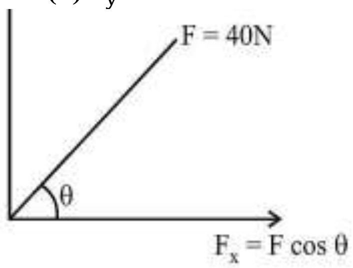
Physics

1. (a) According to Faraday's law of EMI, when we move a conductor in magnetic field region, there will be induced current in the conductor and the same phenomenon is used in case of electric generator.

2. **(a)** In retardation, the speed reduces continuously and as the body is moving in a straight line, Therefore, displacement will be equal to distance covered by body.

3. **(a)** Time of flight (T) $= 2u\sin\theta/g = \frac{2\times 50\times \sin 30^\circ}{10} = 5\text{ s}$

4. **(b)** $F_y = F\sin\theta \uparrow$



Clearly, $40\cos\theta = 20\sqrt{3}$

$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30$

and $F_y = 40\sin 30 = 20\text{ N}$

5. **(b)** Case 1: $s = 0.t_1 + 1/2gt_1{}^2 \Rightarrow s = \frac{1}{2}gt_1^2$

$[\because a = g]$

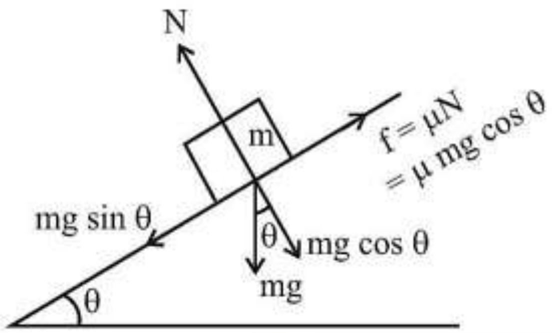
If a_{net} is net acceleration of ball, then in case 2 when lift is going up with acceleration a

$a_{net} = (g + a) \Rightarrow s = \frac{1}{2}(g + a)t_2^2$

From eqs. (A) and (B), we get

$t_1 > t_2$

6. **(a)**



Force along the plane, $mg\sin\theta - f = ma$

$\Rightarrow mg\sin\theta - \mu N = ma$ .... (i)

Since, $N = mg\cos\theta$ ..... (ii)

From eqs. (i) and (ii), we get

$mg\sin\theta - \mu mg\cos\theta = ma$

$a = g(\sin\theta - \mu\cos\theta)$

7. **(b)** After collision, the two balls will stick together and their combined velocity be v_0

Now, by using law of conservation of momentum,

$m_1u_1 + m_2u_2 = (m_1 + m_2)v_0$

$\Rightarrow v = \frac{2\times 10 + 3\times 0}{5} = 4\text{ ms}^{-1}$

$= \left(\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right) - \frac{1}{2}(m_1 + m_2)V_0^2$

$= \left(\frac{1}{2}\times 2\times 10^2 + \frac{1}{2}\times 3\times 0^2\right) - \frac{1}{2}(2 + 3)4^2$

$= 100 - 40 = 60\ \text{J}$

8. (b) Given, mass of body, $m = 8$ kg

Equation of displacement, $s = t^2/4$

$$V = \frac{ds}{dt} = \frac{t}{2}$$

By work energy theorem

$W = \Delta K$

$$K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \frac{1}{2} \times 8 \times \left(\frac{4}{2}\right)^2 = 16\ \text{J}$$

9. (b) $\int_{p_1}^{p_2} dp = \int_0^T F dt =$ Area under graph

$$= \frac{1}{2} \times \frac{T}{4} \times F_0 + \frac{2T}{4} \times F_0 + \frac{1}{2} \times \frac{T}{4} \times F_0 = \frac{3F_0 T}{4}$$

So, $p_2 - p_1 = \frac{3F_0T}{4} \Rightarrow mv - 0 = \frac{3F_0T}{4} \Rightarrow F_0 = \frac{4mv}{3T}$

10. (a) Moment of inertia (I) for following bodies are

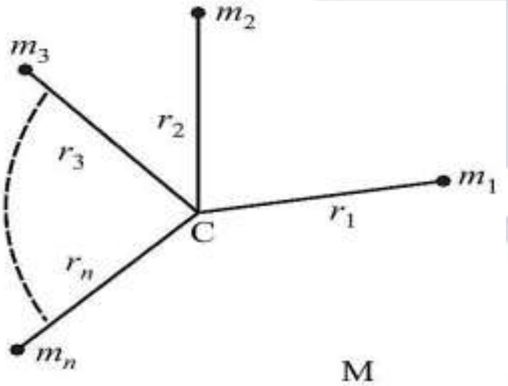
(i) Ring $= MR^2$

(ii) Angular disc $= \frac{M}{2}(R^2 - r^2)$

(iii) Solid disc $= MR^2/2$ (iv) Cylindrical disc $= MR^2/2$

Clearly, MoI of ring is maximum

11. (b) as $r = \frac{m_1r_1 + m_2r_2 + m_3r_3 + \cdots + m_nr_n}{M}$



If CoM lies at origin then, $\vec{r} = 0$

$\therefore$ From eq. (i), we get

$m_1r_1 + m_2r_2 + m_3r_3 + \cdots m_nr_n = 0$ .... (ii)

Net moment of all particles in the system about centre of mass C .

$\tau_c = m_1gr_1 + m_2gr_2 + m_3gr_3 + \cdots m_ngr_n$

$= g[m_1r_1 + m_2r_2 + m_3r_3 + \cdots m_nr_n]$

$= g \times 0 = 0$

[from Eq. (ii)]

Hence, sum of moments of all the particles in a system about its COM is always zero.

12. (d) As we know that,

Earth rotates from West to East

$\therefore$ The train going towards East will have high speed with respect to earth from the train going towards West

$N \propto V^2$

and speeds of different trains are different. Hence, magnitude of normal reaction is different.

$\therefore$ Assertions will be false and reason will be true because their relative speeds are different.

But actual speeds of both the trains are same.

13. (c) Since, force $F = mg = k\Delta x$

$$k = \frac{mg}{\Delta x}$$

$$\Rightarrow k = \frac{m_1 g}{\Delta x_1} = \frac{0.6 \times 10}{0.4} = 15\ \text{N/m}$$

and time period,

$T_2 = 2\pi\sqrt{\frac{m_2}{k}} T_2 = 2\pi\sqrt{\frac{0.255}{15}}$

$= 2\pi\sqrt{0.017} = 0.82$ s

14. (d) As $T = 2\pi\sqrt{\frac{m}{k}}$.

So T depends on m&k which remain unchanged So, time period will remain same.

15. (c) Work done per unit mass $= \frac{W}{M_1}$

$\therefore \frac{W}{M_1} = \frac{GM_2}{r} \left[\because\ W = V = \frac{GM_1M_2}{r}\right]$

$= \frac{6.67 \times 10^{-11} \times 1000}{1} = 6.67 \times 10^{-8}$ J/kg

16. (c) $a_n = \frac{GM}{(R+h)^2} = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2} = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$\frac{a_n}{n} = \frac{R}{20} = \frac{g}{\left(1+\frac{1}{20}\right)^2} = \frac{20^2 g}{21^2} \Rightarrow g = \frac{20^2}{21^2} g$

$\Rightarrow g = \frac{21^2}{20^2} \times 9\ \text{m/s}^2$

$a_{d=\frac{R}{20}} = g\left(1 - \frac{d}{R}\right)$

$= \frac{21^2}{20^2} \times 9\left(1 - \frac{1}{20}\right) = \frac{21^2}{20^2} \times 9 \times \frac{19}{20} = 9.426$

$\approx 9.5\ \text{m/s}^2$

17. (a) Here, $\Delta l = \frac{mgl}{2AY} \Rightarrow Y = \frac{mgl}{A\Delta l} \times \frac{1}{2}$

$\therefore Y = \frac{\rho(A \cdot l) \cdot g \cdot l}{2A \cdot \Delta l}$ $[\because M = \rho A l]$

where, A is area of cross-section

$\therefore Y = \frac{\rho l^2 g}{2\Delta l} \Rightarrow \Delta l = \frac{\rho l^2 g}{2Y}$

$= \frac{1.5 \times (12 \times 10^{-2})^2 \times 10}{2 \times 5 \times 10^8} = 2.16 \times 10^{-10}$ m

18. (d) As, $H = \frac{2\,T\cos\theta}{\rho g R}$

$\Rightarrow R = \frac{2\,T\cos\theta}{\rho g H} = \frac{2 \times 7.5 \times 10^{-2} \times \cos 0°}{1000 \times 10 \times 7.5 \times 10^{-2}}$

$= 2 \times 10^{-4}$ m $= 0.2$ mm

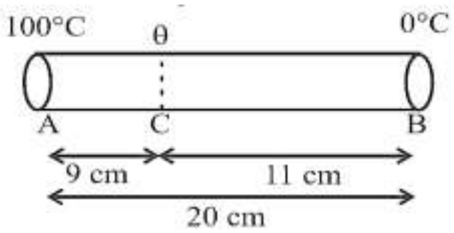
19. (a) As h = constant

$P + \frac{1}{2}\rho v^2$

$\Rightarrow$ PW $\Rightarrow$ VM

So, where pressure is least, velocity is maximum

20. (a) As the flow is steady,



$\therefore \dfrac{KA(100-\theta)}{9} = \dfrac{KA(\theta-0)}{11}$

$\Rightarrow 1100 - 11\theta = 9\theta \Rightarrow \theta = 55^\circ C$

21. (d) As, $\Delta\ell = \ell_0 \alpha \Delta T$

let final heat coefficient be α''

$\therefore (L + 2L)\alpha''\Delta T = L\alpha\Delta T + 2L.2\alpha \cdot \Delta T$

$\Rightarrow 3\alpha'' = \alpha + 4\alpha = 5\alpha$

$\Rightarrow \alpha'' = 5\alpha/3$

22. (c) By using 1st law of thermodynamics,

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = \Delta Q - \Delta W$

In both path ΔU is same

So, $\Delta Q_1 - \Delta W_1 = \Delta Q_2 - \Delta W_2$

or, $Q_1 - W_1 = Q_2 - W_2$

23. (d) Since, $pV^\gamma =$ constant

$\therefore p_1 V_1^y = p_2 V_2^\gamma$

$\Rightarrow p_1/p_2 = (V_2/V_1)^y = \left(\dfrac{1}{4}\right)^{3/2}$

$[\because V_1 = yV_2]$

$\Rightarrow p_1/p_2 = \frac{1}{8}$ Hence, $p_2 : p_1 = 8 : 1$

24. (c) Radius of piston, $r_p = (7.5 - 0.008) \times 10^{-2}$ m $= 7.492 \times 10^{-2}$ m

Let, Δr_c and Δr_p is increase in radius of cylinder and piston respectively, for fully filled piston in cylinder.

