Physics

1. **(a)** We have

$N_A = 4,\ N_B = 4\ \&\ N_c = 4$

So, $N_A = N_B = N_C$.

2. **(a)**

We have, $t_{av} = \frac{l}{V_1} + \frac{l}{V_2}$

$\Rightarrow \frac{2l}{V_{av}} = \frac{l}{V_1} + \frac{l}{V_2} \Rightarrow \frac{2}{48} = \frac{1}{60} + \frac{1}{V}$

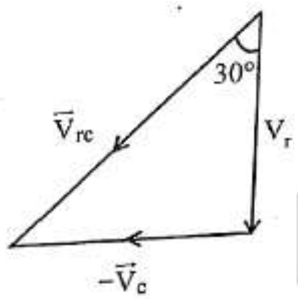
$\Rightarrow \frac{1}{24} = \frac{1}{60} + \frac{1}{V} \Rightarrow -\frac{1}{V} = \frac{1}{60} - \frac{1}{24}$

$\Rightarrow -\frac{1}{V} = \frac{2-5}{120}$

$\Rightarrow -V = -40$

$\Rightarrow\ V = 40\ \text{km/hr}$

3. **(c)**



We can see from diagram

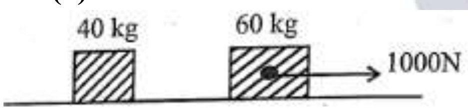
$\sin 30° = \frac{V_c}{V_{rc}} = V_{rc} = \frac{V}{\sin 30°} = 2\ V_c = 2 \times 40$

$= 80\ \text{km/hr}$

4. **(d)** $h = \frac{u^2 \sin^2\theta}{2\ g} = \frac{50^2 \sin^2 60°}{2\times10}$

$= \frac{50 \times 50 \times 3}{4 \times 20} = 93.75\ \text{m}$

5. **(b)**



Taking both mass of single system, we get

$1000 = (40 + 60)a$

$\Rightarrow a = 10\ \text{m/s}^2$

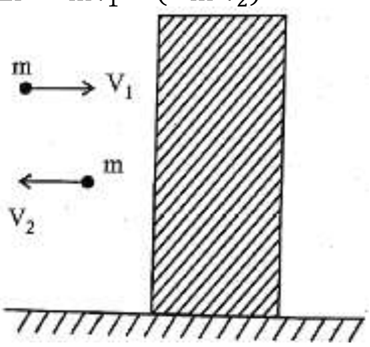
for 60 kg block

$1000 - T = 60a$

$1000 - T = 600 \Rightarrow\ T = 400\ \text{N}$

6. **(a)** We have

$\Delta P = mV_1 - (-m\ V_2)$



$\Rightarrow 8 = m(V_1 + V_2)$
$\Rightarrow 8 = 0.5(10 + V_2)$
$\Rightarrow 16 = 10 + V_2$
$\Rightarrow V_2 = 6\ m/s$

7. (a) Let x be the maximum compression. Now, potential energy gain by spring = Potential energy loss by mass of 1 kg

$$\frac{1}{2}kx^2 = mg(1+x)$$
$$\frac{1}{2} \times 15 \times x^2 = 1 \times 10 \times (1+x)$$
$$\Rightarrow 15x^2 = 20(1+x)$$
$$\Rightarrow 3x^2 = 4(1+x)$$
$$\Rightarrow 3x^2 - 4x - 4 = 0$$
$$\Rightarrow 3x^2 - 6x + 2x - 4 = 0$$
$$\Rightarrow 3x(x-2) + 2(x-2) = 0$$
$$\Rightarrow x = -2/3 \text{ or } x = 2$$

-ve value of ' x ' means spring moves upward, so it is impossible. So $x = 2$

8. (c)

We know that

$$P = FV$$
$$\Rightarrow P = maV$$
$$P \Rightarrow P = m\frac{dV}{dt}V$$
$$\Rightarrow \int_0^t \frac{Pdt}{m} = \int_0^V VdV$$
$$\Rightarrow \frac{Pt}{m} = \frac{V^2}{2}$$
$$\Rightarrow V = \sqrt{\frac{2Pt}{m}} \Rightarrow V = \sqrt{\frac{2 \times 1.2 \times 6}{0.4}} = 6\ m/s$$

9. (a)

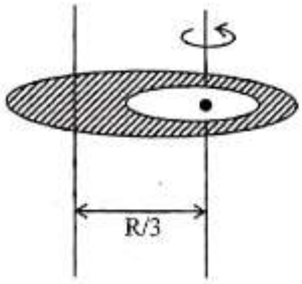
$$\overrightarrow{r_{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r_2} + \cdots.}{m_1 + m_2 + \cdots.}$$

$$\left[m\left(\frac{1}{3}\right)^2 \cdot \frac{1}{2}\right] \times 2 + \left[m\left(\frac{1}{3}\right)^3 \cdot \frac{1}{3}\right] \times 3 + \left[m\left(\frac{1}{3}\right)^4 \cdot \frac{1}{4}\right] \times 4 + \cdots$$
$$M$$

$$= \frac{m\left(\frac{1}{3}\right)^2 + m\left(\frac{1}{3}\right)^3 + m\left(\frac{1}{3}\right)^4 + \cdots}{M}$$

$$= \frac{m}{M} \cdot \frac{\left(\frac{1}{3}\right)^2}{1 - \frac{1}{3}} = \frac{m}{M} \times \frac{1}{\frac{9}{2}} = \frac{m}{6M}$$

10. (b)



We have, $\sigma=\frac{m}{4\pi R^2}$

Fill the hole with mass. Now consider the system as (complete disk + hole filled with negative mass)

About centre, $I_{net}=I_{disk}+I_{hole}$

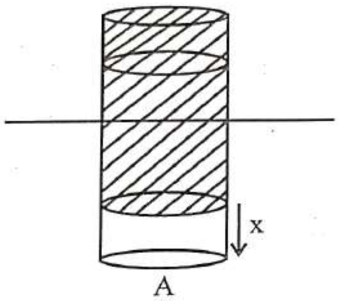
$$=\frac{1}{2}mR^2+\frac{1}{2}(-\sigma)\left[4\pi\left(\frac{R}{3}\right)^2\right]\left[\frac{R}{3}\right]^2-\sigma\left[4\pi\left(\frac{R}{3}\right)^2\right]\left[\frac{R}{3}\right]^2$$

$$=\frac{1}{2}mR^2-\frac{3}{2}\left(\frac{m}{4\pi R^2}\right)\left(4\pi\frac{R^2}{9}\right)\left(\frac{R^2}{9}\right)$$

$$=\frac{1}{2}mR^2-\frac{3}{2\times 81}mR^2=\frac{mR^2}{2}\left(1-\frac{1}{27}\right)$$

$$=\frac{mR^2}{2}\times\frac{26}{27}=\frac{13mR^2}{27}$$

11. (a)



We have $|F_{res}|$ = Extra buyonat force that acts on it after dipping ' x ' distance as shown in diagram $=\rho_l\, A\times g$

Now, $\vec{F}_{res}$ is proportional to x and it acts opposite to ' x '. So we will have a SHM

In SHM, $F_{res}=m\omega^2x$

So, $\rho_l Axg=m\omega^2x$

$$\omega=\sqrt{\frac{plAg}{m}}\Rightarrow T=2\pi\sqrt{\frac{m}{plAg}}=2\pi\sqrt{\frac{m}{p(\pi r^2)g}}$$

12. (b) To move from one point of zero velocity to next such point, time taken $=\frac{T}{2}$

