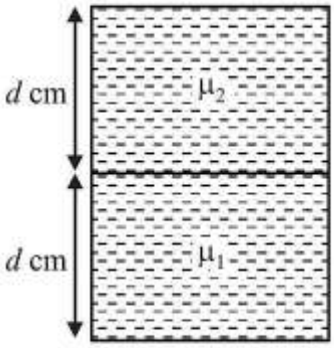
Physics

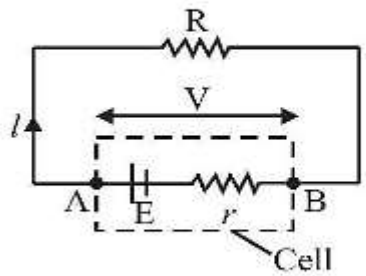
1. **(c)** Microwaves have short wavelength. So energy dispersion during long distance communication is very low. So, we use it in rooar system.
2. **(b)** $(d_1)_{app} = \frac{d}{\mu_1}$

$(d_2)_{\text{app}} = \frac{d}{\mu_2}$



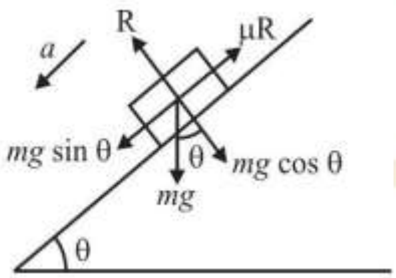
So, $d_{\text{app}} = \frac{d}{\mu_1} + \frac{d}{\mu_2} = d\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)$

3. **(a)** The energy stored in the coil is in the form of magnetic energy and is equal to $E = \frac{1}{2} LI^2$
4. **(b)**



When cell is discharging, then terminal potential difference across the cell is given as
$V = E - Ir$
[By using KVL between A and B]
where I = current drawn by cells.

5. **(a)**

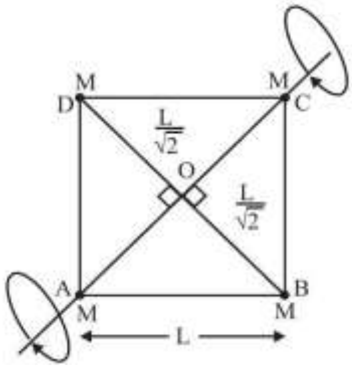


According to Newton's second law of motion,
$mg\sin\theta - \mu R = ma$
$\Rightarrow mg\sin\theta - \mu mg\cos\theta = ma$
$\Rightarrow a = g\sin\theta - \mu g\cos\theta$
$\Rightarrow a = g(\sin\theta - \mu\cos\theta)$

6. **(b)** Numerical aperture, NA = 0.8

Wavelength, $\lambda = 0.6 \mu$ m
$\therefore$ Limit of resolution
$= \frac{\lambda}{2(\text{NA})} = \frac{0.6}{2 \times 0.8} = \frac{0.3}{0.8} = \frac{3}{8} \mu$ m

7. **(b)** The given masses system are shown in figure.



$\therefore$ Moment of inertia of given system about the diagonal AC

$I_{AC} = I_A + I_B + I_C + I_D$

$= 0 + M\left(\frac{L}{\sqrt{2}}\right)^2 + 0 + M\left(\frac{L}{\sqrt{2}}\right)^2$

$= \frac{ML^2}{2} + \frac{ML^2}{2} = ML^2$

8. **(a)** Angular displacement in $t = 10$ s is given as

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ [$\because$ $\omega_0 = 0$ and $\alpha = 2\text{rad/s}^2$]

$= 0 \times 10 + \frac{1}{2} \times 2 \times 10^2 = 100\text{rad.}$

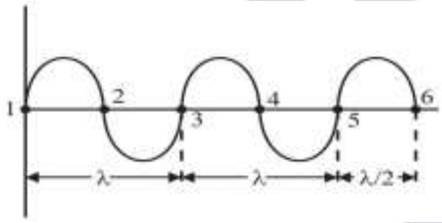
Number of revolution $= \frac{\theta}{2\pi} = \frac{100}{2\pi} \approx 16$

9. **(b)** According to given diagram, net charge enclosed through the surface S_2 is zero.

Net flux through surface,

$S_2 = \frac{\text{Charge inclosed}}{\varepsilon_0} = $ By gauss law

10. **(b)** Since, distance between 6 successive nodes is equivalent to $2\lambda - \frac{\lambda}{2} = \frac{5\lambda}{2}$



$\therefore d = \frac{5\lambda}{2} = 0.85$

$\Rightarrow \lambda = \frac{0.85 \times 2}{5} = 0.34$ m

$\therefore$ Speed of sound in tube,

$v = f\lambda = 1000 \times 0.3$

$= 340\ \text{ms}^{-1}$

11. **(c)** Let T be equilibrium temp.

Then, by the principle of calorimetry

$H_{gain} = H_{loss}$

$\Rightarrow 5c(T - 20) = 10c(60 - T)$

$\Rightarrow T - 20 = 120 - 2T$

$\Rightarrow 3T = 140$

$T = 46.67^\circ \approx = 47^\circ C$

12. **(a)** For surface e^-

$E_{photon} = \phi + (\text{K.E.})_{max}$

So, $(\text{K.E.})_{max} = \frac{1}{2}mv_{max}^2 = E - \phi = 6.2 - 4.2 = 2\text{eV}$

$= 2 \times 1.6 \times 10^{-19}$ J

$\Rightarrow \frac{1}{2}mv_{max}^2 = 3.2 \times 10^{-19}$ J

13. (c) We have

$^{11}_{6}C \rightarrow {}^{11}_{5}B + \beta + X$

When positron (β) is emitted from a nuclei, then atomic number decreases by one unit, while the mass number remains the same. This is β^+decay therefore.

So, X is neutrina.

14. (c) Heat lost by steam,

$Q' = mL_f + m_s c\Delta t$

$= 10^3 \times 540 + 10^3 \times 1 \times (150 - 90)$

$= 54 \times 10^4 + 6 \times 10^4$

$= 10^4(54 + 6) = 60 \times 10^4$cal

Mass of 20 L water, $m_w = 20$ kg $= 20 \times 10^3$ g

Heat gained by water, $Q'' = 20 \times 10^3 \times 1 \times \Delta T$

Now, Heat lost = Heat gained

$60 \times 10^4 = 20 \times 10^3 \times \Delta T$

$\Rightarrow \Delta T = \frac{60 \times 10^4}{20 \times 10^3} = 30°C$

15. (d) As $W = q(V_1 - V_2) = q[0] = 0$

16. (a) $\gamma = a\sin(\omega t - kx)$

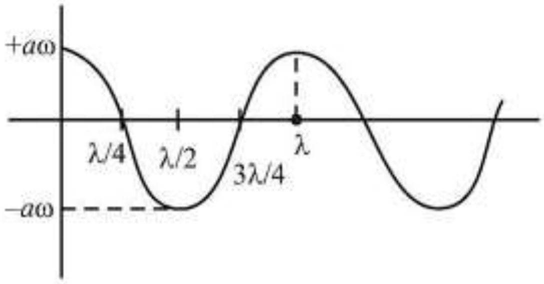
$V_P = \frac{d\gamma}{dt} = a\omega\cos(\omega t - kx)$

$V_P|_{t=0} = a\omega\cos(-kx)$

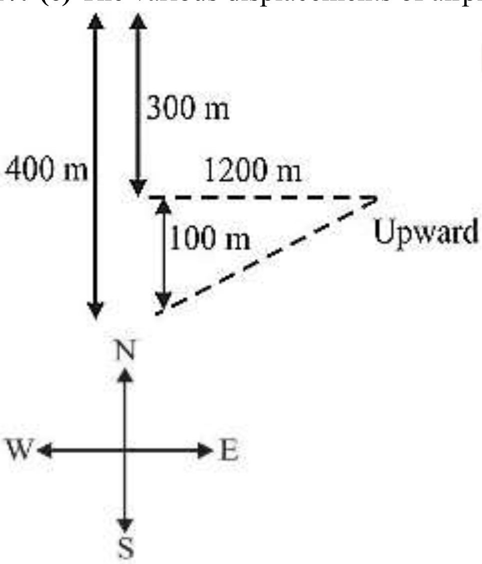
$= a\omega\cos(kx)$

when $x = 0$

$V_P = a\omega$ so the graph will be



17. (c) The various displacements of airplane is shown below



Net displacement
$d = \sqrt{100^2 + 1200^2}$
$= \sqrt{1450000} = 120416 \text{ m} = 1200 \text{ m}$

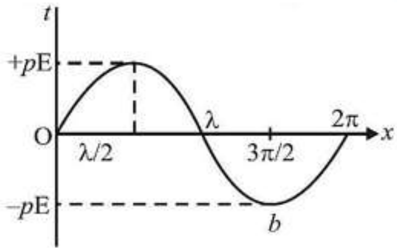
18. **(a)** Reynold's number determines the type of flow.
(i) If $R < 2100$, then flow is laminar.
(ii) If $2100 < R < 4000$, then flow is stream line.
(iii) If $R > 4000$, then flow is turbulent.

19. **(c)** Work done by a constant force is given as
$W = Fs\cos\theta$
where, $\theta =$ angle between the direction of force and displacement.
When, $\theta = 0°$, then $W = Fs$ (positive)
When $\theta = 90°$, then $W = Fs\cos 90°$
$= 0$ (zero)
When, $\theta = 180°$, then $W = Fs\cos 180°$
$= -Fs$ (negative)
Hence, work done may be positive, negative or zero depending on angle between force and displacement.

20. **(c)** At NTP,
Temperature = 300 K
Pressure = 76 cm of Hg
By ideal gas equation, we get
$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$
$\Rightarrow V_2 = \frac{p_1V_1T_2}{p_2T_1}$
$= \frac{78 \times 251 \times 300}{76 \times 293} = 263.8 \text{ cm}^3$

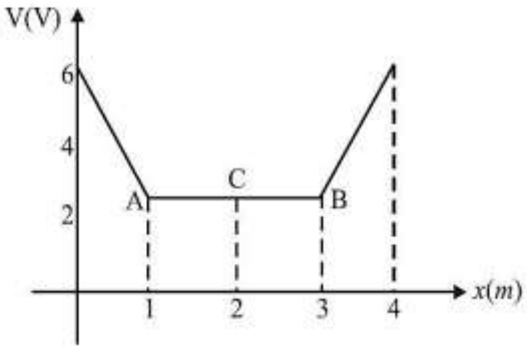
21. **(c)** Frank and Hertz's experiment confirms the prediction of quantum theory that electrons occupy only discrete, quantised energy states.

