0,8) respectively.

24. Vectors $\vec{a} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the vectors representing its diagonals and hence find their lengths.

25. **(A)** Simplify : $\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right), 0 < x < \frac{\pi}{4}$.

OR

(B) Evaluate : $\tan\left(\sin^{-1}1 - \cos^{-1}\left(-\frac{1}{2}\right)\right)$

SECTION – C

26. Evaluate: $\int_0^1 x\tan^{-1}x\, dx$.

27. **(A)** Find $\int \sqrt{\frac{x+2}{x-2}}\, dx$

OR

(B) Find: $\int \frac{x^2}{(x^2+9)(x^2+16)}\, dx$

28. If $I_1 = \int_{-\pi/4}^{\pi/4} \frac{dx}{1+\cos 2x}$ and $I_2 = \int_{-1/2}^{1/2} |x|dx$, then show that $I_1 - 4I_2 = 0$.

29. **(A)** Find the general solution of the following differential equation :

$$x^2\frac{dy}{dx} = x^2 + xy + y^2$$

OR

(B) Find the particular solution of the differential equation $xy\frac{dy}{dx} = (x+2)(y+2)$, given that $y(1) = -1$.

30. Solve the following linear programming problem graphically :

Minimize $Z = 13x - 15y$

Subject to constraints

$x + y \leq 7$

$2x - 3y + 6 \geq 0$

$x \geq 0, y \geq 0$

31. **(A)** Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.

OR

(B) The probability of simultaneous occurrence of atleast one of the two events X and Y is a . If the probability that exactly one of the events X, Y occurs is b, prove that $P(X') + P(Y') = 2 - 2a + b$.

SECTION – D

32. **(A)** A relation R is defined on Z , the set of integers, as $R = \{(x, y): |x - y| \text{ is divisible by a prime number ' p '}, x, y \in Z\}$ check whether R is an equivalence relation or not.

OR

(B) A function $f: R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ is defined as $f(x) = \frac{3x+2}{5x-3}$. Show that f is one-one and onto.

33. **(A)** If $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & -1 & -2 \\ 1 & -1 & 0 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$-2y + z = 7, 2x - y - z = 8, x - 2y = 10$

OR

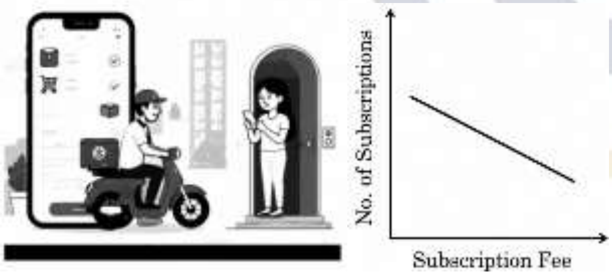
(B) If $\begin{bmatrix} 3 & -1 & \sin 3x \\ -7 & 4 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ is a singular matrix, then find all values of x where $x \in \left[0, \frac{\pi}{2}\right]$.

34. If $x = \cos t, y = \cos mt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$.

35. Check whether the lines given by $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ are parallel or not. If parallel, find the distance between them, otherwise find their point of intersection, if the lines are intersecting.

SECTION – E

36. An online delivery company in a city has 5000 subscribers and collects annual subscription fees of ₹ 300 per subscriber for unlimited free deliveries.



The company wishes to increase the annual subscription fee. It is predicted that, for every increase of ₹ 1 , ten subscribers will discontinue. Assume that the company increased the annual fee by ₹x.

Based on the given information, answer the following questions :

(i) How many subscribers will discontinue after an increase of ₹x in annual fee?

(ii) If R(x) denotes the total revenue collected after the increase of ₹x in subscription fee, express R(x) as a function of x.

(iii) **(A)** Find the value of x for which R(x) is maximum.

OR

(B) Find the sub-intervals of (0,5000) in which R(x) is increasing and decreasing.

37. In an online jackpot, there is one first prize of ₹3,00,000, two second prizes of ₹ 2,00,000 each and three third prizes of ₹ 50,000 each.

A total of 1,00,000 jackpot tickets each costing ₹ 100 were sold there by raising a fund of ₹ 1,00,00,000.
Rohan bought one ticket.
Based on given information, answer the following questions :
(i) What are the possible amounts, the person can win?
(ii) (A) What is the probability that the person wins atleast ₹ 2,00,000 ?
OR
(B) What is the probability that the person does not win any amount?
(iii) In another jackpot, Rohan also bought a ticket having a prize money of ₹ 5,00,000. The chances of winning the jackpot are 1 in 1,00,000. Find the probability that on exactly one of tickets he wins the jackpot.

38. Roundabouts are often made on busy roads to ease the traffic and avoid red lights.

One such round-about is made such that equation representing its boundary is given by $C_1; x^2 + y^2 = 64$.
There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2: x^2 + y^2 = 4$.
Based on the given information, answer the following questions :
(i) Represent the given equations C_1 and C_2 with the help of a diagram.
(ii) Express y as a function of x, $(y = f(x))$, for both C_1 an C_2.
(iii) (A) Using integration find the area of region covered by the roundabout.
OR
(B) Using integration, find the area of region covered by circular pond.

General Instructions :

Read the following instructions very carefully and strictly follow them:
(i) This question paper contains 38 questions. All questions are compulsory.
(ii) This question paper is divided into five Sections $-$A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculator is not allowed.

SECTION – A

39. A relation R on set $A = \{1, 2, 3\}$ defined as $R = \{(1, 1), (2, 2), (1, 2)\}$ is
(a) Reflexive only
(b) Reflexive and Transitive
(c) Symmetric and Transitive
(d) Transitive only

40. If A and B are square matrices of same order, then which of the following statements is/are always true ?

(I) $(A+B)(A-B) = A^2 - B^2$

(II) $AB = BA$

(III) $(A+B)^2 = A^2 + AB + BA + B^2$

(IV) $AB = 0 \Rightarrow A = 0$ or $B = 0$

(a) Only (I) and (III) (b) Only (II) and (III)

(c) Only (III) (d) Only (III) and (IV)

41. If $A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$ is a symmetric matrix, then the value of $3a + b + c$ is

(a) 2 (b) 6

(c) 4 (d) 0

42. If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$ and $A + A' = I$, then the value of $x \in \left[0, \frac{\pi}{2}\right]$ is

(a) 0 (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

43. For a square matrix A, $(3\,A)^{-1} =$

(a) $3\,A^{-1}$ (b) $9\,A^{-1}$

(c) $\frac{1}{3}\,A^{-1}$ (d) $\frac{1}{9}\,A^{-1}$

44. If $\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -86$, then the sum of all possible values of a is

(a) 4 (b) 5

(c) -4 (d) 9

45. If $e^{-x} + e^{-y} = 2$, then $\frac{dy}{dx}$ is

(a) e^{x-y} (b) e^{y-x}

(c) $-e^{x-y}$ (d) $-e^{y-x}$

46. For $f(x) = x + \frac{1}{x} (x \neq 0)$

(a) local maximum value is 2

(b) local minimum value is -2

(c) local maximum value is -2

(d) local minimum value < local maximum value

47. If $\int_0^{2a} \frac{1}{1+4x^2}\, dx = \frac{\pi}{6}$, then the value of a is

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$ (d) $2\sqrt{3}$

48. Which of the following expressions will give the area of region bounded by the curve $y = x^2$ and line $y = 16$?

(a) $\int_0^4 x^2 dx$ (b) $2\int_0^4 x^2 dx$

(c) $\int_0^{16} \sqrt{y} dy$ (d) $2\int_0^{16} \sqrt{y} dy$

49. The general solution of the differential equation $\frac{dy}{dx} = \frac{\sqrt{y}}{\sqrt{x}}$ is

(a) $\log\sqrt{y} = \log\sqrt{x} + C$ (b) $\sqrt{y} + \sqrt{x} = C$

(c) $\sqrt{y} - \sqrt{x} = C$ (d) $\log\sqrt{y} + \log\sqrt{x} = C$

50. The integrating factor of the differential equation $2x\frac{dy}{dx} - y = 3$ is

(a) $\sqrt{x}$ (b) $\frac{1}{\sqrt{x}}$

(c) e^{x} (d) e^{-x}

51. If $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$, then the sum of greatest and the smallest value of $|\lambda\vec{a}|$ is