New radius will be $r'_c = r_c + \Delta r_c$

$= r_c + r_c\alpha_c\Delta T = r_c(1 + \alpha_c\Delta T)$

$= 7.5 \times 10^{-2}[1 + 1.2 \times 10^{-5}(T - 303)]$

Similarly, new radius of piston,

$r_p = 7.492 \times 10^{-2}[I + 1.6 \times 10^{-5}(T - 303)]$

Now, for fully fitted piston, $r_c = r_p$

$7.5 \times 10^{-2}[1 + 1.2 \times 10^{-5}(T - 303)]$

$= 7.492 \times 10^{-2}[1 + 1.6 \times 10^{-5}(T - 303)]$

$\Rightarrow 1.0011 + 1.2013 \times 10^{-5}(T - 303)$

$= 1 + 1.6 \times 10^{-5}(T - 303)$

$\Rightarrow 1.1 \times 10^{-3} = (T - 303) \times 10^{-5}(1.6 - 1.2013)$

$= (T - 303) \times 10^{-5} \times 0.3987$

$\Rightarrow T - 303 = 275$

$\Rightarrow T = 578\ K = 305^\circ C$

25. (d) For an ideal gas, we have

$\frac{pV}{T} =$ constant $\Rightarrow \frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$

$\Rightarrow V_2 = \dfrac{p_1V_1T_2}{T_1p_2} \Rightarrow V_2 = \dfrac{4 \times 1500 \times 270}{300 \times 2} = 2700\ m^3$

26. (d) We have

$$\lambda=\frac{(\text{ speed })v}{(\text{ frequency })}f=\frac{350}{350}=1\text{ m}=100\text{ cm}$$

$$\therefore\ \Delta\phi=\frac{2\pi}{100}\times 25=\frac{\pi}{2}\ \left[\because \Delta\phi=\frac{2\pi}{\lambda}\Delta x\right]$$

As, resultant amplitude,

$$a_{net}=\sqrt{a_1^2+a_2^2+2a_1a_2\cos\phi}$$

$$\Rightarrow\ a_{net}=\sqrt{0.3^2+0.4^2+2\times 0.3\times 0.4\cos\frac{\pi}{2}}$$

$$=\sqrt{\frac{9}{100}+\frac{16}{100}+0}=\sqrt{\frac{25}{100}}=0.5\text{ mm}$$

27. (b) Let first secondary maximum occur at angle θ_2.

$\because n\lambda=a\sin\theta_1$

$$\Rightarrow a=\frac{n\lambda}{\sin 30^\circ}=2\lambda\ [\because n=1]$$

and for maxima and $\theta_1=30^\circ$

$$(2n+1)\frac{\lambda}{2}=a\sin\theta_2\Rightarrow(2+1)\frac{\lambda}{2}=a\sin\theta_2$$

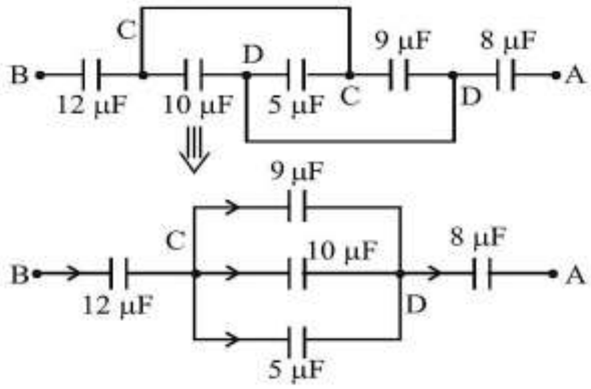
$$\Rightarrow \sin\theta_2=\frac{3\lambda}{2a}$$

Substituting the value of $a=2\lambda$ in Eq. (i), we get

$$\therefore\ \sin\theta_2=\frac{3\lambda}{2\times 2\lambda}=\frac{3}{4}\ \Rightarrow\theta_2=\sin^{-1}\left(\frac{3}{4}\right)$$

28. (d) Statement (d) is incorrect because electric field lines cannot form closed loop.

29. (a) According to given circuit diagram.



Parallel equivalent capacitance,

$C'_{eq}=C_1+C_2+C_2$

$\therefore\ C'_{eq}=9+10+5=24\mu\text{ F}$

and series equivalent capacitance,

$$\frac{1}{C_{eq}}=\frac{1}{C_a}+\frac{1}{C_b}+\frac{1}{C_c}$$

$$\Rightarrow\frac{1}{C_{eq}}=\frac{1}{12}+\frac{1}{24}+\frac{1}{8}\Rightarrow\frac{1}{C_{eq}}=\frac{2+1+3}{24}$$

$$\Rightarrow C_{eq}=\frac{24}{6}\mu\text{ F}=4\mu\text{ F}$$

Since, $C=\frac{Q}{V}\ \therefore Q=CV$

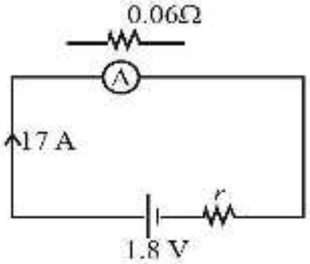
$=4\times 10^{-6}\times 80=320\mu C$

Now, $9\text{ V}+10\text{ V}+5\text{ V}=320$

$\Rightarrow V=\frac{40}{3}$ volt. So, $q_{10\mu\text{ F}}=10\times\frac{40}{3}=133.3\mu C$

30. **(a)** Clearly, $1.8 = 17(0.06 + r)$

$\Rightarrow \frac{1.8}{17} = 0.06 + r$



$\Rightarrow r = \frac{1.8}{17} - 0.06 = 0.046\Omega$

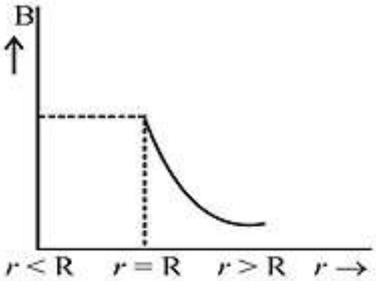
31. **(c)** $F = Bqv\sin\theta$, clearly if $V = 0 \Rightarrow F = 0$

32. **(a)** Here, $B(r < R) = 0$

$B(r = R) = \frac{1}{2\pi}\frac{\mu_0 I}{R}$

and $B\ (r > R) = \frac{1}{2\pi}\frac{\mu_0 I}{r}$

So, these variations are correctly shown as below.



33. **(c)** Here, first apparent dip $= \delta_1$

Second apparent dip $= \delta_2$

Declination $= \theta$

Then, $\tan\theta = \frac{\tan\delta_1}{\tan\delta_2} \Rightarrow \theta = \tan^{-1}\left(\frac{\tan\delta_1}{\tan\delta_2}\right)$

34. **(a)** As, we know that,

emf induced, $\varepsilon = -\frac{Nd\phi}{dt}$

and, A is area of cross-section.

$\therefore \varepsilon = -\frac{NBA\cos\theta}{t} \Rightarrow \varepsilon \propto N$

$\therefore$ If n increases, emf also increases and this back emf resist motion of magnet.

35. **(d)** L_{eq_1} (series) $= L_1 + L_2$

$\therefore L_{eq_1} = L_1 + L_2 = 8 \quad \ldots.(i)$

and $\frac{1}{L_{eq2}}$ (parallel) $= \frac{1}{L_1} + \frac{1}{L_2} \Rightarrow L_{eq_2} = \frac{L_1L_2}{L_2+L_1}$

$\Rightarrow \frac{3}{2} = \frac{L_1(8 - L_1)}{8}$

$\Rightarrow L_1{}^2 - 8\,L_1 + 12 = 0$

$\Rightarrow L_1(\,L_1 - 6) - 2(\,L_1 - 6) = 0$

$\therefore L_1 = 2H$ or $6\,H$

and $L_2 = (8 - 2)$ or $(8 - 6)$

$= 6H$ or $2\,H$

$\therefore$ Difference in inductance is

$(L_1 - L_2)$ or $(L_2 - L_1) = 6 - 2 = 4H$

36. **(a)** $f_r \propto \frac{1}{\sqrt{C}} \Rightarrow f_{r_2} = f_{r_1}\sqrt{\frac{C_1}{C_2}}$

$$\Rightarrow f_{r_2} = \frac{f_{r_1}}{2}$$

$[\because C_2 = yC_1]$

37. **(c)** According to modified Ampere's law or Maxwell Ampere's law,

$\vec{\nabla} \times \vec{B} = \mu_0(\vec{J} + \vec{J}_d)$

Where $\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

38. **(b)** Since $I = \frac{E}{At}$

So, energy fall $= IA$

per sec $= 100 \times 2 \times 10^{-4}$

$= 2 \times 10^{-2}$ J/s

Energy of 1 photon $= \frac{12400}{3000}$eV

$= 4.13\text{eV} = 6.61 \times 10^{-19}$ J

Number of photon falling per sec $= \frac{2\times10^{-2}}{6.61\times10^{-19}}$

$= 3.02 \times 10^{16}$

Number of photoelecton emitted

$= 0.02 \times 3.02 \times 10^{16} = 6.04 \times 10^{14}$

39. **(d)** Force, $F = \frac{-dU}{dR}$

$$\therefore F = -\frac{Ke^2}{3}\frac{d}{dR}R^{-3} \Rightarrow \frac{mv^2}{R} = -\frac{Ke^2}{3}.(-3)R^{-4}$$

$$\therefore \frac{mv^2}{R} = \frac{Ke^2}{R^4}$$

$$\Rightarrow R^3 = \frac{Ke^2}{mv^2}$$

By Bohr's quantization rule

$$L = mvR = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mR}$$

Substituting the value in Eq. (i), we get

$$R^3 = \frac{Ke^2}{m \cdot \frac{n^2h^2}{4\pi^2m^2R^2}} \Rightarrow R^3 = \frac{Ke^2 \cdot 4\pi^2m^2R^2}{mn^2h^2}$$

$$\therefore R = \frac{4\pi^2 \cdot Ke^2 \cdot m}{n^2h^2}$$

40. **(a)**

Chemistry

41. **(b)** $\lambda_A = 2 \times \lambda_B; 2 \times \frac{h}{m_Bv_B} = \frac{h}{m_Av_A}$

Given, $v_A = v_B; \frac{2}{m_B} = \frac{1}{m_A} \Rightarrow m_A = \frac{m_B}{2}$

42. **(a)** Number of electrons in Li^+ =Number of electrons in helium = 2.

