PART - I

1. (b)
2. (c)
3. (b)
4. (c)
5. (b)
6. (c)
7. (b)
8. (a)
9. (c)
10. (d)
11. (c)
12. (a)
13. (a)
14. (c)
15. (d)
16. (b)
17. (b)
18. (d)
19. (c)
20. (c)

PART - II

21. We are asked to prove that: -

$$z=(2+3i)(1-i) \Longrightarrow z^{-1}=\frac{5}{26}-i\frac{1}{26}.$$

Let's solve step by step.

Step (1): Multiply $(2+3i)(1-i)$

$(2+3i)(1-i)=2(1-i)+3i(1-i)=2-2i+3i-3i^2$

$=2-2i+3i-3(-1)$ (since $i^2=-1$)

$=2+i+3=5+i$

So $z=5+i$.

Step (2): Recall formula for inverse of a complex number

If $z=x+iy$, then

$$z^{-1}=\frac{1}{z}=\frac{1}{x+iy}=\frac{x-iy}{x^2+y^2}$$

Step (3): Apply formula

Here $x=5, y=1$. Then

$$z^{-1}=\frac{5-i}{5^2+1^2}=\frac{5-i}{25+1}=\frac{5-i}{26}$$

$$z^{-1}=\frac{5}{26}-i\frac{1}{26}$$

Step (4): Result

$$z^{-1}=\frac{5}{26}-i\frac{1}{26}$$

22. We are asked to prove that if α and β are the roots of

$x^2-5x+6=0$,

then

$\alpha^2-\beta^2=\pm 5$.

Step (1): Sum and product of roots

For a quadratic $x^2-5x+6=0$ by Vieta's formulas

$\alpha+\beta=5, \alpha\beta=6$

Step (2): Factor $\alpha^2-\beta^2$

$\alpha^2 - \beta^2 = (\alpha - \beta)(\alpha + \beta) = (\alpha - \beta) \cdot 5$
Step (3): Find $\alpha - \beta$
For a quadratic
$\alpha - \beta = \pm\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$
Substitute the value:
$\alpha - \beta = \pm\sqrt{5^2 - 4 \cdot 6} = \pm\sqrt{25 - 24} = \pm 1$
Step (4): Compute $\alpha^2 - \beta^2$
$\alpha^2 - \beta^2 = 5 \cdot (\alpha - \beta) = 5 \cdot (\pm 1) = \pm 5$
Step (5): Conclusion
$\alpha^2 - \beta^2 = \pm 5$

23. We want $\sin x = \arcsin x$.
(i) Domain: $\arcsin x$ is defined only for $-1 \leq x \leq 1$.
(ii) Check obvious value: $x = 0$
$\sin 0 = 0, \arcsin 0 = 0 \Longrightarrow$ Equation satisfied.
(iii) Check endpoints: $x = 1$ or -1 does not satisfy the equation.
Solution: -
$x = 0$

24. Three vectors $\vec{A}, \vec{B}, \vec{C}$ are coplanar if their scalar triple product is zero:
$\vec{A} \cdot (\vec{B} \times \vec{C}) = 0$
Given:
$\vec{A} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}, \vec{B} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}, \vec{C} = 3\hat{\imath} + \hat{\jmath} + 3\hat{k}$
Compute scalar triple product: -
$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

25. We want to prove:
$$\lim_{x \to \infty} \frac{e^x}{x^m} = \infty, m \in \mathbb{Z}^+$$
Step (1): Compare growth rates
- Exponential e^x grows much faster than any polynomial x^m as $x \to \infty$.
Step (2): Use L'Hôpital's Rule (m times)
$$\lim_{x \to \infty} \frac{e^x}{x^m} \overset{\text{L'Hôpital}}{=} \lim_{x \to \infty} \frac{e^x}{m x^{m-1}} \overset{\text{repeat}}{=} \cdots = \lim_{x \to \infty} e^x = \infty$$
Step (3): Conclusion
- So $\lim_{x \to \infty} \frac{e^x}{x^m} = \infty$

26. We are asked: $g(x) = x^2 + \sin x$, find dg.
Step (1): Recall differential
$dg = g'(x)dx$
Step (2): Differentiate $g(x)$
$$g'(x) = \frac{d}{dx}(x^2 + \sin x) = 2x + \cos x$$
Step (3): Multiply by dx
$dg = (2x + \cos x)dx$

