Physics

1. (a) We have

$$R_{cq} = \frac{R_1R_2}{R_1 + R_2} = \frac{60 \times 30}{60 + 30} = 20\Omega$$

and tolerance value is

$$\Rightarrow \Delta R_P = R_{eq}\left\{\frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} - \frac{\Delta R_1 + \Delta R_2}{R_1 + R_2}\right\}$$

$$= 20\left\{\frac{0.36}{60} + \frac{0.09}{30} - \frac{0.36 + 0.09}{90}\right\}$$

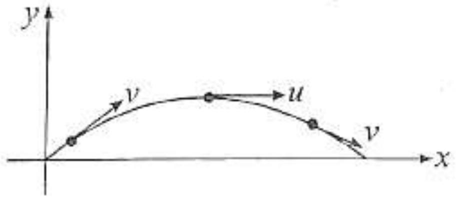
$$= 20\{0.006 + 0.003 - 0.005\} = 0.085\Omega$$

So, resistance in parallel is $R_p = 20 \pm 0.08$

2. (d) Trajectory of a projectile is of form

$$y = f(x) = x\tan\theta - \frac{gx^2}{2\pi^2\cos^2\theta} \therefore \frac{dy}{dx}$$

Slope of $y - x$ curve, which do not gives velocity. So, Assertion (A) is incorrect.
Also, velocity of a projectile is always along tangent to the trajectory (shown)



Hence, reason (R) is correct.

3. (c)

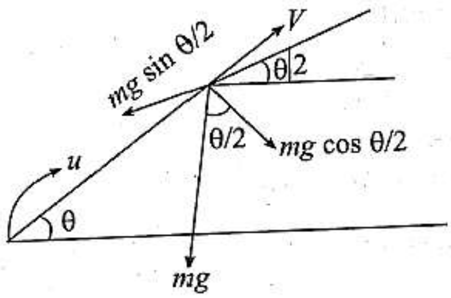
$$\text{Required ratio } = \frac{H_{max}}{R} = \frac{\left(\frac{u^2\sin^2\theta}{2g}\right)}{\left(\frac{u^2\sin 2\theta}{g}\right)} = \frac{\tan\theta}{4}$$

$$= \frac{8}{7} \times \frac{1}{4} = \frac{2}{7}$$

4. (d) Let velocity of projectile is v at an angle $\frac{\theta}{2}$ with horizontal

$$v\cos\frac{\theta}{2} = u\cos\theta$$

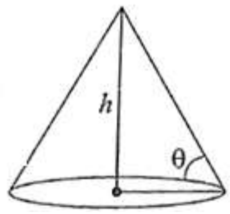
$$\text{or } v = \frac{u\cos\theta}{\cos\frac{\theta}{2}}$$



Now, $\frac{mv^2}{r} = mg\cos\frac{\theta}{2} \Rightarrow r = \frac{v^2}{g\cos\frac{\theta}{2}}$

$$\frac{\frac{u^2\cos^2\theta}{\left(\cos\frac{\theta}{2}\right)^2}}{g\cos\frac{\theta}{2}} \Rightarrow r = \frac{u^2\cos^2\theta \cdot \sec^3\left(\frac{\theta}{2}\right)}{g}$$

5. (d)



$\tan\theta_{\max} = \mu$

$$\frac{h_{\max}}{R} = \mu$$

$$h_{\max} = \mu R$$

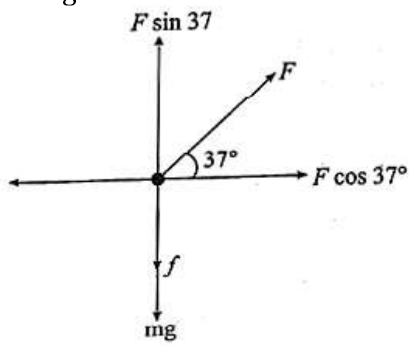
Now, $V_{\max} = \frac{1}{3}\pi R^2 h_{\max}$

$= \dfrac{1}{3}\pi R^2 - \mu R = \dfrac{1}{3}\pi\mu R^3$

6. **(b)** $\mathrm{mg} = 20 \times 10 = 20\ \mathrm{N}$

$\mathrm{Fsin}\,37 = 90 \times \frac{3}{5} = 54\ \mathrm{N}$

as. $\mathrm{mg} < \mathrm{Fsin}\,37$



So, friction force will act downward

So, $\mathrm{Fsin}\,37 - f - \mathrm{mg} = \mathrm{mcl}$

$\mathrm{Fsin}\,37 - \mu\cos 37 - \mathrm{mg} = \mathrm{mcl}$

Putting the value of $F_1\mu$ and m we get $\mathrm{a} = 8\ \mathrm{m/s^2}$

7. **(a)** $\mathrm{E_i} = \frac{1}{2}\mathrm{mv}^2 = \frac{1}{2} \times 2 \times 8^2 = 64\ \mathrm{J}$

$\mathrm{E_f} = \mathrm{mgh} = 2 \times 10 \times 3 = 60\ \mathrm{J}$

Work done against air friction

$=$ Loss of energy $= 64 - 60 = 4\ \mathrm{J}$

8. **(a)** $3\mathrm{g} - \mathrm{T} = 3\mathrm{a}$ ......(i)

$\mathrm{T} - 2\mathrm{g} = 2\mathrm{a}$ ....(ii)

From (i) and (ii)

$\mathrm{a} = \dfrac{\mathrm{g}}{\mathrm{s}} = 2\ \mathrm{m/s^2}$

Now, $\mathrm{s} = \mathrm{ut} + \frac{1}{2}\mathrm{at}^2 = 0 + \frac{1}{2} \times 2(2)^2$

$\Rightarrow \mathrm{s} = 4\ \mathrm{m}$

Work done on block of 3 kg by gravity

$\mathrm{W} = \mathrm{Mgs} = 3 \times 10 \times 4 = 120\ \mathrm{J}$

9. **(d)** Taken, $v = 1$

$v' = 1.09$

and $\Delta v = 0.09$

$$\Rightarrow \omega \propto \frac{1}{I}\left[J = I\omega \Rightarrow I \propto \frac{1}{\omega}\right]$$

But $I \propto r^2$ and $\omega \propto v^{1/3}$

$\omega \propto \frac{1}{v^{2/3}}(\text{ or } v^{-2/3})$

$\Rightarrow \omega = \mathrm{kv}^{-2/3}, k = \mathrm{cons}$

Change in angular speed

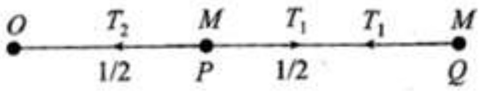
$$= \frac{\Delta\omega}{\omega} \times 100 = -\frac{2}{3}\frac{\Delta v}{v} \times 100$$

$$= -\frac{2}{3} \times 0.09 \times 100 = -6\%$$

So decrease in angular speed is 6%

10. (b) $T_2 - T_1 = m\omega^2 \frac{1}{2}$

$T_1 = m\omega^2 1$



So, $T_2 = m\omega^2 \frac{3}{2}$

So, $\frac{T_1}{T_2} = \frac{m\omega^2 1}{m\omega^2 \frac{3}{2}} = \frac{2}{3}$

11. (a) Given, total energy $= 9$ J

PE at mean position $= 5$ J, At mean position $K \cdot E$ is maximum

So, maximum KE $= 9\text{ J} - 5\text{ J} = 4$ J

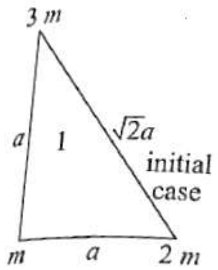
So, $\frac{1}{2}MA^2\omega^2 = 4 \Rightarrow \frac{1}{2}MA^2 \times \frac{4\pi^2}{T^2} = 4$

$$T^2 = \frac{MA^2 \times 4\pi^2}{8}$$

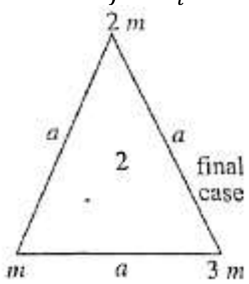
Put $A = 10^{-2}$ m

Then, $T = \pi/100$sec

12. (d) Wext $= \Delta U = f - V_i$



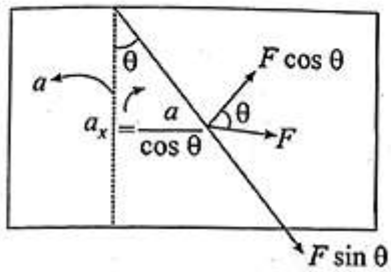
$$W_{ext} = \Delta U$$
$$= f - v_i$$



$$= -\left[-\frac{Gm^2}{a}\left(2 + 3 + \frac{6}{\sqrt{2}}\right) - \left(-\frac{Gm^2}{a}(2 + 3 + 6)\right)\right]$$

$$= -\frac{Gm^2}{a}\left(6 - \frac{6}{\sqrt{2}}\right)$$

13. (a)



Shearing stress $= -\frac{F\sin\theta}{a_x} = -\frac{F}{a}\sin\theta\cos\theta$

Magnitude of shearing stress $= \frac{F}{a}\sin\theta\cos\theta$

14. (c) Density of water - Density of air $\propto T_{water} - T_{air}$

Density of liquid - Density of water $\propto T_{liquid} - T_{water}$

Now, from given values,

$T_{water} - T_{air} = T_{liquid} - T_{water}$

Air density $= 1\ \mathrm{kg/m^3}$

and water density $= 1000\ \mathrm{kg/m^3}$

So, liquid density $= 2000\ \mathrm{kg/m^3}$

15. (b) Let mass of the steam condensed in M.

Heat released = Heat gained by water of steam

$$\Rightarrow M \times 540 + M \times 1 \times (100 - 90)$$
$$= 1 \times 1 \times (90 - 9) + 0.1 \times 1 \times (90 - 9)$$
$$\Rightarrow 540x + 10x = 81 + 8.1$$
$$\Rightarrow x = \frac{89.1}{550} = 0.162\ \mathrm{kg}, x = 162\ \mathrm{g}$$

16. (c) Suppose temp. of middle plate is T_0

Heat gained by first surface = Heat lost by third surface.

So, $\sigma A[(3T)^4 - T_0^4] = \sigma A[T_0^4 - (2T)^4]$

$\Rightarrow 81T^4 - T_0^4 = T_0^4 - 16T^4$

$$\Rightarrow T_0^4 = \frac{97}{2}T^2 \Rightarrow T_0 = \left(\frac{97}{2}\right)^{1/4} T$$

17. (d) Let there are n moles of Nitrogen gas in the cylinder. As all of K.E appear in form of heat

$S_0, n\left(\frac{1}{2}Mv^2\right) = \frac{f}{2}nR\Delta T$ ....(i)

Here, $M = 28g = 28 \times 10^{-3}$ kg, $f = 5$

Also, $\Delta p = \frac{nR\Delta T}{V}$

So, $\frac{\Delta p}{p} = \frac{\left(\frac{nR\Delta T}{V}\right)}{\left(\frac{nRT}{V}\right)} = \frac{nR\Delta T}{nRT}$

$\Rightarrow \frac{\Delta p}{p} = \frac{nMv^2}{fnRT} = \frac{Mv^2}{fRT}$ [from (i)]

So, percentage change in pressure is,

$$\frac{\Delta p}{p} \times 100 = \frac{28 \times 10^{-3} \times 100 \times 100}{5 \times 8.3 \times 300} \times 100$$
$$= 2.25\%$$

18. (b) (i) Zeroth law of thermodynamics states about thermal equilibrium of different states in contact.

(ii) First law is based on energy conservation law.

(iii) In free expansion of gases, work done is always zero as no resistance is there.

(iv) Second law of thermodynamics discusses about direction of heat flow.

19. (a) Mean free path,

$\lambda = \frac{1}{\sqrt{2}\pi n d^2}$,

$n =$ number density, $d =$ diameter

$\Rightarrow d^2 = \frac{1}{\sqrt{2}\pi n \lambda} = \frac{1 \times \pi}{\sqrt{2} \times \pi \times 2\sqrt{2} \times 10^8 \times 10^{-2}}$

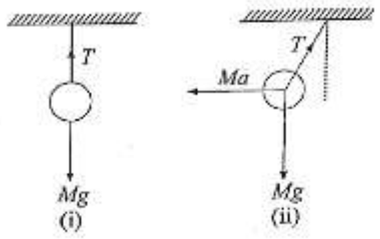
$\Rightarrow d = \frac{1}{2} \times 10^{-3}$ cm $\Rightarrow d = 5 \times 10^{-4}$ cm

20. (c) When car is at rest, tension in string is $T = mg$.