22. **(c)** Torque on electric dipole placed in uniform electric field E is given as
$\tau = pE\sin\theta$
Hence, graph between τ and θ is given as



23. **(b)** Let the final temperature be T.
Then, $H_{gain} = mg\Delta T = 50s(T - 10)$
$H_{loss} = mg\Delta T = 50s(100 - T)$
Now, Heat gain = Heat loss
$\Rightarrow H_{gain} = H_{loss}$
$\Rightarrow 50s(T - 10) = 50s(100 - T)$
$\Rightarrow 2T = 110$
$T = 55°C$

24. **(a)** Potential is constant in region AB as shown in diagram



i.e., V = constant

∴ Electric field at x = 2 m (at point C)

$E=\frac{-dV}{dx}=0 \quad [\because V=\text{constant}]$

25. (b) Let water equivalent of the calorimeter is W_{gram} .

Heat lost by warm water = Heat gained by cold water + Heat gained by the calorimeter

$\Rightarrow m_2(T_2-T_3)=m_1(T_3-T_1)+W(T_3-T_1)$

$\Rightarrow 300(60-40)=500(40-30)+W(40-30)$

$\Rightarrow 10\ W=1000$

$\Rightarrow W=100\ g=\frac{100}{1000}=0.1\ kg$

26. (c) We have

$g_{eff}=g-a_0$ .... (i)

Hence, the pressure by the fluid at the bottom of vessel is given as

$p=p_0+pgh=p_0+\rho(g-a_0)h$ (form eq (i)

27. (b) Weak nuclear force always operates between heaviour elementary particles.

28. (c) $\frac{\text{Lateral strain}}{\text{Longitudinal strain}}=\text{Poisson's ratio}=0.26$

Or, Lateral strain $=0.26\times$ longitudinal strain

$=0.26\times\frac{\Delta l}{l}=0.26\times\frac{3\times10^{-3}}{3}$

$=0.26\times10^{-3}$

29. (b) Rms value of AC, $V_{rms}=220\ V$

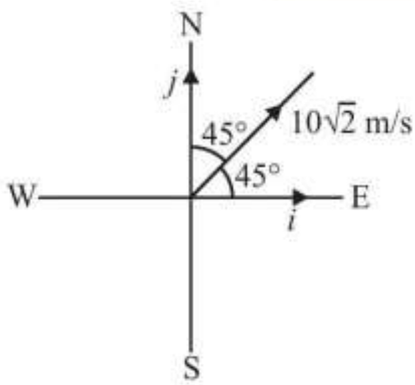
Peak value of AC,

$V_0=\sqrt{2}V_{rms}=\sqrt{2}\times220=311\ V$

$V_{DC}=220\ V$

Since, peak value of 220 VAC is 311 V which is much more than 220 DC . Hence, 220 V AC is more dangerous than 220 V DC.

30. (a)



Initial velocity

$u=10\sqrt{2}\cos45°\hat{i}+10\sqrt{2}\cos45°\hat{j}$

$=10(itg)$

Acceleration applied

$a_s = -2\hat{j}$

$\therefore$ Velocity after 5 s is

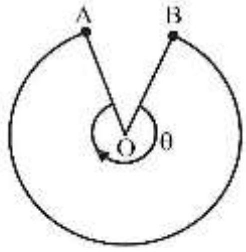
$v = u + a_s t$

$= 10(\hat{i} + \hat{j}) + (-2\hat{j}) \times 5 = 10\hat{i} + 10\hat{j} - 10\hat{j} = 10\hat{i}$

Hence, body will move towards east with the velocity 10 ms^{-1}.

31. (a) Magnetic field at the centre due to current carrying circular loop is given as

$$B_C = \frac{\mu_0 I}{2r}$$



Magnetic field due to current carrying circular arc making an angle θ at the centre, is given as

$B = \frac{\theta}{2\pi} \cdot B_C = \frac{\theta}{2\pi} \cdot \frac{\mu_0 I}{2r}$ [From Eq. (i)]

$B = \frac{\mu_0 I \theta}{4\pi r} = \frac{\mu_0 I}{r}\left(\frac{\theta}{4\pi}\right)$

32. (c) Induced emf,

$$e = L\frac{dI}{dt} = L\frac{\Delta I}{\Delta t} = 8 \times 10^{-3} \times \frac{2}{0.1} = 16 \times 10^{-2}\ \text{V}$$

33. (b) Acceleration due to gravity above the surface (at altitude) is given as

$$g' = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

From Eq. (i), it is clear that acceleration due to gravity decreases with altitude.

34. (c) $F_c = \frac{mv^2}{r}$

$$\frac{F_2}{F_1} = \frac{m_2}{m_1} \cdot \left(\frac{v_2}{v_1}\right)^2 \cdot \left(\frac{r_1}{r_2}\right)$$

When, mass, speed and radius of circular path of particle increases by 100%, i.e., these become double.

Hence, $m_2 = 2m_1, V_2 = 2V_1, r_2 = 2r_1$

$\therefore$ From Eq. (i), we have

$$\frac{F_2}{F_1} = \frac{2m_1}{m_1}\left(\frac{2v_2}{v_1}\right)^2\left(\frac{r_1}{2r_2}\right)$$

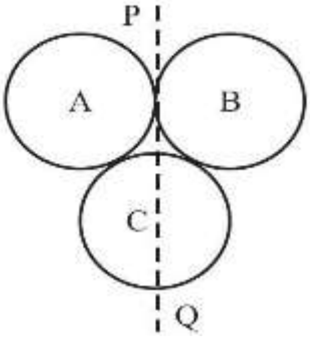
$\Rightarrow F_2 = 4F_1$

Percentage increase

$$\frac{\Delta F}{F_1} = \frac{F_2 - F_1}{F_1} \times 100 = \frac{4F_1 - F_1}{F_1} \times 100 = 300\%$$

35. (a) Moment of inertia of each solid sphere,

$$I = \frac{2}{5}mr^2$$



where, m = mass and r = radius

The moment of inertia of the system about axis PQ as shown in the figure

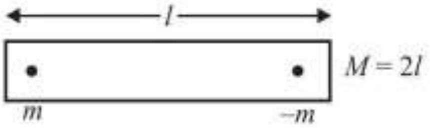
$I_{PQ} = I_A + I_B + I_C$

$= (I_{cm} + mr^2) + (I_{cm} + mr^2) + \frac{2}{5}mr^2$

$= \left[\left(\frac{2}{5}mr^2\right) + mr^2\right] + \left[\left(\frac{2}{5}mr^2\right) + mr^2\right] + \left[\frac{2}{5}mr^2\right]$

$= \frac{16}{5}mr^2$

36. **(d)** Dipole moment of a magnet,



$M = IA$

$\Rightarrow 2ml = IA$ [m = pole strength]

$\Rightarrow m = \frac{IA}{2l}$ $\quad [\because\ M = m(2l)]$

Unit of pole strength $(m) = \frac{Am^2}{m} = Am$

37. **(c)** $m = -\frac{P}{V}x$

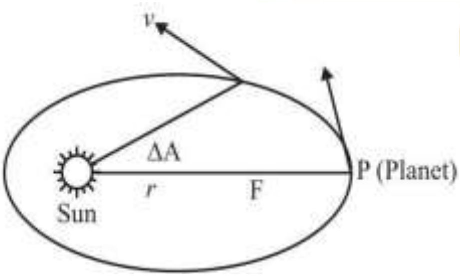
$x(\text{ isothermal }) = 1$

$x(\text{ adiabatic }) = \gamma$

So, $\frac{m_{iso}}{m_{adi}} = \frac{-\frac{P}{V}\cdot 1}{-\frac{P}{V}\cdot \gamma} = \frac{1}{P}$

38. **(c)** According to Kepler's second law of planetary motion, the radius vector joining planet to the sun sweeps out equal areas, in equal interval of time.

i.e., $\frac{\Delta A}{\Delta t} = \text{constant}$



According to figure, r be the position vector of the planet w.r.t. sun and F be the gravitational force. on the planet due to the sun. Then, torque exerted on the planet by this force about the sun is

$\tau = r \times F = 0$ [$\because$ r and F are oppositely directed]

But $\tau = \frac{dL}{dt}$

$\therefore\ \frac{dL}{dt} = 0 \Rightarrow L = \text{constant}$

Angular momentum = constant

Hence, kepler's second law (law of areas) is equivalent to law of conservation of angular momentum.

39. **(c)** We have $= 560\ \text{cm}, R = 10\Omega$.
So, $l_2 = l_1 = 60$
$\Rightarrow l_2 = 560 - 60 = 500\ \text{cm}$
$\therefore$ Internal resistance of cell is given by
$r = \left(\frac{l_1 - l_2}{l_2}\right) R = \left(\frac{560 - 500}{500}\right) 10$
$= \frac{60}{500} \times 10 = \frac{6}{5} = 1.2\Omega.$

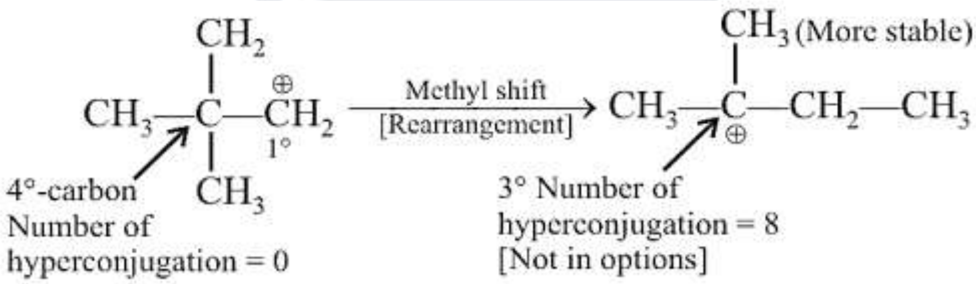
40. **(a)** $H = \frac{R}{4} \tan\theta \Rightarrow H = \frac{2H}{4} \tan\theta \Rightarrow \tan\theta = 2$
So, $\sin\theta = \frac{2}{\sqrt{5}}$ and $\cos\theta = \frac{1}{\sqrt{5}}$
Then, Range $= \frac{v^2 \sin 2\theta}{g} = \frac{v^2 .2 \sin\theta \cos\theta}{g}$
$= \frac{v^2 \times 2 \times \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}}}{g} = \frac{4v^2}{g}.$

Chemistry

41. **(a)** For isoelectronic set of ions greater the charge on cation smaller will be its size and greater the size on anion larger will be its size. Hence, the order of ionic radii will be
$Mg^{2+} < Na^+ < Cl^- < S^{2-}$.

42. **(c)** Antibiotics have either killing effect or a static (inhibitory) effect on microbes.
Bactericidal antibiotics are:
Penicillin ofloxacin etc.
Bacteriostatic antibiotics are:
tetracycline, chloramphenicol etc.

