PART - I

1. The area between $y^2 = 4x$ and its latus rectum is:

(a) $\frac{8}{3}$ (b) $\frac{2}{3}$

(c) $\frac{5}{3}$ (d) $\frac{4}{3}$

2. The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is :

(a) $\frac{3\pi a^2}{8}$ (b) $\frac{\pi a^3}{16}$

(c) $\frac{3\pi a^4}{8}$ (d) $\frac{3\pi a^4}{16}$

3. If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3, 0)$ and $F_2(-3, 0)$, then $PF_1 + PF_2$ is :

(a) 10 (b) 8

(c) 12 (d) 6

4. If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$ then the value of $|z_1 + z_2 + z_3|$ is :

(a) 3 (b) 1

(c) 4 (d) 2

5. The number of rows in the truth table of $(p \vee q) \wedge (p \vee r)$ is :

(a) 6 (b) 9

(c) 3 (d) 8

6. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then :

(a) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ (b) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$

(c) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$ (d) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$

7. The differential equation of the family of curves $y = Ae^x + Be^{-x}$, where A and B are arbitrary constants is :

(a) $\frac{dy}{dx} + y = 0$ (b) $\frac{d^2y}{dx^2} + y = 0$

(c) $\frac{dy}{dx} - y = 0$ (d) $\frac{d^2y}{dx^2} - y = 0$

8. The horizontal asymptote of $f(x) = \frac{1}{x}$ is :

(a) $x = C$ (b) $y = 0$

(c) $y = c$ (d) $x = 0$

9. If $f(x) = \frac{x}{x+1}$, then its differential is given by :

(a) $\frac{1}{x+1}\,dx$ (b) $\frac{-1}{(x+1)^2}\,dx$

(c) $\frac{-1}{x+1}\,dx$ (d) $\frac{1}{(x+1)^2}\,dx$

10. If $(1 + i)(1 + 2i)(1 + 3i) \dots (1 + ni) = x + iy$ then $2 \cdot 5 \cdot 10 \dots (1 + n^2)$ is :

(a) $x^2 + y^2$ (b) 1

(c) $1 + n^2$ (d) i

11. The number given by the Rolle's theorem for the function $x^3 - 3x^2, x \in [0, 3]$ is :

(a) $\frac{3}{2}$ (b) 1

(c) 2 (d) $\sqrt{2}$

12. The type of conic section for $x^2 - 3 = 5x + 3y$ is :

(a) hyperbola (b) ellipse

(c) circle (d) parabola

13. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $\left(A^T\right)^{-1} =$

(a) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$

14. The angle between the line $\vec{r} = (\vec{i} + 2\vec{j} - 3\vec{k}) + t(2\vec{i} + \vec{j} - 2\vec{k})$ and the plane $\vec{r} \cdot (\vec{i} + \vec{j}) + 4 = 0$ is :

(a) $45°$ (b) $0°$

(c) $90°$ (d) $30°$

15. If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then x is equal to :

(a) $\frac{2}{\sqrt{5}}$ (b) $\frac{1}{2}$

(c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{5}}$

16. If α, β and γ are zeros of $x^3 + px^2 + qx + r$ then $\sum \frac{1}{\alpha}$ is :

(a) $\frac{q}{r}$ (b) $-\frac{q}{r}$

(c) $-\frac{CI}{p}$ (d) $-\frac{p}{r}$

17. The value of Var(3) is :

(a) 0 (b) 3

(c) Var(3) (d) 9

18. The random variable X has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is :

(a) 3 (b) 6

(c) 2 (d) 4

19. If A, B and C are invertible matrices of some order, then which one of the following is not true ?

(a) $\det A^{-1} = (\det A)^{-1}$

(b) $\operatorname{adj} A = |A|A^{-1}$

(c) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

(d) $\operatorname{adj}(AB) = (\operatorname{adj}A)(\operatorname{adj}B)$

20. A zero of $x^3 + 64$ is:

(a) $4i$ (b) 0

(c) -4 (d) 4

PART - II

21. Simplify : $\sum_{n=1}^{12} i^n$

22. If α and β are the roots of the quadratic equation $2x^2 - 7x + 13 = 0$, construct a quadratic equation whose roots are α^2 and β^2.

23. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 3$ and $dx = 0.02$.

24. Find the differential equation for the family of all straight lines passing through the origin.

25. For the random variable X with the given probability mass function.

$$f(x) = \begin{cases} 2(x-1) & 1 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the Mean.

26. Find the general equation of a circle with centre $(-3, -4)$ and radius 3 units.

27. Find the rank of the matrix $\begin{bmatrix} -1 & 3 \\ 4 & -7 \\ 3 & -4 \end{bmatrix}$.

28. Evaluate: $\int_0^{\frac{\pi}{2}} \sin^{10} x \, dx$

29. Evaluate: $\lim_{x\to 1} \frac{x^2-3x+2}{x^2-4x+3}$

30. Show that the vectors $2\vec{i} - \vec{j} + 3\vec{k}, \vec{i} - \vec{j}$ and $3\vec{i} - \vec{j} + 6\vec{k}$ are coplanar.

PART - III

31. Show that $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right) = \sec^{-1} x, |x| > 1$.

32. Find the equation of tangent and normal to the parabola $x^2 + 6x + 4y + 5 = 0$ at $(1, -3)$.

33. Prove that $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}] = 0$.

34. If $u(x, y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$.

35. Find two positive numbers whose sum is 12 and their product is maximum.

36. Evaluate: $\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \frac{1}{1+\sqrt{\tan x}} dx$

37. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

38. Solve : $(1 + x^2)\frac{dy}{dx} = 1 + y^2$

39. Three fair coins are tossed simultaneously. Find the probability mass function for number of heads occurred.

40. If $A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$, then find $|\text{adj}(\text{adj}A)|$.

PART – IV

41. **(a)** Find the angle between the curves $y = x^2$ and $y = (x - 3)^2$.

OR

(b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

42. **(a)** A six sided die is marked ' 1 ' on one face, ' 3 ' on two of its faces, and ' 5 ' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find :

(i) the probability mass function

(ii) the cumulative distribution function

(iii) $P(4 \leq X < 10)$

OR

(b) If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$.

43. (a) A conical water tank with vertex down of 12 meters height has a radius of 5 meters at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep ?

OR

(b) Prove by vector method that $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$.

44. **(a)** Assume that water issuing from the end of horizontal pipe, 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground ?

OR

(b) Solve the Linear differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$.

45. **(a)** The rate of increase in the number of bacteria in a certain bacteria culture is proportional to the number present. Given that the number triples in 5 hours, find how many bacteria will be present after 10 hours ?

OR

(b) Find the vector and Cartesian equations of the plane containing $\frac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{3}$ and parallel to the line $\frac{x+1}{3} = \frac{y-1}{2} = \frac{z+1}{1}$.

46. **(a)** Find the area of the region bounded by the ellipse

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

OR

(b) Find the vertex, focus and equation of the directrix of the parabola $y^2-4y-8x+12=0$.

47. (a) Show that

$p \leftrightarrow q \equiv ((\sim p) \vee q) \wedge ((\sim q) \vee p)$

OR

(b) Solve the system of linear equations by Cramer's Rule.

$$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0$$
$$\frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$$
$$\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$$