$T = Mg$ and $V = \sqrt{\frac{T}{\mu}}$ ... ... (i)

$\frac{V_1}{V_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{g}{(a^2 + g^2)^{1/2}}}$

$\left(\frac{60}{66}\right) = \sqrt{\frac{g}{(a^2 + g^2)^{1/2}}}$



$\left(\frac{10}{11}\right)^2 = \frac{g}{(a^2 + g^2)^{1/2}}$

$\left(\frac{100}{121}\right)^2 = \frac{g^2}{a^2 + g^2}$

Solving, we get $a = 6.8 \text{ m/s}^2$

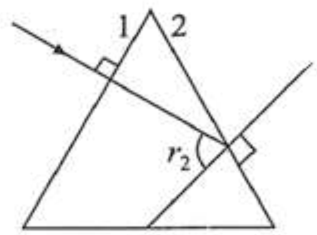
21. (c) Wavelength of the reflected wave is

$\lambda' = \left(\frac{v - v_s}{v + v_s}\right)\lambda = \left(\frac{v - v_s}{v + v_s}\right)\frac{v}{v}$

$= \frac{340 - 20}{340 + 20} \times \frac{340}{160} = \frac{120}{360} \times \frac{340}{160}$

$\lambda' = \frac{17}{9}$ m

22. (a) Given, $i = 0°$



So, $\frac{\sin i}{\sin r_1} = \sqrt{2}, r_1 =$ angle of refraction at surface 1

$\frac{\sin 0°}{\sin r_1} = \sqrt{2}$

$\sin r_1 = 0$ or $r_1 = 0$

Now; $\frac{\sin r_2}{\sin e} = \frac{1}{\mu}$,

$r_2 =$ angle of refraction at surface 2

$\Rightarrow \sin e \doteq \sqrt{2} \sin r_2$

But, $r_1 + r_2 = 60^\circ$ $[\because r_1 + r_2 = A$
So, $\sin e = \sqrt{2}\sin 60^\circ$
$= \sqrt{2} \times \sqrt{3} \cdot /2 > 1$
So, light is incident at more than critical angle.
So, it will totally reflect fact
Deviation angle $= (i + e) - (r_1 + r_2)$
$= 0 - 60^\circ = 60^\circ$ (in magnitude)

23. **(a)** When unpolarised light passes through first polariser, it becomes plane polarised and its intensity becomes half. Therefore, after first polariser, intensity $I_1 = I_0/2$

After second polariser, intensity $I_2 = \frac{I_0}{2}\cos^2\theta$

$\theta = \theta\ \theta = \pi/2$
$I\left|\frac{I_0}{2} I_2\right| I_3$
(Malus law)
After third polariser, intensity
$$I_3 = \frac{I_0}{2}\cos^2\theta\cos^2(90^\circ - \theta)$$
(because angle b/w second and third is $(90 - \theta)$)
$$\Rightarrow\ I_3 = \frac{I_0}{2}\cos^2\theta\sin^2\theta = \frac{I_0}{8}\sin^2 2\theta.$$

24. **(d)** When dielectric of thickness t is introduced in two charges at distance r, the effective force between the charges is given by
$$F = \frac{q_1q_2}{4\pi\varepsilon_0[r - t + t\sqrt{K}]^2}$$
where, K = dielectric constant of medium
In first case, $t = r/2$ and $K = 4$
$$F_1 = \frac{Kq_1q_2}{\left[r - r/2 + \frac{r}{2}\sqrt{4}\right]^2} = \frac{Kq_1q_2}{\frac{9}{4}r^2} = \frac{4}{9}\frac{Kq_1q_2}{r^2}$$
In second case, $t = r/3$ and $K = 9$
$$F_2 = Kq_1q_2.$$
$$\left[r - \frac{r}{3} + \frac{r}{3}\sqrt{3}\right]^2 = \frac{9Kq_1q_2}{25}$$
So, $\frac{F_1}{F_2} = \frac{\frac{Kq_1q_2}{r^2}\times\frac{4}{9}}{\frac{Kq_1q_2}{r_2}\times\frac{9}{25}} = \frac{100}{81}$

25. **(a)** Since $a = \frac{q}{m}E$
$$= \frac{2\times10^{-6}}{0.04}\times 4.2\times10^4\ m/s^2$$
$= 2.1\ m/s^2$(downward)
So, effective acceleration on bob,
$a_e = a + g = 12.1\ m/s^2$
$$\frac{T}{T'} = \sqrt{\frac{g + a_e}{g}} \Rightarrow \frac{T}{T'} = \sqrt{\frac{12.1}{10}} = \frac{11}{10} \Rightarrow T' = \frac{10}{11}T$$
Given, $T = \frac{44}{20} \Rightarrow T' = \frac{10}{11}\times\frac{44}{20} = 2s$
So, time taken in 15 oscillations $= 2\times15 = 30$ s

26. **(d)** Charge stored in the presence of air,
$q_{air} = C_{air}\times V = 80\times30\times10^{-6} = 2400\mu C$
Charge stored in presence of dielectric medium,
$q_d = C_{dielectric}\times\ V = 1600\times30\times10^{-6} = 48000\mu C$
$[C_{dielectric} = 20\times80\times10^{-6} = 1600\mu\ F]$
When dielectric is removed, the charge passing through wire is,

$q = q_d - q_{air}$

$\Rightarrow q = (48 - 2.4) \times 10^{-3} C \Rightarrow = q = 45.6 \times 10^{-3} C$

27. (a) As $Q_i = 0$

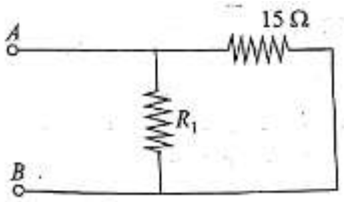
$Q_f = C_1(V_1 - V_0), Q_2 = C_2(V_2 - V_0)$

$Q_3 = C_3(V_3 - V_0)$ and $Q_i = Q_f$

$0 = C_1V_1 + C_2V_2 + C_3V_3 - V_0(C_1 + C_2 + C_3)$

$$\Rightarrow V_0 = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$$

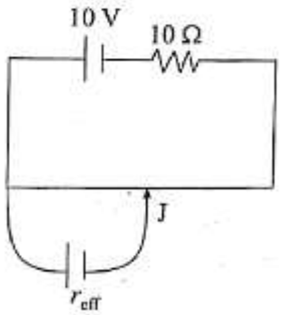
28. (b) Here 2Ω and 3Ω resistances are short circuited, so circuit can be reduced as



$$\text{Req.} = \frac{15R_1}{15 + R_1} = 6$$

$\Rightarrow 15R_1 - 6R_1 = 15 \times 6 \Rightarrow 9R_1 = 90 \Rightarrow R_1 = 10\Omega$

29. (c)



$$\varepsilon_{\text{eff}} = \frac{\varepsilon_1 t_1 + \varepsilon_2 r_2}{r_1 + r_2} = \frac{2 \times 2 + 3 \times 3}{2 + 3} = \frac{13}{5} \text{ V}$$

$$I_{AB} = \frac{10}{10 + 10} = 0.5 \text{ A}$$

$$V_{AB} = \frac{5}{1} \times x = 5x$$

But, $V_{AB} = r_{\text{eff}}$

$$5x = \frac{13}{5}$$

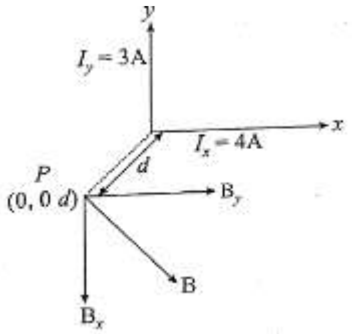
$$x = \frac{13}{25} x = 0.52 \text{ m} = 52 \text{ cm}$$

30. (d) Magnetic field at point P due to I_x

$$\boldsymbol{B}_x = -\frac{\mu_0 \cdot 4}{2\pi d}\hat{\mathbf{j}}$$

magnetic field at point P due to $I_{y'}$.

$\mathbf{B}_y = \frac{\mu_0 I_y}{2\pi d}$ (X = direction)



$\mathbf{B}_y = \frac{\mu_0 \cdot 3}{2\pi d} a\hat{i}$

Resultant magnetic field,

$$B = \sqrt{B_x^2 + B_y^2}$$

$$\Rightarrow B = \frac{\mu_0}{2\pi d}\sqrt{4^2 + 3^2} \Rightarrow B = \frac{5\mu_0}{2\pi d}$$

31. (a) Current sensitivity, $I_S = \frac{NBA}{k}$

So, $\frac{I_{S_1}}{I_{S_2}} = \frac{N_1B_1A_1k_2}{N_2B_2A_2k_1}$

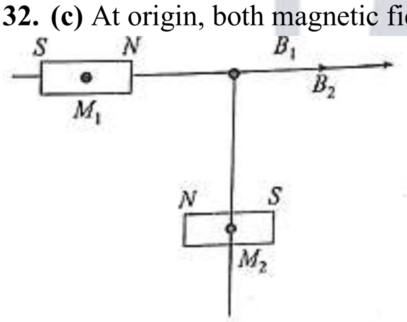
$= \frac{30\times0.25\times4.8\times10^{-3}}{45\times0.5\times24\times10^{-3}} \times \left(\frac{1}{1}\right)$ (Assume $K_1 = K_2$)

$\frac{I_{S_1}}{I_{S_2}} = \frac{2}{3}$

Voltage sensitivity, $V_S = NBA/RK = IS/R$

So, $\frac{V_{S_1}}{V_{S_2}} = \frac{I_{S_1}}{I_{S_2}} \times \frac{R_2}{R_1} = \frac{2}{3} \times \frac{14}{10} = \frac{14}{15}$

32. (c) At origin, both magnetic fields will be in same direction.



$$B_{\text{net}} = B_1 + B_2 = \frac{\mu_0 m}{4\pi r^3}[2+1]$$

$$= \frac{3\mu_0 m}{4\pi r^3} = \frac{3\times10^{-7}\times9}{(3\times10^{-2})^3} = 0.1\text{ T}$$

33. (a) $V = vBl$ and $q = CV$

So, $a = C(vBl) = 10\times10^{-6}\times2\times4\times1$

$q = 80\mu C$

So, charges on plates are $\pm80\mu C$. By Lenz law direction of induced current will be ACW .

So $q_A = +80\mu C$ and $q_B = -80\mu C$.

34. (a) In the shown figure, current is ahead of voltage, so its a RC circuit, so P is a resistor and Q is a capacitor.

As $\cos\phi = \frac{R}{Z}$

$R = Z\cos\phi = 1000\sqrt{2}\times\cos 45 = 1000\text{ohm}$

Now, $Z^2 = X_C^2 + R^2$

$X_C = \sqrt{Z^2 - R^2}$

$= \sqrt{(1000\sqrt{2})^2 - 1000^2} = 1000\Omega$

$\frac{1}{\omega C} = 1000 \Rightarrow C = \frac{1}{100 \times 1000}$ $(\omega = 100)$

$\Rightarrow C = 10\mu$ F

35. (c) Energy density due to electric field is

$U = \frac{1}{2}\varepsilon_0 E^2$

Here, $E = \frac{E_0}{\sqrt{2}} = \frac{40}{\sqrt{2}}$

$U = \frac{1}{2} \times 8.85 \times 10^{-12} \times \frac{40 \times 40}{2} = 3.54 \times 10^{-9}\ J/m^3$

36. (a) Energy of photons of

H_α line $= \Delta E(3 \rightarrow 2) = 13.6\left(\frac{1}{4} - \frac{1}{9}\right)$

$= \frac{13.6 \times 5}{36} = 1.89eV$

Energy of photons of H_β line $= \Delta E(4 \rightarrow 2)$

$= 13.6\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{12 \times 13.6}{64} = 2.55eV$

Energy of photons of H_∞ line $= \Delta E(\infty \rightarrow 2)$

$= \frac{13.6}{4} = 3.4eV$

Ratio of kinetic energy of emitted photons

$= \frac{255 - 1.89}{3.4 - 1.89} = \frac{0.66}{1.51} \approx \frac{0.7}{1.6} \approx \frac{7}{16}$

37. (d) $\lambda\alpha\frac{1}{V}$ and $V\alpha\frac{1}{n}$ $\left[\lambda = \frac{h}{p}\right]$

So $\lambda \propto n$

38. (a) Number of nuclei required

$= \frac{E_{total}}{E_1} = \frac{1000\ J}{200MeV} = 31.25 \times 10^{13}$

39. (c) Diode, D_1 = reverse biased (OFF) → will act like open switch and diode, D_2 = forward biased (ON) → will act like wire Because diodes are ideal, so voltage drop across D_2 is zero

$R_{eff} = 3 + 2 + 3 + 2 = 10\Omega$

$I = V/R = 20/10 = 2A$

40. (c) Modulation index, $M = \frac{(V_n)\ \text{signal}}{(V_m)\ \text{carrier}}$

Let's (V_m) carrier $= V$

In first case, $0.6 = \frac{12}{V} \Rightarrow V = 20\ V$

In second case, $0.75 = \frac{12}{V'} \Rightarrow V' = \frac{12}{0.75} = 16\ V$

Change in peak voltage of carrier wave

$\Delta V = 20 - 16 = 4V$

% change $= \frac{4}{20} \times 100\% = 20\%$(decrement)

Chemistry

41. (c) Given, $r_n = 476.1pm$

$r_1 = 52.9pm$

$r_n = \sum_{k-1}^{10} \int_0^1 f(k - 1 + x)dx$ [Z for hydrogen $= 1$]

$n = 3$

Now, $E_n = -2.18 \times 10^{-18} \cdot \frac{Z^2}{n^2}$

and $E_n = E_3 = -\frac{2.18Z^2 \times 10^{-18}\,J}{9} = -2.42 \times 10^{-19}$ J

42. **(b)** $\frac{hv_1 - hv_0}{hv_2 - hv_0} = \frac{K \cdot E_1}{K \cdot E_2}$

$$\frac{h(1.6 \times 10^{16} - v_0)}{h(1.0 \times 10^{16} - v_0)} = \frac{2\text{ K.E.}}{\text{K.E.}}$$

$1.6 \times 10^{16} - v_0 = 2 \times 1.0 \times 10^{16} - 2v_0$

$v_0 = 1.6 \times 10^{16} - 2(1.0 \times 10^{16}) = v_0 = 4 \times 10^{15}$ Hz

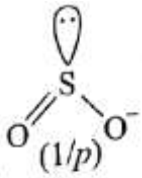
43. **(a)** Electronic configuration $= 1s^2, 2s^2, 2p^6, 3s^2, 3p^6$

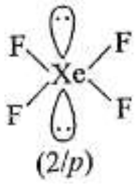
$= 18$ electrons

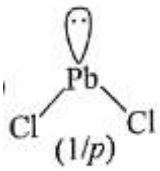
Element is in -2 oxidation state so, it has 16 electrons.

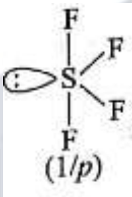
$(Z = 16)$ refer to group 16 and period 3 i.e. sulphur, Sulphur show -2 oxidation state.

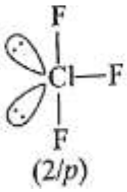
44. **(a)** $SO_2, PbCl_2$ and SF_4 contain one lone pair.

(i) 

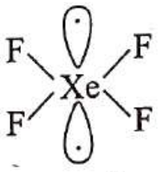
(ii) 

(iii) 

(iv) 

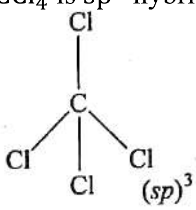
(v) 

45. **(c)** XeF_4 is sp^3d^2 hybridised and due to presence of 2 lone pair of electrons its shape is square planar.



sp^3d^2

CCl_4 is sp^3 hybridised due to 4 bonds. Its geometry is tetrahedral



46. **(c)** Moles of $H_2 = 16/2 = 8$

Moles of He $= 16/4 = 4$

Moles of $O_2 = 16/32 = 0.5$

Total moles at pressure 10 atm .