43. **(d)** For first orbit $(n = 1), r_1 = \left(\frac{1}{z}\right) \times 0.529\text{Å}$ For third orbit $(n = 3), r_3 = \left(\frac{9}{z}\right) \times 0.529\text{Å}$

Hence, $r_3 = 9r_1$

44. **(c)** For electronic configuration $= [Ar]3d^54s^2$ Group = number of valence electrons $= 5 + 2 = 7$

45. **(a)** $EN = \frac{IE+EA}{2}$ .... (i)

$$EN = \frac{13+4}{2} = \frac{17}{2} = 8.5\text{eV}$$

46. **(b)** Due to small size of F atom, it has lower EGE than that of Cl atom

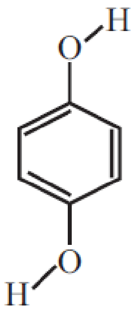
Order of increasing electron gain enthalpy

$N < P < F < Cl$

47. (d) In NH_3, the high dipole moment is due to same direction of dipole moments due to lp and bp of electrons.
Dipole moment order:
$\underset{(1.4D)}{NH_3} > \underset{(0.8D)}{NBr_3} > \underset{(0.6D)}{NCl_3} > \underset{(0.24D)}{NF_3}$

48. (c) The central atoms in BF_3 and $NO_2^{\ -}$ are sp^2 hybridised.

49. (a) The $O-H$ and $S-H$ bond dipoles in (iii) and (iv) molecules do not cancel each other as they exist in different conformations.



50. (b) $pV =$ constant at cons. T and n.

51. (a) $pV = nRTV = nRT/p$
Density, $D = M/V$ (for 1 mole)
$\Rightarrow D = \frac{M}{RT/p} \Rightarrow D = \frac{pM}{RT}$

52. (b)
$\underset{\substack{\text{Calcium}\\\text{carbonate}\\(100g)}}{CaCO_3(s)} + \underset{\substack{\text{Hydrochloric}\\\text{acid}\\(73\ g)}}{2HCl} \rightarrow \underset{\substack{\text{Calcium}\\\text{chloride}}}{CaCl_2(aq)} + H_2O(l) + \underset{(44\ g)}{CO_2(g)}$
As amount of $CaCO_3$ and HCl are same, $CaCO_3$ will be the limiting reagent.
$\therefore$ 100 $g_{\text{of } CaCO_3 \text{ gives}}$ = 44 g_g of CO_2 gas
$\therefore$ 20 g of $CaCO_3$ will give $= \frac{44}{100} \times 20$ g of $CO_2 = 8.8$ g of CO_2

53. (d) Pink colour of MnO_4^- converts to colourless MnO_2 after reduction.

54. (c) $\Delta G = \Delta H - T\Delta S$
If ΔH and ΔS are +ve then, ΔG will be positive till $\Delta H > T\Delta S$.
When the temperature is increased further, then
$\Delta H < T\Delta S$ and ΔG becomes -ve. Which is the essential condition for spontaneous reaction.

55. (b) $\frac{1}{2}X_2 + \frac{3}{2}Y_2 \rightleftharpoons XY_3; \Delta H = -30$ kJ
$X_2 + 3Y_2 \rightleftharpoons 2XY_3; \Delta H = -60$ kJ
$\Delta S_r = 2 \times 50 - 3 \times 40 - 1 \times 60 = -80 JK^{-1}\ mol^{-1}$
$\Delta G = \Delta H - T\Delta S$
When, $\Delta G = 0$
$0 = \Delta H - T\Delta S \Rightarrow \Delta H = T\Delta S$
$1000 \times (-60) = T \times (-80) \Rightarrow T = 750$ K.

56. (b) $SO_2 + \frac{1}{2}O_2(g) \rightleftharpoons SO_3(g)$
When pressure is increased forward reaction is favoured because product side has lower no. of moles.
From p_1 to p_3, yield of SO_3 is increasing because higher pressure favours forward reaction.

57. (d) $2(A)g \rightleftharpoons B(g) + C(g)$
$\Delta n = 2 - 2 = 0$
$K_p = K_C(RT)^0 \Rightarrow K_P = K_C$

58. (a) Peroxides such as BaO_2
on treatment with dilute H_2SO_4 give H_2O_2.

$$\underset{\text{Barium peroxide}}{BaO_2} + H_2SO_4 \longrightarrow \underset{\text{Barium sulphate}}{BaSO_4} + \underset{\text{Hydrogen peroxide}}{H_2O_2}$$

59. (c) K, Rb and Cs form superoxides when they are burned in the air. As we move down the group, the size of an atom from K to Cs increases. Bigger cation can stabilize bigger O_2^- more easily Hence, stability of superoxide increases.

60. (b)

$$\underset{\text{Borax}}{Na_2B_4O_7} + 7H_2O \longrightarrow \underset{\text{Sodium hydroxide}}{2NaOH} + \underset{\text{Boric acid}}{4H_3BO_3}$$

61. (b) $\underset{\text{methyl chloride}}{2CH_3Cl} + Si \xrightarrow[570\,k]{Cu-powder} \underset{\text{Dimethyl silanes}}{(CH_3)_2} - \underset{\text{dichloro(P)}}{SiCl_2}$

$$\xrightarrow[-2HCl]{2H_2O} \underset{(Q)}{(CH_3)_2Si(OH)_2} \longrightarrow -O\left(\begin{array}{c}CH_3\\|\\Si\\|\\CH_3\end{array}-O\right)\begin{array}{c}CH_3\\|\\Si\\|\\CH_3\end{array}-$$

62. (a)

63. (c) Kolbe's electrolysis

$2CH_3COONa + 2H_2O \rightarrow CH_3 - CH_3 + 2CO_2 + 2NaOH + H_2$

Reaction at anode :

$2CH_3CO\dot{O} \longrightarrow 2\dot{C}H_3 + 2CO_2$

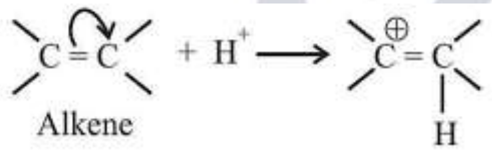
$2\dot{C}H_3 \longrightarrow C_2H_6$

$\dot{C}H_3$ radical is produced at anode.

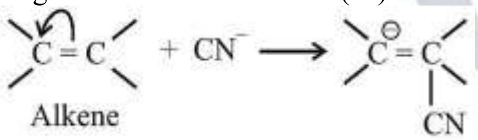
64. (c) Fractional distillation is used for the separation of a mixture of miscible liquids for which the difference in boiling points is less than 25°C.

65. (d) HNO_3 to remove NaCN(X) and Na_2S (Y) so that they do not interfere with the test for halogen.

66. (b) Positive electromeric effect (+E)



Negative electromeric effect (-E)



67. (d) Benzene and toluene have very similar molecular structure. Thus, they form an ideal solution.

68. (b) $\Delta T_b = K_b \times m$

$\Rightarrow K_b = \frac{\Delta T_b}{m}$

69. (a) Time, t = 1.608 min or 96.5 s

Formula, $W_{Al} = \frac{z \times i \times t}{96500}$

$\Rightarrow W_{Al} = \frac{27}{3} \times \frac{10 \times 96.5}{96500}$

$\Rightarrow W_{Al} = 0.09$ g

70. (b) $\lambda^\circ{}_m(NH_3) = \lambda^\circ(H^+) + \lambda^\circ(NH^-{}_2)$

$\Rightarrow \lambda^\circ{}_m(NH_3) = \lambda^\circ(NH_2^-) + \lambda^\circ(H^+) + \lambda^\circ(KBr) - \lambda^\circ$ (KBr)

$= \lambda^\circ(KNH_2) + \lambda^\circ(HBr) - \lambda^\circ(KBr)$

$= 90.48 + 420.6 - 120.5$

$= 390.5\ \mathrm{S\ cm^2\ mol^{-1}}$

71. (c) For first order reaction;

$$t = \frac{2.303}{k}\log\left(\frac{a}{a-x}\right)$$

$$t = \frac{2.303}{2303 \times 10^{-3}}\log\left(\frac{4}{0.2}\right)$$

$\Rightarrow t = 10^3 \times \log 20$

$\Rightarrow t = 1301\ \mathrm{s} = 0.36\mathrm{hr}$

72. (c) $\Rightarrow \log\frac{x}{m} = \log k + \frac{1}{n}\log p$

Slope (m) $= 1/n$

73. (c)

74. (d) Co binds with two bidentate ligands en and two monodentate ligand.