27. We are asked to solve: -
$$\frac{dy}{dx} = \sqrt{\frac{1 - y^2}{1 - x^2}}$$
Step (1): Separate variables
$$\frac{dy}{\sqrt{1 - y^2}} = \frac{dx}{\sqrt{1 - x^2}}$$
Step (2): Integrate both sides

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$$

$$\sin^{-1} y = \sin^{-1} x + C$$

or equivalently:

$$\sin^{-1} x = \sin^{-1} y + C$$

28. Given c.d.f.:

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}(x^2 + x) & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

Step (1): p.d.f. is derivative of c.d.f:

$$f(x) = \frac{dF(x)}{dx}$$

Step (2): Differentiate piecewise

$$f(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}(2x + 1) & 0 \leq x < 1 \\ 0 & x \geq 1 \end{cases}$$

29. We are given:

$$f(x) = \begin{cases} kxe^{-2x}, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

and need to find k

Step (1): Use the property of a p.d.f.

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

Here, $f(x) = 0$ for $x \leq 0$, so

$$\int_{0}^{\infty} \text{laxe}^{-2x} dx = 1$$

Step (2): Factor out k

$$t \int_{0}^{\infty} xe^{-2x} dx = 1$$

Step (3): Recall standard integral

$$\int_{0}^{\infty} xe^{-\alpha x} dx = \frac{1}{a^2} \ (a > 0)$$

Here $a = 2$, so

$$\int_{0}^{\infty} xe^{-2x} dx = \frac{1}{2n} = \frac{1}{4}$$

Step (4): Solve for k

$$k \cdot \frac{1}{4} = 1 \Longrightarrow k = 4$$

30. Step (1): Differentiate y

$$y' = \frac{dy}{dx} = A\cos x$$

Step (2): Express A in terms of y

$$y = A\sin x \Longrightarrow A = \frac{y}{\sin x}$$

Step (3): Substitute A into y'

$$y' = A\cos x = \frac{y}{\sin x} \cdot \cos x = y\cot x?$$

Careful:

$\cot x = \frac{\cos x}{\sin x}$, so

$$y' = \frac{y\cos x}{\sin x} = y\cot x$$

But the question asks $y = y' \tan x \rightarrow$ multiply both sides by $\tan x$:

$$y = y' \tan x$$

This is the differential equation.

PART – III

31. Matrix:

$$A = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$$

Step (1): Swap $R_1 \leftrightarrow R_3$ to get leading nonzero:

$$\begin{bmatrix} 8 & 1 & 0 & 2 \\ 0 & 2 & 4 & 3 \\ 0 & 1 & 2 & 1 \end{bmatrix}$$

Step (2): Eliminate below pivot in column 2:

$$R_3 \to R_3 - \frac{1}{2}R_2 = [0,0,0,-1/2]$$

Step (3): Echelon form has 3 nonzero rows, so

$\operatorname{rank} A = 3$

32. Given:

$$A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$$

We need to verify:

$A(\operatorname{adj}A) = (\operatorname{adj}A)A = |A|I$

Step (1): Find determinant

$|A| = (8)(3) - (-4)(-5) = 24 - 20 = 4$

Step (2): Find adjoint of A

$\operatorname{adj}A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}$ (swap diagonal, change sign of off-diagonal)

Step (3): Multiply $A \cdot \operatorname{adj}A$

$$A(\operatorname{adj}A) = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix} = \begin{bmatrix} 8\cdot 3 + (-4)\cdot 5 & 8\cdot 4 + (-4)\cdot 8 \\ -5\cdot 3 + 3\cdot 5 & -5\cdot 4 + 3\cdot 8 \end{bmatrix}$$

$$= \begin{bmatrix} 24-20 & 32-32 \\ -15+15 & -20+24 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 4I$$

Step (4): Multiply adj A - A

$$\operatorname{adj}A \cdot A = \begin{bmatrix} 3 & 4 \\ 5 & 8 \end{bmatrix}\begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} 3\cdot 8 + 4\cdot(-5) & 3\cdot(-4) + 4\cdot 3 \\ 5\cdot 8 + 8\cdot(-5) & 5\cdot(-4) + 8\cdot 3 \end{bmatrix}$$

$$= \begin{bmatrix} 24-20 & -12+12 \\ 40-40 & -20+24 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = 4I$$

