Physics

1. **(c)** We have

$$p=\frac{F}{A}=\frac{F}{\pi R^2}$$

$$\text{So, } \frac{\Delta p}{p}\times 100=\frac{\Delta F}{F}\times 100+2\frac{\Delta R}{R}\times 100$$

$$=5+2(3)=5+6=11$$

$$\Rightarrow \frac{\Delta p}{p}\times 100=11\%$$

2. **(b)**

$$K_f+P_f=K_i+P_i$$

$$0-\frac{GMm}{R+h}=\frac{1}{2}m\left(\frac{V_e}{2}\right)^2-\frac{GMm}{R}$$

$$-\frac{GMm}{1}\left[-\frac{1}{R}+\frac{1}{R+h}\right]=\frac{1}{8}mV_e^2$$

$$-GMm\left[\frac{-h}{R(R+h)}\right]=\frac{1}{8}mV_e^2$$

$$-GM\left[-\frac{h}{R(R+h)}\right]=\frac{1}{8}\times\left(\sqrt{2R\cdot\frac{GM_e}{R^2}}\right)^2$$

$$\frac{h}{R^2+Rh}=\frac{1}{4R}\Rightarrow 4Rh=R^2+Rh$$

$\Rightarrow$ $R^2=3Rh$

$\Rightarrow$ $h=R/3$

3. **(d)** As Range $\propto \sin 2\theta$

So, $\frac{R-10}{R+15}=\frac{\sin 2\theta_1}{\sin 2\theta_2}=\frac{\sin 30^\circ}{\sin 90^\circ}=\frac{1}{2}$

$\Rightarrow 2R-20=R+15\Rightarrow R=35$

For the range to be maximum $\sin 2\theta=\sin 90^\circ=1$

So, $R_{max}=\frac{u^2}{g}$

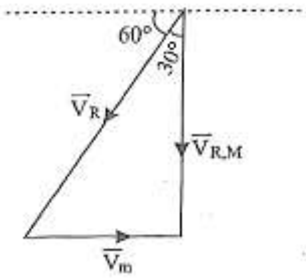
As $R_1=\frac{u^2\sin 2\theta}{g}\Rightarrow 50=\frac{u^2}{g}\Rightarrow u^2=50\text{ g}$

and $35=\frac{50g\times\sin 2\theta}{g}$ $[\because u=50g]$

$\therefore\ \sin 2\theta=\frac{35}{50}=\frac{7}{10}\Rightarrow 2\theta=\sin^{-1}\left[\frac{7}{10}\right]$

$\Rightarrow\ \theta=\frac{1}{2}\sin^{-1}\left[\frac{7}{10}\right]$

4. **(d)** Consider the situational diagram as shown below

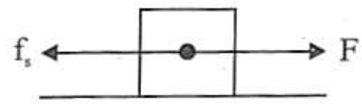


In $\triangle$ OAP, $\tan 30^\circ = \dfrac{v_m}{v_{rm}}$

$$\frac{1}{\sqrt{3}} = \frac{\text{Velocity of man}}{\text{Velocity of rain with respect to man}}$$

$$\Rightarrow v_{rm} = \frac{5}{1/\sqrt{3}} = 5\sqrt{3}\ \text{km/h}$$

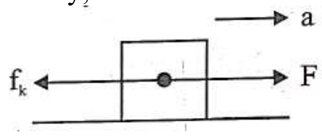
5. **(c)** Initially



$F = f_s = \mu_s N$

$= 0.4 \times 4 \times 10 = 32\ \text{N}$

Finally,



$F - f_k = ma$

$32 - \mu_k mg = ma$

$\Rightarrow 32 - 0.6 \times 40 = 4a$

$\Rightarrow 8 = 4a$

$\Rightarrow a = 2\ \text{m/s}^2$

6. **(b)** $|F| = F = \sqrt{4+1+1} = \sqrt{6}$

$v = \sqrt{16+4+4} = \sqrt{24}$

Now, $F = \frac{dp}{dt} = m\left(\frac{v-u}{t}\right)$

where, m = mass of body

$$\Rightarrow \sqrt{6} = m\left(\frac{\sqrt{24}-0}{20}\right) \Rightarrow m = \frac{\sqrt{6}\times 20}{\sqrt{24}} = 10\ \text{kg}$$

7. **(c)**

$P = w/t$

$w = p \times t$

$\Rightarrow (m+M)gh = 2000 \times 10$

$\Rightarrow (m+50)200 = 2000 \times 10$

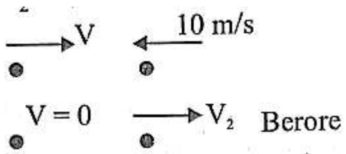
$\Rightarrow m + 50 = 100$

$\Rightarrow m = 50\ \text{kg}$

8. **(b)** From conservation of moment,

$m(v-10) = mv_2$

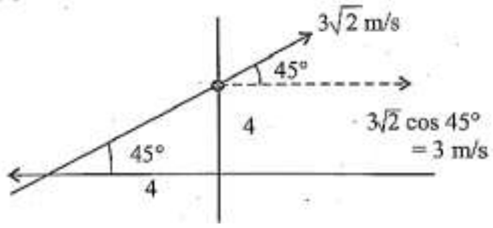
$v_2 = (v-10)$



The coefficient of restitution,

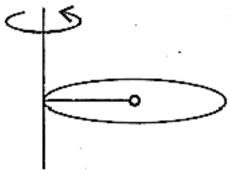
$$e = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$
$$= \frac{v_2 - v_1}{u_1 + u_2} = \frac{(v-10)-0}{(v+10)}$$
$$\Rightarrow 0.6 = \frac{v-10}{v+10}$$
$$\Rightarrow 0.6v + 6 = v - 10$$
$$\Rightarrow 0.4v = 16 \Rightarrow v = 40\ ms^{-1}$$

9. (b)



$$L = mV \perp r$$
$$= 5 \times 3 \times 4 = 60 \text{ units}$$

10. (c)



$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{3}{2}mr^2 4\pi^2 f^2$$
$$\Rightarrow I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$
$$\Rightarrow KE = 3mr^2 \times \pi^2 f^2 = 3 \times 5 \times 1 \times 10 \times 1$$
$$KE = 150\ J$$

11. (a)

$$K \cdot E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$
$$P.E = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2 x^2$$

Therefore, the ratio

$$\frac{KE}{PE} = \frac{\frac{1}{2}m\omega^2(A^2 - x^2)}{\frac{1}{2}m\omega^2 x^2} \Rightarrow \frac{KE}{PE} = \frac{A^2 - x^2}{x^2}$$
$$\Rightarrow \frac{10^2 - 4^2}{4^2} = \frac{84}{16}$$
$$\Rightarrow \frac{KE}{PE} = \frac{21}{4} = 5.25$$

12. (c) The relation between two velocities is given by the relation

$$v_e = (\sqrt{2} - 1)v_0$$
$$= (1.414 - 1)v_0$$
$$= 0.414 \times 10 = 4.14\ km/s$$

13. (c) We have, $F_1 \propto (l_1 - l)$,

where ℓ = original length

Similarly, $F_2 \propto (l_2 - l)$,

The ratio, $\frac{F_1}{F_2} = \frac{l_1 - l}{l_2 - l}$

Solving, we get

$$\Rightarrow l=\frac{F_2l_1-F_1l_2}{F_2-F_1}$$

14. (b) We have,

$$\frac{\Delta V}{V}=\frac{h\rho g}{B}\left[B=\frac{\Delta P}{\Delta V/V}\Rightarrow\frac{\Delta V}{V}=\frac{\Delta P}{B}=\frac{h\rho g}{B}\right]$$

$$\Rightarrow\frac{\Delta V}{V}=\frac{3\times10^3\times10^3\times9.8}{2.2\times10^9}=1.34\times10^{-2}$$

15. (c) $\frac{E_1}{E_2}=\frac{T_1^4-T_S^4}{T_2^4-T_S^4}[\because E=\sigma AT^4]$

$$\Rightarrow\frac{E_1}{E_2}=\frac{(600)^4-(300)^4}{(900)^4-(300)^4}=\frac{3}{16}$$

16. (d) As $C_V=12.6\ \mathrm{J\ mol^{-1}\ K^{-1}}$ is given in SI unit

So, $R=8314\ \mathrm{J\ mol^{-1}\ K^{-1}}$

Then, $C_p=C_V+R=12.6+8.314=21$

17. (b) From first law of thermodynamics,

$dQ=dU+dW$

$\Rightarrow dU=-dW$ [In adiabatic process $dQ=0$]

$dU=-4.5\ \mathrm{J}$

As, the temperature falls so internal energy will decrease by 4.5 J

18. (b) $\eta=\frac{1}{1+\alpha}$. This is the required relation.

19. (a) The average kinetic energy of molecules is given by

$KE_{av}=\frac{3}{2}KT$. So it depends only on temperature

$KE_1:KE_2=1:1$

20. (b) We have $y=2\sin(30t-40x)$

The standard transverse wave is

$y=A\sin(\omega t-kx)$

So, $\omega=30$ and $k=40$

On comparing, we get

$$v=\frac{\omega}{k}=\frac{30}{40}=0.75\ \mathrm{m/s}$$

21. (a) The fundamental frequency,

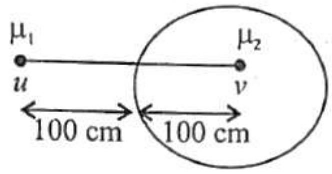
$$f=\frac{v}{4l}=\frac{1}{4e}\sqrt{\frac{\gamma RT}{M_0}}$$

$$\ell\propto\frac{1}{\sqrt{M}}\Rightarrow\frac{\ell_1}{\ell_2}=\sqrt{\frac{M_1}{M_2}}=\sqrt{\frac{2}{32}}=\frac{1}{4}$$

So, $\ell_1:\ell_2=1:4$

22. (a) The refraction formula is

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$



$\mu_2=1.5,\mu_1\ (\text{air})=1,u=-100\ \mathrm{cm},v=100\ \mathrm{cm}$

$$\Rightarrow\frac{1.5}{100}+\frac{1}{100}=\frac{1.5-1}{R}\Rightarrow\frac{25}{100}=\frac{0.5}{R}\Rightarrow R=20\ \mathrm{cm}$$

23. (c) $\frac{I_{max}}{I_{min}}=\left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2=\left(\frac{\sqrt{9}+\sqrt{4}}{\sqrt{9}-\sqrt{4}}\right)^2$

$= \left(\frac{3+2}{3-2}\right)^2 = 25$

24. (c)

$$E = \frac{1}{2}CV^2 \Rightarrow C \propto \frac{1}{d}$$

$$\Rightarrow \frac{C_1}{C_2} = \frac{d_1}{d_2} = \frac{2d}{d} = 2\left[\because C = \frac{A\epsilon_0}{d}\right]$$

Now, $\frac{E_1}{E_2} = \frac{C_1}{C_2} = \frac{C}{C/2} = \frac{2}{1}$

$\Rightarrow \frac{E}{E_2} = \frac{2}{1} \Rightarrow E_2 = \frac{E}{2}$

25. (d) When an amount of charge $= 10\mu C$ is added to both of the charges then, these becomes

$q_1 = -2\mu C$ and $q_2 = +2\mu C$

$F \propto |q_1 q_2|$

$\Rightarrow F_1 \propto |q_1 q_2|$ and $F_2 \propto |q_1^1 q_2^1|$

and $\frac{F_1}{F_2} = \frac{q_1 q_2}{q_1 q_2} \Rightarrow \frac{48}{F_2} = \frac{8 \times 12}{2 \times 2}$

