Physics

1. **(c)** Suppose μ_0 is related with fundamental quantities as shown
$\mu_0 \propto e^a m^b c^c h^d \Rightarrow [\mu_0] = [e]^a[m]^b[c]^c[h]^d$
$[\mu_0] = [MLT^{-2}\ A^{-2}]$,
$[e] = [AT]$,
$[m] = [M]$,
$[c] = [LT^{-1}]$,
and $[h] = [ML^2\ T^{-1}]$
Using dimension of above physical quantities in Eq.
$[MLT^{-2}\ A^{-2}] = [AT]^a[M]^b[\ L\ T^{-1}]^c[ML^2\ T^{-1}]^d$
$[MLT\ ^{-2}\ A^{-2}] = [M]^{b+d}[\ L]^{c+2d}[\ T]^{a-c-d}[\ A]^a$
Comparing dimensions of M, L, T and A on the both sides, we get
$b + d = 1$
$c + 2d = 1$
$a - c - d = -2$
$a = -2$
After solving above equations we get
$a = -2, b = 0, c = -1, d = 1$
Putting values of a, b, c, d in eqn., we get
$$\mu_0 = \frac{h}{ce^2}$$

2. **(c)** Given, velocity, $v = 6t - 3t^2$
Velocity (v)
$$v = \frac{dx}{dt} \Rightarrow dx = vdt$$
Total displacement of the particle.
$$\int dx = \int vdt$$
$$\Rightarrow \Delta x = \int_0^2 vdt = \int_0^2 (6t - 3t^2)dt$$
$$= \left[\frac{6t^2}{2}\right]_0^2 - \left[\frac{3t^3}{3}\right]_0^2$$
$$= [3(2)^2 - 3(0)^2] - [(2)^3 - (0)^2] = 4\ m$$
Average velocity,
$$v_{avg} = \frac{\text{Total displacement}}{\text{Time interval}} = \frac{4}{2} = 2\ m/s$$

3. **(c)** Given,
Distance between gun and target $= 600$ m
Velocity of bullet $= 500\ ms^{-1}$
$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{600}{500} = 1.2\ s$$
Vertical displacement equals height.
$$h = 0 \times (1.2) + \frac{1}{2} \times (-10) \times (1.2)^2$$
$$h = \frac{1}{2} \times (-10)(1.2 \times 1.2), h = -7.2\ m$$

4. **(c)** Given, angle of projection, $\theta = 60°$ and velocity, $u = 140\ ms^{-1}$ = initial velocity.
Horizontal component, $u_x = u\cos 60° = 140\cos 60°$ and Vertical component, $u_y = u\sin 60° = 140\sin 60°$
Suppose after time t, the inclination of particle with horizontal be 45° and at time t velocity along $x = v_x$ and along $y = v_y$.

$\tan 45^\circ = \frac{v_y}{v_x} = 1 \Rightarrow v_x = v_y$

The horizontal component of velocity does not change $\Rightarrow u\cos 60^\circ = 140\cos 60^\circ = v\cos 45^\circ = v_y = v_x$

Using 1st eqn. of motion in vertical direction

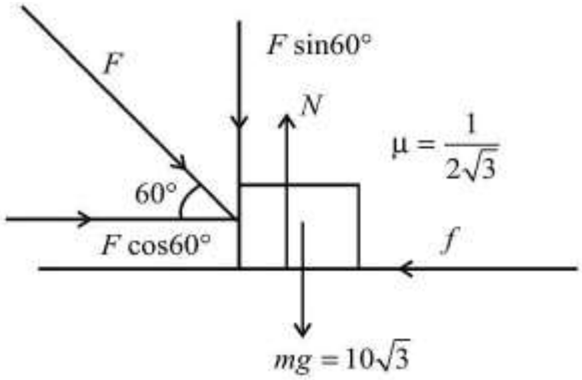
$v_y = u_y - gt$

$140\cos 60^\circ = 140\sin 60^\circ - 10t\ [\because v_y = v_x]$

$\Rightarrow 140 \times \frac{1}{2} = 140 \times \frac{\sqrt{3}}{2} - 10t$

$\Rightarrow 7 \times (0.7320) = 5.124$ s.

5. **(a)** Free body diagram



$N = F\sin 60^\circ + mg$

Force of friction, $f = \mu N$ [Limiting]

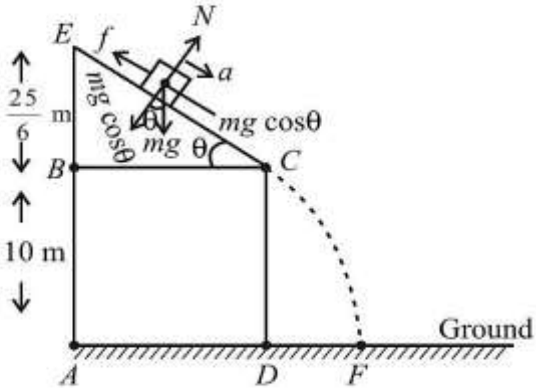
$f = \mu(mg + F\sin 60^\circ)$

$F\cos 60^\circ = \frac{1}{2\sqrt{3}}(10\sqrt{3} + F\sin 60^\circ)$

$\Rightarrow F \times \frac{1}{2} = \frac{1}{2\sqrt{3}}\left(10\sqrt{3} + F \times \frac{\sqrt{3}}{2}\right)$

$\Rightarrow F = 20$ N.

6. **(d)** According to question, a small block in slide down from top E of inclined plane as shown in figure,



From newton's 2^{nd} law

$\Rightarrow mg\sin\theta - f = ma$ .... (i)

Due to slipping, $f = \mu_k N$

or $f = \mu_k(mg\cos\theta)$ (From figure)

Putting value of f in eq^n. (1)

$\Rightarrow mg\sin\theta - \mu_k mg\cos\theta = ma$

$\Rightarrow a = 10\sin\theta - 1/8 \times 10\cos\theta$

$\because$ Given, angle of the inclined plane, $\theta = \sin^{-1}(0.6)$

or $\sin\theta = 0.6$ and $\mu_k = \frac{1}{8}$

We know $\cos\theta = \sqrt{1 - \sin^2\theta} = \sqrt{1 - (0.6)^2}$

$\cos\theta = 0.8$

Putting values in eqn. (2)

$a = 10(0.6) - \frac{1}{8} \times 10(0.8)$ or $a = 5\ ms^{-2}$

Let v be velocity at point C then from third equation of the motion

$v^2 = u^2 + 2as$

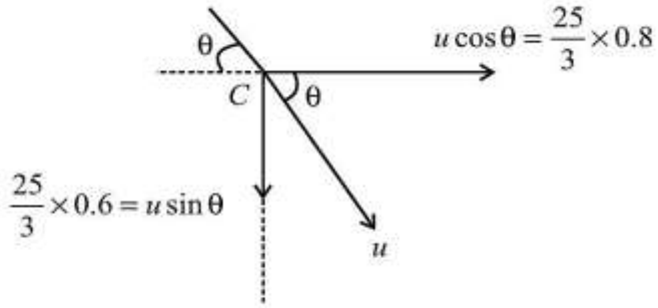
or $v = \sqrt{2as}$ $[u = 0]$

From $\triangle$ CBE, $CE = \frac{EB}{\sin\theta} \Rightarrow s = \frac{25/6}{0.6} = \frac{25}{6\times 0.6}$

$s = \frac{125}{18}$ m = CE

$\Rightarrow V = \sqrt{2 \times 5 \times \frac{125}{18}} = \frac{25}{3}\ ms^{-1}$

Resolving velocity at point C



Using equation of motion in vertical direction

$\Rightarrow \Delta y = h = ut + \frac{1}{2}gt^2$

$\Rightarrow 10 = u\sin\theta t + \frac{1}{2}gt^2$

$\Rightarrow 10 = \frac{25}{3} \times 0.6t + \frac{1}{2} \times 10 \times t^2$ [as $\sin\theta = 0.6$]

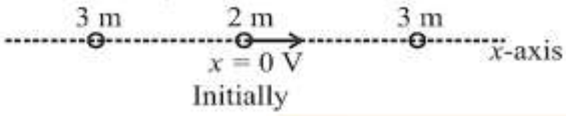
$\Rightarrow \frac{25}{3} \times \frac{6}{10}t + \frac{1}{2} \times 10 \times t^2 = 10$

$\Rightarrow 5t + 5t^2 = 10 \Rightarrow t^2 + t - 2 = 0$ or $t = 1sec$

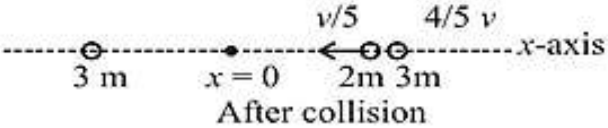
Now, again from second Eqs. of motion in x-direction,

$\Rightarrow DF = v\cos\theta \cdot t + 0$ or $DF = \frac{25}{3} \times 0.8 = \frac{20}{3}$ m

7. (a) According to question, three particles are situated as shown below,



After Ist collision velocity of balls will be as shown. 2 m will slow down to $\frac{4}{5}$v will collide with 3 m on left. After 2nd collision 2 m will further slow down and will not collide again.



So, collision of R and Q

Total no. of collision = 2

8. (c) Given,

Depth of a well, d = 30 m

Water quantity per minute = 1800 litre

Cross-section area of pipe, $A = 30\ cm^2$

$= 30 \times 10^{-4}\ m^2$

Area × Velocity = Volume of water per second

$\Rightarrow$ velocity of water a jet $= \frac{1800\times10^{-3}}{(30\times10^{-4})\times60} = 10\ \text{m/s}$

and work done by engine

= Change in P.E. + Change in K.E.

$= mgd + \frac{1}{2}mv^2$

$= 1800 \times 10 \times 30 + \frac{1}{2} \times 1800 \times (10)^2$

$= 630000\ \text{J}$

Power of engine, $P = \frac{W}{t} = \frac{630000}{60}$

$= 10500\ \text{W} = 10.5\ \text{kW}$

9. (a) Given, mass of solid sphere or disc M = 100 kg

Radius of solid sphere, R = 10 m

Radius of circular disc, r = 20 m

and time = 1 hour = 60 minute = 60 × 60sec

Moment of inertia of the solid sphere,

$I_S = \frac{2}{5}MR^2 = \frac{2}{5} \times 100 \times (10)^2 = 4000\ \text{kg} - \text{m}^2$

Similarly,

Moment of inertia of the disc,

$I_c = \frac{1}{2}Mr^2$

$= \frac{1}{2} \times 100 \times (20)^2 = 20{,}000\ \text{kg} - \text{m}^2$

Rate of change of moment of inertia

$= \frac{I_c - I_S}{t} = \frac{20000 - 4000}{60 \times 60} = \frac{16000}{60 \times 60} = \frac{160}{36}$

$= \frac{40}{9}\ \text{kg} - \text{m}^2\ \text{s}^{-1}$

10. (d) Given,

Mass (plate) = m

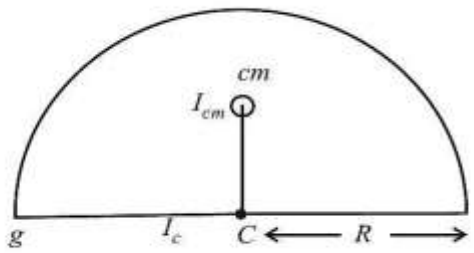
Radius (plate) = R

Using parallel axis theorem,

$I_c = I_{cm} + mx^2$

$\frac{mR^2}{2} = I_{cm} + mx^2$

Using $I_c = \frac{mR^2}{2}$



$\Rightarrow\ I_{cm} = \frac{mR^2}{2} - mx^2$

11. (b) Loss in potential energy = gain in KE

Loss in PE $= \frac{Gm_1m_2}{r^2} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$ ….. (i)

If v_1 and v_2 are their respective velocities when they are a distance r apart then, from law of conservation of momentum.

$m_1v_1 = m_2v_2$

From Eq^n. (i) and Eq^n. (ii),

$$v_1=\sqrt{\frac{2Gm_2^2}{(m_1+m_2)r}}, v_2=\sqrt{\frac{2Gm_1^2}{(m_1+m_2)r}}$$

Therefore, their relative velocity of approach is

$$v_1+v_2=\sqrt{\frac{2Gm_2}{r}}+\sqrt{\frac{2Gm_1}{r}}=\sqrt{\frac{2G}{r}(m_1+m_2)}$$

$$=\left(\frac{2G(m_1\times m_2)}{r}\right)^{\frac{1}{2}}$$

12. (b) As there is no external torque on planet, angular momentum (J) of a planet is constant.

$\Rightarrow mu_Ar_A\sin\theta_A=mu_Br_B\sin\theta_B$

$r_A=90\times10^6$ km, $r_B=60\times10^6$ km

$\theta_A=30°, \theta_B=60°$

Putting values of given quantities we get $\frac{u_A}{u_B}=\frac{2}{\sqrt{3}}$

13. (c) Volume of solid copper cube $=l^3=7^3\times10^{-6}$ m

Increase in pressure, $p=8000\text{kPa}=8000\times10^3$ Pa

Bulk modulus of copper, $\beta=140\text{Gpa}=140\times10^9$ Pa

Bulk modulus $B=\frac{p}{\left(\frac{\Delta V}{V}\right)}$

$$\Rightarrow\Delta V=\frac{pV}{B}=\frac{8000\times10^3\times(l)^3}{140\times10^9}=19.6\times10^{-3}\text{ cm}^3$$

14. (a) Given, diameter of a pinhole, $d=0.2$ mm

So, radius of pinhole,

$$r=\frac{0.2}{2}=0.1\text{ mm}=0.1\times10^{-3}\text{ m}$$

Pressure due to depth must be equal to excess pressure

$$\Rightarrow h\rho g=\frac{2S}{r}\Rightarrow h=\frac{2T}{\rho gr}=\frac{2\times0.007}{10^3\times10\times0.1\times10^{-3}}$$

$=0.14\text{ m}=14$ cm.

15. (d) Given, focal length of spherical mirror, $f=150$ cm coefficient of linear expansion of steel,

$\alpha=12\times10^{-60}C^{-1}$

As temperature changes by dt Radius R changes by dR we can define coefficient of linear exapnsion,

$$\alpha=\frac{dR/R}{dt}\Rightarrow\frac{dR}{R}=\alpha T$$

As $\frac{dR}{R}=\frac{df}{f}$ [As $R=2f$]

$$\Rightarrow\frac{df}{f}=\alpha dT\Rightarrow df=f\alpha dT$$

$f'-f=f\alpha\Delta T$

where, $f=$ focal length of the mirror at temperature t and

$f'=$ final focal length of mirror at $t+dt$

$f'=f+f\alpha dT$

$\Rightarrow f'=f(1+\alpha dT)$

$\Rightarrow f'=f(1+\alpha\Delta T)$ (for large change in temperature)

$\Rightarrow f'=150(1+12\times10^{-6}\times200)$[As $\Delta T=200$]

$f'=150.36$ cm.