Step (5): Conclusion

$A(\operatorname{adj}A) = (\mathrm{ad},\downarrow\perp)A = |A|I = 4I$

33. We are asked to show that the points

$$A = 1, B = -\frac{1}{2} + \frac{\sqrt{3}}{2}i, C = -\frac{1}{2} - \frac{\sqrt{3}}{2}i$$

form an equilateral triangle with side length $\sqrt{3}$.

Step (1): Compute distances between points

Distance formula in complex plane:

$|z_1 - z_2|$

(i) $AB = |A - B| = \left|1 - \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)\right| = \left|\frac{3}{2} - \frac{\sqrt{3}}{2}i\right|$

$$AB = \sqrt{\left(\frac{3}{2}\right)^2 + \left(-\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$$

(ii). $BC = |B - C| = \left|\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) - \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\right| = |i\sqrt{3}| = \sqrt{3}$

(iii). $CA = |C - A| = \left|\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) - 1\right| = \left|-\frac{3}{2} - \frac{\sqrt{3}}{2}i\right| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$

Step (2): Conclusion

All sides are equal: $AB = BC = CA = \sqrt{3}$

Hence, the points form an equilateral triangle of $\downarrow\downarrow\sqrt{3}$.

34. We are given: -

$V_{\text{cubold}} - V_{\text{cube}} = 52$
Step (1): Expand the cuboid volume
$(x+1)(x+2)(x+3) = (x+1)(x^2+5x+6) = x^3+6x^2+11x+6$
Step (2): Subtract cube volume
$(x^3+6x^2+11x+6) - x^3 = 6x^2+11x+6 = 52$
$6x^2+11x+6-52 = 6x^2+11x-46 = 0$
Step (3): Solve the quadratic
$6x^2+11x-46 = 0$
- Using factorization or quadratic formula:
$x = 2$ (positive root)
Step (4): Compute cuboid volume
$V_{\text{cubold}} = (x+1)(x+2)(x+3) = 3 \cdot 4 \cdot 5 = 60$

35. We are asked: parabola with focus $F(4,0)$ and directrix $x = -4$.
Step (1): Use definition of parabola
A point $P(x,y)$ lies on a parabola if its distance from focus = distance from directrix:
$$PF = PD$$
$\sqrt{(x-4)^2+(y-0)^2} = |x-(-4)| = |x+4|$
Step (2): Square both sides
$(x-4)^2+y^2 = (x+4)^2$
$x^2-8x+16+y^2 = x^2+8x+16$
$y^2-16x = 0 \Longrightarrow y^2 = 16x$

36. We are asked to find work done:
Work $W = \vec{F} \cdot \vec{d}$
Step (1): Given
$\vec{F} = 13\hat{\imath}+10\hat{\jmath}-3\hat{k}, \vec{r}_1 = 4\hat{\imath}-3\hat{\jmath}-2\hat{k}, \vec{r}_2 = 6\hat{\imath}+\hat{\jmath}-3\hat{k}$
Step (2): Displacement vector
$\vec{d} = \vec{r}_2 - \vec{r}_1 = (6-4)\hat{\imath}+(1-(-3))\hat{\jmath}+(-3-(-2))\hat{k} = 2\hat{\imath}+4\hat{\jmath}-1\hat{k}$
Step (3): Work done
$W = \vec{F} \cdot \vec{d} = (13)(2)+(10)(4)+(-3)(-1) = 26+40+3 = 69$

37. We are asked to find the point on the curve
$y = x^2-5x+4$
where the tangent is parallel to the line $3x+y=7$.
Step (1): Find slope of the given line
$3x+y=7 \Longrightarrow y=-3x+7 \Longrightarrow$ slope $m=-3$
Step (2): Slope of tangent to the curve
$\frac{dy}{dx} = 2x-5$
Set it equal to $\mathbf{-3}$ for parallelism:
$2x-5=-3 \Longrightarrow 2x=2 \Longrightarrow x=1$
Step (3): Find corresponding y-coordinate
$y = 1^2-5(1)+4 = 1-5+4 = 0$
Step (4): Point on the curve
$(1,0)$

