PART - I

1. (b)
2. (c)
3. (d)
4. (d)
5. (d)
6. (a)
7. (b)
8. (a)
9. (c)
10. (a)
11. (d)
12. (d)
13. (d)
14. (c)
15. (a)
16. (a)
17. (a)
18. (a)
19. (c)
20. (b)

PART – II

21. Let $z = x + iy$ and $\bar{z} = x - iy$, where $x = \mathrm{Re}(z)$ and $y = \mathrm{Im}(z)$.

(i) Real part:

$$\frac{z+\bar{z}}{2} = \frac{(x+iy)+(x-iy)}{2} = \frac{2x}{2} = x = \mathrm{Re}(z)$$

(ii) Imaginary part:

$$\frac{z-\bar{z}}{2i} = \frac{(x+iy)-(x-iy)}{2i} = \frac{2iy}{2i} = y = \mathrm{Im}(z)$$

Hence proved: -

$$\mathrm{Re}(z) = \frac{z+\bar{z}}{2}, \mathrm{Im}(z) = \frac{z-\bar{z}}{2i}$$

22. Step (1): -Rational coefficients $\Longrightarrow$ include conjugate

If a polynomial has rational coefficients, then the conjugate $2+\sqrt{3}$ must also be a root.

So roots: $2-\sqrt{3}, 2+\sqrt{3}$.

Step (2): Form the quadratic

$(x-(2-\sqrt{3}))(x-(2+\sqrt{3})) = 0$

Step (3): Expand

$(x-2+\sqrt{3})(x-2-\sqrt{3}) = (x-2)^2 - (\sqrt{3})^2 = (x-2)^2 - 3$

$(x-2)^2 - 3 = x^2 - 4x + 4 - 3 = x^2 - 4x + 1$

23. We are asked to find the principal value of

$\tan^{-1}(\sqrt{3})$

Step (1): Recall the principal value of $\tan^{-1} x$

$\tan^{-1} x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Step (2): Find the angle

$\tan\theta = \sqrt{3} \Longrightarrow \theta = \frac{\pi}{3}$

(because $\tan \pi/3 = \sqrt{3}$ and it lies in $(-\pi/2, \pi/2)$)

24. Step (1): Slope of the tangent

The slope of the tangent is given by $y' = \frac{dy}{dx}$.

$y' = 3x^2 - 6x + 1$
For the tangent to be parallel to $y = x$, slope $= 1$:
$3x^2 - 6x + 1 = 1$
$3x^2 - 6x = 0$
$3x(x - 2) = 0$
$x = 0$ or $x = 2$
Step (2): Find corresponding y-coordinates
- For $x = 0 : y = 0^3 - 3(0)^2 + 0 - 2 = -2 \rightarrow$ Point: $(0, -2)$
- For $x = 2 : y = 8 - 12 + 2 - 2 = -4 \rightarrow$ Point: $(2, -4)$

25. We are asked to find the differential df for
$f(x) = x^2 + 3x$
and evaluate it at $x = 2$ and $dx = 0.1$.
Step (1): Recall differential formula
$df = f'(x)dx$
Step (2): Compute derivative
$f'(x) = 2x + 3$
So, the differential is:
$df = (2x + 3)dx$
Step (3): Substitute $x = 2, dx = 0.1$
$df = (2(2) + 3)(0.1) = (4 + 3)(0.1) = 7 \cdot 0.1 = 0.7$

26. Step (1): Differentiate y with respect to x
$$\frac{dy}{dx} = Ae^x - Be^{-x}$$
$$\frac{d^2y}{dx^2} = Ae^x + Be^{-x}$$
Step (2): Express second derivative in terms of y
$$\frac{d^2y}{dx^2} = Ae^x + Be^{-x} = y$$
Step (3): Differential equation
$$\frac{d^2y}{dx^2} - y = 0$$

27. We are asked to solve the differential equation:
$$\frac{dy}{dx} = \frac{\sqrt{1 - y^2}}{\sqrt{1 - x^2}}.$$
Here's a short solution:
Step (1): Separate variables
$$\frac{dy}{\sqrt{1 - y^2}} = \frac{dx}{\sqrt{1 - x^2}}$$
Step (2): Integrate both sides
$$\int \frac{dy}{\sqrt{1 - y^2}} = \int \frac{dx}{\sqrt{1 - x^2}}$$
$\arcsin y = \arcsin x + C$
Step (3): Solve for y
$y = \sin(\arcsin x + C)$

