PART - I

1. Subtraction is not a binary operation in : -

(a) N (b) R
(c) Q (d) Z

2. Suppose that X takes on one of the values 0,1 and 2. If for some constant k, $2\,P(X=i)=kP(X=i-1)$ for $i=1,2$ and $P(X=0)=\frac{1}{7}$, then the value of k is :

(a) 3 (b) 1
(c) 4 (d) 2

3. If A is a non-singular matrix of order 3×3 and $|A|=5$ then $\left|A^{-1}\right|$ is :

(a) 5^2 (b) 5
(c) $\frac{1}{5^2}$ (d) $\frac{1}{5}$

4. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x=80t-16t^2$. The stone reaches the maximum height in time t seconds is given by :

(a) 3 (b) 2
(c) 3.5 (d) 2.5

5. The order and degree of the differential equation $\sqrt{\frac{dy}{dx}}-4\frac{dy}{dx}-7x=0$ are respectively :

(a) 1,2 (b) 2,1
(c) 2,2 (d) 1,1

6. If $A=\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ be such that $\lambda A^{-1}=A$, then λ is :

(a) 19 (b) 17
(c) 21 (d) 14

7. The slope at any point of a curve $y=f(x)$ is given by $\frac{dy}{dx}=3x^2$ and it passes through $(-1,1)$. Then the equation of the curve is :

(a) $y=3x^3+4$ (b) $y=x^3+2$
(c) $y=x^3+5$ (d) $y=3x^2+4$

8. The domain of the function defined by $f(x)=\sin^{-1}\sqrt{x-1}$ is :

(a) $[0,1]$ (b) $[1,2]$
(c) $[-1,0]$ (d) $[-1,1]$

9. If $u(x,y)=e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to :

(a) x^2u (b) $e^{x^2+y^2}$
(c) y^2u (d) $2xu$

10. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x-2\sin^2 x+1$ is :

(a) 1 (b) 2
(c) ∞ (d) 4

11. The square root of i are :

(a) $\pm\frac{1}{2}(1+i)$ (b) $\pm\frac{1}{\sqrt{2}}(1+i)$
(c) $\pm\frac{1}{2}(1-i)$ (d) $\pm\frac{1}{\sqrt{2}}(1-i)$

12. The value of $\sum_{n=1}^{13}\left(i^n+i^{n-1}\right)$ is:

(a) 1 (b) $1+i$
(c) 0 (d) i

13. If in 6 trials, X is a binomial variable which follows the relation $9P(X=4)=P(X=2)$, then the probability of success is:

(a) 0.375 (b) 0.125
(c) 0.75 (d) 0.25

14. The angle between the lines $\frac{x-2}{3}=\frac{y+1}{-2}, z=2$ and $\frac{x-1}{1}=\frac{2y+3}{3}=\frac{z+5}{2}$ is :

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$

(c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

15. The point of inflection of the curve $y=(x-1)^3$ is :

(a) (1,0) (b) (0,0)

(c) (1,1) (d) (0,1)

16. The value of $\int_0^{\frac{2}{3}} \frac{dx}{\sqrt{4-9x^2}}$ is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{6}$

(c) π (d) $\frac{\pi}{2}$

17. The volume of solid of revolution of the region bounded by $y^2=x(a-x)$ about x-axis is :

(a) $\frac{\pi a^3}{5}$ (b) πa^3

(c) $\frac{\pi a^3}{6}$ (d) $\frac{\pi a^3}{4}$

18. An ellipse has OB as semi minor axes, F and F′ its foci and the angle FBF′ is a right angle. Then the eccentricity of the ellipse is :

(a) $\frac{1}{4}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{2}$

19. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath}+\hat{\jmath}, \hat{\imath}+2\hat{\jmath}, \hat{\imath}+\hat{\jmath}+\pi\hat{k}$ is :

