Physics

1. **(c)** (A) The value of Boltzmann constant $= 1.38 \times 10^{-23}$ kg $m^2 k^{-1}$

So dimensions of Boltzmann constant $= [ML^2\ T^{-2}\ K^{-1}]$

(B) From stokes' law, $F = 6\pi\eta v r$

Coefficient of viscosity $\eta = \frac{F}{6\pi v r}$

Coefficient of viscosity $= [ML^{-1}\ T^{-1}]$

(C) Water equivalent (W) is the mass of water which would absorb or evolve the same amount of heat as in done by the body in rising or falling through the same range of temperature.

Water equivalent $= [ML^0\ T^0]$

(D) Since $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$

Coefficient of thermal conductivity

$K = \frac{Q.l}{A(\theta_1 - \theta_2)t}$ so

Coefficient of thermal conductivity $= [MLT^{-3}\ K^{-1}]$

Coefficient of thermal

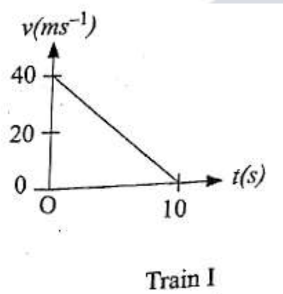
2. **(b)** Given initial distance between trains = 300 m.

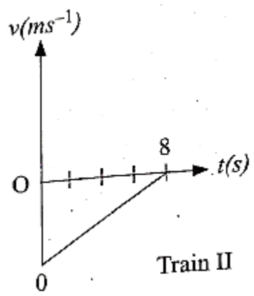
From given graph and as per question two trains are moving in opposite direction. So the separation between the trains when both have stopped

the trains when both

$$= 300 - \left(\frac{1}{2} \times 10 \times 40 + \frac{1}{2} \times 9 \times 20\right)$$

$$= 300 - 200 + 80 = 20 \text{ m}$$





Given θ is the angle between the total acceleration and tangential acceleration.

3. **(d)** $\tan\theta = \frac{a_{radial}}{a_{tangential}}$

$$= \frac{\frac{V^2}{R}}{a_{tangential}}$$

Givn $V = K\sqrt{s}$

$$\Rightarrow \tan\theta = \frac{1}{a_{tangential}}\left[\frac{1}{R} \times K^2 s\right]$$

$$\Rightarrow \tan\theta = \frac{1}{a_{tangential}}\left[\frac{K^2 s}{R}\right]$$

Now, $a_{tangential} = \frac{dv}{dt} = \frac{d}{dt} = [K\sqrt{s}]$

or $a_{tangential} = K \times \frac{1}{2\sqrt{s}} \times \frac{ds}{dt}$

or $a_{tangential} = \frac{K}{2\sqrt{s}} \times V$

$$= \frac{K}{2\sqrt{s}} \times K\sqrt{s} = \frac{K^2}{2}$$

$$\therefore \tan\theta = \frac{\frac{K^2 s}{R}}{\frac{K^2}{2}} \Rightarrow \tan\theta = \frac{2s}{R}$$

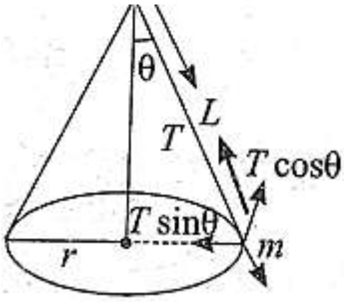
4. **(d)** Total time of flight (t) $= \frac{2u}{g}$

or, $9 = \frac{2u}{g}$ or, $u = \frac{9g}{2}$

or, $u = \frac{9\times10}{2}$ or, $u = 45\ m/s$

Since, in covering the vertical distance, g becomes - (ve) using $h = ut - \frac{1}{2}gt^2 = 45 \times (3) - \frac{1}{2} \times 10 \times (3)^2 =$ $135 - 45 = 90\ m$

5. **(a)** From the diagram,



$$\frac{T\sin\theta}{T\cos\theta} = \frac{F_{Centripetal}}{weight} = \frac{mv^2}{r} \times \frac{1}{mg}$$

$$\Rightarrow \tan\theta = \frac{v^2}{rg}$$

Also, $\tan\theta = \frac{t}{\sqrt{L^2-r^2}}$

$$\frac{v^2}{rg} = \frac{t}{\sqrt{L^2-r^2}}$$

or, $v = r\sqrt{\frac{g}{\sqrt{L^2-r^2}}}$

6. **(b)**

Using $h_{max} = \left(1 - \frac{1}{\sqrt{\mu^2+1}}\right)R$

$$= \left(1 - \frac{1}{\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}}\right) \times R \left(\because \mu = \frac{1}{\sqrt{3}}\right)$$

$$= \left(1 - \frac{1}{\sqrt{\frac{1}{3}+1}}\right) \times R = \left(1 - \frac{1}{\sqrt{\frac{1+3}{3}}}\right) \times R$$

or, $h_{max} = \left(1 - \frac{\sqrt{3}}{2}\right)R$

7. **(c)** Using $\Delta KE = \frac{m_1m_2}{2(m_1+m_2)}(1-e^2)(u_1+u_2)^2$

Given: $m_1 = 1$ kg

$m_2 = \frac{1}{2}$ kg;

$u_1 = 6\ ms^{-1}$; $u_2 = 9\ ms^{-1}$ and $e = \frac{1}{3}$

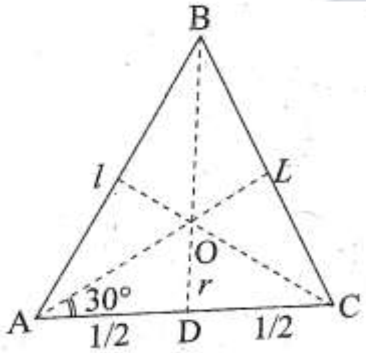
$$\Delta KE = \frac{1\times\frac{1}{2}}{2\left(1+\frac{1}{2}\right)}\left[1-\left(\frac{1}{3}\right)^2\right](6+9)^2$$
$$= \frac{1}{2}\times\frac{2}{6}\times\frac{8}{9}\times(15)^2$$
$$= \frac{1}{6}\times\frac{8}{9}\times 225$$
$$= 33.33\text{ J}$$

8. **(d)** As per question, the ball hits the ground loses $\frac{1}{3}$rd of its total mechanical energy and rebounds back to the same height.

i.e., $\frac{2}{3}\left(\frac{1}{2}mv^2+mgh\right) = mgh$

$$\Rightarrow \frac{2}{3}\left(\frac{1}{2}v^2+gh\right) = gh$$
$$\Rightarrow \frac{v^2}{3} = gh - \frac{2gh}{3}$$
$$\Rightarrow \frac{v^2}{3} = \frac{gh}{3} \Rightarrow v = \sqrt{gh}$$
$$v = \sqrt{400} = 20\text{m/s}$$

9. **(b)** Let the length of the rod be l.



The moment of inertia of one rod separately about an axis passing through the centre of the rod and perpendicular to its length, $I = I_{Ac} + mr^2$

$$= \frac{1}{12}ml^2 + m\left(\frac{l}{2\sqrt{3}}\right)^2$$
$$= \frac{ml^2}{12} + \frac{ml^2}{12} = \frac{2ml^2}{12}$$

Moment of inertia of the system of 3rods

$$= 3\times\frac{2ml^2}{12} = 6\times\frac{ml^2}{12} = 6l$$

According to the question $\frac{6ml^2}{12} = l = n.\left(\frac{ml^2}{12}\right) n = 6$

10. **(d)** If lower prism moves through a horizontal distance k since horizontal position of centre of mass of the two prims system remains unchanged

$$3mk = m[(a-b)-k]$$
$$\Rightarrow 3k = (a-b)$$
$$\Rightarrow 4k = a-b$$
$$\text{or } k = \frac{a-b}{4}$$

11. **(c)** As per question,

$|A\omega^2| - |A\omega| = 4$ [$\because$ A = 2m]

$2\omega^2 - 2\omega - 4 = 0$

$\Rightarrow \omega = 2\text{rad/s} \Rightarrow \frac{2\pi}{T} = 2$

$\Rightarrow T = \pi = \frac{22}{7}\text{s}$

Velocity, $v = \omega\sqrt{A^2 - y^2} = 2\sqrt{(2)^2 - (1)^2}$

$= 2\sqrt{4-1} = 2\sqrt{3}\ \text{ms}^{-1}$

12. (b) Rom the body of mass m, the distance of the point where gravitational field is zero

$$x = \frac{d}{\sqrt{\frac{m_2}{m_1}} + 1} \Rightarrow x = \frac{r}{\sqrt{\frac{9m}{m^-}} + 1} \quad \therefore x = \frac{1}{4}$$

Again $(r - x) = r - \frac{r}{4} = \frac{3r}{4}$

Potential, $v = v_1 + v_2$

$$= \frac{Gm_2}{X} - \frac{Gm_1}{r - x} = -\frac{Gm}{r/4} - \frac{G(9m)}{3r/4}$$

$$= -\frac{4Gm}{r} - \frac{12Gm}{r} = -\frac{16Gm}{r}$$

13. (d)

Natural length (l) $= \frac{l_1T_2 - l_2T_1}{T_2 - T_1}$

$$= \frac{101 \times 100 - 102 \times 80}{(100 - 80)}$$

$$= \frac{101 \times 100 - 102 \times 80}{20}$$

$$= \frac{101 \times 100}{20} - \frac{102 \times 80}{20}$$

$= 505 - 408 = 97$ cm

Again $\frac{T_1}{T_2} = \frac{4}{l_3 - 97}$

$\frac{80}{160} = \frac{4}{l_3 - 97}$ or, $l_3 - 97 = 8$

$l_3 = 105$ mm or 10.5 cm

14. (d) Let r be the radius of cavity

Volume of cavity $= \frac{4}{3}\pi r^3$

Now, $\frac{4}{3}\pi(R^3 - r^3)dmg = \frac{4}{3}\pi R^3\ d_w g$

$\Rightarrow 1 - \frac{R^3}{r^3} = \frac{1}{8} \Rightarrow \frac{R^3}{r^3} = 1 - \frac{1}{8}$

$\Rightarrow \frac{R^3}{r^3} = \frac{7}{8}$

$\Rightarrow r^3 = \frac{8}{7}R^3 = \frac{7}{8}(2)^3 = 7$

There Volume of cavity $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 7 = \frac{88}{3}$

15. (a) As per question $\frac{1}{4}$ th of heat is absorbed by the obstacle so, $\frac{3}{4}\left(\frac{1}{2}mv^2\right) = ms\Delta\theta + mL$

$\frac{3}{4}v^2 = 0.03 \times 4200 \times 300 + 6 \times 4200$

$\Rightarrow v^2 = \frac{8}{3}[0.01 \times 4200 \times 300 + 2 \times 4200]$

$\Rightarrow v^2 = 40 \times 4200$

$\Rightarrow v = \sqrt{168000} = 410 \text{ m/s}$

16. (c) Specific heat of water $= 1\text{calg}^{-10}\text{C}^{-1}$

Latent heat of fusion of ice $= 80\text{calg}^{-1}$

Latent heat of steam $= 540\text{calg}^{-1}$

$\text{m} \times 80 + \text{m} \times 1 \times \text{t} = (\text{M} - \text{m}) \times 1 \times (100 - \text{t}) + 540(\text{M} - \text{m})$

$\Rightarrow \text{m} \times 80 + \text{mt} = (\text{M} - \text{m}) \times 100 - (\text{M} - \text{m})\text{t} + 540\text{M} - \text{m} \times 540$

$\Rightarrow 80\text{m} + \text{mt} = \text{M} \times 100 - \text{m} \times 100 - \text{Mt} + \text{mt} + 540\text{M} - 540 \text{ m}$

$\Rightarrow 80\text{m} + 100\text{m} + 540\text{m} = 640\text{M} - \text{Mt}$

or, $720\text{m} = \text{M}(640 - \text{t})$

$\therefore \frac{\text{m}}{\text{M}} = \frac{640 - \text{t}}{720}$

17. (a)

For isobarically expansion, $\frac{\text{dW}}{\text{dQ}} = 1 - \frac{1}{\gamma}$

$\frac{300}{\text{dQ}} = 1 - \frac{1}{14}$ [$\because$ For diatomic gas $\gamma = 1.4$&$\text{dw} = 300$ J]

$\Rightarrow \frac{300}{\text{dQ}} = \frac{14-1}{14} = \frac{0.4}{14}$

$\text{dQ} = \frac{300 \times 14}{0.4} = 1050 \text{ J}$

18. (d) According to the question, initially

$\eta = \frac{100 - \text{T}_2}{100}$

$= \eta + \eta \times \frac{80}{100} = 1.8\eta = \frac{125 - \text{T}_2}{125}$

$\frac{\eta}{\eta} = \frac{100 - \text{T}_2}{100} \times \frac{125}{125 - \text{T}_2}$