$(P_1) = 8 + 4 + 0.5 = 12.5$

Total moles at $(P_2) = 4.5$ (only He and O_2)

$$\frac{P_1V_1}{P_2V_2}=\frac{n_1RT}{n_2RT}\Rightarrow\frac{10}{P_2}=\frac{12.5}{4.5}$$
$P_2=3.6$

47. (d) When $0.15 M Na_2CO_3$ (1L) mixed with 0.2 M $K_2Cr_2O_7$(0.5 L) than total molarity (M).
$M_1\ V_1(Na_2SO_3)+M_2\ V_2(K_2Cr_2O_7)\equiv M_3\ V_3$ (total)
$$M_3=\frac{(0.15\times1)+(0.2\times0.5)}{1.5}$$
$=0.15$ mol/L
Remaining mole of $K_2Cr_2O_7$
$=0.2-0.15=0.05$ mol/L

48. (a) $CH_3OH+3/2O_2\rightarrow CO_2+2H_2O$,
$\Delta H=-726$ kJ/mol ....(i)
($H_2+1/2O_2\rightarrow H_2O, \Delta H=-286$ kJ/mol) ....(ii)
$C+O_2\rightarrow CO_2, \Delta H=-393$ kJ/mol ...(iii)
From Eq. (ii) $\times 2+$ (iii) $-$ Eq. (i)
$\Delta_fH_{(CH_3OH)}=-286\times2-393-(-726)$
$=-239$ kJ/mol

49. (c) Given, $K_C=4\times10^{-6}$ mol/L
$(\Delta n=n_P-n_R\Rightarrow 3-2=1)$
$K_p=K_C\times(RT)^{\Delta ng}$
$=4\times10^{-6}$ mol L^{-1} $\times$ (0.083 Lbark^{-1} mol^{-1} $\times$ 1000 K)^1
$K_p=3.32\times10^{-4}$ bar

50. (a) pH of the solution of salts of weak acid e.g. acetic acid and weak base e.g. dimethylamine can be calculated as
$$pH=7+\frac{1}{2}[pK_a-pK_b]$$
Here, $pK_a=4.76, pK_b=3.26$
$$=7+\frac{1}{2}[4.76-3.26]$$
$$=7+\frac{1}{2}[1.50]$$
$=7+0.75=7.75$

51. (d) (i) $NaH+H_2O\xrightarrow{\Delta}NaOH+H_2$
(ii) Ammonia (NH_3) is electrons rich hydride due to presence of lone pair at N -atom
(i), (ii) both statements are correct.
(iii) Nickle cannot form saline hydride.

52. (b) (a) $4LiNO_3\xrightarrow[\Delta]{}\Delta Li_2O+4NO_2+O_2$
(b) $2NaNO_3\xrightarrow[\Delta]{}2NaNO_2+O_2$
(Thermal stable)
(c) $2Ba(NO_3)_2\xrightarrow[\Delta]{}2BaO+4NO_2+O_2$
(d) $2Be(NO_3)_2\xrightarrow[\Delta]{}\Delta BeO+4NO_2+O_2$
$NaNO_3$ is thermally stable and does not gives its oxide.

53. (c) $2BF_3+6NaH\xrightarrow{450\text{ K}}6NaF+\underset{(X)}{B_2H_6}$

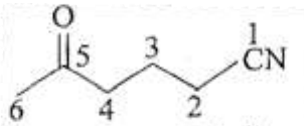
$$\underset{\text{Diborane}}{B_2H_6}+2LiH\xrightarrow{\text{Diethyl ether}}\underset{\substack{(Y)\\\text{Lithiumborohydride}}}{2LiBH_4}$$

54. (b) Assertion and Reason both are correct but Reason
(R) is not the correct explanation of (A).

SiF_6^{2-} is formed but $SiCl_6^{2-}$ is not because Si can accomodate
six F^-due to its smaller ^{6} size but it can not accomodate six Cl^-due to its larger size and steric hindrance.
Electronegativity of F is higher than Cl

55. (c) Hydrogen peroxide is now a days used for bleaching in the presence of catalyst the paper due to ecofriendly nature.

56. (b)



5-Oxohexanenitrile

57. (c) (i) Petrol and CNG operated automobiles causes less pollution.
(ii) Alkane having tertiary hydrogen therefore oxidise in presence of $KMnO_4$ into alcohol.
(iv) Alkyl halide on reduction with zinc and dilute HCl gives alkane.

$$\underset{\text{Alkyl Chloride}}{R-Cl} \xrightarrow{Zn+HCl} \underset{\text{Alkane}}{R-H}$$

58. (a) $\underset{\text{Pent-2-ene}}{CH_3-CH_2-CH_2=CH-CH_3} \xrightarrow{O_3} \underset{\text{(Y) Propanal}}{CH_3CH_2CHO} + \underset{\text{(X) Ethanal}}{CH_3CHO}$

59. (d) 3 body centred lattices possible among the 14 Bravais lattices. Which are in Simple cubic, Tetragonal and Orthorhombic

60. (b) Given, $\pi = 3.735 \times 10^{-3}$bar
Mass of $K_2SO_4 = 17.4$mg
Molar mass $(M)H_2SO_4 = 174, i = 3.0$
Volume $= 2$ L
From osmotic pressure of solution.

$\pi = i\ CRT$
$i =$ van't-Hoff factor
$i = \pi/CRT$

$$= \frac{\pi \times M \times V}{W \times R \times T} = \frac{3.735 \times 174 \times 2}{17.4 \times 0.083 \times 300} = 3.0$$

61. (b) Given,
Mass of solute $(W) = 120$ g
Molar mass of solute $(M) = 60$
Mass of solvent $(w) = 1000$ g
Total mass of solution $= 1000 + 120 = 1120$ g

$$d = \frac{\text{Mass}}{\text{Volume (V)}}$$

Total volume $(V) = \frac{1120}{1.12} = 1000$ mL

Molarity $= \frac{w}{M} \times \frac{1000}{V}$

Molarity $= \frac{120}{60} \times \frac{1000}{1000} = 2.0$

62. (c) When an aqueous solution of $CuCl_2$ is electrolysed using pt inert electrodes. The chloride ion is oxidised to chlorine (Cl_2) at anode and Cu^{2+} ion is reduced to Cu at cathode.

At anode: $2Cl^- \xrightarrow{-2e^-} Cl_2(g)$
At cathode: $Cu^{2+} + 2e^- \longrightarrow Cu$

63. (b) Final quantity $(a-x) = (100-90) = 10$
Initial quantity $(a) = 100\%$
$k = 4.606 \times 10^{-3}\ s^{-1}$
First order reaction

$k = \frac{2.303}{t} \log \frac{a}{(a-x)}$

$k = 4.606 \times 10^{-3}\ s^{-1}$

$t = \frac{2.303}{4.606 \times 10^{-3}} \log \frac{100}{10}$

$t = 500\ s$

64. (a) A mixture of N_2 and O_2 gases at room temperature form gaseous homogeneous mixture but do not form aerosol.

65. (b) In Ellingham diagram, the plot is drawn between temperature and ΔG°.

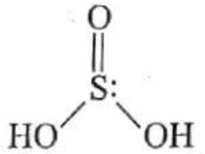
66. (c) (a) $2NaN_3 \xrightarrow{\Delta} 2Na + 3\ N_2$

(b) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} Cr_2O_3 + N_2 + 4H_2O$

(c) $2NH_4Cl + Ca(OH)_2 \longrightarrow CaCl_2 + 2NH_4OH$

(d) $Ba(N_3)_2 \longrightarrow Ba + 3\ N_2$

67. (d) Sulphur atom contains lone pair in only in sulphurous acid among the given acids.



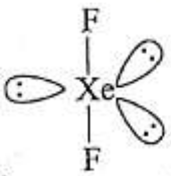
Sulphurous acid

68. (d) (d) Helium has lowest boiling point among all the noble gases. Due to minimum Vander Waal's force of attraction.

(a) $Xe(g) + 3\ F_2(\ g) \xrightarrow[60-70\ bar]{573\ K} XeF_6\ (s)$

(b) Ar used in electric bulbs.

(c) 3 lone pairs are present in XeF_2.



(sp^3d) Linear shape

69. (b) The relation between octahedral splitting energy (Δ_0) and tetrahedral splitting energy (Δ_t) is

$\Delta_t = \frac{4}{9}\Delta_0 \ \therefore 9\Delta_t = 4\Delta_0$

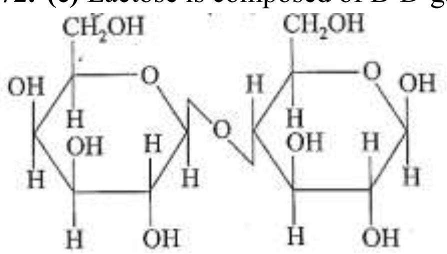
70. (b) Cerium exihibit +4 oxidation state.

E.C. of $Ce(58) = [Xe]4f^1 5d^1 6s^2$

$Ce^{+4} = [Xe]4f^0 5d^0 6s^0$ (stable configuration)

71. (d) Alkyl lithium used as initiators for anionic poly-merisation.

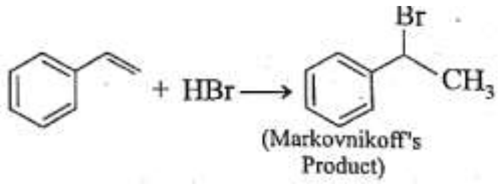
72. (c) Lactose is composed of B-D-galactose (C-1) linked to B-D glucose at (C-4).

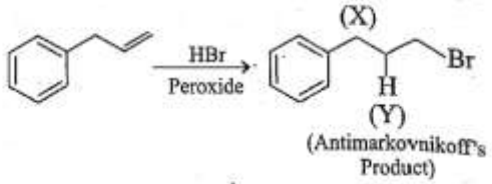


73. (c) (ii) Snape of receptor changes after attachment of chemical messenger.

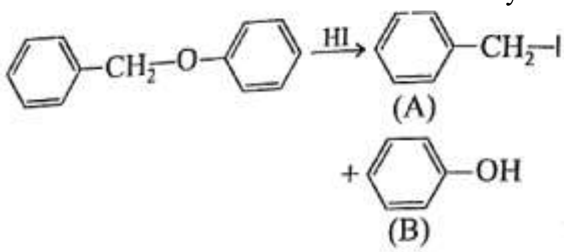
(ii) When stearic acid reacts with polythene glycol non- ionic detergent is formed.

74. (c)

(i)


(ii)


75. **(b)** Aryl oxygen bond is more stable due to resonance. Hence bond will cleaved at benzyl oxygen bond which leads to the formation more stable benzyl carbocation that combine with $I^-$to form benzyl iodide.



76. **(c)**

77. **(a)** A-(iii), B-(iv), C-(ii), D-(i)

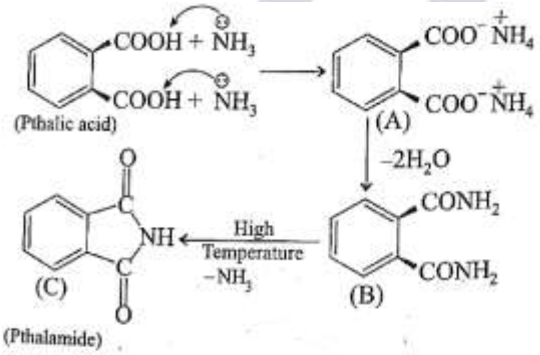
(A) Anhydrous $ZnCl_2$ + conc HCl is known as Lucas reagent and used to distinguish between primary, secondary, tertiary alcohols.

(B) Zn − Hg/HCl is called Clemmensen reagent. Thus this reagent is used in conversion of carbonyl into alkane i.e., Clemmensen reaction.

(C) Tollen's reagent $[Ag(NH_3)_2]^+$used to distinguish aldehyde and ketones as oxidising reagent.

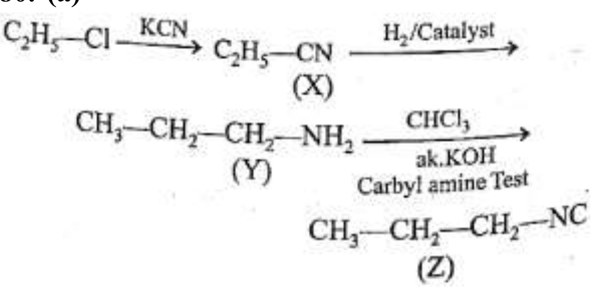
(D) Sephen reagent $SnCl_2$ + HCl used in reduction of nitrogen compounds.

78. **(d)**



79. **(c)**

80. **(a)**



Mathematics

81. **(d)** According to the given data,

$x = [x] + \{x\}$

$[2x] = 2[x] + 2[x]$

$$[2x] = \begin{cases} 2[x] + 0, & 0 < \{x\} < \frac{1}{2} \\ 2[x] + 1, & \frac{1}{2} \le \{x\} < 1 \end{cases}$$

$$[2x] - 2[x] = \begin{cases} 0, & 0 \le \{x\} < \frac{1}{2} \\ 1, & \frac{1}{2} \le \{x\} < 1 \end{cases}$$

So, the range of f is $\{0,1\}$

82. (d) We have,

$f(x) = \log\{ax^3 + (a+b)x^2 + (b+c)x + c\}$

$= \log\{(ax^2 + bx + c)(x+1)\}$

For f(x) to be defined

$(ax^2 + bx + c)(x+1) > 0$

$\Rightarrow\ x + 1 > 0$

$\{\because a > 0 \text{ and } b^2 = 4ac\}$

and $x \neq -\frac{b}{2a}$

So, $\{D = x : x \in (-1, \infty) \text{ and } x \neq -\frac{b}{2a}\}$

or $D = R - \left\{-\frac{b}{2a}\right\} \cup (-\infty, -1]$

83. (d) Given that, $(15 \times 5^{2n}) + (2 \times 2^{3n})$

Put $n = 1$,

Thus, we have $15 \times 5^2 + 2 \times 2^3$

$= 15 \times 25 + 2 \times 8 = 375 + 16 = 391$

which is divisible by 17

$(15 \times 5^{2n}) + (2 \times 2^{3n})$ is divisible by $17, \forall n \in N$.

84. (c) Given matrix,

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$$

$= 3(-3+4) + 3(2-0) + 4(-2-0) = 1 \neq 0$

$$\operatorname{adj}(A) = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|} \operatorname{adj} A = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$ ....(i)

$$A^2 = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 3 & -4 & 4 \\ 0 & -1 & 0 \\ -2 & 2 & -3 \end{bmatrix}\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \quad \ldots..(ii)$$

From Eqs. (i) and (ii), we get $A^{-1} = A^3$

85. (d) We have,

$A = \begin{bmatrix} k/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & m/4 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

We know that, $AA^{-1} = I$

$$\begin{bmatrix} k/2 & 0 & 0 \\ 0 & l/3 & 0 \\ 0 & 0 & m/4 \end{bmatrix}\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} k/4 & 0 & 0 \\ 0 & l/9 & 0 \\ 0 & 0 & m/16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing the respective terms, we get

$$\frac{k}{4} = 1, \frac{l}{9} = 1 \text{ and } \frac{m}{16} = 1$$

$$\Rightarrow k = 4, l = 9, m = 16$$

$$k + l + m = 4 + 9 + 16 = 29$$

86. (a) We have given that,

$$x + 2y + z = 1; x + 3y + 4z = k; x + 5y + 10z = k^2$$

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 3 & 4 \\ 1 & 5 & 10 \end{vmatrix}$$

$$= 1(30 - 20) - 2(10 - 4) + 1(5 - 3) = 10 - 12 + 2 = 0$$

$$\Delta = 0$$

Given system of equation is consistent.