(a) -5 (b) 5

(c) 10 (d) 15

52. Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z axis is

(a) $\hat{\imath} + 2\sqrt{2}\hat{\jmath}$ (b) $3\hat{k}$

(c) $\frac{3\sqrt{2}}{2}\hat{\imath} + \frac{3\sqrt{2}}{2}\hat{\jmath}$ (d) $\sqrt{3}\hat{\imath} + \sqrt{3}\hat{\jmath} + \sqrt{3}\hat{k}$

53. Direction cosines of the line given by equations: $\frac{2x-1}{4} = \frac{1-y}{3} = \frac{-z}{6}$ are

(a) $2, -3, -6$ (b) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$

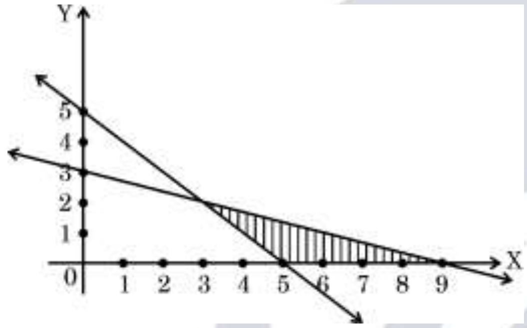
(c) $\frac{2}{7}, \frac{-3}{7}, \frac{6}{7}$ (d) $\frac{4}{\sqrt{61}}, \frac{-3}{\sqrt{61}}, \frac{-6}{\sqrt{61}}$

54. In a linear programming problem, the linear function which has to be maximized or minimized is called

(a) a feasible function (b) an objective function

(c) an optimal function (d) a constraint

55. For the feasible region shown below, the non-trivial constraints of the linear programming problem are



(a) $x + y \leq 5, x + 3y \leq 9$

(b) $x + y \leq 5, x + 3y \geq 9$

(c) $x + y \geq 5, x + 3y \leq 9$

(d) $x + y \geq 5, 3x + y \leq 9$

56. For two events A and B such that $P(A) \neq 0$ and $P(B) \neq 1$, $P(A'/B') =$

(a) $1 - P(A/B)$ (b) $1 - P(A'/B)$

(c) $\frac{1-P(A\cap B)}{P(B')}$ (d) $\frac{1-P(A\cup B)}{P(B')}$

Direction : Question numbers 19 and 20 are Assertion (A) and Reason (R) based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

57. For two vectors $\vec{a}$ and $\vec{b}$

Assertion (A) : $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

Reason (R): : $|\vec{a} \times \vec{b}| = (\vec{a} \cdot \vec{b})\tan\theta, \left(\theta \neq \frac{\pi}{2}\right)$

58. Assertion (A) : A line can have direction cosines $\langle 1,1,1 \rangle$

Reason (R) : $\cos\theta = 1$ is possible for $\theta = 0$.

SECTION – B

59. (A) Check whether $f: R - \{3\} \rightarrow R$ defined as $f(x) = \frac{x-2}{x-3}$ is onto or not.

OR

(B) Check whether f: $Z \times Z \rightarrow Z \times Z$ (where Z is the set of integers) defined as $f(x, y) = (2y, 3x)$ is injective or not.

60. If $x = a\sin^3 t, y = b\cos^3 t$, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

61. **(A)** Find the absolute maximum value of $f(x) = \cos x + \sin^2 x, x \in [0, \pi]$

OR

(B) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.

62. If $\overrightarrow{AB} = \hat{j} + \hat{k}$ and $\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$ represent the two vectors along the sides AB and AC of $\triangle$ ABC, prove that the median $\overrightarrow{AD} = \frac{\overrightarrow{AB}+\overrightarrow{AC}}{2}$, where D is midpoint of BC.

Hence, find the length of median AD.

63. Find the co-ordinates of the point on the line $\vec{r} = -\hat{j} + 3\hat{k} + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ such that the sum of co-ordinates is 3 .

SECTION – C

64. Find : $\int \frac{x+2}{\sqrt{9x-x^2}} dx$

65. **(A)** Evaluate: $\int_{\frac{\pi}{12}}^{\frac{5\pi}{12}} \frac{dx}{1+\sqrt{\cot x}}$

OR

(B) Evaluate: $\int_{\frac{-\pi}{6}}^{\frac{\pi}{2}} (\sin|x| + \cos|x|)dx$

66. If $\frac{d}{dx}(F(x)) = \frac{1}{e^x+1}$, then find $F(x)$ given that $F(0) = \log\frac{1}{2}$.

67. **(A)** Solve the following differential equation :

$x\frac{dy}{dx} = y - x\sin^2\left(\frac{y}{x}\right)$, given that $y(1) = \frac{\pi}{6}$

OR

(B) Find the general solution of the differential equation: $y\log y\frac{dx}{dy} + x = \frac{2}{y}$.

68. Solve the following linear programming problem graphically :

Maximize $Z = 10500x + 9000y$

Subject to constraints

$x + y \leq 50$

$2x + y \leq 80$

$x, y \geq 0$

69. **(A)** The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed.

The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that

(i) target is hit

(ii) atleast one shot misses the target.

OR

(B) Mother, Father and Son line up at random for a family picture. Let events E : Son on one end and F : Father in the middle. Find P(E/F).

SECTION – D

70. **(A)** If P $= \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and Q $= \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find (QP) and hence solve the following system of equations using matrices:

$x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7$

OR

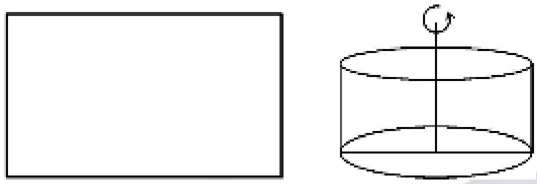
(B) Obtain the value of $\Delta = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$ in terms of x, y and z.

Further, if $\Delta = 0$ and x, y, z are non-zero real numbers, prove that $x^{-1} + y^{-1} + z^{-1} = -1$.

71. **(A)** Find the sub intervals in which $f(x) = \cot^{-1}(\sin x + \cos x), x \in (0, \pi)$ is increasing and decreasing.

OR

(B) A rectangle of perimeter 36 cm is revolved around one of its sides to sweep out a cylinder of maximum volume.



Find the dimensions of the rectangle.

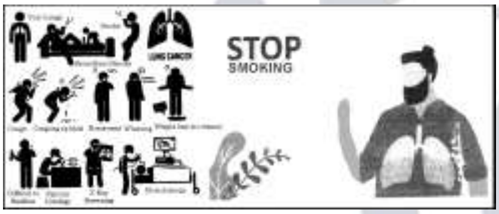
72. Find the domain of $g(x) = \cos^{-1}(x^2 - 1)$. Hence, find the value of x for which $g(x) = \frac{\pi}{3}$.

Also, write the range of $\cos^{-1} x$ other than its principal branch.

73. A line passing through the points A(1,2,3) and B(5,8,11) intersects the line $\vec{r} = 4\hat{\imath} + \hat{\jmath} + \lambda(5\hat{\imath} + 2\hat{\jmath} + \hat{k})$. Find the co-ordinates of the point of intersection. Hence, write the equation of a line passing through the point of intersection and perpendicular to both the lines.