28. Step (1): Use the property of PMF
For a PMF, the sum of all probabilities $= 1$:
$$\sum f(x) = 1$$
$k + 2k + 6k + 5k + 6k + 10k = 1$
Step (2): Add coefficients
$k(1 + 2 + 6 + 5 + 6 + 10) = k(30) = 1$
Step (3): Solve for k
$$k = \frac{1}{30}$$

29. When three fair coins are tossed, the random variable X (number of tails) can take values 0, 1, 2, or 3, with corresponding reverse image sizes (number of outcomes) being 1 (for $X=0$), 3 (for $X=1$), 3 (for $X=2$), and 1 (for $X=3$), respectively, out of a total of 8 possible outcomes (HHH, HHT, etc.).

Values of Random Variable X (Number of Tails)

- $X=0$: (HHH) $-$ 0 tails.
- $X=1$: (HHT, HTH, THH) - 1 tail.
- $X=2$: (HTT, THT, TTH) - 2 tails.
- X = 3: (TTT) - 3 tails.

Number of Points (Outcomes) in the Reverse Images of X

- For $X=0$: {HHH} $-$ 1 outcome.
- For X = 1: {HHT, HTH, THH}- 3 outcomes.
- For X = 2: {HTT, THT, TTH}- 3 outcomes.
- For $X=3$: {TTT} $-$ 1 outcome.

30. We are asked to find the distance from the origin to the plane

$3x+6y+2z+7=0$

Step (1): Recall the formula for distance from a point to a plane

If a plane is

$Ax+By+Cz+D=0$

and a point is (x_0, y_0, z_0), the perpendicular distance is:

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}$$

Step (2): Substitute values

- Plane: $A=3, B=6, C=2, D=7$
- Point: Origin (0,0,0)

$$d=\frac{|3\cdot 0+6\cdot 0+2\cdot 0+7|}{\sqrt{3^2+6^2+2^2}}$$

$$d=\frac{|7|}{\sqrt{9+36+4}}=\frac{7}{\sqrt{49}}=\frac{7}{7}=1$$

PART - III

31. Given matrix:

$$A=\begin{bmatrix}1 & 2 & -1\\ 3 & -1 & 2\\ 1 & -2 & 3\\ 1 & -1 & 1\end{bmatrix}$$

Step (1): Take a 3×3 submatrix

Choose the first 3 rows and 3 columns:

$$M=\begin{bmatrix}1 & 2 & -1\\ 3 & -1 & 2\\ 1 & -2 & 3\end{bmatrix}$$

Step (2): Compute its determinant

$\det(M)=1((-1)(3)-2(-2))-2(3\cdot 3-2\cdot 1)+(-1)(3\cdot -2-(-1)\cdot 1)$

Step by step: -

(i) $1((-3)-(-4))=1(1)=1$

(ii) $-2(9-2)=-2(7)=-14$

(iii) $-1(-6-(-1))=-1(-5)=5$

Add: $1-14+5=-8\neq 0$

Step (3): Conclusion

- A 3×3 submatrix has non-zero determinant, so rank ≥ 3
- The matrix has 4 rows and 3 columns, so rank ≤ 3

$\Rightarrow \text{rank}(A)=3$

32. We are asked to solve the system using the matrix inversion method:

$$\begin{cases}5x+2y=3\\ 3x+2y=5\end{cases}$$

Step (1): Write in matrix form

$\begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

Let $A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

$AX = B \Longrightarrow X = A^{-1}B$

Step (2): Find A^{-1} for 2×2 matrix

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Here $a = 5, b = 2, c = 3, d = 2$

$\det(A) = (5)(2) - (3)(2) = 10 - 6 = 4$

$A^{-1} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$

Step (3): Multiply $A^{-1}B$

$X = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 \cdot 3 + (-2) \cdot 5 \\ -3 \cdot 3 + 5 \cdot 5 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6 - 10 \\ -9 + 25 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 16 \end{bmatrix}$

$X = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$

33. Step (1): Distance formula in the complex plane

For two complex numbers $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$, the distance is:

$|z_1 - z_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Step (2): Distance to $10 - 8i$

$|(10 - 8i) - (1 + i)| = |9 - 9i| = \sqrt{9^2 + (-9)^2} = \sqrt{81 + 81} = \sqrt{162} = 9\sqrt{2}$

Step (3): Distance to $11 + 6i$

$|(11 + 6i) - (1 + i)| = |10 + 5i| = \sqrt{10^2 + 5^2} = \sqrt{100 + 25} = \sqrt{125} = 5\sqrt{5}$

Step (4): Compare distances

$9\sqrt{2} \approx 12.73, 5\sqrt{5} \approx 11.18$

$\Rightarrow 11 + 6i$ is closer to $1 + i$.