$\Rightarrow \frac{1}{18} = \frac{100 - \text{T}_2}{100} \times \frac{125}{125 - \text{T}_2}$

or, $\frac{5}{9} = \frac{100 - \text{T}_2}{100} \times \frac{5}{(125 - \text{T}_2)}$

or, $9(100 - \text{T}_2) = 500 - 4 \text{ T}_2$

or, $900 - 9 \text{ T}_2 = 500 - 4 \text{ T}_2$

or, $400 = 5 \text{ T}_2$

or, $\text{T}_2 = \frac{400}{5} = 80$

The new efficiency,

$\eta'' = 1 - \frac{\text{T}_2}{\text{T}_1} = 1 - \frac{80}{125}$

$\frac{125 - 80}{125} = \frac{45}{125} = \frac{45}{125} \times 100 = 36\%$

19. (a) Amount of heat needed

$\text{dQ} = \eta \text{C}_\text{v} \text{dT} \eta \left(\frac{3}{2}\text{R}\right) \text{dt} \left[\because \text{Q} = \frac{3}{2}\text{R}\right]$

$= \frac{67.2}{22.4} \times \frac{3}{2} \times \text{R} \times \text{dT}$

$= 3 \times \frac{831 \times 3}{2} \times 20 = 784 \text{ J}$

20. **(b)** For closed organ pipe only odd harmonies are possible.
Fundamental frequency
Fundamental frequency for a closed organ pipe

$$v_0 = \frac{v_0}{4l} \Rightarrow 85 = \frac{340}{4 \times l}$$

$$\left[\because v_0 = \frac{595 - 425}{2} = \frac{765 - 595}{2} = 85 \text{ Hz}\right]$$

$$l = \frac{340}{4 \times 85} = 1 \text{ m}$$

21. **(d)** From Doppler's effect, apparent frequency,

$$n' = \left(\frac{v + v_0}{v - v_0}\right) n$$

or, $n' = \left(\frac{340+2}{340-2}\right) \times 170$

or, $n' = \frac{342}{338} \times 170$

or $n' = 172.01 = 172$ Hz

Therefore, beat frequency observed

$(172 - 170) = 2$ Hz

22. **(d)** Using mirror formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $\frac{-1}{f} = \frac{-1}{u} + \frac{1}{v}$ [∴ u& f negative for concave mirror]

$$\Rightarrow \frac{1}{v} = \frac{1}{u} - \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{f - u}{uf}$$

$$v = \frac{uf}{u - f}$$

Length of image,

$$L = |v| - |f| = v = \frac{uf}{u - f} - f$$

$$L = \frac{uf - f(u - f)}{u - f}$$

$$= \frac{uf - uf + f^2}{u - f} = \frac{f^2}{u - f}$$

23. **(b)** Fringe width, $\beta_G = n\frac{\lambda D}{d}$

And for n number of fringes,

$$\beta_G = n\frac{\lambda D}{d} \text{ or, } \beta_R = n\frac{\lambda D}{d}$$

For $(n + 1)$ th number of fringes,

$$\beta_G = (n + 1)\frac{\lambda D}{d}$$

$$\because \beta_R = \beta_G$$

$$\text{or, } n\frac{\lambda_R D}{d} = (n + 1)\frac{\lambda_R D}{d}$$

$$n.6000 \times 10^{-10}\frac{D}{d} = (n + 1)5000 \times 10^{-10}\frac{D}{d} \text{ or,}$$

$$\Rightarrow 6n = (n + 1)5$$

$$\Rightarrow 6n = 5n + 5 \text{ or, } 6n - 5n = 5$$

$$n = 5$$

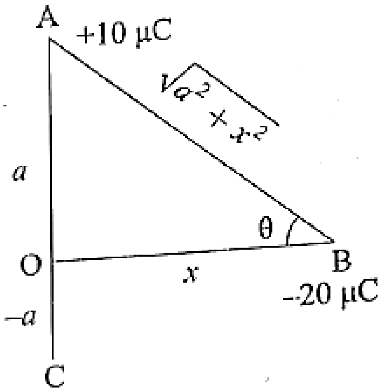
24. **(c)** If force acting on each charge $+10\mu C$ be F and x be the displaced position

∴ Net force $F_{net} = F\cos\theta + F\cos\theta$

$= 2\,F\cos\theta$

From coulomb's law,

$F = k\frac{q_1 q_2}{r^2}$



$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9 Nm^2/C^2$$

$$F = 9\times10^9 \times \frac{10\times10^{-6}\times20\times10^{-6}}{\left(\sqrt{a^2+x^2}\right)^2}$$

$$= 9\times10^9 \times \frac{200\times10^{-12}}{(a^2+x^2)}$$

$$F_{net} = 2\,F.\cos\theta = 2\times10\times10^9\times\frac{200\times10^{-12}}{(a^2+x^2)}\cos\theta$$

$$F_{net} = 2\times9\times\frac{2\times10^{-1}}{(a^2+x^2)}\left(\frac{x}{\sqrt{a^2+x^2}}\right)$$

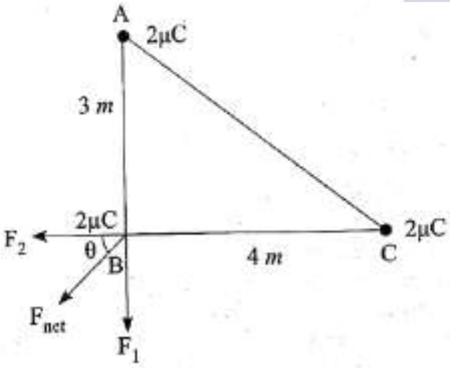
$$\left(\because \cos\theta = \frac{x}{\sqrt{a^2+x^2}}\right)$$

$$= 36\times10^{-1}\frac{x}{(a^2+x^2)^{3/2}}$$

Given, x << a so, neglecting x^2

$$F_{net} = 36\frac{x}{(a^2)^{3/2}} \approx \frac{36x}{a^3}\ N$$

25. (c) From figure the net force F_{net} due to F_1 and F_2 makes an angle θ with force F_2



$$\tan\theta = \frac{F_1}{F_2}$$

Also, $F_1 = k \cdot \frac{q_1 q_2}{(3)^2}$

$\because q_1 = q_2 = q_3 = 2\mu C$

$\Rightarrow F_2 = k \cdot \frac{q_1 q_3}{(4)^3}$

$\therefore \tan\theta = k \cdot \frac{q_1 q_2/(3)^2}{q_1 q_3/(4)^2} = \frac{(4)^2}{(3)^2} = \frac{16}{9}$

$\therefore \theta = \tan^{-1}\left(\frac{16}{9}\right)$

26. (d) Potential $v = \frac{kq}{r}$

For 30 V equipotential surface,

$30 = \frac{kq}{t}$, here $r = 20\text{ cm} = 20 \times 10^{-2}\text{ m}$

$30 = \frac{kq}{20 \times 10^{-2}}$

$\therefore kq = 60 \times 10 \times 10^{-2} = 6$

Therefore, electric field at distance r from charge q.

$E = \frac{kq}{r^2} = \frac{6}{r^2}\text{Vm}^{-1}$

27. (d) Using $E = \frac{dV}{dr}$ or, $dV = Edr$

Displacement, $dr = r_B - r_A$

$= (2\hat{i} + \hat{j} + 3\hat{k}) - (\hat{i} + 2\hat{j})$

$= (10\hat{i} + 30\hat{j})(\hat{i} - \hat{j} + 3\hat{k})$

$= 10\hat{i} \cdot \hat{i} - 30\hat{i} \cdot \hat{j} + 0$

$= 10 - 30 = -20$

Work done, $W = q \times |dV| = 0.8(20) = 16\text{ J}$

28. (a) The drift velocity,

$v_a = \frac{1}{neA} = \frac{V}{neA}\left(l = \frac{V}{R}\right)$

$v_d = \frac{V}{\rho\frac{l}{A}neA}\left(R = \rho\frac{l}{A}\right)$

$v_d = \frac{V}{\rho l n e}$

As drift velocity is independent of cross-sectional area (A) so no change in drift velocity.

29. (c) Since $R \propto l$

$\frac{R_L}{R_R} = \frac{60}{100-60} = \frac{60}{40} = \frac{6}{4} = \frac{3}{2}$ ….. (i)

$\Rightarrow \frac{R_L}{144R_R} = \frac{l}{100-l}$ ….. (ii)

From eqs. (i) and (ii)

$\frac{144R_R}{R_R} = \frac{3/2}{l/100-l} = \frac{3}{2} \times \frac{100-l}{l}$

$\Rightarrow \frac{100-l}{l} = \frac{2}{3} \times 144 = 2 \times 0.48 = 0.96$

$100 - i = 0.96$

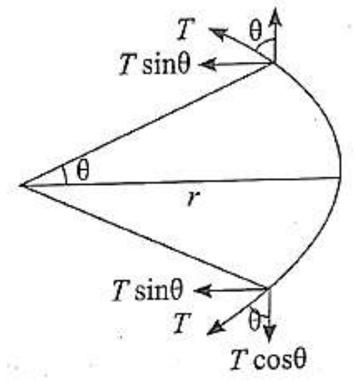
or, $1.96l = 100$

or,$l = \frac{100}{196} = 51.02\text{ cm} = 51\text{ cm}$

30. (c) Let the tension developed is T .

Let radius of loop be r.

For smaller element



$$BI(dl) = 2\,T\sin\left(\frac{d\theta}{2}\right)$$

$$\Rightarrow BI(r\,d\theta) = 2\,T\left(\frac{d\theta}{2}\right)$$

θ is small $\sin \simeq \theta$

$$T = ir\,B = \frac{ilB}{2\pi} = \frac{1.1\times 1\times 2.0}{2\times 3.14} = 0.35\text{ N}$$

31. **(a)** Magnetic dipole moment

$M = niA, n = 1, M = iA$

The current in the ring, $i = q/t = q\times f$

Where, $f =$ frequency of charge

$$i = q\frac{\omega}{2\pi}\left(\because \omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi}\right)$$

$$\therefore\ M = q\cdot\frac{\omega}{2\pi}\cdot\pi R^2 = \frac{1}{2}q\omega R^2$$

32. **(d)** When magnetic dipole is released from E-W a torque acts on it. So, in the displacement from E-W to N-S work is done by the torque.

$$KE = \text{ work done \& } W = \int_{\theta_1}^{\theta_2}\tau\cdot d\theta$$

$$= MB(\cos\theta_1 - \cos\theta_2)$$

$$\therefore KE = MB\cos 0^\circ$$

$$= 2.5\times 3\times 10^{-5}$$

$$= 7.5\times 10^{-5}\text{ J} = 75\times 10^{-6}\mu\text{ J}$$

33. **(a)** Voltage of source, $V = 4$ V

$$V_L = L\cdot\frac{dl}{dt} = 9\times 10^{-3}\times 10^3\text{ V} = 9\text{ V}$$

$$V_{AB} - V_R + V + V_L = 0$$

$$\text{or, } V_{AB} - 14 + 4 + 9 = 0$$

$$\Rightarrow\ V_{AB} - 1 = 0$$

$$\therefore\ V_{AB} = 1\text{ V}$$

34. **(c)** The natural frequency, $f_N = \frac{1}{2\pi\sqrt{KLC}} = Fe$

When capacitor is totally filled with dielectric material of dielectric constant K then capacitance $C' = KC$

$$f_{C'} = \frac{1}{2\pi\sqrt{KLC}}$$

$$\frac{f_C}{f_{C'}} = \frac{1/2\pi\sqrt{LC}}{1/2\pi\sqrt{KLC}} = \sqrt{K}$$

$$\Rightarrow\ \frac{125\times 10^3}{100\times 10^3} = \sqrt{K} \Rightarrow \frac{S}{4} = \sqrt{K}$$

$$[f_c = 125\text{kHz} - 25\text{kHz} = 100\text{kHz} = 100\times 10^3\text{ Hz}]$$

$$K = \left(\frac{5}{4}\right)^2 = \frac{25}{16} = 1.562$$

35. **(b)** The correct sequence of the radiation source in increasing order of the wavelength is Radioactive source ($\sim 10^{-12}$ m) → X-ray tube ($\sim 10^{-10}$ m) → Sodium lamp ($\sim 10^{-9}$ m) → Magnetron value ($\sim 10^{-3}$ m).

36. **(b)** de-Broglie wavelength

$$\lambda_0 = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}}$$

And $E_K = hv = \frac{hc}{\lambda}$

$$\lambda_v = \frac{h}{\sqrt{2m \times \frac{hc}{\lambda}}} = \sqrt{\frac{\lambda h}{2mc}}$$

37. **(c)** As the radius of the orbit will change due to transition from a higher energy level to a lower energy level. Therefore angular momentum L = mvr will not remain constant.