SECTION – E

74. Smoking increases the risk of lung problems.



A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems. In a colony, 50 people were found to be smokers of which 30 are males.

A person is selected at random from these 50 people and tested for lung related problems.

Based on the given information, answer the following questions :

(i) What is the probability that selected person is a female?

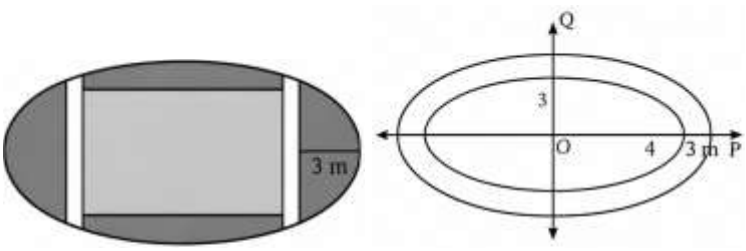
(ii) If a male person is selected, what is the probability that he will not be suffering from lung problems?

(iii) **(A)** A person selected at random is detected with lung complications. Find the probability that selected person is a female.

OR

(B) A person selected at random is not having lung problems, find the probability that the person is a male.

75. A racing track is build around an elliptical ground whose equation is given by $9x^2 + 16y^2 = 144$. The width of the track is 3 m as shown below :



Based on given information, answer the following questions :

(i) Express y as a function of x from the given equation of ellipse.

(ii) Integrate the function obtained in (i) with respect to x.

(iii) (A) Find the area of the region enclosed within the elliptical ground excluding the track using integration.

OR

(B) Write the co-ordinates of the points P and Q where the outer edge of the track cuts x axis and y axis in first quadrant and find the area of the triangle formed by points P, O, Q using integration.

76. Sports car racing is a form of motorsport which uses sports car prototypes. The competition is held on special tracks designed in various shapes.

The equation of one such track is given as follows:

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \le x < 3 \\ x^2 + 40, & x \ge 3 \end{cases}$$

Based on given information, answer the following questions :

(i) Find $f'(x)$ for $0 < x < 3$.

(ii) Find $f'(4)$

(iii) (A) Test for continuity of $f(x)$ at $x = 3$.

OR

(B) Test for differentiability of $f(x)$ at $x = 3$.

General Instructions :

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) This question paper is divided into five Sections —A, B, C, D and E.

(iii) In Section A, Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is not allowed.

SECTION A

77. The domain of $f(x) = \cos^{-1}(2x - 5)$ is :

(a) $[-1,1]$ (b) $[4,6]$

(c) $[-7,-3]$ (d) $[2,3]$

78. If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is :

(a) -1 (b) 1

(c) $\frac{5}{3}$ (d) 7

79. If A and B are skew-symmetric matrices of same order, then $AB' + BA'$ is a/an :

(a) symmetric matrix (b) skew-symmetric matrix

(c) null matrix (d) identity matrix

80. If $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$, then order of A must be :

(a) 3×1 (b) 1×3

(c) 1×1 (d) 3×3

81. If a square matrix A is such that $A^2 = A$ and $(I - A)^3 = xA + I$, then value of x must be :

(a) 7 (b) 5
(c) -7 (d) -1

82.

If $B(\text{adj}B) = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$, then the value of $\det(B^{-1}) =$

(a) $\frac{1}{3}$ (b) $\frac{1}{9}$
(c) 3 (d) 9

83. The value of k for which the function $f(x) = \begin{cases} x^2 \sin\frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$

is a continuous function, is:

(a) $\frac{1}{4}$ (b) 2
(c) $\frac{1}{2}$ (d) 0

84. If $\sin^{-1}x = y$, then $\frac{dy}{dx}$ is :

(a) $\cos^{-1}x$ (b) $\cos y$
(c) $\frac{1}{1-x^2}$ (d) $\sec y$

85. The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm , is :

(a) $400\pi\ cm^3/cm$ (b) $100\pi\ cm^3/cm$
(c) $50\pi\ cm^3/cm$ (d) $25\pi\ cm^3/cm$

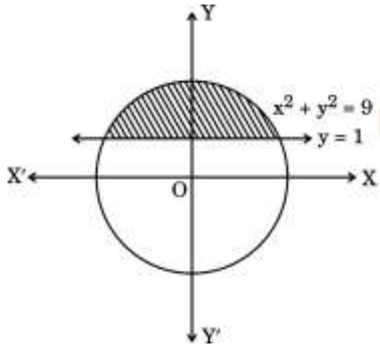
86. $\int \frac{dx}{2^x + 2^{-x}}$ is equal to:

(a) $\tan^{-1}(2^x) + C$ (b) $\tan^{-1}(2^{-x}) + C$
(c) $\frac{\tan^{-1}(2^x)}{\log 2} + C$ (d) $(\log 2)\tan^{-1}(2^x) + C$

87. $\int_{-1}^{1} (1 - |x|)dx$ is equal to:

(a) $2\int_0^1 (1+x)dx$ (b) $2\int_{-1}^0 (1+x)dx$
(c) 0 (d) $2\int_{-1}^0 (1-x)dx$

88. The area of the shaded region of the circle given below is equal to :



(a) $\int_1^3 \sqrt{9-y^2}dy$ (b) $2\int_1^3 \sqrt{9-y^2}dy$
(c) $\int_0^3 \sqrt{9-x^2}dx$ (d) $2\int_0^3 \sqrt{9-x^2}dx$

89. $\frac{dy}{dx} = F(x,y)$ will be a homogeneous differential equation for which of the following functions?

(I) $F(x,y) = 3x + 2y$

(II) $F(x,y) = \sin\frac{y}{x} + \log y - \log x$

(III) $F(x,y) = e^{y/x} + 1$

(IV) $F(x, y) = \sqrt{x^2 + y^2} - y$

(a) (I) and (II) (b) (I), (II) and (III)

(c) (II), (III) and (IV) (d) (II) and (III)

90. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true?

(a) $\vec{a} \cdot \vec{b} \leq |\vec{a}||\vec{b}|$ (b) $|\vec{a} + \vec{b}| \geq |\vec{a}| + |\vec{b}|$

(c) $|\vec{a} - \vec{b}| = |\vec{a}| - |\vec{b}|$ (d) $|\vec{a} \times \vec{b}| \geq |\vec{a}||\vec{b}|$

91. If $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 198$ and $|\vec{a}| = 10|\vec{b}|$, then:

(a) $|\vec{a}| = \sqrt{2}$ (b) $|\vec{b}| = \sqrt{2}$

(c) $|\vec{b}| = 10\sqrt{2}$ (d) $|\vec{a}| = \frac{10}{\sqrt{2}}$

92. If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then :

(a) $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

(b) $\sin\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

(c) $\tan\theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$

(d) $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$

93. Direction ratios of lines l_1 and l_2 are $\langle 12, -3, 9\rangle$ and $\langle 4, q, -p\rangle$ respectively. The values of p and q for which l_1 and l_2 are parallel are respectively :

(a) $-1, 3$ (b) 3,1

(c) $-3, -1$ (d) $-1, -3$

94. If E and F are two independent events such that $P(E) = \frac{3}{10}, P(E \cup F) = \frac{1}{2}$, then $P(E \mid F) - P(F \mid E)$ is equal to :

(a) $\frac{2}{7}$ (b) $\frac{3}{35}$

(c) $\frac{1}{70}$ (d) $\frac{1}{7}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

95. Assertion (A) : One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$.

Reason (R): $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.

96. Assertion (A) : The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq \vec{0}$ are collinear vectors.

Reason (R): $\vec{a} \cdot (-2\vec{a}) = 0$.