$\therefore$ Its coordination number $= 4 + 2 = 6$

75. (c)

$$\underset{(B)}{Zn(OH)_2} \xrightarrow{\Delta} \underset{(A)}{ZnO} + H_2O$$

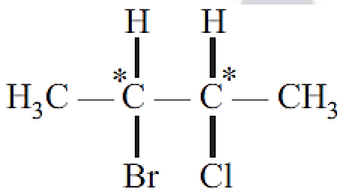
$$\underset{(B)}{Zn(OH)_2} + 2NaOH \longrightarrow Na_2[Zn(OH)_4]$$

$$\xrightarrow[H_2O]{} \underset{(C)}{Na_2ZnO_2}$$

$$\underset{\text{Zinc oxide}}{ZnO} + C \longrightarrow \underset{(D)}{Zn} + \underset{\substack{(E)\\ \text{Carbon}\\ \text{monoxide}}}{CO}$$

76. (b) $\underset{\text{Starch}}{(C_6H_{10}O_5)_n} + nH_2O \xrightarrow[343\ K, 2-3\ atom]{H^+} \underset{\text{Glucose}}{nC_6H_{12}O_6}$

77. (c)



Number of optical isomers $= 2^n$

$S = 2^2 = 4$

78. (b) $\underset{\text{Glucose}}{C_6H_{12}O_6} + \text{Zymase} \rightarrow \underset{\text{Ethanol}}{2C_2H_5OH} + \underset{\text{Carbon dioxide}}{2CO_2}$

79. (b) When CH_3COCl react with $(CH_3)_2Cd$, it will form acetone as a product (P). On further step, acetone (P) react with acetaldehyde to give aldol condensation reaction and formed α,β-unsaturated alkene product.

$$\underset{\text{Methyl chloride}}{CH_3-\overset{\overset{O}{\|}}{C}-Cl} \xrightarrow{(i)(CH_3)_2Cd}$$

$$\underset{\text{Acetone}(P)}{CH_3-\overset{\overset{O}{\|}}{C}-CH_3} + CH_3CdCl$$

$$\underset{(P)}{CH_3-\overset{\overset{O}{\|}}{C}-CH_3} + \underset{\text{Acetaldehyde}}{CH_3-\overset{\overset{O}{\|}}{C}-H}$$

$$\Delta \downarrow NaOH$$

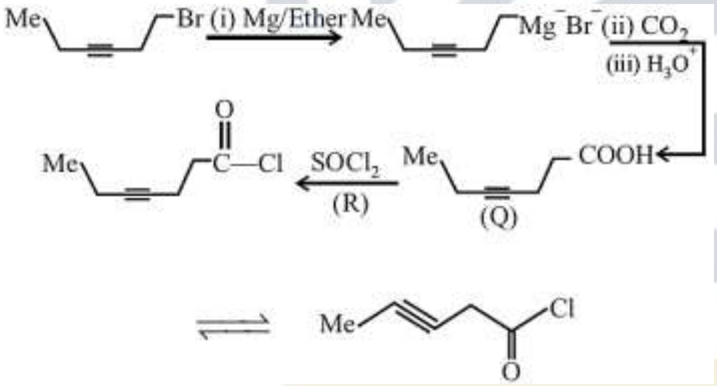
$$\underset{\substack{(\text{cis}/\text{trans})\\(2\text{ products})}}{CH_3-\overset{CH_3}{\overset{|}{C}}=\overset{H}{\overset{|}{C}}-\overset{\overset{O}{\|}}{C}-H} +$$

$$\underset{\substack{(\textit{Cis}/\textit{trans})\\(2\text{ products})}}{CH_3-\overset{\overset{O}{\|}}{C}-CH=CH-CH_3}$$

$$1 + CH_3-CH=CH-CHO + \begin{matrix}H_3C\\H_3C\end{matrix}\!\!>C=CH-\overset{\overset{O}{\|}}{C}-CH_3$$

Hence, total number of products = 4 (without considering isomers).

80. (a)



Mathematics

81. (b) Given, $f(x) = 2x + 1, g(x) = x^2 - 2$

Since, $g \circ f(x) = g(f(x)) = g(2x + 1) = (2x + 1)^2 - 2$
$= 4x^2 + 4x + 1 - 2 = 4x^2 + 4x - 1$

82. (c) Given, $f(x) = \frac{a^x + a^{-x}}{2}$

Then, $f(x + y) + f(x - y)$
put $x + y \to x$ and $x - y \to x$

$$= \frac{a^{x+y} + a^{-(x+y)}}{2} + \frac{a^{x-y} + a^{-(x-y)}}{2}$$

$$= \frac{(a^x + a^{-x})(a^y + a^{-y})}{2}$$

$= 2\cdot\frac{(a^x+a^{-x})}{2}\cdot\frac{(a^y+a^{-y})}{2} = 2f(x)f(y)$

83. (a) We know that

$\{x\} = x-[x]$ and $f:(0,1)\to(0,1)$

Defined by $f(x)=\min\{x-[x],-x-[x]\}$ which is an identity function, as $x\in(0,1)$ and $\{x\}\in(0,1)$

Where, $\{.\}$isfractional partfunction.

$\Rightarrow (f\circ fofof)(x)=x$

84. (c) Given the statement $8n+16\le 2^n, n\in N$

$\Rightarrow 8(n+2)\le 2^n \Rightarrow n+2\le 2^{n-3}$

Clearly, for $n=6$

$6+2\le 2^{6-3}$ or $8\le 8$ is true.

85. (d) Since if we put $x=-1$

We can see R_1 and R_3 became identical so $A=0$

Let $A=\begin{vmatrix}1 & -3 & 1\\ 1 & 6 & 4\\ 1 & 3x & x^2\end{vmatrix}=0$

Hence $x=-1$ is a root of the given equation

$\Rightarrow$ from option, $x^2-x-2=0$

86. (b) Given $A=\begin{bmatrix}a & -b\\ b & a\end{bmatrix}$ and, $ab=\frac{5}{2}$

Then, $A^T=\begin{bmatrix}a & b\\ -b & a\end{bmatrix}$

$\because AA^T=20I$

$\Rightarrow \begin{bmatrix}a & -b\\ b & a\end{bmatrix}\begin{bmatrix}a & b\\ -b & a\end{bmatrix}=20\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\therefore a^2+b^2=20\Rightarrow (a+b)^2-2ab=20$

$\Rightarrow (a+b)^2=20+2\times\frac{5}{2}\ \left[\because ab=\frac{5}{2}\right]$

or $a+b=\pm 5$

Equation whose roots are a and b is given by $x^2+(a+b)x+ab=0$ or $x^2\pm 5x+\frac{5}{2}=0$ or $2x^2\pm 10x+5=0$

87. (c) Since $A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$ Hence, $|A|=10$ …. (i)

and $10B=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & \alpha\\ 1 & -2 & 3\end{bmatrix}$

$\therefore |10\,B|=4(2\alpha)-2(-15-\alpha)+2(10)=10\alpha+50$

$\Rightarrow 10^3|\,B|=10\alpha+50\ [\because |kA|=k^n|\,A|]$ ... (ii)

$\because B=A^{-1}$

$\Rightarrow |\,B|=|A^{-1}|=|A|^{-1}\Rightarrow |B|=\frac{1}{|A|}=\frac{1}{10}$

From Eq. (ii), we get

$1000\cdot\frac{1}{10}=10\alpha+50$ or $\alpha=\frac{50}{10}=5$

88. (a) Given $A=\begin{bmatrix}4 & 2 & 1-x\\ 5 & k & 1\\ 6 & 3 & 1+x\end{bmatrix}$

and, rank of matrix A is 1 .

$\Rightarrow$ All the minors of order 2 are zero.

$\therefore M_{33}=0\Rightarrow\begin{vmatrix}4 & 2\\ 5 & k\end{vmatrix}=0$

or $4k-10=0$ or $k=\frac{5}{2}$

and $M_{11}=\begin{vmatrix}k & 1\\ 3 & 1+x\end{vmatrix}=0$

or $\begin{vmatrix} \frac{5}{2} & 1 \\ 3 & 1+x \end{vmatrix} = 0$

or $\frac{5}{2}x = \frac{1}{2}$ or $x = \frac{1}{5}$

89. (d) Given $a_1, a_2, \ldots, a_9$ are in GP.

Then, $\frac{a_2}{a_1} = \frac{a_3}{a_2} = \frac{a_9}{a_8} = \lambda$ (say) [common ratio]

Now, $\begin{vmatrix} \log a_1 & \log a_2 & \log a_3 \\ \log a_4 & \log a_5 & \log a_6 \\ \log a_7 & \log a_8 & \log a_9 \end{vmatrix}$

$C_3 \to C_3 - C_2, C_2 \to C_2 - C_1$

$\begin{vmatrix} \log a_1 & \log \frac{a_2}{a_1} & \log \frac{a_3}{a_2} \\ \log a_4 & \log \frac{a_5}{a_4} & \log \frac{a_6}{a_5} \\ \log a_7 & \log \frac{a_8}{a_7} & \log \frac{a_9}{a_8} \end{vmatrix}$

or $\begin{vmatrix} \log a_1 & \log \lambda & \log \lambda \\ \log a_4 & \log \lambda & \log \lambda \\ \log a_7 & \log \lambda & \log \lambda \end{vmatrix} = 0$

[$\because$ By property of determinant C_2 and C_3 are identical]

90. (d) Given $(\sin\theta - i\cos\theta)^3 = (-i)^3(\cos\theta + i\sin\theta)^3 = (-i)^3(\cos 3\theta + i\sin 3\theta)$ [by De-Moivre theorem]

91. (b) Let the real part of $(\cos 4 + i\sin 4 + 1)^{2020}$

$= (2\cos^2 2 + i2\sin 2\cos 2)^{2020}$

$= 2^{2020}\cos^{2020}(2)(\cos 2 + i\sin 2)^{2020}$

$= 2^{2020}\cos^{2020}(2)(\cos 4040 + i\sin 4040)$

$= 2^{2020}\cos^{2020} 2.\cos 4040[z + \bar{z} = 2\mathrm{Re}(z)]$

92. (b) Given equation $(x^2 + 5x + 5)^{x+5} = 1$

If $x + 5 = 0 \Rightarrow x = -5$

and $x^2 + 5x + 5 = 1$ or $x^2 + 5x + 4 = 0$

$\Rightarrow x = -1, -4$

$x = -1, -4$ and -5 are the three integers satisfying given equation.