38. **(a)** Using radius of a nucleus, $R \propto A^{1/3}$

Where, A = number of nucleons

$$\frac{R_{Ge}}{R_{Be}} = \left(\frac{A_{Ge}}{A_{Be}}\right)^{1/3}$$

$$2 = \left(\frac{A_{Ge}}{A_{Be}}\right)^{1/3} \text{ or, } 2^3 = \frac{A_{Ge}}{A_{Be}} (\because R_{Ge} = 2R_{Be})$$

$$2^3 = \frac{A_{Ge}}{9} \; [\because A_{Be} = 9]$$

$$\therefore A_{Ge} = 2^3 \times 9 = 8 \times 9 = 72$$

39. **(b)**

Using $l_E = l_B + l_C$

$\Rightarrow l_B = l_E - l_C$

$= 6.6 \times 10^{-3} - 60l_B$

$\Rightarrow l_B + 60l_B = 6.6 \times 10^{-3}$

or, $61l_B = 6.6 \times 10^{-3}$

$\therefore l_B = \frac{6.6 \times 10^{-3}}{61} \simeq 0.108$ mA

$= 0.108 \times 10^{-3}$

$= 0.108$ mA

40. **(a)**

Coverage area $A = \pi d^2$ and $d = \sqrt{2Rh}$

$= \pi(\sqrt{2Rh})^2 = \pi \times 2Rh$

Here $h = 105$ m, $R = 6.4 \times 10^6$ m

$\therefore A = 3.14 \times 2 \times 6.4 \times 10^6 \times 105 \text{ m}^2$

$= 314 \times 2 \times 64 \times 10^6 \times 105 \times 10^3 \text{ m}^2$

$= 4220.6 \text{ km}^2 = 4224 \text{ km}^2$

Chemistry

41. **(c)** According to uncertainty principle,

$$\Delta P . \Delta x \geq \frac{h}{4\pi}$$

$$\Rightarrow m \cdot \Delta v \times \Delta x \geq \frac{h}{4\pi}$$

$$\Rightarrow \Delta v \times \Delta x \geq \frac{h^2}{4\pi \times m}$$

42. **(c)** Electronic configuration of the element in +2 oxidation state is $[Ar]3d^1$. Its electronic configuration in neutral state:

$[Ar]\, 3d^1 4s^2$

Period = Max.principal quantum no. n = 4

Group = Total number of valence shell electrons = 3

43. (b) In PCl_5 bond length of axial bond is greater than the length of equatorial bond.

44. (d) XeF_2 has 2 Bond pairs and 3 lone pairs

45. (b) Number of moles of hydrogen

$$= \frac{3g}{2g\ mol^{-1}} = 1.5$$

Number of moles of oxygen

$$= \frac{80\ g}{32\ g\ mol^{-1}} = 2.5$$

Total number of moles in gaseous mixture of hydrogen and oxygen,

$$n = 1.5 + 2.5 = 4$$

$$KE = \frac{3}{2}nRT = \frac{3}{2} \times 4 \times RT = 6RT$$

46. (d) C has highest critical temperature, i.e., 405.5 K .

Hence, it gets liquified first.

47. (a)

$$2KMnO_4 + 3H_2SO_4 + 5(COOH)_2 \rightarrow K_2SO_4 + 2$$
$$MnSO_4 + 8H_2O + 10CO_2$$

$$\frac{M_1V_1}{n_1} = \frac{M_2V_2}{n_2}$$

$$\Rightarrow \frac{x \times 40mL}{2} = \frac{0.02 \times 200mL}{5}$$

$$\Rightarrow x = 0.04M$$

48. (b)

$$C(s) + O_2(g) \rightarrow CO_2(g)$$
$$\Delta H^\circ = -x\ kJ\ mol^{-1} \quad \ldots\ldots (i)$$
$$2CO(g) + O_2(g) \rightarrow 2CO_2(g)$$
$$\Delta H^\circ = -y\ kJ\ mol^{-1} \quad \ldots (ii)$$

by applying eq. (i) $-\frac{1}{2} \times$ Eq. (ii)

$$C(s) + \frac{1}{2}O_2\ g \rightarrow CO(g)$$

$$\therefore\ \Delta fH(CO) = -x - \left(-\frac{y}{2}\right)$$

$$= -x + \frac{y}{2} = \frac{y - 2x}{2}$$

49. (b)

	$N_2O_4 \rightleftharpoons$	NO_2
Initial moles	1	0
At equilibrium	$(1 - 0.2)$	$2 \times 0.2 = 0.4$

Total moles = 0.4 + 0.8 = 1.2

$$K_P = \frac{(P_{NO_2})^2}{(P_{N_2O_4})} \Rightarrow K_P = \frac{\left(\frac{0.4}{1.2} \times 600\right)^2}{\left(\frac{0.8}{1.2} \times 600\right)} = 100$$

50. (c) Dissolution $NH_4Cl \rightarrow NH_4^+ + Cl^-$

Cl^-will form $Cl^-(aq)$ and will not undergo hydrolysis.

Hydrolysis $NH_4^+ + H_2O \rightleftharpoons NH_3 + H_3O^+$

51. (c)

52. (b) Li is the only alkali metal which reacts directly with N_2 to give nitride (Li_3N)

$6Li + N_2 \rightarrow 2Li_3N$

53. (a) $AlCl_3$ exists as a dimer through halogen bridged bonds. In dimer Al_2Cl_6 aluminium completes its octet through coordinates bond by chlorine atom.

54. (c)

55. (d) The general formula for this series is C_nH_{2n-2}.Hence $C_6H_{10}O_2$ N belongs to the given homologous series.

56. (b) In Dumas method,

$$\% \text{ of nitrogen } = \frac{28 \text{ V} \times 100}{22400 \times \text{W}} = \frac{28 \times 45 \times 100}{22400 \times 0.3} = 18.75$$

57. (a)

58. (b)

59. (d) For equimolar solution,

$$\chi_b = \chi_t = 0.5$$
$$p_b = \chi_b \times p_b^0 = 0.5 \times 160$$
$$= 80 \text{ mm}$$
$$p_t = \chi_t \times p_t^0 = 0.5 \times 60$$
$$= 30 \text{ mm}$$
$$p_{total} = 80 + 30 = 110 \text{ mm}$$

Mole fraction of benzene in vapour phase

$$= \frac{p_b}{p_{total}} = \frac{80}{110} = 0.727 = 0.73$$

60. (d) $\Delta T_f = K_f m$

$$0.93 = \frac{1.86 \times 6 \times 1000}{M \times 100} \Rightarrow M \approx 120$$

61. (b) $K_2SO_4 \rightarrow 2\ K^{+1} + SO_4^{2-}$

Due to platinum electrodes, self -ionization of water will take place.

$H_2O \rightarrow 2H^+ + OH^-$

At cathode: $2H^+ + 2e \longrightarrow H_2$

At anode: $OH \rightarrow 2H_2O + O_2 + 4e$

Due to lower electrode potentials, H_2 gas will generate at cathode and O_2 gas will generate at anode.

62. (c) Arrhénius equation,

$k = Ae^{-E_a'/RT}$

On taking log on both sides, we get

$$\ln k = \ln A - \frac{E_a}{RT}$$

A graph between in k and $1/T$ is a straight line with $-\frac{E_a}{R}$ slope.

63. (c) Multilayered adsorption is seen in physical adsorption process.

64. (b) Siderite is $FeCO_3$

65. (b) In oxygen, the bond is pure double bond but in ozone, it is partial single and double bond.

66. (d) Oxidise cannot oxidise $F^-$to F_2.

67. (a) At high temperature about 1000 k , sulphur consists of mixture of forms S_2, S_4, S_6, S_8, etc.

68. (d) $XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$

69. (c)

70. (d) Catalytic activity of transition elements is due to their variable oxidation states and to form complexes.

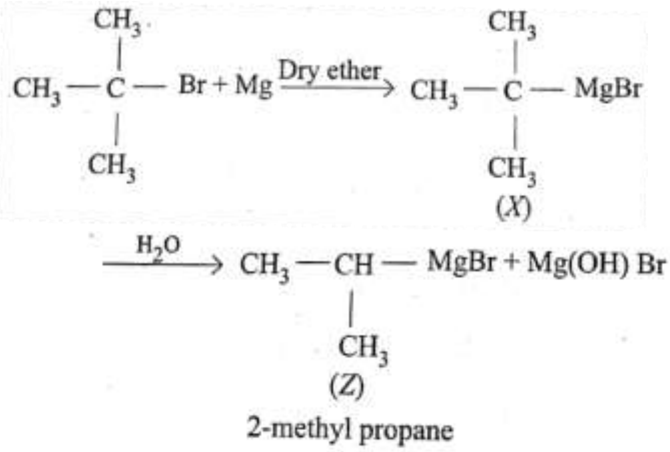
71. (b)

72. (d)

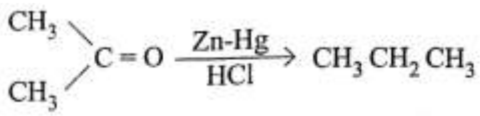
73. (b)

74. (a)

75. (c)



76. (b) Clemmensen reduction:



77. (d)

78. (c)

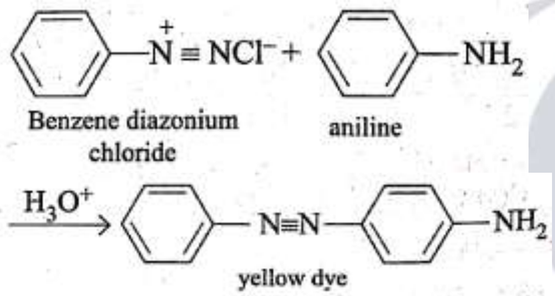
$CH_3OH \xrightarrow{PCl_3} \underset{(A)}{CH_3Cl} \xrightarrow{KCN} \underset{(B)}{CH_3CN}$

$\xrightarrow{H_3O^+} \underset{(C)}{CH_3COOH}$

Hence, C is CH_3COOH.

79. (c) $(CH_3)_2NH > CH_3NH_2 > (CH_3)_3\ N > NH_3 > C_6H_5NH_2$

80. (a)



Mathematics

81. (d) We have, $f(x) = \sqrt{\log_{0.5} x!}$

So, f(x) will be defined when,

$\log_{0.5} x! \geq 0 \Rightarrow x! \leq (0.5)^0$

$x! \leq 1 \ \therefore x \in \{0,1\}$

82. (c) We have,

$f(x) = |x-1| + |x-2| + |x-3|$

$$\Rightarrow f(x) = \begin{cases} 6-3x, & x < 1 \\ 4-x, & 1 < x < 2 \\ x, & 2 < x < 3 \\ 3x-6, & x > 3 \end{cases}$$

For $2 < x < 3$, then $f(x) = x$

So, f(x) is one-one and onto function.

It implies that f(x) is a bijection.

83. (b) Given, $(n+16)(n+17)(n+18)(n+19)$

As we can observe that these numbers are th product of four consecutive natural numbers.

This number gets divided by $4! = 24$.

84. (c) Let $A=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ $[C_1+C_1+C_2+C_3]$

$$=\begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

$$=(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix}$$

$[R_2 \rightarrow R_2-R_1$ and $R_3 \rightarrow R_3-R_1]$

$$=(a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix}$$

$$=(a+b+c)[(c-b)(b-c)-(a-c)(a-b)]$$

$$=-(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$=-\frac{1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2]$$

Since it is given that a, b, c are distinct positive numbers. S_0, the value of determinant A is less than 0 .

85. (d) It is given that x_1, x_2, x_3 and y_1, y_2, y_3 are in GP with the same common ratio.

Let r be the common ratio.

$x_1=x_1, x_2=xr$ and $x_3=xr^2$

Similarly, $y_1=y, y_2=yr$ and $y_3=yr^2$

Area of $\Delta=\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_1 & 1 \\ x_3 & y_1 & 1 \end{vmatrix}=\frac{1}{2}\begin{vmatrix} x & y_1 & 1 \\ xr & yr & 1 \\ xr^2 & yr^2 & 1 \end{vmatrix}$

$=\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ r & r & 1 \\ r^2 & r^2 & 1 \end{vmatrix}=\frac{1}{2}\times 0=0$ [$\because C_1$ and C_2 are identical]

The given points do not form any triangle. They are collinear.

86. (b) Given equations, $x-y+2z=4, 3x+y+4z=6, x+y+z=1$.

Let $\Delta=\begin{vmatrix} 1 & -1 & 2 \\ 3 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix}$

$$=1(1-4)+1(3-4)+2(3-1)$$

$$=-3-1+4=0$$

and $\Delta_1=\begin{vmatrix} 4 & -1 & 2 \\ 6 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix}$

$$=4(1-4)+1(6-4)+2(6-1)$$

$$=-12+2+10=0$$

Now, $\Delta=0$ and $\Delta_1=0$

These equations have infinitely many solutions.

87. (c) Let the complex number, $z=x+iy$

Now, $|z+3|^2-|z-3|^2=15$

$$|x+iy+3|^2-|x+iy-3|^2=15$$

$$(x+3)^2+y^2-(x-3)^2-y^2=15$$

$$x^2+6x+9-x^2+6x-9=15$$

$$12x=15 \Rightarrow 4x=5$$

It represents a straight line.