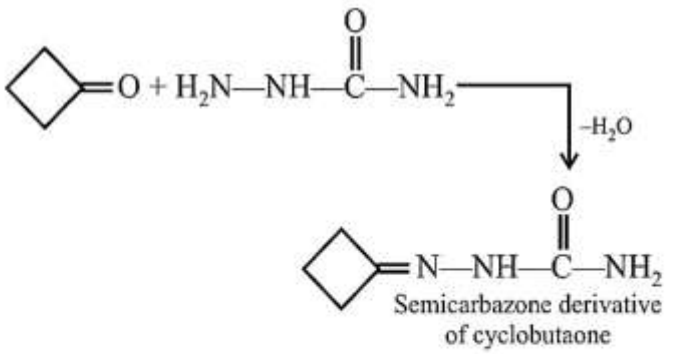
43. **(d)**



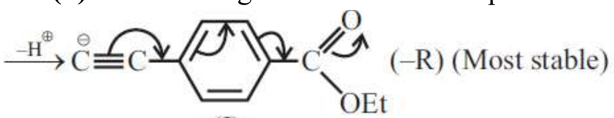
44. **(d)** Sucralose is trichloro derivative of sucrose. It is an artificial sweetening agent which is stable at cooking temperature and does not provide calories.

45. **(d)** Bond order, $= \frac{N_b - N_a}{2}$
Bond order of $N_2(14e^-)$: $BO = \frac{10-4}{2} = 3$
Bond order of $O_2^{2-}(18e^-)$: $BO = \frac{10-8}{2} = 1$
Bond order of $NO^+(14e^-)$: $BO = \frac{10-4}{2} = 3$
Bond order of $F_2(18e^-)$: $BO = \frac{10-8}{2} = 1$
Bond order of $N_2^{2-}(16e^-)$: $BO = \frac{10-6}{2} = 2$
Bond order of $O_2^-(17e^-)$: $BO = \frac{10-7}{2} = 1.5$
Bond oder of $CO(14e^-)$: $BO = \frac{10-4}{2} = 3$
Hence, (N_2, CO, NO^+)have similar bond order, 3 .

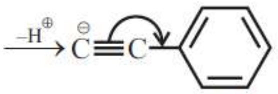
46. **(b)**



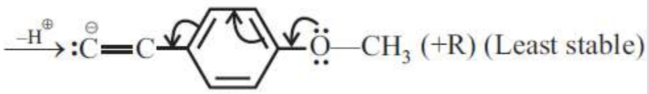
47. **(b)** Acidic strength of an acid can be predicted by determining the stability of its conjugated base.



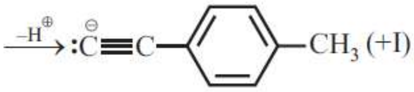
(I)



(II)



(III)



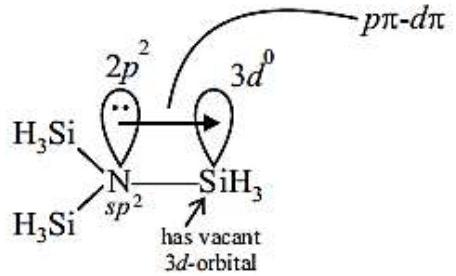
(IV)

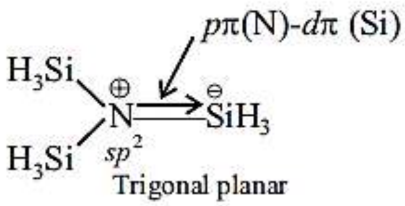
Electron with drawing group $(-R)$ stabilise carbanions but Electron donating groups $(+R > +I)$ destabillise carbanions.
A stronger acid produces stable conjugate base. So, the order of acidity will be
(i) > (ii) > (iv) > (iii).

48. **(b)** Strength of hydrogen bonding depends on the electronegativity of elements. Here, only H_2O will give strongest H -bonding.
HI and H_2S cannot make H -bonds. In HCl, H-bonding is very weak in nature.

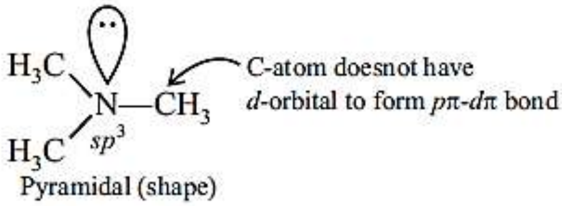
49. **(b)**

$N(SiH_3)_3 \Rightarrow$

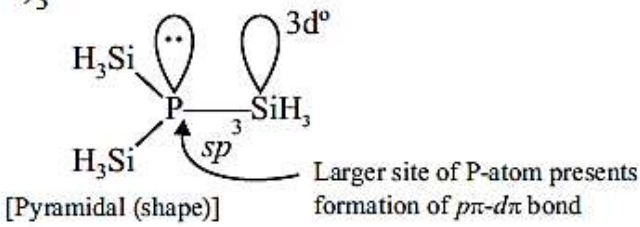




$Me_3N \Rightarrow$



$(SiH)_3P \Rightarrow$



50. (a) $p_{total} = p_A + p_B + p_C$ (According to Dalton's law)

$\Rightarrow p_C = p_{total} - (p_A + p_B) = 10 - (3 + 1) = 6$ atm

$\Rightarrow p_C = X_C \cdot p_{total}$

$\Rightarrow p_C = \dfrac{n_C}{n_A + n_B + n_C} \times p_{total}$

$\Rightarrow \dfrac{6}{10} \times 10 = n_C$

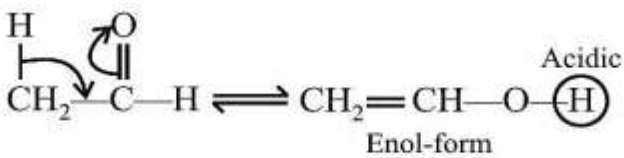
$\Rightarrow n_C = 6.$

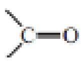
Weight of C is the mixture $= n_C \times$ molar mass of C $= 6 \times 2 = 12$ g.

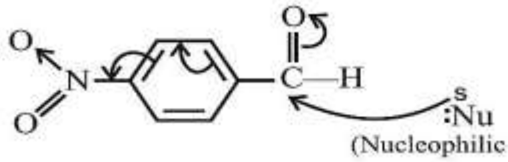
51. (d) (A) Reactivity of aromatic aldehyde and ketones is less than aliphatic carbonyl compound due to steric hindrance caused due to bulkier aryl group.

(B) Fehling's solution is being a very mild oxidising agent, benzaldehyde does not respond to it.

(C) Ethanal on tautomerisation produces its enol form.



(D) Because of the presence of $-NO_2$ group at para position ($-R$ effect), the carbon atom. (sp^2) of  group becomes more susceptible for nucleophilic attack because the sp^2 carbon gains more partial positive charge.



(Nucleophilic addition)

52. **(c)** $Ni(OH)_2 \rightleftharpoons \underset{s\ mol^2}{Ni^{2+}} + \underset{2s\ mol/L}{2OH^-}$

$\Rightarrow K_{sp} = [Ni^{2+}][OH^-]^2 = s \times (2s)^2 = 4s^3$

$2 \times 10^{-15}(\text{ mol/L})^3 = 4\ s^3$

$\Rightarrow s = \left(\frac{2}{4} \times 10^{-15}\right)^{1/3} = 7.9 \times 10^{-6} = 8 \times 10^{-6}\ \text{mol/L}$

$[OH^-] = 2s = 2 \times 8 \times 10^{-6}\ \text{mol/L}$

$pOH = 6 - \log 16 = 4.983 \simeq 5$

$\therefore pH = 14 - pOH = 14 - 5 = 9$ at 25°C.

53. **(c)** Electronic configuration of boron ($Z = 5$); $[He]2s^2 2p^1$. First ionisation will take place from unpaired p^1 electron. $2p^1$-electron of B experience lesser nuclear attractive force for ionisation.

Removal of electron takes place from paired $2s^2$-electrons which requires more energy.

Electronic configuration of $Be(Z = 4)$: $[He]2s^2$.

54. **(d)** PbO_2 is weakly basic in nature. In PbO_2, Pb is in +4 which is less stable than its +2-oxidation state because of inert pair effect. Hence, Pb^{4+} of PbO_2 has a tendency to be reduced into more stable $Pb^{2+}(PbO)$ showing strong oxidising nature of PbO_2.

55. **(a)** 1 mole of O_2 gas $= 6.022 \times 10^{23}$ molecules of O_2

$= 32$ g of O_2

$= 22.4\ L^2$ of O_2 at STP.

56. **(d)** The temperature dependence of the rate of a chemical reaction can be explained by Arrhenius equation, $k = A \cdot e^{E_a/RT}$

Taking ln on both side

$\Rightarrow \ln k = \ln A - \frac{E_a}{RT}$

$\Rightarrow \frac{E_a}{RT} = \ln A - \ln k.$

57. **(c)** For orange radiation of $\lambda = 6300$Å

$= 63 \times 10^{-8}$ m

Wave number,

$\bar{v} = \frac{1}{\lambda} = \frac{1}{63 \times 10^{-8}\ m} = 1.587 \times 10^6\ m^{-1}$

Frequency, $v = \frac{c}{\lambda} = \frac{3\times10^8\ m\ s^{-1}}{63\times10^{-8}\ m}$

$= 4.761 \times 10^{14}\ s^{-1}$.

58. **(b)** Carbon free radical is sp^2 hybridised and is triangular planar. the given reaction is the chain termination step of a free radical substitution reaction. Here, the trigonal planar free radical $(C_2H_5 - \dot{C}H - CH_3)$ gets attracked by $Cl^-$with equal ease of retention and inversion in configuration. So, the final product is the equimolar mixture of d - and l-forms called racemic mixture, which is optically inactive.

59. **(c)** Relative lowering in vapour pressure

$= \frac{(p^\circ - p)}{p^\circ} = X_A$

[Given $p^\circ = 760$ torr. Let, d water = 1 g/mL $\therefore$ 54 mL = 54 g water]

$\Rightarrow X_A = \frac{n_A}{n_A + n_{H_2O}}$

$$\Rightarrow \frac{760-p}{760} = \frac{\frac{160}{160}}{\frac{160}{160}+\frac{54}{18}} = \frac{1}{4}$$

$\Rightarrow p = \frac{3}{4} \times 760 = 570$ torr.

60. (c) According to Mendeleev's periodic law "The properties of elements are periodic functions of their atomic weights."

61. (c) Given, density (d) $= (1.2 \text{ g mL}^{-1})$,

Vol. of sol. $= \frac{\text{mass}}{\text{density}} = \frac{100}{1.2}$ mL

$\Rightarrow$ Molarity $= \frac{\text{moles of solute}}{\text{Vol. of solution (in litre)}}$

$$M = \frac{\text{Given mass } 1000}{\text{Molarmass } V_I(\text{ in Litre })}$$

$M = \frac{40 \times 1.2 \times 1000}{36.5 \times 100} = 13.1M.$

62. (b) Copper matter is composed of copper sulphite Cu_2 S and (Iron Sulphide) FeS.