SECTION B

97. If $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$, find $\frac{dy}{dx}$ at $t = 2$.

98. Find the sub-interval(s) of $\left(0, \frac{\pi}{2}\right)$ in which $f(x) = \tan x - 4x$ is increasing.

99. (A) Find the value of $\sin\left[\cot^{-1}\sqrt{2}\left(\cos(\tan^{-1}1)\right)\right]$.

OR

(B) A relation R on $A = \{1,2,3\}$ is defined as $R = \{(1,1)(3,3), (1,2)\}$. Is R a symmetric relation ? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set {1,2,3}.

100. If for two unit vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + 2\vec{b}| = |2\vec{a} - \vec{b}|$, then find the angle between $\vec{a}$ and $\vec{b}$.

101. **(A)** If the lines $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ and $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are perpendicular to each other, then find the value(s) of p.

OR

(B) Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$.

SECTION C

102. **(A)** Find :

$$\int \frac{dx}{x^{1/2} + x^{1/3}}$$

OR

(B) Find :

$$\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$$

103. Evaluate :

$$\int_0^{\pi} \frac{\sin^{2026}x}{\sin^{2026}x + \cos^{2026}x} dx$$

104. **(A)** Find :

$$\int \frac{\cos x}{(2+\sin x)(4+\sin x)} dx$$

OR

(B) Find :

$$\int \frac{x+3}{x^2+4x+5} dx$$

105. **(A)** Find the general solution of the differential equation $2x^2\frac{dy}{dx} = y^2 + 2xy$.

OR

(B) Find a particular solution of the differential equation $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$, given that $y = 0$ when $x = 0$.

106. If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.

107. A survey was conducted on the patients who have undergone knee replacement surgeries.

It was found that, Robotic Knee replacement surgeries have 90% success rate.
On a particular day, robotic surgery was performed on three patients, A, B and C , one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that :
(i) exactly one surgery is successful,
(ii) at most two surgeries are successful.

SECTION D

108. Show that $f: R \to R$ defined as $f(x) = \frac{x}{\sqrt{1+x^2}}$ is one-one but not onto.

109. Solve the following Linear Programming Problem graphically :

Maximise $Z = 600x + 400y$
subject to the constraints
$x + 2y \le 12$
$4x + 5y \ge 20$
$2x + y \le 12$
$x, y \ge 0$

110. **(A)** On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of ₹ 1,000 and ₹ 500 were given to the lucky draw winners.

A total of 60 vouchers were given on the day. The number of ₹ 1,000 vouchers added to 3 times the number of ₹ 500 vouchers, gives 100 . Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method.

OR

(B) Given that $P = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, Q = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $R = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix S such that PQ − RS is a null matrix

111. **(A)** Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not.

$$l_1: \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$$
$$l_2: \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$$

OR

(B) Opposite sides of a square are along the lines:

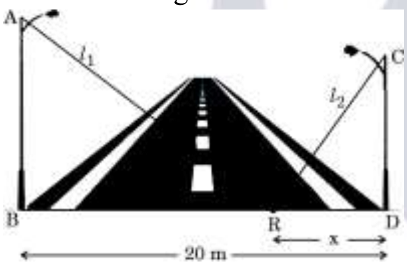
$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$
$$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$$

Find the area of the square if direction ratios of other pair of opposite sides of the square are given by $< -3, 6, p >$. Also, find the value of p.

SECTION E

Case Study - 1

112. Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.



Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole.

Based on the above information, answer the following questions :

(i) Express $p(x) = l_1 + l_2$ in terms of x .

(ii) Find $p'(x)$.

(iii) **(A)** Find the value of x for which $l_1^2 + l_2^2$ is minimum.

OR

(B) If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum?

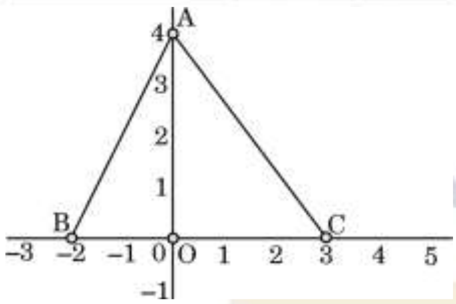
Case Study – 2

113. A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.

As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII.
Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination.
Based on the above information, answer the following questions.
(i) Find the probability that a student selected at random is a regular student.
(ii) A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination?
(iii) (A) A student selected at random qualified the examination. Find the probability that student is not a dropout.
OR
(B) A student selected at random did not qualify the examination. Find the probability that the student was a regular student.

Case Study - 3

114. There is a triangular park in the society. The park is divided into two sections as shown in the figure.



In the region OAC , children are allowed to play games like cricket, football, while in the region AOB , activities which involve running are not allowed.
The vertices of the triangular park ABC are $A(0,4), B(-2,0)$ and $C(3,0)$. Based on the above information, answer the following questions:
(i) Write the equation of the boundary line AB of the park.
(ii) Write the equation of the boundary line AC of the park.
(iii) (A) Using integration, find the area of region OAC , in which children are allowed to play cricket, football.
OR
(B) Using integration, find the area of region AOB .

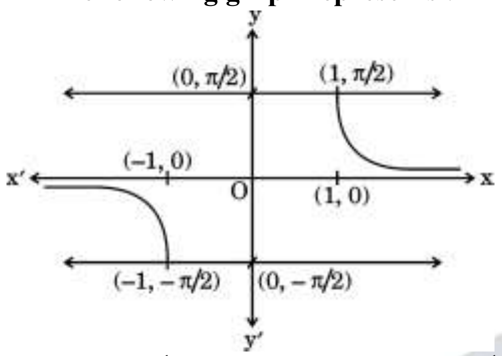
General Instructions :

Read the following instructions very carefully and strictly follow them:
(i) This question paper contains 38 questions. All questions are compulsory.
(ii) This question paper is divided into five Sections $-A, B, C, D$ and E.
(iii) In Section A, Questions no. 1 to 18 are Multiple Choice Questions ($MCQs$) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculator is not allowed.

SECTION A

115. The following graph represents :



(a) $y = \cos^{-1}x$ (b) $y = \sec^{-1}x$
(c) $y = \tan^{-1}x$ (d) $y = \text{cosec}^{-1}x$

116. If $A = [a_{ij}]$ is a 2×2 matrix whose elements are given by $a_{ij} = \frac{|i-3j|}{2}$, then A' is:

(a) $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & \frac{1}{2} \\ \frac{5}{2} & 1 \end{bmatrix}$

117. The principal value of $\sec^{-1}(\sqrt{2}) + 2\text{cosec}^{-1}(-\sqrt{2})$ is :

(a) $-\frac{\pi}{2}$ (b) $-\frac{\pi}{4}$
(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

118. If points $(2,3), (0,4)$ and $(p,2)$ are collinear, then the value of p is :

(a) $\frac{4}{7}$ (b) $-\frac{3}{7}$
(c) 4 (d) -4

119. Differential of e^e with respect to x is :

(a) logx (b) e^{e^x}
(c) $e^x e^{e^x}$ (d) $(e^x)^2$

120. The surface area of a sphere when its volume changes at the same rate as its radius is :

(a) 4π sq. units (b) 1 sq. unit
(c) 4 sq. units (d) π sq. units

121. If $f(x) = \begin{cases} \frac{\sin x}{x} + \cos x, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$, then the value of k is:

(a) 0 (b) -2
(c) -1 (d) 2

122. The greatest integer function, $f(x) = [x], 0 < x < 3$ is not differentiable at how many points?

(a) At only one point (b) At only two points
(c) At no point (d) At three points

123. $\int \frac{dx}{\sqrt{25-16x^2}}$ **is equal to :**

(a) $\frac{1}{5}\sin^{-1}4x + C$ (b) $\frac{1}{25}\sin^{-1}16x + C$

(c) $\frac{1}{4}\sin^{-1}\frac{4x}{5} + C$ (d) $\frac{1}{16}\sin^{-1}\frac{4x}{5} + C$