38. Step (1): Formula for volume of a spherical shell
$V_{\text{shell}} = \frac{4}{3}\pi(R^3-r^3)$
Where:
- $R = 4.2$ mm (outer radius)
- $r = 4$ mm (inner radius)
Step (2): Use approximation for thin shell
If $R = r+\Delta r$ and Δr is small,
$R^3-r^3 \approx 3r^2\Delta r$
Here, $r=4, \Delta r=0.2$
$R^3-r^3 \approx 3 \cdot 4^2 \cdot 0.2 = 3 \cdot 16 \cdot 0.2 = 9.6$

Step (3): Multiply by $\frac{4}{3}\pi$

$V_{\text{shell}} \approx \dfrac{4}{3}\pi \cdot 9.6 = 12.8\pi \text{ mm}^3$

39. We are given an operation on $\mathbb{Q}$:

$a * b = \dfrac{a+b}{2}, a, b \in \mathbb{Q}$

Step (1): Closure

- If $a, b \in \mathbb{Q}$, then $a+b \in \mathbb{Q}$ and $\frac{a+b}{2} \in \mathbb{Q}$

- Hence, operation is closed on $\mathbb{Q}$.

Step (2): Commutativity

$a * b = \dfrac{a+b}{2} = \dfrac{b+a}{2} = b * a$

- Hence, operation is commutative.

40. We are asked to evaluate:

$I = \displaystyle\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}+\sqrt{x}} dx$

Step (1): Substitution $x = 1-t$

$dx = -dt, x = 0 \Longrightarrow t = 1, x = 1 \Longrightarrow t = 0$

$I = \displaystyle\int_1^0 \frac{\sqrt{1-t}}{\sqrt{t}+\sqrt{1-t}}(-dt) = \int_0^1 \frac{\sqrt{1-t}}{\sqrt{t}+\sqrt{1-t}} dt$

Step (2): Add original and substituted integral

$I + I = \displaystyle\int_0^1 \frac{\sqrt{x}+\sqrt{1-x}}{\sqrt{1-x}+\sqrt{x}} dx = \int_0^1 1 dx = 1$

$2I = 1 \Longrightarrow I = \dfrac{1}{2}$

PART - IV

41. (a) We are asked to solve

$\begin{cases} x - y + 2z = 2 \\ 2x + y + 4z = 7 \\ 4x - y + z = 4 \end{cases}$

using Cramer's rule

Step (1): Write coefficient matrix

$D = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 4 \\ 4 & -1 & 1 \end{vmatrix}$

$D = 1\begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix} - (-1)\begin{vmatrix} 2 & 4 \\ 4 & 1 \end{vmatrix} + 2\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix}$

$D = 1(1-(-4)) + 1(2-16) + 2(-2-4) = 5 - 14 - 12 = -21$

Step (2): Replace columns to find x, y, z

(i) For x :

$D_x = \begin{vmatrix} 2 & -1 & 2 \\ 7 & 1 & 4 \\ 4 & -1 & 1 \end{vmatrix} = -21$

$\Longrightarrow x = D_x/D = -21/-21 = 1$

(ii) For g -

$D_t = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 7 & 4 \\ 4 & 4 & 1 \end{vmatrix} = 14$

$\Longrightarrow y = D_v/D = 14/-21 = -2/3$

(iii) For a:

$D_z = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 1 & 7 \\ 4 & -1 & 4 \end{vmatrix} = -7$

$\Longrightarrow z = D_x/D = -7/-21 = 1/3$

Step (3): Solution

$x=1, y=-\frac{2}{3}, x=\frac{1}{3}$

OR

(b) A camera falls from a height $h=400\text{ft}$ with displacement:
$s=-16t^2$
Step (1): Time to hit the ground
- Ground is at $s=-400$ (taking downward as negative):
$$-16t^2=-400 \Longrightarrow t^2=\frac{400}{16}=25 \Longrightarrow t=5\text{sec}$$
Camera hits the ground at $t=5\text{sec}$
Step (2): Velocity when hitting the ground
$$v=\frac{ds}{dt}=\frac{d}{dt}(-16t^2)=-32t$$
At $t=5$:
$v=-32\cdot 5=-160\text{ft/sec}$
- Negative sign indicates downward.