$\Rightarrow F_2 = 2\text{ N}$

26. (a) The dielectric constant permittivity and susceptibility are relative as,

$K = 1 + \frac{\chi_e}{\varepsilon_0} \Rightarrow \frac{4}{3} = 1 + \frac{\chi_e}{\varepsilon_0}$

$$\Rightarrow \chi_e = \varepsilon_0\left(\frac{4}{3} - 1\right)$$

$$\Rightarrow \chi_e = \frac{\varepsilon_0}{3}$$

The electric susceptibility, $\chi_e = \frac{\varepsilon_0}{3}$

27. (b) $a = \frac{F}{m_e} = \frac{eE}{m_e}$

As, $S = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m_c}t^2$

28. (b) $v_d \propto \frac{1}{l}\frac{v_{d_1}}{v_{d_2}} = \frac{I_1}{I_2} = \frac{4I}{I} \Rightarrow v_{d_2} = \frac{v_{d_1}}{4}$

So drift velocity will decrease by 4 times

29. (b) (Initial part)

$\frac{x}{100} - x = \frac{2}{3} \Rightarrow x = 40\text{ cm}$

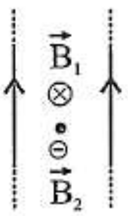
If there is shifting by 22.5 cm . Then, to obtain the balance point in meter bridge

$$\frac{2(3+x)}{3x} = \frac{62.5}{37.5} \Rightarrow (6+2x)37.5 = 62.5 \times 3x$$

$$\Rightarrow 225 + 75x = 187.5x$$

$$187.5x - 75x = 225 \Rightarrow 112.5x = 225 \Rightarrow x = 2$$

30. (c)



At the midpoint of two conductors, the magnetic induction. $B_1 = B_2 = B$ and $\vec{B}_1$ is anti $\|^r$ to $\vec{B}_2$.
Therefore, the net magnetic induction due to two wire at the mid points is

$B_N = B_1 - B_2 = 0$

31. (b) $eVB = eE$

where, $e =$ electronic charge and

$v =$ velocity of electron

$\Rightarrow\ v = \frac{E}{B} v = \frac{20 \times 10^3}{2.0} = 10 \times 10^3\ ms^{-1}$

32. (c)

Magnetic flux $\phi = BA = \mu HA$

$= \mu_0(1 + \chi_m)HA[\because \mu = \mu_0(1 + \chi_m)]$

$= 4\pi \times 10^{-3} \times (1 + 313) \times 1000 \times 0.25 \times 10^{-4}$

$= 9.87 \times 10^{-6}$ Wb

33. (a) $e = -L\frac{dI}{dt} \Rightarrow e = -30 \times 10^{-3} \times \left(\frac{2-6}{2}\right)$

$= +30 \times 10^{-3} \times \frac{4}{2} = +0.06$ V

34. (a) $P = E_{mms} V_{rms} \cos\theta = \frac{E_0 V_0}{2} \cos\phi$

So, $P = \frac{50 \times 50 \times 10^{-3} \times 1}{2 \times 2} = 0.625$ W

35. (c) We have total momentum as $\Delta p = \left(\frac{IA}{c}\right)t$ for non -reflecting surface,

$\Delta p = \left(\frac{9 \times 10^4 \times 20 \times 10^{-4}}{3 \times 10^8} \times 3600\right)$

$= 216 \times 10^{-3} kgms^{-1}$

36. (a) As, we know that de-Broglie wave lengths

$\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{m}$ So, $\frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e}$

37. (b)

$\frac{1}{\lambda_{BL}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36} R \Rightarrow \lambda_B = \frac{36}{5R}$ ... ... (A)

$\frac{1}{\lambda_{PL}} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{7}{144} R \Rightarrow \lambda_P = \frac{144}{7R}$ ... ... (B)

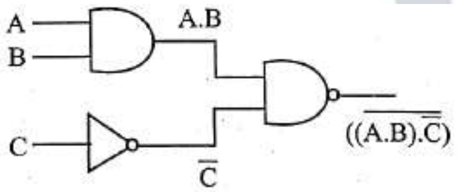
On dividing Eq. (B) by Eq. (A), we get

$\frac{\lambda_{BL}}{\lambda_{PL}} = \frac{7}{20}$

38. (d) Here $\frac{n}{p} \downarrow\downarrow$.

So it is β^-decay. So, x stands for antinutrino.

39. (d) The output of the circuit is $Y = \bar{A} + \bar{B} + C$



$Y = \overline{A \cdot B} + \bar{\bar{C}} = \bar{A} + \bar{B} + C$

If the output is zero,

i.e. $Y = 0$ then $A = 1$

$B = 1$ and $C = 0$

40. (c) We have, $m = \frac{A_m}{A_c} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}}$

Here, $E_{max} = 16$ V

and $E_{min} = 4$ V

$\Rightarrow m = \frac{16 - 4}{16 + 4}$

$= \frac{12}{20} = \frac{3}{5} = 0.6$

Chemistry

41. (c) For 3d-orbitals; $n = 3, \ell = 2$. m can take values from $-\ell$ to $+\ell$, i.e., -2 to +2 . s can take values $+\frac{1}{2}$ or $-\frac{1}{2}$

42. (d) $\lambda \propto \frac{1}{\sqrt{K.E.}} \Rightarrow \frac{\lambda_1}{\lambda_2} = \sqrt{\frac{(KE)_2}{(KE)_1}}$

$KE_2 = \frac{(KE)_1}{2}$

Thus, $\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{1}{2}} \Rightarrow \lambda_2 = \sqrt{2}\lambda_1$

43. (c) As we move from left to right within a period, the acidic character increases. As we move down a group, the acidic character decreases. Order of acidic strength:

$P_4O_{10} < N_2O_5 < SO_3 < Cl_2O_7$

44. (b)

45. (c) Nitric oxide (NO) has 11 electrons in its valence shells and its bond order is 2.5 . Hence, it does not contain triple bond between the atoms.

46. (a) Central I in I_3^-undergoes sp^3d hybridisation giving a linear shape with three lone pairs at equatorial positions. Beryllium chloride has linear structure with sp hybridisation of Be atom.

47. (d) As we go higher in altitude, the atmospheric pressure decreases. We know that, $P \propto T$. This results in decreasing the boiling point at higher altitude.

48. (b) When surface area decreases, less liquid will be exposed to air and therefore, less amount of water can evaporate in a given time period.

49. (c)

Element	% of element	Atomic weight	Relative number	Mole Ratio
Carbon	12.8	12	$\frac{12.8}{12} = 1.06$	$\frac{1.06}{1.06} = 1$
Hydrogen	2.1	1	$\frac{2.1}{1} = 2.1$	$\frac{2.1}{1.06} = 2$
Bromine	85.1	80	$\frac{85.1}{80} = 1.06$	$\frac{1.06}{1.06} = 1$

Empirical formula $= CH_2Br$

Empirical formula mass $= 12 + 2 + 80 = 94$

Factor, $n = \frac{\text{Molecular mass}}{\text{Empirical formula mass}}$

$= \frac{187.9}{94} = 2$

Molecular formula $= (CH_2Br)_2 = C_2H_4Br_2$

50. (a) From Avogadro's law,

Avogadro's number = Atomic mass of the element

3.011×10^{22} atoms of an element weigh $= 1.15$ g

6.023×10^{23} atoms weigh

$= \frac{1.15 \times 6.023 \times 10^{23}}{3.011 \times 10^{22}} = 23$

51. (b) $\Delta E = q + W$

For an adiabatic reaction,

$\Delta q = 0$

So, $\Delta E = \Delta W$

52. (b) For exothermic reaction, k_f decreases and k_b increases with increase of temperature so that K_{eq} decreases. For endothermic reaction, k_f increases and k_b decreases with. increase of temperature so that K_{eq} increases.

53. (d) Molecular mass of NaOH $= 23 + 16 + 1 = 40$

$$\text{Molarity} = \frac{\text{Mass of solute (in g)}}{\text{Molecular mass of the solute} \times \text{Volume of solution (in L)}}$$

$$= \frac{0.04 \times 1000}{40 \times 100} = 10^{-2}$$

$OH^- = 10^{-2}$ mol/L

So, $pOH = -\log[OH]$

$pOH + pH = 14$ or $pH = 14 - 2 = 12$

54. (d)

55. (d) Melting and boiling points of alkali metals are very low due to weak metallic bond.

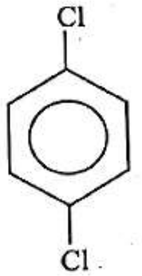
56. (a) Due to inert pair effect, +2 O.S. is more stable than +4 O.S. for lead.

57. (d) Boiling points of noble gases are low due to only acting weak van der Waal's forces of attraction between the atoms.

58. (d) Ocean absorb CO_2 via photochemical and biological process. (d) is also correct.

59. (a) Due to repulsion between two $-CH_3$ groups, the bond angle is more than the tetrahedral angle of $109°28'$.

60. (a) $\mu = 0$



1, 4-dichlorobenzene

61. (b) With cold dilute alkaline $KMnO_4$ (Baeyer's reagent), alkenes give addition products and the purple colour of $KMnO_4$ disappears.

62. (c) Doping of silicon with group 15 elements such as phosphorus give rise to n-type semiconductor.

63. (d) From Rault's law for non-volatile solute, we know that

$$\frac{p^\circ - p_s}{p^\circ} = \frac{n_2}{n_1 + n_2}$$

For dilute solution, relative lowering of vapour pressure,

$$n_1 + n_2 \cong n_1$$

$$n_2 = \frac{18}{180} = 0.1$$

$$n_1 = \frac{90}{18} = 5$$

$$\frac{p^\circ - p_s}{p^\circ} = \frac{0.1}{5} = 0.02$$

64. (c) From Henry's law, if we increase the pressure then the mole fraction of gas in the water will increase by the same multiple. Hence, mole fraction of gas = 0.04. Mole fraction of water = 0.96

65. (b) $E^\circ_{cell} = E^\circ_C - E^\circ_A = 0.80 - (-0.76) = 1.56$ V

From the equation $\Delta G^\circ = -nFE^\circ$

$$= -2 \times 96500 \times 1.56 (n = 2)$$

$$= -301080 \text{ J/mol}$$

$$= -301 \text{ kJ/mol}$$

66. (a) For the first order reaction,

$$t = \frac{2303}{k} \log \frac{a}{a - x}$$

Now putting the values

$$t_{90\%} = t = \frac{2303}{k}\log\left(\frac{100}{100-90}\right)$$
$$t_{99\%} = \frac{2303}{k}\log\left(\frac{100}{100-99}\right)$$
$$\frac{t}{t_{99\%}} = \frac{\frac{2303}{k}\log\left(\frac{100}{10}\right)}{\frac{2303}{k}\log\left(\frac{100}{1}\right)} = \frac{1}{2}$$
$$t_{99\%} = 2t$$

67. (d) Ferric hydroxide solution, is a negatively charged sol.

$KCl \rightleftarrows K^+ + Cl^-$
$KNO_3 \rightleftarrows K^+ + NO_3^-$
$K_2SO_4 \rightleftarrows 2\,K^+ + SO_4^{2-}$
$K_3[Fe(CN)_6] \rightleftarrows 3\,K^+ + [Fe(CN)_6]^{3-}$

Hence, $K_3[Fe(CN)_6]$ will be most effective in causing coagulation of ferric hydroxide solution.

