PART - I

1. (a)
2. (c)
3. (d)
4. (d)
5. (a)
6. (a)
7. (b)
8. (b)
9. (d)
10. (b)
11. (b)
12. (b)
13. (d)
14. (c)
15. (a)
16. (b)
17. (c)
18. (b)
19. (a)
20. (b)

PART - II

21. We know:

$$A^{-1} = \frac{\text{adj}A}{|A|}, \text{ and } |\text{adj}A| = |A|^2$$

(i) Compute $|\text{adj}A|$:

$|\text{adj}A| = 9 \Longrightarrow |A| = \sqrt{9} = 3$

(ii) Then:

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} -1/3 & 2/3 & 2/3 \\ 1/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \end{bmatrix}$$

22. Step (1): Write $1/z$ in standard form

$$\frac{1}{z} = \frac{1}{x+iy} \cdot \frac{x-iy}{x-iy} = \frac{x-iy}{x^2+y^2}$$

Step (2): Take the real part

$$\text{Re}\left(\frac{1}{z}\right) = \frac{x}{x^2+y^2}$$

23. We are asked:

$\arctan(-\sqrt{3})$

Step (1): Recall standard values

$$\tan\frac{\pi}{3} = \sqrt{3} \Longrightarrow \tan\left(-\frac{\pi}{3}\right) = -\sqrt{3}$$

Step (2): Use the principal value of arctan

- Principal value of $\arctan x$ is in $(-\pi/2, \pi/2)$

$-\pi/3 \in (-\pi/2, \pi/2)$

$$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$$

24. Find c if $y = 4x + c$ is tangent to the circle $x^2 + y^2 = 9$.

Step (1): Condition for tangency

For a line $y = mx + c$ and circle $x^2 + y^2 = r^2$, the tangency condition is:

$|c| = r\sqrt{1+m^2}$

Here:

$m = 4, r = 3$
$|c| = 3\sqrt{1+16} = 3\sqrt{17}$
Step (2): Find $\boldsymbol{c}$
$c = \pm 3\sqrt{17}$

25. For $f(x) = \frac{x^2-6x+7}{x+5}$, divide numerator by denominator:

$$x^2 - 6x + 7 \div (x+5) = x - 11 + \frac{62}{x+5}$$

- As $x \to \infty$, remainder $\frac{62}{x+5} \to 0$

Slant asymptote:
$y = x - 11$

26. Step (1): Replace (x, y) by (tx, ty)

$$F(tx, ty) = \frac{(tx)^2 + 5(tx)(ty) - 10(ty)^2}{3(tx) + 7(ty)}$$

$$= \frac{t^2(x^2 + 5xy - 10y^2)}{t(3x + 7y)}$$

$$F(tx, ty) = t \cdot \frac{x^2 + 5xy - 10y^2}{3x + 7y} = tF(x, y)$$

Step (2): Conclude
Since $F(tx, ty) = tF(x, y)$, degree $= 1$.

27. Step (1): Separate variables

$$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

Step (2): Integrate both sides

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$$

$\arcsin y = \arcsin x + C$
Step (3): Solve for y
$y = \sin(\arcsin x + C)$

28. Step (1): Use the total probability condition

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

$$\int_1^4 Cx^2 dx = 1$$

Step (2): Integrate

$$C\int_1^4 x^2 dx = C\left[\frac{x^3}{3}\right]_1^4 = C\left(\frac{64}{3} - \frac{1}{3}\right) = C \cdot \frac{63}{3} = 21C$$

$$21C = 1 \Longrightarrow C = \frac{1}{21}$$

29. Step (1): Use the conjugate root theorem
- If a polynomial has rational coefficients, then complex roots occur in conjugate pairs.
- So, if $i - 2$ is a root, then its conjugate $-i - 2$ is also a root.