So, $\frac{T}{2}=0.5\Rightarrow T=1\text{sec}$

$$\Rightarrow\frac{2\pi}{\omega}=1\Rightarrow\omega=2\pi\text{rad/s}$$

13. (a) By law of conservation of mechanical energy

$$\frac{1}{2}mV^2-\frac{GMm}{R}=\frac{-GMm}{R+h}$$

$$\frac{1}{2}mV^2=GMm\left(\frac{1}{R}-\frac{1}{R+h}\right)$$

$$\frac{1}{2}V^2=\frac{GMh}{R(R+h)}$$

$$\frac{1}{2}\times\alpha^2\times\left(\frac{2GM}{R}\right)=\frac{GMh}{R(R+h)}$$

$$\alpha^2=\frac{h}{R+h}\Rightarrow\alpha^2=\frac{800}{7200}=\frac{1}{9}\Rightarrow\alpha=\frac{1}{3}$$

14. (a) We have

$$\Delta l=\frac{Tl}{\gamma A}\left[\lambda=\frac{Fl}{A\Delta l}\right]$$

$$\Rightarrow\Delta l\propto\frac{l}{A}\Rightarrow\Delta l\propto\frac{l}{r^2}\Rightarrow\Delta l=\frac{Kl}{r^2}$$

For $l=0.5$ m and $d=0.5$ mm

$\Delta l = \dfrac{K \times 0.5}{(0.25 \times 10^{-3})^2} = 8 \times 10^6 k$

For $l = 200$ cm and $d = 2$ mm.

$\Delta l = \dfrac{K \times 2}{(1 \times 10^{-3})^2} = 2 \times 10^6 k$

For $l = 300$ cm and $d = 3$ mm

$\Delta l = \dfrac{K \times 3}{(1.5 \times 10^{-3})^2} = 1.33 \times 10^6 k$

For $l = 100$ cm and $d = 1$ mm

$\Delta l = \dfrac{K \times 1}{(0.5 \times 10^{-3})^2} = 4 \times 10^6 k$

As Δl is highest for $1 = 0.5$ and $d = 0$ mm

So, (a) is correct.

15. (a)

16. (c) For gas, viscosity increases with increase in temperature.

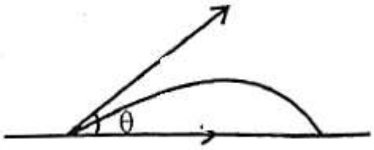
But for liquid viscosity increase with increasing temperature.

So (A) is correct.

Water does not wet an oily glass because adhesive force dominate the cohesive force here.

So (B) is incorrect.

If liquid wet the surface, angle of contact is less than 90°. So, (C) is incorrect.



17. (c)

$$\left(\frac{dQ}{dt}\right)_{net} = \in 0^{-} A(T_0^4 - T^4)$$

$$= 0.5 \times 5.67 \times 10^{-8} \times 4 \times (400^4 - 200^4) = 2721.6 \text{ Watt}$$

18. (c) We have

$\eta = 1 - \dfrac{T_2}{T_1}$

So, initially

$0.4 = 1 - \dfrac{300}{T_1} \Rightarrow \dfrac{300}{T_1} = 0.6 \Rightarrow T_1 = 500 \text{ K}$

Finally,

$$0.5 = 1 - \frac{300}{T_1 + \Delta T} \Rightarrow \frac{300}{T_1 + \Delta T} = 0.5 \Rightarrow T_1 + \Delta T = 600 \text{ K}$$

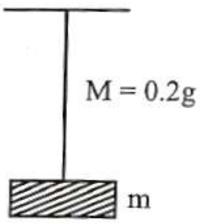
$$\Rightarrow 500 + \Delta T = 600 \Rightarrow \Delta T = 100 \text{ K}$$

19. (a)

20. (b) A diatomic molecule has 1 vibrational DOF.

21. (d) We have

$f_o = \dfrac{1}{2l}\sqrt{\dfrac{T}{\mu}}$



$$\Rightarrow f_o = \frac{1}{2l}\sqrt{\frac{Tl}{m}}$$

$$\Rightarrow 100 = \frac{1}{2 \times 1}\sqrt{\frac{(m + 2 \times 10^{-3})g \times 1}{2 \times 10^{-3}}}$$

$$\left[T = (m + M)g \text{ and } \mu = \frac{m}{l}\right]$$

$$\Rightarrow 10^4 = \frac{1}{4} \times \frac{(2 \times 10^{-3} + m)10}{2 \times 10^{-3}}$$

$$\Rightarrow \frac{10^4 \times 4 \times 2 \times 10^{-3}}{10} = m + 2 \times 10^{-3}$$

$$\Rightarrow 8 = m + 0.002$$

$$m \simeq 8 \text{ kg}$$

22. (d) We have

$$F_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{(r - t + t\sqrt{K})^2}$$

Initially,

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_2}{r^2} \; [t = 0]$$

Finally,

$$t = \frac{r}{5}, K = 9$$

So, $F^1 = \frac{1}{4\pi\epsilon_0}\frac{q^2}{\left(r - \frac{r}{5} + \frac{r}{5}\sqrt{9}\right)^2}$

$$= \frac{1}{4\pi\epsilon_0}\frac{q^2}{\left(\frac{7r}{5}\right)^2} = \frac{25}{49} \cdot \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = \frac{25}{49}F$$

23. (a) We have

$$\sin C = \frac{\mu_2}{\mu_1} \Rightarrow \sin C = \frac{1}{2}$$

$$\Rightarrow C = 30^\circ$$

24. (c) We have

$$\tau = PE\sin\theta$$

$$= 5 \times 10^{-7} \times 2 \times 10^4 \times \sin 60^\circ$$

$$= 5 \times 10^{-7} \times 2 \times 10^4 \times \frac{\sqrt{3}}{2} = 8.66 \times 10^{-3} \text{ N/m}$$

25. (a) We have

$$q_i = q_f$$

$$\Rightarrow 10^{-6} \times 9 = (C_1 + C_2 + C_3)V$$

$$\Rightarrow 9 \times 10^{-6} = 6 \times 10^{-6} \text{ V}$$

$$\Rightarrow V = 1.5 \text{ V}$$

So, $q_3 = C_3$ V

$$= 3 \times 10^{-6} \times 1.5 = 4.5\mu C$$

26. (b) We have $r_i = 10$ cm and $r_f = 4$ cm

Work done $= \Delta U = U_f - U_i$

$$= 9 \times 10^9 \times 10 \times 10^{-6} \times 12 \times 10^{-6}\left[\frac{1}{0.04} - \frac{1}{0.1}\right] = 16.2 \text{ J}$$

27. (a) We have

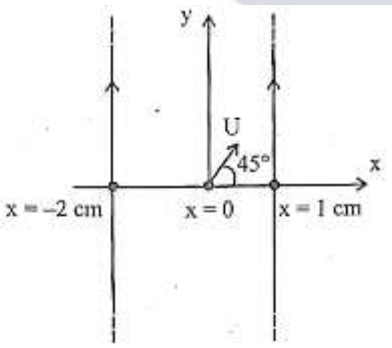
$$I = \int JdA$$
$$= \int_0^R \beta(r^2 + r_0^2 + 2rr_0) \times 2\pi r dr$$
$$= 2\pi\beta\left[\frac{R^4}{4} + \frac{r_0^2 \cdot R^2}{2} + 2r_0 \cdot \frac{R^3}{3}\right]$$

So, $II_{\text{through given section}}$
$$= \frac{\left(\frac{\pi}{6} + \frac{\pi}{6}\right)}{2\pi} \times 2\pi\beta\left[\frac{R^4}{4} + \frac{r_0^2R^2}{2} + \frac{2r_0R^3}{3}\right]$$
$$= \frac{\pi}{3}\beta\left[\frac{R^4}{4} + \frac{r_0^2R^2}{x} + \frac{2r_0R^3}{3}\right]$$