34. Step (1): Bring equation to standard form

$2x^3 - 9x^2 + 10x - 3 = 0$

Step (2): Factor out ($x - 1$) using synthetic division or factorization

Divide $2x^3 - 9x^2 + 10x - 3$ by $x - 1$:

Synthetic division:

1	2	-9	10	-3
		2	-7	3
	2	-7	3	0

Remainder $= 0$, quotient: $2x^2 - 7x + 3$

$2x^3 - 9x^2 + 10x - 3 = (x - 1)(2x^2 - 7x + 3)$

Step (3): Solve the quadratic $2x^2 - 7x + 3 = 0$

$x = \frac{7 \pm \sqrt{(-7)^2 - 4 \cdot 2 \cdot 3}}{2 \cdot 2} = \frac{7 \pm \sqrt{49 - 24}}{4} = \frac{7 \pm \sqrt{25}}{4}$

$x = \frac{7 \pm 5}{4} \Longrightarrow x = 3 \text{ or } x = \frac{1}{2}$

Step (4): All roots

$x = 1, x = 3, x = \frac{1}{2}$

35. We are asked to find the torque about the point $(2,0,-1)$ due to the force

$\mathrm{F} = 2\mathrm{i} + \mathrm{j} - \mathrm{k}$

whose line of action passes through the origin.

Step (1): Torque formula

$\tau = \mathrm{r} \times \mathrm{F}$

where r is the position vector from the point to a point on the line of action.

- Take point online of action: origin $O(0,0,0)$
- Point about which torque is calculated: $P(2,0,-1)$

r = vector from P to O = O − P = 0 − 2,0 − 0,0 − (−1) = −2i + 0j + 1k

Step (2): Cross product

$$\tau = r \times F = \begin{vmatrix} i & j & k \\ -2 & 0 & 1 \\ 2 & 1 & -1 \end{vmatrix}$$

Compute determinant:

$$i\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - j\begin{vmatrix} -2 & 1 \\ 2 & -1 \end{vmatrix} + k\begin{vmatrix} -2 & 0 \\ 2 & 1 \end{vmatrix}$$

Step by step: -

(i) i: $(0)(-1) - (1)(1) = -1$

(ii) −j: $(-2)(-1) - (1)(2) = 2 - 2 = 0 \Rightarrow -0 = 0$

(iii) k: $(-2)(1) - (0)(2) = -2 - 0 = -2$

$\tau = -i + 0j - 2k = -i - 2k$

Step (3): Magnitude of torque

$|\tau| = \sqrt{(-1)^2 + 0^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$

Step (4): Direction cosines

Direction cosines (l, m, n) are:

$$l = \frac{\tau_x}{|\tau|} = \frac{-1}{\sqrt{5}}, m = \frac{\tau_y}{|\tau|} = \frac{0}{\sqrt{5}} = 0, n = \frac{\tau_z}{|\tau|} = \frac{-2}{\sqrt{5}}$$

36. We are asked to evaluate:

$$\lim_{x\to\infty} \frac{2x^2 - 3}{x^2 - 5x + 3}$$

Step (1): Factor out the highest power of x in numerator and denominator

$$\frac{2x^2 - 3}{x^2 - 5x + 3} = \frac{x^2(2 - 3/x^2)}{x^2(1 - 5/x + 3/x^2)} = \frac{2 - 3/x^2}{1 - 5/x + 3/x^2}$$

Step (2): Take the limit as $x \to \infty$

$$\lim_{x\to\infty} \frac{2 - 3/x^2}{1 - 5/x + 3/x^2} = \frac{2-0}{1-0+0} = \frac{2}{1} = 2$$

37. Step (1): Recall formula for area of a circle

$A = \pi r^2$

Small change in area:

$dA \approx 2\pi r dr$

Step (2): Substitute values

- Initial radius: $r = 2$ mm
- Increase in radius: $dr = 2.1 - 2 = 0.1$ mm

$dA \approx 2\pi(2)(0.1) = 0.4\pi \text{ mm}^2$

Step (3): Numerical value

$dA \approx 0.4 \times 3.1416 \approx 1.256 \text{ mm}^2$

38. Step (1): Substitution

Let

$u = \sec x \Rightarrow du = \sec x \tan x dx$

Then the integral becomes:

$$I = \int_{x=0}^{x=\pi/3} \frac{du}{1+u^2}$$

Step (2): Integrate

$$\int \frac{du}{1+u^2} = \tan^{-1} u$$

Step (3): Change limits

- When $x = 0, u = \sec 0 = 1$
- When $x = \pi/3, u = \sec(\pi/3) = 2$

So:

$I = \tan^{-1}(2) - \tan^{-1}(1)$

Step (4): Simplify

$\tan^{-1}(1) = \frac{\pi}{4} \Rightarrow I = \tan^{-1}(2) - \frac{\pi}{4}$

39. Step (1): Check if * is a binary operation

A binary operation on $\mathbb{R}$ must satisfy:

$a * b \in \mathbb{R}\ \forall a, b \in \mathbb{R}$

Here:

$a * b = a + b + ab - 7$

- Sum and product of real numbers are real
- Subtracting 7 is real

So $a * b \in \mathbb{R}$ for all $a, b \in \mathbb{R}$

Conclusion: * is a binary operation on $\mathbb{R}$.

Step (2): Compute $3 * \left(-\frac{7}{15}\right)$

$$3 * \left(-\frac{7}{15}\right) = 3 + \left(-\frac{7}{15}\right) + 3 \cdot \left(-\frac{7}{15}\right) - 7$$

Step by step: -

(i) $3 + (-7/15) = 45/15 - 7/15 = 38/15$

(ii) $3 \cdot (-7/15) = -21/15 = -7/5 = -21/15$

(iii) Add: $38/15 - 21/15 = 17/15$

(iv) Subtract 7: $17/15 - 105/15 = -88/15$

40. Step (1): Midpoint = center of the circle

$$\text{Center } C = \left(\frac{-4 + (-1)}{2}, \frac{-2 + (-1)}{2}\right) = \left(-\frac{5}{2}, -\frac{3}{2}\right)$$

Step (2): Radius = half the distance between A and B

$AB = \sqrt{(-1 + 4)^2 + (-1 + 2)^2} = \sqrt{3^2 + 1^2} = \sqrt{10}$

$$r = \frac{AB}{2} = \frac{\sqrt{10}}{2}$$

Step (3): General equation of a circle

$(x - h)^2 + (y - k)^2 = r^2$

$$\left(x + \frac{5}{2}\right)^2 + \left(y + \frac{3}{2}\right)^2 = \left(\frac{\sqrt{10}}{2}\right)^2 = \frac{10}{4} = \frac{5}{2}$$

Step (4): Expand

$$x^2 + 5x + \frac{25}{4} + y^2 + 3y + \frac{9}{4} = \frac{5}{2} = \frac{10}{4}$$

$$x^2 + y^2 + 5x + 3y + \frac{25 + 9 - 10}{4} = 0$$

$$x^2 + y^2 + 5x + 3y + \frac{24}{4} = 0$$

$x^2 + y^2 + 5x + 3y + 6 = 0$

PART - IV

41. (a) Step (1): Recall the condition for Cramer's rule

Cramer's rule can be applied only if the determinant of the coefficient matrix is non-zero.

Coefficient matrix A is:

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & -3 & 2 \\ 7 & -1 & 4 \end{bmatrix}$$

Step (2): Compute $\det(A)$

$$\det(A) = 3\begin{vmatrix} -3 & 2 \\ -1 & 4 \end{vmatrix} - 1\begin{vmatrix} 1 & 2 \\ 7 & 4 \end{vmatrix} + 1\begin{vmatrix} 1 & -3 \\ 7 & -1 \end{vmatrix}$$

Compute each minor:

(i) $3((-3)(4) - (2)(-1)) = 3(-12 + 2) = 3(-10) = -30$

(ii) $-1((1)(4) - (2)(7)) = -1(4 - 14) = -1(-10) = 10$

(iii) $1((1)(-1) - (-3)(7)) = 1(-1 + 21) = 20$

Add: $-30 + 10 + 20 = 0$

Step (3): Conclusion
$\det(A) = 0 \Longrightarrow$ coefficient matrix is singular.
Cramer's rule is not applicable because the determinant of the coefficient matrix is zero.