88. (a) We have,

$$\frac{(1+i)^{2016}}{(1-i)^{2014}}=\frac{[(1+i)^2]^{1008}}{[(1-i)^2]^{1007}}=\frac{(1+2i+i^2)^{1008}}{(1-2i+i^2)^{1007}}$$

$$=\frac{(2i)^{1008}}{(-2i)^{1007}}=\frac{(2i)^{1008}}{(-1)^{1007}(2i)^{1007}}=-2i$$

89. (d) We have,

$|z_1| = 1, |z_2| = 2, |z_3| = 3$ and
$|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$
Since, we know $|z|^2 = z\bar{z}$
Now, $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$
$$\left| \begin{array}{c} |z_3|^2 z_1z_2 + |z_2|^2 z_1z_3 \\ +|z_1|^2 z_2z_3 \end{array} \right| = 12$$
$$\Rightarrow \left| \begin{array}{c} z_3\bar{z}_3z_1z_2 + z_2\bar{z}_2z_1z_3 \\ +z_1\bar{z}_1z_2z_3 \end{array} \right| = 12$$
$\Rightarrow |z_1z_2z_3(\bar{z}_3 + \bar{z}_2 + \bar{z}_1)| = 12$
$\Rightarrow |z_1z_2z_3||\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12$
$\Rightarrow |z_1||z_2||z_3||\bar{z}_1 + \bar{z}_2 + \bar{z}_2| = 12$
$\Rightarrow 1 \times 2 \times 3|\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 12$
$\Rightarrow |\bar{z}_1 + \bar{z}_2 + \bar{z}_3| = 2$
$\Rightarrow |z_1 + z_2 + z_3| = 2$

90. (c) We have given that, $1, z_1, z_2, z_3, \ldots\ldots zn_{-1}$ are the nth root of unity.
root of unity.
$zn - 1 = (z-1)(z-z_1)(z-z_2), \ldots., (z - z_n - 1)$
$\Rightarrow (z-1)(z^{n-1} + z^{n-2} + \cdots. + z^2 + z + 1)$
$= (z-1)(z-z_1)\ldots..(z - zn - 1)$
$\Rightarrow (z^{n-1} + z^{n-2} + \cdots.. + z^2 + z + 1)$
$= (z - z_1)$
$(z - z_2)\ldots..(z - zn - 1)$
Put $z = 1$, we get
$\Rightarrow (1 + 1 + \cdots. + n \text{ times})$
$= (1 - z_1)(1 - z_2)\ldots.(1 - z_{n-1})$
$\Rightarrow n = (1 - z_1)(1 - z_2)\ldots(1 - z_{n-1})$

91. (b) We have, $(12)^{4+2x^2} = (24\sqrt{3})^{3x^2-2}$
$(12)^{4+2x^2} = (12)^{\frac{3}{2}(3x^2-2)}$
$$4 + 2x^2 = \frac{3}{2}(3x^2 - 2) \therefore x = \pm\sqrt{\frac{14}{5}}$$

92. (a) We have, $|x|^2 - 5|x| - 24 = 0$
$\Rightarrow (|x| - 8)(|x| + 3) = 0$
$\Rightarrow |x| = 8$ or $|x| = -3$ [rejected].
$\Rightarrow |x| = 8 \Rightarrow x = \pm 8$
The roots of this equation. $ax = 8$ and -8 .
Hence, product of roots $8 \times (-8) = -64$
and sum of roots $= +8 - 8 = 0$

93. (a)
Let $f(x) = x^5 + 3x^3 + 4x + 30$
$\Rightarrow f'(x) = 5x^4 + 9x^2 + 4$
As $f'(x)$ consist of the terms which has even powers of x.
$f'(x) > 0$ for all $x \in R$
Hence, the $f(x) = 0$ has only one real root.

94. (b) Given equation,
$x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{144} = 0 \quad \ldots\ldots(i)$
The equation whose roots are k times the roots of the eq. (i), is
$$\left(\frac{x}{k}\right)^3 + \frac{1}{4}\left(\frac{x}{k}\right)^3 - \frac{1}{16}\left(\frac{x}{k}\right) + \frac{1}{144} = 0$$
$$\Rightarrow x^3 + \frac{k}{4}x^2 - \frac{k^2}{16}x + \frac{k^3}{144} = 0$$

Also given that, coefficients of the above equation are integer.
k = 12

95. (c) Given digits are 2,3,4,5,6
Sum of all 4-digit numbers formed by sing the digits 2, 3, 4, 5,6 without repetition
5,6 without repetition

$$= {}^4P_3(2+3+4+5+6)(10^3+10^2+10^1+10^0)$$
$$= 24(20)(1000+100+10+1)$$
$$= (480)(1111) = 533280$$

96. (c) Given set A has 5 elements, there are two subsets P and Q which are mutually disjoint. So, the required number of ways of selecting two subsets will be $3^5 = 243$.

97. (a)
In the expansion $(1-x+x^2-x^3)^4$

$$= (1+x^2-x-x^3)^4 = [(1+x^2)-x(1+x^2)^4]$$
$$= [(1+x^2)(1-x)^4] = (1+x^2)^4(1-x)^4$$
$$= (1+4x^2+1+6x^4+4x^6+x^8)$$
$$(1-4x^2+6x^2-4x^3+x^4)$$
$$\therefore \text{ Coefficient of } x^4 = 1+4\times 6+6$$
$$= 1+24+6 = 31$$

98. (a) In the expansion of $(1+x)^2n$, middle term is ${}^2nCn\ xn$.
Since, middle term is the greatest term.

$$\therefore\ {}^{2n}C_n x^n > {}^{2n}Cn_{-1}x^{n-1}$$
$$\text{and } {}^{2n}Crx^n > {}^{2n}Cn_{+1}x^{n+1}$$
$$\Rightarrow x > \frac{{}^{2n}C_{n-1}}{{}^{2n}C_n} \text{ and } x < \frac{{}^{2n}C_n}{{}^{2n}C_{n+1}}$$

Hence, $x \in \left(\frac{{}^{2n}C_n}{{}^{2n}C_{n+1}}, \frac{{}^{2n}C_n}{{}^{2n}C_{n+1}}\right)$

$$x \in \left(\begin{matrix}\frac{(2n)!}{(n-1)!\,(2n-n+1)} \times \frac{n!\,(2n-n)!}{(2n)!} \\ \frac{(2n)!}{n!\,(2n-n)!} \times \frac{(n-1)!\,(2n-n+1)}{(2n)!}\end{matrix}\right)$$

$$x \in \left(\frac{n(n-1)!\,n!}{n(n-1)!\,(n+1)n!}, \frac{(n+1)n!\,(n-1)!}{n(n-1)!\,n!}\right)$$

$$\therefore\ x \in \left(\frac{n}{n+1}, \frac{n+1}{n}\right)$$

99. (c) Given, $\frac{3x}{(x-2)(x-1)}$ can be written as $\frac{6}{x-2} - \frac{3}{x-1}$

$$\therefore \frac{3x}{(x-2)(x-1)} = \frac{6}{x-2} - \frac{3}{x-1}$$
$$= 6(x-2)^{-1} - 3(x-1)^{-1}$$
$$= -3\left(1-\frac{x}{2}\right) + 3(x-1)^{-1}$$

It is valid iff $\left|\frac{x}{2}\right| < 1$ and $|x| < 1$

$$\Rightarrow |x| < 2 \text{ and } |x| < 1$$
$$\Rightarrow x \in (-2,2) \text{ and } x \in (-1,1)\ \therefore\ x \in (-1,1)$$
$$\text{Hence, } -1 < x < 1$$

100. (a)

Given that $(1+\tan\alpha)(1+\tan 4\alpha)=2, \alpha \in \left(0, \frac{\pi}{16}\right)$

$\Rightarrow 1+\tan\alpha+\tan 4\alpha+\tan\alpha\tan 4\alpha=2$
$\Rightarrow \tan\alpha+\tan 4\alpha=1-\tan\alpha\tan 4\alpha$
$\Rightarrow \dfrac{\tan\alpha+\tan 4\alpha}{1-\tan\alpha+\tan 4\alpha}=1 \Rightarrow \tan 5\alpha=\tan\dfrac{\pi}{4}$
$\Rightarrow 5\alpha=\dfrac{\pi}{4} \Rightarrow \alpha=\dfrac{\pi}{20}$

101. **(a)** We have, $\cos\theta=\frac{\cos\alpha-\cos\beta}{1-\cos\alpha\cos\beta}$

Applying componendo and dividendo, we get

$$\frac{\cos\theta+1}{\cos\theta-1}=\frac{\cos\alpha-\cos\beta+1-\cos\alpha\cos\beta}{\cos\alpha-\cos\beta-1+\cos\alpha\cos\beta}$$
$$\frac{\cos\theta+1}{\cos\theta-1}=\frac{(\cos\alpha+1)(1-\cos\beta)}{(\cos\alpha-1)(\cos\beta+1)}$$
$$\frac{1+\cos\theta}{1-\cos\theta}=\frac{(1+\cos\alpha)(1-\cos\beta)}{(1-\cos\alpha)(1+\cos\beta)}$$
$$\frac{2\cos^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}=\frac{2\cos^2\frac{\alpha}{2}}{2\sin^2\frac{\alpha}{2}}\frac{2\sin^2\frac{\beta}{2}}{2\cos^2\frac{\beta}{2}}$$
$$\cot^2\frac{\theta}{2}=\cot^2\frac{\alpha}{2}\tan^2\frac{\beta}{2}$$
$$\tan\frac{\theta}{2}=\tan\frac{\alpha}{2}\cot\frac{\beta}{2}$$

102. **(d)**

103. **(b)** Given that, terms $\frac{1}{6}\sin\theta, \cos\theta$ and $\tan\theta$ are in GP.

$$\cos^2\theta=\frac{1}{6}\sin\theta\times\tan\theta$$
$\Rightarrow \cos^2\theta=\dfrac{\sin^2\theta}{6\cos\theta}$
$\Rightarrow 6\cos^3\theta-\sin^2\theta=0$
$\Rightarrow 6\cos^3\theta+\cos^2\theta-1=0$
$\Rightarrow 6\cos^3\theta+4\cos^2\theta-3\cos^2\theta+2\cos\theta-2\cos\theta-1=0$
$\Rightarrow 6\cos^3\theta-3\cos^2\theta+4\cos^2\theta-2\cos\theta+2\cos\theta-1=0$
$\Rightarrow 3\cos^2\theta(2\cos\theta-1)+2\cos\theta(2\cos\theta-1)+1(2\cos\theta-1)=0$
$\Rightarrow (2\cos\theta-1)(3\cos^2\theta+2\cos\theta+1)=0$

For $3\cos^2\theta+2\cos\theta+1=0$ value of $\cos\theta$ will be imaginary so only $2\cos\theta-1=0$ will be considered to find $\cos\theta$.

$\Rightarrow \quad 2\cos\theta-1=0 \Rightarrow \cos\theta=\frac{1}{2} \Rightarrow \cos\theta=\cos\frac{\pi}{3}$

$\theta=2n\pi\pm\frac{\pi}{3}$

104. **(a)** We have, $x=\sin(2\tan^{-1}2)$ and $y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$

$\Rightarrow x=\sin(2\tan^{-1}2)$

Let $\tan^{-1}2-\alpha \Rightarrow \tan\alpha=2$

$$x=\sin 2\alpha=\frac{2\tan\alpha}{1+\tan^2\alpha}=\frac{2(2)}{1+(2)^2}=\frac{4}{5}$$

Now, $y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$

Let $\tan^{-1}\frac{4}{3}=\beta$

$$\tan\beta=\frac{4}{3}, \cos\beta=\frac{1}{\sqrt{1+\tan^2\beta}}=\frac{3}{5}$$

Then, $y=\sin\left(\frac{\beta}{2}\right)$

$$= \sqrt{\frac{1-\cos\beta}{2}} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{1}{5}}$$

$x > y$

105. **(c)** Given, $\cos h(x) = \frac{5}{4}$

Now, $\cos h(3x) = 4\cos h^3(x) - 3\cos h(x)$

$$= 4\left(\frac{5}{4}\right)^3 - 3 \times \frac{5}{4} = \frac{125}{16} - \frac{15}{4} = \frac{65}{16}$$

106. **(b)** In $\triangle ABC$, given

$$x = \tan\left(\frac{B-C}{2}\right)\tan\frac{A}{2}$$

$$y = \tan\left(\frac{C-A}{2}\right)\tan\frac{B}{2}$$

and $y = \tan\left(\frac{A-B}{2}\right)\tan\frac{C}{2}$

Since, $\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\tan\frac{A}{2}$

$$\Rightarrow x = \frac{b-c}{b+c}$$

Similarly, $y = \frac{c-a}{c+a}$ and $z = \frac{a-b}{a+b}$

Now, by componendo and dividendo

$$\frac{1+x}{1-x} = \frac{b+c+b-c}{b+c-b+c} = \frac{b}{c}$$

Similarly, $\frac{1+y}{1-y} = \frac{c}{a}$ and $\frac{1+z}{1-z} = \frac{a}{b}$

$$\therefore \left(\frac{1+x}{1-x}\right)\left(\frac{1+y}{1-y}\right)\left(\frac{1+z}{1-z}\right) = \frac{b}{c} \times \frac{c}{a} \times \frac{a}{b} = 1$$

$$\Rightarrow (1+x)(1+y)(1+z)$$

$$= (1-x)(1-y)(1-z)$$

$$\Rightarrow 1 + x + y + z + xy + yz + zx + xyz$$

$$= 1 - (x+y+z) + xy + yz + zx - xyz$$

$$\Rightarrow x + y + z = -xyz$$

107. **(d)** In $\triangle ABC$, sides a, b and c are in GP.