63. (d) $\therefore$ Molarity $= \frac{W_{NaOH} 1000}{M_{NaOH} V_{solution} (\text{ in L})}$

$= \frac{10 \times 1000}{40 \times 500} M = 0.5M.$

64. (b) Molecules in gases are in constant random motion with high velocity. Hence, there is a complete disorder of molecules in gases.

65. (b) Let us complete the reactions:

(i) $Fe^{2+} + H_2O_2 \longrightarrow Fe^{3+} + H_2O$

(ii) $Mn^{7+}O_4^- + H^+ + H_2O_2^- \longrightarrow Mn^{2+} + O_2^0 + H_2O$

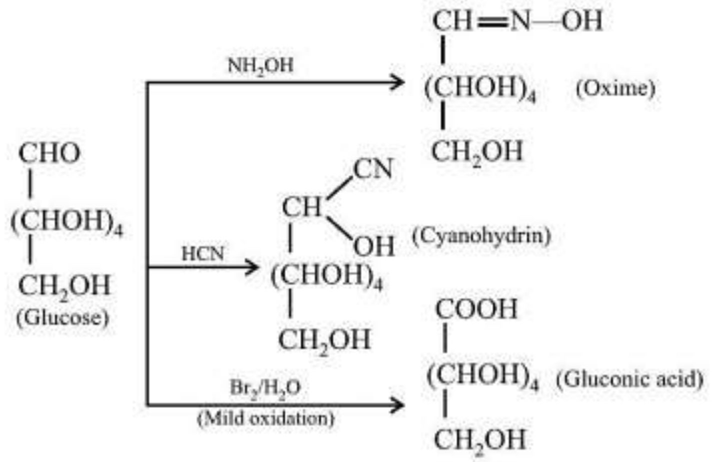
(iii) $I_2^0 + H_2O_2^- + OH^- \longrightarrow I^- + O_2^0 + H_2O$

(iv) $Mn^{2+} + H_2O_2^- \longrightarrow Mn^{4+} + OH^{2-}$

In reactions, (i) and (iv)H_2O_2, acts as oxidising agent and itself gets reduced to H_2O.

66. (d) Glucose does not react with $NaHSO_3$, to form hydrogen sulphide addition product.

Glucose reacts with NH_2OH, HCN and Br_2/H_2O and give oxime, cyanohydrin, gluconic acid respectively.



67. (b) The correct cell representation is

$Cd/Cd^{2+}(0.01M)||Cu^{2+}(0.01M) \mid Cu$

Given $E_{Cu^{2+}/Cu} = 0.35$ V

$E_{Cd^{2+}/Cd} = -0.4$ V

$E^\circ_{Cell} = E_{Cu^{2+}/Cu} - E_{Cd^{2+}/Cd} = 0.35 - (-0.4) = 0.75$ V

$$E_{Cell} = E^\circ_{Cell} - \frac{0.059}{2} \log \frac{[Cd^{2+}]}{[Cu^{+2}]}$$

$$E_{Cell} = 0.75 - \frac{0.059}{2} - \log \frac{[0.01]}{[0.01]} \quad (\because \log 1 = 0)$$

$E_{Cell} = 0.75.$

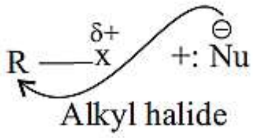
$\Rightarrow E_{cell\,(\,Observed\,)} = E_{internal} - E_{external}$

$= 0.75 - (\text{ resistance} \times \text{current}) \begin{Bmatrix} E = V \\ V = IR \end{Bmatrix}$

$= 0.75 - (4 \times 0.15) = (0.75 - 0.60)V = 0.15\ V$

68. **(a)** Alkyl halides readily undergo nucleophilic substitution like S_N2 and S_N1.

Here a stronger nucleophile, $Nu^{\ominus}$ substitutes a weaker nucleophile (leaving group: X^-) from an alkyl halide.



$R{-}Nu + X^-$
Nucleophilic
Substitution

69. **(b)** Extensive properties depend on mass of substance volume, enthalpy etc.

Intensive properties does not depend on mass of substance like temperature, pressure and density.

where, density $= \frac{\text{mass}}{\text{volume}} = \frac{\text{extensive property}}{\text{extensive property}}$

$=$ Intensive property.

70. **(a)** $Cr^{2+} \Rightarrow [Ar]3d^4, E^0_{Cr^{3+}/Cr^{2+}} = -0.41\ V$

$Mn^{3+} \Rightarrow [Ar]3d^4, E^0_{Mn^{3+}/Mn^{2+}} = +1.51\ V$

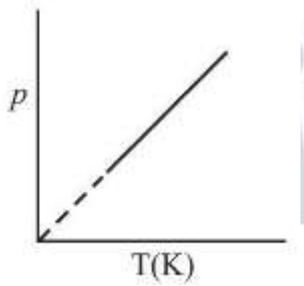
So, reducing power $Cr^{2+} > Mn^{2+}$.

71. **(c)** Li^+among group- 1 metals and Mg^{2+} of group -2 metals have similar ionic radii Li and Mg react with oxgen to give monoxide Li_2O and MgO respectively with nitrogen they give Li_3 N and Mg_3 N_2 respectively.

72. **(a)** Isochores are drawn at constant volume between pressure vs temperature (T, in K) as;

For 1 mole of an ideal gas, $pV = RT$

At isochoric condition, $p \propto T$.

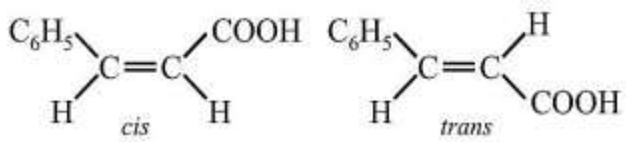


73. **(b)** In hydroxyl ion, $H - \ddot{\underset{..}{O}}:$

3 lone pair of electrons are present on oxygen atom of hydroxyl ion.

74. **(c)** The insecticide, DDT is a non-biodegradable pollutant.

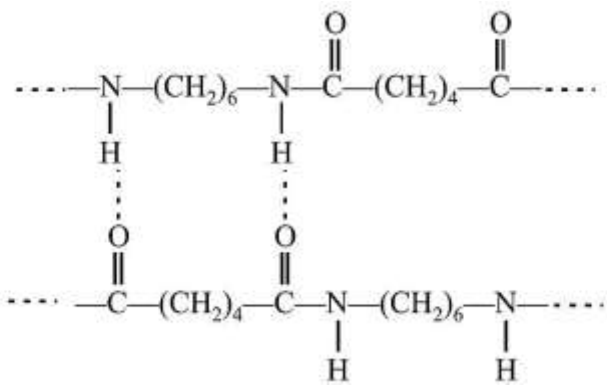
75. **(d)** Cinnamic acid is able to show geometrical isometrism.



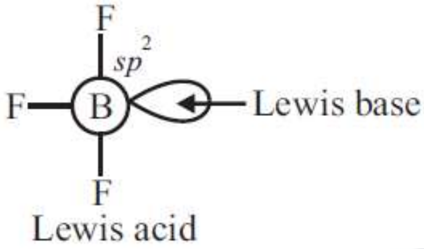
No olefinic $C = C$ is present in butyric acid, aspartic acid and palmitic acid.

76. **(b)** Polyvinyl chloride, neoprene and teflon are addition homopolymers. They do not have suitable N -atoms to show H -bonding.

Nylon-6, 6 is a polyamide formed by condensation polymerisation of hexamethylene diamine and adipic acid. In which H -bonds are present:



77. **(c)** B in BF_3 has incomplete octet and thus it is electron deficiency due to which it can accept lone-pair of electrons and acts as Lewis acid.



78. **(b)** The field strength order of the ligands is $H_2O < NH_3 < NO_2^-$

So, the order of CFSE values:

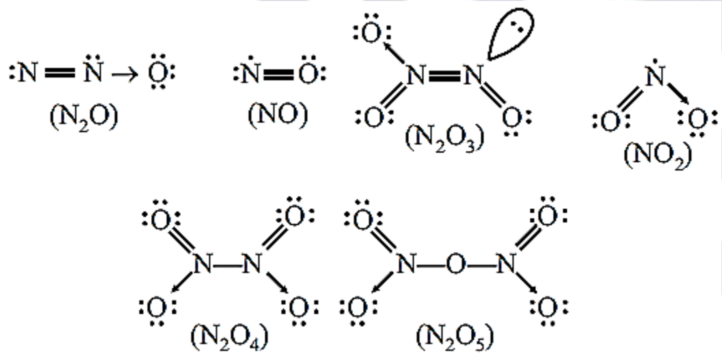
$[Ni(H_2O)_6]^{2+} < [Ni(NH_3)_6]^{2+} < [Ni(NO_2)_6]^{4-}$

We know, $E = \frac{hc}{\lambda}$ or $E \propto \frac{1}{\lambda}$

Hence, the order of wavelength of absorption in visible region will be,

$[Ni(H_2O)_6]^{2+} > [Ni(NH_3)_6]^{2+} > [Ni(NO_2)_6]^{4-}$

79. **(b)** NO and NO_2 are



$N_2O, N_2O_3,\ N_2O_4,\ N_2O_5$ do not have unpaired electrons. So, they are diamagnetic

80. **(c)** BF_3 is used as a catalyst in several industrial processes due to its strong Lewis acid nature.

BF_3 has sp^2 hybridisation and is electron incomplete octet.

Mathematics

81. **(d)** $I = \int \left(\frac{x^3-1}{x^3+x}\right) dx = \int \left(1 - \frac{x+1}{x^3+x}\right) dx$

$\Rightarrow I = \int 1 \cdot dx - \int \frac{(x+1)}{x^3+x} dx = x - \int \frac{(x+1)}{x(x^2+1)} dx$

Let $\frac{x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$\Rightarrow (x+1) = A(x^2+1) + (Bx+C)x$

$\Rightarrow (x+1) = (A+B)x^2 + Cx + A$

On comparing coefficient, we get

$A + B = 0, C = 1, A = 1$

$\Rightarrow B = -1$

$\therefore I = x - \int \frac{1}{x}dx - \int \frac{(1-x)}{x^2+1}dx$

$\Rightarrow I = x - \log|x| - \tan^{-1}x + \frac{1}{2}\log(x^2+1) + C$

82. **(c)** Let $y = f(x) = x^3 + 3x^2 - 2$

$\frac{dy}{dx} = 3x^2 + 6x$

For stationary points $\frac{dy}{dx} = 0$

$\Rightarrow 3x(x+2) = 0$

$\Rightarrow x = 0, -2$

83. **(a)** We have,

In radius $(r) = \frac{\Delta}{s}$, where Δ = Area, S = Semi-perimeter

$\Rightarrow \Delta = (r.s) \Rightarrow \Delta = 8 \times \frac{36}{2} = 144\ cm^2$

84. **(b)** Let $y = f(x) = ([x]^2 - [x] - 2)^{-1/2}$

$\Rightarrow y = \frac{1}{\sqrt{[x]^2 - [x] - 2}}$

For real valued

$[x]^2 - [x] - 2 > 0$

$\Rightarrow \{[x]-2\} \cdot \{[x]+1\} > 0$

$[x] \in R - [-1,2]$

So, $[x] \in R - [-1,3)$ is correct option.