28. (d)

We have
$$i = \frac{E}{R + r}$$
So, $1 = \dfrac{E}{2.5 + r}$ and $0.5 = \dfrac{E}{10 + r}$
$$\Rightarrow 2.5 + r = 0.5(10 + r)$$
$$\Rightarrow 2.5 + r = 5 + 0.5r$$
$$\Rightarrow 0.5r = 2.5$$
$$\Rightarrow r = 5\Omega.$$

29. (d)



$$\vec{B}_{\text{origin}} = \frac{\mu_0 i}{2\pi}\left[\frac{1}{0.01}\hat{k} - \frac{1}{0.02}\hat{k}\right] = B_o\hat{k} - \frac{B_o}{2}\hat{k} = \frac{B_o}{2}\hat{k}$$
$$\vec{F} = e(\vec{V} \times \vec{B})$$
$$= e\left[\frac{U}{\sqrt{2}}\hat{i} + \frac{U}{\sqrt{2}}\hat{j}\right] \times \frac{B_0}{2}\hat{k}$$
$$= \frac{-eUB}{2\sqrt{2}}[-\hat{j} + \hat{i}]$$

30. (a) We know that
$$T = \frac{2\pi\, m}{qB} \Rightarrow f = \frac{qB}{2\pi\, m}$$
So, frequency is independent of kinetic energy of electron, therefore $f_1 = f_2$.

31. (a) We have that
$$T = 2\pi\sqrt{\frac{I}{MB_H}}$$
$$T \propto \frac{1}{\sqrt{B_H}}$$
$$T \propto \frac{1}{\sqrt{B\cos\theta}} \Rightarrow B \propto \frac{1}{T^2\cos\theta}$$

So, $\frac{B_1}{B_2} = \left(\frac{T_2}{T_1}\right)^2 \times \frac{\cos\theta_2}{\cos\theta_1} = \left(\frac{30}{20}\right)^2 \times \frac{\sqrt{3}\times\sqrt{2}}{2\times1}$

$= \frac{9}{4} \times \frac{\sqrt{3}\times\sqrt{2}}{2} = \frac{9\sqrt{3}}{4\sqrt{2}}$

32. (a) We have

$B_o = \frac{E_0}{C} = \frac{8.7}{3\times10^8} = 2.9\times10^{-8}\ T$

33. (d) We have

$\tan\phi = \frac{x_L}{R} = \frac{2\pi f L}{R} = \frac{2\pi\times350\times0.1}{110} \simeq 2$

So, $\phi = \tan^{-1}(2)$

34. (c) We have

$I = \frac{B_0{}^2 C}{2\mu_o}$

$B_o = \sqrt{\frac{2\mu_0 I}{C}} = \sqrt{\frac{2\times4\pi\times10^{-7}\times9240}{3\times10^8}} = 8.8\mu\ T$

35. (d) For completely reflecting surface, where light falls at θ°.

$\left(\frac{\Delta P}{\Delta t}\right) = \frac{2IA}{C}\cos\theta$

$\Rightarrow\ F = \frac{2\times10^{-3}\times20\times10^{-4}}{3\times10^8} \times \frac{1}{\sqrt{2}}$

$\Rightarrow\ F = 9.4\times10^{-15}\ N$

36. (d) Among all the available metal, platinum has highest ionisation energy.
So, it will have highest work function.

37. (b)

(a) $\Delta E = -13.6\left[\frac{1}{3^2} - \frac{1}{1^2}\right]$

$= -13.6\times\left[\frac{1}{9} - 1\right] = -13.6\times\frac{-8}{9}$

$\simeq 12eV$

(b) $\Delta E = -13.6\left[\frac{1}{\infty^2} - \frac{1}{1^2}\right]$

$= 13.6eV$

38. (a) We have

$R = R_0 A^{1/3}$

$\Rightarrow \frac{R}{R_o} = A^{1/3} \Rightarrow \ln\left(\frac{R}{R_o}\right) = \frac{1}{3}\ln A$

So, graph between $\ln\left(\frac{R}{R_o}\right)$ and ln A is straight line.

39. (d)

40. (b)

Chemistry

41. (c) Given condition, $\Delta x = \Delta v$

$\Delta p.\Delta x = \frac{h}{4\pi}$:

or, $m\Delta v.\Delta v = \frac{h}{4\pi}$

or $\Delta v = \sqrt{\frac{h}{4\pi m}} = \frac{1}{2}\sqrt{\frac{h}{\pi m}}$

Uncertainty in momentum,

$\Delta p = m\Delta v = \frac{m}{2}\sqrt{\frac{h}{\pi m}} = \frac{1}{2}\sqrt{\frac{hm}{\pi}}$

42. (a) Radial nodes $= n - l - 1$

Angular nodes = the value of ' l '

4f orbital, $n = 4, l = 3$

Radial nodes $= 4 - 3 - 1 = 0$

and angular nodes $= l = 3$

43. (a) $\frac{Li}{Na} \searrow \frac{Be}{Mg} \searrow \frac{B}{Al}$ Diagonal relationship

44. (a) The metallic character increases down the group and decreases from left to right of a period.

45. (c) Small size and high positive charge of a cation highly polarizes the large sized, high negative charged anion. Therefore, enhances the posibility of covalency.

46. (a)

M.O. : $B_2(10)$

KK*

$= \sigma 2\ s^2(\sigma 2\ s^*)^2(\pi^1 2px = \pi^1 2py);$ B.O. $= \frac{1}{2}(4-2) = 1$

$C_2(12) = KK^* \sigma 2\ s^2(\sigma 2\ s^*)^2(\pi^2 2px = \pi^2 2py)$

B.O. $= \frac{1}{2}(6-2) = 2$

$N_2(14) = KK * \sigma 2\ s^2(\sigma 2\ s^*)^2(\pi^2 2py = \pi^2 2py)\sigma^2 2p_z$

B.O. $= \frac{1}{2}(5-2) = 3$

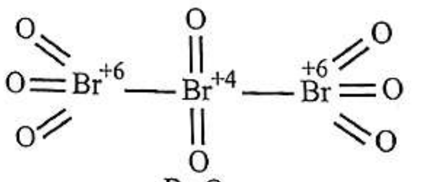
Greater the bond order shorter is the bond length.

$X_3 > X_1 > X_2$.

47. (a) $z > 1$ for a gas in all pressure is not compressible easily.

48. (b) Irrespective of source, a given compound is made up of same element with fixed composition.

49. (b)



Br_3O_8.

50. (c) Entropy increases from liquid phase to gaseous phase.

51. (d) $W = -2.30nRT\log\frac{P_1}{P_2}$

From $1 \longrightarrow 2$

$W_1 = -2.303nRT\log\frac{15}{10}$

$W_1 = -1011.4 \quad \ldots(i)$

From $2 \longrightarrow 3$

$W_2 = -2.303nRT\log\left(\frac{10}{5}\right)$

$W_2 = -1729$

From $3 \longrightarrow 2$

$W_3 = -2.303nRT\log\left(\frac{5}{10}\right) = +1011.4$

From $2 \longrightarrow 1$

$W_4 = -2.303nRT\log\left(\frac{10}{15}\right) = +1011.4$

$W_{Total} = W_1 + W_2 + W_3 + W_4$

$-1011.4 + (-1729) + 1729 + 1011.4 = 0.$

52. (d) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g) + \Delta H$ – Exothermic reaction. According to Le Chatelier's principle, the reaction will proceed in backward reaction as the temperature increases.