93. (b) Given $f(x) = x^3 + ax^2 + bx + c$

Let roots of $f(x)$ are α, β, γ

since roots are in AP. $\Rightarrow 2\beta = \alpha + \gamma$

Now, sum of roots

$\alpha + \beta + \gamma = -a \Rightarrow \beta = -\frac{a}{3}$

It is given that, roots are integers.

$\therefore$ β is an integer when a is multiple of 3 .

Option (a), (c) and (d) are multiple of 3.

$\Rightarrow a \neq 1214$

94. (b) Given $e^{4t} - 10e^{3t} + 29e^{2t} - 22e^t + 4 = 0$

Let $e^t = x$

then, $x^4 - 10x^3 + 29x^2 - 22x + 4 = 0$

Here, roots are x_1, x_2, x_3, x_4

Hence product of roots $x_1 \cdot x_2 \cdot x_3 \cdot x_4 = 4$

$\Rightarrow e^{t_1} \cdot e^{t_2} \cdot e^{t_3} \cdot e^{t_4} = 4 \Rightarrow e^{t_1+t_2+t_3+t_4} = 4$

$\Rightarrow t_1 + t_2 + t_3 + t_4 = \log_e 4 = \log_e 2^2 = 2\log_e 2$

Sum of roots $= 2\log_e 2$

95. (b) Given total number of coins = 3

The number of different sums = ${}^3C_1 + {}^3C_2 + {}^3C_3$

$= 3 + 3 + 1 = 7$

96. **(c)** Given number of white balls $= 7$ and number of black balls $= 3$
Here, B - B - B - B - B - B - B - B
are the places in row where black balls can be placed.
So, number of arrangements $= {}^{8}C_3 = \frac{8\times7\times6}{1\times2\times3} = 56$

97. **(a)** Since, each girl is to given at least one ring so, 3 rings to give 3 girls in one way since rings are identical, so, balls can be arranged in ${}^{n-1}C_{r-1}$ ways. Then, number of ways is ${}^{7}C_2 = 21$.

98. **(b)** Given $\frac{x^4}{(x-1)(x-2)} = f(x) + \frac{A}{x-1} + \frac{B}{x-2}$

$\frac{x^4}{(x-1)(x-2)} = \frac{x^4}{x^2-3x+2}$ is an improper fraction.

$$\therefore \frac{x^4}{(x-1)(x-2)} = x^2 + 3x + 7 + \frac{15x - 14}{(x-1)(x-2)}$$
$$= x^2 + 3x + 7 + \frac{A}{x-1} + \frac{B}{x-2}$$

On comparing with given equation,
$\frac{x^4}{(x-1)(x-2)} = f(x) + \frac{A}{x-1} + \frac{B}{x-2}$, we have
$f(x) = x^2 + 3x + 7$

99. **(c)** Let, $A = 2\alpha, B = 30° - \alpha, C = 60° - \alpha$
then $A + B + C = 2\alpha + 30° - \alpha + 60° - \alpha = 90°$
$\because \tan(A + B + C)$
$$= \frac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$$
If $A + B + C = 90°$, then
$1 - \tan A \tan B - \tan B \tan C - \tan C \tan A = 0$
or $\tan A \tan B + \tan B \tan C + \tan C \tan A = 1$
$\therefore \tan 2\alpha \tan(30° - \alpha) + \tan 2\alpha \cdot \tan(60° - \alpha)$
$+\tan(30° - \alpha)\tan(60° - \alpha) = 1$

100. **(b)** Given $\sin\alpha - \cos\alpha = m$ and $\sin 2\alpha = n - m^2$
$\therefore (\sin\alpha - \cos\alpha)^2 = m^2$
$\sin^2\alpha + \cos^2\alpha - 2\sin\alpha\cos\alpha = m^2$
or $1 - \sin 2\alpha = m^2$
or $\sin 2\alpha = 1 - m^2$
But $\sin 2\alpha = n - m^2$ (given)
$\Rightarrow n = 1$

101. **(c)** Given $3\operatorname{cosec} x = 4\sin x$
or $\sin^2 x = 3/4$
or $\sin^2 x = \sin^2\frac{\pi}{3} \Rightarrow x = n\pi \pm \frac{\pi}{3}, n \in Z$
or $x = \pm\frac{\pi}{3}$

102. **(d)** Given $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \cdots$
$$+\tan^{-1}\left(\frac{}{1+n(n+1)}\right) = \tan^{-1}()$$
$$\Rightarrow \tan^{-1}\left(\frac{2-1}{1+1.2}\right) + \tan^{-1}\left(\frac{3-2}{1+2.3}\right) + \cdots$$
$$+\tan^{-1}\left(\frac{n+1-n}{1+n(n+1)}\right) = \tan^{-1}(x)$$
$\Rightarrow \tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}(3) - \tan^{-1}(2) + \cdots$
$+\tan^{-1}(n+1) - \tan^{-1}(n) = \tan^{-1}(x)$
$\Rightarrow \tan^{-1}(n+1) - \tan^{-1}(1) = \tan^{-1}(x)$
$$\left\{\because \tan^{-1}(A) - \tan^{-1}(B) = \tan^{-1}\left(\frac{A-B}{1+A\cdot B}\right)\right\}$$

$\Rightarrow \tan^{-1}\left(\frac{n}{n+2}\right) = \tan^{-1}(x) \Rightarrow x = \frac{n}{n+2}$

103. **(b)** Since, given $\sinh u = \tan\theta$

$\Rightarrow \tan\theta = \sinh u$

$\Rightarrow \tan\theta = \sqrt{\cosh^2 u - 1}$ $[\cosh^2 x - \sinh^2 x = 1]$

$\Rightarrow \cosh u = \sqrt{1+\tan^2\theta} \Rightarrow \cosh u = \sec\theta$

104. **(c)** Given in $\triangle ABC$, $a = 3, b = 4, \sin A = 3/4$

$\therefore \cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

Since, $\cos A = \frac{b^2+c^2-a^2}{2bc} \Rightarrow \frac{\sqrt{7}}{4} = \frac{4^2+c^2-3^2}{2.4.c}$

$\Rightarrow c^2 - 2\sqrt{7}c + 7 = 0 \Rightarrow (c-\sqrt{7})^2 = 0$

$\Rightarrow c = \sqrt{7}$

So, $\cos B = \frac{c^2+a^2-b^2}{2ac}$

$\Rightarrow \cos B = \frac{7+9-16}{2\cdot 3\cdot\sqrt{7}} = 0 \Rightarrow B = 90^\circ \;\therefore \angle CBA = 90^\circ$

105. **(c)** Since $A = 75^\circ, B = 45^\circ$, then $C = 60^\circ$

$\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is circum radius

$\Rightarrow a = 2R\sin A, b = 2R\sin B, c = 2R\sin C$

Then, $b + c\sqrt{2} = 2R(\sin B + \sqrt{2}\sin C)$

$= 2R(\sin 45^\circ + \sqrt{2}\sin 60^\circ)$

$= 2R\left(\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{\sqrt{2}}\right) = 2R\left(\frac{\sqrt{3}+1}{\sqrt{2}}\right)$

$= 2.\,(2R\sin 75^\circ) = 2(2R\sin A) = 2a$

106. **(b)** In $\triangle ABC$,

Sides opposite to $\angle A$, and $\angle B$ can be given as

$a = 2R\sin A = 4R\sin\frac{A}{2}\cos\frac{A}{2}$

$b = 2R\sin B = 4R\sin\frac{B}{2}\cos\frac{B}{2}$

$r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

$r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

$r_2 = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

$r_3 = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$

Now, $ab - r_1r_2 = 16R^2\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{A}{2}\cos\frac{B}{2}\left(1-\cos^2\frac{C}{2}\right)$

$= 16R^2\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{A}{2}\cos\frac{B}{2}\sin^2\frac{C}{2}$

$\therefore \frac{ab - r_1r_2}{r_3} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = r$

107. **(c)** Here $\cos\alpha = \cos\beta$ and $\cos\gamma = 0$ $(\because \gamma = 90^\circ)$

Hence, $\cos^2\alpha + \cos^2\alpha + \cos^2\gamma = 1$

$\Rightarrow 2\cos^2\alpha = 1 \Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \alpha = 45°$ or $135°$

108. **(c)** P.V. of pt A is $5\hat{i} - 4\hat{j} - 3\hat{k}$

and P.V. of x-axis i.e. $(\hat{i} + 0\hat{j} + 0\hat{k})$

Hence, angle,

$$\cos\theta = \frac{5\cdot 1 + (-4)\cdot 0 + (-3)\cdot 0}{\sqrt{5^2 + (-4)^2 + (-3)^2}\sqrt{1^2 + 0^2 + 0^2}}$$

$$= \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}} \quad \therefore \theta = \frac{\pi}{4}$$

109. **(a)** Let P.V. of A, B, C is represented as $\vec{a}, \vec{b}, \vec{c}$ respectively.

Mid-points D, E, F is $\frac{\vec{a}+\vec{b}}{2}, \frac{\vec{a}+\vec{c}}{2}, \frac{\vec{b}+\vec{c}}{2}$ respectively.