85. **(c)** Given straight lines

$2x^2 + 5xy + 3y^2 + 6x + 7y + 4 = 0$

We know that angle between straight lines represented by $ax^2 + by^2 + 2gx + 2fy + c + 2hxy = 0$ is

$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

$\tan\theta = \frac{2\sqrt{\left(\frac{5}{2}\right)^2 - 6}}{5} = \pm\frac{1}{5}$

$\Rightarrow \theta = \tan^{-1}k = \tan^{-1}\left(\pm\frac{1}{5}\right)$

$\Rightarrow \left(k = \pm\frac{1}{5}\right)$

86. **(d)** Case I: When $x^2 - x - 6 \geq 0$ i.e., $x \in (-\infty, -2] \cup [3, \infty)$

$\therefore x^2 - x - 6 = x + 2$

$\Rightarrow x^2 - 2x - 8 = 0 \Rightarrow x = -2, 4$

Case II: When $x^2 - x - 6 < 0$ i.e., $x \in (-2, 3)$

$x^2 - x - 6 = -x - 2$

$\Rightarrow x^2 = 4 \Rightarrow x = 2$

Hence, $x = \{-2, 2, 4\}$.

87. **(a)** A set formed with whose order of occurrence of real numbers a, b, c is preassigned is called an ordered triad.

88. **(b)** Given that $3f(x) - 2f\left(\frac{1}{x}\right) = x,$

Differentiate both sides w.r.t x

$\Rightarrow 3f'(x) - 2f'\left(\frac{1}{x}\right)\left(-\frac{1}{x^2}\right) = 1$

Put $x = 2$

$\Rightarrow 3f'(2) + \frac{1}{2}f'\left(\frac{1}{2}\right) = 1$ …. (i)

Put $x = \frac{1}{2}$

$3f'\left(\frac{1}{2}\right)+8f'(2)=1$ .... (ii)

On solving equations (i) and (ii), we get

$\Rightarrow f'(2)=\frac{1}{2}$

89. **(a)** Let $I=\int_1^{e^2}\frac{dx}{x(1+\log x)^2}$

Let $1+\log x=t$

Differentiate w.r.t x

$\frac{1}{x}dx=dt$

$I=\int_1^3\frac{dt}{t^2}=\left[\frac{t^{-1}}{-1}\right]_1^3=\left(\frac{3^{-1}}{-1}+\frac{1^{-1}}{1}\right)$

$I=-\frac{1}{3}+1=\frac{2}{3}$

90. **(c)** Given expansion is $\left(\frac{x}{2}+\frac{1}{x}+\sqrt{2}\right)^5=I$ (say)

$\Rightarrow I=\left(\sqrt{\frac{x}{2}}+\frac{1}{\sqrt{x}}\right)^{10}=\left(\frac{x+\sqrt{2}}{\sqrt{2x}}\right)^{10}$

$\Rightarrow I=\frac{(x+\sqrt{2})^{10}}{32x^5}$

This have constant term in the expansion

$=\frac{\text{Coefficient of } x^3 \text{ in } (x+\sqrt{2})^{10}}{32}$

$\Rightarrow\frac{{}^{10}C_5\times 4\sqrt{2}}{32}=\frac{a\sqrt{2}}{2}$

$\Rightarrow (a=63)$

91. **(a)** Given expression is

$\cos^2 x+\cos^2\left(x+\frac{\pi}{3}\right)+\cos^2\left(x-\frac{\pi}{3}\right)$

Since $\cos^2 x=\frac{\cos 2x+1}{2}$

$\Rightarrow\left[\frac{1+\cos 2x}{2}\right]+\left[\frac{1+\cos\left(2x+\frac{2\pi}{3}\right)}{2}\right]$

$+\left[\frac{1+\cos\left(2x-\frac{2\pi}{3}\right)}{2}\right]$

$\Rightarrow\frac{1}{2}\left[1+\cos 2x+1+\cos\left(2x+\frac{2\pi}{3}\right)\right.$

$\left.+1+\cos\left(2x-\frac{2\pi}{3}\right)\right]$

$\Rightarrow\frac{1}{2}\left[3+\cos 2x+2\cos 2x\cdot\cos\frac{2\pi}{3}\right]$

$\Rightarrow\frac{1}{2}\left[3+\cos 2x+2\cos 2x\left(-\frac{1}{2}\right)\right]$

$\Rightarrow\frac{1}{2}[3+\cos 2x-\cos 2x]=\left(\frac{3}{2}\right)$

92. **(c)** Given, circle $x^2+y^2-4x-6y+11=0$

$\Rightarrow$ Centre $=(-g,-f)=(2,3)$

One end of diameter $=(3,4)$(Given)

So, $\frac{h+3}{2}=2,\frac{k+4}{2}=3$

$h = 1, k = 2 \Rightarrow (1,2)$

93. (d) Given $\cos 48^\circ \cos 12^\circ$

$$= \frac{1}{2}(2\cos 48^\circ \cos 12^\circ)$$

$$= \frac{1}{2}(\cos 60^\circ + \cos 36^\circ) = \frac{1}{2}\left[\frac{1}{2} + \frac{\sqrt{5}+1}{4}\right]$$

$$= \left(\frac{3+\sqrt{5}}{8}\right)$$

94. (d) Given $\sin\theta + \operatorname{cosec}\theta = 2$

Maximum value of $\sin\theta = 1$

So, $\sin^{2020}\theta + \operatorname{cosec}^{2020}\theta = 1 + 1 = 2$

95. (b) Given $\frac{\sqrt{1+x}+(1-x)^{3/2}}{(1+x)+\sqrt{1+x}}$

Here $|x|$ is very small, x^2 is negligible.

$$\frac{1+x+(1-x)^{3/2}}{(1+x)+\sqrt{1+x}} = \frac{1+\frac{1}{2}x+1-\frac{3}{2}x}{1+x+1+\frac{x}{2}}$$

$$= \frac{2-x}{2+\frac{3x}{2}} = \frac{4-2x}{4+3x}$$

$$= \frac{(4-2x)(4-3x)}{(16-9x^2)} = \frac{16-20x+6x^2}{16-9x^2}$$

$= 1 - \frac{5x}{4}$ $[\because x^2 \text{ negligible}]$

96. (d) Given $f: Z \to Z$

$f(x+y) = f(x) + f(y); x, y \in Z$

$\therefore f(x) = kx$

So, there are infinitely many bijections.

97. (c) Given matrix

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ null matrix

So, A is nilpotent matrix.

98. (c) Given parabolas,

$$169[(x-1)^2 + (y-3)^2] = (5x - 12y + 17)^2$$

$$\Rightarrow (x-1)^2 + (y-3)^2 = \left(\frac{5x-12y+17}{13}\right)^2$$

$\Rightarrow (SP = PM)$

where S is focus and M is foot of perpendicular

Here, focus is $S(1,3)$

and directrix $(5x - 12y + 17) = 0$

$\therefore$ Distance of focus from directrix

$$\Rightarrow 2a = \left|\frac{5-36+17}{\sqrt{25+144}}\right|$$

$\Rightarrow 2a = \frac{14}{13}$

$\therefore$ Latus rectum $= 4a = \frac{28}{13}$

99. (b) Given $\tan h(x) = \frac{1}{3}$ $\left\{\because \tan h\,(x) = \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)\right\} \Rightarrow \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{3} \Rightarrow 2e^x = 4e^{-x} \Rightarrow (e^{2x} = 2)$ and $\tan h(3x) =$ $\frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \frac{(e^{2x})^3 - 1}{(e^{2x})^3 + 1} = \frac{8-1}{8+1} = \frac{7}{9}$

100. **(c)** Let $A(0,0), B(1,0), C(1,1)$ and $D(0,1)$ be vertices of the square, which is formed by given lines.

Here equation of AC,

$y = \left(\frac{1-0}{1-0}\right)x \Rightarrow (y = x)$

Equation of BD

$y - 0 = -1(x - 1)$

$\Rightarrow y = -x + 1 \Rightarrow x + y = 1$

101. **(d)** Given centre $(5,4)$ of the circle touches Y-axis.

So, radius $= 5$ units

Equation of circle

$\Rightarrow (x-5)^2 + (y-4)^2 = 5^2$

$\Rightarrow x^2 + y^2 - 10x - 8y + 16 = 0$

102. **(a)** Since, according to question

Tossing of three coins

So, $\rho(X^{-1}(2))$ means probability of getting two tails when three coins are tossed.

$\Rightarrow \rho(X^{-1}(2)) = \frac{3}{8}$

103. **(b)** Since, according to question

$p = \frac{|0+0-k|}{\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta}}$,

$q = \frac{|0+0-k\cos^2\theta|}{\sqrt{\cos^2\theta + \sin^2\theta}}$

$\Rightarrow p^2 = \frac{k^2}{\sec^2\theta + \operatorname{cosec}^2\theta}, q^2 = k^2\cos^2 2\theta$

$\Rightarrow p^2 = \frac{k^2}{4}\sin^2 2\theta, q^2 = k^2\cos^2 2\theta$

Thus, $p^2 + \frac{q^2}{4} = \frac{k^2}{4} \Rightarrow 4p^2 + q^2 = k^2$

104. **(c)** Given circle $x^2 + y^2 + 6x + 2ky + 25 = 0$ touches Y-axis centre $(-3, -k)$ and radius $= 3$ units

Also radius $= \sqrt{9 + k^2 - 25}$

$\Rightarrow \sqrt{k^2 - 16} = 3$

$\Rightarrow k^2 = 25 \Rightarrow k = \pm 5$

105. **(a)** Number of ways $= {}^8C_4 \times {}^5C_3 = 700$

106. **(b)** Here $\left(\frac{x^4}{x^3 - 3x + 2}\right)$ is a improper fraction since (degree of number $\geq$ degree of denominator) for a improper fraction.