16. (d) Given,

Length of rod, $l=10\text{ cm}=0.1$ m

Area of cross-section of rod, $A=2.8\times10^{-4}\text{ m}^2$

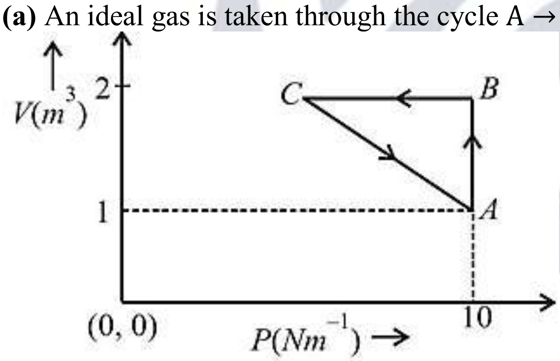
Temperature difference $=\Delta T=80-0=80°C$

Quantity of melted ice, $m=20$gm

Total time, $t = 5 \text{ min} = 300\text{sec}.$
and latent heat of ice, $s = 80\text{calg}^{-1}$
Heat flow per second $= \frac{mL\times4.184}{t}$ J/s
$= \frac{20\times80\times4.184}{300} = 22.314$ J/s (i)
Heat current, $\frac{\Delta Q}{\Delta t} = \frac{kA\cdot\Delta T}{l}$
$= \frac{k(2.8\times10^{-4})\times80}{0.1}$ .... (ii)
Equating (1) and (2),
$\Rightarrow 22314 = \frac{k(2.8\times10^{-4})\times80}{0.1_1{}^{-1}\text{ K}^{-1}}$
$\therefore$ $k = 99.61 \approx 100\text{Js}^{-1}\text{ m}^{-1}\text{ K}^{-1}$

17. (d) Given, $V = kT^{2/3}$
Work done in changing volume by
$dV = dW = PdV = \frac{RT}{V}dV\left[\text{As } P = \frac{RT}{V}\right]$
$= \frac{RT}{kT^{2/3}}dV\left[\text{Put } V = kT^{2/3}\right]$
Taking derivative of $V = kT^{2/3}$ on the both sides, we get
$dV = K\frac{2}{3}T^{-1/3}dT$
Putting value of dV in expression for dW
$dW = \frac{2}{3}RdT$
$\Rightarrow W\int dW = \frac{2}{3}R\int_{T_1}^{T_2} dT = \frac{2}{3}R(T_2 - T_1)$
$= \frac{2}{3}R(60-0) = \frac{2}{3}\times R\times60 = 40R$

18. (a) An ideal gas is taken through the cycle $A \to B \to C \to A$ as shown in figure,



$W_{AB} = P\times\Delta V = 10\times(2-1) = 10$ J
$W_{BC} = PdV. W_{BC} = P\times(0) = 0$ J as $dV = 0$
Applying first law of thermodynamics
$Q = \Delta U + W$
Here, $\Delta U = 0$ as cyclic process
$Q = W$
$\Rightarrow 5 = W_{AB} + W_{BC} + W_{CA}$
$\Rightarrow 5 = 10 + 0 + W_{CA}$
$|W_{CA}| = 5$ J.

19. (d) Given, Energy $0.69\text{eV} = 0.69\times1.6\times10^{-19}$ V
Also average translational kinetic energy $= \frac{3}{2}kT$
$= \frac{3}{2}\times1.38\times10^{-23}T = 0.69\times1.6\times10^{-19}$
$\Rightarrow T = 5333\text{ K} = (5333-273)°\text{C} = 5060°\text{C}.$

20. (c) Let the distance of epicenter of earthquake from point of observation be d.

speed of S-wave, $v_S = 4.5\ km^{-1}\ S$

speed of P-wave, $v_P = 8\ km^{-1}\ S$

then, $d = V_P t_P = V_S t_S$

or, $8t_P = 4.5t_S$

$$t_P = \frac{4.5}{8} t_S$$

The first P-wave arrives 3.5 min earlier than the first S-wave.

Hence, $t_S - t_P = 3.5 \times 60$

$t_S - t_P = 210$ ..... (ii)

From Eq. (1), we get

$$t_S - \frac{4.5}{8} T_s = 210$$

$$\Rightarrow \frac{8t_S - 4.5t_S}{8} = 210$$

$$\Rightarrow 3.5t_S = 210 \times 8$$

$$\Rightarrow t_S = \frac{210 \times 8}{3.5} = 480\ s$$

Now, $d = v_S t_S = 4.5 \times 480 = 2160$ km.

21. (a) Apparent frequency is given by,

$$v' = \left(\frac{v + v_0}{v - v_s}\right) v_0$$

Put $v_s = 0 \Rightarrow v' = \left(\frac{v+v_0}{v}\right) v_0$

Putting $v_0 = \frac{v}{5}$

$$v' = \left(\frac{v + \frac{v}{5}}{v}\right) v_0 = 1.2v_0$$

Apparent wavelength does not change by speed of observer.

22. (b) Given,

Focal length of convex lens, f = 20 cm

Object distance, $u = -0.1\ m = -10\ cm$

Radius of curvature of silvered surface, R = 22 cm

Focal length of the lens is 20 cm .

So, the power of lens, $P_1 = \frac{1}{20} D$

and focal length of concave mirror,

$f = \frac{R}{2} = -11$ cm [As R = 2f] .... (i)

Therefore, the power of the mirror,

$P_2 = -\frac{1}{f_m} \Rightarrow P_m = \frac{1}{11} D$ .... (ii)

Total power P = Power of lens + Power of mirror

Light first enters lens then reflected back by mirror finally it passes through lens again be for forming image.

$P = P_1 + P_2 + P_1$ ....(iii)

From Eqs. (i), (ii) and (iii), we get

$$P = \frac{1}{20} + \frac{1}{11} + \frac{1}{20}$$

$$\Rightarrow P = \frac{21}{110}$$

The focal length of equivalent mirror for whole image formation,

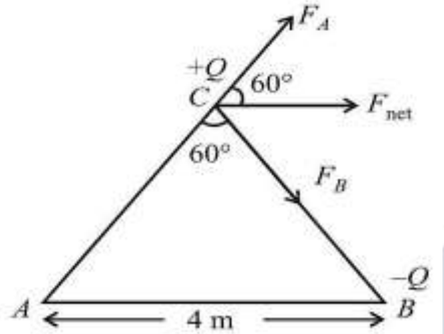
$$f = -\frac{110}{21}\ cm$$

Using mirror formula,

$\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \Rightarrow \frac{-21}{110}=\frac{1}{v}-\frac{1}{10}$
$\Rightarrow v=-11 \text{ cm}$
Hence, the distance of final image from the silvered surface is 11 cm .

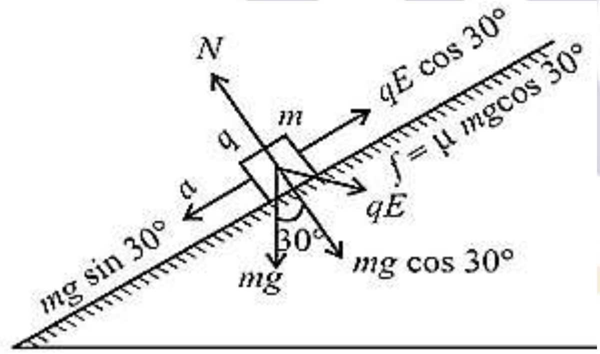
23. (c) Condition for maxima is, $d\sin\theta=n\lambda$
$\Rightarrow 2\lambda\sin\theta=n\lambda$ [As $d=2\lambda$]
$\Rightarrow 2\sin\theta=n$
Maximum value of n is when $\sin\theta=1$
$n_{max}=2$
Possible values of n are $-2,-1,0,1,2$. So, 5 maxima.

24. (a) $Q=100\mu C$
F_A is force on charge at C due to A and F_B is force on charge at C dec. to B.



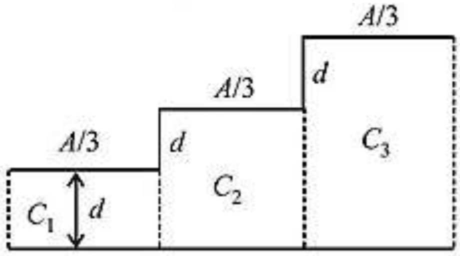
$|F_A|=|F_B|=\frac{KQ^2}{r^2}$
$|F_{net}|=|F_A|=|F_B|$
As angle between F_A and F_B is 120°.
Putting the given values in F_{net}, we get
$=\frac{9\times10^9\times(100\times10^{-6})^2}{(4)^2}$
$F_{net}=5.625 \text{ N}, 60^\circ$

25. (b) Electric field $=100 \text{v/m}$



Writing forces along inclined plane and $F=ma$ From fig.
$mg\sin30^\circ-\mu mg\cos30^\circ-qE\cos30^\circ=ma$
Given,
$\mu=0.2, m=1 \text{ kg}, q=0.01\text{C}$ and $h=1$ m and $E=100$ V/m
Putting these values, we get in above eq ^{n}
$10\times\frac{1}{2}-0.2\times10\times\frac{\sqrt{3}}{2}-0.01\times100\times\frac{\sqrt{3}}{2}=a$
$\Rightarrow a=2.3 \text{ m/s}^2$

26. (d) 3 capacitors have been arranged like this with capacitances $C_1, C_2: C_3$ respectively.



For parallel plate capacitors capacitance $= \frac{\varepsilon_0 A}{d}$

Capacitance of capacitor 1, $C_1 = \frac{\varepsilon_0 A}{3d}$

Capacitance of capacitor 2, $C_2 = \frac{\varepsilon_0 A}{3(2d)} = \frac{\varepsilon_0 A}{6d}$

Capacitance of capacitor 3, $C_3 = \frac{\varepsilon_0 A}{3(3d)} = \frac{\varepsilon_0 A}{9d}$

Net cpacitance $C_{net} = C_1 + C_2 + C_3$

$$C_{eq} = \frac{\varepsilon_0 A}{3d} + \frac{\varepsilon_0 A}{6d} + \frac{\varepsilon_0 A}{9d}$$

$$\Rightarrow C_{eq} = \frac{(6\varepsilon_0 + 3\varepsilon_0 + 2\varepsilon_0)A}{18d} = \frac{11\varepsilon_0 A}{18d}$$

Hence, the capacitance of the arrangement is $\frac{11\varepsilon_0 A}{18d}$.

27. (d) ∵ Pressure difference due to surface tension

$$p_i - p_0 = \frac{4S}{r}$$

Electrostatic pressure due to change on the soap bubble,

$$= \frac{\sigma^2}{2\varepsilon_0}$$

Here, σ = surface charge density and ε_0 = permittivity of the free space

Electric potential on spherical shell, $V = \frac{kq}{r}$

$\Rightarrow V = \frac{k(\sigma A)}{r}$ [as $q = \sigma \cdot A$]

Putting, $A = 4\pi r^2$ and $k = \frac{1}{4\pi\varepsilon_0}$

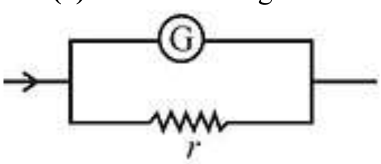
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{\sigma 4\pi r^2}{r}\right) = \frac{\sigma r}{\varepsilon_0} \Rightarrow \sigma = \frac{\varepsilon_0 V}{r}$$

Pressure difference due to surface tension must cancel pressure difference due to charge.

$$\Rightarrow \frac{4S}{r} = \frac{\sigma^2}{2\varepsilon_0} \Rightarrow \frac{4S}{r} = \frac{\left(\frac{\varepsilon_0 V}{r}\right)^2}{2\varepsilon_0} \Rightarrow V = \sqrt{\frac{8Sr}{\varepsilon_0}}$$

So, the potential of the bubble is $\frac{\sqrt{8Sr}}{\varepsilon_0}$.

28. (a) Resistance of galvanometer, $G = 112\Omega$



$$\frac{\text{Heat through shunt}}{\text{Heat through galvanometer}} = \frac{7}{5}$$

$\Rightarrow \frac{\frac{V_2}{r}}{\frac{V^2}{G}} = \frac{7}{5}$ Putting $G = 112\Omega$

$\Rightarrow r = \frac{5}{7} \times 112 = 80\Omega$

$\Rightarrow$ Shunt resistance $= 80\Omega$.

29. (c) Given, $V_{rms} = 220$ V, supply frequency, f = 50 Hz rms current, $I_{rms} = 440$ mA

Capacitance reactance, $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

$X_C = \frac{\pi\times10^6}{2\pi\times50\times50}$ As $C = \frac{50}{\pi}\mu F]$

$\Rightarrow X_C = \dfrac{10^6}{2500 \times 2} \Rightarrow X_C = 200\Omega$

Inductive reactance, $X_L = \omega L$

$X_L = 2\pi f \times L$ (As $\omega = 2\pi f$)

$= 2\pi \times 50 \times \dfrac{6}{\pi} = 600\Omega \left[\text{As } L = \dfrac{6}{\pi}H\right]$

$\therefore X_L = 600\Omega$

Now, impedance of LCR circuit,

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

Putting the given values, we get

$500 = \sqrt{R^2 + (600 - 200)^2}$

Squaring both sides

$250000 = R^2 + (400)^2$

$R^2 = 250000 - 160000R^2 = 90000$

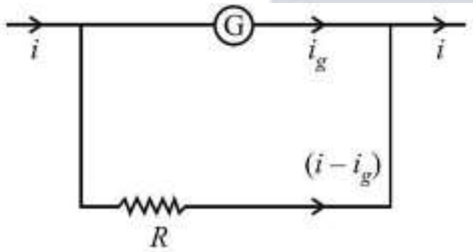
$\Rightarrow R = 300\Omega$

Impedance, $Z = \frac{V_{rms}}{I_{rms}} = \frac{220}{440} \times 10^3 \Rightarrow Z = 500\Omega.$

30. (d) Neutron is neutral so it cannot move in circular path in magnetic field. Only charged particle can do so.

31. (d) According to the question,

Case **a**



$v_G = v_{R_1}$

$\Rightarrow i_G G = (i - i_G)R_1$

$\Rightarrow \dfrac{i}{51}G = \left(i - \dfrac{i}{51}\right)R_1$

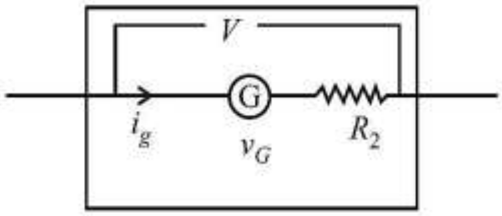
$\Rightarrow R_1 = \dfrac{G}{50}$

$\Rightarrow v_G = \dfrac{V}{11}$

$\Rightarrow v_{R_2} = \dfrac{10}{11}V$

$\Rightarrow \dfrac{v_G}{v_{R_2}} = \dfrac{1}{10}$

Case **b**



$v_G = i_g G \Rightarrow v_{R_2} = i_g R_2$

$\Rightarrow \dfrac{v_G}{v_{R_2}} = \dfrac{i_g G}{i_g R_2} = \dfrac{G}{R_2} \Rightarrow \dfrac{1}{10} = \dfrac{G}{R_2}$

$\Rightarrow R_2 = 10G$

Now, from Eqn. (i) and (ii), we get

$$\frac{R_2}{R_1} = \frac{10G}{\frac{G}{50}} = 500 \therefore \frac{R_2}{R_1} = 500$$

32. (b) Magnetic field at C = magnetic field at C due to straight wire + Magnetic field due to semi circular ring

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4r} = \frac{\mu_0 i}{4\pi r}(1+\pi)$$

33. (b) Given,

Number of turns in the coil, $N = 30$ turns

Area of the coil, $A = 2.5\ cm^2 = 2.5 \times 10^{-4}\ m^2$

Total charge flowing through the coil,

$Q_{Net} = 7.5 \times 10^{-3} C$

and total resistance of wire and galvanometer,

$R = 0.3\Omega$

We know that,

Charge flown $= \frac{\text{Charge in flux}}{\text{Resistance}}$

$$Q_{net} = \frac{\Delta\phi}{R}$$

Magnetic flux, $\phi = NBA$

Charge flown $= \frac{BNA}{R}$ [As $\Delta\phi = NBA$]

Putting the given values, we get

$$7.5 \times 10^{-3} = \frac{B \times 30 \times (2.5 \times 10^{-4})}{0.3}$$

$$7.5 \times 10^{-3} = \frac{B \times 7.5 \times 10^{-3}}{0.3}$$

$B = 0.3$ T

Hence, the magnitude of the magnetic field, $B = 0.3$ T.

34. (d) Maximum energy stored on capacitor $= \frac{1}{2}\frac{Q^2}{C}$

When energy is distributed equally between Electric field and magnetic field, energy in capacitor $= \frac{1}{2} \times \frac{Q^2}{2C}$

$\Rightarrow \frac{1}{2} \times \frac{1}{2}\frac{Q^2}{C} = \frac{q^2}{2C}$ [q is charge at that time]

$\Rightarrow q^2 = \frac{2Q^2}{4} \Rightarrow q = \frac{Q}{\sqrt{2}}$.

