PART - I

1. A square matrix A of order n has inverse if and only if:

(a) $\rho(A) > n$ (b) $\rho(A) = n$
(c) $\rho(A) \neq n$ (d) $\rho(A) < n$

2. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is:

(a) 2 (b) 0
(c) 3 (d) 1

3. If $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$,

(a) $x \in \left(\frac{1}{2}, 1\right)$ (b) $x \in \left[\frac{1}{2}, 1\right]$
(c) $x \in (-\infty, 1]$ (d) $x \in \left[\frac{1}{2}, \infty\right)$

4. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is:

(a) $y = kx$ (b) $xy = k$
(c) $\log y = kx$ (d) $y = k\log x$

5. The number of normals that can be drawn from a point to the parabola $y^2 = 4ax$ is:

(a) 3 (b) 2
(c) 0 (d) 1

6. If $\vec{a}$ and $\vec{b}$ are parallel vectors then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to:

(a) 1 (b) 2
(c) 0 (d) -1

7. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is:

(a) 1 (b) 2
(c) ∞ (d) 4

8. Suppose that X takes on one of the values $0, 1, 2$. If for some constant k, $P(X = i) = kP(X = i - 1)$ for $i = 1, 2$ and $P(X = 0) = \frac{1}{7}$, then the value of k is:

(a) 3 (b) 1
(c) 4 (d) 2

9. The maximum value of the function $x^2 e^{-2x}, x > 0$ is:

(a) $\frac{1}{e^2}$ (b) $\frac{1}{e}$
(c) $\frac{4}{e^4}$ (d) $\frac{1}{2e}$

10. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on:

(a) R (b) Q^+
(c) C (d) Z

11. The area between $y^2 = 4x$ and its latus rectum is:

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$
(c) $\frac{5}{3}$ (d) $\frac{4}{3}$

12. Angle between the curves $y^2 = x$ and $x^2 = y$ at the origin is:

(a) $\frac{\pi}{2}$ (b) $\tan^{-1}\left(\frac{3}{4}\right)$
(c) $\frac{\pi}{4}$ (d) $\tan^{-1}\left(\frac{4}{3}\right)$

13. $|\text{adj}(\text{adj}A)| = |A|^{16}$, then the order of the square matrix A is:

(a) 2 (b) 3
(c) 5 (d) 4

14. The value of $\left(\frac{1+i}{\sqrt{2}}\right)^{8}+\left(\frac{1-i}{\sqrt{2}}\right)^{8}$ is:

(a) 8 (b) 4
(c) 2 (d) 6

15. If $|z|=1$, then the value of $\frac{1+z}{1+\bar{z}}$ is:

(a) $\frac{1}{z}$ (b) z
(c) 1 (d) $\bar{z}$

16. The abscissa of the point on the curve $f(x)=\sqrt{8-2x}$ at which the slope of the tangent is -0.25?

(a) -2 (b) -8
(c) 0 (d) -4

17. The value of $\int_0^{\pi/3}\tan x\,dx$ is:

(a) $-\log 2$ (b) $\log 2$
(c) $-\log 3$ (d) $\log 3$

18. The number of positive zeros of the polynomial $\sum_{r=0}^{n} {}^{n}C_r(-1)^r x^r$ is:

(a) < n (b) 0
(c) r (d) n

19. The principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is:

(a) $\frac{-\pi}{6}$ (b) 0
(c) $\frac{-\pi}{2}$ (d) $\frac{\pi}{2}$

20. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is:

(a) $\sqrt{ab}$ (b) 2ab
(c) $\frac{a}{b}$ (d) ab

PART - II

21. If $|z|=2$, show that $3\le|z+3+4i|\le 7$

22. If p and q are the roots of the equation $1x^2+nx+n=0$, show that $\sqrt{\frac{p}{q}}+\sqrt{\frac{q}{p}}+\sqrt{\frac{n}{l}}=0$

23. If $y=4x+c$ is a tangent to the circle $x^2+y^2=9$, find c.

24. If the radius of a sphere with radius 10 cm, has to decrease by 0.1 cm, approximately how much will its volume decrease?

25. Evaluate: $\int_b^{\infty}\frac{1}{a^2+x^2}\,dx$, a > 0, b ∈ R.

26. Find the vector equation of a plane which is at a distance of 7 units from the origin having $3,-4,5$ as direction ratios of a normal to it.

27. Let $A=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ be any two Boolean matrices of the same type. Find $A\vee B$ and $\boldsymbol{A}\wedge B$.

28. Prove that $\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ is orthogonal.

29. Find the equation of tangent to the curve $y=x^2+3x-2$ at the point (1,2).

30. Express $e^{\cos\theta+i\sin\theta}$ in $a+ib$ form.

PART – III

31. Find the equation of the parabola with vertex $(-1,-2)$, axis parallel to y-axis and passing through (3,6).

32. The maximum and minimum distances of the Earth from the Sun respectively are 152×10^6 km and 94.5×10^6 km. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.

33. For what value of x, the inequality $\frac{\pi}{2}<\cos^{-1}(3x-1)<\pi$ holds?

34. Find the angle made by the straight line $\frac{x+3}{2}=\frac{y-1}{2}=-z$ with coordinate axes.

35. Use the linear approximation to find an approximate value of $(123)^{2/3}$.

36. Solve: $x\cos y\,dy = e^x(x\log x + 1)dx$.

37. If $F(\alpha) = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1} = F(-\alpha)$

38. Show that $p \to q$ and $q \to p$ are not equivalent.

39. If $z = (2+3i)(1-i)$, then find z^{-1}.

40. If $a+b+c=0$ and a, b, c are rational numbers then, prove that the roots of the equation $(b+c-a)x^2 + (c+a-b)x + (a+b-c) = 0$ are rational numbers.

PART - IV

41. (a) Solve the equation $z^3 + 8i = 0$, where $z \in \mathbb{C}$.

OR

(b) Solve: $(1 + x + xy^2)\frac{dy}{dx} + (y + y^3) = 0$.

42. (a) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

OR

(b) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function of random variable X is $f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$.

Find (i) the value of k

(ii) the distribution function

(iii) the probability that daily sales will fall between 300 litres and 500 litres.

43. (a) Identify the type of conic and find centre, foci and vertices of $18x^2 + 12y^2 - 144x + 48y + 120 = 0$

OR

(b) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

44. (a) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8)$, $(-2,-12)$ and $(3,8)$. He wants to meet his friend at P(7,60). Will he meet his friend? (Use Gaussian Elimination method)

OR

(b) Prove that the ellipse $x^2 + 4y^2 = 8$ and the hyperbola $x^2 - 2y^2 = 4$ intersect orthogonally.

45. (a) Find the parametric form of Vector equation and Cartesian equations of the plane containing the line $\vec{\mathbf{r}} = (\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \mathbf{t}(2\hat{\imath} - \hat{\jmath} + 4\hat{k})$ and perpendicular to the plane $\vec{r} \cdot (\hat{\imath} + 2\hat{\jmath} + \hat{k}) = 8$.

OR

(b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

46. (a) Prove that $p \to (\neg q \lor r) \equiv \neg p \lor (\neg q \lor r)$ using truth table.

OR

(b) Suppose a person deposits ₹ 10,000 in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

47. (a) Find the maximum value of $\frac{\log x}{x}$

OR

(b) Find the area of the region common to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$.