Therefore, $\Delta_1 = 0$

$$\Delta_1 = \begin{vmatrix} 1 & 2 & 1 \\ k & 3 & 4 \\ k^2 & 5 & 10 \end{vmatrix} = 0$$

$$1(30 - 20) - 2(10k - 4k^2) + (5k - 3k^2) = 0$$

$$5k^2 - 15k + 10 = 0 \Rightarrow (k - 2)(k - 1) = 0 \Rightarrow k = 2,1$$

Hence, the real values of k i.e.

$A = 2$ and $B = 1$

$A + B = 2 + 1 = 3$

87. (d) Given $z = x + iy$ be a complex number,

Here $\frac{z-1}{z+i}$

$$= \frac{x + iy - 1}{x + iy + i} = \frac{(x - 1) + iy}{x + (y + 1)i}$$

$$= \frac{(x - 1) + iy}{x + (y + 1)i} \times \frac{x - (y + 1)i}{x - (y + 1)i}$$

$$= \frac{x(x - 1) + y(y + 1)}{x^2 + (y + 1)^2} + \frac{[xy - (x - 1)(y + 1)]i}{x^2 + (y + 1)^2}$$

Also given, $\text{Re}\left(\frac{z-1}{z+i}\right) = 1$

$$\frac{x(x - 1) + y(y + 1)}{x^2 + (y + 1)^2} = 1$$

$$x(x - 1) + y(y + 1) = x^2 + (y + 1)^2$$

$$\Rightarrow x^2 - x + y^2 + y = x^2 + y^2 + 2y + 1$$

$$\Rightarrow x + y + 1 = 0$$

lies on $x + y + 1 = 0$

88. (a) Given that,

$$13e^{i\tan^{-1}\frac{5}{12}} = a + ib$$
$$\Rightarrow 13\left[\cos\left(\tan^{-1}\frac{5}{12}\right) + i\sin\left(\tan^{-1}\frac{5}{12}\right)\right] = a + ib$$
$$\Rightarrow 13\left[\cos\left(\cos^{-1}\frac{12}{13}\right) + i\sin\left(\sin^{-1}\frac{5}{13}\right)\right] = a + ib$$
$$\left(\tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}} \text{ and } \tan^{-1}x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}\right).$$
$$\Rightarrow 13\left[\frac{12}{13} + i\frac{5}{13}\right] = a + ib$$
$$\Rightarrow 12 + 5i = a + ib$$

On comparing both the sides, we get

$$a = 12, b = 5$$
$$(a, b) = (12,5)$$

89. (d) Given that,

$z_1 = 1 - 2i, z_2 = 1 + i, z_3 = 3 + 4i$

Now, $\left(\frac{1}{z_1} + \frac{3}{z_2}\right)\frac{z_3}{z_2} = \left(\frac{1}{1-2i} + \frac{3}{1+i}\right)\left(\frac{3+4i}{1+i}\right)$

$$= \frac{(1+i)+3(1-2i)}{(1-2i)(1+i)} \times \frac{3+4i}{1+i} = \frac{4-5i}{3-i} \times \frac{3+4i}{1+i}$$
$$= \frac{12 + 16i - 15i + 20}{3 + 3i - i + 1} = \frac{32 + i}{4 + 2i} = \frac{32 + i}{2(2 + i)}$$
$$= \frac{(32 + i)}{2(2 + i)} \times \frac{2 - i}{2 - i} = \frac{65 - 30i}{10} = \frac{13}{2} - 3i$$

90. (d) Here, $1, \omega$ and ω^2 are the cube roots of unity

$$\frac{1}{1 + 2\omega} + \frac{1}{2 + \omega} - \frac{1}{1 + \omega}$$
$$= \frac{2 + \omega + 1 + 2\omega}{(1 + 2\omega)(2 + \omega)} - \frac{1}{(1 + \omega)}$$
$$= \frac{3 + 3\omega}{(1 + 2\omega)(2 + \omega)} - \frac{1}{(1 + \omega)}$$
$$= \frac{(3 + 3\omega)(1 + \omega) - (1 + 2\omega)(2 + \omega)}{(1 + 2\omega)(2 + \omega)(1 + \omega)}$$
$$= \frac{3 + 3\omega + 3\omega + 3\omega^2 - (2 + \omega + 4\omega + 2\omega^2)}{(1 + 2\omega)(2 + \omega)(1 + \omega)}$$
$$= \frac{1 + \omega + \omega^2}{(1 + 2\omega)(2 + \omega)(1 + \omega)}$$

According to the property of cube roots of unity,

$$\omega^2 + \omega + 1 = 0$$
$$\frac{1}{1 + 2\omega} + \frac{1}{2 + \omega} + \frac{1}{1 + \omega} = 0$$

91. (d) Given, $5x - 1 < (x + 1)^2 < 7x - 3$

$5x - 1 < (x + 1)^2$

$\Rightarrow \quad 5x - 1 < x^2 + 2x + 1$

$\Rightarrow \quad (x - 1)(x - 2) > 0$

$\Rightarrow \quad x \in (-\infty, 1) \cup (2, \infty) \quad \ldots\ldots(i)$

Similarly, $(x + 1)^2 < 7x - 3$

$\Rightarrow x^2 + 2x + 1 < 7x - 3$

$\Rightarrow (x - 1)(x - 4) < 0$

$\Rightarrow x \in (1,4) \quad \ldots\ldots(ii)$

From Eqs. (i) and (ii), we get

$x = 3$

92. (a) We have, $f(x) = x^2 + 2bx + 2c^2$

$= (x + b)^2 + 2c^2 - b^2$

Minimum value, $f(x) = 2c^2 - b^2$

Now, $g(x) = -x^2 - 2cx + b^2$

$= -[x^2 + 2cx - b^2]$

$= -[x^2 + 2cx + c^2 - b^2 - c^2]$

$= -[(x+c)^2 - b^2 - c^2] = -(x+c)^2 + b^2 + c^2$

Maximum value, $g(x) = b^2 + c^2$

As given in the question,

$\text{Min}[f(x)] > \text{Max}[g(x)]$

$2c^2 - b^2 > b^2 + c^2 \Rightarrow c^2 > 2b^2$

93. **(a)** Given equation, $x^3 + qx + r = 0$

Since, a, b and c are the roots of equation $a + b + c = 0$

$ab + bc + ca = q$ and $abc = -r$

As we have, $a + b + c = 0$

$(a+b+c)^2 = 0$

$a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = 0$

$a^2 + b^2 + c^2 = -2q$

Now, $(a-b)^2 + (b-c)^2 + (c-a)^2$

$= a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ca$

$= 2a^2 + 2b^2 + 2c^2 - 2(ab + bc + ca)$

$= 2(-2q) - 2(q)$ [From Eq. (i)]

$= -4q - 2q = -6q$

94. **(a)** Let the two roots of equation is a and $-a$.

Now, sum of three roots $= 2p$

So , third root will be 2p,

Product of two consecutive roots $= 3q$

$a \times (-a) + a \times (2p) + (-a) \times 2p = 3q$

$\Rightarrow -a^2 = 3q$

As we know, product of roots $= 3q$

$a \times (-a) \times 2p = 4r \Rightarrow 3q \times 2p = 4r$

$\Rightarrow r = \dfrac{3pq \times 2}{4} \Rightarrow r = \dfrac{3pq}{2}$

95. **(b)** Given digits are 0,3,5,4. Then four -digit numbers which are even and without repetition are 3054, 3504, 5034,5304,3450,3540,4350,4530,5340,5430

Required sum

$= 3054 + 3504 + 5034 + 5304 + 3450 + 3540 + 4350 + 4530 + 5340 + 5430 = 43536$

96. **(b)** For x number of ways, 6 Boys can be seated in a row in ${}^6P_6$ ways = 6 !

Now, in the 7 gaps 6 girls can be arranged in ${}^7P_6$ ways.

$x = 6! \times {}^7P_6 = 6! \times 7!$

For y number of ways. 6 Boys can be seated in a circle in $(6-1)$! ways = 5 !

Now, in the 6 gaps 6 girls can be arranged in ${}^6P_6$ ways.

$y = 5! \times {}^6P_6 = 5! \times 6!$

$\dfrac{x}{y} = \dfrac{6! \times 7!}{5! \times 6!}$

$x : y = 42 : 1$

97. **(c)** Given word 'SARANAM' there are 7 letters in this word in which it has 3 A and all other are distinct.

The five letter words may consist of

(i) all different letters (using S, A, R, N, M)

Number of words

$= 5! = 120$

(ii) 2 alike and other different (using 2A and 3 other)

Number of words

$= {}^4C_3 \times \frac{5!}{2!} = 4 \times \frac{120}{2} = 240$

(iii) 3 alike and other different (using 3A and 2 other)

Number of words

$= {}^4C_2 \times \frac{5!}{3!} = \frac{4 \times 3}{2 \times 1} \times \frac{120}{6} = 120$

Hence, total words

$= 120 + 240 + 120 = 480$

98. **(c)** In the binomial expansion of $(4\sqrt{5} + 5\sqrt{4})^{100}$,

General term, $T_{r+1} = {}^{100}C_r 5^{\frac{100-r}{4}} 4^{\frac{r}{5}}$

Clearly, T_{r+1} will be an integer if $\frac{100-r}{4}$ and $\frac{r}{5}$ are integers. This is possible when $100 - r$ is a multiple of 4 and r is a multiple of 5

$\Rightarrow\ 100 - r = 0,4,8,12,\ldots,96,100$

and $r = 0,5,10,\ldots,100$

$\Rightarrow\ r = 0,4,8,12,\ldots,100$

and $r = 0,5,10,100$

$\Rightarrow\ r = 0,20,40,60,80,100$

Hence, there are 6 rational terms.

99. **(d)** In the expansion of $(2a - 3b)^9 = (2a)^{19}\left(1 - \frac{3b}{2a}\right)^{19}$.

We know that, the r^{th} term is the greatest term of expansion $(1 + x)^n$, then

$(1 + x)^n = \left[\frac{(n+1)|x|}{1 + |x|}\right]$

Here, $n = 19, x = -\frac{3b}{2a}$.

$$r = \left[\frac{(20)\left|\frac{3b}{2a}\right|}{1 + \left|\frac{3b}{2a}\right|}\right] = \left[\frac{20}{1+4} \times 4\right] = 16.$$

$\left[b = \frac{2}{3}, a = \frac{1}{4}\right]$

greatest term of $(2a - 3b)^{19} = 2^{19}a^{19}\left(1 - \frac{3b}{2a}\right)^{19}$ is

$$= {}^{19}C_{16} \times 2^{19} \cdot a^{19}\left(\frac{3b}{2a}\right)^{16}$$

$$= {}^{19}C_3 \times 2^{19} \times \left(\frac{1}{4}\right)^{19} \times (4)^{16}\ [{}^{19}C_{16} = {}^{19}C_3]$$

$$= {}^{19}C_3 \times 2^{19} \times \frac{1}{2^{38}} \times 2^{32} = {}^{19}C_3 \times 2^{13}$$

100. **(b)** Given that,

$$\frac{x^2 + 5x + 7}{(x-3)^3} = \frac{A}{x-3} + \frac{B}{(x-3)^2} + \frac{C}{(x-3)^3}$$

$\Rightarrow x^2 + 5x + 7 = A(x-3)^2 + B(x-3) + C \quad \ldots\ldots(i)$

At $x = 3$

$\Rightarrow 9 + 15 + 7 = C \Rightarrow C = 31$

At $x = 0$

$\Rightarrow 7 = 9A - 3B + 31$

$\Rightarrow 3A - B = -8 \quad \ldots\ldots(ii)$

At $x = 1$

$\Rightarrow 1 + 5 + 7 = 4A - 2B + 31$

$\Rightarrow 2A - B = -9 \quad \ldots\ldots(iii)$

On solving Eqs. (ii) and (iii), we get

$A = 1$ and $B = 11$

Equation of line having slope A and passing through the points (B, C) is

$y - C = A(x - B)$

$y - 31 = 1(x - 11)$

$x - y + 20 = 0$

101. **(d)** We have given that,

$\cos\left(x - \frac{\pi}{3}\right), \cos x, \cos\left(x + \frac{\pi}{3}\right)$ are in H.P.

$$\frac{2}{\cos x} = \frac{1}{\cos\left(x - \frac{\pi}{3}\right)} + \frac{1}{\cos\left(x + \frac{\pi}{3}\right)}$$

$$\Rightarrow \cos x = \frac{2\left(\cos^2 x - \sin^2 \frac{\pi}{3}\right)}{2\cos x\cos\frac{\pi}{3}}$$

$$\Rightarrow \cos^2 x\cos\frac{\pi}{3} = \cos^2 x - \sin^2\frac{\pi}{3}$$

$$\Rightarrow \cos^2 x\left(1 - \cos\frac{\pi}{3}\right) = \left(1 - \cos^2\frac{\pi}{3}\right)$$

$$\Rightarrow \cos^2 x\left(1 - \cos\frac{\pi}{3}\right) = \left(1 - \cos\frac{\pi}{3}\right)\left(1 + \cos\frac{\pi}{3}\right)$$

$$\Rightarrow \cos^2 x = 1 + \cos\frac{\pi}{3} \Rightarrow \cos^2 a = 1 + \frac{1}{2}$$

$$\cos^2 x = \sqrt{\frac{3}{2}}.$$

102. **(c)** We have given, $\cos^3 10 + \cos^3 110 + \cos^3 130°$

As we know,

$$\cos^3 x + \cos^3(120 - x) + \cos^3(120 + x) = \frac{3}{4}\cos 3x$$

Here, put $x = 10$, we get

$$\cos^3 10° + \cos^3(120° - 10°)\cos^3(120° + 10°)$$

$$= \left(\frac{3}{4}\right)\cos(3 \times 10°)$$

$$\cos^3 10 + \cos^3 110° + \cos^3 130° = \frac{3}{4}\cos 30° = \frac{3\sqrt{3}}{8}$$

103. **(b)** Given trigonometric equation, $\sin 5x = \cos 2x$

$$\Rightarrow \sin 5x = \sin\left(\frac{\pi}{2} - 2x\right)$$

Comparing with the eqn. $\sin x = \sin y$, we get general solution, $x = n\pi + (-1)^n y$

$$5x = n\pi + (-1)^n\left(\frac{\pi}{2} - 2x\right)$$

$$5x + (-1)^n(2x) = \frac{\pi}{2}\{2n + (-1)^n\}$$

$$x(5 + (-1)^n 2) = \frac{\pi}{2}(2n + (-1)^n)$$

$$x = \frac{\pi}{2}\left(\frac{2n + (-1)^n}{5 + 2(-1)^n}\right)$$

On comparing with $x = a_n \cdot \frac{\pi}{2}$, we get

$$a_n = \frac{2n + (-1)^n}{5 + 2(-1)^n}$$

104. **(d)** Given, $ax + b\sec(\tan^{-1} x) = c$ and $ay + b\sec(\tan^{-1} y) = c$