124. **If** $\int_0^1 \frac{dx}{e^x+e^{-x}} = \tan^{-1}e + k$**, then the value of** k **is :**

(a) e (b) $\frac{\pi}{4}$

(c) 0 (d) $-\frac{\pi}{4}$

125. **The area of the region bounded by the curve** $y = x$ **and** x**-axis, between** $x = 0$ **and** $x = 2$ **is :**

(a) 2 sq. units (b) $\frac{1}{2}$ sq. unit

(c) 1 sq. unit (d) 4 sq. units

126. **Product of the order and degree of differential equation** $1 + \left(\frac{dy}{dx}\right)^3 = \lambda\left(\frac{d^3y}{dx^3}\right)^2$ **is :**

(a) 5 (b) 6

(c) 2 (d) 3

127. **Which of the following is not a Linear Differential Equation?**

(a) $(1 + x^2)dy + 2xydx = \cot x dx$

(b) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

(c) $x(1 + y^2)dx - y(1 + x^2)dy = 0$

(d) $ydx - (x + 3y^2)dy = 0$

128. **The region represented by the system of inequations** $3x + y \geq 3$, $2x - y \geq -5$, $x, y \geq 0$ **is:**

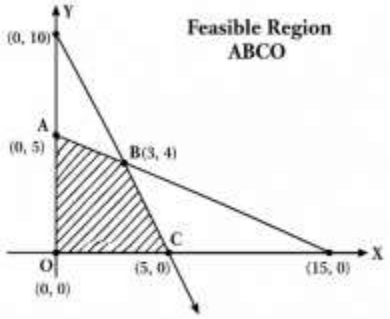
(a) unbounded in 1st quadrant

(b) bounded in 1st quadrant

(c) unbounded in 2nd quadrant

(d) bounded in 2nd quadrant

129. **In the graph, the feasible region representing the Linear Programming Problem for maximising objective function** $Z = px + qy$, $p, q > 0$ **is shaded. If all points on segment** AB **give max (** Z **), then which of the following is true?**



(a) $p = 2q$ (b) $p = 3q$

(c) $q = 3p$ (d) $q = 2p$

130. **If** $(3\hat{\imath} - 2\hat{\jmath} + 5\hat{k}) \times (4\hat{\imath} + p\hat{\jmath} + q\hat{k}) = \vec{0}$**, then the values of** p **and** q **are :**

(a) $p = -\frac{2}{3}, q = \frac{5}{3}$ (b) $p = -\frac{8}{3}, q = \frac{20}{3}$

(c) $p = \frac{20}{3}, q = -\frac{8}{3}$ (d) $p = 0, q = 0$

131. **Three points** $A(0, 1, 1)$, $B(2, 0, -1)$ **and** $C(1, 0, 3)$ **form** $\triangle ABC$**. The ar(**$\triangle ABC$**) is :**

(a) $\frac{\sqrt{53}}{2}$ sq. units (b) $\sqrt{53}$ sq. units

(c) $\frac{\sqrt{11}}{2}$ sq. units (d) $\sqrt{11}$ sq. units

132. A box contains 4 red, 5 blue and 1 green marble. A child randomly takes out a marble from the box, notes down the colour and puts it back in the box. If the activity is repeated 3 times, what is the probability that at least one marble is red?

(a) $\frac{27}{125}$ (b) $\frac{8}{125}$

(c) $\frac{2}{125}$ (d) $\frac{98}{125}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(c) Assertion (A) is true, but Reason (R) is false.

(d) Assertion (A) is false, but Reason (R) is true.

133. Assertion (A) : If A and B are two square matrices such that AB and BA are defined, then it is not necessary that $AB = BA$.

Reason (R) : Product of two diagonal matrices of same order is commutative.

134. Assertion (A) : A function $f: N \to N$ given by $f(x) = x^3 + 2, \forall x \in N$ is one-one but not onto.

Reason (R) : Since $\forall y \in N$ (Codomain), there does not exist $x = (y-2)^{1/3}$ in N (Domain) such that $f(x) = x^3 + 2 = y$.

SECTION B

135. Evaluate:

$$\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$$

136. (A) Show that the function $f(x) = \begin{cases} \frac{\cos x}{-x+\frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

OR

(B) Find whether the function $f(x) = \begin{cases} x-1, & x < 2 \\ 2x-3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.

137. Find the values of x for which $f(x) = x^x, x > 0$ is increasing.

138. If the position vectors of three points A, B and C are $3\hat{\imath} + \hat{\jmath}$, $5\hat{\imath} + 6\hat{\jmath} - 3\hat{k}$ and $4\hat{\jmath}$ respectively, then show that they form an isosceles triangle.

139. (A) Let two rods placed on the ground be represented by vectors $4\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $-2\hat{\imath} + \hat{\jmath} - 2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.

OR

(B) A unit vector $\vec{a}$ is such that it makes an angle $\frac{\pi}{4}$ with x-axis, $\frac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.

SECTION C

140. (A) Let $A = R - \{3\}$ and $B = R - \{1\}$. A function $f: A \to B$ is defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Find whether f is one-one and onto.

OR

(B) Let n be a fixed positive integer. A relation R is defined in set Z such that $R = \{(x, y): (x-y)$ is divisible by $n, x, y \in Z\}$. Determine if R is an equivalence relation.

141. If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then compute $A^2 - 7A + 10I$.

142. (A) If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.

OR

(B) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ with respect to $\cos^{-1}x^2$.

143. Solve the following Linear Programming Problem graphically :

Maximise $Z = 200x + 120y$

subject to the constraints

$x + y \leq 300$

$3x + y \leq 600$

$x - y \geq -100$

$x, y \geq 0$

144. **(A)** Find a point on the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z-3}{2}$ at a distance of $\sqrt{2}$ units from the point (1,2,3).

OR

(B) Find the shortest distance between the lines

$\vec{r} = (4+\lambda)\hat{\imath} + (2\lambda - 1)\hat{\jmath} - 3\lambda\hat{k}$

$\vec{r} = (1+2\mu)\hat{\imath} + (2-5\mu)\hat{k} + (4\mu - 1)\hat{\jmath}$

145. In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams, then find the probability that one will be a girl and the other a boy.

SECTION D

146. **(A)** Find :

$$\int \frac{x}{(x-1)(x^2+4)} dx$$

OR

(B) Evaluate :

$$\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}} dx$$

147. Using integration, find the area of the region enclosed by the curve $y = |x-6|$, the x-axis, and between $x = 4$ and $x = 8$.

148. **(A)** Solve the differential equation $ye^y dx = (y^3 + 2xe^y)dy$, when $y(0) = 1$.

OR

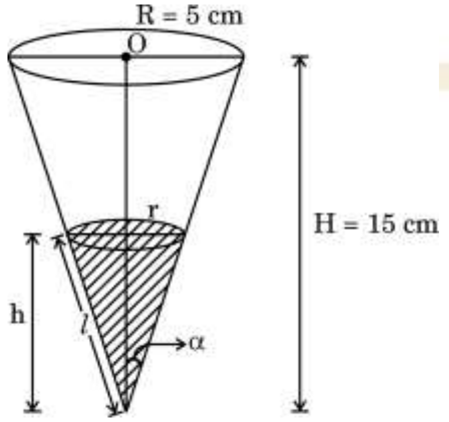
(B) Find the general solution of the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$.

149. Find the equation of a line (in vector and cartesian form) that passes through the point of intersection of lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ and is parallel to the vector $3\hat{\imath} + 2\hat{\jmath} - 8\hat{k}$.

SECTION E

Case Study - 1

150. At a birthday party, children are being served orange juice in conical cups, as shown in the figure.



Each cup is 15 cm deep and has a radius 5 cm . The juice is being poured into this cup at a rate of $0.1\ cm^3/s$.