68. (d) Levigation or gravity separation is the process of separating the light impure particles from the heavier ore particles by washing in current of water.

69. (a) Due to +I effect of three methyl groups, the correct basic order is: $P(CH_3)_3 > PH_3$

70. (b) It means that there is only one counter ion, i.e., the possible complex is $[Co(NH_3)_4Cl_2]Cl$. Hence, X is 4 .

71. (c) Let oxidation state of Fe = x

For $Fe(CO)_5, x + (5 \times 0) = 0$

$\Rightarrow x = 0$

72. (d)

73. (c) In place of nitrogen, helium is mixed with oxygen because helium is not soluble in blood even under high pressure.

74. (d)

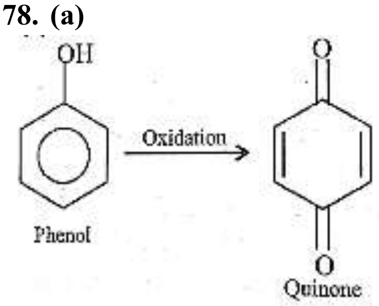
75. (a)

76. (d)

77. (b)

$$C_2H_6OH + 4I_2 + 3Na_2CO_3 \longrightarrow \underset{\text{Triiodomethane}}{CHI_3} + \underset{\text{Sodium formate}}{HCOONa} + 5NaI + 3CO_2 + 2H_2O$$

78. (a)



79. (a)

$$\underset{\text{Acetic acid}}{CH_3COOH} \xrightarrow{SOCl_2} \underset{\text{Acetyl chloride}}{CH_3COCl} \xrightarrow[pd]{H_2-BaSO_4} \underset{\text{Acetaldehyde}}{CH_3CHO}$$

80. (c) The sequence of reaction is

$$Ar_2 - NH_2 \underset{(X)}{\xrightarrow{NaNO_2+HCl}} Ar - \overset{+}{N} \equiv NCl^- \underset{(Y)}{\xrightarrow{Cu/HCl}} Ar - Cl + N_2$$

Mathematics

81. (b) Given, $f(x) = 5x - 3$ and $g(x) = x^2 + 3$

Let, $y = f(x), \therefore y = 5x - 3$

$y + 3 = 5x \Rightarrow x = \frac{y+3}{5}$

$\therefore \ f^{-1}(y) = \frac{y+3}{5} \Rightarrow f^{-1}(x) = \frac{x+3}{5}$

Now, $g(x) = x^2 + 3$;

So, $g \circ f^{-1}(3) = g[f^{-1}(3)]$

$= g\left(\frac{3+3}{5}\right) = g\left(\frac{6}{5}\right) = \frac{(6)^2}{(5)^2} + 3 = \frac{36}{25} + 3 = \frac{111}{25}$

82. (a) Given, $f(x) = \sin x - x$

Here, $\frac{\pi}{4} \leq x \leq \frac{\pi}{3}$

$f\left(\frac{\pi}{4}\right) \geq f(x) \geq f\left(\frac{\pi}{3}\right)$

($\because$ f(x) is decreasing function in $x \in \left[\frac{\pi}{4}, \frac{\pi}{3}\right]$)

$\sin\frac{\pi}{4} - \frac{\pi}{4} \geq f(x) \geq \sin\frac{\pi}{3} - \frac{\pi}{3}$

$\frac{1}{\sqrt{2}} - \frac{\pi}{4} \geq f(x) \geq \frac{\sqrt{3}}{2} - \frac{\pi}{3}$

$f(x) \in \left[\frac{\sqrt{3}}{2} - \frac{\pi}{3}, \frac{1}{\sqrt{2}} - \frac{\pi}{4}\right]$

83. (d) Let given series be

$S = 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + 3 \cdot 4 \cdot 5 + \cdots$ n terms

Now, general term

$T_n = \{1 + (n-1) \cdot 1\}\{2 + (n-1) \cdot 1\}\{3 + (n-1) \cdot 1\}$

$= n(n+1)(n+2) = n^3 + 3n^2 + 2n$

Now, Sum of the series $S = \Sigma T_n = \Sigma(n^3 + 3n^2 + 2n)$

$= \sum n^3 + 3\sum n^2 + 2\sum n$

$= \frac{n^2(n+1)^2}{4} + \frac{3n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$

$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + (2n+1) + 2\right]$

$= \frac{n(n+1)(n^2+5n+6)}{4} \Rightarrow S = \frac{n(n+1)(n+2)(n+3)}{4}$

84. (c) Let $D = \begin{vmatrix} b^2 - ab & b - c & bc - ac \\ ab - a^2 & a - b & b^2 - ab \\ bc - ac & c - a & ab - a^2 \end{vmatrix}$

$= \begin{vmatrix} b(b-a) & b-c & c(b-a) \\ a(b-a) & a-b & b(b-a) \\ c(b-a) & c-a & a(b-a) \end{vmatrix}$

$= (b-a)(b-a)\begin{vmatrix} b & b-c & c \\ a & a-b & b \\ c & c-a & a \end{vmatrix}$

$= (b-a)^2\begin{vmatrix} b & 0 & c \\ a & 0 & b \\ c & 0 & a \end{vmatrix} = 0$ [using $C_2 \to C_2 - (C_1 - C_3)$]

85. (c) Given matrix A is of order 3.

We know that,

$|\text{adj}(\text{adj} A)| = |A|^{(n-1)^2}$

Similarly, $|\text{adj}(\text{adj} A^2)| = |A^2|^{(3-1)^2}$ [$\because n = 3$]

$= |A^2|^{2^2} = |A^2|^4 = |A|^8$

86. **(d)** Given, $2x + 3y + z = 5$,

$3x + y + 5z = 7$

$x + 4y - 2z = 3$

The given system can be written as $AX = B$, where

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 3 & 1 & 5 \\ 1 & 4 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 & 1 \\ 3 & 1 & 5 \\ 1 & 4 & -2 \end{vmatrix}$$

$$= 2(-2 - 20) - 3(-6 - 5) + 1(12 - 1) \Rightarrow |A| = 0$$

As $|A| = 0$, this system can have no solution (or) infinitely many solutions. So we check $(\text{adj}\, A)B$.

$$\text{adj}\, A = \begin{bmatrix} -22 & 10 & 14 \\ 11 & -5 & -7 \\ 11 & -5 & -7 \end{bmatrix}$$

$$(\text{adj}\, A)B = \begin{bmatrix} -22 & 10 & 14 \\ 11 & -5 & -7 \\ 11 & -5 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -110 + 70 + 42 \\ 55 - 35 - 21 \\ 55 - 35 - 21 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence, there exist no solution.

87. **(d)** We have, $\sum_{k=1}^{6} \left[\sin\frac{2k\pi}{7} - i\cos\frac{2\pi k}{7}\right]$

$$= \sum_{k=1}^{6} \left[(-i)\left(\cos\frac{2k\pi}{7} + i\sin\frac{2\pi k}{7}\right)\right]$$

$$= (-i)\sum_{k=1}^{6} \left(\cos\frac{2k\pi}{7} + i\sin\frac{2\pi k}{7}\right) = (-i)\sum_{k=1}^{6} \alpha^k$$

Let $\alpha = \cos\frac{2\pi k}{7} + i\sin\frac{2\pi k}{7}$

$= (-i)(\alpha + \alpha^2 + \alpha^3 + \cdots + \alpha^5)$

Here, $\alpha + \alpha^2 + \alpha^3 + \cdots + \alpha^6$ follows G.P. so its sum.

$$S = \frac{\alpha(1 - \alpha^6)}{1 - \alpha} = \frac{\alpha - \alpha^7}{1 - \alpha} = \frac{\alpha - 1}{1 - \alpha} \quad = i$$

$[\because \alpha^7 = \cos 2\pi + i\sin 2\pi = 1]$

Thus, $\alpha = -i(-1) \Rightarrow \alpha = i$

88. **(b)** Here, $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \cdots \infty$

Follows infinite G.P. series so its sum,

$$S_\infty = \frac{a}{1 - r} = \frac{\frac{1}{3}}{1 - \frac{2}{3}} = \frac{1}{3} \times \frac{3}{1} = 1$$

$$\therefore \omega^{\left(\frac{1}{3}+\frac{2}{9}+\frac{4}{27}+\cdots\infty\right)} = \omega$$

Now, $\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \cdots .. + \infty$

also follows infinite G.P. series so its sum

$$S_\infty = \frac{a}{1 - r} = \frac{\frac{1}{2}}{1 - \frac{3}{4}} = \frac{1}{2} \times \frac{4}{1} = 2$$

$$\therefore \omega^{\left(\frac{1}{2}+\frac{3}{8}+\cdots+\infty\right)} = \omega^2$$

Hence, $\omega^{\left(\frac{1}{3}+\frac{2}{9}+\cdots+\infty\right)} + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\cdots+\infty\right)} = \omega + \omega^2 = -1$

89. **(a)** Given equation, $z^3 + 2z^2 + 2z + 1 = 0$

$(z+1)(z^2+z+1)=0$

Its roots are $-1, \omega$ and ω^2.

Let $f(z)=z^{2014}+z^{2015}+1=0$

Put $z=-1, \omega$ and ω^2 respectively, we get

$f(-1)=(-1)^{2014}+(-1)^{2015}+1=0=1\neq 0$

Therefore, -1 is not a root of the equation $f(z)=0$

$$\text{Again, } f(\omega)=(\omega)^{2014}+(\omega)^{2015}+1$$
$$=(\omega^3)^{671}\cdot\omega+(\omega^3)^{671}\cdot\omega^2+1=\omega+\omega^2+1=0$$

Therefore, ω is a root of the equation $f(z)=0$

$$\text{Similarly, } f(\omega^3)=(\omega^2)^{2014}+(\omega^3)^{2015}+1$$
$$=(\omega^3)^{1342}\cdot\omega^2+(\omega^3)^{1343}\cdot\omega+1=\omega^2+\omega+1=0$$

Hence ω and ω^2 are the common roots of

$z^{2014}+z^{2015}+1=0$

90. (a) Given that the roots of equation

$(b-c)x^2+(c-a)x+(a-b)=0$ are equal, so

$$D=0$$
$$(c-a)^2-4(a-b)(b-c)=0$$
$$c^2+a^2+2ac+4b^2-4b(c+a)=0$$
$$(c+a)^2+(2b)^2-2\cdot 2b(c+a)=0$$
$$[(c+a)-(2b)]^2=0$$
$$2b=a+c$$

Hence, we can conclude that a, b and c are in AP.

91. (b) Given equation, $x^3-kx^2+14x-8=0$

Roots of the above equation is in G.P. So, let $\frac{a}{r}$, a and ar are the roots of equation.

Then, product of roots

$$\frac{a}{r}\cdot a\cdot ar=\frac{D}{A}\Rightarrow\frac{a}{r}\cdot a\cdot ar=8\Rightarrow a^3=8\Rightarrow a=2$$

Therefore, at $a=2$ is the root of equation.

$$\therefore\ x^3-kx^2+14x-8=0$$
$$(2)^3-k(2)^2+14(2)-8=0$$
$$8-4k+28-8=0\Rightarrow k=7$$

92. (c) Given equation $\sqrt{2}x^2-bx+(8-2\sqrt{5})=0$

Let the roots of the given equation are α and β.