35. (c) Given,

Amplitude of the electric field, $E_M = 4Vm^{-1}$

Energy density (energy/volume) is given by

$= \frac{1}{2}\varepsilon_0 E_m^2$ where, $\varepsilon_0 = 8.8 \times 10^{-2} C^2\ N^{-1}\ m^{-2}$)

Putting the given values, we get energy density

$$= \frac{1}{2} \times 8.8 \times 10^{-12} \times (4)^2\ J/m^3$$

$\therefore\ \mu = 70.4 \times 10^{-12} Jm^{-3}$.

36. (b) Say threshold wavelength of emitter plate $= \lambda$

Energy of photon in first case is E.

CASE; I When switch S_1 is closed and switch S_2 is open,

So, $E = h\nu_0 + (5+1)eV \quad \ldots. (1)$

Case II : When switch S_1 is open and switch S_2 is closed and frequency of incident light is doubled.

Then, $2E = hc/\lambda + (20-5)eV \quad (2)$

From Eqs. (1) and (2), we get

$$\Rightarrow 2\left(\frac{hc}{\lambda} + 6eV\right) = \frac{hc}{\lambda} + 15eV$$

$$\Rightarrow 2\frac{hc}{\lambda} + 12eV = \frac{hc}{\lambda} + 15eV$$

$\Rightarrow \frac{hc}{\lambda} = 3eV$

$\Rightarrow \frac{c}{\lambda} = \frac{3 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} = 7.25 \times 10^{14}$ Hz

$\Rightarrow \lambda = \frac{3 \times 10^8}{7.25 \times 10^{14}} = 4133.3Å$

Hence, the threshold wavelength of the emitter plate is 4133.3Å.

37. (c) Cocilombatration = Centripetal force

$\frac{1}{4\pi\varepsilon_0}\frac{2qq}{r^2} = mr\omega^2 \quad \ldots$ (i)

From quantization,

$mvr = \frac{nh}{2\pi} \Rightarrow mr^2\omega = \frac{nh}{2\pi}$ ...(ii) [as $v = \omega r$]

Solving Eqn. (i) and (ii),

$\omega = \frac{2\pi mq^4}{\varepsilon_0^2 h^3}$

Hence, the orbital angular velocity of the particle A is $\frac{2\pi mq^4}{\varepsilon_0^2 h^3}$. (Take $n = 1$ for nearest orbit)

38. (b) Given,

Nuclear reactor the activity of a radioactive substance,

$\frac{dN}{dt} = 2000/s$

and mean-life of the products = 50 min.

$= 50 \times 60sec = 3000$ s

Also, mean life $= \frac{1}{\lambda} \Rightarrow 3000 = \frac{1}{\lambda}$

$\Rightarrow \lambda = \frac{1}{3000}\ s^{-1}$

$\therefore$ The rate of decay, $\frac{dN}{dt} = \lambda N$

$\Rightarrow 2000 = \frac{1}{3000} \times N$

$\Rightarrow N = 60 \times 10^5$

39. (d) p-type semiconductor donor energy level,

$E = 50meV = 50 \times 10^{-3}eV$

According to the Planck's quantum theory,

$\therefore E = h\nu$ or $E = \frac{hc}{\lambda}$

$\Rightarrow \lambda = \frac{1242\text{ nm}}{E(eV)}$

$\Rightarrow \lambda = \frac{1242\text{ nm}}{50 \times 10^{-3}}$

$\Rightarrow \lambda = 24.8\mu$ m.

So, the maximum wavelength of light photon required is 24.8μ m.

40. (a) Given, frequency of signal, $f_S = 10kHz$

Carrier frequency, $f_c = 1mHz$

Side bank frequency are given by,

$f_c \pm f_S = 1 \times 10^6 \pm 10 \times 10^3$ Hz

$= (1000 \pm 10)kHz$

$\therefore$ 1010kHz and 990kHz.

Chemistry

41. (b) $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{m \times \lambda}$

$\Rightarrow v = \frac{6.626 \times 10^{-34} Js}{9.1 \times 10^{-31}\text{ kg} \times 182 \times 10^{-9}\text{ m}}$

$= 4 \times 10^3$ m/s

$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (4 \times 10^3)^2$

$= 7.28 \times 10^{-24}$ J.

42. (c) Given,
Orbit radius = 52.9pm
Ground state energy of electron in hydrogen atom
$= -2.18 \times 10^{-18}$ J

$r_n = r_0 \times \frac{n^2}{Z}$

$52.9\text{pm} = 52.9\text{pm} \times \frac{n^2}{Z} \Rightarrow Z = n^2$

Now, $E_n = E_0 \times \frac{Z^2}{n^2}$

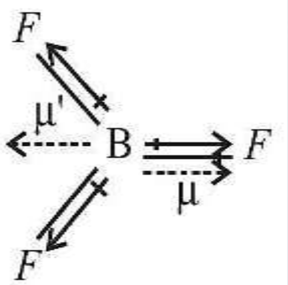
$\Rightarrow E_n = -2.18 \times 10^{-18} \times \frac{n^4}{n^2}$

$\Rightarrow E_n = -2.18 \times 10^{-18} \times n^2$

When $n = 1, E_1 = -2.18 \times 10^{-18}$ J
Which does not match with any option.
When $n = 2, E_2 = 4 \times (-2.18 \times 10^{-18})$
$= -8.72 \times 10^{-18}$ J

43. (a) On moving down, the group electron gain enthalpy decreases. Due to e^-, e^-repulsions in relatively small 2p-subshell of O atom, its e^-gain enthalpy is lowest.

44. (b) (I) In BF_3, electronegativity of F -atom is greater than B.



(II) Covalent bond length increases as
$C - O > N - O > O - H$
Due to increase in size of atom.

(III) Bond order $= \frac{N_b - N_a}{2}$

$C_2 = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^2 \pi 2p_y^2$

B.O. $= \frac{8-4}{2} = 2$

$B_2 = \sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \pi 2p_x^1 \pi 2p_y^1$

B.O. $= \frac{6-4}{2} = 1$

$He_2 = \sigma 1s^2 < \sigma * 1s^2$

B.O. $= \frac{2-2}{2} = 0$

Hence, Bond order:
$C_2 > B_2 > He_2$.

45. (d)

46. (c) $KE = \frac{3RT}{2}$ and $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{rms} = \sqrt{\frac{2KE}{M}}$

$= \sqrt{\dfrac{2 \times 4 \times 10^3 \text{ J}}{32 \times 10^{-3} \text{ kg mol}^{-1}}}$

$= 5 \times 10^2$ m/s $= 5 \times 10^4$ cm/s.

47. (d) $2\overset{+7}{K}MnO_4 + 10I^- \longrightarrow 5\overset{0}{I_2} + 2Mn^{+2}$

n-factor of $KMnO_4 = 7 - 2 = 5$

I^-convert into I_2 then, n-factor $= 1 - 0 = 1$

Number of equivalents of $KMnO_4$

= Number of equivalents of I^-

n-factor × moles of $KMnO_4$ = n-factor × moles of I^-

$\Rightarrow V \times 0.02 \times 5 = 60 \times 0.01 \times 1$

$\Rightarrow V = 6$ mL.

48. (a) In bomb calorimeter,

$\Delta H = \Delta U = q$

Given, $\Delta H = -248$ kJ/mol

For, 12 g of graphite energy released = 248 kJ/mol.

For, 6 g of graphite energy released $= \frac{248 \times 6}{12}$

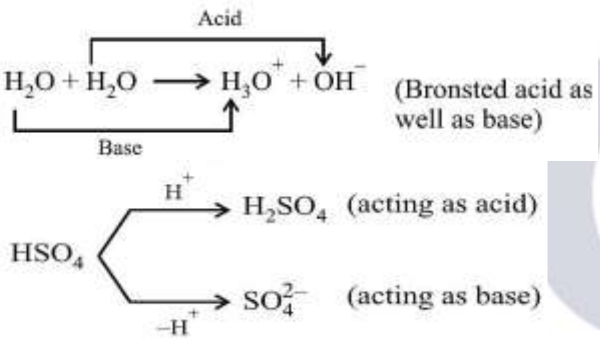
From, $q = C_v \Delta T$

$\Rightarrow \dfrac{248 \times 6}{12} = C_v \times (31 - 25)$

$\Rightarrow C_v = \dfrac{248}{2 \times 6} = 20.6$ kJ/K

49. (b) Solubility of AgCl in 0.1 M KCl solution will be minimum because of common ion effect of Cl^-.

50. (d)



51. (c) Bond lengths of H_2 and D_2 are same because of same size.

52. (c) (II) On moving down in the group the size of the atom increases. The bigger atom can stabilise more the bigger superoxide ion effectively.

(III) Lattice energy of LiF is more than the hydration energy. Hence, LiF has very low solubility.

53. (a) Gallium (Ga) exist in liquid phase at room temperature due to presence of Ga_2 dimers in solid phase.

54. (a) $HCOOH \xrightarrow{\text{Conc. } H_2SO_4} \underset{(B)}{CO} + \underset{(A)}{H_2O}$

$\underset{(B)}{3CO} + Fe_2O_3 \longrightarrow \underset{(D)}{2Fe} + \underset{(C)}{3CO_2}$

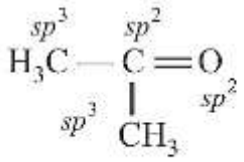
$CaCO_3 + 2HCl \text{ (dil.)} \longrightarrow \underset{(C)}{CO_2} + H_2O + CaCl_2$

55. (c)

56. (b) (a) C_6H_6 All six C -atoms show sp^2-hybridisation, i.e., 3-hybrid orbitals by each C -atom.

Total number of hybrid orbitals $= 6 \times 3 = 18$

(b) $(CH_3)_4C$ All five C -atoms show sp^3 - hybridisation, i.e., 4-orbitals by each C -atom. Total number of hybrid orbitals $= 5 \times 4 = 20$.

(c) 

Thus, total number of hybrid orbitals
$= 4 + 4 + 3 + 3 = 14.$

(d) $\underset{sp^3}{\overset{4}{CH_3}} - \underset{sp^3}{\overset{4}{CH_3}} = \underset{sp^2}{\overset{2}{C}}H - \underset{sp}{C} \equiv \underset{sp}{N}$

- C-1 show sp-hybridisation (i.e. 2-hybrid-orbitals)
- C-2 and 3 show sp^2-hybridisation (i.e. 3-hybrid orbital by each C -atom)
- C-4 show sp^3-hybridisation (i.e. 4-hybrid orbitals)
- N-atom show sp-hybridisation (i.e. 2-hybrid orbitals) Thus, total number of hybrid orbitals

$= 2 + 2 + 3 + 3 + 4 = 14.$

57. (a) $\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

$\Rightarrow \frac{725 \times 50}{300} = \frac{760 \times V_2}{273}$

$\Rightarrow V_2 = \frac{725 \times 50 \times 273}{300 \times 760} = 43.4 \text{ mL}$

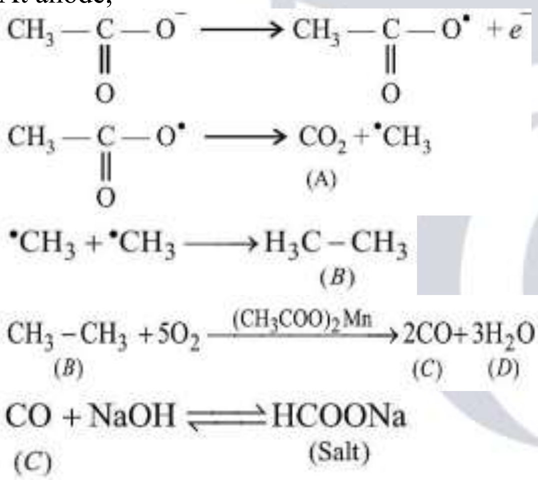
$\Rightarrow 22400 \text{ mL of } N_2 \equiv 28 \text{ g of } N_2$ at STP

$\Rightarrow 43.4 \text{ mL of } N_2 \equiv \frac{28 \times 43.4}{22400} = 0.054 \text{ g of } N_2$

% mass of $N_2 = \frac{0.054 \text{ g}}{1 \text{ g}} \times 100 = 5.4\%.$

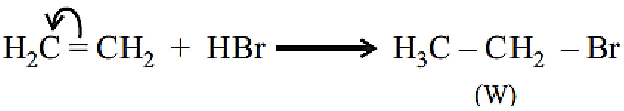
58. (b) Kolbe electrolysis of $CH_3COO^-Na^+$.

At anode,



Hence, A is CO_2 and D is H_2O.

59. (d)



$$\xrightarrow{\text{(i) Mg/dry ether}} H_3C-CH_2-MgBr$$

$$\xrightarrow{\text{(ii) } O=C=O} H_3C-CH_2-\underset{\underset{O}{\|}}{C}-OMgBr$$

$$\xrightarrow{\text{(iii) } H_3O^+} \underset{(X)}{CH_3-CH_2-COOH} + Mg\begin{matrix}OH\\Br\end{matrix}$$

$$\underset{(X)}{CH_3-CH_2-COOH} \xrightarrow[\text{electrolysis}]{\text{NaOH}} \underset{(Z)}{CH_3-CH_2-CH_2-CH_3}$$

$$\underset{(W)}{H_3C-CH_2-Br} \xrightarrow[\text{dry/ether}]{\text{Na}} \underset{(Z)}{CH_3-CH_2-CH_2-CH_3}$$

60. (c) For fcc unit cell, Z = 4

Edge length = xÅ

Atomic mass, $M = 63.5 \text{ g mol}^{-1}$

$\because$ Density, $d = \frac{Z\times M}{x^3\times N}$

$$= \frac{4\times 63.5}{(x\times 10^{-8})^3\times 6.023\times 10^{23}}$$

$d = \frac{423}{x^3}$

61. (a) I. Moles of solute NaOH

$= \text{molarity} \times \text{volume of solution (in L)}$

$$= \frac{0.2\times 500}{1000} = 0.1$$

II. Molarity of $H_2SO_4 = \frac{0.1}{2} = 0.05M$

Moles of $H_2SO_4 = \frac{0.05\times 200}{1000} = 0.01$

III. Moles of area $= \frac{\text{Mass of urea}}{\text{Molecular mass of urea}} = \frac{6}{60} = 0.1$

62. (b) $\Delta T_f = K_f \times m$

$\Rightarrow 0.70 = 5.1\times m$

$$\Rightarrow m = \frac{0.70}{5.1} = \frac{\text{Moles of solute}}{\text{Total mass of solvent (kg)}}$$

Lets, assume x g napthalene is present.

$$0.137 = \frac{\left(\frac{x}{m_{C_{10}H_8}} + \frac{6-x}{m_{C_{14}H_{10}}}\right)\times 1000}{300}$$

$$\Rightarrow 0.041 = \frac{x}{128} + \frac{6-x}{178} = 3.4 \text{ g}$$

Mass of anthracene $= 6 - x = 6 - 3.4 = 2.6$ g

63. (a) $E^\circ_{cell} = E^\circ_C - E^\circ_A = 0.34 - (-0.76)V = 1.1$ V

$$\Rightarrow E_{cell} = E^\circ_{Cell} - \frac{2.303RT}{nF}\log\left(\frac{Zn^{2+}}{Cu^{2+}}\right)$$

$$\Rightarrow E_{cell} = 1.1 - \frac{0.059}{n}\log\frac{C_2}{C_1}\ \begin{pmatrix}Zn^{2+} = C_2\\ Cu^{2+} = C_1\end{pmatrix}$$

E_{Cell} value will be maximum when, $\log\frac{C_2}{C_1}$ will be minimum.