Step (2): Form the polynomial
$(x - (i-2))(x - (-i-2)) = (x + 2 - i)(x + 2 + i)$
$= (x+2)^2 - i^2 = (x+2)^2 + 1$
$(x+2)^2 + 1 = x^2 + 4x + 5$
(polynomial of minimum degree):
$\boxed{x^2 + 4x + 5 = 0}$

30. $\int_0^{\pi} \sin x dx = \int_0^{\pi/2} \sin x dx + \int_{\pi/2}^{\pi} \sin x dx$

Substitute $x=\pi-t$ in the second integral:

$$\int_{\pi/2}^{\pi} \sin x dx = \int_{0}^{\pi/2} \sin t dt$$

$$\int_{0}^{\pi} \sin x dx = 2\int_{0}^{\pi/2} \sin x dx$$

PART - III

31. Step (1): Write in matrix form

$$\begin{bmatrix}2 & 5\\1 & 2\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}-2\\-3\end{bmatrix}$$

$$AX=B, A=\begin{bmatrix}2 & 5\\1 & 2\end{bmatrix}, B=\begin{bmatrix}-2\\-3\end{bmatrix}$$

Step (2): Find A^{-1} for 2×2 matrix

$$A^{-1}=\frac{1}{\det A}\begin{bmatrix}2 & -5\\-1 & 2\end{bmatrix}$$

$$\det A=(2)(2)-(5)(1)=4-5=-1$$

$$A^{-1}=\frac{1}{-1}\begin{bmatrix}2 & -5\\-1 & 2\end{bmatrix}=\begin{bmatrix}-2 & 5\\1 & -2\end{bmatrix}$$

Step (3): Solve $\mathrm{X}=A^{-1}\mathrm{B}$

$$X=\begin{bmatrix}-2 & 5\\1 & -2\end{bmatrix}\begin{bmatrix}-2\\-3\end{bmatrix}=\begin{bmatrix}(-2)(-2)+5(-3)\\(1)(-2)+(-2)(-3)\end{bmatrix}=\begin{bmatrix}4-15\\-2+6\end{bmatrix}=\begin{bmatrix}-11\\4\end{bmatrix}$$

32. Step (1): Use the triangle inequality

$$|z+(6+8i)|\le|z|+|6+8i|, |z+(6+8i)|\ge||6+8i|-|z||$$

Step (2): Compute $|6+8i|$

$$|6+8i|=\sqrt{6^2+8^2}=\sqrt{36+64}=\sqrt{100}=10$$

Step (3): Apply inequalities

$$|z+(6+8i)|\le|z|+|6+8i|=2+10=12$$

$$|z+(6+8i)|\ge||6+8i|-|z||=|10-2|=8$$

Step 4: Conclusion

$$8\le|z+6+8i|\le 12$$

33. $7x^3-43x^2=43x-7\Rightarrow 7x^3-43x^2-43x+7=0$

Factor by grouping:

$$(7x^3-43x^2)-(43x-7)=x^2(7x-43)-1(43x-7)=(7x-43)(x^2-1)$$

$$x^2-1=(x-1)(x+1)$$

$$x=-1,1,\frac{43}{7}$$

34. Step (1): Use the formula

$$\arctan a+\arctan b=\arctan\frac{a+b}{1-ab}\quad \text{if } ab<1$$

Here:

$$a=\frac{2}{11}, b=\frac{7}{24}$$

Step (2): Compute numerator and denominator

$$a+b=\frac{2}{11}+\frac{7}{24}=\frac{48+77}{264}=\frac{125}{264}$$

$$1-ab=1-\frac{2}{11}\cdot\frac{7}{24}=1-\frac{14}{264}=\frac{250}{264}$$

$$\frac{a+b}{1-ab}=\frac{125/264}{250/264}=\frac{125}{250}=\frac{1}{2}$$

Step (3): Conclude

$$\arctan\frac{2}{11}+\arctan\frac{7}{24}=\arctan\frac{1}{2}$$

35. Use linearity of scalar triple product $[,,] and the fact that [\mathrm{u},\mathrm{v},\mathrm{v}]=0$:

$$[\mathrm{a}+\mathrm{c},\mathrm{a}+\mathrm{b},\mathrm{a}+\mathrm{b}+\mathrm{c}]=[\mathrm{a},\mathrm{b},\mathrm{c}]$$

because all terms containing repeated vectors vanish.