On the basis of the above information, answer the following questions:

(i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α.

(ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep ?

(iii) (A) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing ?

OR

(B) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing.

Case Study - 2

151. A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm . Its breadth added to 7 times its height is 1 cm less than 3 times its length.

On the basis of the above information, answer the following questions:

(i) Write the equations representing the various dimensions and express them as the matrix equation $AX = B$.

(ii) Find if A^{-1} exists. Justify your answer.

(iii) (A) Find A^{-1}.

OR

(B) Find $A^2 + 7I$.

Case Study - 3

152. An NGO organises a charity event in which they decide to distribute woollen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, and Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where $i = 1,2,3$.

Based on the above information, answer the following questions:

A person selects a cap.

(i) What is the probability that he selects a red cap?

(ii) If he selects a green cap, what is the probability that the cap has come from Box II?

General Instructions :

Read the following instructions very carefully and strictly follow them:

(i) This question paper contains 38 questions. All questions are compulsory.

(ii) This question paper is divided into five Sections $-A, B, C, D$ and E.

(iii) In Section A, Questions no. 1 to 18 are Multiple Choice Questions ($MCQs$) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.

(v) In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.

(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is not allowed.

SECTION A

153. **If matrix $A = \begin{bmatrix} -p & q \\ r & p \end{bmatrix}$ is such that $A^2 = I$, then :**

(a) $1 + p^2 + qr = 0$(b) $1 - p^2 - qr = 0$

(c) $1 - p^2 + qr = 0$(d) $1 + p^2 - qr = 0$

154. **If A is a square matrix such that $A^2 = A$, then $(A - I)^3 - A$ is equal to :**

(a) I (b) -I

(c) A (d) A^2

155. **For the inverse trigonometric functions, which of the following Principal Value Branch is not correctly defined?**

(a)$\tan^{-1}: R \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(b)$\sec^{-1}: R - (-1,1) \rightarrow [0, \pi] - \left\{\frac{\pi}{2}\right\}$

(c)$\cot^{-1}: R \rightarrow (0, \pi)$

(d)$\text{cosec}^{-1}: R - (-1,1) \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

156. Let A = $\begin{bmatrix} 0 & -3 & 4 \\ 1 & 0 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} -3 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix}$. If A + B + C = O, then matrix C is :

(a) $\begin{bmatrix} -3 & -3 & 5 \\ 3 & 4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & 3 & 5 \\ -3 & -4 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 3 & -5 \\ -3 & -4 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & -3 & -5 \\ 3 & 4 & 2 \end{bmatrix}$

157. If A is a non-singular matrix, then which of the following is not true ?

(a) adjA is singular (b) $(\text{adjA})^{-1} = (\text{adjA}^{-1})$

(c) $|A| \neq 0$ (d) A^{-1} exists

158. If $f(x) = \begin{cases} \frac{x^2-4x-5}{x+1}, & x \neq -1 \\ k, & x = -1 \end{cases}$ is continuous at $x = -1$, then the value of k is :

(a) Any real value (b) 6

(c) -1 (d) -6

159. If the area of $\triangle$ ABC with vertices A$(3, 1)$, B$(-2, 1)$ and C$(0, k)$ is 5 sq. units, then values of k are :

(a) 3,1 (b) -1,3

(c) -1,2 (d) 0,2

160. Derivative of $\cos^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right), -\frac{\pi}{4} < x < \frac{\pi}{4}$ with respect to x is :

(a) -1 (b) 1

(c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

161. Absolute minimum value of $f(x) = (x-2)^2 + 5$ in the interval $[-3, 2]$ is :

(a) -3 (b) 2

(c) 5 (d) 30

162. $\int \frac{1}{\sqrt{1+\cos 2x}}$dx is equal to :

(a) $\log\cos x + C$

(b) $\frac{1}{\sqrt{2}}\log|\sec x + \tan x| + C$

(c) $\frac{1}{\sqrt{2}}\log|\sec x - \tan x| + C$

(d) $\log\sin 2x + C$

163. The value of $\int_{-5}^{-1} \frac{1}{x} dx$ is equal to:

(a) $-\log 5$ (b) x^6

(c) $\log(-5)$ (d) x^{-6}

164. An ant is observed crawling on a sheet of paper along a straight line given by equation y = 2x − 4. Area of the surface covered by the ant bounded by y -axis, x -axis and x = 1 is :

(a) 1 sq. unit (b) 3 sq. units

(c) 2 sq. units (d) 4 sq. units

165. The order and degree of the differential equation $1 + \left(\frac{d^3y}{dx^3}\right)^3 = \lambda\frac{d^2y}{dx^2}$ is :

(a) Order = 3, Degree = 3

(b) Order = 2, Degree = 2

(c) Order = 3, Degree = 1

(d) Order = 2, Degree = 1

166. The general solution for the differential equation $\frac{dy}{dx} = e^{3x-y}$ is :

(a) $3e^y = e^{3x} + C$ (b) $\log(3x - y) = C$

(c) $e^{3x-y} = C$ (d) $-e^y + 3e^{3x} = C$

167. The corner points of the feasible region determined by the system of linear constraints are $(0, 0), (0, 40), (20, 40) (60, 20)$ and $(60, 0)$. If the objective function of an LPP is $Z = 4x + 3y$, then the maximum value is:

(a) 200 (b) 300
(c) 240 (d) 120

168. If position vector $\vec{p}$ of a point $(24, n)$ is such that $|\vec{p}| = 25$, then the value of n is :

(a) ± 49 (b) ± 5
(c) ± 1 (d) ± 7

169. If vectors $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + \lambda\hat{k}$ and $\vec{b} = 2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$, represent the two strips of the Red Cross sign placed outside a doctor's clinic, then the value of λ is :

(a) 1 (b) $\frac{5}{2}$
(c) $\frac{2}{5}$ (d) 0

170. If $3P(A) = P(B) = \frac{3}{5}$ and $P(A \mid B) = \frac{2}{3}$, then $P(A \cup B)$ is :

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$
(c) $\frac{2}{15}$ (d) $\frac{2}{5}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

171. Assertion (A) : A relation R on the set $\{1,2,3\}$ defined as $R = \{(1,1), (1,2), (2,1), (2,2), (3,3)\}$ is an equivalence relation.

Reason (R): A relation that is reflexive, symmetric and transitive is an equivalence relation.

172. Assertion (A) : Consider a Linear Programming Problem with minimise $Z = x + 2y$ subject to constraints $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0$ which gives minimum Z at infinitely many points. The corner points of feasible region are $(0,3)$ and $(6,0)$.

Reason (R) : If two corner points produce the same minimum value of the objective function, then every point on the line segment joining the points will give the same minimum value.

SECTION B

173. Evaluate $\sin\left[\tan^{-1}\tan\left(\frac{3\pi}{4}\right)\right]$.

174. (A) Differentiate x^x with respect to $x\log x$.

OR

(B) If $y = P\cos ux + Q\sin ux$, show that $\frac{d^2y}{dx^2} + u^2y = 0$.

175. Determine the values of x for which $f(x) = \frac{x-3}{x+1}, x \neq -1$ is an increasing function.

176. (A) Three honey bees were found flying along the vectors $\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}, \vec{b} = 4\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 3\hat{\imath} + 2\hat{k}$ respectively.

Find the value of λ such that the path for $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$.

OR

(B) If A, B and C be three non-collinear points such that $\overrightarrow{AB} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\overrightarrow{AC} = 2\hat{\imath} - 3\hat{\jmath}$, then find the area of $\triangle ABC$.