Sum of roots, $\alpha+\beta=\frac{-b}{a}=-\left(\frac{-b}{\sqrt{2}}\right)=\frac{b}{\sqrt{2}}$

and product of roots, $\alpha\beta=\frac{c}{a}=\frac{8-2\sqrt{5}}{\sqrt{2}}$

Now, as given in the question

$$\frac{2\alpha\beta}{\alpha+\beta}=4\Rightarrow\frac{2\left(\frac{8-2\sqrt{5}}{\sqrt{2}}\right)}{\frac{b}{\sqrt{2}}}=4$$

$$\Rightarrow\ 2\left(\frac{8-2\sqrt{5}}{\sqrt{2}}\right)=b\Rightarrow b=4-\sqrt{5}$$

93. (d)

$$\text{Let } y=\frac{x^2+2x+1}{x^2+2x-1}$$
$$yx^2+2xy-y=x^2+2x+1$$
$$(y-1)x^2+2(y-1)x-y-1=0$$

For real values of $x, b^2-4ac\geq 0$

$[2(y-1)]^2+4(y-1)(y+1)\geq 0$

$8y(y-1)\geq 0$

At $y=0, x^2+2x+1=0$

$(x+1)^2=0\Rightarrow x=-1\in R$

At $y=1, x^2+2x+1=x^2+2x-1$

$1\neq -1\Rightarrow y\neq 1$

$y\in(-\infty,0]\cup(1,\infty)$

94. (b) Given, T_m = Number of triangles formed with the vertices of a polygon of m sides.

Also, $T_{m+1}-T_m=15$

$\Rightarrow {}^{m+1}C_3-{}^{m}C_3=15 \quad \ldots\ldots(i)$

As we know,

$${}^{n}C_r+{}^{n}C_{r+1}={}^{n+1}C_{r+1}$$

$$\therefore\ {}^{m}C_2+{}^{m}C_3={}^{m+1}C_{2+1}$$

$${}^{m}C_2+{}^{m}C_3=15+{}^{m}C_3\Rightarrow {}^{m}C_2=15 \quad \text{[From eq. (i)]}$$

$$\frac{m!}{2!\,(m-2)!}=15\Rightarrow m(m-1)=30$$

$$(m-6)(m+5)=0\Rightarrow m=6-5$$

$$\therefore\ m=6 \quad [m\neq -5]$$

95. (b) $\frac{3x}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1} \quad \ldots\ldots.(i)$

$3x=A(x+1)+B(x-2)$

At $x=2$,

$$A=\frac{3(2)}{2+1}\Rightarrow A=\frac{3(2)}{3}\Rightarrow A=2$$

At $x=-1, B=\frac{3(-1)}{-1-2}\Rightarrow B=1$

Substitute the values of A and B in Eq. (i), we get

$$\frac{3x}{(x-2)(x+1)}=\frac{2}{x-2}+\frac{1}{x+1}$$

$$=\frac{2}{-2\left(1-\frac{x}{2}\right)}+\frac{1}{(x+1)}=-\left(1-\frac{x}{2}\right)^{-1}+(1+x)^{-1}$$

$$=-\left[1+\frac{x}{2}+\left(\frac{x}{2}\right)^2+\cdots+\left(\frac{x}{2}\right)^5+\cdots.\right]$$

$+(1-x+x^2-x^3+x^4-x^5+\cdots.)$

Coefficient of $x^5=-\left(\frac{1}{2}\right)^5-1=\frac{-1}{32}-1=-\frac{33}{32}$

96. (c) Given that coefficient of x^9, x^{10} and x^{11} in the expansion of $(1+x)^n$ are in A.P.

It means ${}^{n}C_9\cdot{}^{n}C_{10}\cdot{}^{n}C_{11}$ are in A.P.

$$\therefore\ 2{}^{n}C_{10}={}^{n}C_9+{}^{n}C_{11}\Rightarrow 2=\frac{{}^{n}C_9}{{}^{n}C_{10}}+\frac{{}^{n}C_{11}}{{}^{n}C_{10}}$$

$$\Rightarrow\ 2=\frac{n!}{9!\,(n-9)!}\times\frac{10!\,(n-10)!}{n!}+\frac{n!}{11!\,(n-11)!}\times\frac{10!\,(n-10)!}{n!}$$

$$\Rightarrow\ 2=\frac{10}{n-9}+\frac{n-10}{11}\ \therefore\ n^2-41n=-398$$

97. (b) We have given,

$\sin\theta+\cos\theta=p \quad \ldots..\text{(i)}$

and $\tan\theta+\cot\theta=q \quad \ldots\ldots.\text{(ii)}$

From Eq. (i)

$$(\sin\theta+\cos\theta)^2=p^2$$

$$\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=p^2$$

$$1+\sin 2\theta=p^2$$

$$\sin 2\theta=p^2-1 \quad \ldots\ldots\text{(iii)}$$

From Eq. (ii), $\tan\theta + \cot\theta = q$

$$\tan\theta + \frac{1}{\tan\theta} = q \Rightarrow \frac{\tan^2\theta + 1}{\tan\theta} = q$$

$$\frac{\tan^2\theta + 1}{2\tan\theta} = \frac{q}{2} \qquad \left[\because \operatorname{cosec} 2\theta = \frac{\tan^2\theta + 1}{2\tan\theta}\right]$$

$$\left[\because \operatorname{cosec} 2\theta = \frac{q}{2}\right.$$

$$\sin 2\theta = \frac{2}{q} \Rightarrow p^2 - 1 = \frac{2}{q} \qquad \text{[From eq. (iii)]}$$

$$q(p^2 - 1) = 2$$

98. **(a)** Given, $\tan\frac{\pi}{5} + 2\tan\frac{2\pi}{5} + 4\cot\frac{4\pi}{5}$

$$= \tan\frac{\pi}{5} + 2\left[2\cot\frac{4\pi}{5} + \tan\frac{2\pi}{5}\right]$$

We know that,

$$2\cot 2A + \tan A = \cot A \quad \ldots\ldots (i)$$

$$= \tan\frac{\pi}{5} + 2\cot\frac{2\pi}{5} = \cot\frac{\pi}{5} \qquad \text{[from Eq. (i)]}$$

99. **(d)** Here, we have the pattern of data in $\cos x$ and $\sin x$, so let's use the method of polar form of complex number.

Polar form of complex number,

$$z = \cos\theta + i\sin\theta \text{ then } \bar{z} = \cos\theta - i\sin\theta$$

$$\text{and } z\bar{z} = 1 \Rightarrow \bar{z} = \frac{1}{z}$$

Let $z_1 = \cos A + i\sin A$, $z_2 = \cos B + i\sin B$

and $z_3 = \cos C + i\sin C$

Then, $\bar{z}_1 = \cos A - i\sin A$

$$\bar{z}_2 = \cos B - i\sin B$$

$$z_3 = \cos C - \sin C$$

Now, to find $\cos(A+B) + \cos(B+C) + \cos(C+A)$

$$\bar{z}_1 + \bar{z}_2 + \bar{z}_3 = (\cos A - i\sin A)$$

$$+(\cos B - i\sin B) + (\cos C - i\sin C)$$

$$= (\cos A + \cos B + \cos C) - i(\sin A + \sin B + \sin C) = 0$$

$$\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} = 0 \quad \left[\because \bar{z} = \frac{1}{z}\right]$$

$$z_1 z_2 + z_2 z_3 + z_3 z_1 = 0$$

$$\Rightarrow \Sigma(\cos A + i\sin A)(\cos B + i\sin B) = 0$$

$$\Rightarrow \Sigma(\cos A\cos B - \sin A\sin B)$$

$$+ \; + i(\sin A\cos B + \cos A\sin B) = 0$$

$$\Rightarrow \sum \cos(A+B) + i\sin(A+B) = 0$$

Comparing the real part, we get

$$\Rightarrow \sum \cos(A+B) = 0$$

$$\cos(A+B) + \cos(B+C) + \cos(C+A) = 0$$

100. **(a)** We have given,

$$\tan\theta \cdot \tan(120^\circ - \theta)\tan(120^\circ + \theta) = \frac{1}{\sqrt{3}}$$

Since, we know

$$\tan\theta\tan(120^\circ - \theta)\tan(120^\circ + \theta) = \tan 3\theta$$

$$\therefore \tan 3\theta = \frac{1}{\sqrt{3}} \Rightarrow \tan 3\theta = \tan\frac{\pi}{6}$$

$$3\theta = n\pi + \frac{\pi}{6} \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{18}, n \in Z$$

101. **(b)** Given that, $\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^2}{4}-\cdots.\infty\right)$

$+\cos^{-1}\left(x-\frac{x^4}{2}+\frac{x^6}{4}-\cdots\infty\right)=\frac{\pi}{2}$

Here, $x-\frac{x^2}{2}+\frac{x^3}{4}-\cdots\infty$

Forms G.P. so its sum can be given as $\frac{x}{1-\frac{x}{2}}$

Similarly, for series

$$x^2-\frac{x^4}{2}+\frac{x^6}{4}-\cdots\infty \Rightarrow \text{ sum } =\frac{x^2}{1-\frac{x^2}{2}}$$

$$\sin^{-1}\left[\frac{x}{1-\frac{x}{2}}\right]+\cos^{-1}\left[\frac{x^2}{1-\frac{x^2}{2}}\right]=\frac{\pi}{2}$$

$$\sin^{-1}\left[\frac{2x}{2-x}\right]+\cos^{-1}\left[\frac{2x^2}{2-x^2}\right]=\frac{\pi}{2}$$

$$\frac{2x}{2-x}=\frac{2x^2}{2-x^2} \Rightarrow (2-x^2)=x(2-x)$$

$$2-x^2=2x-x^2 \Rightarrow 2x=2 \Rightarrow x=1$$

102. **(a)** $2\sinh^{-1}\left(\frac{a}{\sqrt{1-a^2}}\right)=\log\left(\frac{1+x}{1-x}\right)$ ….. (i)

As we know,

$\sin h^{-1} x=\log x+\sqrt{1+x^2}$

So, $2\sin\ h^{-1}x=2\log\left(x+\sqrt{1+x^2}\right)=\log\left(x+\sqrt{1+x^2}\right)^2$

$\Rightarrow 2\sin\ h^{-1}x=\log\left[2x^2+1+2x\sqrt{1+x^2}\right]$

Now, put $x=\frac{a}{\sqrt{1-a^2}}$

$$2\sinh^{-1}\left(\frac{a}{\sqrt{1-a^2}}\right)=\log\left[\frac{2a^2}{1-a^2}+1+2\times\frac{a}{\sqrt{1-a^2}}\times\sqrt{1+\frac{a^2}{1-a^2}}\right]$$

$$=\log\left[\frac{2a^2}{1-a^2}+1+\frac{2a}{1-a^2}\right]=\log\left[\frac{a^2+2a+1}{1-a^2}\right]$$

Now, by using Eq. (i),

$$\log\left[\frac{a^2+2a+1}{1-a^2}\right]=\log\left[\frac{1+x}{1-x}\right]$$

$$\frac{a^2+2a+1}{1-a^2}=\frac{1+x}{1-x}$$

[by using componendo and dividendo rule]

$$\frac{a^2+2a+1+1-a^2}{a^2+2a+1-1+a^2}=\frac{1+x+1-x}{1+x-1+x} \Rightarrow \frac{2+2a}{2a^2+2a}=\frac{1}{x}$$

$$\Rightarrow x=a$$

103. **(b)** Given that, $r_1=2r_2=3r_3$

$$\frac{\Delta}{s-a}=\frac{2\Delta}{s-b}=\frac{3\Delta}{s-c}=\frac{1}{K}$$

Then , $s-a=\Delta K$ ….. (i)