$b^2 = ac$ ....(i)

Given, largest angle exceeds the smallest angle by $60°$.

$C - A = 60°$ ....(ii)

$\cos(C - A) = \cos 60°$

$$\cos C\cos A + \sin C\sin A = \frac{1}{2}$$

We know, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k$

$$\Rightarrow 2\cos C\cos A + 2\sin C\sin A = 1$$

$$\Rightarrow \cos(C+A) + \cos(C-A) + 2k^2ac = 1$$

$$\Rightarrow \cos(\pi + B) + \cos 60° + 2k^2b^2 = 1 \; [\because ac = b^2]$$

$$\Rightarrow -\cos B + \frac{1}{2} + 2\sin^2 B = 1 \; [\because bk = \sin B]$$

$$\Rightarrow -2\cos B + 1 + 4(1 - \cos^2 B) = 2$$

$$\Rightarrow 4\cos^2 B + 2\cos B - 3 = 0$$

$$\therefore \cos B = \frac{-2 \pm \sqrt{4+48}}{2 \times 4} = \frac{-2 \pm 2\sqrt{13}}{8}$$

$$= \frac{\sqrt{13}-1}{4} \text{ or } \frac{-\sqrt{13}-1}{4}$$

108. **(c)** Given, $\angle A = 90°$

In$\triangle$ ABC, we know that

$$r_2 + r_3 = 4R\cos^2\frac{A}{2} \Rightarrow r_2 + r_3 = 4R\cos^2 45°$$

$$\Rightarrow r_2 + r_3 = 4R \times \frac{1}{2} \Rightarrow r_2 + r_3 = 2R$$

$$\cos^{-1}\left(\frac{R}{r_2+r_3}\right) = \cos^{-1}\left(\frac{R}{2R}\right) = \cos^{-1}\left(\frac{1}{2}\right) = 60°$$

109. **(d)**

$$\gamma = (1+\lambda-\mu)\hat{i} + (2-\lambda)\hat{j} + (3-2\lambda+2\mu)\hat{k}$$

$$= \hat{i} + \lambda\hat{i} - \mu\hat{i} + 2\hat{j} - \lambda\hat{j} + 3\hat{k} - 2\lambda\hat{k} + 2\mu\hat{k}$$

$$= (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} - 2\hat{k}) + \mu(-\hat{i} + 2\hat{k})$$

Equation of plane is given by

$$\{r - (\hat{i} + 2\hat{j} + 3\hat{k})\} + [(\hat{i} - \hat{j} - 2\hat{k}) \times (-\hat{i} + 2\hat{k})] = 0$$

$$\Rightarrow \{r - (\hat{i} + 2\hat{j} + 3\hat{k})\} \cdot (-2\hat{i} - \hat{k}) = 0$$

$$\Rightarrow r(2\hat{i} + \hat{k}) = 5$$

Now, equation of plane in cartesian form

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{j} + \hat{k}) = 5$$

$$2x + z = 5$$

110. **(b)** For three vectors p, q and r,

$$r = 3p + 4q \quad \ldots\ldots (i)$$

$$2r = p - 3q \quad \ldots\ldots (ii)$$

From eqs. (i) and (ii), we get

$$r = 3(2r + 3q) + 4q$$

$$r = 6r + 9q + 4q$$

$$\Rightarrow 5r = -13q \Rightarrow r = \frac{13}{5}q \Rightarrow |r| = \frac{13}{5}|q|$$

$\therefore$ r and q have opposite directions

Hence, $|r| > 2|q|$

111. **(a)** We have, $a = 2\hat{i} + 3\hat{j} - 5\hat{k}$

$b = m\hat{i} + n\hat{j} + 12\hat{k}$

Since, $a \times b = 0 \Rightarrow a \parallel b$

$$\frac{2}{m} = \frac{3}{n} = \frac{-5}{12}$$

$$\Rightarrow m = -\frac{24}{5} \text{ and } n = -\frac{36}{5}$$

Thus, $(m, n) = \left(-\frac{24}{5}, -\frac{36}{5}\right)$

112. **(d)** We have, $|a| = 3, |b| = 4$

$$|4a + 3b|^2 = (4a + 3b)(4a + 3b)$$

$$= 16|a|^2 + 9|b|^2 + 24a \cdot b$$

$$= 16|a|^2 + 9|b|^2 + 24|a| \cdot |b|\cos\theta$$

$$= 16(3)^2 + 9(4)^2 + 24 \times 3 \times 4 \times \cos 120°$$

$$= 16 \times 9 + 9 \times 16 + 24 \times 12 \times \left(\frac{-1}{2}\right) = 144$$

So, $|4a + 3b| = \sqrt{144} = 12$

113. **(a)** Here, a, b and c are non-zero vectors

$c \perp a \Rightarrow c.a = 0$ and θ is the angle between the vectors b and c.

$$(a \times b) \times c = \frac{1}{3}|b| \cdot |c|a$$

$$(c \cdot a)b - (c \cdot b)a = \frac{1}{3}|b||c|a$$

$$\Rightarrow\ 0 - (|c||b|\cos\theta)a = \frac{1}{3}|b||c|a$$

$$\Rightarrow \cos\theta = -\frac{1}{3} \Rightarrow \cos^2\theta = -\frac{1}{9}$$

$$\Rightarrow\ 1 - \sin^2\theta = \frac{1}{9} \Rightarrow \sin^2\theta = 1 - \frac{1}{9} = \frac{8}{9}$$

$$\therefore \sin\theta = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

114. **(c)** Given that,
$a(\alpha \times \beta) + b(\beta \times \gamma) + c(\gamma \times \alpha) = 0$
On taking dot product of α with the given cross product, we get $a\alpha \cdot (\alpha \times \beta) + b\alpha \cdot (\beta \times \gamma) + c\alpha \cdot (\gamma \times \alpha) = 0$
$a[\alpha\alpha\beta] + b(\alpha\beta\gamma) + c(\alpha\gamma\alpha) = 0$
$[xxy] = [xyx] = [yxx] = 0$
$\Rightarrow b[\alpha\alpha\beta] = 0$
Since, $b \neq 0 \therefore [\alpha\alpha\beta] = 0$
Hence, α, β, γ are coplanar.

115. **(c)** Given, number of observation, $n = 10$,
Mean, $\bar{x} = 50$ and $\sum|xi - \bar{x}|^2 = 250$
Variance, $\sigma^2 = \sum_{i=1}^{n} \frac{|x_i-\bar{x}|^2}{n} \Rightarrow \sigma^2 = \sum_{i=1}^{10} \frac{|x_i-\bar{x}|^2}{10} = \frac{250}{10} = 25 \Rightarrow \sigma = 5$
Coefficient of variation,
C.V. $= \frac{\sigma}{x} \times 100 = \frac{5}{50} \times 100 = 10$

116. **(b)** Given, 50 even natural numbers are 2,4,6, ….100

$$\text{Mean, } \bar{x} = \frac{\sum xi}{n} = \frac{2+4+6+\cdots.+98+100}{50}$$

$$\bar{x} = \frac{2(1+2+3+\cdots.+49+50)}{50} = 51$$

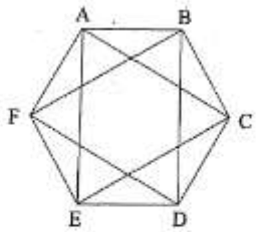
$$\text{Variance, } \sigma^2 = \frac{\sum x_1^2}{n} - (\bar{x})^2$$

$$= \frac{(2^2+4^2+6^2+\cdots.+98^2+100^2)}{50} - (51)^2$$

$$= \frac{2^2(1^2+2^2+3^2+\cdots.+49^2+50^2) - (51)^2 \times 50}{50}$$

$$= \frac{4(50)(51)(101) - (51)\times(51)\times 50 \times 6}{6 \times 50} = 833$$

117. **(c)** Give that, 3 out of 6 vertices of hexagon are chosen.
Total number of outcomes $= {}^6C_3 = \frac{6!}{3!3!} = 20$



Only two equilateral triangles are possible of regular hexagon, i.e. $\triangle$ AEC and $\triangle$ BFD.
Probability $= \frac{2}{20} = \frac{1}{10}$

118. **(a)** Let Event A: A speaks the truth
Event B: B speaks the truth

$$P(A) = 75\% = \frac{75}{100} = \frac{3}{4}$$

and $P(B) = 80\% = \frac{80}{100} = \frac{4}{5}$

Required probability

$$P(A\bar{B}) + P(\bar{A}B) = P(A) \times P(\bar{B}) + P(\bar{A}) \times P(B)$$
$$= P(A) \times [1 - P(B)] + P(A) \times P(B)$$
$$= \frac{3}{4} \times \left(1 - \frac{4}{5}\right) + \left(1 - \frac{3}{4}\right) \times \frac{4}{5} = \frac{7}{20}$$

119. **(d)** For a binomial distribution, Mean, $\mu = np = 4$ ......(i)

Variance, $\sigma^2 = npq = 2$ ....(ii)

On solving eqs. (i) and (ii), we get

$n = 8, p = \frac{1}{2}, q = \frac{1}{2}$

Probability of getting 2 successes,

$p(X = 2) = {}^8C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^6 = \frac{{}^8C_2}{2^8} = \frac{28}{256} = \frac{7}{64}$

120. **(a)** For poisson distribution,

$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}$

Where λ = mean of distribution = np

k = probability of success

Here, 10 accidents take place in 50 days.

So, $p = \frac{10}{50} = \frac{1}{5}$ and $n = 1$

$\lambda = 1 \times \frac{1}{5} = 0.2$

Probability that three or more accidents occur in a day,

$$P(x \geq 3) = P(x = 3) + P(x = 4) + \cdots$$
$$= \sum_{k=3}^{\lambda} \frac{e^{-\lambda}\lambda^k}{k!}, \lambda = 0.2$$

121. **(a)** Let A, B and C the vertices of triangle

$A = (a\cos k, a\sin k)$

$B = (b\sin k - b\cos k)$

$C = (1,0)$

Let $G(x, y)$ be the centroid,

$$x = \frac{a\cos k + b\sin k + 1}{3}$$
$$\Rightarrow 3x - 1 = a\cos k + b\sin k \quad \ldots\ldots (i)$$
$$\text{and } y = \frac{a\sin k - b\cos k + 0}{3}$$
$$\text{and } 3y = a\sin k - b\cos k \quad \ldots\ldots (ii)$$

On squaring and then adding Eqs. (i) and (ii) we get

$$(3x - 1)^2 + 3y^2 = a^2(\sin^2 k + \cos^2 k) + b^2(\sin^2 k + \cos^2 k)$$
$$(3x - 1)^2 + 9y^2 = a^2 + b^2$$
$$(1 - 3x)^2 + 9y^2 = a^2 + b^2$$

122. **(c)** Given equation, $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$

Coordinate axes are rotated through an angle of $\frac{\pi}{6}$ about the origin

Now, axis is $x\cos\frac{\pi}{6} - y\cos\frac{\pi}{6}$

and $x\cos\frac{\pi}{6} + y\cos\frac{\pi}{6}$

Thus, equation becomes

$\sqrt{3}\left(x\cos\frac{\pi}{6} - y\cos\frac{\pi}{6}\right)^2$

$-4\left(x\cos\frac{\pi}{6} - y\cos\frac{\pi}{6}\right)$

$\left(x\cos\frac{\pi}{6} + y\cos\frac{\pi}{6}\right) + \sqrt{3}\left(x\cos\frac{\pi}{6} + y\cos\frac{\pi}{6}\right)^2 = 0$

$\Rightarrow \sqrt{3}\left(\frac{\sqrt{3}x}{2} - \frac{1}{2}y\right)^2 - 4\left(\frac{\sqrt{3}x - y}{2}\right)$

$\left(\frac{x + \sqrt{3}y}{2}\right) + \sqrt{3}\left(\frac{x + \sqrt{3}y}{2}\right)^2 = 0$

$\Rightarrow \sqrt{3}(3x^2 - 2\sqrt{3}xy + y^2) - 4(\sqrt{3}x^2 + 3xy - xy - \sqrt{3}y^2)$