177. Find the angle between the following pair of lines:

$\frac{x-2}{3} = \frac{y+5}{2} = \frac{1-z}{-6}$ and $\frac{x-7}{1} = \frac{y}{2} = \frac{6-z}{-2}$

SECTION C

178. A spherical balloon loses its volume due to escape of air from it in such a way that decrease of volume at any instant is proportional to its surface area. Show that the radius is decreasing at a constant rate.

179. (A) Find:

$$\int \frac{x - \sin x}{1 - \cos x} dx$$

OR

(B) Evaluate :

$$\int_0^2 \frac{1}{\sqrt{x^2 + 2x + 3}} dx$$

180. Solve the differential equation $(x + 2y^3)dy = ydx$.

181. Solve the following Linear Programming Problem graphically:

Maximize $Z = \frac{2x}{5} + \frac{3y}{10}$

subject to constraints

$2x + y \leq 1000$

$x + y \leq 800$

$x, y \geq 0$.

182. **(A)** Let three toys A, B and C be placed in the same straight line. If the position vectors of A, B and C are $55\hat{\imath} - 2\hat{\jmath}, 5\hat{\imath} + 8\hat{\jmath}$ and $a\hat{\imath} - 52\hat{\jmath}$ respectively, find the value of ' a '.

OR

(B) If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors, then prove that $|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 \leq 9$.

183. **(A)** A die is rolled. Consider events :

$A = \{1,2,5\}, B = \{3,5\}, C = \{2,3,4,5\}$

and hence find:

(i) P(A | C) and P(C | A)

(ii) P(A ∩ B | C) and P(A ∪ B | C)

OR

(B) A box contains 6 cards numbered 1 to 6. A student is asked to pick up two cards, one by one after replacement and note down the numbers on the cards. Let A be the event of getting sum of the numbers on two cards as 10 , and B , the event of a number other than 4 on the first card selected.

Find P(A and B) and find whether the events A and B are independent events or not.

SECTION D

184. A man goes to buy fruits from the market. The shopkeeper informs him that 4 apples, 3 oranges and 2 bananas cost ₹ 60 ; 2 apples, 4 oranges and 6 bananas cost ₹ 90 ; whereas 6 apples, 2 oranges and 3 bananas cost ₹ 70 . Using matrix method, find the cost of one fruit of each kind.

185. **(A)** If $y\sqrt{x^2 + 1} = \log\sqrt{x^2 + 1} - x$, show that $(x^2 + 1)\frac{dy}{dx} + xy + 1 = 0$.

OR

(B) Find the differential of $x^{\cot x} + \frac{2x^2 - 3}{2x^2 - x + 2}$ with respect to x.

186. Sketch the curve $\{(x, y): 100x^2 + 25y^2 = 2500\}$ and find the area of the region enclosed by it, using integration.

187. **(A)** Find the foot of the perpendicular from the point $(0,2,3)$ on the line $\frac{-x-3}{-5} = \frac{1-y}{-2} = \frac{3z+12}{9}$ and hence find the length of the perpendicular.

OR

(B) Find the value of p if the shortest distance between the lines

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k}) + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = (p\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ is $\frac{3}{\sqrt{2}}$ units.

SECTION E

Case Study - 1

188. A school wants the students of class XII to do a project on 'Sustainability' keeping the world environment in mind. They select the student participants on the basis of an essay writing competition.

7 students out of 80 are selected for the project and are categorized into two sets such that:

Girl students belong to Set A = $\{G_1, G_2, G_3, G_4\}$,

Boy students belong to Set B = $\{B_1, B_2, B_3\}$.

Based on the above information, answer the following questions:

(i) How many relations are possible from Set A → Set B ?

(ii) Let R be a relation from $A \rightarrow B$ such that
$R = \{(G_1, B_1), (G_2, B_2), (G_3, B_2), (G_4, B_3), (G_1, B_2)\}$.
Is R an injective function? Justify your answer.
(iii) (A) Let the relation R from $A \rightarrow A$ be such that $R = \{(x, y), x, y \in A, x$ and y are students from the same colony in the city$\}$
Verify if R is an equivalence relation.

OR

(B) Verify if any function $f: B \rightarrow A$ is bijective. Give reason to support your answer.

Case Study - 2

189. There are three types of vaccines A_1, A_2, A_3, available in the market to protect the population of the country from spread of certain infection. According to a survey conducted, it was found that 25% of the population was given Vaccine A_1, 35% of the population was given Vaccine A_2 and 40% of the population was given Vaccine A_3. The survey also stated that the probabilities that Vaccines A_1, A_2 and A_3 would protect against the infection were 60%, 55% and 50% respectively.
Based on the above information, answer the following questions:
Find the probability that:
(i) The person taking vaccine A_2 will get infected.
(ii) If a person is chosen randomly, he/she will be protected from the infection.
(iii) (A) The person was given Vaccine A_1, given that the randomly chosen person is infected.

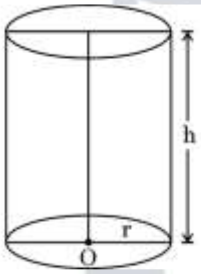
OR

(B) The person was given Vaccine A_3, given that the randomly chosen person is not infected.

Case Study - 3

190. A company produces cylindrical tumblers, open from the top. Since they want uniformity in the product, they fix the surface area of the tumblers produced.

Tumbler



Based on the above information, answer the following questions:
If for a tumbler, V is its volume, h the height and r the radius of the circular base, then :
(i) Differentiate its volume with respect to radius of the base, where the surface area is constant.
(ii) If the company wants to maximize the volume of each tumbler, then establish a relation between its height and the radius of the base.

General Instructions :

Read the following instructions very carefully and strictly follow them:
(i) This question paper contains 38 questions. All questions are compulsory.
(ii) This question paper is divided into five Sections $-$A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are Multiple Choice Questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are Long Answer (LA) type questions carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
(ix) Use of calculator is not allowed.

SECTION – A

191. **Let $f: R \rightarrow R$ be a function defined as $f(x) = x^3$, then f is :**
(a) one-one onto function
(b) many-one function

(c) one-one but not onto function
(d) neither one-one nor onto function

192. The number of all possible matrices of order 2×3 with each entry 1 or 2 is:
(a) 16 (b) 81
(c) 64 (d) 32

193. If $A = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$, then $A + A' = I$, if α is equal to :
(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$
(c) $\frac{\pi}{2}$ (d) $\frac{3\pi}{2}$

194. If the area of a triangle is 15 sq. units with vertices $(3, -7), (6, 3)$ and $(k, 3)$, then the value(s) of k is/are :
(a) 3 (b) 9
(c) $-9, -3$ (d) 9,3

195. The interval in which $y = x^3 \cdot e^{-3x}$ is an increasing function is :
(a) $(-1,2)$ (b) $(-\infty, 2)$
(c) $(1, \infty)$ (d) $(0,1)$

196. The value of k for which $f(x) = \begin{cases} \frac{\sin 7x}{5x}, & x \neq 0 \\ k, & x = 0 \end{cases}$ is continuous at $x = 0$ is :
(a) $\frac{5}{7}$ (b) $\frac{7}{5}$
(c) 0 (d) $\frac{1}{5}$

197. If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is equal to :
(a) 1 (b) 2
(c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

198. $\int \frac{\cos 2x}{\sin^2 x \cos^2 x} dx$ is equal to :
(a) $\cot x + \tan x + C$ (b) $-\cot x + \sec x + C$
(c) $-\cot x + \tan x + C$ (d) $-(\tan x + \cot x) + C$

199. $\int \frac{dx}{x^2+6x+10}$ is equal to :
(a) $x\tan^{-1}(x+3) + C$ (b) $\tan^{-1}(x+3) + C$
(c) $\frac{1}{3}\tan^{-1}(x+3) + C$ (d) $\tan^{-1}x + C$