OR

(b) Step (1): Find derivative
$$f'(x) = \frac{d}{dx}(4x^6 - 6x^4) = 24x^5 - 24x^3 = 24x^3(x^2 - 1) = 24x^3(x-1)(x+1)$$
Set $f'(x) = 0$:
$x = 0, x = 1, x = -1$
Step (2): Use second derivative test
$$f''(x) = \frac{d}{dx}f'(x) = \frac{d}{dx}(24x^5 - 24x^3) = 120x^4 - 72x^2 = 24x^2(5x^2 - 3)$$
- At $x = 1 : f''(1) = 24(1^2)(5-3) = 24 \cdot 2 = 48 > 0 \Longrightarrow$ local minimum
- At $x = -1: f''(-1) = 24(1)(5-3) = 48 > 0 \Longrightarrow$ local minimum
- At $x = 0: f''(0) = 0 \rightarrow$ test inconclusive

Check $f(x)$ near $x = 0$:
$f(-0.1) \approx -6(0.1)^4 = -0.0006 < 0, f(0.1) \approx -0.0006$
So $x = 0$ is a local maximum, not minimum.

42. **(a)** We are asked to find the locus of points $z = x + iy$ satisfying:
- $|z + i| = |z - 1|$

Step (1): Write in terms of x and y
$$z + i = x + i(y+1) \Longrightarrow |z + i| = \sqrt{x^2 + (y+1)^2}$$
$$z - 1 = (x-1) + iy \Longrightarrow |z - 1| = \sqrt{(x-1)^2 + y^2}$$
Equation becomes: -
$$\sqrt{x^2 + (y+1)^2} = \sqrt{(x-1)^2 + y^2}$$
Step (2): Square both sides
$$x^2 + (y+1)^2 = (x-1)^2 + y^2$$
$$x^2 + y^2 + 2y + 1 = x^2 - 2x + 1 + y^2$$
$$2y = -2x \Longrightarrow x + y = 0$$

OR

(b) Step (1): Use substitution $x \rightarrow a - x$
Let $u = a - x \Longrightarrow dx = -du.$
Change limits:
- When $x = 0 \Longrightarrow u = a$
- When $x = a \Longrightarrow u = 0$

So:
$$\int_0^a \frac{f(x)}{f(x) + f(a-x)}dx = \int_{u=a}^{u=0} \frac{f(a-u)}{f(a-u) + f(u)}(-du) = \int_0^a \frac{f(a-u)}{f(a-u) + f(u)}du$$
Step (2): Add the two integrals
Let
$$I = \int_0^a \frac{f(x)}{f(x) + f(a-x)}dx$$
Then from step 1:
$$I = \int_0^a \frac{f(a-x)}{f(a-x) + f(x)}dx$$
Add both expressions:
$$2I = \int_0^a \frac{f(x)}{f(x) + f(a-x)} + \frac{f(a-x)}{f(x) + f(a-x)}dx = \int_0^a 1dx = a$$
$$\Longrightarrow I = \frac{a}{2}$$

43. **(a)** Step (1): Use the definition of a parabola
A parabola is the locus of points equidistant from the focus and the directrix:
Distance from focus = Distance from directrix
Let a point on the parabola be (x, y). Then:

$\sqrt{(x+\sqrt{2})^2+(y-0)^2} = |x-\sqrt{2}|$
Step (2): Square both sides
$(x+\sqrt{2})^2 + y^2 = (x-\sqrt{2})^2$
$x^2 + 2\sqrt{2}x + 2 + y^2 = x^2 - 2\sqrt{2}x + 2$
$y^2 + 2\sqrt{2}x + 2 = -2\sqrt{2}x + 2$
$y^2 + 2\sqrt{2}x = -2\sqrt{2}x$
$y^2 =$-$4\sqrt{2}x$