$+ \sqrt{3}(x^2 + 2\sqrt{3}xy + 3y^2) = 0$

$\Rightarrow 3\sqrt{3}x^2 - 6xy + \sqrt{3}y^2 - 4\sqrt{3}x^2 - 12xy$

$+ 4xy + 4\sqrt{3}y^2 + \sqrt{3}x^2 + 6xy + 3\sqrt{3}y^2 = 0$

$\Rightarrow 8\sqrt{3}y^2 - 8xy = 0$

$\therefore \sqrt{3}y^2 - xy = 0$

123. **(d)** Given that, $x + 3y - 9 = 0,\ 4x + by - 2 = 0$

$2x - y - 4 = 0$ are concurrent.

$$\begin{vmatrix} 1 & 3 & -9 \\ 4 & b & -2 \\ 2 & -1 & -4 \end{vmatrix} = 0$$

$1(-4b - 2) - 3(-16 + 4) - 9(-4 - 2b) = 0$

$-4b - 2 + 36 + 36 + 18b = 0$

$14b = -70 \Rightarrow b = -5$

From equations $x + 3y - 9 = 0$ and $2x - y - 4 = 0$

$\Rightarrow x = 3$ and $y = 2$

Concurrency point is $(3,2)$

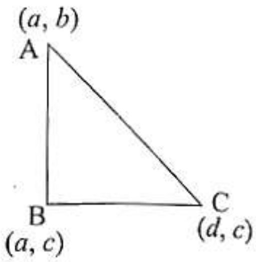
Equation of line passing through $(-5,0)$ and $(3,2)$ is

$y - 0 = \frac{2 - 0}{3 + 5}(x + 5)$

$y = \frac{1}{4}(x + 5) \qquad \Rightarrow 4y = x + 5$

$\Rightarrow x - 4y + 5 = 0$

124. **(a)** The vertices of the triangle are given as $A(a, b), B(a, c)$ and $C(d, c)$



Centroid of ΔABC

$= \left(\frac{a + a + d}{3}, \frac{b + c + c}{3}\right)$

$\equiv \left(\frac{2a + d}{3}, \frac{2c + b}{3}\right)$

Since, x-coordinate of point A and B are same. Also y-coordinate of point B and C are same so it forms a right angle triangle.

Orthocentre of $\triangle ABC$ is (a, c)

Mid-point of centroid and orthocentre is

$$= \left(\frac{\frac{2a+d}{3}+a}{2}, \frac{\frac{2c+b}{3}+c}{2}\right) = \left(\frac{5a+d}{6}, \frac{5c+d}{6}\right)$$

125. **(c)** As we know that the image of (1,1) with respect to line $x+y+5=0$ is

$$\frac{x-1}{1} = \frac{y-1}{1} = \frac{2(1+1+5)}{1+1}$$

$\Rightarrow x-1=-7, y-1=-7$

$\Rightarrow x=-6, y=-6$

Image of point (1,1) is $(-6,-6)$

Now, distance from origin

This is the required transformed equation.

$D = \sqrt{(0+6)^2+(0+6)^2} = 6\sqrt{2}$

126. **(d)** Given, $x^2+y^2=9$ and $x+y=3$

Equation of pair of lines.

$$x^2+y^2 = 9\left(\frac{x+y}{3}\right)^2$$

$\Rightarrow x^2+y^2=(x+y)^2$

$\Rightarrow x^2+y^2=x^2+y^2+2xy \Rightarrow xy=0$

127. **(a)** Given that, $x+y=1$ and $2y^2-xy-6x^2=0$

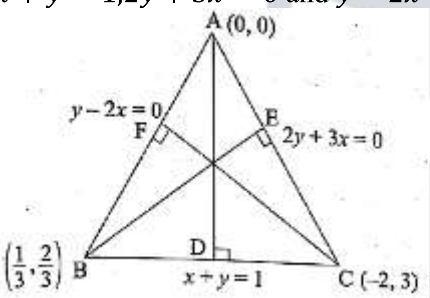
$\Rightarrow 2y^2-4xy+3xy-6x^2=0$

$\Rightarrow (2y-3x)(y-2x)=0$

$\Rightarrow 2y+3x=0$ and $y-2x=0$

Equation of sides are

$x+y=1, 2y+3x=0$ and $y-2x=0$



Solving these equations simultaneously, we get the coordinate of the points A(0,0), B$\left(\frac{1}{3},\frac{2}{3}\right)$ and C$(-2,3)$ Equation of altitude AD ,

$x-y=0$ .....(i)

Equation of altitude CF,

$x+2y=\lambda$ ....(ii)

Since, this passes through $(-2,3)$

$-2+6=\lambda \Rightarrow \lambda=4$

So, equation of altitude CF $x+2y=4$

On solving eqs. (i) and (ii), we get $x=\frac{4}{3}, y=\frac{4}{3}$

Orthocentre of the $\triangle$ ABC is $\left(\frac{4}{3},\frac{4}{3}\right)$

128. **(b)** Given that, $2x^2-3xy-2y^2+10x+5y=0$

$(2x+y)(x-2y+5)=0$

$2x+y=0$ and $x-2y+5=0$

Now, equation of line passing through origin is

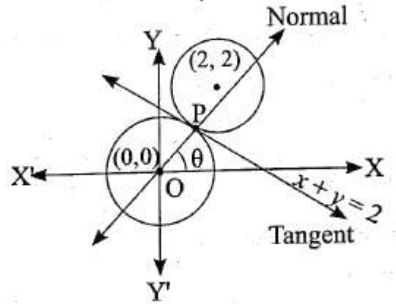
$2x^2+y=0 \Rightarrow m_1=-2$

Since, this line is perpendicular to the line

$kx+y+3=0 \Rightarrow m_2=-k$

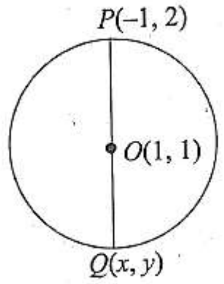
$$m_1 \times m_2 = -1 \Rightarrow k = -\frac{1}{2}$$

129. **(c)** Equation of line at $P(1,1)$ is $x+y-2=0$ Slope of this line is -1 .
So, slope of line perpendicular to this line is 1
$\tan\theta=1\ \theta=\frac{\pi}{4}$.



Let, centre of circle (h,k)
i.e. $x=h\pm r\cos\theta$ and $y=k\pm r\sin\theta$
As, it passes through $(1,1)$.
$h=1\pm\sqrt{2}\cos\frac{\pi}{4}, k=1\pm\sqrt{2}\sin\frac{\pi}{4}$
$\Rightarrow\ h=1\pm\frac{\sqrt{2}}{\sqrt{2}}$ and $k=1\pm\frac{\sqrt{2}}{\sqrt{2}}$
$\Rightarrow h=2,0$ and $k=2,0$
Centres are $(2,2)$ or $(0,0)$
Hence, equation of circle $x^2+y^2=2$ or $(x-2)^2+(y-2)^2=2$
Length of tangent from $(1,2)$
$=\sqrt{1^2+2^2-2}=\sqrt{3}$

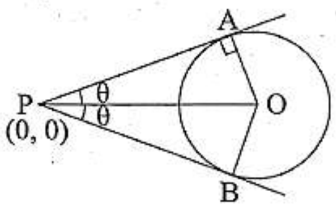
130. **(a)** Equation of circle,
$x^2+y^2+2x-2y-3=0$



Its centre is $(1,1)$
Given that,
PQ being normal to the circle represent the diameter of circle and O is the mid-point of
$\frac{x-1}{2}=1, \frac{y+2}{2}=1$
$x=3, y=0$. So, coordinate of $Q=(3,0)$

131. **(b)** The lines $kx+2y-4=0$ and $5x-2y-4=0$ are conjugate with respect to the circle
$x^2+y^2-2x-2y+1=0$
Then, necessary condition
$r^2(a_1a_2+b_1b_2)=(a_1g+b_1f-c_1)(a_2g+b_2f-c_2)$ ....(i)
From the above equation,
$a_1=k,\quad b_1=2\ c_1=-4$
$a_2=5,\quad b_2=-2\ c_2=-4$
$g=-1,\quad f=-1\ c=1$
$r^2=g^2+f^2-c=1+1-1=1$
Now, by using equation (i)
$1(5k-1)=(-k-2+1)(-5+2+4)$
$5k-4=-k+2\Rightarrow k=1$

132. **(c)** We have,
$x^2+y^2+4x-6y+4=0$



Centre $\equiv (-g, -f) \equiv (-2,3)$

Radius of circle, $OA = \sqrt{g^2 + f^2 - c}$

$= \sqrt{(2)^2 + (-3)^2 - 4}$

$= \sqrt{4 + 9 - 4} = 3$

Length of tangent

$PA = PB$

$= \sqrt{(0)^2 + (0)^2 + 4(0) - 6(0) + 4}$

$= \sqrt{4} = 2$

In $\triangle$ OAP, $\tan\theta = \frac{OA}{PA} = \frac{3}{2}$

Now, the angle between the tangent drawn from origin

$$2\theta = \tan^{-1}\left(\frac{2\tan\theta}{1 - \tan^2\theta}\right)$$

$$2\theta = \tan^{-1}\left(\frac{2 \times \frac{3}{2}}{1 - \frac{9}{4}}\right) \Rightarrow 2\theta = \tan^{-1}\left(\frac{-12}{5}\right)$$

133. **(d)** We have,

$C_1: x^2 + y^2 - 2x - 4y + c = 0$ ……(i)

$C_2: x^2 + y^2 - 4x - 2y + 4 = 0$ ……(ii)

So, centre of circle $C_1 = (1,2), r_1 = \sqrt{5 - c}$

and centre of circle $C_2 = (2,1), r_2 = 1$

Now, angle between the two circles,

$$\cos\theta = \frac{(C_1C_2)^2 - (r_1^2 + r_2^2)}{2r_1r_2}$$

$$\cos\theta = \frac{[(2-1)^2 + (1-2)^2] - (5 - c + 1)}{2\sqrt{(5-c)}(1)}$$

Given, $\theta = 60^\circ$

$$\cos 60^\circ = \frac{1 + 1 - 5 + c - 1}{2\sqrt{5 - c}}$$

$$\frac{1}{2} = \frac{c - 4}{2\sqrt{5 - c}} \Rightarrow \sqrt{5 - c} = c - 4$$

On squaring both sides, we get

$$5 - c = c^2 - 8c + 16 \Rightarrow c = \frac{7 \pm \sqrt{5}}{2}$$

134. **(a)** Let the circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$

$x^2 + y^2 - 4x - 2y + 4 = 0$

$x^2 + y^2 - 2x - 4y + 1 = 0$

and $x^2 + y^2 + 4x + 2y + 1 = 0$, respectively.

$2g(2) + 2f(1) = c + 4$

$\Rightarrow 4g + 2f = c + 4$ ….(i)

$2g(1) + 2f(2) = c + 1$

$\Rightarrow 2g + 4f = c + 1$ ……(ii)

and $2g(-2) + 2f(-1) = 1 + c$

$\Rightarrow -4g - 2f = 1 + c$ ……(iii)

On solving eqs. (i), (ii) and (iii), we get

$g=\frac{3}{4}, f=-\frac{3}{4}, c=-\frac{10}{4}$

Equation of circle,

$$S=x^2+y^2+\frac{3}{2}x-\frac{3}{2}y-\frac{10}{4}=0$$

$$\therefore \text{ Radius } =\sqrt{\left(\frac{3}{4}\right)^2+\left(\frac{-3}{4}\right)^2+\frac{10}{4}}=\sqrt{\frac{29}{8}}$$

135. **(b)** Given equation of parabola

$x^2-2x+3y-2=0$

$\Rightarrow x^2-2x+1=-3y+2+1$

$\Rightarrow (x-1)^2=-3(y-1)$

Distance between the vertex and focus of parabola

$$=\frac{1}{4}\times \text{ Length of latusrectum } =\frac{1}{4}\times 4|a|=\left|\frac{-3}{4}\right|=\frac{3}{4}$$

136. **(c)** Given parabola $y^2=5x$

Let (x_1,y_1) and (x_2,y_2) be the end points of a focal chord.