107. **(a)** Number of ways $= {}^6P_4 = 360$

108. **(b)** Given $[a, b]$ is range of $\frac{x+2}{2x^2+3x+6}$ and $x \in R$

Let $y = \frac{x+2}{2x^2+3x+6}$

$\Rightarrow 2yx^2 + 3xy + 6y = x + 2$

or $2yx^2 + (3y-1)x + 6y - 2 = 0, x \in R$. So, $D \geq 0$

$\Rightarrow (3y-1)^2 - 4(6y-2)(2y) \geq 0$

$\Rightarrow 39y^2 - 10y - 1 \leq 0 \Rightarrow (3y-1)(13y+1) \leq 0$

$\Rightarrow yy \in \left[-\frac{1}{13}, \frac{1}{3}\right]$ So, $a = -\frac{1}{13}, b = \frac{1}{3}$

$\therefore a < 0, b > 0$

109. **(c)** Let the distance $P(h,k)$ on the circle

So, given equation $\frac{|4h+3k-12|}{5} = \frac{4}{5}$

$\Rightarrow (4h + 3k = 16) \Rightarrow (4h + 3k = 8)$

Since (h,k) lies on circle so

$h^2 + k^2 = 4$

$\Rightarrow \left[h^2 + \left(\frac{16-4h}{3}\right)^2 = 4\right]$ or $\left[h^2 + \left(\frac{8-4h}{3}\right)^2 = 4\right]$

$\Rightarrow (25h^2 - 128h + 220 = 0)$ or $(25h^2 - 64h + 28 = 0)$

So, $25h^2 - 64h + 28 = 0$ will be considered and $\left(h = 2, \frac{14}{25}\right)$

At $h = 2, k = 0$

At $h = \frac{14}{25}, k = \frac{48}{25}$

So, $(h,k) = (2,0)$ or $\left(\frac{14}{25}, \frac{48}{25}\right)$

110. **(c)** Since, we know that

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

or $\cos 60° = \frac{a^2+b^2-c^2}{2ab}$

$$\Rightarrow \frac{1}{2} = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow a^2 + b^2 = ab + c^2$$

Now, $\frac{c(a+b)+(a^2+b^2)}{(b+c)(c+a)} = \frac{ca+bc+ab+c^2}{bc+ab+c^2+ac} = 1$

111. **(d)** Given $f(0) = f(1) = 3f(2) = -3$

Let $f(x) = ax^2 + bx + c$ is a two degree polynomials

$\Rightarrow c = -3, a + b + c = -3$

$\Rightarrow a + b = 0$ and $4a + 2b + c = -1$

$\Rightarrow 4a + 2b = 2 \Rightarrow a = 1, b = -1$

$$\int \frac{f(x)}{x^3 - 1} dx = \int \frac{x^2 - x - 3}{x^3 - 1} dx$$

Now $\frac{x^2-x-3}{x^3-1} = \frac{A}{x-1} + \left(\frac{Bx+C}{x^2+x+1}\right)$

Comparing coefficients, we get

$A + B = 1, A - B + C = -1, A - C = -3$

we get, $A = -1, B = 2, C = 2$

$$\int \left(\frac{x^2 - x - 3}{x^3 - 1}\right) dx = \int \frac{-1}{(x-1)} dx + \int \frac{2x + 2}{(x^2 + x + 1)} dx$$

$$= -\log|x + 1| + \int \frac{(2x + 1)}{x^2 + x + 1} dx + \int \frac{1dx}{x^2 + x + 1}$$

$$= -\log|x - 1| + \log(x^2 + x + 1) + \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \left(\sqrt{\frac{3}{2}}\right)^2}$$

$$= \log\left(\frac{x^2 + x + 1}{|x - 1|}\right) + \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) + C$$

112. **(a)** $\sqrt{x + 1} - |\sqrt{x - 1}| = \sqrt{4x - 1}$

Here $x \geq 1$ to define each

Now, $\sqrt{x + 1} - \sqrt{4x - 1} = |\sqrt{x - 1}|$

On squaring on both sides, we get

$\Rightarrow x + 1 + 4x - 1 - 2\sqrt{(x + 1)(4x - 1)} = x - 1$

$\Rightarrow 4x + 1 = 2\sqrt{(x + 1)(4x - 1)}$

Again, squaring both the sides, we get

$\Rightarrow 16x^2 + 1 + 8x = 4(4x^2 + 3x - 1)$

$\Rightarrow 4x = 5 \Rightarrow x = \frac{5}{4}$

113. **(d)** According to the given conditions (i) and (ii) both are correct.

114. **(b)** Given function

$f(x) = \cos^{-1}\left(\frac{x-3}{2}\right) - \log_{10}(4-x)$

Here, domain of $\log_{10}(4-x) \Rightarrow 4 - x > 0$

$\Rightarrow x < 4$

and domain of $\cos^{-1}\left(\frac{x-3}{2}\right)$

$\Rightarrow -1 \leq \frac{x-3}{2} \leq 1 \Rightarrow 1 \leq x \leq 5$

Now domain of $f(x)$ will be $x \in [1,4]$

115. **(c)** Given $\int \left(1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots \infty\right) dx$

$= \int e^x \cdot dx = (e^x + c)$

116. **(b)** Given $|a| = 2, |b| = 3$ and $(a, b) = \frac{\pi}{6}$

$|\vec{a} \times \vec{b}|^2 = [|\vec{a}||\vec{b}|\sin\theta]^2 = \left(2 \times 3 \times \frac{1}{2}\right)^2 = 9$

117. **(a)** Given $\cos x - \sin x = 0$

$\Rightarrow \tan x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}; (n \in Z)$

118. **(b)** Given $\sum_{i=1}^{n=1} \frac{1}{2-a^i}$

$1, \alpha, \alpha^2, \ldots, \alpha^{n-1}$ are the nth root of unity.

$\because x^n - 1 = 0$

$\Rightarrow x^n - 1 = (x-1)(x-\alpha)(x-\alpha^2) \ldots (x - \alpha^{n-1})$

Taking log both side

$\Rightarrow \log(x^n - 1) = \log(x-1) + \log(x-\alpha)$

$+ \cdots \log(x - \alpha^{n-1})$

Differentiating w.r.t. ' x ', we get

$$\Rightarrow \frac{nx^{n-1}}{x^n - 1} = \frac{1}{(x-1)} + \frac{1}{(x-\alpha)} + \cdots + \frac{1}{(x-\alpha^{n-1})}$$

At$(x = 2)$

$$\Rightarrow \frac{n \cdot 2^{n-1}}{2^n - 1} = 1 + \frac{1}{2-\alpha} + \frac{1}{2-\alpha^2} + \cdots + \frac{1}{2-\alpha^{n-1}}$$

$$\Rightarrow \sum_{i=1}^{n=1} \frac{1}{2-a^i} = \left(\frac{n \cdot 2^{n-1}}{2^n - 1} - 1\right) = \frac{(n-2)2^{n-1} + 1}{2^n - 1}$$

119. **(c)** Given differential equation $(e^{y-x})dy = (e^x - e^y)dx$

$\Rightarrow e^y \cdot \frac{dy}{dx} = e^{2x} - e^x \cdot e^y \Rightarrow e^y \cdot \frac{dy}{dx} + e^x \cdot e^y = e^{2x}$

Let $e^y = t$

$\Rightarrow e^y \cdot \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dt}{dx} + e^x \cdot t = e^{2x}$

$IF = e^{\int e^x dx} = e^{e^x}$

$\therefore$ Solution is

$t \cdot e^{e^x} = \int e^{2x} \cdot e^{e^x} \cdot dx + c$

Let $e^x = u \Rightarrow e^x dx = du$

$\Rightarrow t \cdot e^{e^x} = \int u \cdot e^u du + c = ue^u - e^u + c$

$\Rightarrow e^y \cdot e^{e^x} = e^{e^x}(e^x - 1) + c$

120. **(b)** Since $(2 + \sqrt{3})^5 = {}^5C_0 \cdot 2^5 \cdot (\sqrt{3})^0 + {}^5C_1 \cdot 2^4 \cdot (\sqrt{3})$

$+ {}^5C_2 \cdot 2^3(\sqrt{3})^2 + {}^5C_3 \cdot 2^2 \cdot (\sqrt{3})^3$
$+ {}^5C_4 \cdot 2 \cdot (\sqrt{3})^4 + {}^5C_5 \cdot 2^0 \cdot (\sqrt{3})^5$
$= 32 + 80\sqrt{3} + 240 + 120\sqrt{3} + 90 + 9\sqrt{3}$
$(2+\sqrt{3})^5 = 362 + 209\sqrt{3} = 723.99$
$\Rightarrow [(2+\sqrt{3})^5] = 723 \Rightarrow 723 = 3^1 \times 241^1$

Now, positive integral divisors of
$723 = (1+1)(1+1) = 4$

121. **(b)** Since direction cosines are $\cos\alpha, \cos\beta, \cos\gamma$
$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$
So, $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

122. **(b)** Since, circle is passing three (0,0) and intercepts are a and b.

$$f(x) = (x-3)(x-5)\begin{vmatrix} 1 & 2(x+3) & 3(x^2+9+3x) \\ 1 & 2(x+5) & 4(x^2+25+5x) \\ 1 & 2 & 3 \end{vmatrix}$$

Hence, f(x)

$$= \begin{vmatrix} x-3 & 2(x-3)(x+3) & 3(x-3)(x^2+9+3x) \\ x-5 & 2(x-5)(x+5) & 4(x-5)(x^2+25+5x) \\ 1 & 2 & 3 \end{vmatrix}$$

$\Rightarrow f(3) = f(5) = 0$
So, $f(1)f(3) + f(3)\cdot f(5) + f(5)f(1) = f(3)$

123. **(a)** Given ellipse is $2x^2 + 3y^2 - 4x - 12y + 13 = 0$

$$\Rightarrow \frac{(x-1)^2}{\left(\frac{1}{\sqrt{2}}\right)^2} + \frac{(y-1)^2}{\left(\frac{1}{\sqrt{3}}\right)^2} = 1$$

Here, centre $(h,k) = (1,2)$

Now, $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{1/3}{1/2}} = \frac{1}{\sqrt{3}}$

$\Rightarrow a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{3}} \Rightarrow ae = \frac{1}{\sqrt{6}}$

Hence, foci $(h \pm ae, k) \equiv \left(1 \pm \frac{1}{\sqrt{6}}, 2\right)$

124. **(a)** Let $P(K) = \frac{k^5}{5} + \frac{k^3}{3} + \frac{7k}{15}, k \in N$

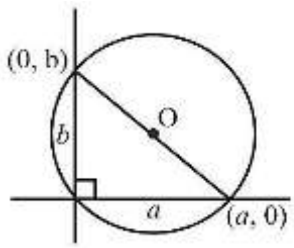
$$= \frac{3k^5}{15} + \frac{5k^3}{15} + \frac{(15-5-3)k}{15}$$

$$= \frac{3}{15}(k^5 - k) + \frac{5}{15}(k^3 - k) + k$$

$$= \frac{1}{5}\underbrace{k(k^2+1)(k+1)(k-1)}_{\text{divisible by 5}} + \frac{1}{3}\underbrace{k(k+1)(k-1)}_{\text{divisible by 3}} + k$$

So, overall it is a natural number.
or for objective point of view you can assume $k = 1$ $(k \in N)$ and solve k.
Now, we will get natural number.