From the given options $\log\frac{0.01}{0.1} = \log 10^{-1}$ will give the maximum value of E_{Cell}

64. (c)
65. (b)
66. (d) (I) Magnetite $\rightarrow Fe_3O_4$
(II) Kaolinite $\rightarrow Al(OH)_4, Si_2O_3$
(III) Siderite $\rightarrow FeCO_3$
(IV) Calamine $\rightarrow ZnCO_3$
(III), (IV) both are carbonate ores.
67. (c) S_8 molecule is monoclinic as well as rhombic.
68. (c) Nobel metal like gold and platinum are soluble in aqua-regia which is a mixture of nitric acid and hydrochloric acid in molar ratio of $1:3$.
69. (d) $Mn_2O_7, CrO_3,\ V_2O_5$ are acidic.
Due to higher oxidation state of central metal, oxides.
70. (c) The wavelength of the light absorbed by the complexes $\propto \frac{1}{\text{strength of ligands}}$.
Strength of ligand : en $> H_2O$.
Hence, order of wavelength :

$$\underset{(\lambda_1)}{[Ni(H_2O)_6]}^{2+} > \underset{(\lambda_3)}{[(Ni(H_2O)_4en)]}^{2+} > \underset{(\lambda_2)}{[Ni(en)_3]^{2+}}$$

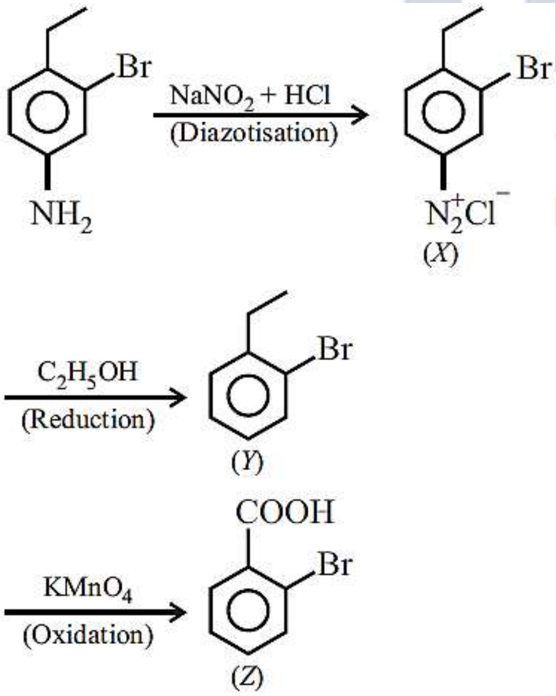
71. (d) In neutral or faintly alkaline solution,

$$8MnO_4^- + 3\,S_2O_3^{-2} + H_2O \longrightarrow$$
$$\longrightarrow 8MnO_2 + 6SO_4^{-2} + 2OH^-$$

In acidic medium,

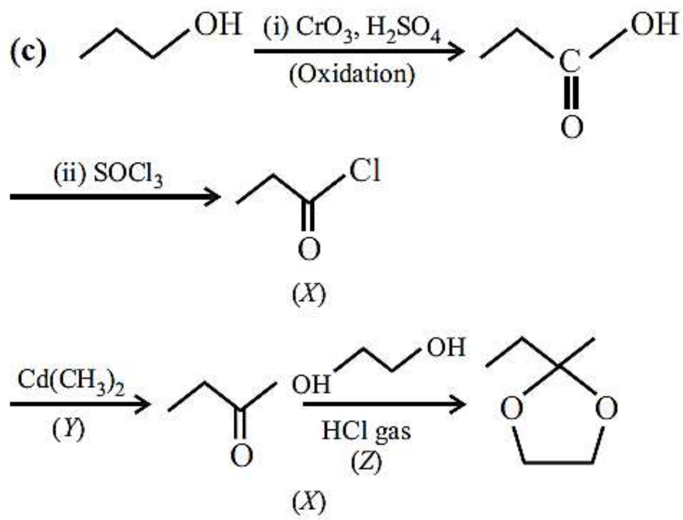
$$2MnO_4^- + 5NO_2^- + 6H^+ \longrightarrow 2Mn^{+2} + 5NO_3^- + 3H_2O$$

72. (c)
73. (b)
74. (c)
75. (d) Rate of S_N2 reaction $\propto \frac{1}{\text{steric hindrance}}$
$\therefore$ Rate is $1° > 2° > 3°$ alkyl halide.
76. (a) The bond angle between $C-O$ and $O-H$ bonds in alcohols is close to $109°$. (i.e. $109°28'$)
77. (d)

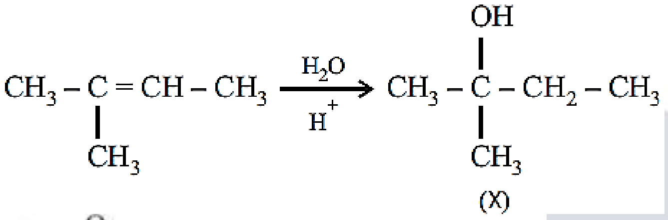


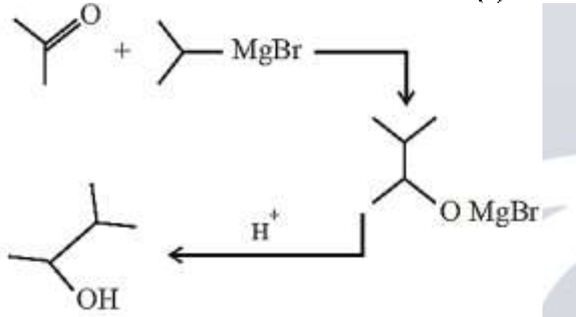
78.

(c)



79. (c)





(isomer of X)

80. (b) $\underset{\text{(Acetic acid)}}{CH_3COOH} \xrightarrow{NH_3} CH_3CO\overset{-}{O}\overset{+}{N}H_4$

$\xrightarrow[-H_2O]{\Delta} \underset{(A)}{CH_3CONH_2}$

$\xrightarrow{LiAlH_4} \underset{(b)}{CH_3-CH_2-NH_2}$

$\xrightarrow{CHCl_3,KOH} \underset{(C)}{CH_3-CH_2-N\equiv C}$

Mathematics

81. (d) Given that function

$f(x)=\frac{x}{e^x-1}+\frac{x}{2}+2\cos^3\frac{x}{2}$ define on $R-\{0\}$.

$\because\ f(-x)=\frac{-x}{e^{-x}-1}-\frac{x}{2}+2\cos^3\left(-\frac{x}{2}\right)$

$=\frac{xe^x}{e^x-1}-\frac{x}{2}+2\cos^3\frac{x}{2}$

$=\frac{x}{e^x-1}+\frac{x}{2}+2\cos^3\frac{x}{2}=f(x)$

$\because\ f(-x)=f(x), \forall x\in R-\{0\}$

$\therefore$ f(x) is an even function.

82. (c) (A) Since, $f(x)=\frac{|x+2|}{x+2}, x\neq -2$

$$= \begin{cases} \frac{x+2}{x+2}, & x > -2 \\ -\frac{x+2}{x+2}, & x < -2 \end{cases} = \begin{cases} 1, & x > -2 \\ -1, & x < -2 \end{cases}$$

So, range of f(x) is $\{-1,1\}$.

(B) Since, $g(x) = |[x]|, x \in R$

We have, $[x] \in I \Rightarrow |[x]| \in W$

So, range of g(x) is W.

(C) Since, $h(x) = |x - [x]|, x \in R \Rightarrow |\{x\}| \in [0,1)$

$[\because \{x\} = x - [x]$ and $\{x\} \in [0,1)]$

So, range of h(x) is $[0,1)$.

(D) Since, $f(x) = \frac{1}{2-\sin 3x}, x \in R$

We have, $-1 \leq \sin 3x \leq 1, \forall x \in R$

$\Rightarrow \ -1 \leq -\sin 3x \leq 1$

$\Rightarrow \ 2 - 1 \leq 2 - \sin 3x \leq 2 + 1$

$\Rightarrow \ \frac{1}{3} \leq \frac{1}{2 - \sin 3x} \leq \frac{1}{1}$

So, range of f(x) is $\left[\frac{1}{3}, 1\right]$.

83. (a) Since, $1 + (1 + 2 + 4) + (4 + 6 + 9) + (9 + 12 + 16) + \cdots + (81 + 90 + 100)$

$= 1 + (1^2 + (2 \times 1) + 2^2) + (2^2 + (2 \times 3) + 3^2) + (3^2 + (3 \times 4) + 4^2) + \cdots + (9^2 + (9 \times 10) + 10^2)$

$= \sum_{k=1}^{10} [(k-1)^2 + k(k-1) + k^2]$

$= \sum_{k=1}^{\alpha} [k^3 - (k-1)^3]$

$= (1^3 - 0^3) + (2^3 - 1^3) + (3^3 - 2^3) + \cdots + (10^3 - 9^3)$

$= 10^3 - 0^3 = 1000$

So, both (A) and (R) are true and (R) is the correct explanation of (A).

84. (d) Since $AA^T = I$, therefore matrix A is orthogonal matrix.

Now, $A^TX^{50}A = A^TX^{49}(APA^T)A$

$= A^TX^{49}AP(A^TA) = A^TX^{49}AP$

$= A^TX^{48}(APA^T)AP = A^TX^{48}AP^2$ ... .

$= A^TAP^{50} = 1P^{50} = P^{50}$

$\because \ P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow P^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \ P^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$ .....

$\Rightarrow \ P^{50} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

So, $A^TX^{50}A = P^{50} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

85. (c) Given that system of equations are

$2x + 3|y| + 5[z] = 0$

$x + |y| - 2[z] = 4$

and $x + |y| + [z] = 1$

According to Cramer's rule,

$x = \frac{\Delta_x}{\Delta}, |y| = \frac{\Delta_y}{\Delta}$ and $[z] = \frac{\Delta_z}{\Delta}$

Now, $\Delta = \begin{vmatrix} 2 & 3 & 5 \\ 1 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = -3.$

$\Delta_x = \begin{vmatrix} 0 & 3 & 5 \\ 4 & 1 & -2 \\ 1 & 1 & 1 \end{vmatrix} = -18 + 15 = -3.$

$$\Delta_y = \begin{vmatrix} 2 & 0 & 5 \\ 1 & 4 & -2 \\ 1 & 1 & 1 \end{vmatrix} = -3$$

and $\Delta_z = \begin{vmatrix} 2 & 3 & 0 \\ 1 & 1 & 4 \\ 1 & 1 & 1 \end{vmatrix} = 3$

Now, $x = \frac{-3}{-3} = 1, |y| = \frac{-3}{-3} = 1$ and $[z] = \frac{-3}{3} = -1$

$\therefore\ x = 1, |y| = 1 \Rightarrow y = \pm 1$ and $[z] = -1$

$\Rightarrow\ z \in [-1, 0)$

Hence, the given system of equations has infinitely many solution.

86. (b) Given that system of linear equations are

$x + 2y + 3z = 6$

$x + 3y + 5z = 9$

and $2x + 5y + \lambda z = \mu$

According to Cramer's rule,

now, $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 2 & 5 & \lambda \end{vmatrix} = \lambda - 8$

$$\Delta_x = \begin{vmatrix} 6 & 2 & 3 \\ 9 & 3 & 5 \\ \mu & 5 & \lambda \end{vmatrix} = \mu - 15$$

$$\Delta_y = \begin{vmatrix} 1 & 6 & 3 \\ 1 & 9 & 5 \\ 2 & \mu & \lambda \end{vmatrix} = 3\lambda - 2\mu + 6$$

and $\Delta_z = \begin{vmatrix} 1 & 2 & 6 \\ 1 & 3 & 9 \\ 2 & 5 & \mu \end{vmatrix} = \mu - 15$

1 Let system of equations has infinite number of solutions.

$\therefore\ \Delta = 0 \Rightarrow \lambda = 8$ and $\Delta_x = 0 \Rightarrow \mu = 15$.

2 Let system of equations has no solutions.

$\therefore \Delta = 0 \Rightarrow \lambda = 8$ and $\Delta_x \neq 0 \Rightarrow \mu \neq 15$

3 Let system of equations has unique solution.

$\therefore\ \Delta \neq 0 \Rightarrow \lambda \neq 8$ and $\Delta_x \neq 0 \Rightarrow \mu \neq 15$

87. (a) Let $z = x + iy, x, y \in R, (x, y) \neq (0, -4)$

$$\frac{2z-3}{z+4i} = \frac{(2x-3)+2iy}{x+i(y+4)} \times \frac{x-i(y+4)}{x-i(y+4)}$$

$$= \frac{(2x^2 - 3x + 2y^2 + 8y) + i(12 + 3y - 8x)}{x^2 + (y+4)^2}$$

So, $\arg\left(\frac{2z-3}{z+4i}\right) = \tan^{-1}\left(\frac{12+3y-8x}{2x^2-3x+2y^2+8y}\right) = \frac{\pi}{4}$

(given)

$$\Rightarrow\ \frac{12+3y-8x}{2x^2-3x+2y^2+8y} = 1$$

$\Rightarrow\ 2x^2 + 2y^2 + 5x + 5y - 12 = 0$

88. (d) Let $z = x + iy$

Now, $\frac{\bar{z}-1}{\bar{z}-i} = \frac{(x-1)-iy}{x-i(y+1)} \times \frac{x+i(y+1)}{x+i(y+1)}$

$$= \frac{[x(x-1) + y(y+1)] + i[(y+1)(x-1) - xy]}{x^2 + (y+1)^2}$$

$\therefore\ \text{Im}\left(\frac{\bar{z}-1}{\bar{z}-i}\right) = \frac{xy-y+x-1-xy}{x^2+(y+1)^2} = 1$ (given)

$\Rightarrow\ x^2 + y^2 - x + 3y + 2 = 0, (x, y) \neq (0, -1)$

89. (b) Given that ω is a complex cube root of unity

$\therefore\ 1 + \omega + \omega^2 = 0$ and $\omega^3 = 1$ …. (i)

$\because\ \left(k+\frac{1}{\omega}\right)\left(k+\frac{1}{\omega^2}\right)=k^2+k\left(\frac{1}{\omega}+\frac{1}{\omega^2}\right)+\frac{1}{\omega^3}$

$=k^2+\left(\frac{\omega^2+\omega}{\omega^3}\right)k+\frac{1}{\omega^3}=k^2-k+1$ [From Eq. (i)]

Now, $\sum_{k=1}^{n}\left(k+\frac{1}{\omega}\right)\left(k+\frac{1}{\omega^2}\right)$

$=\sum_{k=1}^{n}(k^2-k+1)=\frac{n(n+1)(2n+1)}{6}-\frac{n(n+1)}{2}+n$

$=\frac{n(n+1)}{6}[2n+1-3]+n=\frac{n(n+1)}{6}(2n-2)+n$

$=\frac{n}{3}(n^2-1)+n=\frac{n}{3}[n^2-1+3]=\frac{n(n^2+2)}{3}$

90. **(d)** Given that ω is a complex cube root of unity, then

$1+\omega+\omega^2=0$ and $\omega^3=1$ ….. (i)

$\because\ r(r+1-\omega)(r+1-\omega^2)$

$=r[(r+1)^2-(\omega+\omega^2)(r+1)+\omega^3]$

$=r[(r+1)^2+(r+1)+1]$ [from Eq. (i)]

$=r^3+3r^2+3r$

Now,

$\sum_{r=1}^{9}r(r+1-\omega)(r+1-\omega^2)=\sum_{r=1}^{9}(r^3+3r^2+3r)$

$=\left(\frac{9(10)}{2}\right)^2+3\frac{9(10)(19)}{6}+3\frac{9\times10}{2}$

$=(45)^2+(45\times19)+(3\times45)=3015.$

91. **(d)** Let $\frac{2\alpha}{3-4\alpha}=y\Rightarrow 2\alpha=3y-4\alpha y$

$\Rightarrow\alpha(2+4y)=3y\Rightarrow\alpha=\frac{3y}{2+4y}$

Given that α is root of quadratic equation

$x^2+7x+3=0,$

$\therefore\left(\frac{3y}{2+4y}\right)^2+7\left(\frac{3y}{2+4y}\right)+3=0$

$\Rightarrow 9y^2+84y^2+42y+48y^2+48y+12=0$

$\Rightarrow 141y^2+90y+12=0$

$\Rightarrow 47y^2+30y+4=0$ ….. (i)

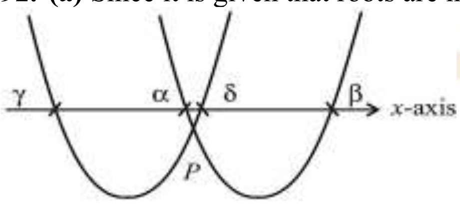
$\because\ y=\frac{2\alpha}{3-4\alpha}$ is root of quadratic equation

So, compare equation (i) with $ax^2+bx+c=0$.