42. (a) We are given:
$$z=x+iy, \left|\frac{z-4i}{z+4i}\right|=1$$
Step (1): Substitute $z=x+iy$
$$\frac{z-4i}{z+4i}=\frac{x+i(y-4)}{x+i(y+4)}$$
Magnitude:
$$\left|\frac{x+i(y-4)}{x+i(y+4)}\right|=\frac{\sqrt{x^2+(y-4)^2}}{\sqrt{x^2+(y+4)^2}}=1$$
Step (2): Equate magnitudes
$$\sqrt{x^2+(y-4)^2}=\sqrt{x^2+(y+4)^2}$$
$$x^2+(y-4)^2=x^2+(y+4)^2$$
$$(y-4)^2=(y+4)^2$$
$$y^2-8y+16=y^2+8y+16 \Longrightarrow -8y=8y \Longrightarrow y=0$$
Step 3: Locus
$y=0$ (the real axis)

OR

(b) We are asked to evcluate:
$$I=\int_0^1 \sqrt{\frac{1-x}{1+x}}dx$$
Step (1): Substitution $x=\sin t$
$dx=\text{costdt}, x=0 \Longrightarrow t=0, x=1 \Longrightarrow t=\pi/2$
$$\sqrt{\frac{1-x}{1+x}}=\sqrt{\frac{1-\text{sint}}{1+\text{sint}}}=\sqrt{\frac{(\cos t)^2}{(1+\sin t)^2}}=\frac{\cos t}{1+\sin t}$$
Step (2): Integral in terms of t
$$I=\int_0^{\pi/2}\frac{\cos t}{1+\sin t}\cdot \cos td?$$
Careful $dx=\text{cost } dlt$, so
$$I=\int_0^{\pi/2}\frac{\cos t}{1+\sin t}\cdot \cos tdt=\int_0^{\pi/2}\frac{\cos^2 t}{1+\sin t}dt$$
Step (3): Simplify using $\cos^2 t=1-\sin^2 t=(1-\sin t)(1+\sin t)$
$$\frac{\cos^2 t}{1+\sin t}=\frac{(1-\sin t)(1+\sin t)}{1+\sin t}=1-\sin t$$
Step (4): Integrate

$$I=\int_0^{\pi/2}(1-\sin t)dt=[t+\cos t]_0^{\pi/2}=\frac{\pi}{2}+\cos(\pi/2)-(0+\cos 0)=\frac{\pi}{2}-1$$

43. (a) We are asked to evaluate:

$\cos^{-1}(\cos(4\pi/3))+\cos^{-1}(\cos(5\pi/4))$

Step (1): Bring angles into principal value $[0,\pi]$

- For $\cos^{-1}(\cos\theta)$, the principal value is in $[0,\pi]$

(i) $4\pi/3>\pi \Longrightarrow \cos^{-1}(\cos 4\pi/3)=2\pi-4\pi/3=2\pi/3$

(ii) $05\pi/4>\pi \Longrightarrow \cos^{-1}(\cos 5\pi/4)=2\pi-5\pi/4=3\pi/4$

Step (2): Add the values

$$\frac{2\pi}{3}+\frac{3\pi}{4}=\frac{8\pi+9\pi}{12}=\frac{17\pi}{12}$$

OR

(b) Step (1): Identify parameters

- Standard form $z^2+\frac{1^2}{2}=1$ with $a^2>b^2$

- Here $a^2=16 \Longrightarrow a=4, b^2=9 \Longrightarrow b=3$

- Centre $C=(0,0)$

Step (2): Eccentricity

$$e=\sqrt{1-\frac{\vartheta^2}{a^2}}=\sqrt{1-\frac{9}{16}}=\sqrt{\frac{7}{16}}=\frac{\sqrt{7}}{4}$$