53. (c) According to Henderson's equation-

$\mathrm{pH}=\mathrm{pK}_{\mathrm{a}}+\log \frac{[\text{salt}]}{[\text{ acid }]}=4.75+\log \frac{0.5}{0.5}=4.75$

54. (d) $CaC_2+2D_2O \rightarrow C_2D_2+Ca(OD)_2$.

$Al_4C_3+D_2O \rightarrow CD_4+4Al(OD)_3$

55. (a) Smaller the size of the cation, greater is the hydration enthalpy.

56. (a) $NaNO_3 \rightarrow NaNO_2+O_2$ but $4LiNO_3 \rightarrow 2Li_2O+4NO_2+O_2$.

57. (d) Boron is highly crystalline black coloured solid. ^{10}B isotopic has high ability to absorb neutrons. That's why metal borides are used in nuclear chemistry as protective shields and control rods. Boron has high melting point, low density and very low conductivity of electricity.

58. (d) PbI_4 does not exist because Pb − I bond initially formed during the reaction does not release enough energy to unpair 6 s^2 electrons and excite them to higher orbital in order to get 4 unpaired electrons around Pb atom.

59. (c) Order of stability of conjugate base: I > II > II Greater the stability of conjugate base higher is the acidity. +I and +R effects of $-CH_3, -OCH_3$, respectively destabilizes the conjugate base.

60. (b) Condition for aromaticity: $(4n+2)$ no. of π electrons should be present in the planar molecule, where n is any positive integer.

Anti-aromatic compound: 4nπ electrons.


$\longrightarrow 4n=4$ (when $n=1$) π electrons

61. (b) Ionic solids: NaCl, CaF_2, MgO, Zns

Network: Diamond, SiO_2, AlN

Metallic: Cu, Ag

H -Bonded Molecular: H_2O (ice)

62. (b) NaCl has monovalent Na^+ion. If $SrCl_2$ remains as impurity in NaCl then one Na^+ion has to be absent in order to maintain electrical neutrality with divalent Sr^{2+}. Thus, it creats cation cation vacancies.

63. (b) Molality of the solution $=\left(\frac{x}{78}\right) \times \frac{1000}{500}=\frac{x}{39}(m)$

$\Delta T_f=m(K_f)_{H_2O}$

$\frac{x}{39}=\frac{1}{1.86}, x=\frac{39}{1.86}=20.96$

64. (c) The osmotic pressure of the fluid inside the blood cells are same as 0.9% (mass/volume) saline water. This particular percentage of saline water is known as normal saline water.

When the concentration of the saline water > 0.9%, then the osmotic pressure becomes more in comparison to that blood cell. Therefore, water starts come out of the cell resulting into the shrinking of cell. And when it is < 0.9%, water enters into the blood cells and thus it swells resulting into the disease edema.

65. (c) Reduction process: $2H^+ + 2e \rightarrow H_2$

$$E=E^{\circ}-\frac{0.059}{2} \log \frac{p_{H_2}}{[H^+]^2}\left[\begin{array}{c}\text{neutral pH at } 25^{\circ}\mathrm{C}, \\ \log[H^+]=7 \\ \therefore [H^+]=10^{-7}\end{array}\right]$$

$$=0.00-\frac{0.059}{2} \log \frac{1}{(10^{-7})^2}=-\frac{0.059}{2} \times 14=-0.413 \text{ V}$$

66. (c) The rate equation for zero order reaction, $[A]_t=[A]_0-kt$ or, $0.075=0.1-0.003t$

or $t=(0.1-0.075)/0.003=\frac{0.025}{0.003}=\frac{25}{3}=8.33$ s

67. (a) Diameter range of colloidal particles = 1 mm to 1000 mm .

68. (a) AgBr emulsion is used for coating on photographic plate.

69. (d) $2\ Pb_3O_4 \longrightarrow 6PbO+O_2$

$x=2, y=6$

70. (b) HCl and H_2SO_4 both are acidic, Therefore, concentrated H_2SO_4 is used for drying HCl gas.

$NH_3(g)+HCl(g) \longrightarrow NH_4Cl(s)$

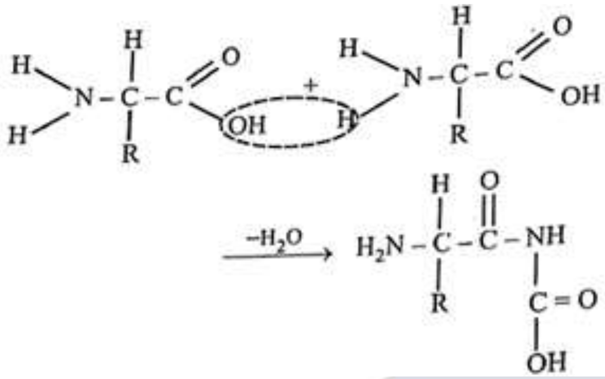
71. **(d)** The catalyst used in manufacturing polyethylene is known as Ziegler-Natta catalyst. It is a mixture of $TiCl_4$, $Al(C_2H_5)_3$ or $Al(CH_3)_3$ works at $333-343$ K at pressure $6-7$ atm pressure in a hydrocarbon solvent.

72. **(a)** $TiCl_3 \rightarrow Ti^{3+}(d'$ system $) \rightarrow$ Due to presence of unpaired e^-in d-orbital $TiCl_3$ will show purple colour solution.

$[Ti(H_2O)_6]Cl_3 \rightarrow$ T Hexacoordinated Ti^{3+} leads to the splitting of degenerated di-orbitals, which is known as crystal field splitting. Thus, $d-d$ transition of d' electron of Ti^{3+} ion takes place from $t_{2\,g}$ to e_g and results in a coloured compound.

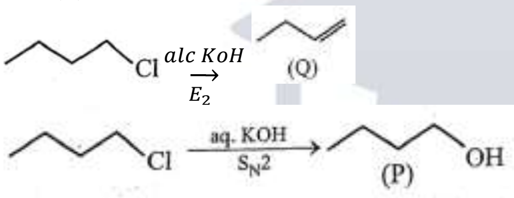
73. **(b)** Glucagon increases the glucose and fatty acid in the blood stream. It is a peptide hormone, produced pancreas.

74. **(a)**

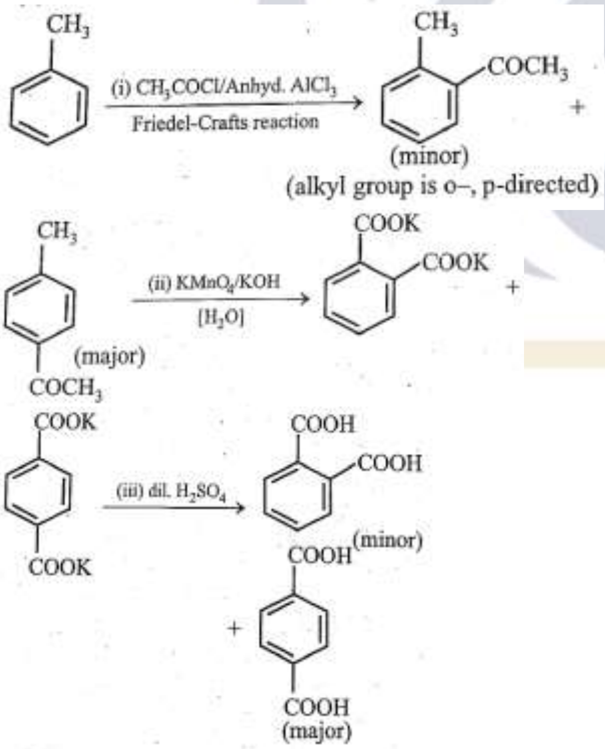


75. **(d)**

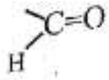
76. **(d)**



77. **(d)**



78. **(a)** Ketone's are more stable than aldehydes. Phenyl ring has $(-R)$ effect nd $(+R)$ effect due to the delocalized π-electrons over the ring. +I effect of substituents $-CH_3 > H > -NO_2$