So, $x_1=\frac{5}{4}t_1^2$ and $y_1=\frac{5}{4}t_1$

Since, $t_1t_2=-1$ as t_1 and t_2 are the end points of a focal chord

$x_2=\frac{5}{4}\left(\frac{1}{t_1^2}\right)$ and $y_2=\frac{5}{2}\left(\frac{-1}{t_1}\right)$

Now, $x_1x_2=\frac{25}{16}$ and $y_1y_2=-\frac{25}{4}$

$$4x_1x_2+y_1y_2=4\left(\frac{25}{16}\right)-\frac{25}{4}=\frac{25}{4}-\frac{25}{4}=0$$

137. **(a)** Given that, $x=3\cos\theta \Rightarrow \frac{x}{3}=\cos\theta$ …..(i)

and $y=4\sin\theta \Rightarrow \frac{y}{4}=\sin\theta$ …..(ii)

On squaring and adding eqs. (i) and (ii) we get

$$\left(\frac{x}{3}\right)^2+\left(\frac{y}{4}\right)^2=\cos^2\theta+\sin^2\theta$$

$$\frac{x^2}{9}+\frac{y^2}{16}=1$$

Distance between their foci,

$f_1f_2=2\sqrt{b^2-a^2}=2\sqrt{16-9}=2\sqrt{7}$

138. **(b)** We have, $\Rightarrow 9x^2+25y^2-36x+50y-164=0$

$\Rightarrow 9(x^2-4x+4)-25(y^2+2y-1)=225$

$$\Rightarrow \frac{(x-2)^2}{25}+\frac{(y+1)^2}{9}=1$$

Eccentricity $e=\sqrt{1-\left(\frac{b}{a}\right)^2}=\sqrt{1-\frac{9}{25}}=\frac{4}{5}$

Equation of the latusrectum,

$x-2=\pm ae$

$$\Rightarrow x-2=\pm 5\times\frac{4}{5}=\pm 4$$

$\Rightarrow x=\pm 4+2 \Rightarrow x=6$ and $x=-2$

$\therefore x-6=0, x+2=0$

139. **(b)** We have, line, $y=mx+2$ ….. (i)

Hyperbola, $4x^2-9y^2=36$ …… (ii)

On solving eqs. (i) and (ii), we get

$x^2(4-9m^2)-36mx-72=0$
Since, this line is a tangent of hyperbola

$D=b^2-4ac=0$

$(36m)^2+4\times 72(4-9m^2)=0$

$\Rightarrow \quad 9m^2=8\Rightarrow m^2=\frac{8}{9}\Rightarrow m=\pm\frac{2\sqrt{2}}{3}$

140. **(b)** Let we points P(2,3,4), A(3, −2,2) and B(6, −17, −4). and P divides AB in the ratio k: 1 then

$$(2,3,4)=\left(\frac{6k+3}{k+1},\frac{-17k-2}{k+1},\frac{-4k+2}{k+1}\right)$$

$$2=\frac{6k+3}{k+1}\Rightarrow k=-\frac{1}{4}$$

Harmonic conjugate Q divides in the ratio −k: 1, So Ratio will be $\frac{1}{4}:1$

Coordinates of Q

$$=\left(\frac{\frac{1}{4}(6)+3}{\frac{1}{4}+1},\frac{\frac{1}{4}(-17)-2}{\frac{1}{4}+1},\frac{\frac{1}{4}(-4)+2}{\frac{1}{4}+1}\right)$$

$$=\left(\frac{18}{5},-5,\frac{4}{5}\right)$$

141. **(b)** Given that, line makes angles α, β, γ, δ with the four diagonals of a cube, then we know that

$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$$

$$1-\sin^2\alpha+1-\sin^2\beta+1-\sin^2\gamma+1-\sin^2\delta=\frac{4}{3}$$

$$\therefore\ \sin^2\alpha+\sin^2\beta+\sin^2\gamma+\sin^2\delta=4-\frac{4}{3}=\frac{8}{3}$$

142. **(c)** Given that equation of plane,

$$56x+4y+9z=2016$$

$$\frac{x}{\frac{2016}{56}}+\frac{y}{\frac{2016}{4}}+\frac{z}{\frac{2016}{9}}=1$$

Also given, this plane meets the coordinate axes at points A, B and C .

Coordinate of A $=\left(\frac{2016}{56},0,0\right)$

Coordinate of B $=\left(0,\frac{2016}{4},0\right)$

Coordinate of C $=\left(0,0,\frac{2016}{9}\right)$

Now, centroid of △ ABC

$$G=\left(\frac{2016}{56\times 3},\frac{2016}{4\times 3},\frac{2016}{9\times 3}\right)=\left(12,168,\frac{224}{3}\right)$$

143. **(b)** Given function,

$$f(x)=\begin{cases}1-x, & x<1\\(1-x)(2-x), & 1\le x\le 2\\3-x & x>2\end{cases}$$

For $f(x)$ to be continuous at $x=1$.

$$\lim_{x\to 1^+}f(x)=\lim_{x\to 1^+}f(x)=f(1)$$

$$\Rightarrow\ \lim_{x\to 1^+}(1-x)=\lim_{x\to 1^+}(1-x)(2-x)$$

$$(1-1)=(1-1)(2-1)=0$$

and $f(1)=(1-1)(2-1)=0$

$f(x)$ is continuous at $x=1$.

Now, at $x=2$, $\lim_{x\to 2^-}f(x)=\lim_{x\to 2^-}f(x)=f(2)$

$\Rightarrow \lim_{x\to 2^-}(1-x)(2-x) = \lim_{x\to 2^+}(3-x)$

$\Rightarrow (1-2)(2-2) = (3-2)$

$\Rightarrow 0 \neq 1$

$f(x)$ is discontinuous at $x = 2$.

144. **(a)** Let,

$$l = \lim_{x\to 0}\frac{6^x - 3^x - 2^x + 1}{x^2} = \lim_{x\to 0}\frac{3^x \cdot 2^x - 3^x - 2^x + 1}{x^2}$$

$$= \lim_{x\to 0}\frac{3^x(2^x-1) - 1(2^x-1)}{x^2} = \lim_{x\to 0}\frac{(2^x-1)(3^x-1)}{x \times x}$$

$$= \lim_{x\to 0}\frac{2^x-1}{x} \times \lim_{x\to 0}\frac{3^x-1}{x} = (\log a2)(\log e3)$$

145. **(a)** Given function,

$$f(x) = \begin{cases} x^2 + bx + c, & x < 1 \\ x, & x \geq 1 \end{cases}$$

$$f'(x) = \begin{cases} 2x + b, & x < 1 \\ 1, & x \geq 1 \end{cases}$$

Since, f(x) is differentiable at x = 1.

$\lim_{x\to 1^+} f'(x) = \lim_{x\to 1^+} f'(x) \Rightarrow \lim_{x\to 1^+}(2x+b) = \lim_{x\to 1^+} 1$

$2 + b = 1 \Rightarrow b = -1$

As form is differentiable at $n = 1$So, it will be continuous at $x = 1$ also.

$\Rightarrow \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} f'(x) = f(1)$

$\Rightarrow \lim_{x\to 1^+} x^2 + bx + c = \lim_{x\to 1^+} x = 1$

$\Rightarrow 1 + b + c = 1 \Rightarrow c = 1$

Hence, $b - c = -1 - 1 = -2$

146. **(d)** Given that, $x = a$ is a root of multiplicity two of a polynomial equation $f(x) = 0$.

Let $f(x) = (x-a)g(x)$

On differentiating w.r.t. x, we get

$\Rightarrow f'(x) = 2(x-a)g(x) + (x-a)^2 g'(x)$

Again, differentiating w.r.t x, we get

Now, $f''(x) = 2g(x) + 2(x-a)g'(x) + 2(x-a)g'(x) + (x-a)^2 g''(x)$

$= 2g(x) + 4(x-a)g'(x) + (x-a)^2 g''(x) + (x-a)^2 g''(x)$

At $x = a$,

At $x = a$

$\Rightarrow f'(a) = 2(a-a)g(a) + (a-a)^2 g'(a) = 0$

$\Rightarrow f''(a) = 2g(a) + 4(a-a)g'(a) + (a-a)^2 g''(a)$

$= 2g(a)$

Hence, $f(a) = f'(a) = 0, f''(a) \neq 0$

147. **(c)** We have, $y = \log_2(\log_2 x)$

$$y = \log_2\left(\frac{\log x}{\log 2}\right) \left\{\because \log_a b = \frac{\log b}{\log a}\right\}$$

$$\therefore y = \frac{\log \frac{\log x}{\log 2}}{\log 2} \Rightarrow y = \frac{\log(\log x) - \log(\log 2)}{\log 2}$$

On differentiating w.r.t x, we get

$$\therefore \frac{dy}{dx} = \frac{1}{\log 2}\left[\frac{1}{\log_e x} \times \frac{1}{x} - 0\right] = \frac{1}{\log 2 \cdot \log_e x \cdot x}$$

$$\frac{dy}{dx} = \frac{1}{(x\log_e x)\log_e 2}$$

148. **(b)** Given curves, $y^2 + x^2 = a^2\sqrt{2}$ .....(i)

and $x^2 - y^2 = a^2$ ......(ii)

On solving eqs. (i) and (ii), we get the point of intersection

$$x = a\sqrt{\frac{\sqrt{2}+1}{2}}, y = a\sqrt{\frac{\sqrt{2}-1}{2}}$$

Now, for curve $-1: y^2 + x^2 = a^2\sqrt{2}$

$$2y\left(\frac{dy}{dx}\right)_1 + 2x = 0 \Rightarrow \left(\frac{dy}{dx}\right)_1 = -\frac{x}{y} = m_1$$

$$m_1 = \frac{-a\sqrt{\frac{\sqrt{2}+1}{2}}}{a\sqrt{\frac{\sqrt{2}-1}{2}}} = -\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$$

and curve -2: $x^2 - y^2 = a^2$

On differentiating

$$\left(\frac{dy}{dx}\right)_2 = \frac{x}{y} \Rightarrow m_2 = -\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}$$

Then, $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

$$= \frac{-\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} - \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}}{1 + \left(-\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}\right)\left(\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}\right)} = \frac{-2\sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}}}{1 - \frac{\sqrt{2}+1}{\sqrt{2}-1}}$$

$$\tan\theta = \frac{-2\sqrt{(\sqrt{2}+1)(\sqrt{2}-1)}}{\sqrt{2}-1-\sqrt{2}-1} = \frac{-2\sqrt{2-1}}{-2} = 1$$

$$\theta = \frac{\pi}{4}$$

149. **(b)** Given, $A + B = \frac{\pi}{3}$

Let $y = \tan A \tan B \Rightarrow y = \tan A \tan\left(\frac{\pi}{3} - A\right)$

Differentiating w.r.t A , we get

$$\Rightarrow \frac{dy}{dA} = \sec^2 A \tan\left(\frac{\pi}{3} - A\right) - \sec^2\left(\frac{\pi}{3} - A\right)\tan A$$

For maxima or minima, $\frac{dy}{dA} = 0$

$$\sec^2 A\tan\left(\frac{\pi}{3}-A\right)-\sec^2\left(\frac{\pi}{3}-A\right)\tan A=0$$
$$\Rightarrow (1+\tan^2 A)\tan\left(\frac{\pi}{3}-A\right)$$
$$=\tan A\left[1+\tan^2 A\left(\frac{\pi}{3}-A\right)\right]$$
$$\Rightarrow \tan\left(\frac{\pi}{3}-A\right)+\tan^2 A\tan\left(\frac{\pi}{3}-A\right)$$
$$=\tan A+\tan A\tan^2\left(\frac{\pi}{3}-A\right)$$
$$\Rightarrow \left[\tan\left(\frac{\pi}{3}-A\right)-\tan A\right]\cdot\left[1-\tan A\tan\left(\frac{\pi}{3}-A\right)\right]=0$$
$$\Rightarrow \tan\left(\frac{\pi}{3}-A\right)=\tan A$$
$$\Rightarrow \frac{\pi}{3}-A=A\Rightarrow A=\frac{\pi}{6}$$
$$\therefore A=B=\frac{\pi}{6}$$

Maximum value of $\tan A\tan B=\tan\frac{\pi}{6}\tan\frac{\pi}{6}=\frac{1}{3}$

150. **(c)** Let $P\left(t,\frac{-1}{t}\right)$ be a point on $xy=-1$

Equation of tangent of P

$$=\frac{1}{2}\left[x\left(\frac{-1}{t}\right)+xy\right]=-1-x+t^2y=-2t$$
$$y=\frac{x}{t^2}+\frac{2}{t}$$

This is also a tangent of $y^2=8x$

i.e. $c=\frac{a}{m}\Rightarrow\frac{2}{t}=\frac{2}{\frac{1}{t^2}}$

$\Rightarrow t^3=1\Rightarrow t=1$

$\Rightarrow t^3=1\Rightarrow t=1$

Hence, the equation of common tangent

$$y=\frac{x}{1}+\frac{2}{1}\Rightarrow y=x+2$$

151. **(a)** We have,

$$f(x)=x(x+3)(x-2),x\in[-1,4]$$
$$\Rightarrow f(x)=x^3+x^2-6x$$

Differentiating w.r.t x, we get

$$\Rightarrow f'(x)=3x^2+2x-6$$

Given $f(c)=10$

$$\Rightarrow 3c^2+2c-6=10$$
$$\Rightarrow (3c+8)(c-2)=0\Rightarrow c=-\frac{8}{3}\text{ or } c=2$$
$$c=2\in[-1,4]$$