OR

(b) Step (1): Evaluate each inverse function
(i) $\cot^{-1}(1)$
$\cot\theta = 1 \Rightarrow \theta = \frac{\pi}{4}\ (0<\theta<\pi)$
(ii) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
$\sin\theta = -\frac{\sqrt{3}}{2} \Rightarrow \theta = -\frac{\pi}{3}\ (-\pi/2 \le \theta \le \pi/2)$
(iii) $\sec^{-1}(-\sqrt{2})$
$\sec\theta = -\sqrt{2} \Rightarrow \cos\theta = -\frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{3\pi}{4}$
(principal value: $0 \le \theta < \pi, \theta \ne \pi/2$)
Step (2): Substitute values
$\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2}) = \frac{\pi}{4} + \left(-\frac{\pi}{3}\right) - \frac{3\pi}{4}$
$= \frac{\pi}{4} - \frac{\pi}{3} - \frac{3\pi}{4} = -\pi/3 - 2\pi/4 = -\pi/3 - \pi/2$
$= -\frac{2\pi}{6} - \frac{3\pi}{6} = -\frac{5\pi}{6}$

44. (a) Step (1): Recall ellipse formula
For an ellipse: -
$r_{\max} = a + c, r_{\min} = a - c$
where:
$a =$ semi-major axis
$c =$ distance from center to each focus
Step (2): Solve for a and c
Add equations: -
$r_{\max} + r_{\min} = (a+c) + (a-c) = 2a$
$a = \frac{r_{\max} + r_{\min}}{2} = \frac{152 + 94.5}{2} \times 10^6$ km
$a = \frac{246.5}{2} \times 10^6 = 123.25 \times 10^6$ km
Subtract equations: -
$r_{\max} - r_{\min} = (a+c) - (a-c) = 2c$
$c = \frac{r_{\max} - r_{\min}}{2} = \frac{152 - 94.5}{2} \times 10^6$
$c = \frac{57.5}{2} \times 10^6 = 28.75 \times 10^6$ km
Step (3): Express in required form
$c = 28.75 \times 10^6$ km $= 287.5 \times 10^5$ km

OR

(b) Step (1): Represent vectors
Let's take two-unit vectors making angles A and B with the x-axis:
$\vec{u} = \hat{\imath}\cos A + \hat{\jmath}\sin A, \vec{v} = \hat{\imath}\cos B + \hat{\jmath}\sin B$
Step (2): Use rotation idea
Rotating vector $\vec{u}$ by angle B gives a vector at angle $A+B$:

$\vec{u}_{\text{rot}} = (\cos B\hat{\imath} - \sin B\hat{\jmath})(\cos A) + (\sin B\hat{\imath} + \cos B\hat{\jmath})(\sin A)$
Step (3): x - and y-components
The y-component of rotated vector $= \sin(A+B)$
y-component $= \sin A \cdot \cos B + \cos A \cdot \sin B$
Step (4): Conclusion
$\sin(A+B) = \sin A \cos B + \cos A \sin B$

45. (a) Step (1): Find a vector in the plane
A vector in the desired plane is calculated using the two given points, A(2,2,1) and B(9,3,6):
$\overrightarrow{AB} = \text{B} - \text{A} = (9-2, 3-2, 6-1) = (7,1,5)$
Step (2): Find the normal of the given plane
The normal vector of the perpendicular plane $2x + 6y + 6z = 9$ is directly obtained from its coefficients:
$\vec{n}_2 = (2,6,6)$
Step (3): Determine the normal of the desired plane
The desired plane's normal vector, $\vec{n}_1$, must be perpendicular to both $\overrightarrow{AB}$ and $\vec{n}_2$, so we find their cross product:
$\vec{n}_1 = \overrightarrow{AB} \times \vec{n}_2 = |\text{i} \quad \text{j} \quad \text{k}7 \quad 1 \quad 52 \quad 6 \quad 6| = (-24, -32, 40)$
We use a proportional, simplified normal vector $\vec{n} = (3,4,-5)$ by dividing by -8 .
Step (4): Form the equation
Using the point A(2,2,1) and the normal vector $\vec{n} = (3,4,-5)$, the Cartesian equation is $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$;
$3(x-2) + 4(y-2) - 5(z-1) = 0 \Longrightarrow 3x + 4y - 5z = 9$

OR

(b) Step (1): Recall formula for angle between curves
If two curves $y = f_1(x)$ and $y = f_2(x)$ intersect, the angle θ between them at a point is:
$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$
where m_1 and m_2 are the slopes of the tangents at the point of intersection.
Step (2): Find slopes of tangents
(i) Curve $y = x^2 \Rightarrow$ slope:
$$m_1 = \frac{dy}{dx} = 2x \ \Rightarrow m_1|_{(1,1)} = 2$$
(ii) Curve $x = y^2 \Rightarrow$ slope:
$$\frac{dx}{dy} = 2y \ \Rightarrow \frac{dy}{dx} = \frac{1}{2y} \ \Rightarrow m_2\Big|_{(1,1)} = \frac{1}{2}$$
Step (3): Apply formula
$$\tan\theta = \left|\frac{2 - 1/2}{1 + 2 \cdot 1/2}\right| = \frac{3/2}{1+1} = \frac{3/2}{2} = \frac{3}{4}$$
Step (4): Angle
$$\theta = \tan^{-1}\frac{3}{4}$$

