PART - I

1. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to :-

(a) y^2u (b) $e^{x^2+y^2}$
(c) $2xu$ (d) x^2u

2. Subtraction is not a binary operation in:

(a) Q (b) R
(c) Z (d) N

3. The value of $\int_0^{\pi} \sin^4 x\, dx$ is:

(a) $\frac{3\pi}{2}$ (b) $\frac{3\pi}{10}$
(c) $\frac{3\pi}{8}$ (d) $\frac{3\pi}{4}$

4. A polynomial equation of degree n always has:

(a) exactly n roots
(b) n distinct roots
(c) n real roots
(d) n imaginary roots

5. If $\rho(A) = \rho([A \mid B])$, then the system $AX = B$ of linear equations is:

(a) inconsistent
(b) consistent and has a unique solution
(c) consistent
(d) consistent and has infinitely many solutions3

6. The vertex of the parabola $x^2 = 8y - 1$ is:

(a) $\left(0, -\frac{1}{8}\right)$ (b) $\left(-\frac{1}{8}, 0\right)$
(c) $\left(\frac{1}{8}, 0\right)$ (d) $\left(0, \frac{1}{8}\right)$

7. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; then $\cos^{-1} x + \cos^{-1} y$ is equal to:

(a) π (b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

8. The value of $\sum_{i=1}^{13} \left(i^{n} + i^{n-1}\right)$ is:

(a) 0 (b) $1 + i$
(c) i (d) 1

9. $\vec{r} = s\hat{\imath} + t\hat{\jmath}$ is the equation of (s, t is parameters):

(a) zox plane
(b) a straight line joining the points $\hat{\imath}$ and $\hat{\jmath}$
(c) xoy plane
(d) yoz plane

10. The order of the differential equation of all circles with centre at (h, k) and radius ' a ', where h, k and a are arbitrary constants, is:

(a) 1 (b) 2
(c) 3 (d) 4

11. $\arg(0)$ is:

(a) ∞ (b) 0
(c) π (d) undefined

12. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=$

(a) $\tan^{-1}\left(\frac{1}{2}\right)$, (b) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$

(c) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (d) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$

13. **The position of a particle moving along a horizontal line of any time t is given by $s(t)=3t^2-2t-8$. The time at which the particle is at rest, is: -**

(a) $t=3$ (b) $t=0$

(c) $t=\frac{1}{3}$ (d) $t=1$

14. **The least possible perimeter (in meter) of a rectangle of area $100\ \text{m}^2$ is:**

(a) 50 (b) 10

(c) 20 (d) 40

15. **A random variable X has binomial distribution with $n=25$ and $p=0.8$, then the standard deviation of X is:**

(a) 2 (b) 6

(c) 4 (d) 3

16. **The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is:**

(a) $\sqrt{11}$ (b) 1

(c) 3 (d) $\sqrt{10}$

17. **The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is:**

(a) $\frac{7}{2\sqrt{2}}$ (b) $\frac{\sqrt{7}}{2\sqrt{2}}$

(c) $\frac{7}{2}$ (d) $\frac{\sqrt{7}}{2}$

18. **If $(AB)^{-1}=\begin{bmatrix}12 & -17\\ -19 & 27\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}$, then $B^{-1}=$**

(a) $\begin{bmatrix}8 & -5\\ -3 & 2\end{bmatrix}$ (b) $\begin{bmatrix}2 & -5\\ -3 & 8\end{bmatrix}$

(c) $\begin{bmatrix}8 & 5\\ 3 & 2\end{bmatrix}$ (d) $\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}$

19. **The value of $\int_0^{2/3}\frac{dx}{\sqrt{4-9x^2}}$ is:**

(a) π (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

20. **The order and degree of the differential equation $\frac{dx}{dy}+\frac{dy}{dx}=0$ are:**

(a) 2, degree not defined (b) 1,2

(c) 2,1 (d) 2,2

PART – II

21. Prove that $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3=-2i$.

22. If $(1+i)(1+2i)$ ____ $(1+ni)=x+iy$, then prove that $2\cdot5\cdot10.$ ____ $(1+n^2)=x^2+y^2$.

23. Find the value of $\sin^{-1}\left[\sin\left(\frac{5\pi}{4}\right)\right]$.

24. Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\hat{\imath}+\hat{\jmath}-\hat{k}$, whose line of action passes through the origin.

