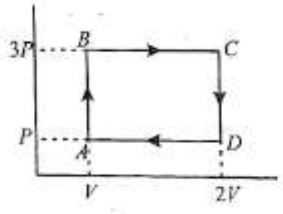
Physics

1. (c)



Work done in complete cycle

$= \text{Area under closed curve} = 2PV$

Heat given to the system from A to $B = nC_V\Delta T$

$= n\frac{3}{2}R\Delta T = \frac{3}{2} \times V \times \Delta P = \frac{3}{2} \times V \times (3P - P) = 3PV$

Similarly, heat given to the system from B to C

$$= nC_P\Delta T$$

$$= n\left(\frac{5}{2}R\right)\Delta T = \frac{5}{2}(3P)(\Delta V)$$

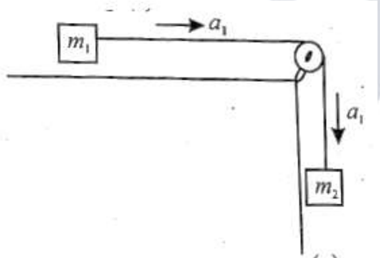
$$= \frac{5}{2} \times (3P) \cdot (2V - V) = \frac{15}{2}PV$$

The heat is released from C to D and D to A.

$$\text{Efficiency } (\eta) = \frac{\text{Work done by gas}}{\text{Heat given to gas}} \times 100$$

$$= \frac{2PV}{3PV + \frac{15}{2}PV} \times 100 = \frac{4}{21} \times 100 = 19.04\%$$

2. (c) For fig. (a)



(a)

$$m_2g - \mathrm{T} = m_2a_1 \quad \ldots..(i)$$

$$T = m_1a_1 \quad \ldots\ldots..(ii)$$

From $(i)\&(ii)$, we get

$$a_1 = \frac{m_2g}{m_1 + m_2}$$

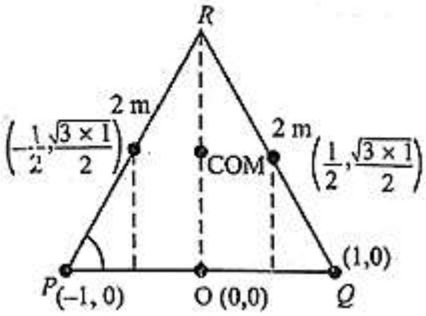
as $m_1 = 3$ kg, $m_2 = 4$ kg

$$a_1 = \frac{4 \times 10}{3 + 4} = \frac{40}{7} \text{ m/s}^2$$

For fig. (b)

$$a = \left(\frac{m_2 - m_1}{(m_1 + m_2)}\right)g = \frac{10}{7} \text{ m/s}^2$$

3. (d)



Initially,

$$\mathbf{r}_{\text{COM}} = \frac{m_1\boldsymbol{r}_1 + m_2\boldsymbol{r}_2 + m_3 r_3}{(m_1 + m_2 + m_3)}$$

$$\Rightarrow \mathbf{r}_{\text{COM}} = \frac{1}{3}\left(\frac{\sqrt{3}}{2} \times 1 \times 2\right)\hat{\mathbf{j}} \Rightarrow \mathrm{r}_{\text{COM}} = \frac{1}{\sqrt{3}}\hat{\mathbf{j}}$$

Final length,

$l_f = l_i(1 + \alpha\Delta T)$

or, $l_f = l_i\left(1 + \frac{\gamma}{3}\Delta T\right)$ $[\gamma = 3\alpha]$

or, $l_f = 2\left(1 + \frac{12\sqrt{3}\times10^{-6}}{3} \times 50\right)$

or, $l_f = 2(1 + 4 \times 50 \times \sqrt{3} \times 10^{-6})$

or, $l_f = 2(1 + 2\sqrt{3} \times 10^{-4})$

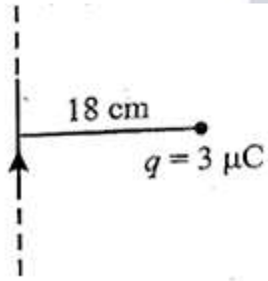
So, $(\mathbf{r}_{\text{COM}})_{\text{final}} = \frac{1}{3} \times \frac{\sqrt{3}}{2} \times (1 + 2\sqrt{3} \times 10^{-4}) \times 2\hat{\mathbf{j}}$

$= \left(\frac{1}{\sqrt{3}} + 2 \times 10^{-4}\right)\hat{\mathbf{j}}$

So, $\Delta y = (\mathrm{r}_{\text{COM}})_{\text{final}} - (\mathrm{r}_{\text{COM}})_{\text{initital}}$

$= 2 \times 10^{-4}\ \mathrm{m} = 0.2\ \mathrm{mm}$

4. (b)



$$E = \frac{\lambda}{2\pi\varepsilon_0 \cdot r} = \frac{2\lambda}{4\pi\varepsilon_0 \cdot r} = \frac{9 \times 10^9 \times 2 \times 1}{3 \times 18 \times 10^{-2}}$$

$$= \frac{1}{3} \times 10^{11}\ \mathrm{N/C}$$

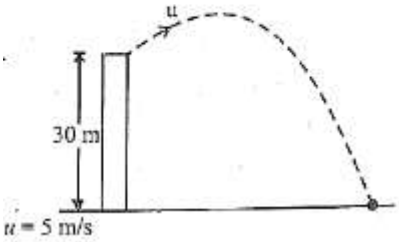
So, $F = qE = 3 \times 10^{-6} \times \frac{1}{3} \times 10^{11} = 10^5\ \mathrm{N}$

5. (a) As, $\mathrm{V} = \frac{\mathrm{kq}}{\mathrm{r}}$

So, $\mathrm{q} = \frac{\mathrm{V}\times\mathrm{r}}{\mathrm{k}} = \frac{25\times10^3\times9\times10^{-2}}{9\times10^9}$

$= 25 \times 10^{-8}\mathrm{C} = 0.25\mu\mathrm{C}$

6. (a)



($\hat{\mathbf{k}}$ is given vertically upward direction)

$a = -10\ m/s^2$

$h = -30\ m$

As, $S = ut + \frac{1}{2}at^2 \Rightarrow -30 = 5t - \frac{1}{2} \times 10 \times t^2$

$\Rightarrow t^2 - t - 6 = 0$

$\Rightarrow t = -2s$ (Not possible)

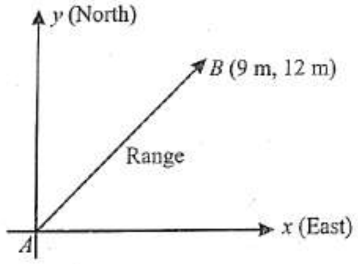
or, $t = 32$ s. This is time in which body reaches the ground

In this time, projectile moving in East and North with speeds $3\ m/s$ and $4\ m/s$. Distances covered in these directions are;

In east (x-coordinate) $= 3 \times 3 = 9$ m and in North (y-coordinate) $= 4 \times 3 = 12$ m

So, Projectile land at $(x, y) \equiv (9\ m, 12\ m)$ mark.

So, horizontal range of body on ground is:



Range $= \sqrt{9^2 + 12^2} = 15$ m

7. (c) When conducting wire is displaced by (x) dx distance, then work done per unit length is;

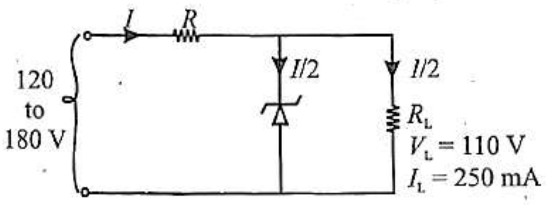
$dW = F/\ell . dx$

or, $dW = \frac{\mu_0 I_1 I_2}{2\pi x} dx \left[\because \frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi x}\right]$

or, $\int dW = \int \frac{\mu_0 I_1 I_2}{2\pi x} dx$ or, $W = \left(\frac{\mu_0 I_1 I_2}{2\pi}\right) \log_e x$

or, $W \propto \log_e x$

8. (b)



We have, $R_L = \frac{V_L}{I_L} = \frac{110}{250 \times 10^{-3}} = 440\Omega$

As current is equally shared by Zener diode and resistance R. So, $I_Z = 250$ mA

Then, $I = 2 \times 250 = 500$ mA $[\because I = I_L + I_Z] = 0.5$ mA

So, $R_s = \frac{V_S}{I} = \frac{(180-110)}{0.5} = \frac{70}{0.5} = 140\Omega$

[when current is maximum $V = 180$ V]

9. (d) We have, $y = Px - Qx^2$ .....(i)

Equation of trajectory is

$y = x\tan\theta - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$ ....(ii)

After comparing Eqs. (i) and (ii), we get

$P = \tan\theta$ ... (iii)

$Q = \dfrac{g}{2u^2\cos^2\theta}$ ... . (iv)

We know that

$H = \dfrac{u^2\sin^2\theta}{2g} = \dfrac{u^2\sin\theta\sin\theta}{2g\times 2\cos\theta}\times 2\cos\theta$

$\Rightarrow H = \dfrac{u^2\sin 2\theta}{g}\times\dfrac{\tan\theta}{4}$

$\Rightarrow H = \dfrac{R\tan\theta}{4} \Rightarrow \dfrac{H}{R} = \dfrac{\tan\theta}{4}$

$\Rightarrow \dfrac{H}{R} = \dfrac{P}{4}$ $[\because \tan\theta = p]$

10. (d) Transverse displacement

$y_{(x,t)} = 0.03\sin\left(\dfrac{2\pi x}{3}\right)\cos 60\pi t.$

This is equation of standing wave.

The standard equation of standing wave is

$y = a\sin kx\cos\omega t$

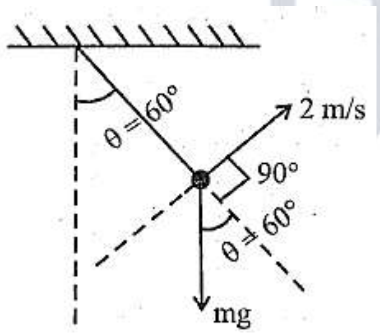
Here, $\omega = 60\pi, k = 2\pi/3$

So, $V = \frac{60\pi}{2\pi}\times 3 = 90$

So, $\sqrt{\frac{T}{\mu}} = 90$

$\Rightarrow T = 90^2\mu \Rightarrow T = 90^2\times 10 = 81$ N

11. (c)

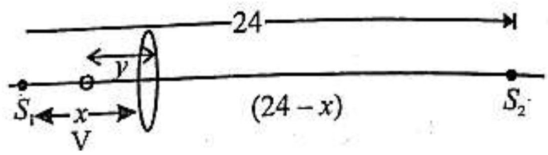


$P = \vec{F}\cdot\vec{V} = mgv\cos 150°$

$= 50\times 9.8\times 2\cos 150° = -490\sqrt{3}$

$P = 490\sqrt{3}$ watt

12. (c)



The given condition will be satisfied only if one source (S_1) placed on one side such that $u < f$ (i.e. it lies under the focus. The other source (S_2) is placed on the other side of the lens such that $u > f$ (i.e. it lies beyond the focus).

If S_1 is the object for the lens, then

$\frac{1}{f} = \frac{1}{-y} - \frac{1}{-x}$ ....(i)

If S_2 is the object for the lens, then

$\frac{1}{f} = \frac{1}{+y} - \frac{1}{-(24-x)}$ ....(ii)

Adding (i) & (ii), we get

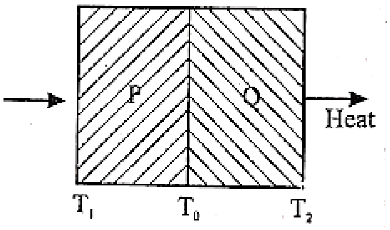
$$\frac{1}{x}+\frac{1}{(24-x)}=\frac{2}{f}=\frac{2}{7}$$
$$\Rightarrow x^2-24x+108=0 \Rightarrow x=18\text{ cm and } x=6\text{ cm}$$
So, the correct answer will be 6 cm .

13. (b) (A) It is more difficult to push a magnet into a coil with large turns because of opposition caused by induced current.

(B) The emf induced in a coil always opposes the motion of a magnet when it is moved towards the coil. It was experimentally verified by denz law.

So both A and R are true. R is correct explanation of A.

14. (d) Here,



We have,
$$K_Q=\frac{K_P}{2} \Rightarrow K_P=2K_Q$$
$$\frac{2K_Q a(T_1-T_0)}{x}=\frac{K_Q a(T_0-T_2)}{x}$$
$$\left[\because \left[\frac{Q}{t}\right]_P=\left[\frac{Q}{t}\right]_Q\right]$$
$$2(T_1-T_0)=T_0-T_2$$
$$\Rightarrow\ 2T_1-3T_0+T_2=0 \qquad \ldots\ldots\ldots\text{(i)}$$
According to the question,
$T_1-T_2=24°\text{C}$
Now, from Eq. (i), ...(ii)
$$2T_1-3T_0+T_1-24=0$$
$$\Rightarrow\ 3T_1-3T_0=24 \Rightarrow T_1-T_0=8°\text{C}$$

15. (b)
$$\text{r}=\text{R}\left(\frac{l_1}{l_2}-1\right)$$
This is value of internal resistance here.
$$\frac{\Delta \text{r}}{\text{r}}\times 100=\frac{\Delta \text{R}}{\text{R}}\times 100+\frac{\Delta\left(\frac{l_1}{l_2}\right)}{\left(\frac{l_1}{l_2}\right)}\times 100$$
$$=4+0.2=4.2\%$$

16. (a)
$$\text{As, K.E.}=\frac{1}{2}\text{mv}^2 \qquad \Rightarrow\ \text{V}=\sqrt{\frac{2\times \text{K}\cdot\text{E.}}{\text{m}}}$$
$$\Rightarrow\ \text{V}=\sqrt{\frac{2\times 4\times 16\times 10^{-19}}{32\times 10^{-21}}}=\sqrt{\frac{2\times 4}{2\times 10^{-2}}}$$
$$\Rightarrow\ \text{V}=20\text{ m/s}$$
Time to travel a distance of 3.6 km is
$$\text{t}=\frac{\text{D}}{\text{v}}=\frac{3.6\times 1000}{20}\text{s}=\frac{3.6\times 1000}{20\times 60}\text{ min}$$
$$=3\text{ min}=3\text{ half-time}$$
In three half life times, the number of particles decayed

$$= N_0 - \frac{N_0}{2^3} = \frac{7}{8}\,N_0$$

$$\%\text{ req.} = \frac{\frac{7}{8}}{N_0}\,N_o \times 100 = 87.5\%$$

17. (b) After modulation, recovery of signal is done by demodulating it.

18. (a) At the position when the gravitational field is zero.

(Distance x is measured from 4 m mass)

$$\frac{G(4\text{ m})}{x^2} = \frac{G(9\text{ m})}{(r-x)^2}$$

$$\frac{4}{9} = \left(\frac{x}{r-x}\right)^2 \Rightarrow \frac{2}{3} = \frac{x}{r-x}$$

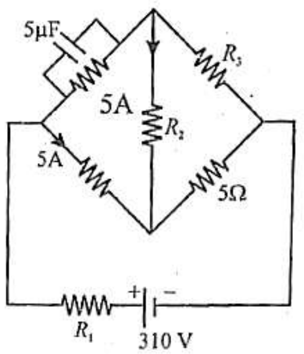
$$2r - 2x = 3x \Rightarrow 2r = 5x \Rightarrow x = \frac{2r}{5}$$

The point P is at a distance $\frac{2r}{5}$ from mass 4 m and $\left(r - \frac{2}{5}r\right) = \frac{3r}{5}$ from mass m.

$$v = -\frac{G(4\text{ m})}{\left(\frac{2}{5}r\right)} - \frac{G(9\text{ m})}{\frac{3r}{5}} = \frac{-5Gm}{r}\left[\frac{4}{2} + \frac{9}{3}\right]$$

$$= \frac{-5Gm}{r}[2+3] = -25\frac{Gm}{r}$$

19. (c)



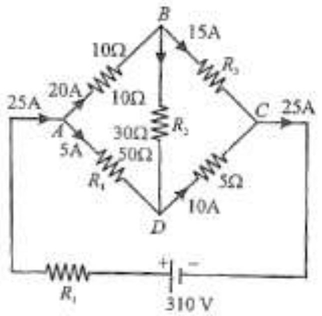
Charge on cap = 1mC

$$V_{\text{capacitor}} = \frac{Q}{C} = \frac{1\times10^{-3}}{5\times10^{-6}} = \frac{1000}{5} = 200\text{ V}$$

So, current through R_5

$$I = \frac{V}{R} = \frac{200}{10} = 20\text{ A}$$

Hence, we have following current distribution.