Then, $\overrightarrow{BE} = \frac{\vec{a}-2\vec{b}+\vec{c}}{2}, \overrightarrow{AF} = \frac{\vec{b}-2\vec{a}+\vec{c}}{2}$

$$\overrightarrow{CD} = \frac{\vec{a} + \vec{b} - 2\vec{c}}{2}$$

$$\overrightarrow{BE} + \overrightarrow{AF} = \frac{2\vec{c} - \vec{a} - \vec{b}}{2} = \frac{-(\vec{a} + \vec{b} - 2\vec{c})}{2}$$

$$= -\overrightarrow{CD} = \overrightarrow{DC}$$

110. **(b)** Let $\vec{a} = \hat{i} + a\hat{j} + \hat{k}, \vec{b} = \hat{j} + a\hat{k} \Rightarrow \vec{a} = a\hat{i} + \hat{k}$

Volume of parallelopiped $= [\vec{a}\vec{b}\vec{c}]$

So, $[\vec{a}\vec{b}\vec{c}] = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$

$\Rightarrow V = a^3 - a + 1$

Differentiating V with respect to a

$$\frac{dV}{da} = 3a^2 - 1$$

For stationary points $\frac{dV}{da} = 0$

$\Rightarrow 3a^2 - 1 = 0$ or $a = \pm\frac{1}{\sqrt{3}}$

At, $a = \frac{1}{\sqrt{3}}$,

$$\because \frac{d^2V}{da^2} = 6a \Rightarrow \frac{d^2V}{da^2} = 6\cdot\frac{1}{\sqrt{3}} > 0$$

$\therefore$ Volume is minimum at $a = \frac{1}{\sqrt{3}}$

111. **(b)** Given $\vec{a} = \frac{3}{2}\hat{k}, \vec{b} = \frac{2\hat{i}+2\hat{j}-\hat{k}}{2}$

$$\vec{a} + \vec{b} = \frac{3}{2}\hat{k} + \frac{2\hat{i} + 2\hat{j} - \hat{k}}{2} = \hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} - \vec{b} = \frac{3}{2}\hat{k} - \frac{2\hat{i} + 2\hat{j} - \hat{k}}{2} = -\hat{i} - \hat{j} + 2\hat{k}$$

Let θ be the angle between $(a + b)$ and $(a - b)$ then

$$= \frac{1(-1) + 1(-1) + 1(2)}{\sqrt{1^2 + 1^2 + 1^2}\sqrt{(-1)^2 + (-1)^2 + 2^2}} = 0$$

$\therefore \theta = \frac{\pi}{2}$ or $90°$

112. **(a)** Given $\bar{a} = \hat{i} + \hat{j} + \hat{k}, \bar{b} = \hat{i} + 3\hat{j} + 5\hat{k}$

$\bar{c} = 7\hat{i} + 9\hat{j} + 11\hat{k}$

Then, $\bar{a} + \bar{b} = 2\hat{i} + 4\hat{j} + 6\hat{k}$

and $\bar{b}+\bar{c}=8\hat{i}+12\hat{j}+16\hat{k}$

Area of parallelogram $=\frac{1}{2}|(\bar{a}+\bar{b})\times(\bar{b}+\bar{c})|$

$=\frac{1}{2}|2(\hat{i}+2\hat{j}+3\hat{k})\times 4(2\hat{i}+3\hat{j}+4\hat{k})|$

$=4\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 3\\ 2 & 3 & 4\end{vmatrix}=4|-\hat{i}+2\hat{j}-\hat{k}|=4\sqrt{6}$ sq. units.

113. **(b)** Given $\vec{a}=2\hat{i}+\hat{j}+3\hat{k}, \vec{b}=\hat{i}+3\hat{j}-\hat{k}, \vec{c}=3\hat{i}-\hat{j}-2\hat{k}$

$\vec{a}\cdot\vec{a}=4+1+9=14$

$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}=2+3-3=2$

$\vec{a}\cdot\vec{c}=\vec{c}\cdot\vec{a}=6-1-6=-1$

$\vec{b}\cdot\vec{b}=1+9+1=11$

$\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{b}=3-3+2=2$

$\vec{c}.\vec{c}=9+1+4=14$

Now, $\begin{vmatrix}\vec{a}\cdot\vec{a} & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c}\\ \vec{b}\cdot\vec{a} & \vec{b}\cdot\vec{b} & \vec{b}\cdot\vec{c}\\ \vec{c}\cdot\vec{a} & \vec{c}.\vec{b} & \vec{c}\cdot\vec{c}\end{vmatrix}=\begin{vmatrix}14 & 2 & -1\\ 2 & 11 & 2\\ -1 & 2 & 14\end{vmatrix}=2025$

114. **(c)** Given $|\vec{a}|=2, |\vec{b}|=3$

$\because (\vec{a}+t\vec{b})\perp(\vec{a}-t\vec{b})$

$\Rightarrow (\vec{a}+t\vec{b})\cdot(\vec{a}-t\vec{b})=0$

$\Rightarrow \vec{a}\cdot\vec{a}-t(\vec{a}\cdot\vec{b})+t(\vec{b}\cdot\vec{a})-t^2(\vec{b}\cdot\vec{b})=0$

$\Rightarrow |\vec{a}|^2-t^2|\vec{b}|^2=0$

$\Rightarrow t^2=\frac{|\vec{a}|^2}{|\vec{b}|^2}\Rightarrow t=\frac{|\vec{a}|}{|\vec{b}|}=\frac{2}{3}$

115. **(c)** Given variates x = 112,116,120,125,132

x	$(x-\bar{x})^2$
112	81
116	25
120	1
125	16
132	121

$\Sigma x=605; \Sigma(x-\bar{x})^2=244$

Mean $=\bar{x}=\frac{605}{5}=121$

Variance $=\frac{\Sigma(x-\bar{x})^2}{n}=\frac{244}{5}=48.8$

116. **(d)** Clearly, difference of values from arithmetic mean is least in option (d).

$\Rightarrow$ Option (d) has least standard deviation.

117. **(a)** Since total distribution $=3^{12}$

Number of distribution when first box contains three balls $={}^{12}C_3\times 2^9$

Required probability $=\frac{{}^{12}C_3\times 2^9}{3^{12}}$

118. **(b)** Since number of letters in the word REGULATIONS

$=11$. So, number of possible arrangements $=11$!

We are required 4 letter between R and E.

So we can select 4 letters out of 9 letters

Favourable number of arrangements
$= {}^9C_4 \times 4! \times 2! \times 6!$

Required probability $= \frac{{}^9C_4 \times 4! \times 2! 6!}{11!} = \frac{6}{55}$

119. **(a)** Since events E = {X is a prime number }

So, $P(E) = P(X=2) + P(X=3) + P(X=5) + P(X=7) = 0.23 + 0.12 + 0.20 + 0.07 = 0.62$

Given $F = \{x < 4\}$

$P(F) = P(X=1) + P(X=2) + P(X=3)$

$= 0.15 + 0.23 + 0.12 = 0.50$

$P(E \cap F) = P(X=2) + P(X=3)$

$= 0.23 + 0.12 = 0.35$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= 0.62 + 0.50 - 0.35 = 0.77$

120. **(d)** Since sample spaces of a dice $S = \{1,2,3,4,5,6\}$

Let E = getting even number $\Rightarrow E = \{2,4,6\}$

Hence, $P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$

If dice is thrown three times

Let X = success, then

X	x = 0	x = 1	x = 2	x = 3
P(X = x)	${}^3C_0\left(\frac{1}{2}\right)^3$	${}^3C_1\left(\frac{1}{2}\right)^3$	${}^3C_2\left(\frac{1}{2}\right)^3$	${}^3C_3\left(\frac{1}{2}\right)^3$

Probability of getting at least 2 successes

$= P(X \geq 2) = P(X=2) + P(X=3)$

$= {}^3C_2\left(\frac{1}{2}\right)^3 + {}^3C_3\left(\frac{1}{2}\right)^3 = \frac{3}{8} + \frac{1}{8} = \frac{1}{2}$

121. **(b)** Given, coordinate $(x, y) = (2\sqrt{2}, -3\sqrt{2})$

Let new coordinates be (x', y').

Then, the new coordinate is given as

$x' = x\cos\theta + y\sin\theta$ and $y' = y\cos\theta - x\sin\theta$

Since, $\theta = 45°$

$\therefore x' = 2\sqrt{2}\cos 45° + (-3\sqrt{2})\sin 45°$

$= 2\sqrt{2} \cdot \frac{1}{\sqrt{2}} - 3\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 2 - 3 = -1$

and $y' = (-3\sqrt{2})\cos 45° - 2\sqrt{2}\sin 45° = -2 - 3 = -5$

New coordinate is $(-1, -5)$.

122. **(a)** Given line, $5x - 2y = 10$

$\Rightarrow \frac{x}{2} + \frac{y}{-5} = 1$

Sum of the squares of the intercepts

$= 2^2 + (-5)^2 = 29$

123. **(c)** Since, a, b and c are three consecutive odd integers.

So, a, b and c will be in A.P.

$\Rightarrow 2b = a + c \Rightarrow a - 2b + c = 0$

which is similar to the line $ax + by + c = 0$ passing through $(1, -2)$.

$\therefore (\alpha, \beta) = (1, -2)$

$\Rightarrow \alpha^2 + \beta^2 = 1^2 + (-2)^2 = 5$

124. **(b)** Let the equation of line parallel to X -axis is $y = k$ Then, point of intersection of the line and the curve $y = \sqrt{x}$ is (k^2, k)

Now, slope of the curve is $y' = \frac{1}{2\sqrt{x}}$

or, $\left(\frac{dy}{dx}\right)_{(k^2,k)} = \tan 45^\circ = 1$

$\Rightarrow \frac{1}{2\sqrt{k^2}} = 1 \Rightarrow k = \frac{1}{2}$ $\therefore$ Line is $y = \frac{1}{2}$

125. **(a)** Since line $2x + 3y + 4 = 0$ is perpendicular bisector of line through $A(1,2)$ and $B(\alpha, \beta)$

Hence, distance of point A from the line will be equal to the distance of B from line.

