PART - I

1. **The inverse of $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ is :**

(a) $\begin{bmatrix} 3 & -1 \\ -5 & -3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 5 \\ 1 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$

2. **The centre of the hyperbola $\frac{(x-1)^2}{16} - \frac{(y+1)^2}{25} = 1$ is :**

(a) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (b) $(-1,1)$

(c) $(1,-1)$ (d) $(0,0)$

3. **The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$ are :**

(a) 2,6 (b) 2,3

(c) 2,4 (d) 3,3

4. **A pair of dice numbered 1, 2, 3, 4, 5, 6 of a six sided die and 1, 2, 3, 4 of a four sided die is rolled and the sum is determined. If the random variable X denote the sum, then the number of elements in the inverse image of 7 is :**

(a) 3 (b) 1

(c) 4 (d) 2

5. **If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is :**

(a) $\frac{1}{z}$ (b) z

(c) 1 (d) $\bar{z}$

6. **The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \cos x\, dx$ is :**

(a) 0 (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) $\frac{1}{2}$

7. **The function $f(x) = x^2$, in the interval $[0, \infty)$ is :**

(a) cannot be determined

(b) increasing function

(c) increasing and decreasing function

(d) decreasing function

8. **The volume of the parallelepiped with its edges represented by the vectors $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}, \hat{i} + \hat{j} + \pi\hat{k}$ is :**

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

9. **In the set R of real numbers ' $*$ ' is defined as follows. Which one of the following is not a binary operation on R ?**

(a) $a * b = a$ (b) $a * b = \min(a, b)$

(c) $a * b = a^b$ (d) $a * b = \max(a, b)$

10. **The position of a particle ' s ' moving at any time t is given by $s(t) = 5t^2 - 2t - 8$. The time at which the particle is at rest, is :**

(a) 1 (b) 0

(c) 3 (d) $t = \frac{1}{5}$

11. **If the function $f(x) = \frac{1}{12}$ for a $< x <$ b, represents a probability density function of a continuous random variable X , then which of the following cannot be the values of a and b ?**

(a) 7 and 19 (b) 0 and 12

(c) 16 and 24 (d) 5 and 17

12. **If $\mathrm{P}(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $\mathrm{F}_1(3,0)$ and $\mathrm{F}_2(-3,0)$, then $\mathrm{PF}_1 + \mathrm{PF}_2$ is :**

(a) 10 (b) 8

(c) 12 (d) 6

13. **If the planes $\vec{r} \cdot (2\hat{i} - \lambda\hat{j} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{i} + \hat{j} - \mu\hat{k}) = 5$ are parallel, then the values of λ and μ are respectively :-**

(a) $-\frac{1}{2}, -2$ (b) $\frac{1}{2}, -2$

(c) $\frac{1}{2}, 2$ (d) $-\frac{1}{2}, 2$

14. **A zero of $x^3 + 64$ is :**

(a) $4i$ (b) 0

(c) -4 (d) 4

15. **The solution of $\frac{dy}{dx} + \mathrm{P}(x)y = 0$ is:**

(a) $x = ce^{-\int \mathrm{P}dy}$ (b) $y = ce^{\int \mathrm{P}dx}$

(c) $x = cc^{\int \mathrm{P}dy}$ (d) $y = ce^{-\int \mathrm{P}dx}$

16. $\int_0^{\frac{\pi}{2}} \sin^7 x\,dx =$

(a) $\frac{\pi}{2}$ (b) $\int_0^{\frac{\pi}{2}} \cos^7 x\, dx$

(c) 0 (d) 1

17. **The value of $\sin^{1}\left(\frac{1}{2}\right) + \cos^{1}\left(\frac{1}{2}\right)$ is :**

(a) 0 (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ (d) π

18. **If A, B and C are invertible matrices of some order, then which one of the following is not true ?**

(a) $\det \mathrm{A}^{-1} = (\det \mathrm{A})^{-1}$

(b) $\mathrm{adj}\, \mathrm{A} = |\mathrm{A}|\mathrm{A}^{-1}$

(c) $(\mathrm{ABC})^{-1} = \mathrm{C}^{-1}\,\mathrm{B}^{-1}\,\mathrm{A}^{-1}$

(d) $\mathrm{adj}(\mathrm{AB}) = (\mathrm{adj}\mathrm{A})(\mathrm{adj}\mathrm{B})$

19. **The value of the complex number $\left(i^{25}\right)^3$ is equal to :**

(a) 1 (b) i

(c) $-i$ (d) -1

20. **If we measure the side of a cube to be 4 cm with an error of 0.1 cm , then the error in calculation of the volume is (in cubic cm) :**

(a) 2 (b) 0.4

(c) 4.8 (d) 0.45

PART - II

21. If $z = (2+3i)(1-i)$ then prove that $z^{-1} = \frac{5}{26} - i\frac{1}{26}$.

22. If α and β are the roots of $x^2 - 5x + 6 = 0$ then prove that $\alpha^2 - \beta^2 = \pm 5$.

23. For what value of x does $\sin x = \sin^{-1} x$?

24. Show that the three vectors $2\hat{\imath}+3\hat{\jmath}+\hat{k}, \hat{\imath}-2\hat{\jmath}+2\hat{k}$ and $3\hat{\imath}+\hat{\jmath}+3\hat{k}$ are coplanar.