In loop $BCDB$,

$$-15R_3 + 10\times5 + 5R_2 = 0 \qquad \text{..... (i)}$$

In loop $ABCDA$,

$$-200 - 15R_3 + 50 + 250 = 0$$

$$15R_3 = 100 \Rightarrow R_3 = \frac{100}{15}\Omega$$

From Eq. (i), we get
$5R_2 = 50 \Rightarrow R_2 = 10\Omega$
and from loop $ADCA$,

$$-250 - 50 + 310 - 25R_1 = 0$$

$$25R_1 = 10 \Rightarrow R_1 = \frac{10}{25}\Omega$$

So, $\frac{R_1 \times R_2}{R_3} = \frac{\frac{10}{25}\times 10}{\left(\frac{100}{15}\right)} = \frac{15}{25} = \frac{3}{5} = 0.6$

20. (a)

$$B_0 = \frac{E_0}{c}\ [\because c = 3\times 10^8\ \mathrm{m/s}]$$

$$= \frac{60}{3\times 10^8} = 20\times 10^{-8} = 2\times 10^{-7}\ \mathrm{T}$$

21. (a) The total time after the two cars which meet

$$= \frac{\text{Total distance}}{\text{Relative velocity of car w.r.t each other}}$$

$$T = \frac{36}{54+36} \Rightarrow T = \frac{36}{90} = 0.4\mathrm{h}$$

In this duration, the distance travel by bird
$= 36 \times 0.4 = 14.4\ \mathrm{km} = 14400\ \mathrm{m}$

22. (d) According to question, the translation KE of a molecule of gas $= \frac{3}{2}\mathrm{kt}$

According to the question,

$$\frac{3}{2}\mathrm{kt} = \mathrm{eV} \Rightarrow \frac{3}{2}\mathrm{kt} = 10\mathrm{e}$$

$$\Rightarrow\ T = \frac{20\mathrm{e}}{3K_B} = \frac{20\times 1.6\times 10^{-19}}{3\times 1.38\times 10^{-23}}$$

$$= 77.3\times 10^3\ \mathrm{K}$$

23. (b) For closed organ pipe, Ist overtone $=$ 3rd harmonic

$$f = f_2 = 3f_1 = \frac{3\times v}{4L} = \frac{3}{4L}\times\sqrt{\frac{\gamma p}{\rho_1}}$$

For open organ pipe, Ist overtone $=$ 2 nd harmonics

$$f = f_2 = 2f_1 = \frac{2v}{2L'} = \frac{v}{L'} = \frac{1}{L'}\times\sqrt{\frac{\gamma p}{\rho_2}}$$

As B is same. So pressure ($\alpha\beta$) is also same for both gas For same sound frequency

$$\frac{3}{4L}\times\sqrt{\frac{\gamma p}{\rho_1}} = \frac{1}{L'}\times\sqrt{\frac{\gamma p}{\rho_2}}$$

$$L' = \frac{4L}{3}\times\sqrt{\frac{\rho_1}{\rho_2}}$$

24. (d) $V = KT^2 \Rightarrow dV = 2KTdT$

So, $W = \int PdV = R\int \frac{T}{v}dV$

$$= R\int \frac{T}{KT^2}2KT = 2R\int_0^{60} dT = 120R$$

25. (c)

$$I^2R = 240\ \mathrm{W}, I = 4\mathrm{A}$$

$$\therefore\ R = \frac{240}{16} = 15\Omega$$

As, $V = IZ = I\sqrt{X_L^2 + R^2} \Rightarrow \frac{V}{I} = \sqrt{X_L^2 + R^2}$

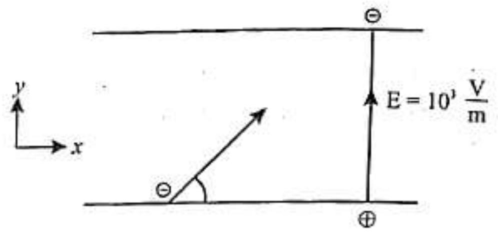
$$\Rightarrow \frac{V^2}{I^2} = X_L^2 + R^2$$

or $X_L^2 = \frac{100\times100}{4\times4} - 225 = 400$

$X_L = 20 = L\omega$

$$L = \frac{20}{2\pi f} = \frac{20}{2\pi \times 50} = \frac{1}{5\pi} H$$

26. (d)



We have,

$$h_{max} = \frac{u^2 \sin^2\theta}{2\left(\frac{qE}{m}\right)} \left[\text{Here } g \to \frac{qE}{m}\right]$$

$$u^2 = \frac{2hmqE}{m\sin^2 45^\circ} u^2 = \frac{2\times 2\times 10^{-2}\times 10^{-6}\times 10^3}{2\times 10^{-3}\times 1/2}$$

$$\Rightarrow u^2 = 0.04 \Rightarrow u = 0.2 \text{ m/s}$$

27. (a) The de-Broglie wavelength of electron

$$\lambda_e = \frac{h}{\sqrt{2mK_e}}$$

K_e = Kinetic energy of electron

$$\lambda_e^2 = \frac{h^2}{2m_eK_e} \Rightarrow K_e = \frac{h^2}{\lambda_e^2 \cdot 2m_e}$$

De-Broglie wavelength of photon is λ_p.

Kinetic energy of photon

$$K_P = h\nu = \frac{hc}{\lambda_P}$$

$$\frac{K_e}{K_P} = \frac{h^2/\lambda_e^2 \cdot 2m_e}{hc/\lambda_P}$$

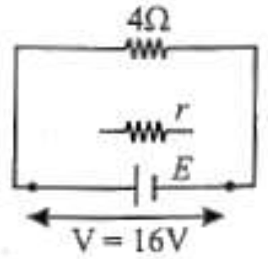
As, $\lambda_e = \lambda_p$

$$\text{Then, } \frac{K_e}{K_P} = \frac{h}{c\lambda_e \times 2m_e}$$

$$= \frac{6.66\times 10^{-34}}{3\times 10^8 \times 1.2\times 10^{-10}\times 2\times 9.1\times 10^{-31}} \cong \frac{1}{100}$$

$K_e : K_P :: 1 : 100$

28. (b) $E = V + Ir$



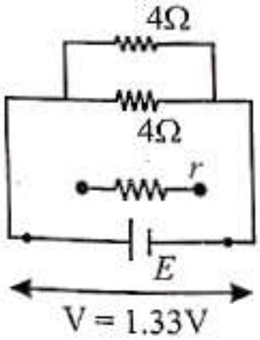
Given, $V = 1.6$ V; $R = 4\Omega$

$$I_1 = \frac{1.6}{4} = 0.4 \text{ A}$$

$$E = 1.6 + 0.4r \quad \dots\dots (i)$$

When a resistance of 4Ω is connected in parallel the resultant resistance

$$\frac{1}{R'} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \Rightarrow R' = 2\Omega$$



$$I_2 = \frac{133}{2} = 0.66 \text{ A}$$

So, E = 1.33 + 0.66r .... (ii)

From Eqs. (i) and (ii), we get

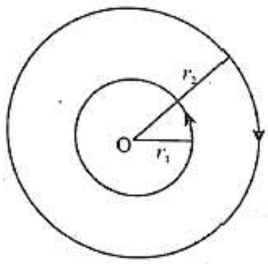
$$1.6 + 0.4r = 1.33 + 0.66r$$

$$\Rightarrow r = \frac{0.27}{0.26} = 1$$

Substituting of r in Eq. (i), we get

E = 1.6 + 0.4 × 1 = 2 V

29. (d)



As $B_{\text{centre}} = \frac{\mu_0 n I}{2r}$

So, Magnetic field at O ,

$$B_O = \frac{\mu_0}{2}\left[\frac{n_1 I_1}{r_1} - \frac{n_2 I_2}{r_2}\right]$$

$$n_1 = n_2 = 20$$

$$B_O = \frac{\mu_0}{4\pi} \cdot 2\pi \times 20\left[\frac{0.4}{30 \times 10^{-2}} - \frac{0.6}{60 \times 10^{-2}}\right]$$

$$= \frac{\mu_0}{2} \times 20\left[\frac{0.8 - -0.6}{60 \times 10^{-2}}\right] = \frac{10}{3}\mu_0$$

30. (b) Length of pendulum $= L$

Time period of simple pendulum

$$T_1 = 2\pi\sqrt{\frac{L}{g}}$$

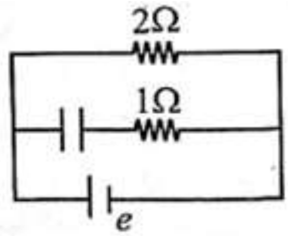
Length of rod $= L$

Time period of rod;

$$T_2 = 2\pi\sqrt{\frac{2L}{3g}}\left[\because T = 2\pi\sqrt{\frac{I}{mgr_{cm}}} \text{ and } r_{cm} = \ell/2\right]$$

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{L/g}}{2\pi\sqrt{2L/3g}} = \sqrt{\frac{3}{2}}$$

31. (c)



$v_C = \frac{Q}{C} = \frac{4 \times 10^{-6}}{2 \times 10^{-6}} = 2\mathrm{v}$

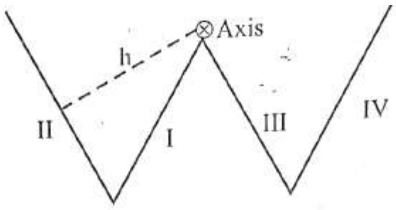
Now, $V_c = V$ (across 2Ω resistor)

$I = \frac{V}{R} = \frac{2}{2} = 1\ \mathrm{A}$

Now, using the relation, $V = E - Ir$

So, $E = V + Ir = 2 + 1 \times 0.5 = 2.5\ \mathrm{V}$

32. **(a)**There are four rods each of mass $m = 400\ \mathrm{g} = 0.4\ \mathrm{kg}$ and each having a length of $\ell = 30\ \mathrm{cm} = 0.3\ \mathrm{m}$ So, we can calculate moment of inertia as follows



$$h^2 = l^2 - \left(\frac{l}{2}\right)^2 = \frac{3}{4} l^2$$

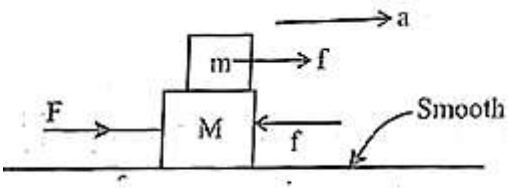
$$\text{M.I. system } = (\text{ MI of I }) \times 2 + (\text{ MI of II }) \times 2$$

$$= \frac{ml^2}{3} \times 2 + 2\left(\frac{ml^2}{12} + mh^2\right)$$

$$= \frac{2}{3} ml^2 + 2\left(\frac{ml^2}{12} + \frac{3}{4} ml^2\right)$$

$$= \frac{7}{3} ml^2 = \frac{7}{3} \times 0.4 \times 0.3 \times 0.3 = 0.084\ \mathrm{kg} - \mathrm{m}^2$$

33. **(c)**



For ' m ' mass $f = ma$ ... (i)

For ' M ' mass $F - f = ma$ …….(ii)

From (i) & (ii), we get

$a = \frac{F}{m+M}$ and $f = \frac{mF}{m+M}$

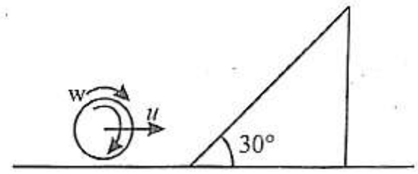
Distances moved in time $t(s)$

$= \frac{1}{2} at^2 = \frac{1}{2}\left(\frac{F}{M+m}\right) t^2$

Work done by friction $= f \times s$

$= \left(\frac{mF}{M+m}\right) \times \frac{1}{2} \frac{Ft^2}{(M+m)} = \frac{mF^2 t^2}{2(M+m)^2}$

34. **(a)**



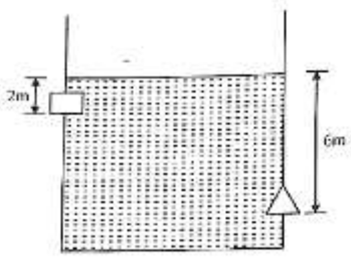
Given, $u = 4\ ms^{-1}$

$$a = \frac{g\sin\theta}{1 + \frac{I}{mR^2}} = \frac{10 \times \sin 30}{1 + \frac{2}{5}} = \frac{5}{\frac{7}{5}} = \frac{25}{7}\ m/s^2$$

As $v^2 = u^2 + 2aS$

$$\Rightarrow |S| = \frac{u^2}{2a} = \frac{4^2}{2 \times \frac{25}{7}} = \frac{16 \times 7}{50} = \frac{112}{50} = 2.24\ m$$

35. (d)



Volume flow rate, R_v

R_v = area × velocity

$R_{v_1} = x^2 \times \sqrt{2 \times g \times 2}$ ... square hole ……(i)

$R_{v_2} = \frac{\sqrt{3}}{4} \times 4^2 \times \sqrt{2 \times g \times 6}$ triangle hole ………(ii)

Equating (i)&(ii) and solving, we get

$x = 2\sqrt{3} \Rightarrow x = 3.46\ cm$

36. (a) $r_1 = 5.5 \times 10^{-11}\ m$

$v_1 = 4 \times 10^6\ m/s$

As $r_n \propto n^2$

So, $r_2 = 5.5 \times 10^{-11} \times 2^2 = 22.0 \times 10^{-11}\ m/s$

As $v_n \propto \frac{1}{n}$

So, $v_2 = \frac{4 \times 10^6}{2} = 2 \times 10^6\ m/s$

The time period of revolution

$$T = \frac{2\pi r_2}{v_2} = \frac{2 \times 3.14 \times 22.0 \times 10^{-11}}{2 \times 10^6}$$

$$= 6.908 \times 10^{-16}\ s$$

37. (a) Given,

$d = 0.5 \times 10^{-9}$ m and $\gamma = 3 \times 10^{-3}$ mm

$\lambda = 500 \times 10^{-9}\ m$

$D = 120\ cm = 120 \times 10^{-2}\ m$

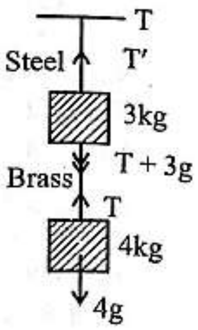
$$\gamma = \frac{\Delta x D}{d}$$

$$\Delta x = \frac{\gamma d}{D}$$

$$\Delta x = \frac{3 \times 10^{-3} \times 0.5 \times 10^{-3}}{1.2} = \frac{5}{4} \times 10^{-6}$$

$$\Delta\phi = \frac{2\pi \times 1.25 \times 10^{-6}}{500 \times 10^{-9}} = 5\pi$$

38. (c) Now,



$T' = T + 3g \Rightarrow T = 4g$
So, $T' = 7g$
So, $\frac{F_S}{F_B} = \frac{T'}{T} = \frac{7}{4}$
We know that,
$Y = \dfrac{F/A}{\Delta L/L} \Rightarrow \Delta L = \dfrac{FL}{AY}$
$\dfrac{\Delta L_S}{\Delta L_B} = \left(\dfrac{F_S}{F_B}\right)\left(\dfrac{L_S}{L_B}\right)\left(\dfrac{A_B}{A_S}\right)\left(\dfrac{Y_B}{Y_S}\right)$
$= \dfrac{7}{4} \times (a)\left(\dfrac{1}{b}\right) \times \left(\dfrac{1}{c}\right)$
or, $\frac{\Delta L_S}{\Delta L_B} = \frac{7a}{4bc}$ or, $\frac{\Delta L_B}{\Delta L_S} = \frac{4bc}{7a}$

39. **(a)** Here $g_{cff} = g - a$
So, $m(g - a) = 150$ N
$60(10 - a) = 150$
$a = 7.5\ m/s^2$

40. **(b)** → We need high retentivity for permanent magnet so that it does not loses its magnetic property even if the external magnetic field is removed.
→ Dimagnetic substance is feebly induced in the direction opposite of magnetic field. So it has negative susceptibility.