$\therefore a=47, b=30$ and $c=4$ and GCD of 47,30,4 is 1 .

$\therefore\ a+b+c=47+30+4=81.$

92. **(a)** Since it is given that roots are in order, $\gamma<\alpha<\delta<\beta$ and, $x^2+b_1x+c_1=0, x^2+bx+c=0$



For x-coordinate of point P, on subtracting given quadratic equations, we get

$x^2+b_1x+c_1=0$

$x^2+bx+c=0$

$(b_1-b)x+(c_1-c)=0$

$\Rightarrow\ x=\left(\frac{c-c_1}{b_1-b}\right)$

Now, with respect to quadratic expression

$f(x)=x^2+bx+c$

$f\left(x=\frac{c-c_1}{b_1-b}\right)<0$

$\Rightarrow \left(\frac{c-c_1}{b_1-b}\right)^2 + b\left(\frac{c-c_1}{b_1-b}\right) + c < 0$
$\Rightarrow (c-c_1)^2 < b(c_1-c)(b_1-b) - c(b_1-b)^2$
$\Rightarrow (c-c_1)^2 < (b_1-b)(bc_1-cb_1)$

93. (a) Given that roots of the quadratic equation
$x^2 + 2(a+b+c)x + 3\lambda(ab+bc+ca) = 0$
are real, so $D \geq 0$
$\Rightarrow 4(a+b+c)^2 - 4 \times 3\lambda(ab+bc+ca) \geq 0$
$\Rightarrow \lambda \leq \frac{(a+b+c)^2}{3(ab+bc+ca)}$
Now, for scalane triangle.
$\because$ For $\triangle ABC |b-c| < a, |c-a| < b$ and $|a-b| < c$
$\Rightarrow (b-c)^2 + (c-a)^2 + (a-b)^2 < a^2 + b^2 + c^2$
$\Rightarrow a^2 + b^2 + c^2 + 2(ab+bc+ca) < 4(ab+bc+ca)$
$\Rightarrow \frac{(a+b+c)^2}{3(ab+bc+ca)} < \frac{4}{3}$
$\therefore \lambda < \frac{4}{3}$

94. (d) Since it is given that, the polynomial equation of degree 4 having real three coefficients of its roots as $2 \pm \sqrt{3}$ and $1+2i$, hence the remaining root is $1-2i$. Now, the quadratic equation whose roots as $2 \pm \sqrt{3}$ is $x^2 - 4x + 1 = 0$, and the quadratic equation whose roots as
$1 \pm 2i$, is
$x^2 - 2x + 5 = 0$
So, the required polynomial equation is
$(x^2 - 4x + 1)(x^2 - 2x + 5) = 0$
$\Rightarrow x^4 - 6x^3 + 14x^2 - 22x + 5 = 0$

95. (d) On arranging the letters of word ANIMAL in alphabetic order, we get AAILMN, now to find the rank of word ANIMAL, x (given)
AA. 4! AI 4!
AL 4!, AM 4!
ANA .... 3!, ANIA .... 2!
ANIL .... 2!, ANIMAL 1
$107 = x$
Similarly for word PERSON on arranging the letters in alphabetic order, we get ENOPRS, now to find the word having 107 ways,
EN .... 4! EO .... 4!
EP .... 4! ER 4!
ESN .... 3! ESON .... 2!
ESOP .... 2! ESORNP 1
107 same as x.
So, ESORNP is required word.

96. (a) Since number of ways of taken book from $(2n+1)$ books can be given as
${}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \cdots \ldots + {}^{2n+1}C_n = x$ (Let)
$\because {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \cdots \ldots + {}^{2n+1}C_n + \cdots.$
$+ {}^{2n+1}C_{2n+1} = 2^{2n+1}$
$\Rightarrow 2x + {}^{2n+1}C_0 + {}^{2n+1}C_{2n+1} = 2^{2n+1}$ $[\because {}^nC_{n-r} = {}^nC_r]$
$\Rightarrow 2x + 2 = 2^{2n+1}$ $[\because {}^nC_0 = {}^nC_n = 1]$
$\Rightarrow x = 2^{2n} - 1 = 255$
(given)
$\Rightarrow 2^{2n} = 256 = 2^8 \Rightarrow n = 4$

97. **(c)** Since it is given that sum of all the coefficients in the binomial expansion of $(1+2x)^n$ is $6561=(1+2)^n$ on putting $x=1$

$\Rightarrow 3^n=6561 \Rightarrow n=8$

Now, at $x=\frac{1}{\sqrt{2}}$, then $R=(1+2x)^n=I+F$

$\Rightarrow\ R=(\sqrt{2}+1)^8=I+F,$

where $I\in N$ and $0<F<1$

$\therefore\ (\sqrt{2}-1)^8=F'$, where $0<F'<1$

$\therefore\ (\sqrt{2}+1)^8+(\sqrt{2}-1)^8=I+(F+F')$

$\Rightarrow\ 2\left[(\sqrt{2})^8+{}^8C_2(\sqrt{2})^6+{}^8C_4(\sqrt{2})^4+{}^8C_6(\sqrt{2})^2+{}^8C_8\right]$

$=I+(F+F')$

For even integer $=I+(F+F')$

$\Rightarrow\ F+F'\in$ Integer

$\because\ 0<F<1$ and $0<F'<1\Rightarrow 0<F+F'<2$

So, $F+F'=1$

$\Rightarrow\ F=1-F'=1-(\sqrt{2}-1)^8$

So, $1-\frac{F}{1+(\sqrt{2}-1)^4}=1-\frac{1-(\sqrt{2}-1)^8}{1+(\sqrt{2}-1)^4}$

$=1-\left[1-(\sqrt{2}-1)^4\right]=(\sqrt{2}-1)^4$

98. **(b)** Since we are given that

$$\frac{(1-px)^{-1}}{(1-qx)}=a_0+a_1x+a_2x^2+a_3x^3+\cdots$$

We know

$(1-px)^{-1}=1+px+p^2x^2+p^3x^3+\cdots+p^nx^n+\cdots$

and $(1-qx)^{-1}=1+qx+q^2x^2+q^3x^3+\cdots+q^nx^n+\cdots$ hence, coefficient of x^n in the expansion of $(1-px)^{-1}(1-qx)^{-1}$

$=p^n+p^{n-1}q+p^{n-2}q^2+p^{n-3}q^3+\cdots+q^n$

$$a_n=\frac{p^n\left(1-\left(\frac{q}{p}\right)^{n+1}\right)}{1-\frac{q}{p}}=\frac{p^n(p^{n+1}-q^{n+1})p}{(p-q)p^{n+1}}$$

So, $a_n=\frac{p^{n+1}-q^{n+1}}{p-q}$

99. **(c)** Since, we are given that

$$\frac{3}{(x-1)(x^2+x+1)}=\frac{1}{x-1}-\frac{x+2}{x^2+x+1}$$

$=f_1(x)-f_2(x)$

and $\frac{x+1}{(x-1)^2(x^2+x+1)}=Af_1(x)+\left(B+\frac{D}{x-1}\right)$

$$f_2(x)+\frac{C}{(x-1)^2}$$

$\Rightarrow\ f_1(x)=\frac{1}{x-1}$ and $f_2(x)=\frac{x+2}{x^2+x+1}$

Therefore, $\frac{x+1}{(x-1)^2(x^2+x+1)}$

$$=\frac{A}{x-1}+\frac{(Bx-B+D)(x+2)}{(x^2+x+1)(x-1)}+\frac{C}{(x-1)^2}$$

$\Rightarrow\ x+1=A(x-1)(x^2+x+1)+B(x-1)$

$(x+2)(x-1)+D(x+2)(x-1)+C(x^2+x+1)$

On comparing we get

coefficient of $x^3=0\Rightarrow A+B=0$

coefficient of $x^2=0\Rightarrow D+C=0$

Hence, $A+B+C+D=0$

100. **(b)** Since, it is given that

$$f(\theta)=\sqrt{a^2\cos^2\theta+b^2\sin^2\theta}+\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}$$

So,
$[f(\theta)]^2 = a^2\cos^2\theta + b^2\sin^2\theta + a^2\sin^2\theta + b^2\cos^2\theta$
$+2\sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)(a^2\sin^2\theta + b^2\cos^2\theta)}$
$\because [f(\theta)]^2$ will be maximum, if $\sin^2\theta = \cos^2\theta = \frac{1}{2}$ and will be minimum, if either $\sin^2\theta = 0$ or $\cos^2\theta = 0$.

$$\therefore\ M = (a^2 + b^2) + 2\sqrt{\left(\frac{a^2}{2} + \frac{b^2}{2}\right)^2} = 2(a^2 + b^2)$$

and $m = (a^2 + b^2) + 2\sqrt{a^2b^2} = a^2 + b^2 + 2ab$
$\therefore\ M - m = 2(a^2 + b^2) - [a^2 + b^2 + 2ab]$
$= a^2 + b^2 - 2ab = (a - b)^2$

101. **(a)** Given, $\cos A = \frac{-60}{61}, \tan B = \frac{-7}{24}$ where A and B do not lie in second quadrant so A and B will be in third and fourth quadrant respectively. Thus, $\frac{B}{2}$ will be in second quadrant.

Now, we know $\tan B = \frac{2\tan\frac{B}{2}}{1-\tan^2\frac{B}{2}} = -\frac{7}{24}$ (given)

$$\Rightarrow\ 7\tan^2\frac{B}{2} - 48\tan\frac{B}{2} - 7 = 0$$

$$\Rightarrow \left(7\tan\frac{B}{2} + 1\right)\left(\tan\frac{B}{2} - 7\right) = 0$$

$\Rightarrow \tan\frac{B}{2} = -\frac{1}{7}$ $\left[\because \frac{B}{2} \in \text{IInd quadrant}\right]$

Now, $\tan\left(A + \frac{B}{2}\right) = \frac{\tan A + \tan\frac{B}{2}}{1 - \tan A \tan\frac{B}{2}} = \frac{\frac{11}{60} - \frac{1}{7}}{1 + \frac{11}{420}}$

$\left[\because \cos A = -\frac{60}{61} \text{ and } A \in \text{IIIrd Quadrant}\right]$

$$\therefore \tan A = \frac{11}{60}$$

$$= \frac{77 - 60}{420 + 11} = \frac{17}{431}$$

$\because A \in$ IIIrd quadrant and $\frac{B}{2} \in \left(\frac{3\pi}{4}, \pi\right)$ and $\tan\left(A + \frac{B}{2}\right)$ is positive.

$\therefore\ (+-) \in$ Ist quadrant.

102. **(a)** Given, $\cos^2 5^\circ - \cos^2 15^\circ - \sin^2 15^\circ + \sin^2 35^\circ$
$+\cos 15^\circ \sin 15^\circ - \cos 5^\circ \sin 35^\circ$
After rearranging the terms
$= \cos 5^\circ(\cos 5^\circ - \sin 35^\circ) - \cos 15^\circ(\cos 15^\circ - \sin 15^\circ)$
$+\sin^2 35^\circ - \sin^2 15^\circ$
$= \cos 5^\circ(\cos 5^\circ - \cos 55^\circ) - \cos 15^\circ(\cos 15^\circ - \cos 75^\circ)$
$+\sin 50^\circ \sin 20^\circ$

$$= -\frac{\cos 15^\circ}{\sqrt{2}} + \cos 5^\circ \sin 25^\circ + \sin 50^\circ \sin 20^\circ$$

$$= -\frac{\cos 15^\circ}{\sqrt{2}} + \frac{1}{2}[\cos 70^\circ + \cos 60^\circ + \cos 30^\circ - \cos 70^\circ]$$

$$= -\frac{\cos 15^\circ}{\sqrt{2}} + \frac{1}{2}[2\cos 45^\circ \cos 15^\circ]$$

$$= -\frac{\cos 15^\circ}{\sqrt{2}} + \frac{\cos 15^\circ}{\sqrt{2}} = 0$$

103. **(b)** It is given that
$\cos\theta \neq 0$ and $\sec\theta - 1 = (\sqrt{2} - 1)\tan\theta$

$\Rightarrow \frac{1-\cos\theta}{\cos\theta} = (\sqrt{2}-1)\frac{\sin\theta}{\cos\theta}$

$\Rightarrow \sin^2\frac{\theta}{2} = (\sqrt{2}-1)\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\Rightarrow$ Either $\sin\frac{\theta}{2} = 0$ or $\tan\frac{\theta}{2} = \sqrt{2}-1$

$\Rightarrow$ Either $\frac{\theta}{2} = n\pi, n \in Z$

or $\theta = 2n\pi + \frac{\pi}{4}$ or $2n\pi, n \in Z$.

104. **(a)** Since, given expression is sum of even natural numbers so,

$\because \sum_{k=1}^{n} 2k = n(n+1)$

$\therefore \cot\left(\sum_{n=3}^{32} \cot^{-1}\left(1+\sum_{k=1}^{n} 2k\right)\right)$

$=\cot\left(\sum_{n=3}^{32} \cot^{-1}(1+n(n+1))\right)$

$\cot\left(\sum^{32} \tan\left(\frac{(n+1)-n}{1+(n+1)n}\right)\right)$

$\because \sum_{n=3}^{32} \tan^{-1}\left(\frac{(n+1)-n}{1+(n+1)n}\right) = \sum_{n=3}^{32} [\tan^{-1}(n+1) - \tan^{-1} n]$

$=(\tan^{-1}4 - \tan^{-1}3 + (\tan^{-1}5 - \tan^{-1}4) + \cdots \ldots..$

$=$

$+(\tan^{-1}33 - \tan^{-1}32)$

$\tan^{-1}33 - \tan^{-1}3 = \tan^{-1}\left[\frac{33-3}{1+99}\right] = \tan^{-1}\left(\frac{3}{10}\right)$

$\therefore \cot\left(\sum_{n=3}^{32} \tan^{-1}\frac{(n+1)-n}{1+(n+1)n}\right) = \cot\left[\tan^{-1}\left(\frac{3}{10}\right)\right]$

$= \cot\left[\cot^{-1}\left(\frac{10}{3}\right)\right] = \frac{10}{3}$

$\left\{\because \tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right) \text{ if } x \in I^{st} \text{ quadrant}\right\}$

105. **(d)** Here, it is given $\tan x = \frac{3}{4}$

$\Rightarrow \sin^2 x = \frac{9}{25}$ and $\cos^2 x = \frac{16}{25}$

and $\sin x\cosh y = \cos\theta, \cos x\sinh y = \sin\theta$

$\because \cos^2\theta + \sin^2\theta = 1$

$\Rightarrow \frac{9}{25}(1+\sinh^2 y) + \frac{16}{25}\sinh^2 y = 1$

$\Rightarrow 9 + 9\sinh^2 y + 16\sin\ h^2 y = 25$

$\Rightarrow \sinh^2 y = \frac{16}{25}$

106. **(c)** Given, in $\triangle ABC, \frac{b+c}{9} = \frac{c+a}{10} = \frac{a+b}{11}$

Let $\frac{b+c}{9} = \frac{c+a}{10} = \frac{a+b}{11} = k$

$\Rightarrow b+c = 9k, c+a = 10k$ and $a+b = 11k$ and

$a+b+c = 15k$

$\therefore\ a = 6k, b = 5k$ and $c = 4k$

$$\because \frac{\cos A+\cos B}{\cos C}=\frac{\frac{b^2+c^2-a^2}{2bc}+\frac{a^2+c^2-b^2}{2ac}}{\frac{a^2+b^2-c^2}{2ab}}$$

$$=\frac{\frac{25+16-36}{40}+\frac{36+16-25}{48}}{\frac{36+25-16}{60}}=\frac{11}{12}.$$