25. Prove that $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right)$, where m is a positive integer, is ∞.

26. If $g(x)=x^2+\sin x$, then find dg.

27. Show that the solution of $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$ is $\sin^{-1}y=\sin^{-1}x+C$ (or) $\sin^{-1}x=\sin^{-1}y+C$.

28. If X is the random variable with distribution function $F(x)$, given by :

$$F(x)=\begin{cases}0 & ;-\infty<x<0\\ \frac{1}{2}(x^2+x) & ;0\leqslant x<1\\ 1 & ;1\leqslant x<\infty\end{cases}$$ then prove that the p.d.f. is $$f(x)=\begin{cases}\frac{1}{2}(2x+1) & ;0\leqslant x<1\\ 0 & ;\text{ otherwise}\end{cases}$$

29. The probability density function of X is given by $f(x)=\begin{cases}kxe^{-2x} & \text{for } x>0\\ 0 & \text{for } x\le 0\end{cases}$ Prove that the value of k is 4.

30. Show that the differential equation corresponding to $y=A\sin x$, where A is an arbitrary constant, is $y=y'\tan x$.

PART - III

31. Show that the rank of the matrix $\begin{bmatrix}0&1&2&1\\0&2&4&3\\8&1&0&2\end{bmatrix}$ is 3.

32. If $A=\begin{bmatrix}8&-4\\-5&3\end{bmatrix}$, verify that $A(\text{adj}A)=(\text{adj}A)A=|A|I$.

33. Show that the points $1,-\frac{1}{2}+i\frac{\sqrt{3}}{2}$ and $-\frac{1}{2}-i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle of side length $\sqrt{3}$.

34. If the sides of a cubic box are increased by 1,2,3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Show that the volume of the cuboid is 60 cubic units.

35. Prove that the equation of the parabola with focus (4,0) and directrix $x=-4$ is $y^2=16x$.

36. A force $13\hat{\imath}+10\hat{\jmath}-3\hat{k}$ acts on a particle which is displaced from the point with position vector $4\hat{\imath}-3\hat{\jmath}-2\hat{k}$ to the point with position vector $6\hat{\imath}+\hat{\jmath}-3\hat{k}$. Show that the work done by the force is 69 units.

37. Show that the point on the curve $y=x^2-5x+4$ at which the tangent is parallel to the line $3x+y=7$, is (1,0).

38. An egg of a particular bird is spherical in shape. If the radius to the inside of the shell is 4 mm and radius to the outside of the shell is 4.2 mm , prove that the approximate volume of the shell is 12.8π mm^3.

39. Define an operation * on Q, the set of all rational numbers, as follows :

$a*b=\left(\frac{a+b}{2}\right)$; $a,b\in Q$. Examine the closure and commutative properties satisfied by * on Q.

40. Show that $\int_0^1\frac{\sqrt{x}}{\sqrt{1-x}+\sqrt{x}}\,dx=\frac{1}{2}$

PART - IV.

41. **(a)** Solve the system of equations $x-y+2z=2, 2x+y+4z=7, 4x-y+z=4$ by Cramer's rule.

OR

(b) A camera is accidently knocked off an edge of a cliff 400 ft . high. The camera falls a distance of $s=-16t^2$ in t seconds. Show that the camera hits the ground when $t=5$ seconds and also prove that the velocity when it hits the ground is -160 ft ./ sec.

42. **(a)** If $z=x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right|=1$, show that the locus of z is real axis or $y=0$

OR

(b) Show that $\int_0^1\sqrt{\frac{1-x}{1+x}}\,dx=\frac{\pi}{2}-1$.

43. **(a)** Prove that $\cos^{-1}\left[\cos\left(\frac{4\pi}{3}\right)\right]+\cos^{-1}\left[\cos\left(\frac{5\pi}{4}\right)\right]=\frac{17\pi}{12}$

OR

(b) Find the eccentricity, centre, vertices and foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and also draw the rough diagram.

44. **(a)** Solve $(e^{y}+1)\cos x\,dx+e^{y}\sin x\,dy=0$

OR

(b) Show that $\neg(p\rightarrow q)\equiv p\wedge(\neg q)$.

45. **(a)** Using Vector method, prove that $\cos(A-B)=\cos A\cos B+\sin A\sin B$.

OR

(b) Find two positive numbers whose product is 20 and their sum is minimum.

46. **(a)** On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Show that the angle of projection is $\tan^{-1}\left(\frac{4}{3}\right)$

OR

(b) A random variable X has the following probability mass function :-

X	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	2 k	3k

Find: (i) the value of k.

(ii) $P(2\leqslant X<5)$.

(iii) $P(3<X)$.

47. **(a)** Show that the area of the region bounded by $3x-2y=0, x=-3$ and $x=1$ is $\frac{15}{2}$.

OR

(b) Show that the Cartesian equation of the plane passing through the points (1,2,3) and (2,3,1) and also perpendicular to the plane $3x-2y+4z+5=0$ is $2y+z-7=0$.