(a)	High retentivity	(i)	Permanent magnet
(b)	High resistivity	(ii)	To decrease eddy current losses
(c)	Low coercivity	(iii)	Telephone diaphragm
(d)	Negative susceptibility	(iv)	Dimagnet

Chemistry

41. **(d)** Required relation
$H_3PO_4(aq) \rightleftharpoons 3H^+(aq) + PO_4^{3-}(aq)$
$$K = \frac{[H^+]^3[PO_4^{3-}]}{[H_3PO_4]}$$
For reaction (*i*)
$K_1 = \frac{[H^+]^+_{(aq)}[H_2PO_4^-]_{(aq)}}{H_3PO_{4(aq)}}$ …… (i)
For reaction (ii)

$K_2 = \frac{[H^+]_{(aq)}[HPO_4^{2-}]_{(aq)}}{[H_2PO_4^-]_{(aq)}}$ …… (ii)

For reaction (iii)

$K_3 = \frac{[H^+]_{(aq)}[PO_4^{3-}]_{(aq)}}{[HPO_4^{2-}]_{(aq)}}$ …….. (iii)

On multiplying (i), (ii) and (iii)

$K = K_1 \times K_2 \times K_3$

42. **(c)** Molecular orbital configuration for

(i) $B_2(10) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi p_x^1 \approx \pi p_y^1$

Two unpaired electrons, paramagnetic.

(ii) $N_2(14)$

$= \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi p_x^2 \approx \pi p_y^2, \sigma 2p_z^2$

No unpaired electrons, diamagnetic.

(iii) $O_2(16)$

$= \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 \approx \pi 2$

$\pi^* 2p_x^1, \pi^* 2p_y^1$

Two unpaired electrons, paramagnetic

(iv) C_2 (12)

$= \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma * 2s^2, \pi 2p_x^2 \approx \pi 2p_y^2$

No unpaired electron, diamagnetic.

43. **(c)** $2M(s) + 2H_2O \longrightarrow 2MOH + H_2(g)$

(Where M = Alkali Metal)

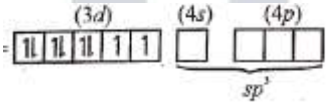
44. **(d)** Bimolecular nucleophilic substitution (S_N ^{2}) reaction

involves reaction between reactant and nucleophile simultaneously without the formation of carbocation. Less hindered species are more reactive towards S_N2 reaction. As CH_3X is least hindered alkyl halide, it will show more fast reaction towards S_N2 reaction.

45. **(d)** (a) Ni in $[Ni(Cl)_4]^{2-}$ exist as Ni^{2+} ion.

Cl is a weak field ligand (high spin). It will not cause pairing of electrons.

Hence, configuration of $[Ni(Cl)_4]^{2-}$ is



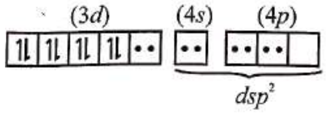
$[Ni(Cl)_4]^{2-} =$

Tetrahedral with two unpaired electrons (i.e., paramagnetic) and has sp^3 hybridisation.

(b) In $[Co(C_2O_4)_3]^{3-}$, $(C_2O_4)_3^{2-}$ is a bidentate ligand thus, give octahedral structure.

(c) In $[Ni(CN)_4]^{2-}$, Ni exist as Ni^{2+} ion.

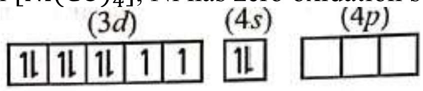
$CN^-$is a strong field ligand (low spin), causes pairing of electrons of $3d$ orbital.



$[Ni(CN)_4]^{2-} =$

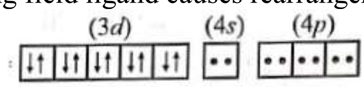
Hence, the structure of $[Ni(CN)_4]^{2-}$ is square planar and it is diamagnetic.

(d) In $[Ni(CO)_4]$, Ni has zero oxidation state. i.e.,



Ni =

CO is a strong field ligand causes rearrangement and pairing of electrons of $3d$ and $4s$ orbital.

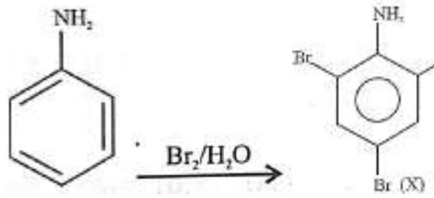


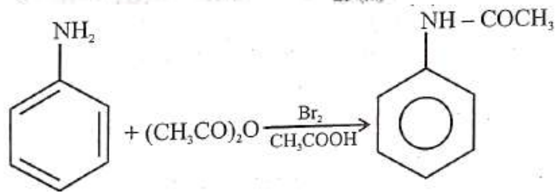
$[Ni(CO)_4] =$

Structure of $Ni(CO)_4$ is tetrahedral with sp^3 hybridisation and it is diamagnetic.

46. **(a)**

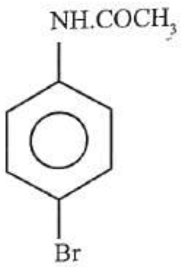
(i) Since H_2O is highly polar in nature and aniline is highly reactive towards electrophilic substitution.
Thus, Br_2 in H_2O gives: 2,4,6 tribromo aniline as main product i.e.





(ii)

and as a result only mono-substituted bromine at p-position is the major product i.e.,



47. (d) Chloro -fluoro carbons (CFCs) such as CF_2Cl_2, $CFCl_3$, hydrofluoro carbons (HFCs) are responsible for ozone layer depletion or ozone holes.

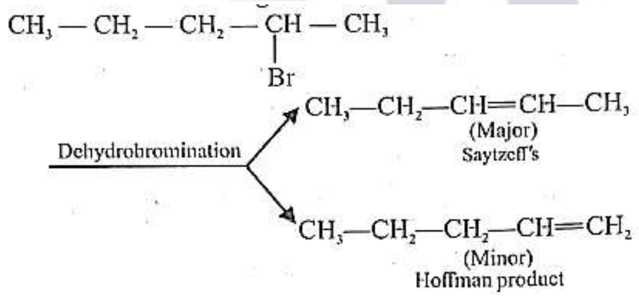
$CF_2Cl_2 \xrightarrow{hv} \dot{C}l + \dot{C}\,F_2Cl$

$\dot{C}l + \frac{3}{2}O_2 \longrightarrow ClO + O_2$

$ClO + O \longrightarrow \dot{C}l + O_2$

48. (d) In ionic polymerisation ions/ion pairs have ends with ionic center $(C_6H_5CO)_2O_2$ is a covalent compound and will not give any ionic center for chain initiation, thus can not be used as an initiator in ionic polymerisation.

49. (a) Elimination of 2-bromopentane follow Saytzeff's rule and give pent-2-ene as major product. Some pent-1ene also form according to Hofmann rule:



50. (d)

$Xe + 3\,F_2 \underset{60-70\text{ bar}}{\overset{573\text{ K}}{\rightarrow}} XeF_1$

(1:20)

51. (d) $\pi = iCRT$.

(a) π for 5.0 M urea is

$\pi = 1 \times 5 \times 0.0821 \times 340\text{ K}$

$\pi = 139.57\text{ atm}$

(b) π for $1.5M_2B_3$ type is $(i = 4)$

$\pi = 4.1 \times 1.5 \times 0.0821 \times 300K$

$\pi = 151.47\text{ atm}$

(c) π for 3.0MAB type is $(i = 1.6)$

$\pi = 1.6 \times 3 \times 0.0821 \times 300\text{ K}$

$\pi = 118.22\text{ atm}$

(d) π for $2.5 MABB_2$ type is $(i = 2.5)$

$\pi = 2.5 \times 2.5 \times 0.0821 \times 330\ K$

$\pi = 169.33$ atm

Hence AB_2 highest osmotic pressure at 57°C shows.

52. (d)

(i) $2Al(s) + 6HCl(aq) \longrightarrow 2AlCl_3(aq) + 3H_2$

(ii) $2Al(s) + 2NaOH(aq) + 6H_2O \longrightarrow$

$2Na[Al(OH)_4] + 3H_2$

Sodium tetrahydroxo aluminate (III)

(iii) $NaBH_4 + I_2 \longrightarrow B_2H_6 + 2NaI + H_2$

53. (b) Given,

W_B (mass of ethylene glycol) $= 31$ g

W_A (mass of water) $= 600$ g

K_f (for water) $= 1.86\ K\ kg\ mol^{-1}$

and M_B (for $C_2H_6O_2$) $= 62$

$$\because\ \Delta T_f = K_f \cdot \frac{W_B}{M_B} \times \frac{1000}{W_A}$$

$$\therefore\ \Delta T_f = \frac{1.86 \times 31 \times 1000}{62 \times 600}$$

$= 1.55$ K

54. (a) Let, number of moles of $SO_2 = x$ moles of $SO_3 = 2x$

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

At equilibrium,

$$K_c = \frac{[SO_3]^2}{[SO_2]^2[O_2]}$$

$$100 = \frac{\left[\frac{2x}{10}\right]^2}{\left[\frac{x}{10}\right]^2\left[\frac{n_{O_2}}{10}\right]}$$

$$100 = \frac{40}{n_{O_2}},\ n_{O_2} = \frac{40}{100} = 0.4$$

Hence, number of moles of oxygen $= 0.4$

55. (d)

For $He^+ : Z = 2, n = 2$

$$E_n = -2.18 \times 10^{-18} \cdot \frac{Z^2}{n^2} \cdot J$$

$$E_n = -2.18 \times 10^{-18} \times \frac{4}{4}$$

$$= -2.18 \times 10^{-18}\ J$$

$$r_n = 52.9 \times \frac{n^2}{Z} pm = 52.9 \times \frac{4}{2} = 105.8 pm$$

56. (b)

(a) $2MnO_4^- + 10I^- + 16H^+ \longrightarrow 2Mn^{2+} + 5I_2 + 8H_2O$

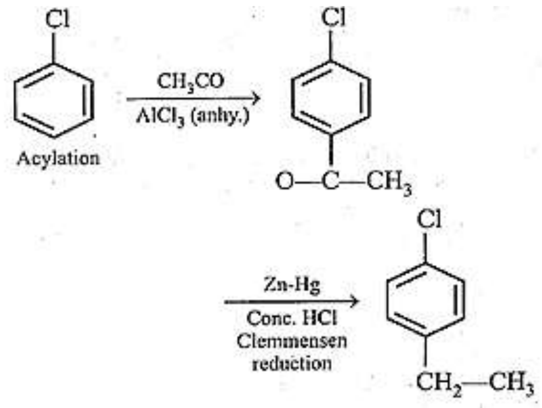
(b) $2MnO_4^- + I^- + H_2O \longrightarrow 2MnO_2 + IO_3^- + 2OH^-$ Y x

Hence, (X) and (Y) are respectively I_2 and IO_3^-.

57. (b) Na^+ and Mg^{2+} are isoelectrionic species having $10e^-$ each. But Mg^{2+} has 12 protons while Na^+ has 11 protons. Hence effective nuclear charge is greater in case of Mg^{2+}.

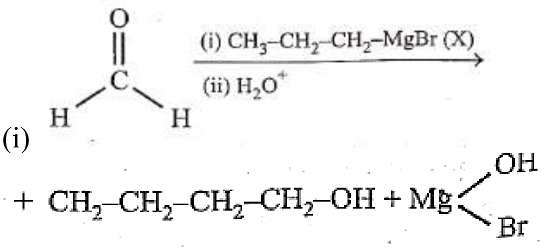
ionic radius $\propto \frac{1}{\text{effective nuclear charge}}$

58. (d)

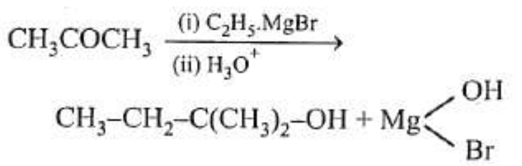


59. (b)

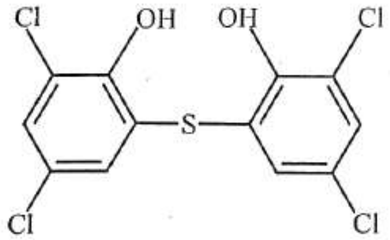
(i)



(ii)



60. (a) Bithionol is added to soap generally to impart antiseptic properties.



Structure of Bithionol

61. (b) Energy of EM wave used

$$E = \frac{hc}{\lambda} = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{300 \times 10^{-9}}$$

$$E = 0.066 \times 10^{-17} = 6.6 \times 10^{-19}\ \text{J}$$

Metals surface which have work function less than or equal to 6.6×10^{-19} J will eject electrons on radiation with light of 300 nm .

Work function of Ag $= 4.3\text{eV} \times 1.6022 \times 10^{-19} = 6.89 \times 10^{-19}$ J

$\text{Mg} = 3.7\text{eV} \times 1.6022 \times 10^{-19} = 5.93 \times 10^{-19}$ J

$\text{K} = 2.25\text{eV} \times 1.6022 \times 10^{-19} = 3.60 \times 10^{-19}$ J

$\text{Na} = 2.30\text{eV} \times 1.6022 \times 10^{-19} = 3.68 \times 10^{-19}$ J

Only Mg, K and Na have their work function less than 6.6×10^{-19} J hence, they will eject the electrons.

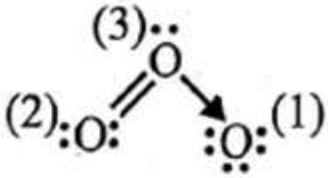
62. (d) (i) NO_2 group increases the acidic nature due to -I effect.

(ii) OCH_3 and CH_3 are electron donating groups (i.e., +I effect), thus decreases the acidic nature, but OCH_3 will decrease acidic nature more than of CH_3 group Hence, correct order is

III < IV < I < II

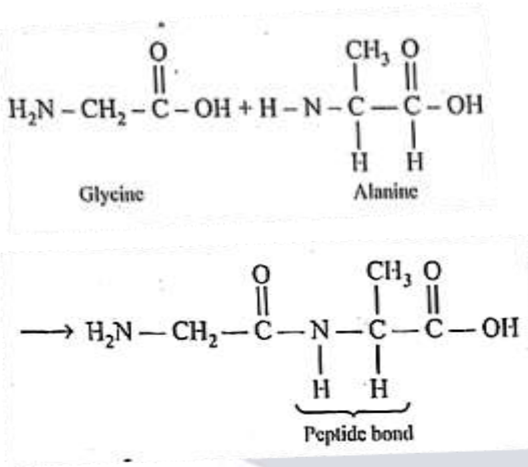
63. (b) Colloidal solution of gold show different colours (like red, purple, blue and golden) due to difference in size gold particles in colloidal solution.

64. **(a)** Formal charge = No. of valency electron in central atom $-$[No. of lone pair $+\frac{1}{2}$ No. of bond pair of electron]

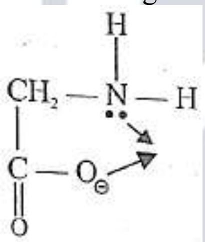


F.C for O labelled as 1 $= 6 - [6 + 1] = -1$
F.C for O labelled as 2 $= 6 - [4 + 2] = 0$
F.C for O labelled as 3 $= 6 - [2 + 3] = +1$

65. **(d)** (i)



Hence- $\overset{O}{\overset{\|}{C}}$ of peptide bond belongs to glycine



(ii) glycylamine is:

(iii) Phosphate group - pentose sugar - Nitrogenous base unit is known as nucleotide.

66. **(d)** Given ΔH_f for $CH_3OH = -239\ kJ\ mol^{-1}$

ΔH_f of $H.CHO = -116\ kJ\ mol^{-1}$

ΔH_f of $H_2O = -286\ kJ\ mol^{-1}$

Required relation:

$2CH_3OH + O_2 \longrightarrow 2HCHO + 2H_2O$

$\Delta_r H = \Sigma(\Delta_f H_{Product}) - \Sigma(\Delta_f H_{Reactant})$

$= (-116 + (-286)) - (-293 + 0) \Rightarrow \dfrac{-326}{2}$

$= -163\ kJ\ mol^{-1}$

67. **(b)**

$$\underset{\substack{\text{Copper pyrites}\\ \text{[ore]}}}{2CuFeS} + O_2 \longrightarrow \underbrace{Cu_2S + 2FeS}_{\text{Matte}} + SO_2(g)$$

68. **(b)**

Given

$T = 27 + 273 = 300K$

$V = 10L$

$W = W_1(\text{He}) + W_2(\text{Ne}) = 4g \quad \ldots..(i)$

Molar mass (He)=4

Molar mass (Ne) = 20

$P = 1.23$ atm
$R = 0.082$ L atm K^{-1} mol^{-1}
(i) $PV = (n_1 + n_2)RT = \left[\frac{W_1}{4} + \frac{W_2}{20}\right]RT$

$$\left[\frac{W_1}{4} + \frac{W_2}{20}\right] = \frac{1.23 \times 10}{0.082 \times 300} = 0.5$$

$$25W_1 + 5W_2 = 50$$

$$5W_1 + W_2 = 10 \quad \ldots\ldots(ii)$$

On solving equation (ii) with the help of eq. (i)

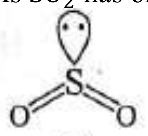
$$5W_1 + 4 - W_1 = 10 \Rightarrow W_1 = 1.5$$

$$W_2 = 2.5$$

Mass % of neon $= \frac{2.5 \times 100}{4} = 62.5\%$

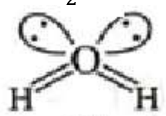
69. (c) $S + 2H_2SO_4$ (conc.) $\longrightarrow \underset{x}{3SO_2} + \underset{y}{2H_2O}$

As SO_2 has one lone pair of electrons on central atom i.e., sulphur.