Step (3): Vertices

- Along x-axis (major axis) $(\pm a,0)=(\pm 4,0)$

Step (4): Foci

$$c=ae=4\cdot\frac{\sqrt{7}}{4}=\sqrt{7}$$

Foci along x-axic: $(\pm c,0)=(\pm\sqrt{7},0)$

(Step 5): Rough diagram

$y=\mathrm{nt}$

```
  y
  |
3 +     *
  |
  |
0 --+------------------ x
  |    *
  |
-3 +    *
```

Major axis along x, minor axis along y

44. (a) We are asked to solve the differential equation:

$(e^y+1)\cos x dx+e^y\sin x dy=0$

Step (1): Rewrite in standard form

$$\frac{dy}{dx}=-\frac{(e^e+1)\cos x}{e^e\sin x}=-\frac{e^y+1}{e^y}\cdot\cot x=-(1+e^{-y})\cot x$$

Step (2): Separate variables

$$\frac{dy}{1+e^{-y}}=-\cot x dx$$

$$\frac{dy}{1+e^{-y}}=-\cot x dx$$

Multiply numerator and denominator by e*:

$$\frac{e^x dy}{e^x+1}=-\cot x dx$$

Step (3): Integrate both sides

- UHS: $\int\frac{x}{x+1}dy=\int\frac{d(x+1)}{d+1}=\ln|e''+1|$

- RHS: $\int -\cot x dx=-\ln|\sin x|$

$\ln|e^x+1| = -\ln|\sin x| + C$

$\ln|e^y+1| + \ln|\sin x| = C \Longrightarrow \ln|(e^y+1)\sin x| = C$

Step (4): Exponentiate

$(e^T+1)\sin x = K, K = e^C$

Step (5): Solution

$(e' + 1)\sin x =$ constant

OR

(b) We are asked to show:

$\neg(p \rightarrow q) \equiv p \wedge \neg q$

Step (1): Recall implication equivalence

$p \rightarrow q \equiv \neg p \vee q$

Step (2): Apply negation

$\neg(p \rightarrow q) = \neg(\neg p \vee q)$

Step (3): Apply De Morgan's law

$\neg(\neg p \vee q) \equiv \neg(\neg p) \wedge \neg q \equiv p \wedge \neg q$

45. **(a)** Consider two unit vectors in the plane:

$\vec{u} = \cos A\hat{\imath} + \sin A\hat{\jmath}, \vec{v} = \cos B\hat{\imath} + \sin B\hat{\jmath}$

Step (1): Dot product

$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$

Here $\theta =$ angle between $\vec{u}$ and $\vec{v} = A - B$, and $|\vec{u}| = |\vec{v}| = 1$

$\vec{u}\cdot\vec{v} = \cos(A-B)$

Step (2): Compute $\vec{u}\cdot\vec{v}$ using components

$\vec{u}\cdot\vec{v} = (\cos A)(\cos B) + (\sin A)(\sin B)$

Step (3): Equate

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

OR

(b) Let the two positive numbers be x and y.

- Given: $xy = 20 \Longrightarrow y = \frac{20}{x}$
- Sum: $S = x + y = x + \frac{20}{x}$

Step (1): Minimize the sum

$$\frac{dS}{dx} = 1 - \frac{20}{x^2} = 0 \Longrightarrow x^2 = 20 \Longrightarrow x = \sqrt{20} = 2\sqrt{5}$$

$$y = \frac{20}{x} = \frac{20}{2\sqrt{5}} = 2\sqrt{5}$$

Step (2): Minimum sum

$S_{\min} = x + y = 2\sqrt{5} + 2\sqrt{5} = 4\sqrt{5}$

46. **(a)** We are given a projectile problem:

- Maximum height $H = 4$ m at $x = 6$ m
- Range $R = 12$ m

We are to find the angle of projection θ.

Step (1): Equation of parabolic path

$$y = x\tan\theta - \frac{g}{2u^2\cos^2\theta}x^2$$

- Let $\frac{n}{2\pi\cos^2\theta} = k$ for simplicib:

$y = x\tan\theta - kx^2$

Step (2): Use maximum height

- At maximum height, $x = \frac{8}{2} = 6$ (since parabolic path is symmetric)