$\therefore \frac{|2+6+4|}{\sqrt{13}} = \frac{|2\alpha + 3\beta + 4|}{\sqrt{13}}$

$\Rightarrow 2\alpha + 3\beta + 4 = \pm 12$

$\Rightarrow 2\alpha + 3\beta = 8$

Also, the line joining A and B is perpendicular to

$2x + 3y + 4 = 0$

$\Rightarrow$ Product of slopes $= -1$

$\therefore \frac{\beta - 2}{\alpha - 1} \times \left(\frac{-2}{3}\right) = -1$

$\Rightarrow 2\beta - 3\alpha = 1$

Solving eqs. (ii) and (iii), we have

$\alpha = \frac{-35}{13}, \beta = \frac{-46}{13}$

$\therefore 13\alpha + 13\beta = -35 - 46 = -81$

126. **(b)** Since the equation of the pair of lines perpendicular to the pair $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and passes through origin is given by $bx^2 - 2hxy + ay^2 = 0$

$\Rightarrow a = 2, b = 2, h = \frac{3}{2}$

So, required pair of lines $2x^2 - 3xy + 2y^2 = 0$

127. **(c)** Given lines,

$x + y = k$

and $2y^2 + 5xy - 3x^2 = 0$

or $(y + 3x)(2y - x) = 0$

$\Rightarrow y + 3x = 0$

$2y - x = 0$

On solving eqs. (i), (ii) and (iii), we get vertices of triangle, which are $(0,0), \left(\frac{2k}{3}, \frac{k}{3}\right)$ and $\left(\frac{-k}{2}, \frac{3k}{2}\right)$

$\therefore$ Centroid $= \left(\frac{k}{18}, \frac{11k}{18}\right)$

Comparing with given centroid $\left(\frac{1}{18}, \frac{11}{18}\right)$, we have $k = 1$

128. **(c)** Given homogeneous equation is $5x^2 - 8xy + 3y^2 = 0$

It can be broken as

$\Rightarrow (5x - 3y)(x - y) = 0$

$\Rightarrow 5x - 3y = 0$ or $x - y = 0$

Slopes $m_1 = \frac{5}{3}, m_2 = 1 \Rightarrow m_1 : m_2 = \frac{5}{3} : 1 = 5 : 3$

129. **(c)** Let m be the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$

Hence, the other line has slope m^2

Now, $m + m^2 = \frac{-2h}{b}$ and $m \cdot m^2 = \frac{a}{b}$

$\because (m + m^2)^3 = m^3 + (m^2)^3 + 3m \cdot m^2(m + m^2)$

$\Rightarrow \frac{-8h^3}{b^3} = \frac{a}{b} + \frac{a^2}{b^2} - \frac{6ah}{b^2}$

$\Rightarrow \frac{a}{b} + \frac{a^2}{b^2} - \frac{6ah}{b^2} + \frac{8h^3}{b^3} = 0$

or $ab(a + b) + 8\,h^3 = 6abh$

or $\frac{a+b}{h}+\frac{8h^2}{ab}=6$

or $\left|\frac{a+b}{h}+\frac{8h^2}{ab}\right|=6$

130. **(a)** Given equation of circle $x^2+y^2=50$ .. (i)

and a line $x+7=0$

On solving both the equations simultaneously

We get $(-7,-1)$ and $(-7,1)$ as pt. of inter-section

$\therefore$ $(-7,1)$ and $(-7,-1)$ are points of contact of eqs. (i) and (ii),

for circle $x^2+y^2=r^2$

Since, equation of tangent at (x_1,y_1) is

$xx_1+yy_1=r^2$

$\therefore$ Equation of tangent of at $(-7,1)$ and $(-7,-1)$ are

$-7x+y=50$ or $7x-y+50=0$

and $-7x-y=50$ or $7x+y+50=0$

131. **(c)** Let be any point on the circle $x^2+y^2=r_1^2$

$\Rightarrow h_1^2+k_1^2=r_1^2$ ... (i)

Equation of chord of contact of tangents from

(h,k) to the circle $x^2+y^2={r_2}^2$ is

$hx+ky={r_2}^2$

$\because$ Eq. (ii) touches the circle, $x^2+y^2=r_3^2$

The length of perpendicular from centre (0,0) on Eq. (ii) is radius $=r_3$

$$\Rightarrow \frac{r_2^2}{\sqrt{h^2+k^2}}=r_3 \Rightarrow \frac{r_2^2}{\sqrt{r_1^2}}=r_3$$

$\Rightarrow r_2^2=r_1r_3 \Rightarrow r_1,r_2,r_3$ are in G.P.

132. **(d)** Since circle touches at (3,0) hence its centre is $(3,k)$ So, equation of circle is $(x-3)^2+(y-k)^2=r^2$

it passes through $(1,-2)$

and $4+k^2+4k+4=k^2$

$\Rightarrow 8+4k=0 \Rightarrow k=-2$

$\therefore$ Required equation of circle

$(x-3)^2+(y+2)^2=2^2$

or $x^2+y^2-6x+4y+9=0$

133. **(b)** Since equation of line passes through origin Let $L_1: y=mx$ given, intercepts made by L_1 and L_2 are equal

$\Rightarrow L_1$ and L_2 are at the same distance from centre of circle $x^2+y^2-x+3y=0$

Now, centre $\left(\frac{1}{2},\frac{-3}{2}\right)$ [$\because$ centre $=(-g,-f)$]

Distance of L_1 from $\left(\frac{1}{2},-\frac{3}{2}\right)=\frac{\left|m\cdot\frac{1}{2}+\frac{3}{2}\right|}{\sqrt{m^2+1}}$

Distance of L_2 from $\left(\frac{1}{2},-\frac{3}{2}\right)=\frac{\left|\frac{1}{2}-\frac{3}{2}-1\right|}{\sqrt{1+1}}$

$\because L_2: x+y-1=0$ [$\because y=mx \Rightarrow mx-y=0$]

$$\Rightarrow \frac{\left|\frac{m+3}{2}\right|}{\sqrt{m^2+1}}=\frac{2}{\sqrt{2}}$$

Squaring both sides, we get

$$\frac{(m+3)^2}{4(m^2+1)}=\frac{4}{2} \Rightarrow 7m^2-6m-1=0$$

$$\Rightarrow m=\frac{6\pm\sqrt{36+28}}{14}=\frac{6\pm 8}{14}=1,\frac{-1}{7}$$

$\therefore L_1$ is $y = x$ and $y = \frac{-1}{7}x$

or $x - y = 0$ or $x + 7y = 0$

134. **(d)** Given, circle

$x^2 + y^2 + 4x + 16y - 30 = 0$ ….. (i)

Where, $g_1 = 2, f_1 = 8, c_1 = -30$

Now equation of circle whose centre at $(1,2)$ is

$x^2 + y^2 - 2x - 4y + c = 0$ …. (ii)

Here, $g_2 = -1, f_2 = -2, c_2 = c$

Since, circles cut each other orthogonally

So, $2(g_1g_2 + f_1f_2) = c_1 + c_2$

$\Rightarrow 2(-2, -16) = -30 + c$ or $c = -6$

From eqs. (ii), we get

$x^2 + y^2 - 2x - 4y - 6 = 0$

Radius $r = \sqrt{(-1)^2 + (-2)^2 - (-6)} = \sqrt{11}$

135. **(b)** Let the circles as S_1, S_2 and S_3

Hence, $S_1 : x^2 + y^2 - 8x + 40 = 0$

$S_2 : x^2 + y^2 - 5x + 16 = 0$

$S_3 : x^2 + y^2 - 8x + 16y + 160 = 0$

To obtain the point which has same power is

$S_2 - S_1 = 0$ and $S_3 - S_1 = 0$

$\Rightarrow 3x - 24 = 0$ and $16y + 120 = 0$

$\Rightarrow x = 8$ and $y = \frac{-15}{2} \Rightarrow 2at = 2$

$\therefore$ Required point is $\left(8, \frac{-15}{2}\right)$.

136. **(c)** Here given the parabola, $y^2 = 8x$

$\Rightarrow a = 2$ and given one end of focal chord $\left(\frac{1}{2}, 2\right)$

Let parametric form of ends of focal chord $= (at_1{}^2, 2at)$

$\Rightarrow 2at = 2$

$\therefore 2.2.t = 2 \Rightarrow t = \dfrac{1}{2}$

Hence, length of focal chord $= a\left(t + \frac{1}{t}\right)^2$

$= 2\left(\dfrac{1}{2} + 2\right)^2 = 2 \cdot \left(\dfrac{5}{2}\right)^2 = \dfrac{25}{2}$

137. **(b)** Given equation of ellipse, $\frac{x^2}{25} + \frac{y^2}{16} = 1$

Centre $C = (0,0)$

If $P(x, y)$ be any point on the ellipse, then maximum value of $CP =$ length of the semi-major axis $= 5$ minimum value of $CP =$ length of semi-minor axis $= 4$

Their sum $= 5 + 4 = 9$

138. **(a)** Since, we know that any tangent to hyperbola forms a triangle with asymptotes which has constant area $=$ ab

Given $ab = a^2 \tan\alpha \Rightarrow \frac{b}{a} = \tan\alpha$

Eccentricity, $e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \tan^2\alpha} = \sec\alpha$

139. **(d)** Let the line joining the points $A(2,4,5)$ and $B(3,5,-4)$ is divided by YZ-plane at $(0, y, z)$ in the ratio $1:\lambda$.

Then, by section formula $\frac{2\lambda+3}{\lambda+1} = 0$

$\Rightarrow \frac{1}{\lambda} = -\frac{2}{3}$ or $2:3$ (externally)

140. **(c)** Since the line makes equal angles with the coordinate axes, then $\cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$

$\Rightarrow \cos^2\alpha+\cos^2\alpha+\cos^2\alpha=1$

or $3\cos^2\alpha=1$ or $\cos\alpha=\pm\frac{1}{\sqrt{3}}$

Direction cosines are $\left(\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}},\pm\frac{1}{\sqrt{3}}\right)$

141. **(a)** Let $P(x,y,z)$ and $O(0,0,0)$

DR's of $OP=\sqrt{(x-0)^2+(y-0)^2+(z-0)^2}$

But it is given that DR's of $OP=(1,-2,-2)$

$\because x=1, y=-2, z=-2$

$\therefore$ Coordinate of $P=(1,-2,-2)$

142. **(c)** Given $\lim_{z\to1}\frac{z^{1/3}-1}{z^{1/6}-1}\left[\frac{0}{0}\text{ form }\right]$

$$=\lim_{z\to1}\frac{\frac{1}{3}z^{-2/3}}{\frac{1}{6}z^{-5/6}}=\frac{6\,(1)^{5/6}}{3\,(1)^{2/3}}=\frac{6}{3}=2$$

143. **(d)** Given f is continuous at $x=0$

$\therefore \lim_{x\to0}f(x)=f(0)$

$$\Rightarrow \lim_{x\to0}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}=k\log 2\log 3$$