$\dot{s}-b=2\Delta K$ …….(ii)

and $s-c=3\Delta K$ …….(iii)

Now, adding Eqs. (i), (ii) and (iii), we get

$(s-a)+(s-b)+(s-c)=\Delta K+2\Delta K+3\Delta K$

$3s-(a+b+c)=6\Delta K$

$3s-2s=6\times\left(\frac{s-b}{2}\right)=3(s-b)$ [Using Eq. (ii)]

$s=3s-3b \Rightarrow 2s=3b$

104. **(a)** Given that, $a_1=2\hat{i}-\hat{j}+\hat{k}, a_2=3\hat{i}-4\hat{j}-4\hat{k}$,

$a_3=-\hat{i}+\hat{j}-\hat{k}, a_4=-\hat{i}+3\hat{j}+\hat{k}$

As M_1, M_2, M_3 and M_4 are the magnitudes of the vectors a_1, a_2, a_3 and a_4 respectively,

$\therefore M_1=|a_1|=\sqrt{(2)^2+(-1)^2+(1)^2}=\sqrt{6}$

$M_2=|a_2|=\sqrt{(3)^2+(-4)^2+(-4)^2}=\sqrt{41}$

$M_3=|a_3|=\sqrt{(-1)^2+(1)^2+(-1)^2}=\sqrt{3}$

$M_4=|a_4|=\sqrt{(-1)^2+(3)^2+(-1)^2}=\sqrt{11}$

Hence, the correct order of magnitudes

$M_3<M_1<M_4<M_2$

105. **(b)** Given that $\hat{a}, \hat{b}, \hat{c}$ are unit vectors and $\hat{a}+\hat{b}+\hat{c}=0$

$|\hat{a}|=|\hat{b}|=|\hat{c}|=1$

Here, $\hat{a}+\hat{b}+\hat{c}=\hat{0}$. Then, $(\hat{a}+\hat{b}+\hat{c})^2=(\hat{0})^2$

$\Rightarrow \hat{a}\cdot\hat{a}+\hat{a}\cdot\hat{b}+\hat{a}\cdot\hat{c}+\hat{b}\cdot\hat{a}+\hat{b}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}+\hat{c}\cdot\hat{b}+\hat{c}\cdot\hat{c}=0$

$\Rightarrow |\hat{a}|^2+\hat{a}\cdot\hat{b}+\hat{c}\cdot\hat{a}+\hat{a}\cdot\hat{b}+|\hat{b}|^2+\hat{b}\cdot\hat{c}$

$+\hat{c}\cdot\hat{a}+\hat{b}\cdot\hat{c}+|\hat{c}|^2=0$

$\Rightarrow (1)^2+2\hat{a}\cdot\hat{b}+2\hat{b}\cdot\hat{c}+2\hat{c}\cdot\hat{a}+(1)^2+(1)^2=0$

$\Rightarrow \hat{a}\cdot\hat{b}+\hat{b}\cdot\hat{c}+\hat{c}\cdot\hat{a}=-\frac{3}{2}$

106. **(d)** Three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are given as,

$\vec{a}=2\hat{i}+\hat{k}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}=4\hat{i}-3\hat{j}+7\hat{k}$

Given condition, $\vec{r}\times\vec{b}=\vec{c}\times\vec{b}$

$(\vec{r}-\vec{c})\times\vec{b}=0 \Rightarrow$ It means, $(\vec{r}-\vec{c}) \parallel \vec{b}$

So, $\vec{r}-\vec{c}=\lambda\vec{b} \Rightarrow \vec{r}=\vec{c}+\lambda\vec{b}$ ……(i)

Also given, $\vec{r}\cdot\vec{a}=0 \Rightarrow (\vec{c}+\lambda\vec{b})\cdot\vec{a}=0$ [using Eq. (i)]

$(4\hat{i}-3\hat{j}+7\hat{k}+\lambda\hat{i}+\lambda\hat{j}+\lambda\hat{k})\cdot(2\hat{i}+\hat{k})=0$

$[(4+\lambda)\cdot\hat{i}+(-3+\lambda)\hat{j}+(7+\lambda)\hat{k}]\cdot(2\hat{i}+\hat{k})=0$

$(4+\lambda)\cdot 2+(7+\lambda)\cdot 1=0 \Rightarrow \lambda=-5$

Put the value of λ in Eq. (i), we get

$\vec{r}=4\hat{i}-3\hat{j}+7\hat{k}-5(\hat{i}+\hat{j}+\hat{k})=-\hat{i}-8\hat{j}+2\hat{k}$

107. **(d)** For three vectors $\vec{a}, \vec{b}$ and $\vec{c}$

$|\vec{a}|=1, |\vec{b}|=2, |\vec{c}|=3$

and $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$

$$\because \left|\left[\vec{a} \quad \vec{b} \quad \vec{c}\right|^2=\begin{vmatrix} a\cdot a & a\cdot b & a\cdot c \\ b\cdot a & b\cdot b & b\cdot c \\ c\cdot a & c\cdot b & c\cdot c \end{vmatrix}\right.$$

$$=\begin{vmatrix} |\vec{a}|^2 & 0 & 0 \\ 0 & |\vec{b}|^2 & 0 \\ 0 & 0 & |\vec{c}|^2 \end{vmatrix} \quad [\because \vec{a}\cdot\vec{a}=|\vec{a}|^2]$$

$$=\begin{vmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 9 & 9 \end{vmatrix}=1(36-0)$$

$\Rightarrow |[\vec{a}\vec{b}\vec{c}]|^2=36 \Rightarrow |[\vec{a}\vec{b}\vec{c}]|=6$

108. **(b)** Given, $[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}]=\lambda[\vec{a}\vec{b}\vec{c}]^2$

As we know, the properties of cross product of three vectors, we have $[\vec{a}\times\vec{b}\vec{b}\times\vec{c}\vec{c}\times\vec{a}]=[\vec{a}\vec{b}\vec{c}]^2$

$\lambda = 1$

109. **(c)** Vector $\vec{b}$ and $\vec{c}$ are given as,

$b = \hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{c} = 3\hat{i} + 2\hat{j} - 5\hat{k}$

Cartesian equation of plane passing through the point

$(3, -2, -1)$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x - 3 & y + 2 & z + 1 \\ 1 & -2 & 4 \\ 3 & 2 & -5 \end{vmatrix} = 0$$

$(x - 3)(10 - 8) - (y + 2)(-5 - 12) + (z + 1)(2 + 6) = 0$

$2x + 17y + 8z + 36 = 0$

110. **(d)** Given observations are $10, 8, 5, a, b$.

Arithmetic Mean, A.M. $= \frac{10+8+5+a+b}{5} = 6$.

$\Rightarrow a + b = 7 \quad \ldots\ldots(i)$

Here,

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
10	4	16
8	2	4
5	-1	1
a	a - 6	$(a - 6)^2$
b	b - 6	$(b - 6)^2$

$\Sigma(x_i - \bar{x})^2 = a^2 + b^2 + 93 - 12(a + b)$

Variance, $\frac{1}{n}\Sigma(x_i - \bar{x})^2 = 6.8$

$$\frac{a^2 + b^2 + 93 - 12(a + b)}{5} = 6.8$$

$a^2 + b^2 + 93 - 84 = 34 \quad [a + b = 7]$

$a^2 + b^2 = 25 \quad \ldots..(ii)$

$\because (a + b)^2 = a^2 + b^2 + 2ab$

$(7)^2 = 25 + 2ab$ [using Eqs. (i) and (ii)]

$\Rightarrow ab = 12$

111. **(b)** Since, there are six observations and are written in ascending order then

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{observation} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

$$\frac{x - 2 + x}{2} = 16 \Rightarrow x = 17$$

So, the observations are 6,7,15,17,18 and 21.

$$\text{Mean} = \frac{6 + 7 + 15 + 17 + 18 + 21}{6} = 14 = \bar{x}$$

$\mathbf{x_i}$	$\mathbf{x_i - \bar{x}}$	$\mathbf{(x_i - \bar{x})^2}$
6	-8	64
7	-7	49
15	1	1
17	3	9
18	4	16
21	7	49

		$\Sigma(x_i-\bar{x})^2 = 188$

Variance $=\frac{1}{n}\Sigma(x_i-x)^2=\frac{188}{6}=\frac{94}{3}=31\frac{1}{3}$

112. **(a)** Here, probability of success, $p=\frac{1}{6}$ and probability of failure. $q=\frac{5}{6}$

Let A starts the game, then

$$A=\frac{1}{6}+\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^2+\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^4+\cdots.$$

$$P(A)=\frac{\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2}=\frac{\frac{1}{6}}{1-\frac{25}{36}}=\frac{\frac{1}{6}}{\frac{11}{36}}=\frac{6}{11}$$

Total probability, $P(A)+P(B)=1$

$$P(B)=1-P(A)=1-\frac{6}{11}=\frac{5}{11}$$

113. **(d)** The given word is 'QUESTION', let E be the event that there are exactly two letters between Q and S.

Here, number of sample space, $n(S)=8\,!$

number of possible events, $n(E)={}^6P_2\times 2!\times 5\,!$

Required probability

$$P(E)=\frac{n(E)}{n(S)}=\frac{{}^6P_2\times 2!\times 5!}{8!}=\frac{6!\times 2!\times 5!}{4!\times 8!}=\frac{5}{28}$$

114. **(a)** Given,

$$0\le\frac{1+3P}{3}\le 1 \text{ and } 0\le\frac{1-2P}{3}\le 1$$

$$\Rightarrow\ 0\le 1+3P\le 3 \text{ and } 0\le 1-2P\le 2$$

$$\Rightarrow\ -1\le 3P\le 2 \text{ and } -1\le -2P\le 1$$

$$\Rightarrow\ -\frac{1}{3}\le P\le\frac{2}{3} \text{ and } -\frac{1}{2}\le P\le\frac{1}{2}$$

$$\therefore\ P\in\left[-\frac{1}{3},\frac{1}{2}\right]$$

115. **(a)** Given, $p=0.2, q=0.8, n=3$

From binomial distribution,

$P(X=r)={}^nC_2P^rq^{n-r}$

Probability that event happens at most once,

$$P(X\le 1)=P(X=0)+P(X=1)$$

$$={}^3C_0(0.2)^0(0.8)^{3-0}+{}^3C_1(0.2)^1(0.8)^{3-1}$$

$$=\frac{3!}{0!\,3!}\times 1\times\left(\frac{8}{10}\right)^3+\frac{3!}{1!\,2!}\left(\frac{2}{10}\right)\left(\frac{8}{10}\right)^2$$

$$=\frac{64}{125}+\frac{48}{125}=\frac{112}{115}=0.896$$

116. **(c)** As we know, $\sum_{i=1}^{8}P(x_i)=1$

$0+K+2K+2K+3K+K^2+2K^2+7K^2+K=1$

$9K+10K^2=1\Rightarrow K=-1,\frac{1}{10}$

As the probability cannot be negative. So K must be greater than 0 .

$$\therefore K=\frac{1}{10}$$

$$P(0<x<5)=P(X=1)+P(X=2)+P(X=3)+P(X=4)$$

$$\Rightarrow K+2K+2K+3K=8K=\frac{8}{10}$$

117. **(b)** Let $P(x,y)$ be the required point whose locus is given by $ax+by+c=0$

Also given that P is equidistant from $A(-2,3)$ and $B(6,-5)$. Then,

$PA = PB$ or $PA^2 = PB^2$
$\Rightarrow (x+2)^2 + (y-3)^2 = (x-6)^2 + (y+5)^2$
$\Rightarrow x^2 + 4x + 4 + y^2 - 6y + 9$
$= x^2 - 12x + 36 + y^2 + 10y + 25 \Rightarrow x - y - 3 = 0$
On comparing with $ax + by + c = 0$, we get

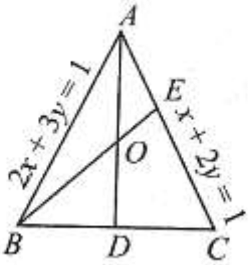
$a = 1, b = -1, c = -3$
Hence, the ascending order is c, b, a.