125. **(d)** Centre $= \left(\frac{a}{2}, \frac{b}{2}\right)$, Radius $= \frac{\sqrt{a^2+b^2}}{2}$



Hence, equation of circle is

$\Rightarrow \left(x-\frac{a}{2}\right)^2+\left(y-\frac{b}{2}\right)^2=\frac{a^2+b^2}{4}$

$\Rightarrow x^2+y^2-ax-bx=0$

126. **(d)** Since equation of angle bisector for $ax^2+2hxy+by^2=0$ is

$\Rightarrow \frac{x^2-y^2}{a-b}=\frac{xy}{h}$

Hence for $x^2-2pxy-y^2=0$

$\Rightarrow \frac{x^2-y^2}{1-(-1)}=\frac{xy}{-p}\Rightarrow x^2-y^2+\frac{2xy}{p}=0$

Given equation of angle bisector is

$x^2-2qxy-y^2=0$

On comparing the coefficients, we get

$\Rightarrow \frac{2}{p}=-2q\Rightarrow pq=-1$

127. **(a)** Given $P^{2006}=O$ and $PQ=P+Q$

where P and Q are square matrix

$\Rightarrow P^{2006}Q=P^{2006}+Q\cdot P^{2005}\Rightarrow O=O+Q\cdot P^{2005}$

$\Rightarrow P^{2005}\cdot Q=O\Rightarrow \det.(P^{2005}\cdot Q)=0$

$\Rightarrow \det P^{2005}(\det Q)=0\Rightarrow \det Q=0$

128. **(b)** Let $I=\int\left(\frac{2\tan x}{1+2\tan^2 x}\right)dx$

$=\int\frac{2\sin x\cdot\cos x}{\cos^2 x+2\sin^2 x}dx=\int\frac{2\sin x\cdot\cos x}{1+\sin^2 x}dx$

Let $1+\sin^2 x=t\Rightarrow 2\sin x.\cos x dx=dt$

$I=\int\frac{dt}{t}=\log|t|+c_1=\log|1+\sin^2 x|+c_1$

$=\log|2\sin^2 x+\cos^2 x|+c_1$

$=\log 2+\left|\sin^2 x+\frac{\cos^2 x}{2}\right|+c_1$

$=\log\left|\frac{\cos^2 x}{2}+\sin^2 x\right|+c\ [\because\ \log 2+c_1=c]$

129. **(a)** Since, given that

$\theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

So, $\cos^{-1}(\sin\theta)\Rightarrow\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\theta\right)\right]=\left(\frac{\pi}{2}-\theta\right)$

130. **(d)** Given $\vec{a}=\alpha\hat{i}+3\hat{j}-6\hat{k},\vec{b}=2\hat{i}-\hat{j}+\beta\hat{k}$

For $(\vec{a})$ and $(\vec{b})$ may be collinear $(\vec{a}\times\vec{b})=0$

$\Rightarrow\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\\alpha&3&-6\\2&-1&\beta\end{vmatrix}=0$

$\Rightarrow\hat{i}(3\beta-6)-\hat{j}(\alpha\beta+12)+\hat{k}(-\alpha-6)=0$

$\Rightarrow(3\beta-6)=0,(\alpha\beta+12)=0$ and $(\alpha+6)=0$

$\Rightarrow(\beta=2),(\alpha=-6)$

131. **(d)** Given $2\alpha=-1-i\sqrt{3},2\beta=-1+i\sqrt{3}$

Now, $5\alpha^4+5\beta^4+7\alpha^{-1}\beta^{-1}$

$\Rightarrow 5(\alpha^4+\beta^4)+\frac{7}{\alpha\beta}$

or $5[(\alpha^2+\beta^2)-2\alpha^2\beta^2]+\frac{7}{\alpha\cdot\beta}$

$= 5\left[\left\{\frac{1}{4}(2\alpha+2\beta)^2 - 2\alpha\beta\right\}^2 - 2\alpha^2\beta^2\right] + \frac{7}{\alpha\cdot\beta}$

$= 5\left[\left(\frac{1}{4}\times 4 - 2\right)^2 - 2\right] + 7$

$= 5[1-2]+7 = 2$

132. **(d)** Let l, m, n be the direction cosines of the required line. Given it is perpendicular to the lines whose direction cosines are proportional to $1,-2,-2$ and $0,2,1$ respectively.

Thus, $l - 2m - 2n = 0$ and $2m + n = 0$

On solving, $\frac{l}{2} = \frac{m}{-1} = \frac{n}{2}$

Direction ratios of required line are proportional to $(2,-1,2)$

Thus Direction cosines are $= \left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right)$

133. **(d)** Given integrals $\int_{\sqrt{2}}^{x} \frac{dt}{|t|\sqrt{t^2-1}} = \frac{\pi}{12}$

$\Rightarrow [\sec^{-1} t]_{\sqrt{2}}^{x} = \frac{\pi}{12} \Rightarrow \sec^{-1} x - \sec^{-1}\sqrt{2} = \frac{\pi}{12}$

$\Rightarrow \sec^{-1} x = \frac{\pi}{12} + \frac{\pi}{4} = \frac{\pi}{3} \Rightarrow x = \sec\left(\frac{\pi}{3}\right) = 2$

134. **(c)** Given equations of lines $2x + 11y - 7 = 0$

and $x + 3y + 5 = 0$

Hence, their slopes are $m_1 = -\frac{2}{11}, m_2 = -\frac{1}{3}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{1}{7}$

$\theta = \tan^{-1}\left(\frac{1}{7}\right)$

135. **(a)** Condition of tangency is $(c^2 = a^2m^2 + b^2)$

For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $y = mx + c$ is tangent.

Here, $\frac{x^2}{5} + \frac{y^2}{3} = 1$ and $y = 2x + k$ is a tangent

$\Rightarrow k^2 = 5\times 4 + 3 \Rightarrow k = \pm\sqrt{23}$

136. **(b)** Since we know that is correct and (ii) is incorrect.

137. **(c)** Given expansion is

$(1 - x + x^2)^{10} = a_0 + a_1x + a_2x^2 + \cdots + a_{20}x^{20}$

On differentiating

$10(1 - x + x^2)^9 \cdot (-1 + 2x)$

$= a_1 + 2a_2x + 3a_3x^2 + \cdots + 20a_{20}x^{19}$

At $x = 1 \Rightarrow a_1 + 2a_2 + 3a_3 + \cdots + 20a_{20} = 10$

At $x = 0 \Rightarrow a_1 = -10$

So, $2a_2 + 3a_3 + \cdots + 20a_{20} = 20$

138. **(c)** Given $y = \sin^{98} x \cdot \cos^{39} x$

Now, $\frac{dy}{dx} = \sin^{98} x \cdot 39\cos^{38} x(-\sin x)$

$+\cos^{39} x \cdot 98\sin^{97} x(\cos x)$

$= 98\sin^{97} x \cdot \cos^{40} x - 39\sin^{99} x \cdot \cos^{38} x$

$= 98\cos^{99} x \cdot \sin^{38} x - 39\sin^{40} x \cdot \cos^{97} x$

139. **(b)** Given inverse trigo functions $\tan^{-1}\left[\frac{x}{1+\sqrt{1-x^2}}\right]$

Let $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right), u = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$

Put $x = \cos\theta \Rightarrow y = \tan^{-1}\left(\frac{\cos\theta}{1+\sin\theta}\right)$

and $u = \sec^{-1}(\sec 2\theta)$

or $y = \tan^{-1}\left(\frac{1}{\sec\theta+\tan\theta}\right)$ and $(u = 2\theta)$

$y = \tan^{-1}(\sec\theta - \tan\theta)$

$\Rightarrow \tan y = \sec\theta - \tan\theta$

$\Rightarrow \sec^2 y\frac{dy}{d\theta} = \sec\theta \cdot \tan\theta - \sec^2\theta$ and $\frac{du}{d\theta} = 2$

or $\frac{dy}{d\theta} = \frac{\sec\theta\cdot\tan\theta-\sec^2\theta}{\sec^2 y}, \frac{du}{d\theta} = 2$

$$\frac{dy}{d\theta} = \frac{\sec\theta \cdot \tan\theta - \sec^2\theta}{1 + (\sec\theta - \tan\theta)^2}, \frac{du}{d\theta} = 2$$

$$\frac{dy}{du} = \frac{1}{2}\left[\frac{\sec\theta \cdot \tan\theta - \sec^2\theta}{1 + \sec^2\theta + \tan^2\theta - 2\sec\theta \cdot \tan\theta}\right]$$

$$= \frac{1}{2}\frac{\sec\theta(\tan\theta - \sec\theta)}{2\sec^2\theta - 2\sec\theta \cdot \tan\theta} \Rightarrow \frac{dy}{du} = \frac{1}{4}$$

140. **(b)** Since, we know in radius

$$= \frac{\Delta}{s} = \frac{\sqrt{s(s-a)(s-b)(s-c)}}{s}$$

$$= \frac{\sqrt{10(4)(5)(1)}}{10} = \sqrt{2}$$

141. **(d)** Given $a + bi = \frac{i}{1-i}$

$$\Rightarrow a + bi = \frac{i(1+i)}{2} = -\frac{1}{2} + \frac{i}{2} \Rightarrow a = -\frac{1}{2}, b = \frac{1}{2}$$

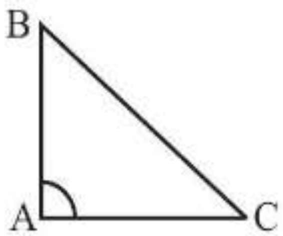
142. **(b)** Given $\overrightarrow{OB} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\overrightarrow{OC} = 5\hat{\imath} + \hat{\jmath} - 3\hat{k}$

Hence, $\overrightarrow{BC} = 2\hat{\imath} + 3\hat{\jmath} - 4\hat{k}$

$\max\{AB, BC, AC\} = BC$

$\therefore \overrightarrow{BC}$ is hypotenuse of $\triangle$ ABC



$\angle A = 90°$

$\therefore \overrightarrow{AB} \cdot \overrightarrow{AC} = 0$

$\overrightarrow{BA} \cdot \overrightarrow{BC} = |\overrightarrow{BA}||\overrightarrow{BC}|\cos B$

$\overrightarrow{CA} \cdot \overrightarrow{CB} = |\overrightarrow{CA}||\overrightarrow{CB}|\cos C$