107. **(c)** Since, given information for $\triangle$ ABC

(A) $r_1r_2\sqrt{\frac{4R-r_1-r_2}{r_1+r_2}}=\left[\frac{\Delta^2}{(s-a)(s-b)}\right.$

$$=\sqrt{\frac{4R-4R\cos\frac{C}{2}\left(\sin\frac{A}{2}\cos\frac{B}{2}+\sin\frac{B}{2}\cos\frac{A}{2}\right)}{4R\cos\frac{C}{2}\left(\sin\frac{A}{2}\cos\frac{B}{2}+\sin\frac{B}{2}\cos\frac{A}{2}\right)}}\ \Bigg]$$

$$=\frac{\Delta^2}{(s-a)(s-b)}\sqrt{\frac{4R\left(1-\cos^2\frac{C}{2}\right)}{4R\cos^2\frac{C}{2}}}$$

$$=\frac{\Delta^2}{(s-a)(s-b)}\tan\frac{C}{2}$$

$$=\frac{\Delta^2}{(s-a)(s-b)}\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}=\frac{\Delta^2}{\Delta}=\Delta$$

(B) $$\frac{r_2(r_3+r_1)}{\sqrt{r_1r_2+r_2r_3+r_3r_1}}=\frac{\frac{\Delta}{s-b}\left(\frac{\Delta}{s-c}+\frac{\Delta}{s-a}\right)}{\sqrt{\frac{(s-a)+(s-b)+(s-c)}{(s-a)(s-b)(s-c)}}}$$

$$=\frac{\frac{\Delta}{(s-b)}\times\frac{2S-a-c}{(s-a)(s-c)}}{\sqrt{\frac{3S-a-b-c}{(s-a)(s-b)(s-c)}}}=\frac{\Delta(b)}{\sqrt{s(s-a)(s-b)(s-c)}}=b$$

(C) Given, $\frac{a}{c}=\frac{\sin(A-B)}{\sin(B-C)}\Rightarrow\frac{\sin A}{\sin C}=\frac{\sin(A-B)}{\sin(B-C)}$

$\Rightarrow\ \sin A\sin(B-C)=\sin(A-B)\sin C$

$\Rightarrow\ \sin A(\sin B\cos C-\sin C\cos B)$

$\Rightarrow\ 2\sin A\cos B\sin C=\sin A\sin B\cos C+\sin B\cos A\sin C$

$$\Rightarrow 2\frac{a}{2R}\times\frac{a^2+c^2-b^2}{2ac}\times\frac{c}{2R}$$

$$=\left(\frac{a}{2R}\times\frac{b}{2R}\times\frac{a^2+b^2-c^2}{2ab}\right)+\left(\frac{b}{2R}\times\frac{b^2+c^2-a^2}{2bc}\times\frac{c}{2R}\right)$$

$\Rightarrow\ 2(a^2+c^2-b^2)=a^2+b^2-c^2+b^2+c^2-a^2$

$\Rightarrow 2a^2+2c^2-2b^2=2b^2\Rightarrow 2b^2=a^2+c^2$

Hence, a^2,b^2,c^2 are in AP.

(D) $bc\cos^2\frac{A}{2}=bc\times\frac{s(s-a)}{bc}=s(s-a)$

108. **(b)** Given, a, b and c are sides of $\triangle$ ABC where $r_1=8$, $r_2=12$ and $r_3=24$ then we have,

$r_1=8=\frac{\Delta}{s-a}$

$r_2=12=\frac{\Delta}{s-b}$

and $r_3=24=\frac{\Delta}{s-c}$

From Eqs. (i) and (ii), we get

$\frac{s-b}{s-a}=\frac{2}{3}\Rightarrow 3s-3b=2s-2a$

$\Rightarrow 5a+c=5b$

From Eqs. (ii) and (iii), we get

$$\frac{s-c}{s-b}=\frac{1}{2}$$

$\Rightarrow 2s-2c=s-b$

and from Eqs. (i) and (iii), we get

$$\frac{s-c}{s-a}=\frac{1}{3}$$

$\Rightarrow 3s-3c=s-a$

$\Rightarrow 2a+b=2c$

On solving Eqs. (iv), (v) and (vi), we get $(a,b,c)=(12,16,20)$

109. **(d)** Given, $\overline{OA}=4\hat{\imath}+7\hat{\jmath}+8\hat{k}, \overline{OB}=2\hat{\imath}+3\hat{\jmath}+4\hat{k}$ and $\overline{OC}=2\hat{\imath}+5\hat{\jmath}+7\hat{k}$

Since we know that bisector of angle A divides the BC in ratio $c:b$

where c is length of side AB $=\sqrt{4+16+16}=6$

and b is length of side AC $=\sqrt{4+4+1}=3$

$\therefore$ Position vector of the point where the bisector of angle A meet $\overline{BC}$ is

$$\frac{6(2\hat{\imath}+5\hat{\jmath}+7\hat{k})+3(2\hat{\imath}+3\hat{\jmath}+4\hat{k})}{6+3}$$

$$=\frac{18\hat{\imath}+39\hat{\jmath}+54\hat{k}}{9}=2\hat{\imath}+\frac{13}{3}\hat{\jmath}+6\hat{k}$$

110. **(c)** Given equation of plane passes through the point $(\hat{\imath}+2\hat{\jmath}-\hat{k})$.

Hence $(1,2,-1)$ having direction ratios to normal are a, b, c is

$a(x-1)+b(y-2)+c(z+1)=0$

$\because$ Plane (i) is perpendicular to line of intersection of planes $\mathbf{r}\cdot(3\hat{\imath}-\hat{\jmath}+\hat{k})=1$ and $\mathbf{r}\cdot(\hat{\imath}+4\hat{\jmath}-2\hat{k})=2$

$\therefore$ $3a-b+c=0$ and $a+4b-2c=0$

$$\Rightarrow \frac{a}{2-4}=\frac{-b}{-6-1}=\frac{c}{12+1}$$

$$\Rightarrow \frac{a}{2}=\frac{b}{-7}=\frac{c}{-13}$$

So, equation of required plane is

$2(x-1)-7(y-2)-13(z+1)=0$

$\Rightarrow 2x-7y-13z=1$

or $\mathbf{r}\cdot(2\hat{\imath}-7\hat{\jmath}-13\hat{k})=1$

111. **(b)** Given, p.v. of the vertices A, B and C are

$\hat{\imath}+2\hat{\jmath}-5\hat{k}, -2\hat{\imath}+2\hat{\jmath}+\hat{k}$ and $2\hat{\imath}+\hat{\jmath}-\hat{k}$.

$\because$ $\overrightarrow{BA}=3\hat{\imath}-6\hat{k}$ and $\overrightarrow{BC}=4\hat{\imath}-\hat{\jmath}-2\hat{k}$

$$\therefore \cos(\angle B)=\frac{|\overrightarrow{BA}\cdot\overrightarrow{BC}|}{|\overrightarrow{BA}|\cdot|\overrightarrow{BC}|}$$

$$=\frac{12+12}{\sqrt{9+36}\sqrt{16+1+4}}=\frac{24}{\sqrt{45\times 21}}=\frac{8}{\sqrt{105}}$$

$$\Rightarrow \angle B=\cos^{-1}\left(\frac{8}{\sqrt{105}}\right)$$

112. **(b)** Here, length of altitude of $\triangle ABC$ drawn from A is $d=\frac{(\text{Area of }\triangle ABC)}{\frac{1}{2}|\overrightarrow{BC}|}=\frac{\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|}{\frac{1}{2}|\overrightarrow{BC}|}$

Now, $\overrightarrow{AB}=-2\hat{\imath}+\hat{\jmath}+\hat{k}, \overrightarrow{AC}=-\hat{\imath}+2\hat{\jmath}-\hat{k}$ and $\overrightarrow{BC}=\hat{\imath}+\hat{\jmath}-2\hat{k}$

$$\text{So, } \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ -2 & 1 & 1\\ -1 & 2 & -1\end{vmatrix}$$

$=\hat{\imath}(-1\cdot 2)-\hat{\jmath}(2+1)+\hat{k}(-4+1)=-3\hat{\imath}-3\hat{\jmath}-3\hat{k}$

$\therefore |\overrightarrow{AB}\times\overrightarrow{AC}|=3\sqrt{3}$ and $|\overrightarrow{BC}|=\sqrt{6}$

$$\therefore d=\frac{3}{\sqrt{2}}$$

113. **(b)** Let the position vectors of the vertices of tetrahedron OABC are OA = a, OB = b and $\overrightarrow{OC} = c$

Since, volume of tetrahedron OABC $= \frac{1}{6}[\vec{a} \quad \vec{b} \quad \vec{c}]$

Now, position vectors of vertices of new tetrahedron are $\frac{\vec{a}+\vec{b}}{3}, \frac{\vec{b}+\vec{c}}{3}, \frac{\vec{c}+\vec{a}}{3}$ and $\frac{\vec{a}+\vec{b}+\vec{c}}{3}$, so, coterminous edge vectors of the new tetrahedron are $\frac{\vec{a}}{3}, \frac{\vec{b}}{3}$ and $\frac{\vec{c}}{3}$.

∴ Volume of new tetrahedron is

$$\frac{1}{6}\left[\frac{\vec{a}}{3} \quad \frac{\vec{b}}{3} \quad \frac{\vec{c}}{3}\right] = \frac{1}{6 \times 27}[\vec{a} \quad \vec{b} \quad \vec{c}]$$

So, the required ratio $= \frac{1}{27}$

114. **(b)** Given, $\vec{A} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{B} = \hat{\imath} + \hat{\jmath}$, so

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = \hat{\imath}(2) - \hat{\jmath}(2) + \hat{k}(2-1)$$

$= 2\hat{\imath} - 2\hat{\jmath} + \hat{k}$

Now $|(\vec{A} \times \vec{B}) \times \vec{C}| = |\vec{A} \times \vec{B}||\vec{C}|\sin 30°$

[as angle between $\vec{A} \times \vec{B}$ and $\vec{C} = 30°$ (given)]

$= \sqrt{4+4+1}|\vec{C}|\left(\frac{1}{2}\right) = \frac{3}{2}|\vec{C}|$

$|\vec{C} - \vec{A}| = 2\sqrt{2}$ (Given)

$\Rightarrow |\vec{C}|^2 + |\vec{A}|^2 - 2\vec{C} \cdot \vec{A} = 8$

$\Rightarrow |\vec{C}|^2 + 9 - 2|\vec{C}| = 8$ [as $\vec{A} \cdot \vec{C} = |\vec{C}|$]

$\Rightarrow (|\vec{C}| - 1)^2 = 0 \Rightarrow |\vec{C}| = 1$

So, $|(\vec{A} \times \vec{B}) \times \vec{C}| = \frac{3}{2}$ [from Eq. (i)]

115. **(c)** Here, mean of given data is

$$\bar{x} = \frac{a_0 + a_1 + a_2 + \cdots + a_{11}}{12} = \frac{a_0 + a_{11}}{2}$$

$$= \frac{a_1 + a_{10}}{2} = \cdots$$

Now, deviations from their mean are

$$|\bar{x} - a_0| = \frac{|a_{11} - a_0|}{2} = \frac{11|d|}{2}$$

$|\bar{x} - a_1| = \frac{|a_{10} - a_1|}{2} = \frac{9|d|}{2}$, ... and so on

So, sum of deviations

$$= 2\left[\frac{11|d|}{2} + \frac{9|d|}{2} + \frac{7|d|}{2} + \frac{5|d|}{2} + \frac{3|d|}{2} + \frac{|d|}{2}\right]$$

$= 36|d|$

∴ Mean deviation from their arithmetic mean is

$\frac{36|d|}{12} = 3|d|$.

116. **(d)** Let table of frequency distribution.

Class Interval	Frequency (f_i)	x_i	$x_i f_i$	$(\bar{x} - x_i)^2$	$f_1(\bar{x} - x_i)^2$
0-10	2	5	10	196	392
10-20	3	15	45	16	48
20-30	4	25	100	36	144
30-40	1	35	35	256	256

	$N = \Sigma f_i = 10$		$\Sigma x_i f_i = 190$		$\Sigma f_i(\bar{x} - x_i)^2 = 840$

$\because\ \bar{x} = \dfrac{\Sigma x_i f_i}{N} = \dfrac{190}{10} = 19$

Variance$(\sigma)^2 = \dfrac{1}{N}\Sigma f_i(\bar{x} - x_i)^2 = \dfrac{1}{10}(840) = 84$

117. **(b)** Since, two particular students are always together so according to question, the required probability

$$= \frac{{}^{73}C_{28} + {}^{73}C_{43}}{{}^{75}C_{30}}$$

$$= \frac{\frac{1}{45 \times 44} + \frac{1}{30 \times 29}}{\frac{75 \times 74}{(30 \times 29)(45 \times 44)}} = \frac{(30 \times 29) + (44 \times 45)}{(75 \times 74)}$$

$$= \frac{29 + (22 \times 3)}{5 \times 37} = \frac{29 + 66}{5 \times 37} = \frac{95}{5 \times 37} = \frac{19}{37}.$$

118. **(c)** Since, bag contains $2n$ coins out of which $n - 1$ are unfair with heads on both sides.

Now, according to question

$$\frac{(n-1) \times 1 + (n+1)\frac{1}{2}}{2n} = \frac{41}{56}$$

$$\Rightarrow \frac{2n - 2 + n + 1}{4n} = \frac{41}{56}$$

$\Rightarrow 42n - 14 = 41n$

$\Rightarrow n = 14$

Hence, number of unfair coins in the bag is $(n - 1) = 13$.

119. **(a)** Since Bag A has 6 Green and 8 Red balls while B has 9 Green and 5 Red and by using other informations we deduce to.