36. $\frac{du}{dt}=\frac{\partial u}{\partial x}\frac{dx}{dt}+\frac{\partial u}{\partial y}\frac{dy}{dt}$

$\frac{\partial u}{\partial x}=2xy+3y^4, \frac{\partial u}{\partial y}=x^2+12xy^3$

$\frac{dx}{dt}=e^t, \frac{dy}{dt}=\cos t$

$x=e^t, y=\sin t \Longrightarrow \frac{du}{dt}=(2e^t\sin t+3\sin^4 t)e^t+(e^{2t}+12e^t\sin^3 t)\cos t$

$\frac{du}{dt}=2e^{2t}\sin t+3e^t\sin^4 t+e^{2t}\cos t+12e^t\sin^3 t\cos t$

37. Step (1): Use the formula

$$\int_0^{\pi/2}\frac{dx}{a+b\cos^2 x}=\frac{\pi}{2\sqrt{a(a+b)}}\ (a>0, a+b>0)$$

Here: $a=1, b=5$

$\sqrt{a(a+b)}=\sqrt{1\cdot(1+5)}=\sqrt{6}$

Step (2): Apply the formula

$I=\frac{\pi}{2\sqrt{6}}$

38. Step (1): List outcomes

- Prize 200 → probability $1/600$
- Prize 100 → probability $4/600=1/150$
- Prize 50 → probability $6/600=1/100$
- No prize → probability 589/600

Cost of ticket = 2 → profit = prize - 2

Step (2): Compute expected profit

$E(\text{ profit })=\frac{1}{600}(200-2)+\frac{4}{600}(100-2)+\frac{6}{600}(50-2)+\frac{589}{600}(-2)$

$E=\frac{198+392+288-1178}{600}=\frac{-300}{600}=-0.5$

39. For $f(x)=x^3+2x+1$, Taylor series about $x=2$:

$f(x)=(2)+f'(2)(x-2)+\frac{f''(2)}{2!}(x-2)^2+\frac{f'''(2)}{3!}(x-2)^3$

$f(2)=13, f'(2)=14, f''(2)=12, f'''(2)=6$

$f(x)=13+14(x-2)+6(x-2)^2+(x-2)^3$

40. Step (1): Write the operation explicitly

$\frac{3}{2}+m-\frac{3}{2}m+7=\frac{87}{10}$

$\frac{17}{2}+m-\frac{3}{2}m=\frac{87}{10}\left(\text{ since } \frac{3}{2}+7=\frac{17}{2}\right)$

Step (2): Simplify

$\frac{17}{2}+m-\frac{3}{2}m=\frac{17}{2}-\frac{1}{2}m=\frac{87}{10}$

$-\frac{1}{2}m=\frac{87}{10}-\frac{17}{2}=\frac{87}{10}-\frac{85}{10}=\frac{2}{10}=\frac{1}{5}$

$\text{m}=-\frac{2}{5}$

PART - IV

41. **(a)** Given:

$$\begin{cases}x_1-x_2=3\\2x_1+3x_2+4x_3=17\\x_2+2x_3=7\end{cases}$$

Determinant

$$\Delta = \begin{vmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{vmatrix} = 6 \neq 0$$

$\Delta_1 = 12, \Delta_2 = -6, \Delta_3 = 24$

By Cramer's rule:

$$x_1 = \frac{12}{6} = 2, x_2 = \frac{-6}{6} = -1, x_3 = \frac{24}{6} = 4$$

$\boxed{x_1 = 2, x_2 = -1, x_3 = 4}$

OR

(b) Given parametric curve:

$x = 7\cos t, y = 2\sin t$

$$\frac{dx}{dt} = -7\sin t, \frac{dy}{dt} = 2\cos t$$

Slope of tangent:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2\cos t}{-7\sin t} = -\frac{2}{7}\cot t$$

At point $(7\cos t, 2\sin t)$,

Equation of Tangent

$$y - 2\sin t = -\frac{2}{7}\cot t(x - 7\cos t)$$

Equation of Normal

Slope of normal $= \frac{7}{2}\tan t$

$$y - 2\sin t = \frac{7}{2}\tan t(x - 7\cos t)$$

42. (a) Given

$(z-1)^3 + 8 = 0$

$(z-1)^3 = -8 = -2^3$

Let $\omega \neq 1$ be a cube root of unity, so the cube roots of -1 are

$-1, -\omega, -\omega^2$

Hence,

$z - 1 = -2, -2\omega, -2\omega^2$

$z = -1, 1 - 2\omega, 1 - 2\omega^2$

$\boxed{\text{Roots are } -1, 1 - 2\omega, 1 - 2\omega^2}$

OR

(b) $y^2 = x$ and $y = x - 2 \Rightarrow x = y + 2$

Points of intersection:

$y^2 = y + 2 \Rightarrow y^2 - y - 2 = 0$

$(y-2)(y+1) = 0 \Rightarrow y = 2, -1$

Area between curves (w.r.t. y):

$$A = \int_{-1}^{2} [(y+2) - y^2]dy$$

$$A = \int_{-1}^{2} (-y^2 + y + 2)dy = \left[-\frac{y^3}{3} + \frac{y^2}{2} + 2y\right]_{-1}^{2}$$

$$A = \left(\frac{10}{3}\right) - \left(-\frac{7}{6}\right) = \frac{27}{6} = \frac{9}{2}$$

$\boxed{\text{Area } = \frac{9}{2} \text{ sq. units}}$

43. (a) Given

$$6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0, \text{ one root } x = \frac{1}{3}$$

Multiply by 3 and factor by grouping:

$(3x-1)(2x^3 - x^2 - 12x - 6) = 0$

Now factor the cubic:

$2x^3 - x^2 - 12x - 6 = x^2(2x-1) - 6(2x+1)$

Trying $x = -\frac{1}{2}$ works, so
$(2x+1)(x^2-6) = 0$
Roots:
$$x = \frac{1}{3}, -\frac{1}{2}, \sqrt{6}, -\sqrt{6}$$
$$\boxed{x = \frac{1}{3}, -\frac{1}{2}, \pm\sqrt{6}}$$

OR

(b) $(x^2 - 3y^2)dx + 2xydy = 0$
This is homogeneous. Put $y = vx \Rightarrow dy = vdx + xdv$.
$[x^2(1-3v^2)]dx + 2x(vx)(vdx + xdv) = 0$
$(1-v^2)dx + 2vxdv = 0$
$$\frac{dx}{x} = \frac{2vdv}{v^2-1}$$
Integrate:
$\ln x = \ln(v^2-1) + C$
$$x = C(v^2-1) = C\left(\frac{y^2}{x^2} - 1\right)$$
$$\boxed{\frac{y^2}{x^3} - \frac{1}{x} = C}$$

44. **(a)** Then the parabola has vertex at $(0,10)$ and opens downward.
So its equation is
$y = 10 - ax^2$
At the bottom, width $= 30$ m $\Rightarrow$ endpoints at $x = \pm 15, y = 0$
$$0 = 10 - a(15)^2 \Rightarrow a = \frac{10}{225} = \frac{2}{45}$$
Hence,
$$y = 10 - \frac{2}{45}x^2$$
At 6 m from the centre $(x = 6)$:
$$y = 10 - \frac{2}{45}(36) = 10 - \frac{72}{45} = 10 - \frac{8}{5} = \frac{42}{5}$$
$y = 8.4$ m