$\therefore \overrightarrow{AB} - \overrightarrow{AC} + \overrightarrow{BA} - \overrightarrow{BC} + \overrightarrow{CA} - CB$

$= 0 + |\overrightarrow{BC}|(|\overrightarrow{AB}|\cos B + |\overrightarrow{CB}|\cos C)$

$= 0 + |\overrightarrow{BC}||\overrightarrow{BC}|$ [$\because$ By projection formula]

$= |\overrightarrow{BC}|^2 - \left(\sqrt{(2)^2 + 3^2 + 4^2}\right)^2$

$= 4 + 9 + 16 + 29$

143. **(d)** Given lines are

$3x + 4y - 5 = 0$

$2x + 3y - 4 = 0$

and $px + 4y - 6 = 0$

From equation (i) and (ii), we get

$x = -1, y = 2$

This should be satisfy III rd equation

So, $-p + 8 - 6 = 0 \Rightarrow p = 2$

144. **(d)** Given $a > 0, n \in R$

$\Rightarrow \lim_{x\to a} x^n = a^n$

145. **(c)** $\int_{-\pi}^{\pi} x^2(\sin x)dx$

$x^2(\sin x)$ is an odd function

By property of odd & even function

For $f(-x) = -f(x)$ then $\int_{-a}^{a} f(x)dx = 0$

$\Rightarrow \int_{-\pi}^{\pi} x^2(\sin x)dx = 0$

146. **(c)** Given differential equation

$\left\{x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right\} ydx$

$= \left\{y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right\} xdy$

Hence, $\frac{dy}{dx} = \left[\frac{x\cos\left(\frac{y}{x}\right)+y\sin\left(\frac{y}{x}\right)}{y\sin\left(\frac{y}{x}\right)-x\cos\left(\frac{y}{x}\right)}\right] \cdot \frac{y}{x}$

Let $\frac{y}{x} = V \Rightarrow \frac{dy}{dx} = V + x\frac{dV}{dx}$

$$\Rightarrow V + x\frac{dV}{dx} = \left(\frac{\frac{1}{V}\cos V + \sin V}{\sin V - \frac{1}{V}\cos V}\right) \cdot V$$

or $V + x\frac{dV}{dx} = \left(\frac{\cos V + V\sin V}{V\sin V - \cos V}\right) V$ or $x\frac{dV}{dx}$

$$= \left(\frac{V\cos V + V^2\sin V - V^2\sin V + V\cos V}{V\sin V - \cos V}\right)$$

or $\frac{(V\sin V - \cos V)}{V\cos V} dV = 2\frac{dx}{x}$

Now, put $V\cos V = t$

$\Rightarrow (\cos V - V\sin V)dV = dt$

So, $\int -\frac{dt}{t} = 2\int \frac{dx}{x} \Rightarrow 2\ln x + c = -\ln t$

or $2\ln x + c = -\ln\left(\frac{y}{x}\cos\frac{y}{x}\right)$

or $\ln\left(xy\cos\frac{y}{x}\right) = -C \Rightarrow xy \cdot \cos\frac{y}{x} = \pm e^{-C}$

147. **(a)** Given $f(x) = \left(\frac{1}{2}\right)^x$

Let $y = \left(\frac{1}{2}\right)^x$ on R

$$\frac{dy}{dx} = \left(\frac{1}{2}\right)^x \log\frac{1}{2} = -\left(\frac{1}{2}\right)^x \log 2$$

$\because \left(\frac{dy}{dx} < 0\right)$ so strictly decreasing.

148. **(b)** Given pair of straight lines is

$6x^2 - 5xy + y^2 = 0$

it can be written as

$$\left(\frac{y}{x}\right)^2 - 5\left(\frac{y}{x}\right) + 6 = 0 \Rightarrow \left(\frac{y}{x} - 3\right)\left(\frac{y}{x} - 2\right) = 0$$

$\Rightarrow y = 3x$ and $y = 2x$ are straight lines

$\tan\alpha = 3, \tan\beta = 2$

$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta} = \frac{3-2}{1+6} = \frac{1}{7}$.

149. **(a)** Given, curve is

$$\left(\frac{x}{31}\right)^n + \left(\frac{y}{1209}\right)^n = 2$$

On differentiating

$$\frac{nx^{n-1}}{31^n}+\frac{ny^{n-1}}{1209^n}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=-\left(\frac{nx^{n-1}\times 1209^n}{ny^{n-1}\cdot 31^n}\right)\Rightarrow\frac{dy}{dx}=-\left(\frac{x}{y}\right)^{n-1}\times(39^n)$$

Hence, slope of tangent at $(31,1209)$

$$=-\left(\frac{1}{39}\right)^{n-1}\times(39^n)=-39$$

150. **(d)** Given hyperbola is $x^2-9y^2=9$

$$\frac{x^2}{9}-\frac{y^2}{1}=1$$

We know that

$y=mx\pm\sqrt{9m^2-1}$ represent tangents which passes through $(3,2)$

So, $m=\frac{5}{12}$

Now, tangents are $12y=5x+9$ and $x=3$

and chord of contact will be $\frac{xx_1}{a^2}-\frac{yy_1}{b^2}=1$

$\Rightarrow\frac{3x}{9}-\frac{2y}{1}=1$ or $x-6y=3$

The vertices of the triangle formed with tangents and chord will be $(3,2),(3,0)$ and $\left(-5,-\frac{4}{3}\right)$.

So, area of $\Delta=\frac{1}{2}\begin{vmatrix}3&2&1\\3&0&1\\-5&-\frac{4}{3}&1\end{vmatrix}=8$ unit ^{2}

151. **(d)** Given standard deviation $\sigma=3.5$

So, mean of the given numbers is

Mean $(\bar{x})=\frac{2+3+2x+11}{4}=\left(4+\frac{x}{2}\right)$

$$\sigma^2=\frac{1}{n}\Sigma(x_i-\bar{x})^2$$

$$\Rightarrow(3.5)^2=\frac{1}{4}\left[\left(-2-\frac{x}{2}\right)^2+\left(-1-\frac{x}{2}\right)^2\right.$$
$$\left.+\left(\frac{3x}{2}-4\right)^2+\left(7-\frac{x}{2}\right)^2\right]$$

$$\Rightarrow 3x^2-16x+21=0\Rightarrow x=\frac{7}{3},3$$

152. **(c)** Given expression $x^{2019}\cdot y^{2020}=(x+y)^{4039}$

On differentiating both the sides, we have

$$\Rightarrow 2020y^{2019}\frac{dy}{dx}x^{2019}+2019x^{2018}\cdot y^{2020}$$
$$=4039(x+y)^{4038}\cdot\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow 2020\frac{(x+y)^{4039}}{y}\frac{dy}{dx}+2019\frac{(x+y)^{4039}}{x}$$
$$=4039(x+y)^{4038}\cdot\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\left(\frac{2020}{y}\frac{dy}{dx}+\frac{2019}{x}\right)(x+y)=4039\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\frac{2020(x+y)}{y}\cdot\frac{dy}{dx}+\frac{2019}{dx}+\frac{2019}{x}(x+y)$$
$$=4039\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\frac{dy}{dx}\left[\frac{2019y-2020x}{y}\right]=\frac{2019(x+y)}{x}-4039$$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}$

153. **(a)** Given $\lambda=3, B=5, G=8$

$P(B)=\frac{5}{16} \Rightarrow P(\bar{B})=\frac{11}{16}$

154. **(a)** Given $y=5x^2+6x+6, x=2$

and $\Delta x=0.001$

Now, $dy=(10x+6)dx$

$\Rightarrow dy=(10\times 2+6)\times 0.001 \Rightarrow dy=0.026$

155. **(a)** Let $S_1 \equiv x^2+y^2+8x+8y-m=0$

$S_2=x^2+y^2-2x+4y+n=0$

So, equation of radical axis (common chord)

$\Rightarrow S_1-S_2=0$

$\Rightarrow 10x+4y-(m+n)=0$

$\Rightarrow 10x+4y=(m+n)$

Centre of bisected circle is $(-4,-4)$ which will lie on radical axis.

So, $\{(m+n)=-56\}$

156. **(a)** Given equation is $x\cos\theta+y\sin\theta=P$

The axis are rotated through angle (θ), then $x=X\cos\theta-Y\sin\theta$ and $y=X\sin\theta+Y\cos\theta$

Put in Eq. (i)

$\cos\theta(X\cos\theta-Y\sin\theta)+\sin\theta(X\sin\theta+Y\cos\theta)$

$\Rightarrow X\cos^2\theta-Y\sin\theta\cdot\cos\theta+X\sin^2\theta$

$+Y\sin\theta\cdot\cos\theta=P$

$\Rightarrow X=P$

157. **(a)** Since irrational roots exist in pair.

So, the degree of $f(x)$ will be an even number.

158. **(c)** $\int \sin^3 x\cdot\cos^3 x dx$

$=\int \sin^3 x\cdot\cos x(1-\sin^2 x)dx$

Let $\sin x=t \Rightarrow \cos x dx=dt$

$\Rightarrow \int t^3\cdot(I-t^2)dt=\int(t^3-t^5)dt$

$\Rightarrow \frac{t^4}{4}-\frac{t^6}{6}+C=\frac{\sin^4 x}{4}-\frac{\sin^6 x}{6}+C$

159. **(d)** Given 5 different coloured flowers.

So, number of different garlands

$=\frac{4!}{2}=12$

160. **(b)** Given $\lim_{x\to 0}\frac{8}{\sin^8 x}\left\{1-\cos\left(\frac{x^2}{2}\right)-\cos\left(\frac{x^2}{4}\right)\right.$

$\left.+\cos\left(\frac{x^2}{2}\right)\cos\left(\frac{x^2}{4}\right)\right\}$

On simplifying, we get

$\Rightarrow \lim_{x\to 0}\frac{8}{\sin^8 x}\left\{\left[1-\cos\left(\frac{x^2}{2}\right)\right]\left[1-\cos\left(\frac{x^2}{4}\right)\right]\right.$

or $\lim_{x\to 0}\frac{8}{\sin^8 x}\left(2\sin^2\left(\frac{x^2}{4}\right)\right)\left(2\sin^2\left(\frac{x^2}{8}\right)\right)$

or $\lim_{x\to 0}\frac{32\sin^2\left(\frac{x^2}{4}\right)\cdot\sin\left(\frac{x^2}{8}\right)}{\sin^8 x}$

$\Rightarrow \lim_{x\to 0}\frac{32\sin^2\left(\frac{x^2}{4}\right)\cdot\sin^2\left(\frac{x^2}{8}\right)\times x^8}{\left(\frac{x^2}{4}\times\frac{x^2}{8}\right)^2\cdot\sin^8 x\times(4\times 8)^2}=\frac{1}{32}$