118. **(c)** Let $P(2,3)$ be the given point and Q be the reflection of point $P(2,3)$ about the line $y = x$. Then, the conordinates of Q are $(3,2)$.
Now, the point Q is translated through a distance of 2 units along the positive direction of X-axis. Let the new position of Q be R. Then, the coordinates of R are, $R = (3+2,2) = (5,2)$

119. **(c)** Here, point A is the intersection of line AB and AC so equation of line passing through A.

$(x + 2y - 1) + \lambda(2x + 3y - 1) = 0 \quad \ldots..(i)$
This line passes through the ortho centre $(0,0)$, then

$-1 - \lambda = 0 \Rightarrow \lambda = -1$
On substituting $\lambda = -1$ in Eq. (i), we get $x + y = 0$ as the equation of AD. Since $AD \perp BC$, therefore



$-1x - -= -1 \Rightarrow a + b = 0 \quad \ldots..(ii)$
Similarly, by applying the condition that BE is perpendicular to CA, we get
$a + 2b = 8 \quad \ldots...(iii)$
Now, solving Eqs. (ii) and (iii), we get $a = -8, b = 8$

120. **(b)** Let the required point be $P(x_1, y_1)$ and it is on the line $4x - y - 2 = 0$. Then, $4x_1 - y_1 - 2 = 0 \quad \ldots..(i)$
Also given, point P is equidistant from $A(-5,6)$ and $B(3,2)$.
$\therefore PA^2 = PB^2$
$(x_1 + 5)^2 + (y_1 - 6)^2 = (x_1 - 3)^2 + (y_1 - 2)^2$
$\Rightarrow 16x_1 - 8y_1 + 48 = 0 \Rightarrow 2x_1 - y_1 + 6 = 0 \quad \ldots\ldots(ii)$
On solving Eq. (i) in Eq. (ii), we get
$x_1 = 4$ and $y_1 = 14$

121. **(c)** Let these three lines be L_1, L_2 and L_3
$L_1 = x + 2ay + a = 0$
$L_2 = x + 3by + b = 0$
$L_3 = x + 2cy + c = 0$
If L_1, L_2 and L_3 are concurrent, then

$$\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$,

$$\begin{vmatrix} 0 & 2a-3b & a-b \\ 0 & 3b-4c & b-c \\ 1 & 4c & c \end{vmatrix} = 0$$

$\Rightarrow\ 1[(2a-3b)(b-c)-(3b-4c)(a-b)] = 0$

$\Rightarrow\ 2ab-2ca-3b^2+3bc = 3ab-3b^2-4ca+4bc$

$\Rightarrow\ ab+bc = 2ca \Rightarrow \frac{1}{a}+\frac{1}{c}=\frac{2}{b}$

Hence, a, b and c are in Harmonic progression.

122. **(a)** Given pair of lines, $ax^2-6xy+y^2=0$

Let the slope of one line be m, then slope of another line will be m^2. We know that

$\Rightarrow\ m+m^2 = \frac{-(-6)}{1} = 6 \quad \ldots\ldots(i)$

$\Rightarrow\ m\cdot m^2 = m^3 = \frac{a}{1} = a \quad \ldots\ldots(ii)$

On cubing Eq. (i) both sides, we get

$(m+m^2)^3 = (6)^3$

$\Rightarrow m^3+m^6+3m^3(m+m^2) = 216$

$\Rightarrow m^3+m^6+18m^3 = 216\ [\because m^3 = a]$

$\Rightarrow a+a^2+18a = 216 \Rightarrow a^2+19a-216=0$

$\therefore\ a = -27, 8$

123. **(d)** Let the given point be $P(4,-3)$ and the given circle is

$x^2+y^2+4x-10y-7=0$

Centre of circle $= C(-2,5)$

Radius $= \sqrt{(-2)^2+(5)^2+7} = 6$

Maximum distance, $a = CP + r$

Minimum distance, $b = CP - r$

Sum of the maximum and minimum distance,

$a+b = CP+r+CP-r = 2CP$

and $CP = \sqrt{(-2-4)^2+(5+3)^2} = 10$

Thus, $a+b = 2CP = 2\times 10 = 20$

124. **(b)** Let the circle be

$x^2+y^2+2gx+2fy+c=0 \quad \ldots\ldots(i)$

Cuts the circle

$x^2+y^2+4x-6y+9=0$ and

$x^2+y^2-5x+4y+2=0$ orthogonally.

For circle

$x^2+y^2+4x-6y+9=0$

$2(g_1g_2+f_1f_2) = c_1+c_2$

$2[(g)(-2)+(f)(3)] = c+9$

$-4g+6f = c+9 \quad \ldots\ldots(ii)$

For circle,

$x^2+y^2-5x+4y+2=0$

$2\left[(g)\left(\frac{5}{2}\right)+(f)(-2)\right] = c+2$

$5g-4f = c+2 \quad \ldots\ldots(iii)$

On subtracting Eq. (iii) from Eq. (ii), we get

$-9g+10f = 7 \Rightarrow 9g-10f = -7$

Replace g and f by x and y to get the locus of centre,

$9x-10y = -7 \Rightarrow 9x-10y+7=0$

125. **(a)** Given that circle, $S = x^2+y^2+y-1=0$

Line, $L: x-y+1=0$

Equation of the circle passing through the intersection of line and circle is given by

$S+\lambda L=0$

$(x^2+y^2+y-1)+\lambda(x-y+1)=0$

$x^2+y^2+\lambda x+(1-\lambda)y+\lambda-1=0$

Centre of above circle $=\left(\frac{-\lambda}{2},\frac{\lambda-1}{2}\right)$

Since, centre lies on $x-y+1=0$

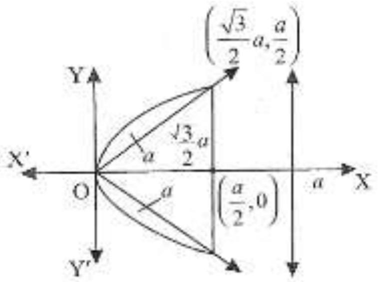
$$\frac{-\lambda}{2}-\left(\frac{\lambda-1}{2}\right)+1=0\Rightarrow\lambda=\frac{3}{2}$$

Now, the required equation of circle

$$x^2+y^2+y-1+\frac{3}{2}(x-y+1)=0$$

$\therefore\ 2(x^2+y^2)+3x-y+1=0$

126. **(b)** Let a be the length of the side of an equilateral triangle.



Then, from above figure, we can say that the point $\left(\frac{\sqrt{3}}{2}a,\frac{a}{2}\right)$ will lie on parabola $y^2=8x$.

So, $\left(\frac{a}{2}\right)^2=8\left(\frac{\sqrt{3}}{2}a\right)\Rightarrow a^2=16\sqrt{3}a\Rightarrow a=16\sqrt{3}$ units

127. **(a)** Given that focus of parabola is (3,4) and equation of directrix is $2x-3y+5=0$

As we know, Length of the latusrectum $=2\times$ Length of perpendicular from focus on directrix

$$=\left|\frac{2\times3-3\times4+5}{\sqrt{2^2+(-3)^2}}\right|=\frac{2}{\sqrt{13}}$$

128. **(b)** Equation of ellipse, $\frac{x^2}{16}+\frac{y^2}{9}=1$

$a^2=16\Rightarrow a=4, b^2=9\Rightarrow b=3$

Here, $a>b$

Now, $e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{9}{16}}=\frac{\sqrt{7}}{4}$

Focus $=(\pm ae,0)=(\pm\sqrt{7},0)$

Since, the circle passes through $P(\pm\sqrt{7},0)$ and centre at $C(0,3)$.

$\therefore$ Radius $=CP=\sqrt{(0-7)^2+(3-0)^2}$

$=\sqrt{7+9}=\sqrt{16}=4$

129. **(c)** As the straight line, $y=mx+4$ touches the circle

$x^2+4y^2=4$

Put $y=4x+m$ in $x^3+4y^2=4$, we get

$x^2+4(4x+m)^2=4$

$\Rightarrow\ x^2+4(16x^2+8mx+m^2)=4$

$\Rightarrow\ 65x^2+32mx+4(m^2-1)=0$

Since, line is tangent to the given curve

$\Rightarrow\ D=0\Rightarrow(32m)^2-4(65)[4(m^2-1)]=0$

$\Rightarrow m=\pm\sqrt{65}$

130. **(b)** Equation of ellipse, $\frac{x^2}{16}+\frac{y^2}{b^2}=1$

Eccentricity, $e=\sqrt{1-\frac{b^2}{16}}=\frac{\sqrt{16-b^2}}{4}$

So, the focus will be $\left(\pm\sqrt{16-b^2},0\right)$

Also, the equation of hyperbola,

$$\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}\Rightarrow \frac{x^2}{\left(\frac{144}{25}\right)}-\frac{y^2}{\left(\frac{81}{25}\right)}=1$$

$$\therefore\ e=\sqrt{1+\frac{b^2}{a^2}}=\sqrt{1+\frac{\frac{81}{25}}{\frac{144}{25}}}=\pm\frac{15}{12}$$

So, the focus will be $\left(\pm\frac{12}{5}\times\frac{15}{12},0\right)$ i.e., $(\pm3,0)$

On comparing the focus of ellipse with hyperbola, we get

$\sqrt{16-b^2}=3\Rightarrow b^2=7$

131. **(c)** Given direction ratios are $(2,-1,2)$ and $(K,3,5)$

Let θ be the angle between two lines, then

$$\cos\theta=\frac{(2,-1,2)(K,3,5)}{\sqrt{4+1+4}\sqrt{K^2+9+25}};\cos45^\circ=\frac{2-3+10}{\sqrt{9}\sqrt{34}}$$

$$\frac{1}{\sqrt{2}}=\frac{2K+7}{3\times\sqrt{K^2+9+25}}$$

$$K^2-56K+208=0\Rightarrow K=52,4$$

$\therefore$ According to the given options, $K=4$

132. **(a)** Here, intercepts made with the coordinate axes

X, Y and Z are $\frac{1}{3},\frac{1}{4}$ and $\frac{1}{5}$ respectively.

Intercept form of a plane is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

We have, $a=\frac{1}{3},b=\frac{1}{4},c=\frac{1}{5}$

$\therefore$ Equation of plane is

$3x+4y+5z-1=0$

Distance from origin

$$=\left|\frac{3\times0+4\times0+5\times0-1}{\sqrt{3^2+4^2+5^2}}\right|=\frac{1}{5\sqrt{2}}$$

133. **(a)** (A) Given coordinates of triangle are

$P(2,3,-1),Q(5,6,3),R(2,-3,1)$

$$\text{Centroid, } G=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$$

$$=\left(\frac{2+5+2}{3},\frac{3+6-3}{3},\frac{-1+3+1}{3}\right)=(3,2,1)$$

(B) Given coordinates of triangle are $P(1,2,3)$, $Q(2,3,1)$ and $R(3,1,2)$

Since, △ PQR is an equilateral triangle.