Let $x = \tan\theta$, then we have

$a\tan\theta + b\sec\theta = c$

$$\Rightarrow \frac{a\sin\theta}{\cos\theta} + \frac{b}{\cos\theta} = c$$

$\Rightarrow a\sin\theta + b = c\cos\theta$
$\Rightarrow c\cos\theta - a\sin b = b$
for some α,
$c = r\cos\alpha, a = r\sin\alpha$
Then, $\tan\alpha = \frac{a}{c}$
Thus, $\cos\alpha\cos\theta - \sin\alpha\sin\theta = \frac{b}{r}$
$\Rightarrow \cos(\alpha+\theta) = \frac{b}{r} \Rightarrow \alpha + \theta = \pm\cos^{-1}\frac{b}{r}$
Similarly, for $ay + b\sec(\tan^{-1} y) = c$
Let $y = \tan\phi$, we get
$\alpha + \phi = \pm\cos^{-1}\frac{b}{r}$
Let $\alpha+\theta$ be the positive solution and $\alpha+\phi$ the negative solution, where

$$y = \tan\phi$$
$$\alpha + \phi = -(\alpha+\theta)$$
$$-2\alpha = \theta + \phi$$
$$\tan(-2\alpha) = \tan(\theta+\phi)$$
$$\frac{-2\tan\alpha}{1-\tan^2\alpha} = \frac{\tan\theta + \tan\phi}{1 - \tan\theta + \tan\phi}$$
$$\frac{-2a/c}{1-a^2/c^2} = \frac{x+y}{1-xy} \Rightarrow \frac{2ac}{a^2-c^2} = \frac{x+y}{1-xy}$$

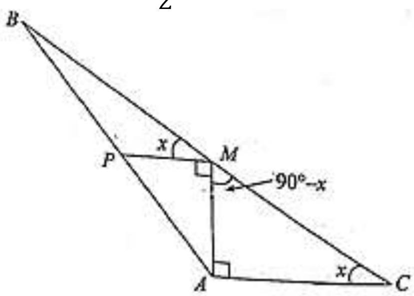
105. **(a)** Since, we know

$$\tan h^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$
$$\coth^{-1}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

Given that, $\tan h^{-1}\left(\frac{1}{2}\right) + \cot h^{-1}(3)$

$$= \frac{1}{2}\ln\left(\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right) + \frac{1}{2}\ln\left(\frac{3+1}{3-1}\right) = \frac{1}{2}\log 3 + \frac{1}{2}\log 2$$
$$= \log\sqrt{3} + \log\sqrt{2} = \log(\sqrt{3}\cdot\sqrt{2}) = \log\sqrt{6}$$

106. **(c)** Since M is mid-point of line
$BM = MC = \frac{1}{2}BC$



In $\triangle$ AMC
$\angle AMC = 90° - x$
M lies on line BC
$\angle BMP + 90° + 90° - x = 180°$
$\Rightarrow \angle BMP = x$
PM ∥ AC [Alternate angles are equal]
$PM = \frac{1}{2}AC$ ....(i)
In $\triangle$ APM
$\angle PAM = A - 90°$

$\tan(A - 90°) = \frac{PM}{AM}$

$\Rightarrow -\cot A = \frac{PM}{AM}$

$\Rightarrow AM = -PM\tan A$ .....(ii)

In $\triangle$ AMC

$\tan C = \frac{AM}{AC} \Rightarrow AM = AC\tan C$ From (ii)

$AC\tan C = -PM\tan A$

$\frac{\tan A}{\tan C} = \frac{-AC}{PM} = -2$ [From (i)]

107. (b)

$$\text{In } \Delta ABC, \tan\frac{A}{2} + \tan\frac{B}{2} = \frac{\Delta}{s(s-a)} + \frac{\Delta}{s(s-b)}$$

$$= \frac{\Delta}{s}\left(\frac{1}{s-a} + \frac{1}{s-b}\right) = \frac{\Delta}{s}\left(\frac{s-b+s-a}{(s-b)(s-a)}\right)$$

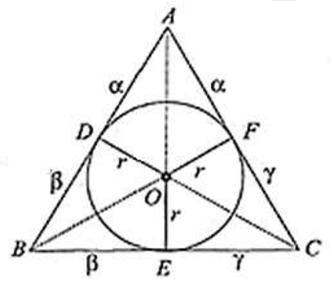
$$= \frac{\Delta}{s}\left(\frac{2s-a-b}{(s-b)(s-a)}\right) = \frac{\Delta}{s}\left(\frac{a+b+c-a-b}{(s-b)(s-a)}\right)$$

$$= \frac{c\Delta}{s(s-a)(s-b)} \qquad (2s = a+b+c)$$

$$= \frac{2c\Delta}{(a+b+c)(s-a)(s-b)}$$

$$= \frac{2c}{(a+b+c)}\sqrt{\frac{s(s-c)}{(s-a)(s-b)}} = \frac{2c\cot\frac{c}{2}}{a+b+c}$$

108. (d) Given that, in $\triangle$ ABC, point D, E and F are the points of contact of the in circle of the sides AB, BC and CA.



From the above figure, Area of $\triangle$ ABC

$= \text{Area of } \triangle AOB + \text{Area of } \triangle BOC + \text{Area of } \triangle COA$

$\Rightarrow A = \frac{1}{2}cr + \frac{1}{2}ar + \frac{1}{2}br.$ [where $ar(\triangle ABC) = A$]

$\Rightarrow A = \frac{1}{2}r(a+b+c)$

$\Rightarrow A = \frac{1}{2}r(2s)$ [where s is semi perimeter]

$\Rightarrow A = rs \Rightarrow r = \frac{A}{s} \Rightarrow r^2 = \frac{A^2}{s^2}$

$\Rightarrow r^2 = \frac{s(s-a)(s-b)(s-c)}{s^2}$

$\Rightarrow r^2 = \frac{\alpha\cdot\beta\cdot\gamma}{\alpha+\beta+\gamma}$ [$\because 2s = 2\alpha + 2\beta + 2\gamma$]

109. (b) Given, a, b and c be three non-coplanar vectors. Equation of line joining the points, $\mathbf{a} + 2\mathbf{b} - 5\mathbf{c}, -\mathbf{a} - 2\mathbf{b} - 3\mathbf{c}$ is

$a + 2b - xc - ab(ab + 4b - 2c)$

$l_1: r = (a + 2b - 5c) + \lambda(2a + 2)$ ... ... (i)

Similarly, equation of the line joining the points $-4c, 6a - 4b + 4c$ is

$l_2: r = -4c + t(6a - 4b + 8c)$ ....(ii)

Now, for point of intersection of lines l_1 and l_2,

$$(2\lambda + 1)a + (4\lambda + 2)b + (-2\lambda - 5)c$$
$$= (6t)a + (-4t)b + (8t + 4)c$$

On comparing, we get

$2\lambda + 1 = 6t$ ..... (iii)

$4\lambda + 2 = -4t$ .... (iV)

and $-2\lambda - 5 = 8t + 4$ ..(v)

From Eqs. (iii), (iv) and (v)

$\lambda = -\frac{1}{2}$ and $t = 0$

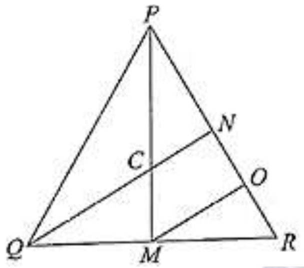
So, the intersection point is $-4c$ and for $\mu = -3$ the line $r = (3a + 6b - c) + \mu(a + 2b + c)$ passes through point $-4c$ only.

110. **(d)** Given that,

M is the mid-point of OR and

C is the mid-point of PM.

Draw MO such that MO is parallel to CN.



Since, N is the mid-point of PO

$CN = \frac{1}{2}MO$ ....(i)

M is the mid-point of QR

MO is parallel to QN

So, $MO = \frac{1}{2}QN$

From Eq. (i),

$$CN = \frac{1}{2}\left(\frac{1}{2}QN\right) CN = \frac{1}{4}QN$$

$$\left|\frac{QN}{CN}\right| = 4$$

111. **(a)** Here, vectors a, b and c are given as

$a = \hat{i} - 2\hat{j} - 3\hat{k}$

$b = 2\hat{i} + \hat{j} - \hat{k}$

$c = \hat{i} + 3\hat{j} - 2\hat{k}$

According to the question,

$(a \times b) \times (b \times c) = [abc]b - [abbb]c = [abc]b.$

$(b \times c) \times (c \times a) = [b \quad c \quad a]c - [b\ c\ c]a = [a \quad b \quad c]c$

$(c \times a) \times (a \times b) = [c\ a\ b]a - [c \quad a \quad a]b = [a \quad b \quad c]a$

Now, $[abc] = \begin{vmatrix} 1 & -2 & -3 \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix}$

$= 1(-2+3) + 2(-4+1) - 3(6-1) = 1 - 6 - 15 = -20$

$[(a \times b) \times (b \times c)(b \times c) \times (c \times a)(c \times a) \times (a \times b)]$

$= [(-20)b(-20)c(-20)a]$

$= \begin{vmatrix} -40 & -20 & 20 \\ -20 & -60 & 40 \\ -20 & 40 & 60 \end{vmatrix},$

$= -40[-3600 - 1600] + 20[-1200 + 800]$

$+20[-800 - 1200]$

$= 208000 - 8000 - 40000 = 160000$

112. **(b)** Given that, n is perpendicular to both the vectors a and b.

$n = a \times b$

$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 2 & 1 \end{vmatrix} = -4\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$

Also given, θ be the angle between c and n. Then

$\sin\theta = \frac{|n \times c|}{|n||c|}$

So, $n \times c = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -4 & -4 & 4 \\ 1 & 2 & -2 \end{vmatrix} = -4\hat{\jmath} - 4\hat{k}$

$\sin\theta = \frac{|-4\hat{\jmath} - 4\hat{k}|}{|-4\hat{\imath} - 4\hat{\jmath} + 4\hat{k}||\hat{\imath} + 2\hat{\jmath} - 2\hat{k}|}$

$= \frac{\sqrt{(-4)^2 + (-4)^2}}{\sqrt{(-4)^2 + (-4)^2 + (4)^2}\sqrt{(1)^2 + (2)^2 + (-2)^2}}$

$= \frac{\sqrt{16+16}}{\sqrt{16+16+16}\sqrt{1+4+4}} = \frac{4\sqrt{2}}{(4\sqrt{3}) \times 3} = \frac{\sqrt{2}}{3\sqrt{3}}$

113. **(b)** Given that vectors a, b and c are mutually perpendicular of same magnitude so,

Let $|a| = |b| = |c| = k$

Now, $|a+b+c|^2 = (a+b+c)\cdot(a+b+c)$

$= |a|^2 + |b|^2 + |c|^2 + 2(a \cdot b + b \cdot c + c \cdot a).$

$= k^2 + k^2 + k^2 + 20(0+0+0)$

$= 3k^2 [a \perp b, b \perp c, c \perp a, \text{So } a \cdot b = b \cdot c = c \cdot a = 0]$

$|a+b+c| = \sqrt{3}k$

Now, let θ be the angle between the vectors a and

$a+b+c$

$\cos\theta = \frac{a\cdot(a+b+c)}{|a||a+b+c|} = \frac{|a|^2}{|a||a+b+c|}$

$= \frac{k^2}{k+\sqrt{3}k} = \frac{1}{\sqrt{3}}$

114. **(b)** Given that vectors **a**, **b** and **c** are non-coplanar vectors.

Let $A(2a+3b-c), B(a-2b+3c), C(3a+4b-2c)$

and $D(ka-6b+6c)$

$AB = B - A = -a - 5b + 4c$

$AC = C - A = a + b - c$

$AD = D - A = (k-2)a - 9b + 7c$

Now, As A, B, C and D coplanar

$AB \cdot (AC \times AD) = 0$

$$\therefore \begin{vmatrix} -1 & -5 & 4 \\ 1 & 1 & -1 \\ k-2 & -9 & 7 \end{vmatrix} = 0$$

$\Rightarrow (k-2)(5-4) + 9'(1-4) + 7(-1+5) = 0$

$\Rightarrow k - 1 = 0 \Rightarrow k = 1.$

115. **(a)** Given that, $n = 8, \bar{x} = 25$ and $\sigma = 5$

Mean $\bar{x} = \dfrac{\Sigma x_i}{n}$

$\Rightarrow \Sigma x_i = n\bar{x} = 8 \times 25 = 200 \Rightarrow \Sigma x_i = 200$

Variance, $\sigma^2 = \frac{\Sigma x_i{}^2}{n} - (\bar{x})^2$

$25 = \dfrac{\Sigma x_i^2}{8} - 625 \Rightarrow \Sigma x_i^2 = 5200$

As, two new observations 15 and 25 are added, then

corrected $\Sigma x_i = 200 + 15 + 25 = 240$

Corrected $\Sigma x_i^2 = 5200 + 225 + 625 = 6050$

Corrected variance $= \dfrac{6050}{10} - \left(\dfrac{240}{10}\right)^2$

$= 605 - (24)^2 = 605 - 576 = 29$

116. **(b)**

x_i	f_i	Cumulative frequency	$d_i = \lvert x_i - 15 \rvert$	$f_i d_i$
6	4	4	9	36
9	5	9	6	30
3	3	12	12	36
12	2	14	3	6
15	5	19	0	0
13	4	23	2	8
21	4	27	6	24
22	3	30	7	21
	$N = \Sigma f_i$ $= 30$			$\Sigma f_i d_i = 161$

The cumulative frequency just greater than $\frac{N}{2}$ is 19 and corresponding value of x is 15 .

Therefore, median $= 15.$

Mean deviation $= \dfrac{\Sigma f_i |x_i - M|}{n} = \dfrac{\Sigma f_i |x_i - 15|}{30}$

$= \dfrac{161}{30} = 5.36$

117. **(b)** If a die is thrown three times,

Total possible outcomes,

$n = 6^3 = 216$

For getting a larger number on its face than the previous number each. Then, favourable outcomes $= {}^6C_3 = 20$

Now, required probability $= \frac{r}{n} = \frac{20}{216} = \frac{5}{54}$

118. **(b)** Let

E_1 : six occurs on the die

E_2 : six does not occurs on the die
A : when man reports that it is six.

