PART - I

1. **If $f(x)=\begin{cases}2x & 0\leqslant x\leqslant a\\ 0 & \text{otherwise}\end{cases}$ is a probability density function of a random variable, then the value of a is :**
   (a) 3 (b) 1
   (c) 4 (d) 2
2. **Which one of the following is not true in the case of discrete random variable X ?**
   (a) $\lim\limits_{x\to\infty} \mathrm{F}(x)=\mathrm{F}(\infty)=1$
   (b) $0\leqslant \mathrm{F}(x)\leqslant 1$ for all $x\in \mathrm{R}$
   (c) $\mathrm{F}(x)$ is real valued decreasing function.
   (d) $\lim\limits_{x\to-\infty} \mathrm{F}(x)=\mathrm{F}(-\infty)=0$
3. **If $f(x)=\frac{x}{x+1}$, then its differential is :**
   (a) $\frac{1}{x+1}\,dx$ (b) $\frac{-1}{(x+1)^2}\,dx$
   (c) $\frac{-1}{x+1}\,dx$ (d) $\frac{1}{(x+1)^2}\,dx$
4. **The value of $\int_0^1 x(1-x)^{99}\,dx$ is :**
   (a) $\frac{1}{10010}$ (b) $\frac{1}{11000}$
   (c) $\frac{1}{10001}$ (d) $\frac{1}{10100}$
5. **The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is :**
   (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$
   (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{6}$
6. **If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $\lambda A^{-1}=A$, then λ is :**
   (a) 19 (b) 17
   (c) 21 (d) 14
7. **If α,β and γ are the zeros of x^3+px^2+qx+r, then $\sum\frac{1}{\alpha}$ is:**
   (a) $\frac{q}{r}$ (b) $-\frac{q}{r}$
   (c) $-\frac{q}{p}$ (d) $-\frac{p}{r}$
8. **If $(1+i)(1+2i)(1+3i)\ldots(1+ni)=x+iy$ then the value $2\cdot5\cdot10\ldots\left(1+n^2\right)$ is :**
   (a) x^2+y^2 (b) 1
   (c) $1+n^2$ (d) i
9. **The minimum value of the function $|3-x|+9$ is :**
   (a) 6 (b) 0
   (c) 9 (d) 3
10. **The value of $\sum_{n=1}^{12} i^n$ is :**
    (a) 0 (b) 1
    (c) -1 (d) i
11. **If the vectors $2\hat{\imath}-\hat{\jmath}+3\hat{k}, 3\hat{\imath}+2\hat{\jmath}+\hat{k}, \hat{\imath}+m\hat{\jmath}+4\hat{k}$ are coplanar, then the value of m is :**
    (a) 2 (b) 3
    (c) -2 (d) -3
12. **The general equation of a circle with centre $(-3,-4)$ and radius 3 units is :**
    (a) $x^2+y^2-6x+8y-16=0$
    (b) $x^2+y^2-6x-8y+16=0$
    (c) $x^2+y^2+6x-8y+16=0$
    (d) $x^2+y^2+6x+8y+16=0$

13. The solution of $\frac{dy}{dx} + p(x)y = 0$ is :

(a) $x = ce^{-\int pdy}$ (b) $y = ce^{\int pdx}$

(c) $x = ce^{\int pdy}$ (d) $y = ce^{-\int pdx}$

14. The value of $\int_0^{\infty} e^{-3x}x^2\,dx$ is :

(a) $\frac{4}{27}$ (b) $\frac{7}{27}$

(c) $\frac{2}{27}$ (d) $\frac{5}{27}$

15. The point of inflection of the curve $y = (x-1)^3$ is :

(a) (1,0) (b) (0,0)

(c) (1,1) (d) (0,1)

16. The angle between the lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$ is :

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{4}$

(c) $\frac{2\pi}{3}$ (d) $\frac{\pi}{3}$

17. Which one of the following is a binary operation on N?

(a) Multiplication (b) Division

(c) Subtraction (d) All the above

18. Which one of the following is incorrect ?

(a) If A is a square matrix of order n , and λ is a scalar, then $\mathrm{Adj}(\lambda A) = \lambda^n(\mathrm{Adj}A)$.

(b) Adjoint of a symmetric matrix is also a symmetric matrix.

(c) $A(\mathrm{Adj}A) = (\mathrm{Adj}A)A = |A|I$.

(d) Adjoint of a diagonal matrix is also a diagonal matrix.

19. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is :

(a) $\tan x$ (b) $\log \sin x$

(c) $\cot x$ (d) $\cos x$

20. The length of the latus rectum of the parabola $x^2 = 24y$ is :

(a) 8 (b) 24

(c) 6 (d) 12

PART – II

21. Prove the following properties :

$\mathrm{Re}(z) = \frac{z+\bar{z}}{2}$ and $\mathrm{Im}(z) = \frac{z-\bar{z}}{2i}$

22. Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.

23. Find the principal value of $\tan^{-1}(\sqrt{3})$.

24. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line $y = x$.

25. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$.

26. Show that the differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary constants, is $\frac{d^2y}{dx^2} - y = 0$.

27. Solve : $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$

28. A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	2k	6k	5k	6k	10k

Find k.

29. X is the number of tails occurred when three fair coins are tossed simultaneously. Find the values of the random variable X and number of points in its reverse images.

30. Show that the distance from the origin to the plane $3x + 6y + 2z + 7 = 0$ is 1 .

PART - III

31. Show that the rank of the matrix $\begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 1 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix}$ is 3.

32. Solve the following system of linear equations, using matrix inversion method : $5x + 2y = 3, 3x + 2y = 5$.

33. Which one of the point $10 - 8i, 11 + 6i$ is closest to $1 + i$.

34. Solve the equation $2x^3 - 9x^2 + 10x = 3$, if 1 is a root, find the other roots.

35. Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\hat{\imath} + \hat{\jmath} - \hat{k}$, whose line of action passes through the origin.

36. Evaluate $\lim_{x\to\infty} \frac{2x^2-3}{x^2-5x+3}$

37. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm , how much is cross-sectional area increased approximately.

38. Show that $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x}\, dx = \tan^{-1}(2) - \frac{\pi}{4}$.

39. Let * be defined on R by $(a^* b) = a + b + ab - 7$. Is * binary on **R** ? If so, find $3^* \left(\frac{-7}{15}\right)$.

40. Prove that the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(-1,-1)$, is $x^2 + y^2 + 5x + 3y + 6 = 0$.

PART - IV

41. **(a)** Cramer's rule is not applicable to solve the system $3x + y + z = 2, x - 3y + 2z = 1, 7x - y + 4z = 5$. Why?

OR

(b) Prove that the local minimum values for the function $f(x) = 4x^6 - 6x^4$ attain at -1 and 1.

42. **(a)** Show that the locus of $z = x + iy$ if $|z + i| = |z - 1|$, is $x + y = 0$.

OR

(b) Show that $\int_0^a \frac{f(x)}{f(x)+f(a-x)} dx = \frac{a}{2}$.

43. **(a)** Show that the equation of the parabola with focus $(-\sqrt{2}, 0)$ and directrix $x = \sqrt{2}$ is $y^2 = -4\sqrt{2}x$.

OR

(b) Find the value of $\cot^{-1}(1) + \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) - \sec^{-1}(-\sqrt{2})$.

44. **(a)** The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Show that the distance from the Sun to the other focus is 575×10^5 km.

OR

(b) Prove by vector method
$\sin(A + B) = \sin A \cdot \cos B + \cos A \cdot \sin B$.

45. **(a)** Find the vector equation (any form) or Cartesian equation of a plane passing through the points $(2,2,1), (9,3,6)$ and perpendicular to the plane $2x + 6y + 6z = 9$.

OR

(b) Show that the angle between the curves $y = x^2$ and $x = y^2$ at $(1,1)$ is $\tan^{-1}\left(\frac{3}{4}\right)$.

46. **(a)** The distribution function of a continuous random variable X is :

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{x-1}{4}, & 1 \leqslant x \leqslant 5 \\ 1, & x > 5 \end{cases}$$

Find (i) $P(X < 3)$ (ii) $P(2 < X < 4)$ (iii) $P(3 \leqslant X)$

OR

(b) Show that the area of the region bounded by $3x - 2y + 6 = 0, x = -3, x = 1$ and x-axis, is $\frac{15}{2}$.

47. (a) Show that the solution of the differential equation $(1+x^2)\frac{dy}{dx} = 1 + y^2$ is $\tan^{-1} y = \tan^{-1} x + C$ (or) $\tan^{-1} x = \tan^{-1} y + C$.

OR

(b) Prove $p \to (q \to r) \equiv (p \wedge q) \to r$ using truth table.