$\Rightarrow K\log 2.\log 3$

$$=\lim_{x\to0}\frac{(9^x-1)(8^x-1)(\sqrt{2}+\sqrt{1+\cos x})}{2-(1+\cos x)}$$

$$=\lim_{x\to0}\left[\frac{(9^x-1)}{x}\cdot\left(\frac{8^x-1}{x}\right)\cdot\frac{x^2}{1-\cos x}\cdot(\sqrt{2}+\sqrt{1+\cos x})\right]$$

$$=\log 9\cdot\log 8.2(\sqrt{2}+\sqrt{2})\left\{\because \lim_{x\to0}\frac{a^x-1}{x}=\log a\right\}$$

$\Rightarrow k\log 3\cdot\log 2=24\sqrt{2}\log 3\cdot\log 2\Rightarrow k=24\sqrt{2}$

144. **(d)**

145. **(c)** Given $y=x+\frac{1}{x}$ …. (i)

On differentiating we get, $y'=1-\frac{1}{x^2}$

or $x^2y'=x^2-1$ ….. (ii)

$\Rightarrow x^2=xy-1\left\{y'=1-\frac{1}{x^2}\right\}$

$\therefore$ From eq. (ii), we have

$x^2y'=xy-1-1$ or $x^2y'-xy+2=0$

146. **(c)** Given $y=\tan^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

Putting $x^2=\cos 2\theta$ (where $x^2\le 1$)

$\Rightarrow \theta=\frac{1}{2}\cos^{-1}(x^2)$

$$\therefore y=\tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}\right)$$

$$=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\theta\right)\right)\Rightarrow y=\frac{\pi}{4}+\theta$$

$$\Rightarrow y=\frac{\pi}{4}+\frac{1}{2}\cos^{-1}(x^2)$$

$$\therefore \frac{dy}{dx}=\frac{1}{2}\cdot\left(\frac{-2x}{\sqrt{1-x^4}}\right)=\frac{-x}{\sqrt{1-x^4}}$$

147. **(c)** Given $3\sin xy + 4\cos xy = 5$
Differentiating with respect to x, we get
$3\cos xy.\left(x\frac{dy}{dx}+y\right) - 4\sin xy\left(x\frac{dy}{dx}+y\right) = 0$
$\Rightarrow x\frac{dy}{dx} + y = 0$ or $\frac{dy}{dx} = \frac{-y}{x}$

148. **(b)** Given $f(x) = \sqrt{x^2+1}, g(x) = \frac{x+1}{x^2+1}$
and $h(x) = 2x - 3$
$\Rightarrow f'(x) = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$
and $h'(x) = 2$ So, $h'(g'(x)) = 2$
$\therefore f'(h'(g'(x))) = f'(2) = \frac{2}{\sqrt{2^2+1}} = \frac{2}{\sqrt{5}}$

149. **(d)** Let radius of circle is r and error is Δr Since area of circle
$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$
We know $\left(\frac{dA}{dr}\right) = \Delta r$
$\Rightarrow (2\pi p)0.05\% \Rightarrow 2.(0.05) = 0.1$
$\therefore$ Error in calculating area $= 0.1\%$

150. **(a)** Given curve is $y = 8x^2 - x^4 - 4$
Differentiating w.r.t. x
$\frac{dy}{dx} = 16x - 4x^3 = 4x(2-x)(2+x)$
For stationary points $\frac{dy}{dx} = 0$
or $4x(2-x)(2+x) = 0$
$\Rightarrow x = 0, x = 2, x = -2$
Putting in the given curve we get $y = 0,2,-2$
Hence, Stationary points are
$(0,-4), (2,12), (-2,12)$.

151. **(a)** Given (i) $f(x) = x|x| = \begin{cases} x(-x), & x < 0 \\ x(x), & x \geq 0 \end{cases}$
$f(x) = \begin{cases} -x^2, & x < 0 \\ x^2, & x \geq 0 \end{cases}$
$f'(x) = \begin{cases} -2x, & x < 0 \\ 2x, & x \geq 0 \end{cases}$
$f'(x) > 0, \forall x \in R - \{0\}$
$\Rightarrow f(x)$ is strictly increasing on $R - \{0\}$.

152. **(d)** $f(x) = -x^3 + 4ax^2 + 2x - 5$
$f'(x) = -3x^2 + 8ax + 2$
For decreasing $f'(x) < 0$
Discriminant of $f'(x) = (8a)^2 - 4(-3)$
$= 64a^2 + 24 > 0, \forall a \in R$
Hence, f(x) is decreasing for no value of a.

153. **(c)** Given curve, $y = e^{2x} + x^2$
At $x = 0, y = e^0 + 0 = 1$
$\Rightarrow \frac{dy}{dx} = 2e^{2x} + 2x$
Let, find slope at (0,1)
and $\frac{dy}{dx} = 2 \cdot e^0 + 0 = 2$
$y - 1 = \frac{-1}{2}(x - 0) \Rightarrow x + 2y - 2 = 0$

Distance between normal to the curve and origin is
$$\frac{|1.0+2.0-2|}{\sqrt{1^2+2^2}}=\frac{2}{\sqrt{5}}$$

154. **(b)** Given $\int \frac{dx}{x\sqrt{x^4-1}}$

$$\Rightarrow \frac{1}{2}\int \frac{2xdx}{x^2\sqrt{x^4-1}}=\frac{1}{k}\sec^{-1}(x^k)$$

Let $x^2=t \Rightarrow 2xdx=dt$

$$=\frac{1}{2}\int \frac{dt}{t\sqrt{t^2-1}}=\frac{1}{2}\sec^{-1}t$$

$$\Rightarrow \frac{1}{k}\sec^{-1}(x^k)=\frac{1}{2}\sec^{-1}(x^2)\Rightarrow k=2$$

155. **(a)** Let $I=\int \frac{e^x(x+3)}{(x+5)^3}dx=\int e^x\left[\frac{1}{(x+5)^2}-\frac{2}{(x+5)^3}\right]dx$

Since $\int [f(x)+f'(x)]e^x dx=e^x f(x)+C$

$$\Rightarrow I=\frac{e^x}{(x+5)^2}+C$$

156. **(b)** Given $\int \frac{(x-1)^2}{(x^2+1)^2}dx=\tan^{-1}(x)+g(x)+k$

$$=\int\left(\frac{1}{x^2+1}-\frac{2x}{(x^2+1)^2}\right)dx=\tan^{-1}(x)-\int \frac{2x}{(x^2+1)^2}dx$$

Let $x^2+1=t\Rightarrow 2xdx=dt$

$$=\tan^{-1}(x)-\int t^{-2}dt=\tan^{-1}(x)+\frac{1}{t}+k$$

$$\Rightarrow \int \frac{(x-1)^2}{(x+1)^2}dx=\tan^{-1}(x)+\frac{1}{x^2+1}+k$$

Comparing with $\int \frac{(x-1)^2}{(x^2+1)^2}dx=\tan^{-1}(x)+g(x)+k$

we have $g(x)=\frac{1}{x^2+1}$

157. **(d)** On solving the given equation we get

$$\frac{1}{A}\log|(\sin x)^{2023}+(\cos x)^{2023}|+C$$

$$=\int \frac{1-(\cot x)^{2021}}{\tan x+(\cot x)^{2022}}dx=\int \frac{1-\frac{(\cos x)^{2021}}{(\sin x)^{2021}}}{\frac{\sin x}{\cos x}+\frac{(\cos x)^{2022}}{(\sin x)^{2022}}}dx$$

$$=\int \frac{\cos x(\sin x)^{2022}-\sin x(\cos x)^{2022}}{(\sin x)^{2023}+(\cos x)^{2023}}dx$$

$$\int \frac{f'(x)}{f(x)}dx=\log|f(x)|+C$$

$$\Rightarrow \frac{1}{A}\log|(\sin x)^{2023}+(\cos x)^{2023}|+C$$

$$=\frac{1}{2023}\log|(\sin x)^{2023}+(\cos x)^{2023}|+C$$

$\Rightarrow A=2023$

158. **(c)** Let $I=\int_2^4(|x-2|+|x-3|)dx$

$$=\int_2^4(x-2)dx-\int_2^3(x-3)dx+\int_3^4(x-3)dx$$

$$=\left(\frac{x^2}{2}-2x\right)_2^4-\left(\frac{x^2}{2}-3x\right)_2^3+\left(\frac{x^2}{2}-3x\right)_3^4$$

$$= \left[\left(\frac{16}{2}-8\right)-\left(\frac{4}{2}-4\right)\right]-\left[\left(\frac{9}{2}-9\right)-\left(\frac{4}{2}-6\right)\right]$$
$$+\left[\left(\frac{16}{2}-12\right)-\left(\frac{9}{2}-9\right)\right]$$
$$= [2+9-8] = 3$$

159. **(c)** Let $I = \int_{-1/2}^{1/2}\left\{[x]+\log\left(\frac{1+x}{1-x}\right)\right\}dx$

By property

$$\because \int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & f(-x)=f(x) \\ 0, & f(-x)=-f(x) \end{cases}$$
$$= \int_{-1/2}^{1/2}[x]dx + \int_{-1/2}^{1/2}\log\left(\frac{1+x}{1-x}\right)dx$$

Let $f(x) = \log\left(\frac{1+x}{1-x}\right)$

$$= \int_{-1/2}^{1/2}[x]dx + 0 = \int_{-1/2}^{0}[x]dx + \int_0^{1/2}[x]dx$$
$$= \int_{-1/2}^{0}(-1)dx + \int_0^{1/2}0dx$$
$$= -(x)_{-1/2}^{0} + 0 = -\left(0+\frac{1}{2}\right) = -\frac{1}{2}$$

160. **(a)** Here the differential equation $\frac{d^2y}{dx^2}+y=0$

A solution is function of x which satisfies given differential equation.

Consider option (a) $y = 3\sin x + 4\cos x$

$$\frac{dy}{dx} = 3\cos x - 4\sin x$$
$$\frac{d^2y}{dx^2} = -3\sin x - 4\cos x = -y$$
$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$