$P(E_1) = \frac{1}{6}, P(E_2) = \frac{5}{6}$

and $P\left(\frac{A}{E_1}\right) = \frac{2}{3}, P\left(\frac{A}{E_2}\right) = \frac{1}{3}$

According to Bayes' theorem,

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1) \times P\left(\frac{A}{E_1}\right)}{P(E_1) \times P\left(\frac{A}{E_1}\right) + P(E_2) \times P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{6} \times \frac{2}{3}}{\frac{1}{6} \times \frac{2}{3} + \frac{5}{6} \times \frac{1}{3}} = \frac{2}{7}$$

Hence, the probability that it is actually six $= \frac{2}{7}$

The probability that it is not actually six $= \frac{5}{7}$

The probability that is actually five $= \frac{1}{5} \times \frac{5}{7} = \frac{1}{7}$

119. **(b)** Given that,

$P(X = k) = a\left(\frac{k+1}{2^k}\right)$

As we know,

$$\sum P(X = k) = 1$$

$$\Rightarrow P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) = 1$$

$$\Rightarrow a + \frac{2a}{2} + \frac{3}{4}a + \frac{4}{8}a + \frac{5}{16}a + \frac{6}{32}a = 1$$

$$\Rightarrow \frac{16a + 16a + 12a + 8a + 5a + 3a}{16} = 1 \Rightarrow a = \frac{4}{15}$$

Required probability

$$= P(X = 2) + P(X = 3) + P(X = 4)$$

$$= \frac{3a}{4} + \frac{1a}{2} + \frac{3a}{16}$$

$$= \frac{12a + 8a + 3a}{16} = \frac{23a}{16} = \frac{23}{16} \times \frac{4}{15} = \frac{23}{60}$$

120. **(b)** For a binomial variable X,

Mean = 6 and variance = 2

$\therefore$ np = 6 and npq = 2

So $q = \frac{1}{3}; p = \frac{2}{3}$

and np = 6; $n \times \frac{2}{3} = 6 \Rightarrow n = 9$

$$P(5 \le X \le 7) = P(X = 5) + P(X = 6) + P(X = 7)$$

$$= {}^9C_5\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^4 + {}^9C_6\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^3 + {}^9C_7\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^2$$

$$= \frac{1}{3^9}\left[\frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2 \times 1} \times 32 + \frac{9 \times 8 \times 7}{3 \times 2 \times 1} \times 64 + \frac{9 \times 8}{2 \times 1} \times 128\right]$$

$$= \frac{1344 + 1792 + 1536}{6561} = \frac{4672}{6561}$$

121. **(d)** It is given that, points are $A(2,3), B(3,-6), C(5,-7)$.

Let point P be (x, y), such that satisfying the given condition

$PA^2 + PB^2 = 2PC^2$
$(x-2)^2 + (y-3)^2 + (x-3)^2 + (y+6)^2$
$= 2[(x-5)^2 + (y+7)^2]$
$\Rightarrow\ x^2 + 4 - 4x + y^2 + 9 - 6y + x^2 + 9 - 6x$
$+ y^2 + 36 + 12y$
$= 2[x^2 + 25 - 10x + y^2 + 14y + 49]$
$\Rightarrow 2x^2 + 2y^2 - 10x + 6y + 58$
$= 2x^2 + 2y^2 - 20x + 28y + 148$
$\Rightarrow 10x - 22y = 90 \Rightarrow 5x - 11y = 45$
By checking options, we get to know that point $(-13, -10)$ lies on the locus of P.

122. **(c)** Let (x, y) are old coordinates and (X, Y) are new coordinates, when axes are rotated through an angle of θ, then
$x = X\cos\theta - Y\sin\theta$ and $y = X\sin\theta + Y\cos\theta$
Put $X = 2, Y = -6$ and $\theta = 135°$
$x = 2\cos 135° - (-6)\sin 135°$
and $y = 2\sin 135° + (-6)\cos 135°$
$x = 2\left(\frac{-1}{\sqrt{2}}\right) + 6\left(\frac{1}{\sqrt{2}}\right)$ and $y = 2\left(\frac{1}{\sqrt{2}}\right) - 6\left(\frac{-1}{\sqrt{2}}\right)$
$x = \frac{4}{\sqrt{2}}$ and $y = \frac{8}{\sqrt{2}}$
$x = 2\sqrt{2}$ and $y = 4\sqrt{2}$
Coordinates of point $P(x, y)$ in the original system are $(2\sqrt{2}, 4\sqrt{2})$.

123. **(d)** Let the equation of the line in intercept form,
$\frac{x}{a} + \frac{y}{b} = 1$
The coordinate of the point $(2, -1)$ which divides the line joining $A(a, 0)$ and $B(0, b)$ in the ratio 3: 2 are
$$(2, -1) = \left(\frac{3 \times 0 + 2 \times a}{3 + 2}, \frac{3 \times b + 2 \times 0}{3 + 2}\right)$$
$$(2, -1) = \left(\frac{2a}{5}, \frac{3b}{5}\right)$$
On comparing both the sides, we get
$$\frac{2a}{5} = 2 \text{ and } \frac{3b}{5} = -1\ \Rightarrow\ a = 5 \text{ and } b = \frac{-5}{3}$$
Hence, the required equation of line $\frac{x}{5} + \frac{y}{\frac{-5}{3}} = 1$
$$\Rightarrow\ \frac{x}{5} - \frac{3y}{5} = 1$$
$$\Rightarrow\ \frac{x}{5} - \frac{3y}{5} = 1$$
$\Rightarrow x - 3y = 5 \Rightarrow x - 3y - 5 = 0$

124. **(b)** Let $l_1: 2x + y - 4 = 0, l_2: x - 3y + 5 = 0$
The equation of a line passing through the intersection of l_1 and l_2,
$(2x + y - 4) + \lambda(x - 3y + 5) = 0$ ... ... (i)
$x(2 + \lambda) + y(1 - 3\lambda) + 5\lambda - 4 = 0$
This line is at a distance of $\sqrt{5}$ units from the origin.
$$D = \left|\frac{5\lambda - 4}{(2+\lambda)^2 + (1 - 3\lambda)^2}\right| = \sqrt{5}$$
$$\frac{(5\lambda - 4)^2}{4 + \lambda^2 + 4\lambda + 1 + 9\lambda^2 - 6\lambda} = 5$$
$25\lambda^2 + 16 - 40\lambda = 50\lambda^2 - 10\lambda + 25$
$$(5\lambda + 3)^2 = 0 \Rightarrow \lambda = -\frac{3}{5}$$
Putting the value of $\lambda = -\frac{3}{5}$ in Eq. (i), we get

$$(2x+y-4)-\frac{3}{5}(x-3y+5)=0$$
$$10x+5y-20-3x+9y-15=0$$
$$7x+14y-35=0 \Rightarrow x+2y-5=0$$

125. **(b)** Let AD and BE are altitudes of the triangle.

Equation of AD is given by

$$y-3=(\text{ Slope of AD})(x+2)$$
$$y-3=\frac{-1}{\text{Slope of BC}}(x+2)$$
$$\Rightarrow y-3=\frac{-1}{\left(\frac{0+1}{4-2}\right)}(x+2)$$
$$y-3=-2(x+2)$$
$$2x+y+1=0 \quad ....(i)$$

Now, equation of BE is given by

$$y+1=(\text{ Slope of BE })(x-2)$$
$$y+1=\frac{-1}{\text{Slope of AC}}(x-2)$$
$$\Rightarrow y+1=\frac{-1}{\left(\frac{0-3}{4+2}\right)}(x-2)$$
$$y+1=2(x-2)$$
$$y=2x-5 \quad \ldots\ldots..(ii)$$

Since, orthocentre is the intersecting point of altitudes.

On solving Eqs. (i) and (ii), we get orthocentre as $(1,-3)$.

Also, centroid of $\triangle$ ABC $=\left(\frac{-2+2+4}{3},\frac{3-1+0}{3}\right)=\left(\frac{4}{3},\frac{2}{3}\right)$

Equation of line joining centroid $\left(\frac{4}{3},\frac{2}{3}\right)$ and orthocentre $(1,-3)$

$$y+3=\frac{-3-\frac{2}{3}}{1-\frac{4}{3}}(x-1)$$
$$\Rightarrow 11x-y-14=0$$

This is required equation of line.

126. **(c)** Given that,

$$23x^2-48xy+3y^2=0$$
$$\Rightarrow 3y^2-48xy+23x^2=0$$

Here, $m_1+m_2=16$ ....(i)

and $m_1m_2=\frac{23}{3}$ ....(ii)

On solving Eqs. (i) and (ii), we get

$m_1=8+\frac{13}{\sqrt{3}}$ and $m_2=8-\frac{13}{\sqrt{3}}$

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{\frac{26}{\sqrt{3}}}{1+\frac{23}{3}}\right|=\sqrt{3} \;\therefore\; \theta=60^\circ$$

Angle between line of slope $8+\frac{13}{\sqrt{3}}$ and $-\frac{2}{3}$ is

$$\tan\alpha=\frac{8+\frac{13}{\sqrt{3}}+\frac{2}{3}}{1+\left(8+\frac{13}{\sqrt{3}}\right)\left(\frac{2}{3}\right)}=\sqrt{3}\Rightarrow\alpha=60^\circ$$

Hence, θ and α both are 60° so these lines from a equilateral triangle.

127. **(c)** Given curve,

$$x^2-xy+y^2+3x+3y-2=0 \quad ......(i)$$

and line, $x + 2y = k \Rightarrow \frac{x+2y}{k} = 1$
Now, By using homogeneous of Eq. (i)

$$x^2 - xy + y^2 + 3x(1) + 3y(1) - 2(1)^2 = 0$$

$$x^2 - xy + y^2 + 3x\left(\frac{x+2y}{k}\right)$$

$$+3y\left(\frac{x+2y}{k}\right) - 2\left(\frac{x+2y}{k}\right)^2 = 0$$

$$k^2x^2 - k^2xy + k^2y^2 + 3kx^2 + 6kxy + 3kxy + 6ky^2 \quad - 2x^2 - 8xy - 8y^2 = 0$$

$$x^2(k^2 + 3k - 2) - (k^2 - 9k + 8)xy + (k^2 + 6k - 8)y^2 = 0$$

Since, $\angle AOB = 90°$, it means that

$$k^2 + 3k - 2 + k^2 + 6k - 8 = 0$$

$$\Rightarrow\ 2k^2 + 9k - 10 = 0$$

128. **(c)** We know that,
If the lines $ax^2 + 2hxy + by^2 = 0$ be two sides of a parallelogram and the line $lx + my = 1$ will be one of its diagonals, then the other diagonal is
$y(bl - hm) = x(am - hl)$ ....(i)
Here, for the given pair of lines, $2x^2 + 3xy - 2y^2 = 0$

$$a = 2, b = -2, h = \frac{3}{2}$$

$$l = -3, m = -1$$

Putting all values in Eq. (i), we get

$$y\left(6 + \frac{3}{2}\right) = x\left(-2 + \frac{9}{2}\right) \Rightarrow y\left(\frac{15}{2}\right) = x\left(\frac{5}{2}\right)$$

$$3y = x \Rightarrow x - 3y = 0$$

129. **(d)** As we know, length of tangent drawn from the point (x_1, y_1) to the circle $x^2 + y^2 + 2gx + 2fy +$

$$\sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

Let point $P(h, k)$ be the point from which the length of tangents are in the ratio of $2:1$.

$$\therefore\ \frac{\sqrt{h^2 + k^2 - 2h + 4k - 20}}{\sqrt{h^2 + k^2 - 2h - 8k + 1}} = \frac{2}{1}$$

$$h^2 + k^2 - 2h + 4k - 20 = 4(h^2 + k^2 - 2h - 8k + 1)$$

$$3h^2 + 3k^2 - 6h - 36k + 24 = 0$$

$$h^2 + k^2 - 2h - 12k + 8 = 0$$

Replace h and k by x and y,
$x^2 + y^2 - 2x - 12y + 8 = 0.$
This is the required locus of P.

130. **(c)** Let the centre of the circle be $c(h_1, k)$ and radius r.
Since, circle touches both coordinates axes, then centre will be (h, h) and radius $= h$

$$\left|\frac{3h - 4h - 12}{\sqrt{(3)^2 + (-4)^2}}\right| = h \Rightarrow \left|\frac{-h - 12}{5}\right| = h$$

$$\Rightarrow -h - 12 = \pm 5h$$

$$\Rightarrow -12 = \pm 5h + h \Rightarrow -12 = 6h \text{ or } -12 = -4h$$

$$\Rightarrow h = -2 \text{ or } 3 \Rightarrow h = 3 \qquad [h > 0]$$

Thus, equation of circle

$$(x - 3)^2 + (y - 3)^2 = 3^2$$

$$x^2 + y^2 - 6x - 6y + 9 = 0$$

This is required equation of circle.

131. **(b)** Given, $L: 9x + y - 28 = 0$
$C: x^2 + y^2 - \frac{3}{2}x + \frac{5}{2}y - \frac{7}{2} = 0$
Let $P(h, k)$ be the pole then, the equation of polar is

$hx + ky - \frac{3}{4}(x+h) + \frac{5}{4}(y+k) - \frac{7}{2} = 0$
$\Rightarrow x(4h-3) + y(4k+5) - 3h + 5k - 14 = 0$
On comparing this line with $9x + y - 28 = 0$
$4h - 3 = 9 \Rightarrow h = 3$
Similarly,
$4k + 5 = 1 \Rightarrow k = -1$
Hence, the pole of the given line is $(3, -1)$

132. **(d)** Given circles,

$C_1: (x+11)^2 + (y-2)^2 = (15)^2$
$C_2: (x-11)^2 + (y+2)^2 = (5)^2$

Centres, $C_1(-11,2)$ and $C_2(11,-2)$
Radius, $r_1 = 15$ and $r_2 = 5$
The direct common tangents to two circles meet on the line of centres and divide it externally in the ratio of the radii centres of the two circles.
Point of intersection

$$= \left(\frac{11 \times 15 - (-11) \times 5}{15 - 5}, \frac{-2 \times 15 - 2 \times 5}{15 - 5}\right) = (22, -4)$$

133. **(b)** We know that, angle between two circles is given by $\cos\theta = \frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}$, where r_1 and r_2 are radius and d is distance between centres.