(a) π (b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{3}$

20. If $f(x)>0$ for all x and $g(x)=\log(f(x))$, then dg is :

(a) $\frac{1}{f(x)}dx$ (b) $\frac{1}{f(x)}f'(x)dx$

(c) $\frac{1}{x}\,dx$ (d) $\frac{1}{x}f(x)dx$

PART - II

21. If $\text{adj}A=\begin{bmatrix}-1 & 2 & 2\\ 1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$, find A^{-1}.

22. If $z=x+iy$, then find Re $\left(\frac{1}{z}\right)$ in rectangular form.

23. Find the value of $\tan^{-1}(-\sqrt{3})$.

24. If $y=4x+c$ is a tangent to the circle $x^2+y^2=9$, find c.

25. Find the slant (oblique) asymptote for the function $f(x)=\frac{x^2-6x+7}{x+5}$.

26. Show that $F(x,y)=\frac{x^2+5xy-10y^2}{3x+7y}$ is a homogeneous function of degree 1.

27. Solve $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

28. Find the constant C such that the function $f(x)=\begin{cases}Cx^2 & 1<x<4\\ 0 & \text{otherwise}\end{cases}$ is a density function of X .

29. Find a polynomial equation of minimum degree with rational coefficients having $i-2$ as a root.

30. If $f(x)=\sin x$, then prove that $\int_0^{\pi} f(x)dx=2\int_0^{\frac{\pi}{2}} f(x)dx$

PART - III

31. Solve the system of linear equations $2x+5y=-2, x+2y=-3$ by matrix inversion method.

32. If $|z|=2$ show that $8\le|z+6+8i|\le12$

33. Solve the equation $7x^3-43x^2=43x-7$

34. Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$

35. If $\vec{a},\vec{b},\vec{c}$ are three vectors, prove that $[\vec{a}+\vec{c},\vec{a}+\vec{b},\vec{a}+\vec{b}+\vec{c}]=[\vec{a},\vec{b},\vec{c}]$

36. If $u(x,y)=x^2y+3xy^4x=e^t$ and $y=\sin t$, find $\frac{du}{dt}$.

37. Evaluate $\int_0^{\frac{\pi}{2}}\frac{dx}{1+5\cos^2 x}$.

38. A lottery with 600 tickets gives one prize of 200, four prizes of 100 and six prizes of 50 If the ticket cost is 2, find the expected profit amount of a ticket.

39. Find the Taylor's series about $x=2$ for $f(x)=x^3+2x+1(-\infty<x<\infty)$

40. Let Q be the set of all Rational numbers. If * is a binary operation defined on Q as $a^{\star}b=a+b-ab+7$ and $\left(\frac{3}{2}\star m=\frac{87}{10}\right.$ then find the value of m.

PART – IV

41. **(a)** Solve, by Cramer's rule, the system of equations $x_1-x_2=3,2x_1+3x_2+4x_3=17.\ x_2+2x_3=7$

OR

(b) Find the equation of tangent and normal to the curve given by $x=7$ cost and $y=2\sin t, t\in$ R at any point on the curve.

42. **(a)** If $\omega\neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3+8=0$ are $-1,1-2\omega,1-2\omega^2$

OR

(b) Find the area of the region bounded by the parabola $y^2=x$ and the line $y=x-2$

43. **(a)** Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.

OR

(b) Solve $(x^2-3y^2)\mathrm{d}x+2xy\ \mathrm{d}y=0$.

44. **(a)** A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre, on either sides.

OR

(b) Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$.

45. **(a)** During war, 1 ship out of 9 was sunk on an average in making a certain voyage. What was the probability that:
(i) Exactly 3 out of a convoy of 6 ships would arrive safely ?
(ii) No ships arrive safely from a convoy of 4 ships.

OR

(b) Find the equation of the ellipse whose Foci are (2, 1), (-2, 1) and the length of the latus rectum is 6

46. **(a)** Find the non-parametric form of Vector equation, and the Cartesian equation of the plane passing through the point $(0,1,-5)$ and parallel to the straight lines.
$\vec{r}=(\hat{\imath}+2\hat{\jmath}-4\hat{k})+s(2\hat{\imath}+3\hat{\jmath}+6\hat{k})$ and $\vec{r}=(\hat{\imath}-3\hat{\jmath}+5\hat{k})+t(\hat{\imath}+\hat{\jmath}-\hat{k})$

OR

(b) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

47. **(a)** A hollow cone with base radius a cm and height b cm is placed on a table. Show that the volume of the largest cylinder that can be hidden underneath is $\frac{4}{9}$ times volume of the cone.

OR

(b) Using truth table, prove that $p\wedge(q\vee r)=(p\wedge q)\vee(p\wedge r)$.