Required probability

$$= \frac{\frac{1}{4}\left(\frac{{}^6C_2 + {}^8C_2}{{}^{14}C_2}\right)}{\frac{1}{4}\left(\frac{{}^6C_2 + {}^8C_2}{{}^{14}C_2}\right) + \frac{3}{4}\left(\frac{{}^9C_2 + {}^5C_2}{{}^{14}C_2}\right)}$$

$$= \frac{(6 \times 5) + (8 \times 7)}{[(6 \times 5) + (8 \times 7)] + 3[(9 \times 8) + (5 \times 4)]} = \frac{43}{181}$$

120. **(c)** For a random variable x, the given probability distribution is,

$x = x_i$	1	2	3	4	5	6
$P(X = x_i)$	0.2	0.3	0.12	0.1	0.2	0.08

and two events $A = \{x_i \mid x_i \text{ is a prime number }\}$
$= \{2,3,5\}$

and $B = \{x_i \mid x_i < 4\} = \{1,2,3\}$

So,

$P(A \cup B) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 5)$

$= 0.2 + 0.3 + 0.12 + 0.2 = 0.82$

121. **(c)** Since the unit mean is $\bar{x} = 1$

Hence $\sum_{x=0}^{\infty} |x - \bar{x}| P(X = x) = \sum_{x=0}^{\infty} |x - 1| \dfrac{e^{-1}}{x!}$

$$= \frac{1}{e}\left[\frac{1}{0!} + \sum_{x=1}^{\infty} \frac{x-1}{x!}\right] = \frac{1}{e}\left[1 + \sum_{x=1}^{\infty} \frac{1}{(x-1)!} - \sum_{x=1}^{\infty} \frac{1}{x!}\right]$$

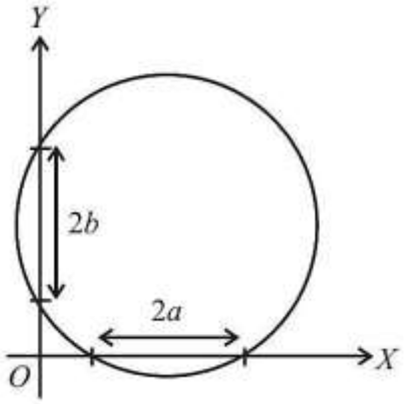
$= \frac{1}{e}[1 + e - (e - 1)] = \frac{2}{e}.$

122. **(d)** Let the equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$

Hence x-intercept and y-intercept are given as,

$\because\ 2\sqrt{g^2-c}=2a$ and $2\sqrt{f^2-c}=2b$



then $g^2-a^2=0$ and $f^2-b^2=0$
so, $g^2-a^2=f^2-b^2$
$\Rightarrow g^2-f^2=a^2-b^2$
Hence locus of the centre $(-g,-f)$, we get $x^2-y^2=a^2-b^2$

123. **(b)** Since coordinate axes are rotates about the origin through angle $\frac{\pi}{4}$

Hence, the new coordinates are (X,Y) have relation with older coordinates (x,y) as

$(x,y)=[(X\cos\theta-Y\sin\theta)\cdot(Y\cos\theta+X\sin\theta)]$

$$\Rightarrow\left(\left(\frac{X}{\sqrt{2}}-\frac{Y}{\sqrt{2}}\right),\left(\frac{Y}{\sqrt{2}}+\frac{X}{\sqrt{2}}\right)\right)\ \left\{\because\theta=\frac{\pi}{4}\right\}$$

Now equation $25x^2+9y^2=225$ becomes

$$25\left(\frac{X-Y}{\sqrt{2}}\right)^2+9\left(\frac{X+Y}{\sqrt{2}}\right)^2=225$$

$\Rightarrow 34X^2+34Y^2-32XY=450$

$\Rightarrow 17X^2+17Y^2-16XY=225$

On comparing, we get

$\alpha=\gamma=17,\beta=-16$ and $\delta=225$

$\therefore\ (\alpha+\beta+\gamma-\sqrt{\delta})^2=(34-16-15)^2=3^2=9$

124. **(c)** Since we know that equation of line through intersection of lines L_1 and L_2 is given as $L_1+\lambda L_2=0$

$\Rightarrow\ (3x-4y+1)+\lambda(5x+y-1)=0$

$\Rightarrow\ (3+5\lambda)x+(\lambda-4)y=(\lambda-1)$

$$\Rightarrow\frac{x}{\frac{\lambda-1}{3+5\lambda}}+\frac{y}{\frac{\lambda-1}{\lambda-4}}=1$$

Since inten. cells are equal

$\frac{\lambda-1}{3+5\lambda}=\frac{\lambda-1}{\lambda-4}$ and $\lambda\neq 1$

$\Rightarrow 4\lambda=-7\Rightarrow\lambda=-\frac{7}{4}$

So, equation of required line is

$$\left(3-\frac{35}{4}\right)x+\left(-\frac{7}{4}-4\right)y=\left(-\frac{7}{4}-1\right)$$

$$\Rightarrow\ -\frac{23}{4}x-\frac{23}{4}y=-\frac{11}{4}$$

$\Rightarrow\ 23x+23y=11$

125. **(a)** Given that a line passes through $P(a,2), a\neq 0$ making an angle 45° with positive direction of the X-axis is :

hence $\frac{x-a}{\frac{1}{\sqrt{2}}}=\frac{y-2}{\frac{1}{\sqrt{2}}}=r\Rightarrow x=a+\frac{r}{\sqrt{2}}$ and $y=2+\frac{r}{\sqrt{2}}$

Hence for point B, $r = -2\sqrt{2}$ (as it is on X-axis)
and $B(a-2,0)$ for point C, $r = -a\sqrt{2}$ (as it is on Y-axis) so, $C(0,2-a)$

$\therefore\ PB = \sqrt{4+4} = 2\sqrt{2}$

$PC = \sqrt{a^2+a^2} = a\sqrt{2}$

For points A and D

$$\frac{\left(a+\frac{r}{\sqrt{2}}\right)^2}{9} + \frac{\left(2+\frac{r}{\sqrt{2}}\right)^2}{4} = 1$$

$$\Rightarrow 4\left(a+\frac{r}{\sqrt{2}}\right)^2 + 9\left(2+\frac{r}{\sqrt{2}}\right)^2 = 36$$

$$\Rightarrow 13\left(\frac{r^2}{2}\right) + \left(\frac{8a}{\sqrt{2}} + \frac{36}{\sqrt{2}}\right)r + 4a^2 = 0,$$

let having roots $r_1 = PA$ and $r_2 = PD$, so $r_1 r_2 = \frac{4a^2}{13/2}$

$$\Rightarrow (PA)(PD) = \frac{8a^2}{13} = (PA)(PD)$$

Therefore PA, PB, PC and PD are in GP. So,

$$(PA)(PD) = (PB)(PC) \Rightarrow \frac{8a^2}{13} = 4a \Rightarrow 2a = 13$$

126. **(a)** Image of A in $x - y + 5 = 0$ is

$$\Rightarrow \frac{x-1}{1} = \frac{x+2}{-1} = -2\frac{1+2+5}{2} = -8$$

$\Rightarrow x = -7, x = 6$

so, $B(-7,6)$.

Image of A in $x + 2y$, is

$$\Rightarrow \frac{x-1}{1} = \frac{y+2}{2} = -2\frac{1-4}{5} = \frac{6}{5}$$

$\Rightarrow x = \frac{11}{5}$ and $y = \frac{2}{5}$

So, $C\left(\frac{11}{5}, \frac{2}{5}\right)$.

Hence equation of line BC is

$$y - 6 = \frac{28}{-46}(x+7)$$

$$\Rightarrow\ y - 6 = \frac{-14}{23}(x+7)$$

$\Rightarrow 14x + 23y = 138 - 98$

$\Rightarrow\ 14x + 23y - 40 = 0$

127. **(b)** Since equation of line passing through origin is $y - mx = 0$.

Hence $4 = \frac{|4-3m|}{\sqrt{1+m^2}}$ where m is slope of line.

or $16 + 16m^2 = 16 + 9m^2 - 24m$

$\Rightarrow 7m^2 + 24m = 0 \Rightarrow m = 0$ or $m = -\frac{24}{7}$

so combined equation of required lines

$$y\left(y + \frac{24}{7}x\right) = 0 \Rightarrow 7y^2 + 24xy = 0$$

128. **(a)** Let homogenisation of the curve $y^2 - 4ax = 0$.
For line $y = mx + C$ where $C = y - mx$
Getting combined equation of straight lines which subtend 90° at centre is

$$y^2 - 4ax\left(\frac{y - mx}{c}\right) = 0$$

$y = m(x - 4a)$, represent family of line passes through $(4a, 0)$. Hence, option (a) is correct.

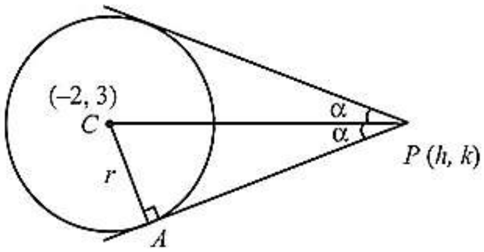
129. **(a)** Given abscissae of points P, Q are roots of x of the equation $2x^2 + 4x - 7 = 0$ and their ordinates are the roots of the equation $3x^2 - 12x - 1 = 0$.

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, then

$x_1 + x_2 = -2$ and $y_1 + y_2 = 4$

Now, centre of the circle with PQ as a diameter is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = (-1,2)$.

130. **(d)** Let a circle C: $(-2,3)$ with centre and tangents drawn from point P is



$\because \tan\alpha = \frac{AC}{PA}$

$\Rightarrow \tan\alpha = \frac{\sqrt{4+9-9\sin^2\alpha - 13\cos^2\alpha}}{\sqrt{h^2+k^2+4h-6k+9\sin^2\alpha+13\cos^2\alpha}}$

$= \frac{\sqrt{13\sin^2\alpha - 9\sin^2\alpha}}{\sqrt{h^2+k^2+4h-6k+9+4\cos^2\alpha}}$

$= \sqrt{\frac{4\sin^2\alpha}{h^2+k^2+4h-6k+9+4\cos^2\alpha}}$

$\Rightarrow \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{4\sin^2\alpha}{h^2+k^2+4h-6k+9+4\cos^2\alpha}$

$\Rightarrow h^2+k^2+4h-6k+9+4\cos^2\alpha = 4\cos^2\alpha$

$\Rightarrow h^2+k^2+4h-6k+9 = 0$

On taking locus of point $P(h,k)$, we get $x^2+y^2+4x-6y+9=0$

131. **(b)** Since, given equation of circle is,

$x^2+y^2-4x-6y-12=0$ with centre $C_1(2,3)$ and radius $r_1 = \sqrt{4+9+12} = 5$.

Let $C_2(h,k)$ and $r = 3$ (given) for required circle touches the given circle at $A(-1,-1)$.

The point $A(-1,-1)$ divides the line joining the centres $C_1(2,3)$ and $C_2(h,k)$ externally in $5:3$ so

$\Rightarrow (-1,-1) = \left(\frac{5h-6}{2}, \frac{5k-9}{2}\right)$

$\Rightarrow h = \frac{4}{5}$ and $k = \frac{7}{5}$

so equation of required circle is

$\left(x-\frac{4}{5}\right)^2 + \left(y-\frac{7}{5}\right)^2 = (3)^2$

$\Rightarrow x^2+y^2-\frac{8x}{5}-\frac{14y}{5}+\frac{65}{25} = 9$

$\Rightarrow 5x^2+5y^2-8x-14y-32=0$

132. **(d)** Since given equation of the circle is, $x^2+y^2+2gx+2fy+c=0$

With centre $(-g,-f)$ lies on the line

$2x+3y-7=0$

will satisfy it

$\Rightarrow 2g+3f+7=0$

Since circle (i) cuts the given circles

$x^2+y^2-4x-6y+11=0$

and $x^2+y^2-10x-4y+21=0$

orthogonally

so, $2g(-2)+2f(-3) = c+11$

$\Rightarrow 4g+6f+c+11=0$

and $2g(-5)+2f(-2) = c+21$

$\Rightarrow 10g + 4f + c + 21 = 0$

From Eqs. (iii) and (iv), we get

$6g - 2f + 10 = 0$

From Eq. (ii) and (v), we get

$11f + 11 = 0 \Rightarrow f = -1$

so, $g = -2 \Rightarrow c = 3$

$\therefore\ 5g - 10f + 3c = -10 + 10 + 9 = 9.$

133. **(a)** Given radical axis of the circles

$x^2 + y^2 + 2gx + 2fy + c = 0$

and $2x^2 + 2y^2 + 3x + 8y + 2c = 0$

is $(4g - 3)x + (4f - 8)y = 0$

Since, the radical axis (i) touches the circle $x^2 + y^2 + 2x + 2y + 1 = 0$, so

$$\sqrt{1 + 1 - 1} = \frac{|-(4g-3) - (4f-8)|}{\sqrt{(4g-3)^2 + (4f-8)^2}}$$

$\Rightarrow (4g-3)^2 + (4f-8)^2 + 2(4g-3)(4f-8)$

$= (4g-3)^2 + (4f-8)^2$

$\Rightarrow 8(4g-3)(f-2) = 0$

$\Rightarrow\ (4g-3)(f-2) = 0$

134. **(b)** Here length of the chord intercepted by the line $y = mx + c$ by the given parabola $x^2 = 4ay$ is given as

$4\sqrt{a(1+m^2)(c+am^2)}$

So, $\sqrt{40} = 4\sqrt{a(1+4)(1+a(4))}$

$\Rightarrow 40 = 16(5a(1+4a))$

or $\quad 1 = 2a(1+4a)$

$\Rightarrow \quad 8a^2 + 2a - 1 = 0$

or $\quad 8a^2 + 4a - 2a - 1 = 0$

$\Rightarrow \quad a = \frac{1}{4}$ or $-\frac{1}{2}$

135. **(d)** Here, equation of normal to parabola $y^2 = 4ax$, having slope ' m ' is $y = mx - 2am - am^3$

It passes through (h, k) so $k = mh - 2am - am^3$ is cubic equation in ' m '. Let having roots m_1, m_2 and m_3.

So, $m_1m_2m_3 = -\frac{k}{a}$

and $m_1m_2 = -1$ due to perpendicular normals.

So $m_3 = \frac{k}{a}$

$$k = \frac{k}{a}h - 2a\frac{k}{a} - a\left(\frac{k}{a}\right)^3 \Rightarrow 1 = \frac{h}{a} - 2 - \frac{k^2}{a^2}$$

$$\Rightarrow \frac{k^2}{a^2} = \frac{h}{a} - 3 \Rightarrow k^2 = a(h - 3a)$$

On taking locus (h, k), we get

$y^2 - ax + 3a^2 = 0$

136. **(c)** Let a parametric point $P(a\cos\theta, b\sin\theta)$ on the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

Hence area of $\Delta PF_1F_2 = \frac{1}{2}(2ae)b|\sin\theta|$

$= aeb|\sin\theta| = A$

For maximum value of $A, \theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

so $A_{max} = aeb.$

137. **(c)** Given ellipse is $\frac{x^2}{25}+\frac{y^2}{9}=1$, so equation of chord joining the points $A(\alpha)$ and $B(\beta)$ on the ellipse is

$\frac{x}{5}\cos\frac{\alpha+\beta}{2}+\frac{y}{3}\sin\frac{\alpha+\beta}{2}=-\cos\frac{\alpha-\beta}{2}$ .... (I)

Hence, obtained chord is the focal chord so, it will pass through focus $(4,0)$.

Thus, $\frac{4}{5}\cos\frac{\alpha+\beta}{2}=\cos\frac{\alpha-\beta}{2}$

$$\Rightarrow 4\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2}-\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$$

$$=5\left(\cos\frac{\alpha}{2}\cos\frac{\beta}{2}+\sin\frac{\alpha}{2}\sin\frac{\beta}{2}\right)$$

$$\Rightarrow 4\left(\cot\frac{\alpha}{2}\cot\frac{\beta}{2}-1\right)=5\left(\cot\frac{\alpha}{2}\cot\frac{\beta}{2}+1\right)$$

$$\Rightarrow \cot\frac{\alpha}{2}\cot\frac{\beta}{2}=-9$$

138. **(a)** Let's deduce the given equation of hyperbola

$16x^2-25y^2-96x+100y-356=0$ in to

$$\frac{(x-3)^2}{25}-\frac{(y-2)^2}{16}=1$$

Now equation of tangent to the hyperbola with slope ' 1 ' is

$y-2=1(x-3)+\sqrt{25(1)-16}$

$\Rightarrow y-2=x-3+3$

or $x-y-4=0$

$\Rightarrow x-y+2=0$

139. **(a)** Given points $P(0,7,10), Q(-1,6,6)$ and $R(-4,9,6)$

Here, we think acc. to options and to find side lengths.

$\therefore\ PO=\sqrt{1+1+16}=\sqrt{18}=3\sqrt{2}$

$QR=\sqrt{9+9+0}=\sqrt{18}=3\sqrt{2}$

and $PR=\sqrt{16+4+16}=\sqrt{36}=6$

$\because\ PQ^2+QR^2=18+18=36=PR^2$ and $PQ=QR$

$\therefore$ PQR is a right angled isosceles triangle.

140. **(a)** Given, points $A(2,3,5), B(\alpha,3,3)$ and $C(7,5,\beta)$ are Vertices of a triangle ABC,

$\therefore$ Mid-point of BC is $D\left(\frac{\alpha+7}{2},4,\frac{3+\beta}{2}\right)$

$\because$ Direction ratios of line joining points $A(2,3,5)$ and

$D\left(\frac{\alpha+7}{2},4,\frac{3+\beta}{2}\right)$ is $\left(\frac{\alpha+3}{2},1,\frac{\beta-7}{2}\right)$.