$t_{\text{tax}} = 4 = 6\tan\theta - k - 6^2 = 6\tan\theta - 36k$

Step (3): Use range

- At $x = R = 12y = 0$

$0 = 12\tan\theta - k\cdot 12^2 = 12\tan\theta - 14k$

$$14k = 12\tan\theta \Longrightarrow k = \frac{\tan\theta}{12}$$

Step (4): Substitute k into maximum height equation

$4 = 6\tan\theta - 36 - \frac{\tan\theta}{12} = 6\tan\theta - 3\tan\theta = 3\tan\theta$

$\tan\theta = \frac{4}{3}$

OR

(b) We are given the probability mass function (PMF) of a discrete random variable **X** :

$X: 1\ 2\ 3\ 4\ 5$

$f(x) = k^2\ 2k^2\ 3k^2\ 2k\ 3k$

Step (1): Use total probability

$\sum f(x) = 1$

$k^2 + 2k^2 + 3k^2 + 2k + 3k = 6k^2 + 5k = 1$

$6k^2 + 5k - 1 = 0$

Solve quadratic

$k = \frac{-5 \pm \sqrt{25+24}}{12} = \frac{-5 \pm 7}{12}$

Positive solutions $k = \frac{-5+7}{12} = \frac{2}{12} = \frac{1}{8}$

Sa $k = \frac{1}{6}$

Step (2): Find $P(2 \le X < 5) = P(X=2) + P(X=3) + P(X=4)$

$P(2 \le X < 5) = 2k^2 + 3k^2 + 2k = 5k^2 + 2k$

$= 5 \cdot \left(\frac{1}{6}\right)^2 + 2 \cdot \frac{1}{6} = \frac{5}{36} + \frac{12}{36} = \frac{17}{36}$

$P(2 \le X < 5) = \frac{17}{36}$

Step (3): Find $P(X > 3) = P(X=4) + P(X=5) = 2k + 3k = 5k$

$5k = 5 - \frac{1}{6} = \frac{5}{6}$

$P(x > 3) = \frac{5}{6}$

47. **(a)** $3x - 2y = 0 \Rightarrow y = \frac{3}{2}x$

The area between $x = -3$ and $x = 1$ (taking absolute value where y is negative):

$A = \int_{-3}^{0} \left|\frac{3}{2}x\right| dx + \int_{0}^{1} \frac{3}{2}x dx = \int_{-3}^{0} -\frac{3}{2}x dx + \int_{0}^{1} \frac{3}{2}x dx$

$A = \frac{3}{2}\left[\frac{x^2}{2}\right]_{-3}^{0} + \frac{3}{2}\left[\frac{x^2}{2}\right]_{0}^{1} = \frac{3}{2} \cdot \frac{9}{2} + \frac{3}{2} \cdot \frac{1}{2} = \frac{27}{4} + \frac{3}{4} = \frac{30}{4} = \frac{15}{2}$

OR

(b) Step (1): Normal vector condition

The plane is perpendicular to $3x - 2y + 4z + 5 = 0$, so its normal vector $\mathrm{n} = (a, b, c)$ satisfies:

$\mathrm{n} \cdot (3, -2, 4) = 0 \Rightarrow 3a - 2b + 4c = 0$

Step (2): Plane passes through points $\rightarrow$ direction vector condition

Vector along line joining the points:

$\overrightarrow{AB} = B - A = (2-1, 3-2, 1-3) = (1, 1, -2)$

Since the plane passes through both points, $\overrightarrow{AB}$ lies in the plane $\rightarrow$ perpendicular to normal:

$(a, b, c) \cdot (1, 1, -2) = a + b - 2c = 0$

Step 3: Solve for normal vector

We have two equations:

(i) $3a - 2b + 4c = 0$

(ii) $a + b - 2c = 0$

From (2): $a = 2c - b$

Plug into (1):

$3(2c - b) - 2b + 4c = 0 \Rightarrow 6c - 3b - 2b + 1 = 0 \Rightarrow 10c - 5b = 0 \Rightarrow 2c - b = 0 \Rightarrow b = 2c$

Then $a = 2c - b = 2c - 2c = 0$

So normal vector: $(a, b, c) = (0, 2, 1)$

Step (4): Equation of the plane

Plane passing through point $A(1,2,3)$:

$0(x-1)+2(y-2)+1(z-3)=0 \Longrightarrow 2(y-2)+(z-3)=0 \Longrightarrow 2y+z-7=0$