(A) Given, $r_1 = \sqrt{2}, r_2 = 1, c_1 = (2,0), c_2 = (2,1)$

$$\cos\alpha = \frac{(\sqrt{2})^2 + (1)^2 - \left[\sqrt{(2-2)^2 + (1-0)^2}\right]^2}{2 \times \sqrt{2} \times 1} = \frac{1}{\sqrt{2}}$$

$\alpha = 45^\circ$ or 135°

(B) Given, $r_1 = 3, r_2 = \sqrt{17}, c_1 = (3,3), c_2 = (2,-2)$

$$\cos\beta = \frac{(3)^2 + (\sqrt{17})^2 - \left[\sqrt{(3-2)^2 + (3+2)^2}\right]^2}{2 \times 3 \times \sqrt{17}}$$

$\beta = 90^\circ$

(C) Given, $r_1 = 5, r_2 = 3, c_1 = (-2,7), c_2 = (-2,0)$

$$\cos\gamma = \frac{(5)^2 + (3)^2 - \left[\sqrt{(-2+2)^2 + (7-0)^2}\right]^2}{2 \times 5 \times 3}$$

$$= \frac{25 + 9 - 49}{30} = \frac{-15}{30} = \frac{-1}{2}$$

So, $\gamma = 120^\circ g$ or 60°.

134. **(a)** Given, $S_1: x^2 + y^2 + 2gx + 2fy + c = 0$

$S_2: 2x^2 + 2y^2 + 3x + 8y + 2c = 0$
$S_3: x^2 + y^2 + 2x + 2y + 1 = 0$

Let point $P(a, b)$, So

$S_1(a,b) = S_2(a,b)$

$$a^2 + b^2 + 2ga + 2fb + c = a^2 + b^2 + \frac{3}{2}a + 4b + c = 0$$

$$a\left(2g - \frac{3}{2}\right) + b(2f - 4) = 0$$

Now, replace a and b by x and y
So, $x\left(2g - \frac{3}{2}\right) + y(2f - 4) = 0$ is radical axis of given circles
This touches the $x^2 + y^2 + 2x + 2y + 1 = 0$
Its radius $= \sqrt{1^2 + 1^2 - 1} = 1$ and centre $= (-1, -1)$
So, radius of circle = distance between centre and touching point of radical axis.

$$I = \frac{\left|\left(\frac{3}{2} - 2g\right) + (2f - 4)\right|}{\sqrt{\left(\frac{3}{2} - 2g\right)^2 + (2f - 4)^2}}$$

$$= \left|\left(\frac{3}{2} - 2g\right) + (2f - 4)\right|$$

Taking square both sides, we get

$$2\left(\frac{3}{2} - 2g\right)(2f - 4) = 0$$

So, $\frac{3}{2} - 2g = 0$ or $2f - 4 = 0$

$2g = \frac{3}{2}$ or $2f = 4$

$\Rightarrow g = \frac{3}{4}$ or $f = 2$

135. **(c)** Given line, $l: y = 6x + 1$ and parabola: $y^2 = 24x$.

The locus of the point of intersection of perpendicular tangent to a parabola is its directrix.

So, required point will be the point of intersection of $y = 6x + 1$ and directrix $x = -6$.

$y = 6(-6) + 1 = -35$

Hence, its coordinates are $(-6, -35)$

136. **(a)** The slope of the line joining the focus $S(1, -1)$ and vertex $A(1,1)$ is $m = \frac{-1-1}{1-1} = 0$

Let $Q(h, k)$ be the point of intersection of the axis AS with the directrix. The $A(1,1)$ will be the mid-point of OS.

$$(1,1) = \left(\frac{h+1}{2}, \frac{K-1}{2}\right)$$

$$\frac{h+1}{2} = 1 \text{ and } \frac{k-1}{2} = 1$$

$$\Rightarrow h = 1 \text{ and } k = 3 \Rightarrow Q(1,3)$$

Su, the directrix passes through the point $(1,3)$ and has the gradient 0 . So directrix is $\dot{y} - 3 = 0$

Let $P(x, y)$ be any point on the parabola and M be the foot of the perpendicular drawn from P on the directrix.

$$PS = PM \Rightarrow PS^2 = PM^2$$

$$(x-1)^2 + (y+1)^2 = \left(\frac{y-3}{\sqrt{1}}\right)^2$$

$$(x-1)^2 = 8(1-y)$$

By checking option (a),

$$(3-1)^2 = 8\left(1 - \frac{1}{2}\right)$$

$$(2)^2 = 8 \times \frac{1}{2} \Rightarrow 4 = 4$$

Hence, point $\left(3, \frac{1}{2}\right)$ lies on the parabola $(x-1)^2 = 8(1-y)$.

137. **(c)** Let the equation of ellipse is

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \ldots\ldots(i)$

Given that eccentricity, $e = \sqrt{\frac{2}{5}}$

We know,

$$b^2 = a^2(1 - e^2)$$

$$a^2 = \frac{5b^2}{3}$$

Now, ellipse passes through the point $(-3,1)$

$$\frac{(-3)^2}{a^2} + \frac{(1)^2}{b^2} = 1 \Rightarrow 9b^2 + a^2 = a^2b^2$$

Put $a^2 = \frac{5b^2}{3}$, we get

$$9b^2+\frac{5b^2}{3}=\frac{5b^4}{3}\Rightarrow\frac{32b^2}{3}=\frac{5b^4}{3}$$
$$b^2=\frac{32}{5}\Rightarrow a^2=\frac{5}{3}\times\frac{32}{3}=\frac{32}{3}$$
From Eq (i), we get
$$\Rightarrow\frac{x^2}{\frac{32}{3}}+\frac{y^2}{\frac{32}{5}}=1$$
$$\Rightarrow 3x^2+5y^2-32=0$$

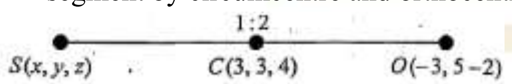
138. **(b)** Given, ellipse $\frac{x^2}{36}+\frac{y^2}{27}=1$

Let $P(6\cos\theta, 3\sqrt{3}\sin\theta)$ be any point on it the equation of the tangent,
$$\frac{x}{6}\cos\theta+\frac{y}{3\sqrt{3}}\sin\theta=1$$
$$\Rightarrow\ 3\sqrt{3}x\cos\theta+6y\sin\theta-18\sqrt{3}=0 \qquad \ldots\ldots(i)$$
Let P be the product of the lengths of the perpendiculars from (3,0) and (−3,0) on Eq. (i) is given by
$$P=\left|\frac{3\times3\sqrt{3}\cos\theta-18\sqrt{3}}{\sqrt{27\cos^2\theta+36\sin^2\theta}}\right|\left|\frac{3\times3\sqrt{3}\cos\theta+18\sqrt{3}}{\sqrt{27\cos^2\theta+36\sin^2\theta}}\right|$$
$$=\frac{36\times27-9\times27\cos^2\theta}{36\sin^2\theta+27\cos^2\theta}=\frac{9\times27(4-\cos^2\theta)}{36-9\cos^2\theta}$$
$$P=\frac{9\times27(4-\cos^2\theta)}{9(4-\cos^2\theta)}=27$$

139. **(b)** Given, asymptotes are
$$3x+4y-2=0 \qquad \ldots\ldots(i)$$
and $2x+y+1=0 \qquad \ldots\ldots.(ii)$

From Eqs. (i) and (ii)
$$(3x+4y-2)(2x+y+1)=0 \qquad \ldots\ldots(iii)$$
As, the equation to the hyperbola will differ from Eq. (iii) only by a constant, so
$$(3x+4y-2)(2x+y+1)=\lambda \qquad \ldots\ldots (iV)$$
This passes through the point (1,1), so
$$(3+4-2)(2+1+1)=\lambda$$
$$(5)(4)=\lambda\ \Rightarrow\ \lambda=20$$
Hence, the equation of the hyperbola will be
$$(3x+4y-2)(2x+y+1)=20$$
$$6x^2+3xy+3x+8xy+4y^2+4y-4x-2y-2=20$$
$$6x^2+11xy+4y^2-x+2y-22=0$$

140. **(a)** We know that, if O is orthocentre, G is centroid and S is circumcentre, then centroid divides the line segment by circumcentre and orthocentre in the ratio of 1: 2. Let the coordinates of circumcentre be (x, y, z).



Coordinates of circumference $=\left(\frac{-3+2x}{1+2},\frac{5+2y}{1+2},\frac{2+2z}{1+2}\right)$
$$(3,3,4)=\left(\frac{-3+2x}{3},\frac{5+2y}{3},\frac{2+2z}{3}\right)$$
$$\frac{-3+2x}{3}=3,\frac{5+2y}{3}=3,\frac{2+2z}{3}=4$$
$$x=6, y=2, z=5$$
Circumcentre be S(6,2,5)

141. **(d)** Equation of the plane which cuts the coordinates axes x, y, and z at a, b and c respectively,
$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$
Centroid

$$G=\left(\frac{a+0+0}{3},\frac{0+b+0}{3},\frac{0+0+c}{3}\right)$$

$$(6,6,3)=\left(\frac{a}{3},\frac{b}{3},\frac{c}{3}\right)$$

Therefore, $a=18, b=18, c=9$

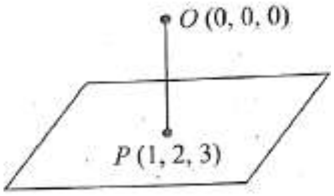
The equation of the plane becomes

$$\frac{x}{18}+\frac{y}{18}+\frac{z}{9}=1 \Rightarrow \frac{x+y+2z}{18}=1$$

$$x+y+2z=18$$

$$x+y+2z-18=0$$

142. **(b)** Here, foot of the perpendicular $P(1,2,3)$ is drawn from the origin $O(0,0,0)$



Direction Ratio's of $OP=\langle 1-0,2-0,3-0\rangle =<1,2,3>$

Since, OP is perpendicular to the plane, therefore OP is normal to the plane.

Equation of plane passing through $(1,2,3)$,

$$(r-a)\cdot n=0$$

$$(x-1,y-2,z-3)\cdot(1,2,3)=0$$

$$1(x-1)+2(y-2)+3(z-3)=0$$

$$x+2y+3z-14=0$$

According to the given options, $(7,2,1)$ lies on the given plane

143. **(b)** We have,

$$\lim_{n\to\infty}\frac{1}{n^3}\{[1^2x]+[2^2x]+[3^2x]+\cdots+[n^2x]\}$$

$$=\lim_{n\to\infty}\frac{1}{n^3}\sum_{r=1}^{n}[r^2x]=\lim_{n\to\infty}\frac{1}{n^3}\sum_{r=1}^{n}\left(r^2x-(r^2x)\right)$$

$$=\lim_{n\to\infty}\frac{1}{n^3}\left(x\sum_{r=1}^{n}r^2-\sum_{r=1}^{n}\{r^2n\}\right)$$

$$=\lim_{n\to\infty}\frac{x\times n(n+1)(2n+1)}{n^3\times 6}-\frac{1}{n^3}\sum_{r=1}^{n}\{r^2n\}$$

$$=\lim_{n\to\infty}\left(\frac{x}{6}\times\frac{n}{n}\times\frac{n+1}{n}\times\frac{(2n+1)}{n}\right)-\lim_{n\to\infty}\frac{1}{n^3}\sum_{r=1}^{n}\{r^2n\}$$

$$=\lim_{n\to\infty}\left[\frac{x}{6}\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)\right]-\lim_{n\to\infty}\frac{1}{n^2}\sum_{r=1}^{n}\{r^2n\}$$

$$=\frac{x}{6}\times 1\times 2-0=\frac{x}{3}$$

144. **(a)** Given that,

$$f(x)=\begin{cases}\dfrac{1-\sqrt{2}\sin x}{\pi-4x}, & x\neq\dfrac{\pi}{4}\\ k, & x=\dfrac{\pi}{4}\end{cases}$$

Since, it is given that $f(x)$ is continuous at $x=\frac{\pi}{4}$

$$f(\pi/4)=\lim_{\pi\to\frac{\pi}{4}}f(x)$$

$\Rightarrow\ k = \lim_{\pi\to\frac{\pi}{4}} \frac{1-\sqrt{2}\sin x}{\pi - 4x}$

[using L′ Hospital rule]

$\Rightarrow k = \frac{\sqrt{2}}{4} \times \frac{1}{\sqrt{2}}$

$\Rightarrow\ k = \frac{1}{4}.$

145. **(d)** Let, $f(x) = x^{\tan^{-1}x}$

Taking log on both the sides, $\log f(x) = \tan^{-1} x \log x$

Differentiating w.r.t x, we get

$$\frac{1}{f(x)} \cdot \frac{d}{dx} f(x) = \frac{1}{1+x^2} \log x + \frac{\tan^{-1} x}{x}$$

$$\Rightarrow \frac{df(x)}{dx} = x^{\tan^{-1}x} \left[\frac{\log x}{1+x^2} + \frac{\tan^{-1} x}{x}\right]$$

Also, Let $g(x) = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

$= \cos^{-1}(2x^2-1) = 2\cos^{-1} x$

Differentiating w.r.t x, we get

$$\frac{d}{dx} g(x) = \frac{-2}{\sqrt{1-x^2}}$$

$$\frac{d}{dg(x)} f(x) = \frac{dx}{\frac{dg(x)}{dx}}$$

$$= \frac{x^{\tan^{-1}x}\left[\frac{\log x}{1+x^2} + \frac{\tan^{-1} x}{x}\right]}{\frac{-2}{\sqrt{1-x^2}}}$$

$$= \frac{-1}{2}\sqrt{1-x^2} \cdot x^{\tan^{-1}x}\left[\frac{\log x}{1+x^2} + \frac{\tan^{-1} x}{x}\right]$$

146. **(d)** Given that,

$x = 3\cos t$ and $y = 4\sin t$

$$\frac{x}{3} = \cos t \Rightarrow \frac{x^2}{9} = \cos^2 t \qquad \ldots\ldots(i)$$

$$\frac{y}{4} = \sin t \Rightarrow \frac{y^2}{16} = \sin^2 t \qquad \ldots\ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = \cos^2 t + \sin^2 t$$

$$\Rightarrow \frac{x^2}{9} + \frac{y^2}{16} = 1$$

Differentiating w.r.t. x, we get

$$\Rightarrow \frac{2x}{9} + \frac{2y}{16}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-16}{9}\frac{x}{y} \qquad \ldots\ldots(iii)$$

Again, differentiating w.r.t. x, we get

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-16}{9}\left[\frac{1\cdot y - x\frac{dy}{dx}}{y^2}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{-16}{9}\left[\frac{y - x\left(\frac{-16}{9}\frac{x}{y}\right)}{y^2}\right]$$

[From Eq. (iii)]

$$=\frac{-16}{9}\left[\frac{9y^2+16x^2}{9y^3}\right]=\frac{-16}{9}\times\frac{144}{9y^3}\left[\because \frac{x^2}{9}+\frac{y^2}{16}=1\right]$$

$$\left(\frac{d^2y}{dx^2}\right)\left(\frac{3\sqrt{2}}{2},2\sqrt{2}\right)=\frac{-16\times 144}{81\times(2\sqrt{2})^3}$$

$$=\frac{-16\times 144}{81\times 16\sqrt{2}}=\frac{-144}{81\times\sqrt{2}}=\frac{-16}{9\sqrt{2}}=\frac{-8\sqrt{2}}{9}$$