Therefore, the stability of transition states of nucelophilic addition to  group decreases and as (a) result reactivity increases.

correct order : $a < b < c < d$

79. (a)

80. (c)

$$pK_b = -\log\frac{[\text{ conjugate acid }]}{[\text{ Base }]}$$

More the production of conjugate acid of amines, i.e., $-\overset{\oplus}{N}H$, less is the value of pK_b. Among (a), (b), (c) and (d), (b) has the highest ability of donating lone pair of electrons of ' N ' towards the proton but the steric factor of conjugate acid. $Me_3\overset{\oplus}{N}H$ destabilizes the conjugate acid (C.A.). Hence, its formation reduces. $\therefore (pK_b)_a < (pK_b)_b$ As the phenyl group exerts (-I) effect the donating capability of lone pairs of ' N ' decreases than alkyl amines. However, the steric reason is also valid for the formation of CA. of (c) and (d) and $[\text{C.A.}]_{(c)} > [\text{C.A}]_{(d)}$

Therefore, $(pK_b)_c < (pK_b)_d$.

Hence, the correct order of pK_b

$\Rightarrow (d) > (c) > (b) > (a)$

Mathematics

81. (d) $f(x) = \sqrt{\frac{x^2+2x+8}{x^2+2x+4}}$

$$= \sqrt{\frac{(x^2+2x+4)}{(x^2+2x+4)} + \frac{4}{(x^2+2x+4)}}$$

$$= \sqrt{1 + \frac{4}{(x^2+2x+1)+3}}$$

$$f(x) = \sqrt{1 + \frac{4}{(x+1)^2+3}}.$$

When $(x+1)^2 + 3$ is minimum $\Rightarrow f(x)$ is maximum $\because$ minimum value of a perfect square quantity is 0 .

Minimum value of $(x+1)^2 + 3 = 3$

Maximum value of $f(x) = \sqrt{1+\frac{4}{3}} = \sqrt{\frac{7}{3}}$

82. (b) We know that domain of addition of two functions is the intersection of domain of both functions

$\text{Dom}(f(x)): 2 - x^2 \geq 0 \Rightarrow x^2 \leq 2$

$\Rightarrow x \in [-\sqrt{2}, \sqrt{2}]$

$\text{Dom}(g(x)): 1 - x > 0 \Rightarrow x < 1 \Rightarrow x \in (-\infty, 1)$

$\text{Dom}((f+g)(x)) = [-\sqrt{2}, 1]$

83. (b) As per A and A^T

$$T^A \cdot A = \begin{bmatrix} u_1^2 + b_1^2 + c_1^2 & a_1a_2 + b_1b_2 + c_1c_2 & a_1a_3 + b_1b_3 + c_1c_3 \\ a_1a_2 + b_1b_2 + c_1c_2 & a_2^2 + b_2^2 + c_2^2 & a_2a_3 + b_2b_3 + c_2c_3 \\ a_1a_3 + b_1b_3 + c_1c_3 & a_2a_3 + b_2b_3 + c_2c_3 & u_3^2 + b_3^2 + \iota_3^2 \end{bmatrix}$$

It is given that $a_i^2 + b_i^2 + c_i^2 = 1$ and $a_ia_j + b_ib_j + c_ic_j = 0, \forall i \neq j$

$$A^T \cdot A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$\det(AA^T) = \det(A^TA) = \det(I)$$

$$\det(AA^T) = 1$$

84. (b) As per A and A^T,

$$A \cdot A^T = \frac{1}{7}\begin{bmatrix} 3 & -2 & 6 \\ -6 & -3 & 2 \\ -2 & 6 & 3 \end{bmatrix} \times \frac{1}{7}\begin{bmatrix} 3 & -6 & -2 \\ -2 & -3 & 6 \\ 6 & 2 & 3 \end{bmatrix}$$

$$= \frac{1}{49}\begin{bmatrix} 9+4+36 & -18+6+12 & -6-12+18 \\ -18+6+12 & 36+9+4 & 12-18+6 \\ -6-12+18 & 12-18+6 & 4+36+9 \end{bmatrix}$$

$$= \frac{1}{49}\begin{bmatrix} 49 & 0 & 0 \\ 0 & 49 & 0 \\ 0 & 0 & 49 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$A \cdot A^T = I, \therefore A^{-1} = A^T$

85. (c) $A = \begin{bmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{bmatrix}, \det(A) = \begin{vmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{vmatrix}$

$= -\alpha^3 - 25$

Also, we know that $\det(A^n) = (\det A)^n (n \in I)$

$$\det(A^{10}) = (\det A)^{10} = 1024$$
$$\Rightarrow (-\alpha^3 - 25)^{10} = 1024 = (2)^{10} \Rightarrow -\alpha^3 - 25 = 2 \Rightarrow \alpha = -3$$

86. (a) $\det(A) = \begin{vmatrix} 5 & \sin^2\theta & \cos^2\theta \\ -\sin^2\theta & -5 & 1 \\ \cos^2\theta & 1 & 5 \end{vmatrix}$

Expanding along R_1,

$$= 5(-25-1) - \sin^2\theta(-5\sin^2\theta - \cos^2\theta) + \cos^2\theta(-\sin^2\theta + 5\cos^2\theta)$$
$$= -130 + \sin^2\theta(5\sin^2\theta + \cos^2\theta) + \cos^2\theta(5\cos^2\theta - \sin^2\theta)$$
$$= -130 + 5[\sin^4\theta + \cos^4\theta]$$
$$= -130 + 5[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$$
$$= -130 + 5\left[1 - \frac{1}{2}\sin^2 2\theta\right] = \det A = 5 - 130 - \frac{1}{2}\sin^2 2\theta$$

for maximum value of $\det(A) \sin 2\theta$ must be minimum i.e.,, $\sin 2\theta = 0$

$(\det A)_{max} = 5 - 130 = -125$

87. (d)

$$iz^3 + z^2 - z + i = 0$$
$$= iz^3 + z^2 + i^2 z + i = 0 \qquad (i^2 = -1)$$
$$= (iz+1)(z^2+i) = 0, iz + 1 = 0 \text{ or } z^2 + i = 0$$
$$iz = -1 \text{ or } z^2 = -i$$
$$i(x+iy) = -1 \text{ or } (x+iy)^2 = -i$$
$$xi - y = -1 \text{ or } x^2 - y^2 + 2xyi = -1$$
$$(x = 0 \text{ and } y = 1) \text{ or } x^2 - y^2 = 0 \text{ and } 2xy = -1$$
$$x = \pm y \text{ and } xy = -\frac{1}{2}, \because xy < 0$$

$x = -y, x \cdot (-x) = \frac{-1}{2}, x^2 = \frac{1}{2} = y^2$

in both cases

$|z| = \sqrt{x^2 + y^2} = \sqrt{0^2 + 1^2}$ or $\sqrt{\frac{1}{2} + \frac{1}{2}} \therefore |z| = 1$

88. (b) $\frac{x-1}{3+i} + \frac{y-1}{3-i} = i$

$$= \frac{(x-1)(3-i) + (y-1)(3+i)}{(3+i)(3-i)} = i$$
$$= \frac{(3x+3y-6)}{10} + \frac{i(y-x)}{10} = i$$

comparing both sides,

$$\frac{3x+3y-6}{10}=0 \text{ and } \frac{y-x}{10}=1$$
$3x+3y-6=0$ and $y-x=10$ ..... (i)
$3(x+y-2)=0$, i.e., $x+y=2$ ... ... (ii)
Equation (i) + (ii)
$2y=12 \Rightarrow y=6$
Put $y=6$ in (ii), $x=-4 \therefore x<0, y>0$