46. (a) Recall: For a continuous random variable X,
$P(a \le X \le b) = F(b) - F(a)$
(i) $P(X < 3)$
$$P(X < 3) = F(3) = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$
So, $P(X < 3) = \frac{1}{2}$
(ii) $P(2 < X < 4)$
$P(2 < X < 4) = F(4) - F(2)$
Compute $F(4)$ and $F(2)$ using CDF formula $(x-1)/4$:
$$F(4) = \frac{4-1}{4} = \frac{3}{4}, F(2) = \frac{2-1}{4} = \frac{1}{4}$$
$$P(2 < X < 4) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$
So, $P(2 < X < 4) = \frac{1}{2}$
(iii) $P(X \ge 3)$

$P(X \geq 3) = 1 - P(X < 3) = 1 - F(3) = 1 - \frac{1}{2} = \frac{1}{2}$

OR

(b) Step (1): Express the line in terms of y and find the x-intercept

The equation $3x - 2y + 6 = 0$ can be rewritten as $y = \frac{3}{2}x + 3$. The line crosses the x axis at $x = -2$, which is within the given interval $[-3,1]$.

Step (2): Calculate the absolute areas of the regions

The total area is the sum of the absolute areas of the two regions formed by the x intercept:

$$A = \left|\int_{-3}^{-2} \left(\frac{3}{2}x + 3\right) dx\right| + \left|\int_{-2}^{1} \left(\frac{3}{2}x + 3\right) dx\right|$$

The indefinite integral is $\frac{3}{4}x^2 + 3x$.

The first area (below axis) is

$$\left|\left[\frac{3}{4}x^2 + 3x\right]_{-3}^{-2}\right| = \left|(3-6) - \left(\frac{27}{4} - 9\right)\right| = \left|-3 - \left(-\frac{9}{4}\right)\right| = \frac{3}{4}.$$

The second area (above axis) is

$$\left[\frac{3}{4}x^2 + 3x\right]_{-2}^{1} = \left(\frac{3}{4} + 3\right) - (3 - 6) = \frac{15}{4} - (-3) = \frac{27}{4}.$$

Step (3): Sum the areas

The total area is the sum of the absolute values of the two sections.

$$A_{\text{total}} = \frac{3}{4} + \frac{27}{4} = \frac{30}{4} = \frac{15}{2}$$

47. **(a)** Given differential equation:

$$(1 + x^2)\frac{dy}{dx} = 1 + y^2$$

Step (1): Separate variables

$$\frac{dy}{1 + y^2} = \frac{dx}{1 + x^2}$$

Step (2): Integrate both sides

$$\int \frac{dy}{1 + y^2} = \int \frac{dx}{1 + x^2}$$

$\tan^{-1} y = \tan^{-1} x + C$

where C is the constant of integration.

Step (3): Alternate form

$\tan^{-1} x = \tan^{-1} y - C = \tan^{-1} y + C''$ (where $C'' = -C$)

OR

(b) Step (1): Recall truth table definitions

- $p \to q \equiv \neg p \vee q$
- $p \wedge q$ is true only if both p and q are true

We want to show:

$p \to (q \to r) \equiv (p \wedge q) \to r$

Step (2): Make columns

We need columns for $p.q.r.q \to r.p \to (q \to r).p \wedge q(p \wedge q) \to r$

p	q	r	q→r	p→(q→r)	p∧q	(p∧q)→r
T	T	T	T	T	T	T
T	T	F	F	F	T	F
T	F	T	T	T	F	T
T	F	F	T	T	F	T
F	T	T	T	T	F	T
F	T	F	F	T	F	T

F	F	T	T	T	F	T
F	F	F	T	T	F	T

Step (3): Compare columns
Column $p \to (q \to r)$ = Column $(p \wedge q) \to r$ for a 8 cases.
Conclusion:
$p \to (q \to r) \equiv (p \wedge q) \to r$