Circumcentre of the triangle,

$$C=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right)$$

$$=\left(\frac{1+2+3}{3},\frac{2+3+1}{3},\frac{3+1+2}{3}\right)=(2,2,2)$$

(C) Given coordinates of triangle are $P(2,1,5)$, $Q(3,2,3)$ and $R(4,0,4)$.

Since, △ PQR is an equilateral triangle.

$\therefore$ Orthocentre of the triangle,

$$O = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}, \frac{z_1+z_2+z_3}{3}\right)$$

$$= \left(\frac{2+3+4}{3}, \frac{1+2+0}{3}, \frac{5+3+4}{3}\right) = (3,1,4)$$

(D) Given coordinates of triangle are P(0,0,0), Q(3,0,0) and R(0,4,0).
Since, $\triangle$ PQR is a right angled triangle.
Here, $a = 5, b = 4, c = 3$
Incentre of the triangle,

$$I = \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}, \frac{az_1+bz_2+cz_3}{a+b+c}\right)$$

$$= \left(\frac{5\times 0+4\times 3+3\times 0}{3+4+5}, \frac{5\times 0+4\times 0+3\times 4}{3+4+5}, \frac{5\times 0+4\times 0+0\times 3}{3+4+5}\right) = (1,1,0)$$

134. **(c)** Given, $g(x) = \frac{x}{[x]}$

When, $x > 2$, then $[x] = 2 \Rightarrow g(x) = \frac{x}{2}$

Now, $\lim_{x\to 2}\frac{g(x)-g(2)}{x-2} = \lim_{x\to 2}\frac{\frac{x}{2}-1}{x-2} = \frac{1}{2}$

135. **(c)** Let $\ell = \lim_{x\to\frac{\pi}{2}}\left(\frac{2x-\pi}{\cos x}\right)$ $\left[\text{form } \frac{0}{0}\right]$

Use L' Hospital rule $= \lim_{x\to\pi/2}\frac{2}{-\sin x} = \frac{2}{-1} = -2$

136. **(b)** $f(x) = \begin{cases} x & \text{for } \quad 0 \le x < 1 \\ 2-x & \text{for } \quad x \ge 1 \end{cases}$

At $x = 1$

$\Rightarrow f(x) = 2 - x \Rightarrow f(1) = 2 - 1 = 1$

LHL $= \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x = 1$

RHL $= \lim_{x\to t^+} f(x) = \lim_{x\to 1^+}(2-x) = 2 - 1 = 1$

LHL $=$ RHL $= f(1)$

$f(x)$ is continuous at $x = 1$

$\Rightarrow$ To check the differentiability of $f(x)$,

$f'(x) = \begin{cases} 1 & \text{for } \quad 0 \le x < 1 \\ -1 & \text{for } \quad x \ge 1 \end{cases}$

LHD $= \lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-} 1 = 1$

RHD $= \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(-1) = -1$

LHD $\neq$ RHD

$f(x)$ is not differentiable at $x = 1$

137. **(b)** We have, ……(i)

$x^2 + y^2 = t + \frac{1}{t}$ ….. (ii)

and $x^4 + y^4 = t^2 + \frac{1}{t^2}$

From Eq. (i); squaring both the sides

$$\Rightarrow (x^2+y^2)^2 = \left(t + \frac{1}{t}\right)^2$$

$$\Rightarrow x^4 + y^4 + 2x^2y^2 = t^2 + \frac{1}{t^2} + 2$$

$\Rightarrow x^4 + y^4 + 2x^2y^2 = x^4 + y^4 + 2$ [using Eq. (ii)]

$\Rightarrow 2x^2y^2 = 2$

Differentiating w.r.t. x, we get

$$2xy^2 + 2x^2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2xy^2}{2x^2y} = -\frac{y}{x}$$

138. **(d)** We have, $x = at^2$

$$\Rightarrow \frac{dx}{dt} = 2at \Rightarrow \frac{dt}{dx} = \frac{1}{2at}$$

and $y = 2at \Rightarrow \frac{dy}{dt} = 2a$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$$

Differentiating w.r.t x, we get

Now, $\frac{d^2y}{dx^2} = \frac{-1}{t^2} \cdot \frac{dt}{dx} = \frac{-1}{t^2} \cdot \frac{1}{(2at)}$ $\left[\because \frac{dt}{dx} = \frac{1}{2at}\right]$

$$= \frac{-1}{2at^3}$$

$$\left(\frac{d^2y}{dx^2}\right)_{t=\frac{1}{2}} = \frac{-1}{2a \times \frac{1}{8}} = \frac{-4}{a}$$

139. **(b)** Let V be the volume, S be the surface area and r be the radius of the sphere.

It is given that, $\frac{dV}{dt} = 1200$ cu cm/s and $r = 10$ cm

Now, Volume of sphere $= V = \frac{4}{3}\pi r^3$

$$\therefore \frac{dV}{dt} = \frac{4}{3}\pi 3r^2\frac{dr}{dt}$$

$$1200 = 4\pi(10)^2\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{3}{\pi}$$

and Surface area of sphere,

$$S = 4\pi r^2 = \frac{dS}{dt} = 4\pi \cdot 2r\frac{dr}{dt}$$

$$\Rightarrow \frac{dS}{dt} = 4\pi \cdot 2(10)\frac{dr}{dt} = 4\pi \cdot 2 \times 10\left(\frac{3}{\pi}\right)$$

$$\frac{dS}{dt} = 240\text{sq cm/s}$$

140. **(c)** We have, $y = \int_0^x \frac{1}{1+t^3}dt$

Differentiating w.r.t. x, we get $\frac{dy}{dx} = \frac{1}{1+x^3}$

At $x = 1, \therefore \left(\frac{dy}{dx}\right)_{x=1} = \frac{1}{1+1} = \frac{1}{2}$

141. **(a)** Given $x^2 + y^2 = 25 \Rightarrow y = \sqrt{25 - x}$

Let $z = 3x + 4y$

$\Rightarrow z = 3x + 4\sqrt{25 - x^2}$

Differentiating w.r.t. x, we get

$$\frac{dz}{dx} = 3 + \frac{4}{2\sqrt{25 - x^2}}(-2x)$$

For maxima put $\frac{dz}{dx} = 0$

$$\Rightarrow 3 = \frac{4x}{\sqrt{25 - x^2}} \Rightarrow 3\sqrt{25 - x^2} = 4x$$

$$x^2 = 9 \Rightarrow x = 3$$

$$\Rightarrow y = \sqrt{25 - x^2} \Rightarrow y = 4$$

Now, $z = 3x + 4y$
$= 3(3) + 4(4) = 25$
So, $\log_5[\max(3x + 4y)] = \log_5[\max(z)]$
$= \log_5[(25)] = \log_5(5)^2 = 2\log_5 5 = 2$

142. **(c)** Let $I = \int \frac{dx}{(x-1)\sqrt{x^2-1}}$

Put $x - 1 = \frac{1}{t} \Rightarrow x = 1 + \frac{1}{t} \Rightarrow dx = \frac{-1}{t^2}dt$

$$\therefore\ I = \int \frac{-\frac{1}{t^2}dt}{\frac{1}{t}\sqrt{\left(1+\frac{1}{t}\right)^2 - 1}} = -\int \frac{dt}{t\sqrt{1 + \frac{1}{t^2} + \frac{2}{t} - 1}}$$

$$= -\int \frac{dt}{t\sqrt{\frac{1+2t}{t^2}}} = -\int \frac{dt}{\sqrt{1+2t}} = -\int (1+2t)^{-\frac{1}{2}}dt$$

$$= -\sqrt{1+2t} + C$$

$$= -\sqrt{1 + 2\left(\frac{1}{x-1}\right)} + C = -\sqrt{\frac{x+1}{x-1}} + C$$

143. **(c)** Let $I = \int e^x \frac{x^2+1}{(x+1)^2}dx = \int e^x\left(\frac{x^2-1+2}{(x+1)^2}\right)dx$

$$= \int e^x\left(\frac{x^2-1}{(x+1)^2} + \frac{2}{(x+1)^2}\right)dx$$

$$= \int e^x\left(\frac{x-1}{x+1} + \frac{2}{(x+1)^2}\right)dx$$

Put $f(x) = \frac{x-1}{x+1}$

$$f'(x) = \frac{(1)(x+1) - (x-1)(1)}{(x+1)^2} = \frac{2}{(x+1)^2}$$

As we know, $\int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C$

$$I = e^x\left(\frac{x-1}{x+1}\right) + C$$

144. **(b)**

$$\text{Let } I = \int \frac{x+1}{x(1+xe^x)}dx$$

Put $xe^x = t$

$$\Rightarrow\ (x+1)dx = \frac{dt}{e^x}$$

$$\Rightarrow\ I = \int \frac{dt}{xe^x(1+xe^x)} = \int \frac{1}{t(1+t)}dt = \int \frac{t+1-t}{t(t+1)}dt$$

$$\therefore\ I = \int \left(\frac{1}{t} - \frac{1}{t+1}\right)dt$$

$$= \log t - \log(t+1) + C = \log\left|\frac{t}{t+1}\right| + C$$

$$= \log\left|\frac{xe^x}{xe^x+1}\right| + C$$

145. **(b)**

Let

$$I = \int \frac{f(x)g'(x) - f'(x)g(x)}{f(x)g(x)} \times [(\log(x) - \log f(x)]dx$$

$$= \int \log\left[\frac{g(x)}{f(x)}\right] \cdot d\left\{\log\frac{g(x)}{f(x)}\right\}$$

Put $t = \log\left[\frac{g(x)}{f(x)}\right] \Rightarrow dt = d\left(\log\left[\frac{g(x)}{f(x)}\right]\right)$

Now, $I = \int t\,dt = \frac{t^2}{2} + C = \frac{1}{2}\left\{\log\frac{g(x)}{f(x)}\right\}^2 + C$

146. **(d)** Let $I = \int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx$

Put $\sin x - \cos x = t$

$(\sin x + \cos x)dx = dt$

$\sin 2x = 1 - (1 - \sin 2x)$

$= 1 - (\sin^2 x + \cos^2 x - 2\sin x\cos x)$

$= 1 - (\sin x - \cos x)^2 = 1 - t^2$

Now, $I = \int_{-1}^{0} \frac{dt}{3+1-t^2} = \int_{-1}^{0} \frac{dt}{(2)^2-(t)^2}$

$= \frac{1}{4}\left[\log\left(\frac{2+t}{2-t}\right)\right]_{-1}^{0} = -\frac{1}{4}\log\frac{1}{3} = \frac{1}{4}\log 3$

147. **(c)** $I = \int_{-1}^{1} \frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}} dx$

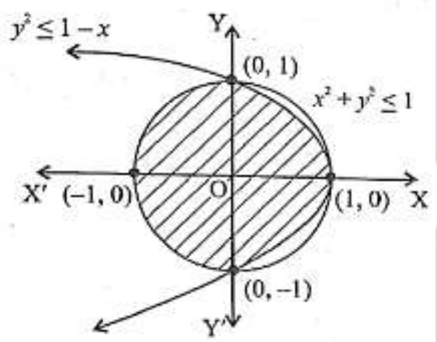
Let $f(x) = \frac{\sqrt{1+x+x^2}-\sqrt{1-x+x^2}}{\sqrt{1+x+x^2}+\sqrt{1-x+x^2}}$

Replacing x by $-x$, we get $\Rightarrow f(-x) = -f(x)$

So, $f(x)$ is an odd function.