152. **(d)** We have given that,

$$\int x^3e^{5x}dx=\frac{e^{5x}}{5^4}[f(x)]+C$$

By using the method of intergration by parts, we get

$$\int x^3e^{5x}dx=x^3\int e^{5x}dx-\int\left\{\frac{dx^3}{dx}\int e^{5x}dx\right\}dx$$
$$=\frac{x^3e^{5x}}{5}-\int\frac{3x^2e^{5x}}{5}dx+C$$

By using the method of integration by parts, we get

$$= \frac{x^3e^{5x}}{5} - \frac{3}{5}x^2\int e^{5x}dx + \frac{3}{5}\int\left\{\frac{dx^2}{dx}\int e^{5x}dx\right\}dx + C$$

$$= \frac{x^3e^{5x}}{5} - \frac{3x^3e^{5x}}{25} + \frac{6}{25}\int xe^{5x}dx$$

$$= \frac{x^3e^{5x}}{5} - \frac{3x^3e^{5x}}{25} + \frac{6x}{25}\int e^{5x}dx - \frac{6}{25}\int\left\{\frac{dx}{dx}\int e^{5x}dx\right\}dx$$

$$= \frac{x^3e^{5x}}{5} - \frac{3x^3e^{5x}}{25} + \frac{6xe^{5x}}{125} - \frac{6}{125}\int e^{5x}dx$$

$$= \frac{x^3e^{5x}}{5} - \frac{3x^3e^{5x}}{25} + \frac{6xe^{5x}}{125} - \frac{6}{125} + C$$

$$= \frac{e^{5x}}{625}[125x^3 - 75x^2 + 30x - 6] + C$$

Comparing the above equation with

$\frac{e^{5x}}{5^4}[f(x)] + C$, we get

$f(x) = 5^3x^3 - 75x^2 + 30x - 6$

153. (c)

$$\text{Let } I = \int \frac{x}{(x^2+2x+2)^2}dx$$

$$I = \int \frac{x}{[(x+1)^2+1]^2}dx$$

Put $x + 1 = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$

$$I = \int \frac{(\tan\theta - 1)\sec^2\theta}{(\tan^2\theta+1)^2}d\theta = \int \frac{(\tan\theta-1)\sec^2\theta}{\sec^4\theta}d\theta$$

$$= \int\left(\frac{\tan\theta}{\sec^2\theta} - \frac{1}{\sec^2\theta}\right)d\theta$$

$$= \int(\sin\theta\cos\theta - \cos^2\theta)d\theta$$

$$= \frac{1}{2}\int(\sin 2\theta - 1 - \cos 2\theta)d\theta$$

$$= \frac{1}{2}\left(\frac{-\cos 2\theta}{2} - \theta - \frac{\sin 2\theta}{2}\right) + C$$

$$= -\frac{1}{4}[\cos 2\theta + \sin^2\theta] - \frac{1}{2}\theta + C$$

$$= -\frac{1}{4}\left[\frac{1-\tan^2\theta}{1-\tan^2\theta} + \frac{2\tan\theta}{1+\tan\theta}\right] - \frac{1}{2}\tan^{-1}(x+1) + C$$

$$= -\frac{1}{4}\left[\frac{1-\tan^2\theta+2\tan\theta}{1+\tan^2\theta}\right] - \frac{1}{2}\tan^{-1}(x+1) + C$$

$$= -\frac{1}{4}\left[\frac{1-(x+1)^2+2(x+1)}{1+(x+1)^2}\right] - \frac{1}{2}\tan^{-1}(x+1) + C$$

$$I = -\frac{1}{4}\left[\frac{x^2-2}{x^2+2x+2}\right] - \frac{1}{2}\tan^{-1}(x+1) + C$$

154. (c) Let $I = \int \log(a^2 + x^2)dx$

By using method of integration by parts, we get

$$= \log(a^2+x^2)\int dx - \int\left\{\frac{d(\log(a^2+x^2)}{dx}\int dx\right\}dx + C$$

$$= \log(a^2+x^2)\int \frac{2x}{x^2+a^2}\cdot xdx + C$$

$$= x\log(a^2+x^2) - 2\int \frac{x^2}{x^2+a^2}dx + C$$

$$= x\log(a^2+x^2) - 2\int \frac{x^2+a^2}{x^2+a^2}dx + 2\int \frac{a^2}{x^2+a^2}dx$$

$$= x\log(a^2+x^2) - \frac{2a^2}{a}\tan^{-1}\frac{x}{a} + C$$

$$I = x\log(a^2+x^2) - 2x + 2a\tan^{-1}\frac{x}{a} + C$$

Comparing the above equation with $\int \log(x^2+a^2)dx = h(x) + C$, we get

$h(x) = x\log(a^2+x^2) - 2x + 2a\tan^{-1}\left(\frac{x}{a}\right)$

155. **(a)** Given that,

$$\int (\log x)^5 dx = x[A(\log x)^5 + B(\log x)^4]$$
$$+C(\log x)^3 + D(\log x)^2 + E(\log x) + F] + C$$

Let $I = \int (\log x)^5 dx$

Put $\log x = t \Rightarrow x = et \Rightarrow dx = etdt$

$I = \int t^5 e^5 dt$

Use by part formula,

$I = et[t^5 - 5t^4 + 20t^3 - 60t^2 + 120t - 120] + C$

Now, put $t = \log x$

$$I = x[(\log x)^5 - 5(\log x)^4 + 20(\log x)^3 - 60(\log x)^2$$
$$+ 120(\log x) - 120] + C$$
$$A + B + C + D + E + F$$
$$= 1 - 5 + 20 - 60 + 120 - 120 = -44$$

156. **(d)** $y = \frac{x^2}{4a}$ ......(i)

$y = \frac{8a^3}{x^2+4a^2}$ ......(ii)

For point of intersection equation (i) and (ii),

$$\frac{x^2}{4a} = \frac{8a^3}{x^2+4a^2}$$
$$\Rightarrow x^4 + 4a^2x^2 - 32a^2 = 0$$
$$\Rightarrow (x^2-4a^2)(x^2+8a^2) = 0$$
$$\Rightarrow x^2 - 4a^2 = 0 \text{ or } x^2 + 8a^2 = 0$$
$$\Rightarrow x^2 = 4a^2\ [x^2 = -8a^2 \text{ is not possible }]$$
$$\Rightarrow x = \pm 2a$$

So, we take limits from 0 to 2a.

Now, Area enclosed by the two curves

$$A = 2\times\int_0^{2a}\left[\frac{8a^3}{(x^2+4a^2)} - \frac{x^2}{4a}\right]dx$$

$$= 2 \times \left[\int_0^{2a} \frac{8a^3}{(x^2+4a^2)} dx - \int_0^{2a} \frac{x^2}{4a} dx\right]$$

$$= 2 \times \left[8a^3 \times \int_0^{2a} \frac{1}{x^2+(2a^2)} dx - \frac{1}{4a}\int_0^{2a} x^2 dx\right]$$

$$= 2 \times \left\{8a^3\left[\frac{1}{2a}\tan^{-1}\left(\frac{x}{2a}\right)\right]_0^{2a} - \frac{1}{4a}\left[\frac{x^3}{3}\right]_0^{2a}\right\}$$

$$= 2 \times \left\{\frac{8a^3}{2a}\left[\tan^{-1}\left(\frac{2a}{2a}\right) - \tan^{-1}\left(\frac{0}{2a}\right)\right]\right.$$

$$\left. - \frac{1}{4a}\left[\frac{(2a)^3}{3} - \frac{(0)^3}{3}\right]\right\}$$

$$= 2 \times \left\{4a^2[\tan^{-1}(1) - \tan^{-1}(0)] - \frac{1}{4a}\left(\frac{8a^3}{3} - 0\right)\right\}$$

$$= 2 \times \left\{4a^2\left(\frac{\pi}{4} - 0\right) - \frac{1}{4a}\left(\frac{8a^3}{3}\right)\right\} = 2\left\{a^2\left(\pi - \frac{2}{3}\right)\right\}$$

157. **(b)** Given that,

$$= \lim_{n\to\infty}\left\{\frac{1}{\sqrt{n^2-1^2}} + \frac{1}{\sqrt{n^2-2^2}} + \cdots + \frac{1}{\sqrt{n^2-(n-1)^2}}\right\}$$

$$= \lim_{n\to\infty}\left\{\frac{1}{n\sqrt{1-\left(\frac{1}{n}\right)^2}} + \frac{1}{\sqrt{1-\left(\frac{2}{n}\right)^2}} + \cdots + \frac{1}{n\sqrt{1-\left(\frac{n-1}{n}\right)^2}}\right\}$$

$$= \lim_{n\to\infty}\left\{\frac{1}{\sqrt{1-\left(\frac{1}{n}\right)^2}} + \frac{1}{\sqrt{1-\left(\frac{2}{n}\right)^2}} + \cdots + \frac{1}{\sqrt{1-\left(\frac{n-1}{n}\right)^2}}\right\}$$

$$= \lim_{n\to\infty} \frac{1}{\sum_{r=1}^{n-1} \frac{1}{n\sqrt{1-\left(\frac{1}{n}\right)^2}}} = \int_0^1 \frac{dx}{\sqrt{1-x^2}} = [\sin^{-1} x]_0^1 = \frac{\pi}{2}$$

158. **(d)** Given, $I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x+\frac{\pi}{4}}{2-\cos 2x} dx$

$$I = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x}{2-\cos 2x} dx + \frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x} dx$$

Let, $I_1 = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{x}{2-\cos 2x} dx$

$$f(-x) = \frac{-x}{2-\cos 2(-x)} = \frac{-x}{2-\cos 2x} = -f(x)$$

$f(x)$ is an odd function. Thus, $I_1 = 0$

Let $I_1 = \frac{\pi}{4}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2-\cos 2x} dx$

$$g(-x) = \frac{\pi}{4(2-\cos 2(-x))} = \frac{\pi}{4(2-\cos 2x)} = g(x)$$

$g(x)$ is an even function

Thus, $I_2 = 2 \times \frac{\pi}{4}\int_0^{\frac{\pi}{4}} \frac{dx}{2-\cos 2x}$

Now, $I = I_2 = \frac{\pi}{4}\int_0^{\frac{\pi}{4}} \frac{dx}{2-\frac{1-\tan^2 x}{1+\tan^2 x}}$

$\mathrm{I}=\frac{\pi}{4}\int_0^{\frac{\pi}{4}}\frac{\sec^2 x}{1+3\tan^2 x}dx$

Put $\tan x=t \Rightarrow \sec^2 x dx=dt$

$\Rightarrow \mathrm{I}=\frac{\pi}{4}\int_0^1\frac{dt}{1+3t^2}=\frac{\pi}{2}\times\frac{1}{\sqrt{3}}\left[\tan^{-1}\sqrt{3}t\right]_0^1$

$I=\frac{\pi}{2\sqrt{3}}\left[\tan^{-1}(\sqrt{3})-\tan^{-1}(0)\right]=\frac{\pi^2}{6\sqrt{3}}$

159. **(c)** Given differential equation,

$$(1+y^2)\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

$$P=\frac{x}{1+y^2}, Q=\frac{e^{\tan^{-1}y}}{1+y^2}$$

Now, integration factor,

$\mathrm{IF}=e^{\int pdy}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$

The solution of differential equation,

$$x.\mathrm{IF}=\int Q.\mathrm{IF}dy+C$$

$$\Rightarrow xe^{\tan^{-1}y}=\int\frac{e^{2\tan^{-1}y}}{1+y^2}dy+C$$

$$\Rightarrow xe^{\tan^{-1}y}=\frac{e^{2\tan^{-1}y}}{2}+C$$

$$\Rightarrow 2xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+C_1, C_1=2C$$

160. **(d)** Given differential equation,

$$(2x-4y+3)\frac{dy}{dx}+(x-2y+1)=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{(x-2y+1)}{2(x-2y+3)} \quad \ldots\ldots(i)$$

Let, $x-2y=v \Rightarrow 1-2\frac{dy}{dx}=\frac{dv}{dx}$

$$\frac{1}{2}\left(1-\frac{dy}{dx}\right)=\frac{dv}{dx}$$

Now, substitute $v=x-2y$ and $\frac{dy}{dx}$ in eq. (i) integration both the sides, we get

$$\therefore \frac{1}{2}\left(1-\frac{dv}{dx}\right)=-\left(\frac{v+1}{2v+3}\right)$$

$$\Rightarrow \frac{dv}{dx}=1+\frac{2v+2}{2v+3}\Rightarrow\frac{2v+2}{4v+5}dv=dx$$

$$\frac{1}{2}\int\frac{4v+6}{4v+5}dv=\int dx$$

$$\Rightarrow\frac{1}{2}\int\left(\frac{4v+6}{4v+5}+\frac{1}{4v+5}\right)dv=\int dx$$

$$\Rightarrow\frac{1}{2}v+\frac{1}{2\times 4}\log[4v+5]=x+C$$

$$\Rightarrow 4v+\log[4v+5]=8x+C$$

$$\Rightarrow 4(x-2y)+\log[4(x-2y)+5]=8x+C$$

$$\Rightarrow \log[4(x-2y)+5]=4(x+2y)+C$$