(X)

And H_2O has two lone pair of electrons on the central atom i.e., oxygen.



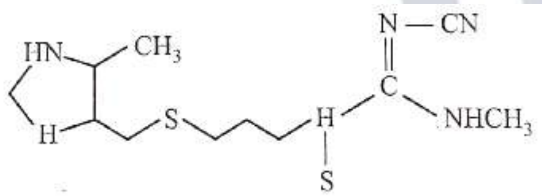
(Y)

70. (d) (i) Electromeric effect is a temporary effect, it takes place only in the presence of reagent.
(ii) Hyper-conjugation is a permanent effect which allows delocalised sigma electrons to adjacent empty r or p-orbital.

71. (d) Carbon monoxide is estimated using I_2O_5 as per the following reaction:
$I_2O_5 + 5CO \longrightarrow \underset{\text{violet}}{I_2} + 5CO_2$

72. (c) $[SiF_6]^{2-}$ is known to exist but due to large size of chlorine (Cl) atom, $[SiCl_6]^{2-}$ ions is not known.

73. (b) Cimetidine is used to prevent the interaction of histamine with the receptors present in the stomach wall, the structure of cimetidine is as follows:



74. (d) For a compound to be aromatic
(i) The ring system must contain $(4n + 2)\pi$ electrons.
(ii) The ring system must be planar.

75. (c) In XO_2, X: O: : 50: 50
Mass of oxygen $= 2 \times 16 = 32$ g
Hence, mass of X $= 32$ g
In XO_n, X: O: : 40: 60
Mass of X, in this case also remains $= 32$ g
$40\% \equiv 32$ g

$$60\% \equiv \frac{32}{40} \times 60 = 48 \text{ g}$$

Mass of X, in this case also remains $= 32$ g
$40\% \equiv 32$ g

$60\% \equiv \frac{32}{40} \times 60 = 48$ g

Mass of oxygen $= 48$ g

$\Rightarrow n = \frac{48}{16} = 3$

Formula of oxide $= XO_3$

76. (d) Given,

Body centered cubic (bcc) $Z = 2$

Edge length (a) $= 400\text{pm} = 400 \times 10^{-10}$ cm

M (Atomic mass) $= 24$ g mol^{-1}

Density (d) $= \frac{Z \times M}{a^3 \times N_A}$

$$d = \frac{2 \times 24}{(400 \times 10^{-10})^3 \times 6.022 \times 10^{23}}$$

$d = 125$ g cm^{-3}

77. (b) For $t = 90$ min (from 10 a.m. to 11:30 a.m.)

As 20% was already completed $\therefore a = 80$

$(a - x) = (100 - 80) = 20$

$$k = \frac{2.303}{t} \log \frac{a}{a - x}$$

$$k = \frac{2.303}{90} \log \frac{80}{20}$$

$$k = \frac{2.303}{90} \log 4 = \frac{2.303 \times 0.6020}{90}$$

$k = 0.015$ min

$$t_{1/2} = \frac{0.693}{k}$$

$$\Rightarrow t_{1/2} = \frac{0.693}{0.015} = 46.2 \approx 45 \text{ min}$$

78. (c) When sodium acetate is electrolysed in aqueous medium, it react as follows

$2CH_3COONa + 2H_2O \longrightarrow 2CO_2 + 2C_2H_6 + 2H_2$

At anode (Y): $C_2H_6 + CO_2$

At cathode (X): H_2

79. (c) Given,

Normality of $KMnO_4 (N_1) = 1$ N

Volume of $KMnO_4 (V_1) = 500$ mL

Normality of $H_2O_2 (N_2) = \frac{\text{Volume strength}}{5.6 \text{ (eq. mass)}} = \frac{20}{5.6}$

$(N_2) = 3.5$

Now, $N_1V_1(KMnO_4) = N_2V_2(H_2O_2)$

$1 \times 500 = 3.57 \times V_2$

$$V_2 = \frac{500}{3.57} = 140 \text{ mL}$$

80. (b) When same amount of electricity is passed through $AgNO_3$ and $CuSO_4$ aqueous solution. Then 1 mole of Ag^+ will deposit and half mole of Cu^{2+} will deposit.

The relations between Ag^+ and Cu^{2+} ion is as follows

1 mol of $Ag^+ = \frac{1}{2}$ mol of Cu^{2+}

Thus for, $\frac{x \text{ mol of } Ag^+}{y \text{ mol of } Cu^{2+}} = \frac{1 \text{ mol}}{1/2 \text{ mol}}$

$x = 2y$

Mathematics

81. (b) We have, $b\cos\theta = c - a \Rightarrow \cos\theta = \frac{c-a}{b}$

$\sin\theta = \dfrac{\sqrt{b^2-(c-a)^2}}{b}$ and $\tan\theta = \dfrac{b}{\dfrac{\sqrt{b^2-(c-a)^2}}{(c-a)}}$

Now, $(c-a)\tan\theta = \sqrt{b^2-(c-a)^2}$

$= \sqrt{b^2-c^2-a^2+2ac} = \sqrt{-(c^2+a^2-b^2)+2ac}$

$= \sqrt{2ac-2ac\cos B}$ [Using cosine rule]

$= \sqrt{2ac}\sqrt{1-\cos B}$

[Using cosine rule]

$= \sqrt{2}\sqrt{ac}\sqrt{2\sin^2\dfrac{B}{2}} = 2\sqrt{ac}\sin\dfrac{B}{2}$

82. (d) For a distribution of random variable x,

$\alpha = P(X^6 < 3) = P(X^6 = 1) + P(X^6 = 2) = \lambda + 2\lambda = 3\lambda$

and $\beta = P(X^6 > 2) = P(X^6 = 3) + P(X^6 = 4)$

$= 3\lambda + 4\lambda = 7\lambda$

$\alpha : \beta = 3 : 7$

83. (c) Given function

$$f(x) = \begin{cases} x-1, & \text{for } x \le 1 \\ 2-x^2, & \text{for } 1 < x \le 3 \\ x-10, & \text{for } 3 < x < 5 \\ 2x, & \text{for } x \ge 5 \end{cases}$$

So, $f(x)$ will be continuous in the intervals $(-\infty,1),(1,3),(3,5)$ and $(5,\infty)$

Now, let us check the continuity at $x = 1,3$ and 5

Here,

(i) At $x = 1$,

$\lim\limits_{x\to1^-} f(x) = 1-1 = 0$ and $\lim\limits_{x\to1^+} f(x) = 2-1 = 1$

therefore F is not continuous at $x = 1$.

(ii) At $x = 3$,

$\lim\limits_{x\to3^-} f(x) = 2-9 = -7$ and

$f(3) = -7$ and $\lim\limits_{x\to3^+} f(x) = 3-10 = -7$

therefore f is continuous at $x = 3$

(iii) At $x = 5, \lim_{x\to5^-} f(x) = 5-10 = -5$ and,

$\lim\limits_{x\to5^+} f(x) = 2\times5 = 10$

therefore f is not continuous at $x = 5$

Hence, points of discontinuity of f are $\{1,5\}$

84. (a) We have, equations of pair of straight lines

$x^2 - 16pxy - y^2 = 0$ ....(i)

and $x^2 - 16qxy - y^2 = 0$ ....(ii)

Now, equations of bisectors of these -line are

$-8px^2 - 2xy + 8py^2 = 0$

i.e. $4px^2 + xy - 4py^2 = 0$ ....(iii)

and $-8qx^2 - 2xy + 8qy^2 = 0$

i.e. $4qx^2 + xy - 4qy^2 = 0$ ......(iv)

According to the given condition in the data, Eqs. (i) and (iv), and Eqs. (ii) and (iii) must be coincident. So,

$\dfrac{1}{4q} = \dfrac{-16p}{1} = \dfrac{-1}{-4q}$

$1 = -64pq \Rightarrow pq = \dfrac{-1}{64}$

85. (d) Normal of the plane P_1 determined by the vectors $\hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$,

$n_1 = [(\hat{j} - \hat{k}) \times (3\hat{j} - 2\hat{k})] = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ 0 & 3 & -2 \end{vmatrix} = \hat{i}$

Normal of the plane P_2 determined by the vectors $2\hat{i} + 3\hat{j}$ and $\hat{i} - 3\hat{j}, \Rightarrow n_2 = [(2\hat{i} + 3\hat{j}) \times (\hat{i} - 3\hat{j})]$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 1 & -3 & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + \hat{k}(-6-3) = -9\hat{k}$

Since, a is parallel to the line of intersection of planes P_1 and P_2.

$a = \pm(n_1 \times n_2) = \pm \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 0 & -9 \end{vmatrix} = \pm 9\hat{j}$

Now, angle θ between $9\hat{j}$ and $\hat{i} + \hat{j} + \hat{k}$ is

$\cos\theta = \pm\left(\frac{9}{9\sqrt{3}}\right) = \pm\left(\frac{1}{\sqrt{3}}\right) \Rightarrow \theta = \cos^{-1}\left(\pm\frac{1}{\sqrt{3}}\right)$

86. (a) Given, set $S = \{1,2,\ldots\ldots 10\}$

Here, three numbers are drawn at random from the given - set. So, total possible outcomes, $n = {}^{10}C_3 = 120$

Let A be the event that minimum of chosen number is 3 .

$n(A) = \{4,5,6,7,8,9,10\} = {}^{T}C_2 = 21$

B be the event that maximum at chosen number is 7 .

$n(B) = \{1,2,3,4,5,6\} = {}^{6}C_2 = 15$

So, $n(A \cap B) = \{4,5,6\} = {}^{3}C_1 = 3$

Hence, required probability

$P = \frac{n(A) + n(B) - n(A \cap B)}{n} = \frac{21 + 15 - 3}{120} = \frac{11}{40}$

87. (a) Let $y = \log\left\{e^x \left(\frac{x-2}{x+2}\right)^{\frac{3}{4}}\right\}$

$= \log e^x + \log\left(\frac{x-2}{x+2}\right)^{\frac{3}{4}} = x\log e + \frac{3}{4}\log\left(\frac{x-2}{x+2}\right)$

$= x + \frac{3}{4}\log\left(\frac{x-2}{x+2}\right)$

Now, differentiating w.r.t. x, we get

$\frac{dy}{dx} = 1 + \frac{3}{4}\frac{d}{dx}\log\left(\frac{x-2}{x+2}\right)$

$= 1 + \frac{3}{4}\left[\frac{x+2}{x-2} \cdot \left\{\frac{(x+2) \cdot 1 - (x-2) \cdot 1}{(x+2)^2}\right\}\right]$

$= 1 + \frac{3}{4} \times \frac{4}{x^2 - 4} = 1 + \frac{3}{x^2 - 4}$

At $x = 3$,

$\left(\frac{dy}{dx}\right)_{x=3} = 1 + \frac{3}{5} = \frac{8}{5}$

88. (d) Given, $y = \frac{\sinh^{-1} x}{\sqrt{1+x^2}}$

$\sqrt{1 + x^2}y = \sinh^{-1} x$

On differentiating w.r.t. x, we get

$\sqrt{1 + x^2}y_1 + y\frac{1}{2\sqrt{1 + x^2}} \cdot 2x = \frac{1}{\sqrt{1 + x^2}}$

$(1 + x^2)y_1 + xy = 1$

Again, on differentiating w.r.t. x, we get

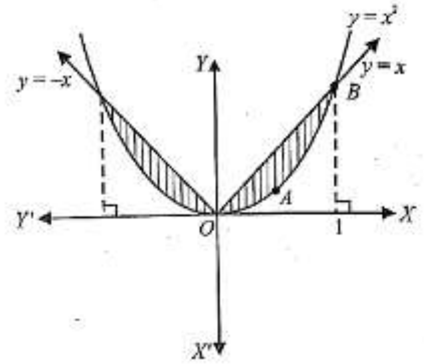
$(1 + x^2)y_2 + y_1 \cdot 2x + xy_1 + y = 0$

$(1 + x^2)y_2 + 3xy_1 + y = 0$

89. (c) Given curves are

$y = x^2$

and $y = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$



On solving Eqs. (i) and (ii), we get $x = 0, x = 1$

Thus, coordinates of point B are (1,1)

Total area = 2 (Area of region OABO)

$$= 2\int_0^1 (x - x^2)dx = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{3}\right]$$

$$= 2 \times \frac{1}{6} = \frac{1}{3} \text{ sq unit}$$

90. (c) Let, $I = \int \frac{5x^2+3}{x^2(x^2-2)} dx$

Put $x^2 = y$. Then,

$I = \int \frac{5y+3}{y(y-2)} dx \quad \ldots\ldots(i)$

Now, $\frac{5y+3}{y(y-2)} = \frac{A}{y} + \frac{B}{y-2} \quad \ldots\ldots.(ii)$

$$5y + 3 = (y - 2)A + yB$$

$$5y + 3 = y(A + B) - 2A$$

On comparing the coefficients, we get

$A + B = 5$ and $3 = -2A$

$A = \frac{-3}{2}$ and $B = \frac{13}{2}$

Thus, $\frac{5y+3}{y(y-2)} = \frac{5x^2+3}{x^2(x^2-2)} = -\frac{3}{2}\frac{1}{x^2} + \frac{13}{2}\frac{1}{x^2-2}$

Now, $I = -\frac{3}{2}\int \frac{dx}{x^2} + \frac{13}{2}\int \frac{dx}{x^2-2}$

$$= -\frac{3}{2}\left(-\frac{1}{x}\right) + \frac{13}{2} \times \frac{1}{2\sqrt{2}} \log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + C$$

$$= \frac{3}{2\pi} + \frac{\sqrt{13}}{4\sqrt{2}} \log\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + C$$

91. (a) Given that,

$$y = \tan^{-1}\left\{\frac{x}{1+\sqrt{1-x^2}}\right\} + \sin\left\{2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right\}$$

Let $x = \cos 2\theta$

$$y=\tan^{-1}\left\{\frac{\cos 2\theta}{1+\sqrt{\sin^2 2\theta}}\right\}+\sin\left\{2\tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}\right\}$$

$$\Rightarrow y=\tan^{-1}\left\{\frac{\cos 2\theta}{1+\sin 2\theta}\right\}+\sin\left\{2\tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}\right\}$$

$$\Rightarrow y=\tan^{-1}\left\{\frac{\cos^2\theta-\sin^2\theta}{(\cos\theta+\sin\theta)^2}\right\}$$
$$+\sin\{2\tan^{-1}(\tan\theta)\}$$

$$\Rightarrow y=\tan^{-1}\left\{\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right\}+\sin 2\theta$$

$$\Rightarrow y=\tan^{-1}\left\{\frac{1-\tan\theta}{1+\tan\theta}\right\}+\sin 2\theta$$

$$\Rightarrow y=\tan^{-1}\left(\tan\left(\frac{\pi}{4}-\theta\right)\right)+\sin 2\theta$$

$$\Rightarrow y=\frac{\pi}{4}-\theta+\sqrt{1-\cos 22\theta}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}\cdot\frac{1}{\sqrt{1-x^2}+\frac{1}{2\sqrt{1-x^2}}(-2x)}$$

Now, on differentiating w.r.t. x, we get

$$\frac{dy}{dx}=\frac{1}{2}\cdot\frac{1}{\sqrt{1-x^2}}+\frac{1}{2\sqrt{1-x^2}}(-2x)=\frac{1-2x}{2\sqrt{1-x^2}}$$

92. (b) Let DR's of normal to the plane is $\langle a,b,c\rangle$. Then, equation plane passing through $(4,4,0)$

$$(r-a).n=0$$

$$a(x-4)+b(y-4)+c(z)=0$$

Since this plane is $\perp$ to the given planes, So, we get

$$2a+b+2c=0 \quad\text{and}\quad 3a+3b+2c=0$$

By using cross-multiplication method,

$$\frac{a}{2-6}=\frac{-b}{4-6}=\frac{c}{6-3}\Rightarrow\frac{a}{-4}=\frac{b}{2}=\frac{c}{3}$$

So, the required equation of plane

$$-4(x-4)+2(y-4)+3(z)=0$$
$$-4x+16+2y-8+3z=0$$
$$4x-2y-3z=8$$

93. (c) We have,

$$\lim_{n\to\infty}\left[\frac{1^k+2^k+3^k+\cdots.+n^k}{n^{k+1}}\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\left(\frac{1}{n}\right)^k+\left(\frac{2}{n}\right)^k+\left(\frac{3}{n}\right)^k+\cdots.+\left(\frac{n-1}{n}\right)^k+1\right]$$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\left(\frac{r}{n}\right)^k+\lim_{n\to\infty}\frac{1}{n}=\int_0^1 x^k dx+0=\left[\frac{x^{k+1}}{k+1}\right]_0^1$$

$$\lim_{n\to\infty}\left[\frac{1^k+2^k+3^k+\cdots.+n^k}{n^{k+1}}\right]=\frac{1}{k+1}$$

94. (a) As we know, coefficient of x^r in the binomial expansion of $(1+x)^n$ is given by ${}^nC_r$.