89. (b)

RHS: $a\cos 4\theta + b\cos 2\theta + c$
$= a[2\cos^2 2\theta - 1] + b[2\cos^2\theta - 1] + c$
$= a[2(2\cos^2\theta-1)^2 - 1] + b(2\cos^2\theta - 1) + c$
$= a[8\cos^4\theta - 8\cos^2\theta + 1] + b(2\cos^2\theta - 1) + c$
$= 8a\cos^4\theta + (2b-8a)\cos^2\theta + (a+c-b)$
Comparing the coeff. of $\cos^4\theta, \cos^2\theta$ and constant term in LHS and RHS
$\cos^4\theta = 8a\cos^4\theta + (2b-8a)\cos^2\theta + (a+c-b)$
$$8a=1 \Rightarrow a=\frac{1}{8}$$
$$2b-8a=0 \Rightarrow b=4a=4\times\frac{1}{8}=\frac{1}{2}$$
$$a+c-b=0, c=b-a=\frac{1}{2}-\frac{1}{8}=\frac{3}{8}$$
$$(a,b,c)=\left(\frac{1}{8},\frac{1}{2},\frac{3}{8}\right)$$

90. (a) $|1-i|=\sqrt{(1)^2+(-1)^2}=\sqrt{2}$
$|1-i|^x=2^x, \Rightarrow (\sqrt{2})^x=2^x \Rightarrow 2^{\frac{x}{2}}=2^x$
$\frac{x}{2}=x$ only possible if $x=0$
Number of integer solution $=1$

91. (a) $f(x)=ax^2+bx+c$ and $f(x+y)=f(x)+f(y)+x.y$
at $x=0, f(0)=a(0)^2+b(0)+c=c$
and $f(0+0)=f(0)+f(0)+0=2f(0)$
$c=2f(0)=2c, c=0 \Rightarrow f(0)=0$
$f(x)=ax^2+bx$
Put $y=-x$
$f(x-x)=f(x)+f(-x)+x(-x)$
$f(0)=ax^2+bx+a(-x)^2+b(-x)-x^2$
$$x^2(2a-1)=0, 2a-1=0 \Rightarrow a=\frac{1}{2}$$
also $f(1)=a(1)^2+b(1)+c=a+b+c=3$
$$a=\frac{1}{2}, b=?, c=0, \therefore \frac{1}{2}+b+0=3 \Rightarrow b=\frac{5}{2}$$
$$f(x)=\frac{1}{2}x^2+\frac{5}{2}x$$
$$\sum_{n=1}^{10} f(n)=\sum_{n=1}^{10}\frac{1}{2}n^2+\sum_{n=1}^{10}\frac{5}{2}n=\frac{1}{2}\sum_{n=1}^{10}n^2+\frac{5}{2}\sum_{n=1}^{10}n$$
$$=\left(\frac{1}{2}\times 10\times\frac{11}{6}\times 21\right)+\left(\frac{5}{2}\times\frac{10\times 11}{2}\right)=330$$

92. (c)

$3^x \cdot 3 + 3^{-x} \cdot 3 = 10$

$3^x \cdot 3 + \frac{3}{3^x} = 10$

Let $3^x = y, 3y + \frac{3}{y} = 10$

$3y^2 + 3 = 10y$

$3y^2 - 9y - y + 3 = 0$

$(y-3)(3y-1) = 0 \Rightarrow y = \frac{1}{3}, 3$

$3^x = \frac{1}{3}$ or $3^x = 3$

$3^x = 3^{-1}$ or $3^x = 3^1, x = -1$ or 1

for positive values, there is only one solution $x = 1$

93. (b) Let $\sqrt{\frac{x}{1-x}} = y, y + \frac{1}{y} = \frac{13}{6}$

$\frac{y^2+1}{y} = \frac{13}{6}$

$6y^2 - 9y - 4y + 6 = 0$

$(2y-3)(3y-2) = 0 \Rightarrow y = \frac{3}{2}, \frac{2}{3}$

$\sqrt{\frac{x}{1-x}} = \frac{3}{2}$ or $\sqrt{\frac{x}{1-x}} = \frac{2}{3}$

$\frac{x}{1-x} = \frac{9}{4}$ or $\frac{x}{1-x} = \frac{4}{9}$

$4x = 9 - 9x$ or $9x = 4 - 4x \Rightarrow x = \frac{9}{13}$ or $x = \frac{4}{13}$

There are 2 real roots

94. (d)

$4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$

$4^x + 2^{2x-1} = 3^{x+\frac{1}{2}} + 3^{x-\frac{1}{2}} \Rightarrow 4^x + \frac{4^x}{2} = 3^x\left(\sqrt{3} + \frac{1}{3}\right)$

$\frac{3}{2}4^x = 3^x \cdot \frac{4}{\sqrt{3}}$

$\left(\frac{3}{4}\right)^x = \frac{3\sqrt{3}}{8} = \frac{3^{3/2}}{4^{3/2}} \Rightarrow \left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^{3/2} \quad \therefore x = \frac{3}{2}$

95. (c) First place can be filled by n ways with n things

1^{st} and 2^{nd} place by $n \times n$ i.e., n^2 ways

$1^{st}, 2^{nd}$ and 3^{rd} place by $n \times n \times n$ i.e., n^3 ways

Similarly

$1^{st}, 2^{nd}, 3^{rd}, \ldots.. r^{th}$ place by

$n \times n \times n \times \ldots.. n$ i.e., n^r ways

Total number of ways =

$n + n^2 + n^3 \ldots\ldots n^r \Rightarrow$ sum of r terms of GP

ith $a = n, r = n \Rightarrow \frac{n(n^r-1)}{n-1}$

96. (b) Number of chords = Number of ways of selecting 2 points out of given 21 points = ${}^{21}C_2$

$= \frac{(21)!}{2!\,19!} = \frac{21 \times 20 \times 19!}{2 \times 19!} = 210$

97. (a) If we select 2 points out of n vertices of polygon by ${}^nC_2$ ways then it will have diagonals and the n sides

Number of diagonal = ${}^nC_2 - n \Rightarrow {}^nC_2 - n = 560$

$\Rightarrow \frac{n!}{2!\,(n-2)!} - n = 560 \Rightarrow \frac{n(n-1)}{2} - n = 560$

$n(n-1) - 2n = 560 \times 2$

$n(n-3) = 1120 = 35 \times 32 \therefore n = 35$

98. (c) Number of dearrangements for n things is given by

$D_n = n! \sum_{K=0}^{n} \frac{(-1)^K}{K!}$

For $n = 6, D_6 = 6! \sum_{K=0}^{6} \frac{(-1)^K}{K!}$

$= 6!\left[\frac{(-1)^0}{0!} + \frac{(-1)^1}{1!} + \frac{(-1)^2}{2!} + \frac{(-1)^3}{3!} + \frac{(-1)^4}{4!} + \frac{(-1)^5}{5!} + \frac{(-1)^6}{6!}\right]$

$^6C_0 \cdot 6!\left[\frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right] = {}^6C_{6-6}D_6$

Similarly, for 5 wrong (5 dearrangements)

$^6C_5 \cdot D_5 = {}^6C_{6-1}D_5$

4 wrong $\rightarrow {}^6C_4 \cdot D_4 = {}^6C_{6-4}D_4$

3 wrong $\rightarrow {}^6C_3 \cdot D_3 = {}^6C_{6-3}D_3$

2 wrong $\rightarrow {}^6C_2 \cdot D_2 = {}^6C_{6-4}D_2$

For at least 2 wrong: Total $= {}^6C_{6-4}D_2 + {}^6C_{6-3}D_3 +$

$^6C_{6-4}D_4 + {}^6C_{6-1}D_5 + {}^6C_{6-6}D_6 = \sum_{r=2}^{6} {}^6C_{6-r}D_r$

99. (c)