Since it is given that the line segment AD is equally inclined with the co-ordinate axes, so

$$\frac{\alpha+3}{2}=1=\frac{\beta-7}{2}$$

$\Rightarrow \alpha=-1$ and $\beta=9$

$$\therefore\ \cos^{-1}\left(\frac{\alpha}{\beta}\right)=\cos^{-1}\left(-\frac{1}{9}\right)$$

141. **(a)** Equation of plane passes through the origin and containing the line

$\vec{r}=(\hat{\imath}+2\hat{\jmath}-3\hat{k})+t(2\hat{\imath}-3\hat{\jmath}+\hat{k})$ is

$(\vec{r}-0)\cdot[(\hat{\imath}+2\hat{\jmath}-3\hat{k})\times(2\hat{\imath}-3\hat{\jmath}+\hat{k})]=0$

$\Rightarrow \vec{r}\cdot[\hat{\imath}(2-9)-\hat{\jmath}(1+6)+\hat{k}(-3-4)]=0$

$\Rightarrow \vec{r}\cdot(-7\hat{\imath}-7\hat{\jmath}-7\hat{k})=0$

$\Rightarrow \vec{r}\cdot(\hat{\imath}+\hat{\jmath}+\hat{k})=0$ or $x+y+z=0$

142. **(d)** Since it is given that,

$\lim_{x\to\infty}\left\{\frac{x^3+1}{x^2+1}-(\alpha x+\beta)\right\}$ exists and equals to 2.

$\lim_{x\to\infty}\left\{\frac{x^3+1}{x^2+1}-(\alpha x+\beta)\right\}=2$

or, $\lim_{x\to\infty}\frac{x^3+1-\alpha x^3-\beta x^2-\alpha x-\beta}{x^2+1}=2$

For the existance of limit, coefficient of $x^3=0$

$\therefore\ \alpha=1$

$\therefore\ \lim_{x\to\infty}\frac{-\beta x^2-x-\beta+1}{x^2+1}=2$

$\Rightarrow \lim_{x\to\infty}\frac{-\beta-\frac{1}{x}-\frac{\beta}{x^2}+\frac{1}{x^2}}{1+\frac{1}{x^2}}=2$

$\Rightarrow -\beta=2\Rightarrow\beta=-2$

$\therefore\ (\alpha,\beta)=(1,-2)$

143. **(d)** Since for $k>0$, given

$$\sum_{x=0}^{\infty}\frac{k^x}{x!}\lim_{n\to\infty}\frac{n!}{(n-x)!}\left(1-\frac{k}{n}\right)^{n-x}\left(\frac{1}{n}\right)^x$$

$$=\lim_{n\to\infty}\sum_{x=0}^{n}\frac{n!}{x!\,(n-x)!}\left(1-\frac{k}{n}\right)^{n-x}\left(\frac{k}{n}\right)^x$$

$$=\lim_{n\to\infty}\sum_{x=0}^{n}{}^nC_x\left(1-\frac{k}{n}\right)^{n-x}\left(\frac{k}{n}\right)^x$$

$$=\lim_{n\to\infty}\left(1-\frac{k}{n}+\frac{k}{n}\right)^n=\lim_{n\to\infty}1^n=1$$

144. **(d)** Since given function $f: R\to R$ in such a case

$$f(x)=\begin{cases}5, & \text{if } x\le 1\\ a+bx, & \text{if } 1<x<3\\ b+5x, & \text{if } 3\le x<5\\ 30, & \text{if } x\ge 5\end{cases}$$

Here we check continuity at all the given points,

$x=1, a+b=5$

At $x=3, a+3b=b+15$

$\Rightarrow a+2b=15$

At $x=5, b+25=30\Rightarrow b=5$

From above equations, $a=5$

Hence given function is continuous at $x=1, x=3$ and $x=5$.

But $a=5, b=5A=a+b=5$

So, $f: R\to R$ is not continuous for any values of a and b.

145. **(d)** The given function $y=[x]+|1-x|$ have point of discontinuity at $x=0,1,2$ and 3 for $-1\le x\le 3$. So function $y=[x]+|1-x|$ is not differentiable at 4 points. Hence, option (d) is correct.

146. **(c)**

147. **(d)** Given, $f(x)$ is a twice differentiable function at point

P, where $\frac{dy}{dx}=4, \frac{d^2y}{dx^2}=-3$

Hence, $\frac{d^2x}{dy^2}=\frac{d}{dy}\left(\frac{dx}{dy}\right)=\frac{dx}{dy}\frac{d}{dx}\left(\frac{1}{\left(\frac{dy}{dx}\right)}\right)$

$$=\left(\frac{dx}{dy}\right)\left(\frac{-\frac{d^2y}{dx^2}}{\left(\frac{dy}{dx}\right)^2}\right)=-\frac{\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{dy}{dx}\right)^3}$$

$\therefore\ \left(\frac{d^2x}{dy^2}\right)_p=-\frac{(-3)}{(4)^3}=\frac{3}{64}.$

148. **(a)** Since from the given formula T is directly proportional to and error is 2 minutes less per day.
Hence, for simple pendulum of length L, the time of oscillation is given as

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow \frac{\Delta T}{T} \times 100 = \frac{1}{2}\frac{\Delta L}{L}$$

$$\Rightarrow \frac{\Delta L}{L}\% = -\frac{2 \times 2}{24 \times 60} \times 100 \ (\because \Delta T = 2 \text{ min})$$

$$= -\frac{5}{18}$$

$\therefore \frac{\Delta L}{L} = -\frac{5}{18}$ percentage.

149. **(d)** Given A $\Rightarrow$ A.M., G $\Rightarrow$ G.M., H $\Rightarrow$ H.M. of S
$\Rightarrow$ Sum of the numbers
The function f(x) is

$$= \sum_{k=1}^{n}(x - a_k)^2 = \sum_{k=1}^{n}\left(x^2 - 2xa_k + a_k^2\right)$$

$$= nx^2 - 2x(a_1 + a_2 + a_3 + \cdots a_n) + (a_1^2 + a_2^2 + \cdots + a_n^2)$$

$\because$ The quadratic expression $ax^2 + bx + c$ has its minimum value at $x = -\frac{b}{2a}$.

$\therefore$ f(x) has it's minimum value at

$$x = -\frac{-2(a_1 + a_2 + a_3 + \cdots + a_n)}{2n}$$

$$= \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n} \Rightarrow x = A$$

150. **(c)** Given at $x = c, m > 1, n > 1$
Since applicability of Rolle's theorem for the function
$f(x) = x^{2m-1}(a - x)^{2n}$ in $(0, a)$. So,
$f'(x) = (2m-1)x^{2m-2}(a-x)^{2n} - 2n(a-x)^{2n-1}x^{2m-1}$
$f'(c) = 0$
$\Rightarrow (2m-1)c^{2m-2} = 2nc^{2m-1}(a-c)^{2n-1}$

$$\Rightarrow \frac{(2m-1)}{c} = \frac{2n}{a-c}$$

$$\Rightarrow c = \frac{a(2m-1)}{2n+2m-1}$$

151. **(d)** Since the given function is defined from $f: [-1,1] \to R$

$$\text{and given as, } f(x) = \begin{cases} 2^x + 1, & \text{for } x \in [-1,0) \\ 1, & \text{for } x = 0 \\ 2^x - 1, & \text{for } x \in (0,1] \end{cases}$$

According to definition,
the given function f(x) in $x \in (-1,0)$ strictly increasing and $f(0^-) \to 2$
but $f(0) = 1$ and in interval $x \in (0,1)$, again it is strictly increasing, but $f(0^+) = 0$.
So, function has neither maximum nor minimum.

152. **(a)** Given $\int \frac{x-1}{(x+1)\sqrt{x^3+x^2+x}}dx$
Multiplying by x in numerator and denominator

$$= \int \frac{x^2 - 1}{(x+1)^2 x\sqrt{1 + \frac{1}{x} + x}} dx$$

$$= \int \frac{1 - \frac{1}{x^2}}{\left(1 + \frac{1}{x} + x\right)\sqrt{1 + \frac{1}{x} + x}} dx = \int \frac{x^2 - 1}{\left(1 + 1 + \frac{1}{x} + x\right)x^2\sqrt{1 + \frac{1}{x} + x}} dx$$

Let $1 + \frac{1}{x} + x = t^2$

$$\Rightarrow \left(1 - \frac{1}{x^2}\right)dx = 2tdt = \int \frac{2tdt}{(1+t^2)t} = 2\int \frac{dt}{1+t^2}$$

$$= 2\tan^{-1}(t) + c = 2\tan^{-1}\left(\sqrt{\frac{1 + x + x^2}{x}}\right) + C$$

153. **(c)** Given integral

$I(x) = \int x^2(\log x)^2 dx$ and $I(1) = 0$

$$\Rightarrow I(x) = \int x^2(\log x)^2 dx = \frac{x^3}{3}(\log x)^2 - \int \frac{x^3}{3}\frac{2\log x}{x} dx$$

[using integration by parts]

$$= \frac{x^3}{3}(\log x)^2 - \frac{2}{3}\left[\frac{x^3}{3}(\log x) - \int \frac{x^3}{3}\left(\frac{1}{x}\right)dx\right]$$

$$= \frac{x^3}{3}(\log x)^2 - \frac{2}{3}\left[\frac{x^3}{3}(\log x) - \frac{1}{3}\frac{x^3}{3}\right] + C$$

$$= \frac{x^3}{27}[9(\log x)^2 - 6(\log x) + 2] + C$$

$\because\ I(1) = 0$

$$\Rightarrow C = \frac{-2}{27}$$

Hence $I(x) = \frac{x^3}{27}[9(\log x)^2 - 6(\log x) + 2] - \frac{2}{27}$

154. **(b)** Let integral

$$I = \int \frac{x^5 dx}{(x^2 + x + 1)(x^6 + 1)(x^4 - x^3 + x - 1)}$$

Here $(x^2 + x + 1)(x^4 - x^3 + x - 1) = (x - 1)(x^3 + 1)$

$(x^2 + x + 1) = (x^3 - 1)(x^3 + 1) = x^6 - 1$

Hence $I = \int \frac{x^5 dx}{(x^6+1)(x^6-1)}$

Let $x^6 = t \Rightarrow 6x^5 dx = dt$

$$I = \frac{1}{6}\int \frac{dt}{(t+1)(t-1)} = \frac{1}{12}\log_e\left|\frac{t-1}{t+1}\right| + c$$

$$= \frac{1}{12}\log_e\left|\frac{x^6 - 1}{x^6 + 1}\right| + c$$

155. **(d)** Let integral, $I = \int \frac{dx}{x + \sqrt{x-1}}$

and, put $x - 1 = t^2 \Rightarrow dx = 2tdt$

then $I = \int \frac{2t}{(t^2+1)+t}dt = \int \frac{(2t+1)-1}{t^2+t+1}dt$

$$= \int \frac{2t+1}{t^2+t+1}dt - \int \frac{dt}{t^2+t+1}$$

$$= \log_e|t^2 + t + 1| - \int \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$= \log_e|t^2+t+1| - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$$

Since, $t = \sqrt{x-1}$,

So, $I = \log_e|x+\sqrt{x-1}| - \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\sqrt{x-1}+1}{\sqrt{3}}\right) + c$

156. **(a)** Let integral

$$I = \int_{\log_e 2}^{x} \frac{dt}{\sqrt{e^t-1}} = \frac{\pi}{6}$$

(given)

$$\Rightarrow \int_{\log_e 2}^{x} \frac{e^{-\frac{1}{2}}}{\sqrt{1-(e^{-t/2})^2}}dt = \frac{\pi}{6}$$

Let $e^{-t/2} = u$, at $t = \log_e 2, u = \frac{1}{\sqrt{2}}$ and at $t = x$,

$u = e^{-x/2}$ and $e^{-t/2}dt = -2du$

So, $I = \int_{\frac{1}{\sqrt{2}}}^{e^{-x/2}} \frac{-2du}{\sqrt{1-u^2}} = \frac{\pi}{6}$

$$\Rightarrow [-2\sin^{-1}u]_{\frac{1}{\sqrt{2}}}^{e^{-\frac{x}{2}}} = \frac{\pi}{6} \Rightarrow \sin^{-1}(e^{-x/2}) - \frac{\pi}{4} = -\frac{\pi}{12}$$

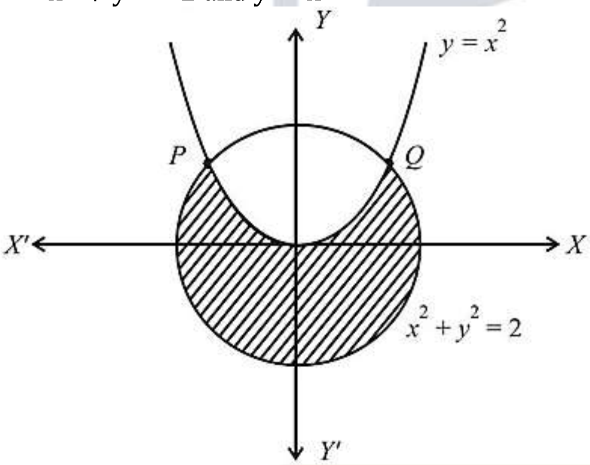
$$\Rightarrow \sin^{-1}(e^{-x/2}) = \frac{\pi}{6} \Rightarrow e^{-x/2} = \sin\frac{\pi}{6} - \frac{1}{2}$$

$$\Rightarrow \quad -\frac{x}{2} = \log_e\left(\frac{1}{2}\right) = -\log_e(2) \Rightarrow x = 2\cdot\log_e(2)$$

157. **(d)**

158. **(a)** Given equations of circle and parabola are

$x^2+y^2 = 2$ and $y = x^2$



On solving the given equation, we get

$y^2+y-2 = 0 \Rightarrow y^2+2y-y-2 = 0$

$\Rightarrow y(y+2) - 1(y+2) = 0$

$\Rightarrow y = 1\ [\because y > 0]$

Hence the required area

$= \pi + 2\int_0^1 \left(\sqrt{(2-y^2)} - \sqrt{y}\right)dy$

$$= \pi + 2\left[\frac{y}{2}\sqrt{2-y^2} + \frac{2}{2}\sin^{-1}\frac{y}{\sqrt{2}} - \frac{2}{3}y^{3/2}\right]_0^1$$

$$= \pi + 2\left[\frac{1}{2} + \frac{\pi}{4} - \frac{2}{3}\right] = \frac{3\pi}{2} + 1 - \frac{4}{3}$$

$= \frac{3\pi}{2} - \frac{1}{3}$ sq. units

159. **(d)** Given, c is a parameter and equation of the family of curves is

$x^2 = c(y+c)^2$

or, $x=\sqrt{c}(y+c)$

On differentiating both sides …. (i)

$1=\sqrt{c}\dfrac{dy}{dx}$

$\Rightarrow\ \sqrt{c}=\frac{dx}{dy}$ ….. (ii)

From Eqs. (i) and (ii) on eliminating ' c ', we get

$x=\left(\dfrac{dy}{dx}\right)\left[y+\left(\dfrac{dx}{dy}\right)^2\right]$

$\Rightarrow x\left(\dfrac{dy}{dx}\right)^3=y\left(\dfrac{dy}{dx}\right)^2+1$

$\Rightarrow x\left(\dfrac{dy}{dx}\right)^3-y\left(\dfrac{dy}{dx}\right)^2-1=0$

160. **(a)** Given $f(x), f'(x)$ are positive functions and $f(0)=1, f'(0)=2$

Since, $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}=0$

(given)

$\Rightarrow\ f(x)f''(x)-(f'(x))^2=0$

$\Rightarrow\ f(x)f''(x)=(f'(x))^2$

$\Rightarrow\ \dfrac{f''(x)}{f'(x)}=\dfrac{f'(x)}{f(x)}$

On, integrating both sides, we get

$\int\dfrac{f''(x)}{f'(x)}dx=\int\dfrac{f'(x)}{f(x)}dx$

$\Rightarrow\ \log_e(f'(x))=\log_e(f(x))+c$

$\because\ f(0)=1$ and $f'(0)=2$

$\therefore\ c=\log_e 2$

$\log_e(f'(x))=\log_e(f(x))+\log_e 2$

$\Rightarrow\ f'(x)=2f(x)$

$\Rightarrow\ \dfrac{f'(x)}{f(x)}=2$

$\Rightarrow\int\dfrac{f'(x)}{f(x)}dx=\int 2dx\Rightarrow\log_e f(x)=2x+c$

$\because\ f(0)=1$, so $c=0$

$\because\ \log_e f(x)=2x\Rightarrow f(x)=e^{2x}$