So, coefficient of x^5 in the binomial expansion of

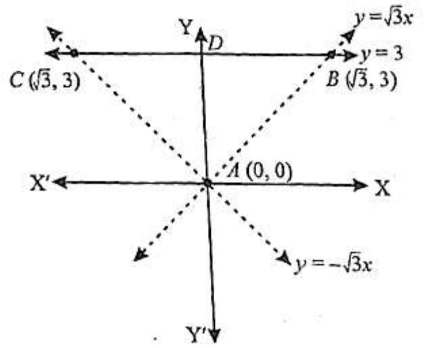
$(1+x)^{21}+(1+x)^{22}+\cdots.+(1+x)^{30}$
$= {}^{21}C_5 + {}^{22}C_5 + \cdots + {}^{30}C_5$
$= ({}^{21}C_6 + {}^{21}C_5 + {}^{22}C_5 + \cdots + {}^{30}C_5) - {}^{21}C_6$
$= ({}^{22}C_6 + {}^{22}C_5 + \cdots + {}^{30}C_5) - {}^{21}C_6$
$[{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r]$
$= ({}^{23}C_6 + {}^{23}C_6 + \cdots + {}^{30}C_5) - {}^{21}C_6$
Similarly solving we get,
$= ({}^{30}C_6 + {}^{30}C_5) - {}^{21}C_6 = {}^{31}C_6 - {}^{21}C_6$

95. (a) Here, function is defined for f: A → R

$$f(x)=\begin{cases}3x-1 & \text{for} \quad x>3\\ x^2+1 & \text{for} \quad -3\le x\le 3\\ 2x-3 & \text{for} \quad x<-3\end{cases}$$

and $A=\{-4,-2,-1,0,3,5\}$
$f(-4)=2(-4)-3=-11$
$(-2)=(-2)+1=4+1=5$
$f(-1)=(-1)^2+1=1+1=2$
$f(0)=0^2+1=1$ and $f(3)=3^2+1=10$
$f(5)=3(5)-1=14$
Hence, range of f is $\{-11,5,2,1,10,14\}$.

96. (a) The triangle formed by the lines $y=\sqrt{3}x, y=-\sqrt{3}x$ and $y=3$ is shown in the figure.



We can observe that the triangle ABC is an isosceles triangle. Therefore, the incentre lie on the median to the base. Since D is mid-point of BC. So, OD is median to the base BC. Thus, incentre of $\triangle ABC$ lie on Y-axis.

97. (b) Here, the differential equation

$$(x-4y^3)\frac{dy}{dx}-y=0,(y>0)$$

$$\Rightarrow (x-4y^3)\frac{dy}{dx}=y \Rightarrow \frac{dx}{dy}=\frac{x-4y^3}{y}$$

$$\Rightarrow \frac{dx}{dy}=\frac{x}{y}-4y^2 \Rightarrow \frac{dx}{dy}-\frac{x}{y}=-4y^2$$

$$P=-\frac{1}{y}, Q=-4y^2$$

Integrating factor,

$$\text{I.F.} = e^{\int P dy} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = e^{\log y^{-1}} = \frac{1}{y}$$

Now, the solution is given by

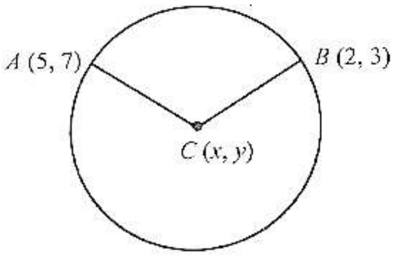
$x \cdot (\text{I.F.}) = \int (\text{I.F.}) \cdot Q dy + C$

$\frac{x}{y} = \int \frac{1}{y} \times (-4y^2) dy + C = \int -4y dy + C$

$\frac{x}{y} = -4\frac{y^2}{2} + C \Rightarrow x = -2y^3 + Cy$

$x + 2y^3 = Cy$

98. **(d)** Let centre of the circle be $c(x, y)$.



Here, CA and CB both represents the radius of the circle.
So, it will be equal.

Then, $CA = CB \Rightarrow CA^2 = CB^2$

$\Rightarrow (5-x)^2 + (7-y)^2 = (2-x)^2 + (3-y)^2$

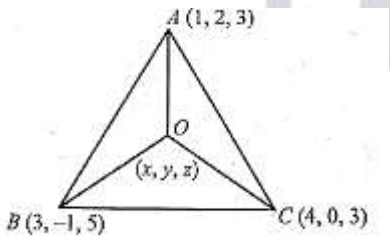
$\Rightarrow 25 + x^2 - 10x + 49 + y^2 - 14y$

$= 4 + x^2 - 4x + 9 + y^2 - 6y$

$\Rightarrow 6x + 8y = 61$

This equation is satisfied by the coordinates given in option (d).

99. **(d)** Let A, B, C be the vertices of the triangle given as,
$A(1,2,3), B(3,-1,5), C(4,0,-3)$



Now, let $O(x, y, z)$ be the circumcentre of $\triangle ABC$.

$OA = OB = OC$

$OA = OB \Rightarrow OA^2 = OB^2$

$\Rightarrow (x-1)^2 + (y-2)^2 + (z-3)^2$

$= (x-3)^2 + (y+1)^2 + (z-5)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 + z^2 - 6z + 9$

$= x^2 - 6x + 9 + y^2 + 2y + 1 + z^2 - 10z + 25$

$\Rightarrow 4x - 6y + 4z - 21 = 0 \quad \ldots\ldots(i)$

Similarly, $OB = OC$

$\Rightarrow (x-3)^2 + (y+1)^2 + (z-5)^2$

$= (x-4)^2 + (y-0)^2 + (z+3)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 + 2y + 1 + z^2 - 10z + 25$

$= x^2 - 8x + 16 + y^2 + z^2 + 6z + 9$

$\Rightarrow x + y - 8z + 5 = 0 \quad \ldots\ldots(ii)$

Similarly, $OA = OC$

$$(x-1)^2+(y-2)^2+(z-3)^2$$
$$=(x-4)^2+(y-0)^2+(z+3)^2$$
$$x^2-2x+1+y^2-4y+4+z^2-6z+9$$
$$=x^2-8x+16+y^2+z^2+6z+9$$
$$\Rightarrow 6x-4y-12z-11=0 \qquad \ldots\ldots\ldots(iii)$$

On solving Eqs. (i), (ii) and (iii), we get

$$x=\frac{7}{2}, y=-\frac{1}{2}; z=1$$

Circumcentre of $\triangle$ ABC is $\left(\frac{7}{2},\frac{-1}{2},1\right)$

100. **(a)** Here, W = White, B = Black. Given that bag P contains 5W marbles and 3B marbles. 4 marbles are drawn from the bag P and A black marble is drawn from bag Q.

Let Event E_1: 1 W and 3 B marbles are transferred

Event E_2: 2 W and 2 B marbles are transferred

Event E_3: 3 W and 1 B marbles are transferred

Event E_4: 4 W and 0 B marbles are transferred and Event A : a black marble is drawn from bag Q.

Then, $P(E_1)=\frac{{}^5C_1\times{}^3C_3}{{}^8C_4}=\frac{5}{{}^8C_4}$

$$P(E_2)=\frac{{}^5C_2\times{}^3C_2C_4}{{}^8C_4}=\frac{30}{{}^8C_4}$$

$$P(E_3)=\frac{{}^5C_3\times{}^3C_1}{{}^8C_4}=\frac{30}{{}^8C_4}$$

$$P(E_4)=\frac{{}^5C_4\times{}^3C_0}{{}^8C_4}=\frac{5}{{}^8C_4}$$

$$P(A/E_1)=\frac{3}{4}, P(A/E_2)=\frac{2}{4}$$

$$P(A/E_3)=\frac{1}{4}, P(A/E_4)=0$$

By using Bayes' theorem,

$$P(E_1/A)=\frac{P(E_1)\cdot P(A/E_1)}{(P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)}$$
$$+P(E_3)P(A/E_3)+P(E_4)\cdot P(A/E_4))$$

$$=\frac{\frac{5}{{}^8C_4}\cdot\frac{3}{4}}{\frac{5}{{}^8C_4}\cdot\frac{3}{4}+\frac{30}{{}^8C_4}\times\frac{2}{4}+\frac{30}{{}^8C_4}\times\frac{1}{4}+0}$$

$$=\frac{15}{15+60+30}=\frac{15}{105}=\frac{1}{7}$$

101. **(a)** We have,

$$\sec h^{-1}\left(\frac{1}{\sqrt{2}}\right)+\operatorname{cosec} h^{-1}(-1)$$

$$=\log_e\left(\frac{1+\sqrt{1-1/2}}{\frac{1}{\sqrt{2}}}\right)+\log_e\left(\frac{1-\sqrt{1+1}}{-1}\right)$$

$$= \log_e\left(\frac{\frac{\sqrt{2}+1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}\right) + \log_e(\sqrt{2}-1)$$

$= \log_e(\sqrt{2}+1) + \log_e(\sqrt{2}-1)$

$= \log_e[(\sqrt{2}+1)(\sqrt{2}-1)] \quad [\log m + \log n = \log(m \times n)]$

$= \log_e((\sqrt{2})^2 - 1) = \log_e(2-1) = \log_e 1$

$\sec h^{-1}\left(\frac{1}{\sqrt{2}}\right) + \operatorname{cosec} h^{-1}(-1) = 0$

102. **(a)** Let A, B, C and D are thepositionvectorsoffourpointsgiven as $A(3\hat{i} - 2\hat{j} - \hat{k}), B(2\hat{i} + 3\hat{j} - 4\hat{k}), C(-\hat{i} + \hat{j} + 2\hat{k})$ and $D(4\hat{i} + 5\hat{j} + \lambda\hat{k})$

Here, $AB = -\hat{i} + 5\hat{j} - 3\hat{k}$ and $BC = -3\hat{i} - 2\hat{j} + 6\hat{k}$ and $CD = 5\hat{i} + 4\hat{j} + (\lambda - 2)\hat{k}$

As given that, these points are coplanar, therefore [AB BC CD] 0

$$\begin{vmatrix} -1 & 5 & -3 \\ -3 & -2 & 6 \\ 5 & 4 & \lambda - 2 \end{vmatrix} = 0$$

$\Rightarrow \quad -1(-2\lambda + 4 - 24) - 5(-3(\lambda - 2) - 30) - 3(-12 + 10) = 0$

$\Rightarrow \quad 2\lambda + 20 + 15(\lambda - 2) + 150 + 36 - 30 = 0$

$\Rightarrow \quad 17\lambda + 20 - 30 + 150 + 6 = 0$

$\Rightarrow \quad 17\lambda + 146 = 0 \Rightarrow \lambda = -\frac{146}{17}$

103. **(a)** Given, equation of tangents to the ellipse are, $y = 2x + \sqrt{76}$ and $2y + x = 8 \Rightarrow y = -\frac{1}{2}x + 4 \Rightarrow m_1 = 2$ and $m_2 = -\frac{1}{2}$

Here, $m_1 \times m_2 = -1$

It means that the given tangents are perpendicular to each other.

Thus, required circle will be the director circle to ellipse. Hence, required equation of circle is

$x^2 + y^2 \quad = a^2 + b^2 = 16 + 12$

$\Rightarrow x^2 + y^2 \quad = 28$

104. **(a)** Given pair of lines, $4xy + 2x + 6y + 3 = 0$

$\Rightarrow 2x(2y + 1) + 3(2y + 1) = 0$

$\Rightarrow (2x + 3)(2y + 1) = 0$

So, equations of lines are $x = \frac{-3}{2}$ and $y = -\frac{1}{2}$

Equations of line passing through (2,1) and perpendicular to the pair of lines $4xy + 2x + 6y + 3 = 0$ are $y - 1 = 0$ and $x - 2 = 0$

Combining the equation of lines are

$(y - 1)(x - 2) = 0 \Rightarrow xy - x - 2y + 2 = 0$

105. **(b)** Let two numbers be a and b

G.M. $= \sqrt{ab} = 2$

$ab = 4 \quad \ldots\ldots(i)$

Now, H.M. $= \frac{2ab}{a+b} = -\frac{8}{5}$

$\frac{2\times4}{a+b} = -\frac{8}{5}$ (from Eq. (i))

$a + b = -5$

Here, roots are twice of these numbers. So,

$(2a)(2b) = 4ab = 4 \times 4 = 16$

$2a + 2b = 2(a + b) = 2 \times -5 = -10$

Quadratic Equation $x^2 + 10x + 16 = 0$

106. **(c)** We have given, $|z| \geq 5$

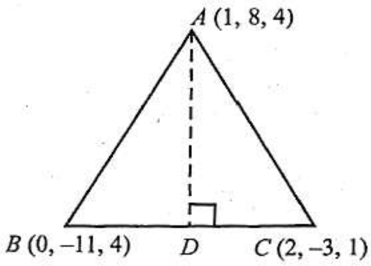
Now, $\left|z + \frac{2}{z}\right| \geq |z| - \left|\frac{2}{z}\right| = \left||z| + \frac{2}{z}\right|$

Thus, the least value of $\left|z+\frac{2}{z}\right|$ is $\frac{23}{5}$.

107. **(b)** The vertices of $\triangle$ ABC are given as $A(1,8,4)$, $B(0,-11,4)$ and $C(2,-3,1)$.

Equation of line BC,

$$\frac{x}{2}=\frac{y+11}{8}=\frac{z-4}{-3}=\lambda(\text{ say })$$



As point D is on line BC, so coordinates of D are

$(2\lambda, 8\lambda-11, -3\lambda+4)$

Since, $AD \perp BC$

$AD \cdot BC = 0$

$(2\lambda-1, 8\lambda-19, -3\lambda)\cdot(2,8,-3)=0$

$2(2\lambda-1)+8(8\lambda-19)+(-3)(-3\lambda)=0$

$\lambda=2$

Hence, the coordinates of D are $(4,5,-2)$

108. **(d)** Given function, $f(x)=(x-1)(x-2)$

$=x^2-3x+2$ and $x\in\left[0,\frac{1}{2}\right]$

By using Lagrange's mean value theorem,

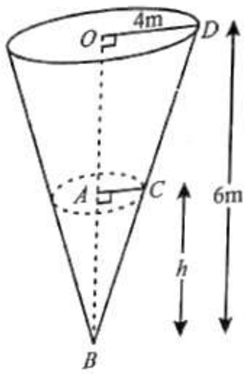
$$f'(c)=\frac{f(b)-f(a)}{b-a}=\frac{f\left(\frac{1}{2}\right)-f(0)}{\frac{1}{2}-0}$$

$$\Rightarrow 2c-3=\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)-(-1)(-2)}{\frac{1}{2}-0}$$

$$\Rightarrow 2c-3=2\left(\frac{3}{4}-2\right)\Rightarrow 2c-3=-\frac{5}{2}$$

$$\Rightarrow 4c-6=-5\Rightarrow c=\frac{1}{4}\in\left(0,\frac{1}{2}\right)$$

109. **(a)** Let V be the volume, r be the radius and h be the height of an inverted cone at any time t. Then



Given, $\frac{dv}{dt}=3\ m^3/\text{min}$

We know that,

$V=\frac{1}{3}\pi r^2 h$ .....(i)

From similarity of triangle

$$\frac{BO}{BA}=\frac{OD}{AC}$$
$$\Rightarrow \frac{6}{h}=\frac{4}{r}\Rightarrow r=\frac{2}{3}h$$
From Eq. (i),
$$V=\frac{1}{3}\pi\left(\frac{2}{3}h\right)^2 h=\frac{4}{27}\pi h^3$$
On differentiating w.r.t. t, we get
$$\Rightarrow \frac{dv}{dt}=\frac{4}{27}\pi 3h^2\frac{dh}{dt}$$
$$\Rightarrow 3=\frac{4}{27}\pi 3h^2\frac{dh}{dt}\Rightarrow \frac{dh}{dt}=\frac{3\times 27}{4\pi 3h^2}$$
$$\Rightarrow \left(\frac{dh}{dt}\right)_{h=3}=\frac{3\times 27}{4\pi 27}=\frac{3}{4\pi}\text{mt/min}$$

110. **(d)** Given equation, $x^3-7x^2+14x-8=0$

Let the roots of this equation are $\frac{a}{r}, a, ar$. Then, we get, Sum of roots $=7$

$\frac{a}{r}+a+ar=7.$ ....(i)

Product of consecutive roots $=14$

$\frac{a}{r}\cdot a+a\cdot ar+ar\cdot\frac{a}{r}=14$ ......(ii)

Product of roots $=8$

$\frac{a}{r}\cdot a\cdot ar=8$ ....(iii)

From Eq. (iii), we get $a=2$

Now, from Eq. (i), we get

$$\frac{2}{r}+2r=5$$
$$\Rightarrow 2+2r^2=5r\Rightarrow r=2 \text{ or } r=\frac{1}{2}$$

Thus, the roots are 1,2,4.