200. The general solution of the differential equation $\frac{dy}{dx} = e^{y-x}$ is :
(a) $e^x - e^y = C$ (b) $e^{-y} + e^x = C$
(c) $e^{-y} - e^{-x} = C$ (d) $e^{-x} + e^{-y} = C$

201. The integrating factor of $(1 + y^2)dx = (\tan^{-1}y - x)dy$ is :
(a) $\tan y$ (b) $\tan^{-1}y$
(c) $e^{\tan^{-1}y}$ (d) $e^{\tan^{-1}x}$

202. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ is the angle between them. Then $\vec{a} + 2\vec{b}$ is a unit vector if :
(a) $\theta = \frac{\pi}{2}$ (b) $\theta = \frac{2\pi}{3}$
(c) $\theta = \frac{3\pi}{4}$ (d) $\theta = \pi$

203. The area (in square units) of a parallelogram, whose diagonals are $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - 3\hat{j} + 4\hat{k}$ is :
(a) $2\sqrt{14}$ (b) $\sqrt{14}$
(c) $\sqrt{38}$ (d) $2\sqrt{6}$

204. The corner points of the feasible region for an LPP are $(0,2)$, $(6,0)$, $(6,8)$ and $(0,5)$. Let $Z = 4x + 6y$ be the objective function. (Maximum of Z) - (Minimum of Z) is equal to :

(a) 60 (b) 48

(c) 42 (d) 18

205. In an LPP, if the objective function has the same minimum value on two corner points of the feasible region, then the number of points at which the minimum value of objective function occurs is :

(a) 0 (b) 2

(c) Infinite (d) Finite

206. If events A and B are such that $P(A \mid B) = P(B \mid A)$, then :

(a) $A \subset B$ but $A \neq B$ (b) $A = B$

(c) $A \cap B = \phi$ (d) $P(a) = P(b)$

207. The probability of getting an odd prime number on each die, when a pair of dice is rolled, is :

(a) $\frac{1}{18}$ (b) $\frac{1}{9}$

(c) $\frac{1}{36}$ (d) $\frac{1}{6}$

208. In a family with three children, the probability that the eldest child is a boy, given that the family has at least one boy, is :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) $\frac{4}{7}$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true, but Reason (R) is false.
(d) Assertion (A) is false, but Reason (R) is true.

209. Assertion (A): $f(x) = e^{-x}$ is a decreasing function, where $x \in R$.

Reason (R) : If $f'(x) < 0$, then $f(x)$ is a decreasing function.

210. Assertion (A) : For two independent events A and B , $P(A \mid B) = P(A)$.

Reason (R): A and B are independent events, then $P(A \cap B) = P(A).P(B)$.

SECTION B

211. **(A)** For the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$, find the numbers a and b so that $A^2 + aA + bI = O$.

OR

(B) Find the equation of line joining the points $(-1,-1)$ and $(1,3)$, using determinants.

212. **(A)** The volume of a spherical balloon is increasing at the rate of $20\ cm^3/s$. Find the rate of change of its surface area when its radius is 4 cm .

OR

(B) Find the intervals on which the function $f(x) = x^3 + 2x^2 - 1$ is strictly increasing.

213. Find :

$$\int \frac{(x-4)e^x}{(x-2)^3}dx$$

214. Find a unit vector perpendicular to both of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$, where A, B and C are $(3,-1,2)$, $(1,-1,-3)$ and $(4,-3,1)$, respectively.

215. Find the image of the point $(2,-1,5)$ in the line $\vec{r} = (11\hat{\imath} - 2\hat{\jmath} - 8\hat{k}) + \lambda(10\hat{\imath} - 4\hat{\jmath} - 11\hat{k})$.

SECTION C

216. **(A)** Find the value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \tan^{-1}\left(\tan\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

OR

(B) Prove that the function $f: N \to N, f(n) = (n^2 + n + 1)$ is one-one but not onto.

217. **(A)** Find :

$$\int \frac{x}{\sqrt{7-6x-x^2}}dx$$

OR

(B) Find:

$$\int \frac{2}{(1-x)(1+x^2)}dx$$

218. **(A)** Solve the differential equation :

$$x\frac{dy}{dx} = y - x\tan\frac{y}{x}$$

OR

(B) Solve the differential equation :

$$\frac{dy}{dx} = e^{x+y} + e^{x-y}$$

219. Evaluate :

$$\int_0^{\pi} \frac{xdx}{1+\sin x}$$

220. If $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

221. The corner points of the feasible region determined by the system of linear constraints in an LPP are $(0,10), (5,5), (15,15)$ and $(0,20)$. Let $Z = px + qy, p, q > 0$, be the objective function, then find the relation between p and q so that the maximum of Z occurs at both the points $(15,15)$ and $(0,20)$.

SECTION D

222. **(A)** If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations :

$$x - y + z = 4$$
$$2x + y - 3z = 0$$
$$x + y + z = 2$$

OR

(B) If $A = \begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 7 \\ 8 & 9 \end{bmatrix}$, then show that $(AB)^{-1} = B^{-1}A^{-1}$.

223. **(A)** Of all the closed right circular cylindrical cans of given volume 200 cm^3, find the dimensions of the can which has the minimum surface area.

OR

(B) Show that the height of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$.

224. Find the area of the region bounded by the line $y = 3x + 2$, the x -axis and the ordinates $x = -1$ and $x = 1$.

225. Find the shortest distance between the lines:

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} - 3\hat{\jmath} + 2\hat{k})$, and $\vec{r} = (4\hat{\imath} + 5\hat{\jmath} + 6\hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$.

SECTION E

Case Study - 1

226. Two friends Ramesh and Divya have started playing Ludo at their house. While rolling the die, they observed each time, that the possible outcomes are $\{1,2,3,4,5,6\}$. Let A be the set of players and B be the set of all possible outcomes, so A = { Ramesh , Divya } and $B = \{1,2,3,4,5,6\}$.

Based on the above information, answer the following questions :

Let relation $R: B \to B$ be defined as

$R = \{(x,y): y \text{ is divisible by } x\}$.

(i) Write all elements of R.

(ii) Find whether R is reflexive or not.

(iii) **(A)** Check whether R in B is symmetric or transitive.

OR

(B) Let $R_1: B \to B$ is defined as
$R_1 = \{(x, y): x = 2y, x, y \in B\}$.
Check whether R_1 is reflexive, symmetric or transitive.

Case Study - 2

227. A Math teacher of a school is teaching the derivative of function in parametric form, to her students. She explains that sometimes the relation between two variables, (say x and y), is neither explicit nor implicit, but the two variables are expressed in a third variable, (say t). So we have $x = f(t)$ and $y = g(t)$. Such form is called parametric form. For finding the derivative in such case we use the chain rule as

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt} \Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}.$$

Based on the above information, answer the following questions :

(i) If $x = at^3, y = at^5$, then find $\frac{dy}{dx}$.

(ii) If $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$, then find $\frac{dy}{dx}$.

(iii) (A) If $x = 2\sin t + \sin 2t$ and $y = 2\cos t + \cos 2t$, then find $\frac{dy}{dx}$ at $t = \frac{\pi}{6}$.

OR

(B) If $x = a\left(\cos t + \log\tan\frac{t}{2}\right), y = a\sin t$, then find $\frac{dy}{dx}$.

Case Study - 3

228. Two bags contain balls such that Bag I contains 5 red and 6 black balls, while Bag II contains 3 red and 4 black balls.

Based on the above information, answer the following questions :

(i) One ball is drawn at random from each of the two bags. Find the probability of getting both red balls.

(ii) One ball is drawn at random from each of the two bags. Find the probability of getting one red and one black ball.

(iii) (A) One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II . Find the probability of getting a red ball.

OR

(B) One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. Find the probability of getting a black ball.