OR

(b) Let
$\vec{a} = \cos\alpha\hat{\imath} + \sin\alpha\hat{\jmath}, \vec{b} = \cos\beta\hat{\imath} + \sin\beta\hat{\jmath}$
Then
$|\vec{a}| = |\vec{b}| = 1$
Dot product:
$\vec{a}\cdot\vec{b} = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
But also,
$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos(\alpha-\beta) = \cos(\alpha-\beta)$
$\boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta}$

45. **(a)** Probability that a ship is sunk $= \frac{1}{9}$

$\Rightarrow$ Probability that a ship arrives safely $(p) = \frac{8}{9}, q = \frac{1}{9}$

(i) Exactly 3 out of 6 arrive safely
This is a binomial distribution:
$$P(X=3) = \binom{6}{3}p^3q^3$$
$$= \binom{6}{3}\left(\frac{8}{9}\right)^3\left(\frac{1}{9}\right)^3$$
$$= \frac{20\times 512}{9^6} = \boxed{\frac{10240}{531441}}$$

(ii) No ships arrive safely out of 4
That means all are sunk:

$$P(X=0)=q^4=\left(\frac{1}{9}\right)^4$$

$$=\boxed{\frac{1}{6561}}$$

OR

(b) Length of latus rectum

$$\frac{2b^2}{a}=6\Rightarrow b^2=3a$$

Also,

$$a^2=b^2+c^2=3a+4$$

$$a^2-3a-4=0\Rightarrow a=4$$

So,

$$b^2=12$$

Equation of ellipse:

$$\boxed{\frac{x^2}{16}+\frac{(y-1)^2}{12}=1}$$

46. (a) Direction vectors of given lines:

$$\vec{a}=2\hat{\imath}+3\hat{\jmath}+6\hat{k},\vec{b}=\hat{\imath}+\hat{\jmath}-\hat{k}$$

Point on plane: $(0,1,-5)$

Vector equation of plane

$$\boxed{\vec{r}=(\hat{\jmath}-5\hat{k})+\lambda(2\hat{\imath}+3\hat{\jmath}+6\hat{k})+\mu(\hat{\imath}+\hat{\jmath}-\hat{k})}$$

Cartesian equation

$$\vec{n}=\vec{a}\times\vec{b}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2 & 3 & 6\\ 1 & 1 & -1\end{vmatrix}=-9\hat{\imath}+8\hat{\jmath}-\hat{k}$$

$$\vec{n}\cdot(\vec{r}-\vec{r}_0)=0\Rightarrow -9x+8(y-1)-(z+5)=0$$

$$\boxed{9x-8y+z+13=0}$$

OR

(b) Let population P grow as

$$\frac{dP}{dt}=kP\Rightarrow P=P_0e^{kt}$$

Given it doubles in 50 years:

$$2P_0=P_0e^{50k}\Rightarrow e^{50k}=2\Rightarrow k=\frac{\ln 2}{50}$$

For triple population:

$$3P_0=P_0e^{kt}\Rightarrow e^{kt}=3\Rightarrow t=\frac{\ln 3}{k}$$

$$t=\frac{50\ln 3}{\ln 2}$$

$$\boxed{t\approx 79.3\text{ years}}$$

47. (a) For a cylinder inside cone, by similarity:

$$r=a\left(1-\frac{h}{b}\right)$$

Volume:

$$V=\pi r^2h=\pi a^2h\left(1-\frac{h}{b}\right)^2$$

Maximize V:

$$\frac{dV}{dh}=0\Rightarrow h=\frac{b}{3}$$

$$r=\frac{2a}{3}$$

$$V_{\max}=\pi\left(\frac{2a}{3}\right)^2\left(\frac{b}{3}\right)=\frac{4}{27}\pi a^2b$$

Volume of cone:

$V_c = \frac{1}{3}\pi a^2 b$

$\boxed{V_{\max} = \frac{4}{9}V_c}$

OR

(b) Truth Table

p	q	r	qVT	p^ (qvr)	pAq	pAr	(p^q)v (pAr)
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Since the columns $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are identical for all cases,

$\boxed{p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)}$