Hence, difference of largest and smallest number

$=4-1=3$

111. **(c)** For a binomial variate x,

Mean, $np=\frac{4}{3}$ and ......(i)

Variance, $npq=\frac{8}{9}$ ......(ii)

From Eqs. (i) and (ii), we have

$q=\frac{2}{3}, p=\frac{1}{3}$ and $n=4$

Now, $P(X=2)={}^4C_2p^2q^2$

$$=6\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^2=\frac{24}{81}=\frac{8}{27}$$

112. **(c)** We have given that α is non-real root of $x^7=1$

$\alpha^7=1$ and $\alpha\neq 1$

Now, $\alpha(1+\alpha)(1+\alpha^2+\alpha^4)$

$=\alpha(1+\alpha^2+\alpha^4+\alpha+\alpha^3+\alpha^5)$

$=\alpha+\alpha^3+\alpha^5+\alpha^2+\alpha^4+\alpha^6$

$=\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\alpha^6$

$=\frac{\alpha(1-\alpha^6)}{1-\alpha}=\frac{\alpha-\alpha^7}{1-\alpha}=\frac{\alpha-1}{1-\alpha}$ $[\because \alpha\neq 1]$

$[\because \alpha^7=1]$

$\alpha(1+\alpha)(1+\alpha^2+\alpha^4)=-1$

113. **(a)** Given that,

$$\cot(\cos^{-1}x)=\sec\left\{\tan^{-1}\left(\frac{a}{\sqrt{b^2-a^2}}\right)\right\}$$

Since, $\cos^{-1} x = \cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$
and $\tan^{-1} x = \sec^{-1}\left(\sqrt{1+x^2}\right)$

$$\Rightarrow \cot\left(\cot^{-1}\frac{x}{\sqrt{1-x^2}}\right) = \sec\left\{\sec^{-1}\sqrt{1+\left(\frac{a}{\sqrt{b^2-a^2}}\right)^2}\right\}$$

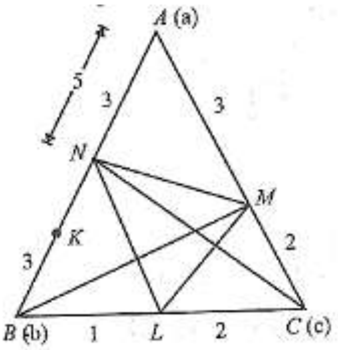
$$\Rightarrow \frac{x}{\sqrt{1-x^2}} = \sqrt{\frac{b^2-a^2+a^2}{b^2-a^2}} \Rightarrow \frac{x}{\sqrt{1-x^2}} = \frac{b}{\sqrt{b^2-a^2}}$$

On squaring both the sides, we get

$$\frac{x^2}{1-x^2} = \frac{b^2}{b^2-a^2} \Rightarrow x^2b^2 - x^2a^2 = b^2 - b^2x^2$$

$$\Rightarrow x^2(2b^2-a^2) = b^2 \Rightarrow x = \frac{b}{\sqrt{2b^2-a^2}}$$

114. **(d)** Given that, in $\triangle$ ABC, L, M, N are points on BC, CA and AB respectively, dividing in the ratio 1: 2,2: 3 and 3: 5. Also, point K divides AB in the ratio 5: 3.



Now, according to the section formula,

$$L = \frac{2\mathbf{b}+\mathbf{c}}{3}, M = \frac{2\mathbf{a}+3\mathbf{c}}{5}$$

$$N = \frac{3b+5\mathbf{a}}{8}, K = \frac{5\mathbf{b}+3\mathbf{a}}{8}$$

$$\overrightarrow{AL} = \frac{2\,b+\mathbf{c}}{3} - \mathbf{a} = \frac{2\,b+\mathbf{c}-3a}{3}$$

$$\overline{BM} = \frac{2a+3c}{5} - b = \frac{2b+3c-5b}{5}$$

$$\overrightarrow{CN} = \frac{3\,b+5a}{8} - c = \frac{3\,b+5a-8c}{8}$$

$$\overrightarrow{CK} = \frac{5\,b+3a}{8} - \mathbf{c} = \frac{5\,b+3a-8c}{8}$$

$$\left|\frac{\overrightarrow{AL}+\overrightarrow{BM}+\overrightarrow{CN}}{\overrightarrow{CK}}\right|$$

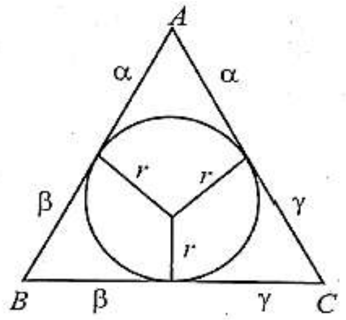
$$= \left|\frac{\frac{2b+c-3a}{3}+\frac{2a+3c-5b}{5}+\frac{3b+5a-8c}{8}}{\frac{5b+3a-8c}{8}}\right|$$

$$\Rightarrow \left|\frac{(5b+3a-8c)}{15(5b+3a-8c)}\right| = \frac{1}{15}$$

115. **(c)** $2x^2+4xy-6y^2+2x+8y+1=0$
is in the form of $ax^2+2hxy+by^2+2gx+2fy+c=0$, we get $a=2, h=2, b=-6, g=1, f=4, c=1$
Required point

$$= \left(\frac{bg-fh}{h^2-ab}, \frac{af-gh}{h^2-ab}\right) = \left(\frac{-6-8}{4+12}, \frac{8-2}{4+12}\right) = \left(\frac{-7}{8}, \frac{3}{8}\right)$$

116. **(a)** It is given that α, β, γ are the length of tangents from the vertices of a triangle to its incircle.



Semi-perimeter of $\triangle$ ABC,

$$\Rightarrow S = \alpha + \beta + \gamma$$

$$\text{Area of } \triangle ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(\alpha+\beta+\gamma)(\alpha)(\beta)(\gamma)}$$

As we know,

$$r = \Delta/s$$

$$r = \frac{\sqrt{(\alpha+\beta+\gamma)(\alpha\beta\gamma)}}{\alpha+\beta+\gamma}$$

$$\Rightarrow \text{Hence, } \alpha+\beta+\gamma = \frac{\alpha\beta\gamma}{r^2}$$

117. **(c)** Given, $\int_0^{10} f(x)dx = 5$ ....(i)

Let $I = \int_0^1 f(k-1+x)dx$

Put $k-1+x = t \Rightarrow dx = dt$

Limits:

$$x = 0 \Rightarrow t = k-1$$

$$x = 1 \Rightarrow t = k$$

$$I = \int_{k-1}^{k} f(t)dt$$

It can also be written as, $I = \int_{k-1}^{k} f(x)dx$

Now, $\sum_{k=1}^{10}\int_0^1 f(k-1+x)dx = \sum_{k=1}^{10}\int_{k-1}^{k} f(x)dx$

$$= \int_0^1 f(x)dx + \int_1^2 f(x)dx + \cdots + \int_9^{10} f(x)dx$$

$$= \int_0^{10} f(x)dx = 5 \text{ (from Eq. (i))}$$

118. **(a)** Given ellipse, $3x^2 + 2y^2 - 5 = 0$

Then, equations of pairs of tangents drawn to the ellipse S from the point (1,2).

$$SS_1 = T^2$$

$$(3x^2+2y^2-5)(3(1)^2-2(2)^2-5) = (3x(1)+2y(2)-5)^2$$

$$(3x^2+2y^2-5)(6) = (3x+4y-5)^2$$

$$18x^2+12y^2-30 = 9x^2+16y^2+25+24xy-40y-30$$

$$9x^2-4y^2-24xy+30x+40y-55 = 0$$

No: , the angle θ between these lines is given by

$$\tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}$$

$$\tan\theta = \frac{2\sqrt{144+36}}{5} = \frac{2\sqrt{180}}{5} = \frac{12\sqrt{5}}{5}$$

$$\theta = \tan^{-1}\left(\frac{12\sqrt{5}}{5}\right)$$

119. **(d)** Given equation of normal, $lx + my = 1$

$y=\frac{-l}{m}+\frac{1}{m}$

Which is of the form $y=mx+c$

Since, the lines $y=mx+c$ will be normal to the hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, if $c^2=\frac{m^2(a^2+b^2)^2}{a^2-b^2m^2}$

$$\left(\frac{1}{m}\right)^2=\frac{\left(-\frac{l}{m}\right)^2(a^2+b^2)^2}{a^2-b^2\left(-\frac{l}{m}\right)^2}$$

$$\frac{1}{m^2}=\frac{\frac{l^2}{m^2}(a^2+b^2)^2}{\frac{m^2a^2-l^2b^2}{m^2}}$$

$$\frac{1}{m^2}=\frac{l^2(a^2+b^2)^2}{m^2a^2-l^2b^2}$$

$$a^2m^2-b^2l^2=l^2m^2(a^2+b^2)^2$$

120. **(c)** Let $I=\int\frac{x^{e-1}+e^{x-1}}{x^e+e^x}dx$ ....(i)

Now, put $x^e+e^x=t$ ....(ii)

$\Rightarrow (ex^{e-1}+e^x)dx=dt$

$\Rightarrow (x^{e-1}+e^{x-1})dx=\frac{dt}{e}$ .....(iii)

Now, putting values from Eqs. (ii) and (iii) in Eq. (i)

$$I=\frac{1}{e}\int\frac{dt}{t}=\frac{1}{e}\log|t|+C=\frac{1}{e}\log|x^e+e^x|+C$$

121. **(a)** Given that,

$$\Delta=\begin{vmatrix}1 & \cos\theta & 1\\ -\cos\theta & 1 & \cos\theta\\ -1 & -\cos\theta & 1\end{vmatrix}$$

$$\Delta=\begin{vmatrix}1 & \cos\theta & 1\\ -\cos\theta & 1 & \cos\theta\\ 0 & 0 & 2\end{vmatrix}\quad [\text{Applying } R_3\to R_3+R_1]$$

Now, on expanding along R_3, we get

$\Delta=2(1+\cos^2\theta)$

As we know, $0\le\cos^2\theta\le1\Rightarrow 1\le1+\cos^2\theta\le2$

$\Rightarrow\ 2\le2(1+\cos^2\theta)\le4$

Thus, Δ lies in the interval [2,4].

122. **(a)** Given, $S_1\equiv x^2+y^2+2x+2y+1=0$

and $S_2\equiv x^2+y^2+4x+6y+4=0$

Now, the equation of common chord of these two circles.

$S_1-S_2=0$

$(x^2+y^2+2x+2y+1)-(x^2+y^2+4x+6y+4)=0$

$\Rightarrow -2x-4y-3=0$ i.e. $2x+4y+3=0$

The equation of required circle,

$S_1+\lambda(S_2-S_1)=0$

$(x^2+y^2+2x+2y+1)+\lambda(2x+4y+3)=0$

$x^2+y^2+2x(1+\lambda)+2y(2\lambda+1)+3\lambda+1=0$

Diameter of the above circle is the common chord of circles S_1 and S_2.

So, $2(-(1+\lambda))+4(-(2\lambda+1))+3=0$

$-2-2\lambda-8\lambda-4+3=0$

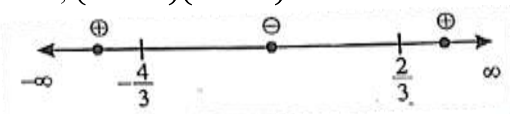
$10\lambda=-3\Rightarrow\lambda=\frac{-3}{10}$

Hence, the required circle

$$x^2 + y^2 + 2x\left(1 - \frac{3}{10}\right) + 2y\left(-\frac{6}{10} + 1\right) - \frac{9}{10} + 1 = 0$$
$$x^2 + y^2 + \frac{14x}{10} + \frac{8y}{10} + \frac{1}{10} = 0$$
$$10x^2 + 10y^2 + 14x + 8y + 1 = 0$$

123. **(a)**

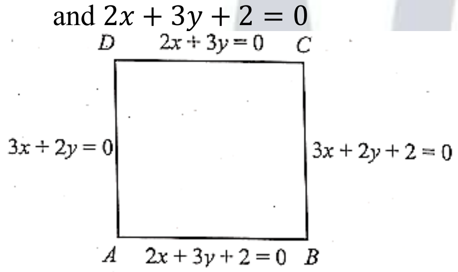
Given that, $\frac{x-1}{3x+4} - \frac{x-3}{3x-2} < 0$
$$\frac{(x-1)(3x-2) - (x-3)(3x+4)}{(3x+4)(3x-2)} < 0$$
$$\frac{(3x^2 - 2x - 3x + 2) - (3x^2 + 4x - 9x - 12)}{(3x+4)(3x-2)} < 0$$
$$\frac{14}{(3x+4)(3x-2)} < 0$$
Thus, $(3x+4)(3x-2) < 0$



x lies in the interval $\left(-\frac{4}{3}, \frac{2}{3}\right)$.

124. **(a)** Given that, there are 10 intermediate stations between two particular stations.
Here, the train stops at only 3 stations and no two of them should be consecutive.
Therefore, there are 8 stations to select these 3 stations.
Hence, number of ways = $^8C_3 = 56$

125. **(c)** Equation of pair of lines,
$$6x^2 + 13xy + 6y^2 = 0$$
$$(3x + 2y)(2x + 3y) = 0$$
$$3x + 2y = 0 \text{ and } 2x + 3y = 0$$
Similarly, for another equation of pairs of lines
$6x^2 + 13xy + 6y^2 + 10x + 10y + 4 = 0$ are $3x + 2y + 2 = 0$
and $2x + 3y + 2 = 0$



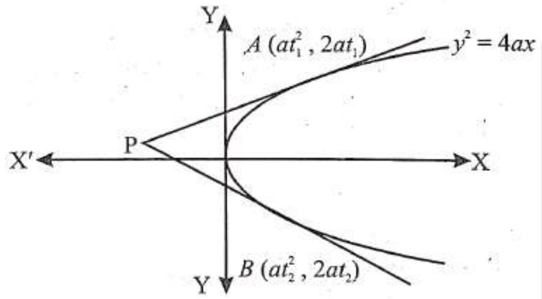
Here, AB is parallel to CD and BC is parallel to AD.
$ABCD$ is a parallelogram.
But also $AB = BC = CD = AD = \frac{2}{\sqrt{13}}$
Hence, $ABCD$ is a rhombus.