$\frac{x^4 + 24x^2 + 28}{(x^2+1)^3} = \frac{(Ax+B)(x^2+1)^2 + (Cx+D)(x^2+1) + (Ex+F)}{(x^2+1)^3}$

$= (Ax+B)(x^4+2x^2+1) + (Cx+D)(x^2+1) + (Ex+F)$

$= Ax^5 + 2\,Ax^3 + Ax + Bx^4 + 2\,Bx^2 + B + Cx^3 + Cx + Dx^2$

$+D + Ex + F$

$x^4 + 24x^2 + 28 = Ax^5 + Bx^4 + (2\,A + C)x^3 + (2\,B + D)x^2 +$

$(A + C + E)x + (B + D + F)$

Comparing coefficient of x and constant term on both sides,

$A + C + E = 0, B + D + F = 28$

$A + B + C + D + E + F = 28$

100. (c)

$\frac{\sin\theta + \sin 3\theta}{\cos\theta + \cos 3\theta} = \frac{\sin\theta + 3\sin\theta - 4\sin^3\theta}{\cos\theta + 4\cos^3\theta - 3\cos\theta}$

$= \frac{4\sin\theta(1-\sin^2\theta)}{2\cos\theta(2\cos^2\theta - 1)} = \frac{2\sin\theta\cdot\cos^2\theta}{\cos\theta(2\cos^2\theta-1)}$

$= \frac{2\sin\theta\cos\theta}{2\cos^2\theta - 1} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$

101. (c)

$\{(1+\tan 1^\circ)(1+\tan 44^\circ)\}\cdot\{(1+\tan 2^\circ)(1+\tan 43^\circ)\}$
$\{(1+\tan 3^\circ)(1+\tan 42^\circ)\}\cdots\ldots\ldots\ldots\ldots(1+\tan 45^\circ)$
$=(1+\tan 1^\circ)[1+\tan(45^\circ-1^\circ)]\cdot(1+\tan 2^\circ)\cdot[1+$
$\tan(45^\circ-2^\circ)](1+\tan 3^\circ)[1+\tan(45^\circ-3^\circ)]\ldots\ldots\ldots$
$(1+\tan 45^\circ)$

$$=(1+\tan 1^\circ)\left[1+\frac{1-\tan 1^\circ}{1+\tan 1^\circ}\right](1+\tan 2^\circ)\left[1+\frac{1-\tan 2^\circ}{1+\tan 2^\circ}\right]$$

$$(1+\tan 3^\circ)\left[1+\frac{1-\tan 3^\circ}{1+\tan 3^\circ}\right]\ldots\ldots\ldots(1+\tan 45^\circ)$$

$$=(1+\tan 1^\circ)\left(\frac{2}{1+\tan 1^\circ}\right)(1+\tan 2^\circ)\left(\frac{2}{1+\tan 2^\circ}\right)$$

$$(1+\tan 3^\circ)\left(\frac{2}{1+\tan 3^\circ}\right)\ldots\ldots\ldots\ldots\ldots(1+1)$$

$=(2\cdot 2\cdot 2\ldots..2)$ - 22 times $\times 2$

$=(2\times 2\times 2\ldots\ldots)$ 23 times $=2^{23}\therefore n=23$

102. **(c)**

$$\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta}=\frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}}+\frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$$

$$=\frac{\cos^2\theta}{\cos\theta-\sin\theta}+\frac{\sin^2\theta}{\sin\theta-\cos\theta}=\frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$$

$$=\frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)}=\cos\theta+\sin\theta$$

103. **(a)** By sine Rule,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=K$$

$a=K\sin A$ and $b=K\sin B$

$$\frac{a\cos A-b\cos B}{a\cos B-b\cos A}+\cos C$$

$$\frac{K\sin A\cos A-K\sin B\cos B}{K\sin A\cos B-K\sin B\cos A}+\cos C$$

$$=\frac{1}{2}\left[\frac{\sin 2A-\sin 2B}{\sin(A-B)}\right]+\cos C\ \{A+B+C=\pi\}$$

$$=\frac{1}{2}\left[\frac{2\cos(A+B)\sin(A-B)}{\sin(A-B)}\right]+\cos(\pi-(A+B))$$

$=\cos(A+B)-\cos(A+B)=0$

104. **(b)**

$$\operatorname{cosec} hx=\frac{1}{\sin hx}$$

$$\frac{4}{5}=\frac{1}{\sin hx}\Rightarrow \sin hx=\frac{5}{4}$$

105. **(c)**

$$\tan hx=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$$\frac{1+\tan hx}{1-\tan hx}=\frac{1+\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)}{1-\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)}=\frac{e^x+e^{-x}+e^x-e^{-x}}{e^x+e^{-x}-e^x+e^{-x}}$$

$$=\frac{2e^x}{2e^{-x}}=e^{2x}$$

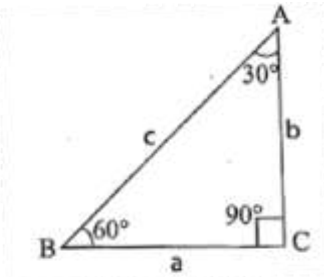
106. **(d)**

$$s=\frac{a+b+c}{2}=\frac{2+3+4}{2}=\frac{9}{2}$$

$$\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\sqrt{\frac{\left(\frac{9}{2}-3\right)\left(\frac{9}{2}-4\right)}{\frac{9}{2}\left(\frac{9}{2}-2\right)}}\sqrt{\frac{\frac{3}{2}\times\frac{1}{2}}{\frac{9}{2}\times\frac{5}{2}}}$$

$$\tan\frac{A}{2}=\sqrt{\frac{1}{15}}$$

107. **(c)** Let the angles are x, 2x and 3x



$x+2x+3x=180^\circ \Rightarrow x=30^\circ$

angles are $30^\circ, 60^\circ$ and 90° By sine Rule,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=K$$

$$a=K\sin A=K\sin 30^\circ=\frac{K}{2}$$

$$b=K\sin B=K\sin 60^\circ=K\cdot\frac{\sqrt{3}}{2}$$

$$c=K\sin C=K\sin 90^\circ=K$$

$$\therefore a:b:c=\frac{K}{2}:\frac{\sqrt{3}}{2}K:K=\frac{1}{2}:\frac{\sqrt{3}}{2}:1$$

$$=1:\sqrt{3}:2$$

108. **(c)** $\frac{\Delta}{s-a}\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}+\frac{\Delta}{s-b}\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}+\frac{\Delta}{s-c}\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$$\Delta\sqrt{s}\left[\frac{1}{\sqrt{(s-a)(s-b)(s-c)}}+\frac{1}{\sqrt{(s-a)(s-b)(s-c)}}+\frac{1}{\sqrt{(s-a)(s-b)(s-c)}}\right]$$

$$=\Delta\sqrt{s}\left[\frac{\sqrt{s}}{\Delta}+\frac{\sqrt{s}}{\Delta}+\frac{\sqrt{s}}{\Delta}\right]\because \Delta=\sqrt{s(s-a)(s-b)(s-c)}$$

$$=3s$$

109. **(d)** For point of intersection

$$2\vec{b}+t(6\vec{c}-\vec{a})=\vec{a}+s(\vec{b}-3\vec{c})$$

$$\Rightarrow 2\vec{b}+6\vec{c}-t\vec{a}=\vec{a}+s\vec{b}-3s\vec{c}$$

$$\Rightarrow (t+1)\vec{a}+(2-s)\vec{b}+(6t+3s)\vec{c}=0$$

It is possible if $t+1=0$ and $2-s=0, t=-1$ and $s=2$

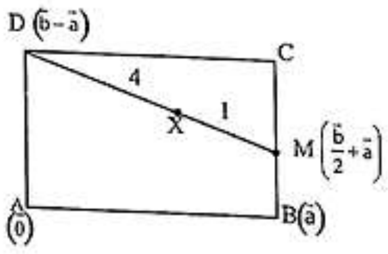
point of intersection is $\vec{r}=2\vec{b}+(-1)(6\vec{c}-\vec{a})$