147. **(b)** Given that,

$$y=\frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left[\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right]$$

Differentiating w.r.t. x, we get

$$\Rightarrow \frac{dy}{dx}=\left[\frac{2}{\sqrt{a^2-b^2}}\cdot\frac{1}{1+\left(\frac{a-b}{a+b}\right)\tan^2\frac{x}{2}}\right]\sqrt{\frac{a-b}{a+b}}\cdot\sec^2\frac{x}{2}\cdot\frac{1}{2}\Bigg]$$

$$=\frac{\sec^2 x/2}{a+b}\frac{a+b}{(a+b)+(a-b)\tan^2 x/2}$$

$$=\frac{\sec^2 x/2}{a+b+a\tan^2\frac{x}{2}-b\tan^2 x/2}$$

$$=\frac{\sec^2 x/2}{\left(1+\tan^2\frac{x}{2}\right)\left[a+b\left(\frac{1-\tan^2 x/2}{1+\tan^2 x/2}\right)\right]}$$

$$\frac{dy}{dx}=\frac{1}{a+b\cos x}\Rightarrow\frac{d^2y}{dx^2}=\frac{b\sin x}{(a+b\cos x)^2}$$

At $x=\pi/2$,

$$\left(\frac{d^2y}{dx^2}\right)_{x=\frac{\pi}{2}}=\frac{b\sin\pi/2}{\left(a+b\cos\frac{\pi}{2}\right)^2}=\frac{b}{a^2}$$

148. **(b)** Given function,

$f(x)=x^3+ax^2+bx+5\sin^2 x$

$\Rightarrow$ since, f(x) is an increasing function $f'(x)>0$

$\Rightarrow 3x^2+2ax+b+10\sin x\cos x>0$

$\Rightarrow 3x^2+2ax+b+5\sin 2x>0$

$\Rightarrow 3x^2+2ax+b-5>0$

$-1\le\sin 2x\le 1$

$\Rightarrow 3x^2+2ax+(b-5)>0$

Here, since, $f'(x)>0\Rightarrow a>0$ and $D<0$

$(2a)^2-4\times 3\times(b-5)<0$

$4a^2-12(b-5)<0$

$a^2-3b+15<0$

149. **(d)** Let $y=\cos x, x=30^\circ=\pi/6$ and $x+\Delta x=31^\circ$

$$(y)_{x=\frac{\pi}{6}}=\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$$

$\Delta x=1^\circ=0.0174$ radian.

Consider the function,

$y=f(x)=\cos x$

Differentiating w.r.t. x,

$$\frac{dy}{dx} = -\sin x$$

$$\left(\frac{dy}{dx}\right)_{x=\frac{\pi}{6}} = -\sin\frac{\pi}{6} = -\frac{1}{2}$$

Let Δy be the change in y due to the change Δx in x.

$$\Delta y = \frac{dy}{dx} \times \Delta x$$

$$= \left(-\frac{1}{2}\right) \times 0.0174$$

$$= (-0.5) \times 0.0174 \approx -0.0087$$

$$f(31^\circ) = f(30^\circ + 1^\circ) = y + \Delta y$$

$$= \frac{\sqrt{3}}{2} - 0.0087 = \frac{1.732}{2} - 0.0087$$

$$= 0.8660 - 0.0087 = 0.8573$$

150. **(d)** Given that, x and y are two positive numbers such that $x + y = 32$.

Let, $s = x^2 + y^2 = x^2 + (32 - x)^2$ [x + y = 32]

$$\therefore \frac{ds}{dx} = 2x + 2(32 - x)(-1)$$

$$= 2x - 2(32 - x) = 2x - 64 + 2x = 4x - 64$$

For maxima or minima, $\frac{ds}{dx} = 0$

$\Rightarrow 4x - 64 = 0 \Rightarrow x = 16$

and $y = 32 - x = 32 - 16 = 16$

Again, $\frac{d^2s}{dx} = 4 > 0$

At $x = 16, y = 16$ s is minimum

Now, minimum value

$s = 16^2 + 16^2 = 256 + 256 = 512$

151. **(b)** Given function, $f(x) = \frac{2x+3}{4x-1}, x \in [1,2]$

Using Lagrange's mean value theorem,

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f'(c) = \frac{f(2) - f(1)}{2 - 1}$$

$$\frac{-14}{(4c-1)^2} = \frac{1 - \frac{5}{3}}{1}$$

$$(4c - 1) = \pm\sqrt{21}$$

$$4c = 1 + \sqrt{21} \text{ or } 4c = 1 - \sqrt{21}$$

$$c = \frac{1+\sqrt{21}}{4} \qquad \left[c = \frac{1-\sqrt{21}}{4} \notin [1,2]\right]$$

152. **(c)** Let $I = \int \frac{\sin 2x dx}{\sin^4 x + \cos^4 x}$

$$I = \int \frac{2\sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

Dividing by $\cos^4 x$ in numerator and denominator, we get

$$I = \int \frac{2\tan x \sec^2 x}{1 + (\tan^2 x)^2} dx$$

Put $\tan^2 x = t \Rightarrow 2\tan x \sec^2 x dx = dt$

$$I = \int \frac{dt}{1+t^2}$$

$I = \tan^{-1} t + C \Rightarrow I = \tan^{-1}(\tan^2 x) + C$

By comparing it with $\tan^{-1} f(x) + C$, we get

$f(x) = \tan^2 x$

Now, At $x = \pi/3$

$$f\left(\frac{\pi}{3}\right) = \left\{\tan\left(\frac{\pi}{3}\right)\right\}^2 = (\sqrt{3})^2 = 3$$

153. **(c)** Let, $I = \int \left\{\frac{\log x - 1}{1+(\log x)^2}\right\}^2 dx$

Put $\log x = t, x = e^t, dx = e^t dt$

$$I = \int e^t \frac{(t-1)^2}{(t^2+1)^2} dt = \int e^t \frac{t^2+1-2t}{(t^2+1)^2} dt$$

$$= \int e^t \left\{\frac{1}{t^2+1} + \frac{-2t}{(t^2-1)^2}\right\} dt$$

$$I = \frac{e^t}{t^2+1} + C \Rightarrow I = \frac{x}{(\log x)^2 + 1} + C$$

154. **(d)** Let, $I = \int \frac{dx}{x^3+3x^2+2x}$

$$= \int \frac{dx}{x(x^2+3x+2)} = \int \frac{dx}{x(x+1)(x+2)}$$

By using the method of partial fraction,

$$\frac{1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2}$$

$1 = A(x+1)(x+2) + Bx(x+2) + Cx(x+1)$

Alt $x = 0 \Rightarrow A = \frac{1}{2}$

At $x = -1 \Rightarrow B = -1$

At $x = -2 \Rightarrow C = \frac{1}{2}$

Now, $I = \int \left(\frac{1}{2x} - \frac{1}{x+1} + \frac{1}{2(x+2)}\right) dx$

$$I = \frac{1}{2}\int \frac{1}{x} dx - \int \frac{1}{x+1} dx + \frac{1}{2}\int \frac{1}{x+2} dx$$

$$= \frac{1}{2}\log x - \log(x+1) + \frac{1}{2}\log(x+2) + C$$

$$= \frac{1}{2}\log\left|\frac{x(x+2)}{(x+1)^2}\right| + C \Rightarrow I = \frac{1}{2}\log\left|\frac{x^2+2x}{(x+1)^2}\right| + C.$$

155. **(b)** Given that,

$$I_n = \int \sec^n x dx$$

Put $n = 2$,

$I_2 = \int \sec^2 x dx = \tan x + c_1$ ......(i)

Put $n = 4$,

$$I_4 = \int \sec^4 x dx = \int \sec^2 x \cdot \sec^2 x dx$$

$$= \int (\tan^2 x + 1)\sec^2 x dx$$

$$I_4 = \frac{\tan^3 x}{3} + \tan x + c_2 \quad \ldots\ldots (ii)$$

From Eqs. (i) and (ii), we get

$$I_4 - \frac{2}{3}I_2 = \frac{\tan^3 x}{3} + \tan x + c_2 - \frac{2}{3}\tan x - \frac{2c_1}{3}$$
$$= \frac{1}{3}\tan^3 x + \frac{1}{3}\tan x + c \left[\text{where } c_2 - \frac{2}{3}c_1 = c\right]$$
$$= \frac{1}{3}\tan x(\tan^2 x + 1) + c$$
$$= \frac{1}{3}\tan x\sec^2 x + c.$$

156. **(d)** Given that,

$$\lim_{x\to\infty}\left[\frac{\sqrt{1} + 2\sqrt{2} + 3\sqrt{3} + \cdots + n\sqrt{n}}{n^{5/2}}\right]$$
$$= \lim_{x\to\infty}\left[\frac{1\sqrt{1} + 2\sqrt{2} + 3\sqrt{3}\cdots n\sqrt{n}}{n\sqrt{n}}\right]\frac{1}{n}$$
$$= \lim_{x\to\infty}\left[\frac{(1)^{3/2} + (2)^{3/2} + (3)^{3/2} + \cdots + (n)^{3/2}}{(n)^{3/2}}\right]\frac{1}{n}$$
$$= \lim_{x\to\infty}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{3/2}\cdot\frac{1}{n}$$
$$= \int_0^1 x^{3/2}dx = \left[\frac{x^{5/2}}{\frac{5}{2}}\right]_0^1 = \frac{2}{5}$$

157. **(d)** Let, $I = \int_0^{\alpha/3}\frac{f(x)}{f(x)+f\left(\frac{\alpha-3x}{3}\right)}dx$ ......(i)

We know, $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

$$I = \int_0^{\alpha/3}\frac{f\left(\frac{\alpha}{3}-x\right)}{f\left(\frac{\alpha}{3}-x\right) + f\left(\frac{\alpha - 3\left(\frac{\alpha}{3}-x\right)}{3}\right)}dx$$

$$\Rightarrow I = \int_0^{\alpha/3}\frac{f\left(\frac{\alpha-3x}{3}\right)}{f\left(\frac{\alpha-3x}{3}\right) + f(x)}dx \quad \ldots\ldots(ii)$$

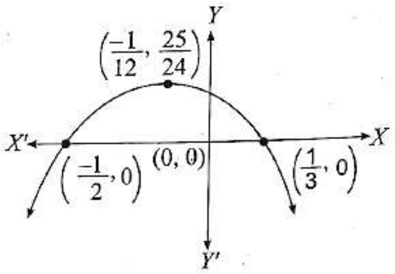
On adding Eqs. (i) and (ii), we get

$$I + I = \int_0^{\alpha/3}\frac{f(x)}{f(x) + f\left(\frac{\alpha-3x}{3}\right)}dx + \int_0^{\alpha/3}\frac{f\left(\frac{\alpha-3x}{3}\right)}{f\left(\frac{\alpha-3x}{3}\right) + f(x)}dx$$
$$2I = \int_0^{\alpha/3} 1dx$$
$$2I = [x]_0^{\alpha/3} \Rightarrow 2I = \frac{\alpha}{3}$$

$$I = \frac{\alpha}{6}$$

158. **(a)** Given curve,

$$y = 1 - x - 6x^2$$
$$y = -[6x^2 + x - 1]$$
$$y = -6\left(x + \frac{1}{12}\right)^2 + \frac{25}{24}$$
$$\left(x + \frac{1}{12}\right)^2 = -\frac{1}{6}\left(y - \frac{25}{24}\right)$$



Required area, $A = \int_{-1/2}^{1/3} y dx$

$$= \int_{-1/2}^{1/3} (1 - x - 6x^2) dx = \left[x - \frac{x^2}{2} - 2x^3\right]_{-1/2}^{1/3}$$

$$= \left(\frac{1}{3} - \frac{1}{18} - \frac{2}{27}\right) - \left(-\frac{1}{2} - \frac{1}{8} + \frac{1}{4}\right)$$

$$\Rightarrow A = \frac{11}{54} + \frac{3}{8} = \frac{44 + 81}{216} = \frac{125}{216} \text{ sq. unit}$$

159. **(c)** The equation of the family of parabolas with focus at the origin and X -axis as its axis is given by

$y^2 = 4a(x + a) = 4ax + 4a^2$ ……(i)

Differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = 4a$$

$$a = \frac{y}{2}\left(\frac{dy}{dx}\right) \quad \dots\dots (i)$$

From Eqs. (i) and (ii), we have

$$y^2 = 2xy\frac{dy}{dx} + y^2\left(\frac{dy}{dx}\right)^2$$

order $= m = 1$ and degree $= n = 2$

Hence, $mn - m + n = 1 \times 2 - 1 + 2 = 3$

160. **(d)** Given differential equation,

$$(1 + e^{x/y})dx + e^{x/y}\left(1 - \frac{x}{y}\right)dy = 0$$

$$(1 + e^{x/y})dx = -e^{x/y}\left(1 - \frac{x}{y}\right)dy$$

$\frac{dx}{dy} = \frac{-e^{x/y}\left(1 - \frac{x}{y}\right)}{(1 + e^{x/y})}$ ……..(i)

Let $\frac{x}{y} = v \Rightarrow x = vy$

Differentiating w.r.t. y, we get

$\frac{dx}{dy} = v\frac{d(y)}{dy} + y\frac{dv}{dy} \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$

Putting value of $\frac{dx}{dy}$ and $v = \frac{x}{y}$ in Eq. (i), we get

$$v + y\frac{dv}{dy} = \frac{-e^v(1 - v)}{1 + e^v} \Rightarrow y\frac{dv}{dy} = -\frac{e^v + ve^v}{1 + e^v} - v$$

$$y\frac{dv}{dy} = \frac{-e^v + ve^v - v - ve^v}{1 - e^v}$$

$$y\frac{dv}{dy} = \frac{-[v + e^v]}{1 + e^v} \text{ or, } \left[\frac{1 + e^v}{v + e^v}\right]dv = -\frac{dy}{y}$$

On integrating both the sides, we get

$\int \frac{1 + e^v}{v + e^v} dv = -\int \frac{dy}{y}$ ……(ii)

Put $v + e^v = t$

$(1 + e^i)dv = dt$

From eq. (ii),

$$\int \frac{dt}{t} = -\int \frac{dy}{y}$$

$\log t = -\log y + \log c$

$\log y(v + e^{v}) = \log c$

Put value of $v = \frac{x}{y} \Rightarrow \log y\left(\frac{x}{y} + e^{x/y}\right) = \log c$

$y\left(\frac{x}{y} + e^{x/y}\right) = c \Rightarrow x + ye^{x/y} = c$