126. **(b)** Given circle, $x^2 + y^2 - 2x - 6y - 8 = 0$
So, chord of contact of two tangents drawn from the point $P(a, b)$ to the given circle is
$$xa + yb - (x + a) - 3(y + b) - 8 = 0$$
$\Rightarrow \quad (a-1)x + (b-3)y - (a + 3b + 8) = 0$
As, this line coincides with $5x + y + 1 = 0$
$$\frac{a-1}{5} = \frac{b-3}{1} = \frac{-(a+3b+8)}{1}$$
$\Rightarrow a - 1 = -5(a + 3b + 8)$ and $b - 3 = -a - 3b - 8$
$\Rightarrow 2a + 5b = -13 \quad$ .....(i)

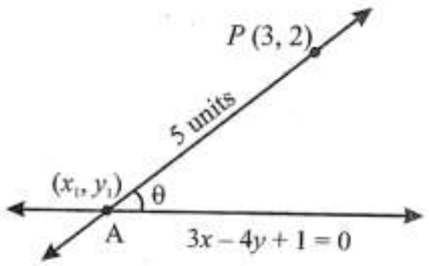
and $a+4b=-5$ ....(ii)
On solving Eqs. (i) and (ii), we get $a=-9$ and $b=1$
Hence, $5a+b=-45+1=-44$
127. **(c)** Given that, a, b and c are non-zero vectors such that **a&b** are not perpendicular.
Also given, $\mathbf{r}\times\mathbf{b}=\mathbf{c}\times\mathbf{b}$
$\Rightarrow \mathbf{a}\times(\mathbf{r}\times\mathbf{b})=\mathbf{a}\times(\mathbf{c}\times\mathbf{b})$
Since, $\mathbf{r}\perp\mathbf{a}$, therefore $\mathbf{a}\cdot\mathbf{r}=0$

$$\Rightarrow (\mathbf{a}\cdot\mathbf{b})\mathbf{r}-(\mathbf{a}\cdot\mathbf{r})\mathbf{b}=\mathbf{a}\times(\mathbf{c}\times\mathbf{b})$$
$$\Rightarrow (\mathbf{a}\cdot\mathbf{b})\mathbf{r}=\mathbf{a}\times(\mathbf{c}\times\mathbf{b})$$
$$\Rightarrow \mathbf{r}=\frac{\mathbf{a}\times(\mathbf{c}\times\mathbf{b})}{\mathbf{a}\cdot\mathbf{b}}=-\frac{[(\mathbf{c}\times\mathbf{b})\times\mathbf{a}]}{\mathbf{a}\cdot\mathbf{b}}$$
$$=-\frac{[-(\mathbf{b}\times\mathbf{c})\times\mathbf{a}]}{\mathbf{a}\cdot\mathbf{b}}=\frac{(\mathbf{b}\times\mathbf{c})\times\mathbf{a}}{\mathbf{a}\cdot\mathbf{b}}$$

128. **(d)** Here, P is intersecting point of tangents at
$A(at_1^2,2at_1)$ and $B(at_2^2,2at_2)$
$P\equiv\left(at_1t_2,a(t_1+t_2)\right)$



Slope of line PA
$$=\tan\theta_1=\frac{2at_1-at_1-at_2}{at_1^2-at_1t_2}=\frac{a(t_1-t_2)}{at_1(t_1-t_2)}=\frac{1}{t_1}$$
Similarly,
Slope of line $PB=\tan\theta_2=\frac{2at_2-at_1-at_2}{at_2^2-at_1t_2}$
$$=\frac{a(t_2-t_1)}{at_2(t_2-t_1)}=\frac{1}{t_2}$$
According to the question, $\tan\theta_1+\tan\theta_2=b$
$$\frac{1}{t_1}+\frac{1}{t_2}=b\Rightarrow t_2+t_1=bt_1t_2$$
$$\frac{y}{a}=\frac{bx}{a}\ [\because x=at_1t_2,y=a(t_1+t_2)]$$
$y=bx$ $\therefore$ P lies on the line $y=bx$
129. **(d)** Let the point $A(x,y)$ on the line $3x-4y+1=0$ is at distance 5 units from the point $(3,2)$.



Equation of line PA $\frac{x_1-3}{\cos\theta}=\frac{y_1-2}{\sin\theta}=\pm5$
$\Rightarrow x_1=3\pm5\cos\theta$ ......(i)
and $y_1=2\pm5\sin\theta$ ....(ii)
Since, (x_1,y_1) lies on the line $3x-4y-1=0$

$$3(3 \pm 5\cos\theta) - 4(2 \pm 5\sin\theta) - 1 = 0$$
$$\Rightarrow \quad 3\cos\theta = \pm 4\sin\theta \Rightarrow \tan\theta = \pm 3/4$$
$$\cos\theta = \pm 4/5 \Rightarrow \sin\theta = \pm 3/5$$

From Eqs: (i) and (ii), we get

$$x_1 = 3 \pm \left(\frac{4}{5}\right)5 = 7, -1$$
$$y_1 = 2 \pm 5\left(\frac{3}{5}\right) = 5, -1$$

Coordinates are $(7,5)$ and $(-1,-1)$

130. (b)

$$\text{Let } l = \lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$$
$$= \lim_{x\to 0}\frac{2\sin^2 x\cdot(3+\cos x)}{x\tan 4x}$$
$$= 2\cdot\lim_{x\to 0}\frac{\sin^2 x}{x^2}\frac{1}{4}\lim_{x\to 0}\frac{4x}{\tan 4x}\cdot\lim_{x\to 0}(3+\cos x)$$
$$= 2\cdot 1\cdot\frac{1}{4}\cdot 1\cdot(3+1) = 2$$

131. (a) Given curves are

$$x^2 = 3y \quad \ldots\ldots(i)$$
$$x^2 + y^2 = 4 \quad \ldots\ldots(ii)$$

On solving Eqs. (i) and (ii), we get

$$x = \pm\sqrt{3}, y = 1$$

Thus, their points of intersection are $(\sqrt{3}, 1)$ and $(-\sqrt{3}, 1)$. Now, from Eq. (i), $\frac{dy}{dx} = \frac{2x}{3}$

and from Eq. (ii), $\frac{dy}{dx} = -\frac{x}{y}$

To calculate angle, any of the point can be taken so we are taking $(\sqrt{3}, 1)$ as the point of intersection.

Now, let m_1 and m_2 be the slope of tangent to the curves at $(\sqrt{3}, 1)$. Then,

$m_1 = \frac{2\sqrt{3}}{3}$ and $m_2 = -\sqrt{3}$

Now, the angle between two curves

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{2\sqrt{3}}{3} - (-\sqrt{3})}{1 + \frac{2\sqrt{3}}{3}\times(-\sqrt{3})}\right|$$
$$= \left|\frac{\left(\frac{5\sqrt{3}}{3}\right)}{(-1)}\right| = \frac{5}{\sqrt{3}} \Rightarrow \theta = \tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$$

132. (c) We have system of equation,

$$(a-1)x - y - z = 0; x - (b-1)y + z = 0$$
$$\text{and } x + y - (c-1)z = 0$$

It is a homogeneous system of equations. Now, for non-trivial solution.

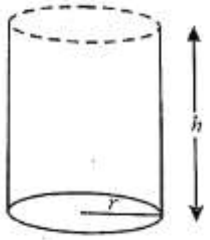
$$\begin{vmatrix} a-1 & -1 & -1 \\ 1 & -(b-1) & 1 \\ 1 & 1 & -(c-1) \end{vmatrix} = 0$$
$$\Rightarrow (a-1)[(1-b)(1-c) - 1] + 1[1 - c - 1]$$
$$-[1 - (1-b)] = 0$$
$$\Rightarrow a - ac - ab + abc - 1 + c + b - bc - a - b - c + 1 = 0$$
$$ab + bc + ca = abc$$

133. (b) Given, White roses = 6 and Red roses = 5

Total number of ways for making garlands such that no two red roses come together is

$= \frac{6! \times 5!}{2} = 43200$

134. (c)



We have given,
$V =$ volume of cylindrical vessel
$r =$ radius and
$h =$ height
As we know for cylindrical vessel
$v = \pi r^2 h \Rightarrow h = \frac{v}{\pi r^2}$ ....(i)
Let S be the area of metal sheet used to form a cylindrical vessel. Then,

$$S = 2\pi rh + \pi r^2 = 2\pi r \frac{V}{\pi r^2} + \pi r^2 \quad \text{[using Eq. (i)]}$$

$$\Rightarrow S(r) = \frac{2V}{r} + \pi r^2$$

$$\Rightarrow S(r) = \frac{2V}{r} + \pi r^2$$

For minimum value of $S(r), S'(r) = 0$

$$\Rightarrow \frac{-2V}{r} + 2\pi r = 0 \Rightarrow \frac{2V}{r^2} = 2\pi r$$

$$\Rightarrow r^3 = \frac{V}{\pi} \Rightarrow r = \sqrt[3]{\frac{V}{\pi}}$$

Now, by using the value of r in Eq. (i)

$$V = \pi \left(\frac{V}{\pi}\right)^{\frac{2}{3}} \cdot h$$

$$\Rightarrow h = \frac{V}{\pi} \times \frac{\pi^{2/3}}{v^{2/3}} \Rightarrow h = \frac{V^{1/3}}{\pi^{1/3}} = \sqrt[3]{\frac{V}{\pi}}$$

Hence, the required radius is $r = \sqrt[3]{\frac{V}{\pi}}$ and height is $h = \sqrt[3]{\frac{V}{\pi}}$.

135. (a)

$$\text{Let } I = \int \frac{x + \sin x}{1 + \cos x} dx$$

$$= \int \frac{x}{1 + \cos x} dx + \int \frac{\sin x}{1 + \cos x} dx$$

$$= \frac{1}{2} \int x\sec^2 \frac{x}{2} dx + \int \frac{2\sin x/2 \cos x/2}{2\cos^2 x/2} dx$$

$$= \frac{1}{2} \int x\sec^2 \frac{x}{2} dx + \int \tan \frac{x}{2} dx$$

$$= \frac{1}{2} \left[x \frac{\tan \frac{x}{2}}{1/2} - \int \frac{\tan x/2}{\frac{1}{2}} dx \right] + \int \tan \frac{x}{2} dx$$

$$= x\tan \frac{x}{2} - \int \tan \frac{x}{2} dx + \int \tan \frac{x}{2} dx = x\tan \frac{x}{2} + C$$

136. (a)

Given, $I_n = \int \frac{\sin nx}{\cos x} dx$ .....(i)

$$= \int \frac{\sin(nx - x + x)}{\cos x} dx$$

$$= \int \frac{\sin[(n-1)x + x]}{\cos x} dx$$

$$= \int \frac{\sin(n-1)x\cos x + \cos(n-1)x\sin x}{\cos x} dx$$

$$= \int \sin(n-1)x dx + \int \frac{\cos(n-1)x\sin x}{\cos x} dx$$

$$= \int \sin(n-1)x dx + \frac{1}{2}\int \frac{2\sin x\cos(n-1)x}{\cos x} dx$$

Since, $2\sin x\cos y = +\sin(x+y) + \sin(x-y)$

$$= -\frac{\cos(n-1)x}{(n-1)} + \frac{1}{2}\int \frac{\sin nx + \sin(2-n)x}{\cos x} dx$$

$$= \frac{-\cos(n-1)x}{(n-1)} + \frac{1}{2}\int \frac{\sin nx}{\cos x} dx - \frac{1}{2}\int \frac{\sin(n-2)x}{\cos x} dx$$

From Eq. (i),

$$I_n = -\frac{\cos(n-1)x}{(n-1)} + \frac{1}{2}I_n - \frac{1}{2}I_{n-2}$$

$$\left(1 - \frac{1}{2}\right) I_n = -\frac{\cos(n-1)x}{(n-1)} - \frac{1}{2}I_{n-2}$$

$$I_n = \frac{-2}{n-1}\cos(n-1)x - I_{n-2}$$

137. **(c)** Given that, $\tan\alpha$ and $\tan\beta$ are the roots of the equation $x^2 + px + q = 0$

$\tan\alpha + \tan\beta = -p$ and $\tan\alpha \cdot \tan\beta = q$

Now, $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{-p}{1-q} = \frac{p}{q-1}$

$$\sec(\alpha+\beta) = \sqrt{1 + \tan^2(\alpha+\beta)} = \sqrt{1 + \frac{p^2}{(q-1)^2}}$$

$$\cos(\alpha+\beta) = \frac{1}{\sqrt{1 + \frac{p^2}{(q-1)^2}}}$$

$$\sin^2(\alpha+\beta) + p\cos(\alpha+\beta)\sin(\alpha+\beta) + q\cos^2(\alpha+\beta)$$

$$= \cos^2(\alpha+\beta)[\tan^2(\alpha+\beta) + p\tan(\alpha+\beta) + q]$$

$$= \frac{1}{1 + \frac{p^2}{(q-1)^2}}\left[\frac{p^2}{(q-1)^2} + \frac{p^2}{q-1} + q\right]$$

$$= \frac{(q-1)^2}{(q-1)^2 + p^2}\left[\frac{p^2 + p^2(q-1) + q(q-1)^2}{(q-1)^2}\right]$$

$$= \frac{p^2 + p^2q - p^2 + q(q-1)^2}{p^2 + (q-1)^2}$$

$$= \frac{q\{p^2 + (q-1)^2\}}{p^2 + (q-1)^2} = q$$

138. **(c)** We have,

$$= \sum_{r=1}^{n-1} r(r+1-w)(r+1-w^2)$$
$$= \sum_{r=1}^{n-1} r[(r+1)^2-(r+1)w^2-w(r+1)+w^3]$$
$$= \sum_{r=1}^{n-1} r\left[(r+1)^2-(r+1)\left((w+w^2)+w^3\right)\right]$$
$$= \sum_{r=1}^{n-1} r[(r+1)^2-(r+1)((-1)+1)]$$
$[\because w^3 = 1 \text{ and } 1+w+w^2=0]$
$$= \sum_{r=1}^{n-1} r(r^2+3r+3) = \sum_{r=1}^{n-1} r^3 + 3\sum_{r=1}^{n-1} r^2 + 3\sum_{r=1}^{n-1} r$$
$$= \left(\frac{(n-1)n}{2}\right)^2 + \frac{3(n-1)(n)(2n-1)}{6} + \frac{3(n-1)n}{2}$$
$$\frac{n^2(n-1)^2}{4} + \frac{n(n-1)(2n-1)}{2} + \frac{3}{2}n(n-1)$$
$$= \frac{n(n-1)}{4}[n(n-1)+2(2n-1)+6]$$
$$= \frac{n(n-1)}{4}[n^2-n+4n-2+6]$$
$$= \frac{n(n-1)}{4}[n^2+3n+4]$$

139. **(d)** Given that,
$$\sigma(n) = \begin{cases} \frac{n}{2} & \text{if n is even} \\ -\frac{(n-1)}{2} & \text{if n is odd} \end{cases}$$
Case-I : If **n** is even.
$\sigma(n) = 1,2,3,4,5, \ldots$
Case-II: If n is odd
$\sigma(n) = 0, -1, -2, -3, -4, \ldots$
Thus, for every value of $n\sigma(n)$ has an unique image.
$\sigma(n)$ is one-one function.
So, range of $\sigma(n) = Z$
Range of $\sigma(n) =$ codomain of $\sigma(n) = Z$.
Hence, $\sigma(n)$ is one-one and onto function.

140. **(d)**

Given that, $^{37}C_4 + \sum_{r=1}^{5} {}^{(42-r)}C_r$
$$= {}^{37}C_4 + {}^{41}C_1 + {}^{40}C_2 + {}^{39}C_3 + {}^{38}C_3 + {}^{37}C_3$$
$$= {}^{37}C_3 + {}^{37}C_4 + {}^{38}C_3 + {}^{39}C_3 + {}^{40}C_3 + {}^{41}C_3$$
As we know,
$${}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r$$
$$= {}^{38}C_4 + {}^{38}C_3 + {}^{39}C_3 + {}^{40}C_3 + {}^{41}C_3$$
$$= {}^{39}C_4 + {}^{39}C_3 + {}^{40}C_3 + {}^{41}C_3$$
$$= {}^{40}C_4 + {}^{40}C_3 + {}^{41}C_3 = {}^{41}C_4 + {}^{41}C_3 = {}^{42}C_4$$

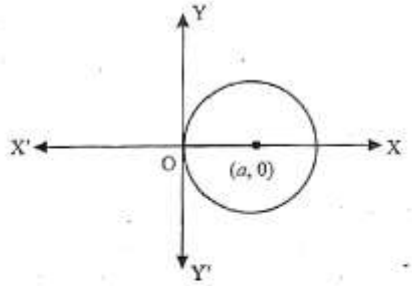
141. **(c)**

x_i	f_i	$d_i = x_i - a$	d_i^2	$f_i d_i$	$f_i d_i^2$

6	2	-12	144	-24	288
10	4	-8	64	-32	256
14	7	-4	16	-28	112
18 $= a$	12	0	0	0	0
24	8	6	36	48	288
28	4	10	100	40	400
30	3	12	144	36	432
	$N = 40$			$\Sigma f_i d_i = 40$	$\sum \mathbf{f_i d_i^2} = 1776$

Now, variance $= \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2 = \frac{1}{40} \times 1776 - 1 = \frac{1736}{40} = 43.4$

142. **(a)** Here, it is given that circle touches the y-axis at origin.



So, equation of family of circles

$$(x-a)^2 + y^2 = a^2$$

$\Rightarrow \quad x^2 + y^2 = 2ax \quad \ldots\ldots(i)$

On differentiating w.r.t. x, we get

$2x + 2y\frac{dy}{dx} = 2a \quad \ldots\ldots(ii)$

Now, from Eqs. (i) and (ii), we get

$$x^2 + y^2 = \left(2x + 2y\frac{dy}{dx}\right)x$$

$$\Rightarrow \quad x^2 + y^2 = 2x^2 + 2xy\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

This is the required differential equation.