$$\int_{-1}^{1} f(x)dx = 0$$

148. **(c)** Region: $\{(x, y) \mid x^2 + y^2 \leq 1 \text{ and } y^2 \leq 1 - x\}$



Required area

$$A = 2\left[\int_{-1}^{0} \sqrt{1-x}\,dx + \int_{0}^{1} \sqrt{1-x}\,dx\right]$$

$$= 2\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1} x\right]_{-1}^{0} + 2\left[\frac{-2}{3}(1-x)^{\frac{3}{2}}\right]_{0}^{1}$$

$$= \frac{\pi}{2} + \frac{4}{3}$$

149. **(a)** Given differential equation, $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$

Dividing the eqn. by e^y, we get

$\Rightarrow e^{-y} = \frac{dy}{dx} + \frac{1}{x}e^{-y} = \frac{1}{x^2}$ ......(i)

Let $e^{-y} = v$

$$\Rightarrow e^{-y}\frac{dy}{dx} = \frac{-dv}{dx}$$

From eqn. (i), we get

$$\frac{dv}{dx} - \frac{1}{x}v = \frac{-1}{x^2}$$

Integrating Factor,

$$IF = e^{\int -\frac{1}{x}dx} = e^{-\log x} = e^{\log\frac{1}{x}} = \frac{1}{x}$$

Now, the required solution is

$$v(IF) = \int -\frac{1}{x^2}(IF)dx + C_1$$
$$v \cdot \left(\frac{1}{x}\right) = \int \frac{-1}{x^2} \cdot \frac{1}{x} dx + C_1$$
$$\frac{v}{x} = -\int x^{-3} dx + C_1$$
$$\frac{v}{x} = \frac{1}{2x^2} + C_1 \Rightarrow \frac{v}{x} = \frac{1 + 2x^2 C_1}{2x^2}$$
$$2xv = 1 + Cx^2 \qquad [\because 2C_1 = C]$$
$$2xe^{-y} = 1 + Cx^2 \qquad [\because v = e^{-y}]$$
$$2x = (1 + Cx^2)e^y$$

150. **(b)** $\frac{dy}{dx} = \frac{1}{ax+by+c} \Rightarrow \left(\frac{dx}{dy}\right) = ax + by + c$

$$\frac{dx}{dy} - ax = by + c$$

Hence, this equation is a linear differential equation in x.

151. **(b)**

$$\left(\frac{1 + \cos\frac{\pi}{8} - i\sin\frac{\pi}{8}}{1 + \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}}\right)^8$$
$$= \left(\frac{2\cos^2\frac{\pi}{16} - 2i\sin\frac{\pi}{16}\cos\frac{\pi}{16}}{2\cos^2\frac{\pi}{16} + 2i\sin\frac{\pi}{16}\cos\frac{\pi}{16}}\right)^8$$
$$= \left(\frac{\cos\frac{\pi}{16} - i\sin\frac{\pi}{16}}{\cos\frac{\pi}{16} + i\sin\frac{\pi}{16}}\right)^8$$
$$= \left(\frac{e^{\frac{\pi}{16}i}}{e^{\frac{\pi}{16}i}}\right)^8 = \left(e^{-\frac{2\pi}{16}i}\right)^8$$
$$= \left(e^{-8\times\frac{2\pi}{16}i}\right) = e^{-\pi i} = -1$$

152. **(b)** Given digits are 0,2,4,5

Total number of four-digits numbers,

$\boxed{3 \quad 3 \quad 2 \quad 1} = 3 \times 3 \times 2 \times 1 = 18$

Total number of four-digit numbers that are divisible by 5 = Number which ends with 0 and 5.

Number ends with 0.

$3 \quad 2 \quad 1 \quad 1 = 3 \times 2 \times 1 \times 1 = 6$

Number ends with 5.

$\boxed{2 \quad 2 \quad 1 \quad 1} = 2 \times 2 \times 1 \times 1 = 4$

Total number of four digits numbers that are not divisible by 5 = $18 - (6 + 4) = 8$

153. **(b)** Given that,

$$x = \frac{1}{5} + \frac{1 \cdot 3}{5 \cdot 10} + \frac{1 \cdot 3 \cdot 5}{5 \cdot 10 \cdot 5} + \cdots$$
$$= \frac{1}{5} + \frac{1 \cdot 3}{2 \times 1}\left(\frac{1}{5}\right)^2 + \frac{1 \cdot 3 \cdot 5}{3 \times 2 \times 1}\left(\frac{1}{5}\right)^3 + \cdots$$
$$= \frac{1}{5} + \frac{1 \cdot 3}{2!}\left(\frac{1}{5}\right)^2 + \frac{1 \cdot 3 \cdot 5}{3!}\left(\frac{1}{5}\right)^3 + \cdots$$

On adding 1 both the sides, we get

$$1 + x = 1 + \frac{1}{5} + \frac{1 \cdot 3}{2!}\left(\frac{1}{5}\right)^2 + \frac{1 \cdot 3 \cdot 5}{3!}\left(\frac{1}{5}\right)^3 + \cdots$$

Now, According to the binomial theorem for any index,

$$\left(1-\frac{2}{5}\right)^{-1/2}=1+\frac{1}{5}+\frac{1\cdot 3}{2!}\left(\frac{1}{5}\right)^2+\frac{1\cdot 3\cdot 5}{3!}\left(\frac{1}{5}\right)^3+\cdots$$

$$1+x=\left(1-\frac{2}{5}\right)^{-1/2}\Rightarrow 1+x=\left(\frac{3}{5}\right)^{-1/2}$$

$$1+x=\left(\frac{5}{3}\right)^{1/2}\Rightarrow (1+x)^2=\frac{5}{3}$$

$$3x^2+6x=2$$

154. **(c)** We have

$$\lambda bc=[(b+c)-a][(b+c)+a]$$

$$\lambda bc=b^2+c^2+2bc-a^2$$

$$\Rightarrow \frac{b^2+c^2-a^2}{bc}=(\lambda-2)$$

$$2\left[\frac{b^2+c^2-a^2}{2bc}\right]=(\lambda-2)$$

$$\lambda-2=2\cos A\Rightarrow \cos A=\frac{\lambda}{2}-1$$

we know that $\cos A$ lies between -1 to 1 .

$$-1\le \cos A\le +1\Rightarrow 0\le\frac{\lambda}{2}\le 2$$

$$0\le\lambda\le 4$$

155. **(d)** Let $I=\frac{a}{\tan A}+\frac{b}{\tan B}+\frac{c}{\tan C}$ ......(i)

we know that

$$R=\frac{a}{2\sin A}=\frac{b}{2\sin B}=\frac{c}{2\sin C}$$

From (i)

$I=2R(\cos R+\cos B+\cos C)$

Now, $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Since, $\frac{r}{R}=4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

Hence, $\cos A+\cos B+\cos C=1+\frac{r}{R}$

$$\Rightarrow I=2R(\cos A+\cos B+\cos C)=2R\left(1+\frac{r}{R}\right)=2(R+r)$$

156. **(c)**

We have $(x^2+y^2)\sin^2\alpha=(x\cos\alpha-y\sin\alpha)^2$

$\Rightarrow (x^2+y^2)\sin^2\alpha=x^2\cos^2\alpha+y^2\sin^2\alpha-2xy\sin\alpha\cos\alpha.$

$\Rightarrow x^2\cos^2\alpha-x^2\sin^2\alpha-xy\sin 2\alpha=0$

$x(x\cos 2\alpha-y\sin 2\alpha)=0$

$$x=0, y=\frac{x\cos 2\alpha}{\sin 2\alpha}$$

Here, $m_1=0, m_2=\cot 2\alpha$

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|\Rightarrow \tan\theta=|-\cot 2\alpha|$$

$$\tan\theta=\cot 2\alpha\Rightarrow \tan\theta=\tan(90^\circ-2\alpha)$$

Required angle between two lines are 2α.

157. **(b)** Given circle passes through (2,0) änd (0,4).

For minimum radius AB should be the diameter and centre lies on AB.

Centre $(h,K)\Rightarrow h=\frac{2+0}{2}=1$ and $K=\frac{0+4}{2}=2$

$(h,K)\Rightarrow(1,2)$

radius of circle :

$$(x-h)^2+(y-K)^2=r^2$$

$$(2-1)^2+(0-2)^2=r^2\Rightarrow r=\sqrt{5}$$

Equation of circle : $(x-h)^2+(y-K)^2=r^2$

$(x-1)^2+(y-2)^2=5$
$x^2+y^2-2x-4y=0.$

158. **(a)** Condition for coaxial system of circles are :
$S+\lambda L=0 \quad ......(i)$
Where $L=x^2+y^2-4x-2y+5-(x^2+y^2-6x-4y-3)=0$
$\Rightarrow L=x+y+4=0$
From (i) $(x^2+y^2+4x-2y+5)+\lambda(x+y+4)=0$
$x^2+y^2+x-(\lambda-4)+y(y-2)+5+4\lambda=0$
Centre $(h,K) \Rightarrow \left(\frac{4-\lambda}{2},\frac{2-\lambda}{2}\right).$
so, $r=\sqrt{g^2+f^2-c}$ with $r=0$
$$\sqrt{\frac{16+\lambda^2-8\lambda+4+\lambda^2-4\lambda-20-16\lambda}{4}}=0$$
$\Rightarrow \lambda=0,\lambda=14$
Centre $(h,K) \Rightarrow \left(\frac{4-14}{2},\frac{2-14}{2}\right)=(-5,-6)$

159. **(c)** We have $f''(x)=-f(x) \quad ......(i)$
Now, $f(x)=\int g(x)dx+K.$
Differentiate both sides $f'(x)=g(x)$
Again differentiate w.r.t. x.
$f''(x)=g'(x) \quad .....(ii)$
From (i) & (ii)
$g'(x)=-f(x) \quad ......(iii)$
Now, $h(x)=\{f(x)\}^2+\{g(x)\}^2$
Differentiate w.r.t. x.
$h'(x)=2f(x)f'(x)+2g(x)g'(x)$
$h'(x)=2f(x)g(x)-2g(x)f(x) \Rightarrow h'(x)=0$
Integrate both sides,
$h(x)=C$
Put $x=0 \Rightarrow h(0)=5=C$
$h(x)=5$
So, $h(2015)-h(2014)=0$

160. **(c)** We have $f(x)=x^3+bx^2+ax$
$f'(x)=3x^3+2bx+a$
$f'(c)=0$
$3c^2+2bc+a=0$
If one root is $c=2+\frac{1}{\sqrt{3}}.=\alpha$ (say)
Then other root is $c=2-\frac{1}{\sqrt{3}}=\beta$ (say)
$$\text{So, } \alpha+\beta=-\frac{2b}{3}$$
$$\left(2+\frac{1}{\sqrt{3}}\right)+\left(2-\frac{1}{\sqrt{3}}\right)=\frac{-2b}{3}$$
$$\Rightarrow 4=\frac{-2b}{3} \Rightarrow b=-6 \Rightarrow \text{ Since } (\alpha\beta)=\frac{a}{3}$$
$$\left(2-\frac{1}{\sqrt{3}}\right)\left(2+\frac{1}{\sqrt{3}}\right)=\frac{a}{3} \Rightarrow a=11$$
It also satisfy $f(1)-f(3)=0$
$1+b+a-27-9b+3a=0$
$-8b-2a-26=0 \Rightarrow a+4b+13=0$
Putting $(a,b)=(11,-6)$
So, $11+4\times(-6)+13=0.$
Therefore, (a,b) is $(11,-6)$.