25. Find the value in the interval $\left(\frac{1}{2},2\right)$ satisfied by the Rolle's theorem for the function

$f(x)=x+\frac{1}{x}, x\in\left[\frac{1}{2},2\right]$

26. For the function $f(x)=x^2+3x$, calculate the differential df when $x=2$ and $dx=0.1$.

27. Prove that $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx = \frac{\pi}{4}$.
28. Find the differential equation of the family of parabolas $y^2 = 4ax$, where $'a'$ is an arbitrary constant.
29. Prove that the identity element is unique if it exists.
30. Find the equation of the parabola if the curve is open leftward, vertex is (2,1) and passing through the point (1,3).

PART – III

31. If $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$ then prove that $(A^T)^{-1} = (A^{-1})^T$.
32. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.
33. A concrete bridge is designed as a parabolic arch. The road over bridge is 40 m long and the maximum height of the arch is 15 m. Write the equation of the parabolic arch. Take (0,0) as the vertex.
34. Find the Vector and Cartesian equations of a straight line passing through the points $(-5,7,-4)$ and $(13,-5,2)$. Find the point where the straight line crosses the xy - plane.
35. Find the critical numbers (orly x values) of the function $f(x) = x^{4/5}(x-4)^2$.
36. If $U = \log(x^3 + y^3 + z^3)$ then find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.
37. A random variable X has the following probability mass function:

X	1	2	3	4	5	6
P(X = x)	k	2 k	6 k	5 k	6 k	10 k

then find P(2 < X < 6).

38. Let X be a continuous random variable and $f(x)$ is defined as:

$f(x) = \begin{cases} kx(1-x)^{10} & ,0 < x < 1 \\ 0 & , \text{otherwise} \end{cases}$ find the value of k.

39. Prove that $p \to q \equiv \neg p \vee q$.
40. If the lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ lie on the same plane, then write the number of ways to find the Cartesian equation of the above plane and explain in detail.

PART - IV

41. **(a)** Test the consistency of the following system of linear equations by rank method.

$x - y + z = -9$
$2x - y + z = 4$
$3x - y + z = 6$
$4x - y + 2z = 7$

OR

(b) If $2\cos\alpha = x + \frac{1}{x}$ and $2\cos\beta = y + \frac{1}{y}$, show that:

(i) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\sin(m\alpha - n\beta)$

(ii) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

42. **(a)** Draw the graph of $\cos x$ in $[0,\pi]$ and $\cos^{-1} x$ in $[-1,1]$.

OR

(b) Find the equation of the circle passing through the points $(1,1), (2,-1)$ and $(3,2)$.

43. **(a)** Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

OR

(b) By vector method, prove that $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

44. **(a)** Find the vector and Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines.

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$

OR

(b) Evaluate: $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$

45. (a) A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east, the police determine with a radar that the distance between the jeep and the car is increasing at $20\ \text{km/hr}$. If the jeep is moving at $60\ \text{km/hr}$ at the instant of measurement, what is the speed of the car?

OR

(b) Find the area of the region bounded by x-axis, the curve $y = |\cos x|$, the lines $x = 0$ and $x = \pi$.

46. (a) A square shaped thin material with area 196 sq. units to make into an open box by cutting small equal squares from the four corners and folding the sides upward. Prove that the length of the side of a removed square is $\frac{7}{3}$ when the volume of the box is maximum.

OR

(b) If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by $M\frac{dV}{dt} = F - kV$, where k is a constant. Prove that $V = \frac{F}{k}\left(1 - e^{\frac{-kt}{M}}\right)$ when $t = 0$ and $V = 0$.

47. (a) In an investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F, prove that the time of death is 5.26 p.m. $\left(5\ \text{hrs}\ 26\ \text{minutes}\right)$ (app.). $\left[\frac{\log(2.43)}{\log(2)} \simeq 1.28\right]$

OR

(b) Three fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred. Verify the results by binomial distribution.