143. **(b)** It is given that $p, x_1, x_2, x_3 \ldots x_n$ and $q, y_1, y_2, y_3 \ldots y_n$ are in A.P. whose common difference are a and b respectively.

$x_1 = p + a, x_n = p + na$

$y_1 = q + b, y_n = q + nb$

Also, given α is A.M. of $x_1, x_2, x_3 \ldots . x_n$

$$\alpha = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

$$\Rightarrow \alpha = \frac{\frac{n}{2}(x_1 + x_n)}{n}$$

$$\Rightarrow \alpha = \frac{x_1 + x_n}{2}$$

Similarly, β is A.M. of $y_1, y_2, y_3 \ldots y_n$. So,

$\beta = \dfrac{y_1 + y_n}{2}$

Thus, substituting the value of x, x_n, y and y_n, we get

$$\alpha = \frac{p + a + p + na}{2} = \frac{2p + a(n+1)}{2} \quad \ldots\ldots (i)$$

$$\beta = \frac{q + a + q + na}{2} = \frac{2q + b(n+1)}{2} \quad \ldots\ldots (ii)$$

From Eqs. (i) and (ii) eliminate (n + 1), we get

$$\frac{2\alpha - 2p}{a} = \frac{2\beta - 2q}{b}$$

$\Rightarrow\ b(\alpha - p) \quad = a(\beta - q)$

Hence, locus of $p(\alpha, \beta)$ is $b(x - p) = a(y - q)$

144. **(a)** Given observations $= \{K\alpha\}$

Here, $K = 1,2,3, \ldots\ldots, 50$

$\{K\alpha\} = \{\alpha, 2\alpha, 3\alpha, \ldots., 50\alpha\}$

Number of observations $= 50$ (Even)

Median of given observation $= \frac{25\alpha + 26\alpha}{2}$

$M = 25.5\alpha$

Now, mean deviation about median

$$\text{M.D. (Me)} = \frac{\sum_{k=1}^{n} |x_i - m|}{n}$$

$$50 = \frac{\sum_{k=1}^{50} |k\alpha - 25.5\alpha|}{50}$$

$$\Rightarrow 50 \times 50 = \sum_{k=1}^{50} |\alpha(K - 25.5)|$$

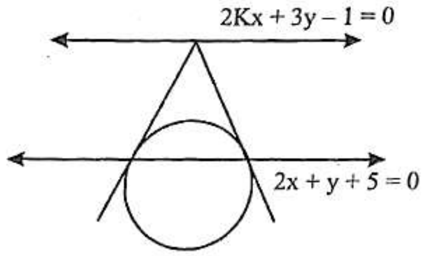
$$\Rightarrow |\alpha| \sum_{k=1}^{50} |k - 25.5| = 2500$$

$$\Rightarrow 2|\alpha|[24.5 + 23.5 + .. + 1.5 + 0.5] = 2500$$

$$\Rightarrow 2|\alpha| \times \frac{25}{2} \times (24.5 + 0.5) = 2500$$

$$\Rightarrow |\alpha| \times 25 = 1400 \Rightarrow |\alpha| = 4$$

145. **(c)** Let point (x_1, y_1) lie on the line $2kx + 3y - 1 = 0$.



$x^2 + y^2 - 2x - 4y - 4 = 0$

Given circle, $x^2 + y^2 - 2x - 4y - 4 = 0$

Equation of chord of contact from (x_1, y_1) to the circle

$$xx_1 + yy_1 - \frac{2(x + x_1)}{2} - \frac{4(y + y_1)}{2} - 4 = 0$$

$$\Rightarrow\ x(x_1 - 1) + y(y_1 - 2) - x_1 - 2y_1 - 4 = 0$$

Now, $2x + y + 5 = 0$ and $2kx + 3y + 1 = 0$ are conjugate line

$2x + y + 5 = 0$ and $x(x_1 - 1) + y(y_1 - 2) - x_1 - 2y_1 - 4 = 0$

are coincide.

$$\therefore \frac{x_1 - 1}{2} = \frac{y_1 - 2}{1} = \frac{x_1 + 2y_1 + 4}{-5} = \lambda$$

$\Rightarrow x_1 = 2\lambda + 1, y_1 = \lambda + 2$

$x_1 + 2y_1 + 4 = -5\lambda$

Put x_1, y_1 in $x_1 - 2y_1 - 4 = -5k$

$2\lambda + 1 + 2\lambda + 4 + 4 = -5\lambda \Rightarrow \lambda = -1$

$x_1 = -1, y_1 = 1$

Now, put $x_1 = -1$ and $y_1 = 1$

$2kx + 3y - 1 = 0$, we get

$-2k + 3 - 1 = 0 \Rightarrow k = 1$

146. **(c)** Given that, $\frac{5x^2+2}{x^3+x} = \frac{A_1}{x} + \frac{A_2x+A_3}{x^2+1}$

$$\Rightarrow \frac{5x^2 + 2}{x(x^2 + 1)} = \frac{A_1(x^2 + 1) + (A_2x + A_3)x}{x(x^2 + 1)}$$

$\Rightarrow 5x^2 + 2 = (A_1 + A_2)x^2 + A_3x + A_1$

On comparing the coefficients of x^2, x and constant term, we get $A_1 = 2; A_2 = 3$ and $A_3 = 0$

147. **(c)** Let, $S_n = \frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \frac{1}{8\cdot11} + \cdots$. upto n terms

$= \frac{1}{3}\left[\frac{1}{2\cdot5} + \frac{1}{5\cdot8} + \frac{1}{8\cdot11} + \cdots\right.$. upto n terms]

$= \frac{1}{3}\left[\frac{5-2}{2\cdot5} + \frac{8-5}{5\cdot8} + \frac{11-8}{8\cdot11} + \cdots\right.$.. upto n terms]

$$= \frac{1}{3}\left[\left(\frac{1}{2} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{8}\right) + \left(\frac{1}{8} - \frac{1}{11}\right) + \left(\frac{1}{3n-1} - \frac{1}{3n+2}\right)\right]$$

$$= \frac{1}{3}\left[\frac{1}{2} - \frac{1}{3n+2}\right] = \frac{1}{3}\left[\frac{3n+2-2}{2(3n+2)}\right] = \frac{n}{2(3n+2)}$$

148. **(b)** Let, general equation of this parabola $x = Ay^2 + By + C$

It is given that, it passes through the points $(-2,1), (1,2)$ and $(-1,3)$ so, we get

$-2 = A + B + C$ ... ... (i)

$1 = 4A + 2B + C$ ... . (ii)

and $-1 = 9A + 3B + C$ ... ... (iii)

On solving Eqs. (i), (ii) and (iii), we get

$A = \frac{-5}{2}, B = \frac{21}{2}$ and $C = -10$

Hence, equation of parabola

$$x = -\frac{5}{2}y^2 + \frac{21}{2}y - 10$$

$\Rightarrow \quad 2x = -5y^2 + 21y - 20$

$\Rightarrow \quad 5y^2 + 2x - 21y + 20 = 0$

149. **(b)** Given that,

$b\cos(C + \theta) + c\cos(B - \theta)$

$= b(\cos C\cos\theta - \sin C\sin\theta) + c(\cos B\cos\theta + \sin B\sin\theta)$

$= b\cos C\cos\theta + c\cos B\cos\theta - b\sin C\sin\theta + c\sin B\sin\theta$

$= \cos\theta(b\cos C + c\cos B) - \sin\theta(b\sin C - c\sin B)$

Since by projection formula and by sine rule

$$\frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow b\sin C - c\sin B = 0$$

$b\cos(C + \theta) + c\cos(B - \theta)$

$= a\cos\theta - \sin\theta \cdot 0$

$= a\cos\theta - \sin\theta \cdot 0 = a\cos\theta$

150. **(c)** We have,

$1+\cos x\cdot\cos 5x=\sin^2 x$
$\Rightarrow\ 1-\sin^2 x+\cos x\cdot\cos 5x=0$
$\Rightarrow\ \cos^2 x+\cos x\cdot\cos 5x=0$
$\Rightarrow\ \cos x(\cos x+\cos 5x)=0$
$\Rightarrow\ \cos x\left[2\cos\left(\frac{x+5x}{2}\right)\cos\left(\frac{x-5x}{2}\right)\right]=0$
$\Rightarrow\ \cos x(2\cos 3x\cos 2x)=0$
$\Rightarrow\ \cos x=0;\ \cos 3x=0$ or $\cos 2x=0$
$\Rightarrow\ x=\frac{\pi}{2},\frac{3\pi}{2}$ or
and $3x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2},\frac{9\pi}{2},\frac{11\pi}{2}$ or
and $2x=\frac{\pi}{2},\frac{3\pi}{2},\frac{5\pi}{2},\frac{7\pi}{2}$ $[\because x\in[0,2\pi]]$
$\Rightarrow x=\frac{\pi}{2},\frac{3\pi}{2},\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6},\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$
Thus, there are 10 solutions in $[0,2\pi]$

151. **(d)** Given that α,β and γ are the roots of the equation

$x^3+px^2+qx+r=0$
$\alpha+\beta+\gamma=-p$ ...... (i)
$\alpha\beta+\beta\gamma+\gamma\alpha=q$ ...... (ii)
and $\alpha\beta\gamma=-r$ .... (iii)

Now, $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$
$=(1+\alpha^2)(1+\beta^2+\gamma^2+(\beta\gamma)^2)$
$=1+\beta^2+\gamma^2+(\beta\gamma)^2+\alpha^2+(\alpha\beta)^2+(\alpha\gamma)^2+(\alpha\beta\gamma)^2$
$=1+(\alpha^2+\beta^2+\gamma^2)+((\alpha\beta)^2+(\beta\gamma)^2+(\gamma\alpha)^2+(\alpha\beta\gamma)^2$
$=1+[(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)]$
$+[(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)]+(\alpha\beta\gamma)^2$

From Eq. (i), (ii) and (iii), we get
$=1+[p^2-2q]+[q^2-2rp]+r^2$
$=1+p^2-2q+q^2-2rp+r^2$
$=(q-1)^2+(r-p)^2$

152. **(a)** We have given a system of linear equations with the three unknowns.

Here, equations are in the matrix form $AX=D$ such that the system is inconsistent.

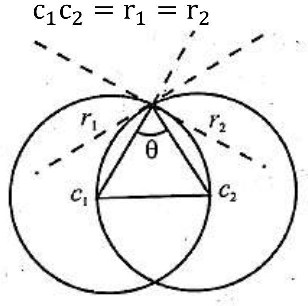
Rank of Augmented matrix AD > Rank of coefficient matrix A

Rank of AD > Rank of A

$\frac{\text{Rank of A}}{\text{Rank of AD}}<1$

153. **(a)** Let θ be the angle between two circles. As each circle passing through the centre at each other. Then,

$c_1c_2=r_1=r_2$



From the figure,

$$\cos\theta = \frac{r_1^2 + r_2^2 - c_1c_2}{2r_1r_2} \Rightarrow \cos\theta = \frac{r_1^2 + r_2^2 - r_1^2}{2r_1r_2}$$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

Hence, angle between two circle is either $\frac{\pi}{3}$ or $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

154. **(c)** Given that, $\log_{\frac{1}{\sqrt{3}}}\left\{\frac{|z|^2-|z|+1}{2+|z|}\right\} > -2$

Since, $\log_a b > c, b < a^c$ if $0 < a < 1$

$$\Rightarrow \frac{|z|^2 - |z| + 1}{2 + |z|} < \left(\frac{1}{\sqrt{3}}\right)^{-2}$$

$$\Rightarrow |z|^2 - |z| + 1 < (\sqrt{3})^2(2 + |z|)$$

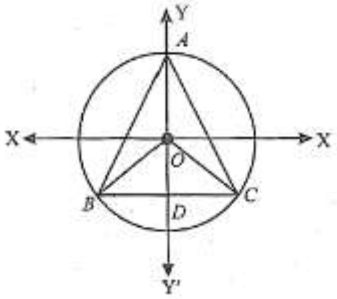
$$\Rightarrow |z|^2 - |z| + 1 < 6 + 3|z|$$

$$\Rightarrow (|z| - 2)^2 < 9 \Rightarrow -3 < |z| - 2 < 3$$

$$\Rightarrow -1 < |z| < 5 \Rightarrow 0 < |z| < 5$$

Hence, z fies inside the circle.

155. **(c)** We have, length of median of $\triangle$ ABC $= 9$



$$AO = \frac{2}{3}AD \Rightarrow AO = \frac{2}{3} \times 9 = 6$$

O is the circumcentre of $\triangle$ ABC.

We know that in equilateral triangle circumcentre, incentre, centroid coincide.

Origin O(0,0) is the centre and AO is radius of circle.

Hence, equation of circle

$$(x - \sigma)^2 + (y - \sigma)^2 = (6)^2$$

$$\Rightarrow \quad x^2 + y^2 = 36$$

156. **(b)** We have,

$$1 + \cos 10^\circ + \cos 20^\circ + \cos 30^\circ$$

$$= (1 + \cos 10^\circ) + (\cos 20^\circ + \cos 30^\circ)$$

$$= 2\cos 5^\circ + 2\cos 25^\circ \cos 5^\circ$$

$$= 2\cos 5^\circ(\cos 5^\circ + \cos 25^\circ)$$

$$= 2\cos 5^\circ 5^\circ(2\cos 15^\circ \cos 10^\circ)$$

$$= 4\cos 5^\circ \cos 10^\circ \cos 15^\circ$$

157. **(b)** $(a - 7)(a - 11) > 0$

$a < 7$ and $a > 11$

158. **(b)** Given that,

$$x = \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \frac{1\cdot 3\cdot 5\cdot 7}{3\cdot 6\cdot 3\cdot 12} + \cdots .\infty \text{ terms}$$

$$\Rightarrow x = \frac{1\cdot 3}{3^2(2!)} + \frac{1\cdot 3\cdot 5}{3^3(3!)} + \frac{1\cdot 3\cdot 5\cdot 7}{3^4(4!)} + \cdots .\infty \text{ terms}$$

$$\Rightarrow x = \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2!}\left(\frac{2}{3}\right)^2 + \frac{\left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)}{3!}\left(\frac{2}{3}\right)^3 + \cdots$$

$$\Rightarrow x = \left[1 + \frac{1}{2}\left(\frac{2}{3}\right) + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)}{2!}\left(\frac{2}{3}\right)^2 + \frac{\frac{1}{2}\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+1\right)}{3!}\left(\frac{2}{3}\right)^3\right] - \left(1+\frac{1}{3}\right)$$

Now, by using binomial expansion for any index

$$\Rightarrow x = \left(1-\frac{2}{3}\right)^{-1/2} - \frac{4}{3} \Rightarrow x = \left(\frac{1}{3}\right)^{-1/2} - \frac{4}{3}$$

$$\Rightarrow x = \sqrt{3} - \frac{4}{3} \Rightarrow 3x + 4 = 3\sqrt{3}$$

On squaring both the sides, we get

$$\Rightarrow (3x+4)^2 = 3(\sqrt{3})^2 \Rightarrow 9x^2 + 24x + 16 = 27$$

$$\Rightarrow 9x^2 + 24x = 11$$

159. **(c)** It is given that,

$$(3\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}}) = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - \hat{\mathbf{k}})x + (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 2\hat{\mathbf{k}})y + (-2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 2\hat{\mathbf{k}})z$$

$$= \hat{\mathbf{i}}(2x + y - 2z) + \hat{\mathbf{j}}(3x - 2y + z) + \hat{\mathbf{k}}(-x + 2y - 2z)$$

On comparing both the sides, we get

$2x + y - 2z = 3$

$3x - 2y + z = -1$

$-x + 2y - 2z = 2$

Now, by checking the options for coordinates (x, y, z), option (c) satisfies them.

160. **(a)** As we know, the volume of tetrahedron formed by the coterminous edges **a**, **b** and **c**,

$$V = \frac{1}{6}[\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}] \Rightarrow 4 = \frac{1}{6}[\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}]$$

$$[\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}] = 24 \qquad \ldots\ldots(i)$$

Now, the volume of parallelopiped formed, by the coterminous edges $\mathbf{a} \times \mathbf{b}, \mathbf{b} \times \mathbf{c}$ and $\mathbf{c} \times \mathbf{a}$.

$$V = [\mathbf{a} \times \mathbf{b} \quad \mathbf{b} \times \mathbf{c} \quad \mathbf{c} \times \mathbf{a}] \Rightarrow V = [\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}]^2$$

$$V = (24)^2 = 576 \quad \text{(from Eq. (i))}$$