Put $t=-1$ in equation

$=2\vec{b}-6\vec{c}+\vec{a}, \vec{r}=\vec{a}+2\vec{b}-6\vec{c}$

110. **(b)** Let $A(\vec{O})$

$$\overrightarrow{AB}=\vec{B}-\vec{A}=\vec{a}$$



Position vector of B $= \vec{a}$
$\overrightarrow{DA}$ = position vector of A − position vector of D
$0 -$ p.v. of $\vec{D} = \vec{a} - \vec{b}$, p.v. of D $= \vec{b} - \vec{a}$
M is mid point of BC

$$\overrightarrow{BM} = \frac{BC}{2} = \frac{\vec{b}}{2}$$

$$\text{p.v. of M} - \text{p.v of B} = \frac{\vec{b}}{2} \Rightarrow \text{p.v. of M} = \frac{\vec{b}}{2} + a$$

$$\overrightarrow{DX} = \frac{4}{5}\overrightarrow{DM}, \therefore DX : XM = 4 : 1$$

$$\text{p.v. of } \vec{X} = \frac{1 \cdot (\vec{b} - \vec{a}) + 4\left(\frac{\vec{b}}{2} + \vec{a}\right)}{1+4} = \frac{3\vec{b} + 3\vec{a}}{5} = \frac{3}{5}(\vec{a} + \vec{b})$$

$$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = \vec{a} + \vec{b}$$

$$\overrightarrow{AX} = \frac{3}{5}(\vec{a} + \vec{b}) = \frac{3}{5}\overrightarrow{AC}$$

A, X, C are collinear.

111. **(a)**

$$(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + \vec{b}) = 0$$

$$6\vec{a} \cdot \vec{a} + 3\vec{a} \cdot \vec{b} - 10\vec{b} \cdot \vec{a} - 5\vec{b} \cdot \vec{b} = 0$$

$$6|\vec{a}|^2 - 7\vec{a} \cdot \vec{b} - 5|\vec{b}|^2 \cdot = 0 \ \ldots \text{(i)} [|\vec{a}|^2 = \vec{a} \cdot \vec{a}]$$

also $(\vec{a} + 4\vec{b}) \cdot (-\vec{a} + \vec{b}) = 0$

$$-\vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} - 4\vec{b} \cdot \vec{a} + 4\vec{b} \cdot \vec{b} = 0$$

$$-|\vec{a}|^2 - 3\vec{a} \cdot \vec{b} + 4|\vec{b}|^2 = 0 \quad \ldots\ldots \text{(ii)}$$

solve (i) and (ii) by cross multiplication

$$\frac{|\vec{a}|^2}{\begin{vmatrix} -7 & -5 \\ -3 & 4 \end{vmatrix}} = \frac{-\vec{a} \cdot \vec{b}}{\begin{vmatrix} 6 & -5 \\ -1 & 4 \end{vmatrix}} = \frac{|\vec{b}|^2}{\begin{vmatrix} 6 & -7 \\ -1 & -3 \end{vmatrix}} = K$$

$$|\vec{a}|^2 = -43\text{ K}, |\vec{b}|^2 = -25\text{ K}, \vec{a} \cdot \vec{b} = -19\text{ K}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$$

$$(-19\text{ K})^2 = (-43\text{ K})(-25\text{ K}) \cdot \cos^2\theta$$

$$\cos\theta = \sqrt{\frac{19^2}{43.25}} = \frac{19}{5\sqrt{43}} \Rightarrow \theta = \cos^{-1}\left(\frac{19}{5\sqrt{43}}\right)$$

112. **(c)** $\vec{a}$ is making angle 135° with z axis i.e., $\hat{k}$

$\vec{a} \cdot \hat{k} = |\vec{a}| \cdot |\hat{k}| \cdot \cos 135°$

$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{k} = 5\sqrt{2} \times 1 \times \left(\frac{-1}{\sqrt{2}}\right) \quad \therefore z = -5$

also $|\vec{a}| = \sqrt{x^2 + y^2 + z^2} = 5\sqrt{2}$

$x^2 + y^2 + z^2 = (5\sqrt{2})^2, x = 2y$ and $z = -5$

$(2y)^2 + y^2 + (-5)^2 = 50 \Rightarrow 5y^2 = 25, y = \sqrt{5}$

$x = 2y = 2\sqrt{5}, \vec{a} = 2\sqrt{5}\hat{i} + \sqrt{5}\hat{j} - 5\hat{k}$

113. **(c)** $\overrightarrow{B'C'}$ is vector passing through $\vec{a}$ and parallel to $\vec{b} - \vec{c}$

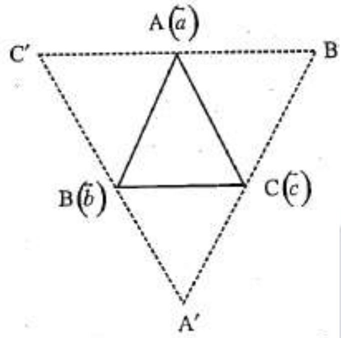
$\overrightarrow{B'C'} = \vec{a} + \lambda(\vec{b} - \vec{c})$ ....(i)

similarly

$\overrightarrow{A'C'} = \vec{b} + \mu(\vec{c} - \vec{a})$ ....(ii)

$\overrightarrow{A'B'} = \vec{c} + t(\vec{a} - \vec{b})$ .... (iii)

For point of intersection



A', B' and C', solving pairwise above equations

From (i) and (ii)

$\vec{a} + \lambda(\vec{b} - \vec{c}) = \vec{b} + \mu(\vec{c} - \vec{a})$

$(1 + \mu)\vec{a} + (\lambda - 1)\vec{b} + (-\mu - \lambda)\vec{c} = 0$

$1 + \mu = 0$ and $\lambda - 1 = 0, \mu = -1$ and $\lambda = 1$

Put $\lambda = 1$ in (i), $\therefore$ Position vector of $C' : \vec{a} + \vec{b} - \vec{c}$

From (ii) and (iii)

$\vec{b} + \mu(\vec{a} - \vec{c}) = \vec{c} + t(\vec{a} - \vec{b})$

$(\mu - t)\vec{a} + (1 + t)\vec{b} + (-\mu - 1)\vec{c} = 0 \Rightarrow \mu - t = 0, \mu = t$

$t = -1$

Substituting values of λ, μ, t in equations.

Position vector of $A' : \vec{a} + \vec{c} - \vec{b}$

Position vector of $B' : \vec{c} - (\vec{a} - \vec{b}), \vec{c} + \vec{b} - \vec{a}$

Position vector of centroid of $A'B'C'$

$$= \frac{\text{sum of position vectors of } \overrightarrow{A'}, \overrightarrow{B'} \text{ and } \overrightarrow{C'}}{3}$$

$$= \frac{(\vec{a} + \vec{b} - \vec{c}) + (\vec{b} + \vec{c} - \vec{a}) + (\vec{c} + \vec{a} - \vec{b})}{3} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

114. **(b)**

x_i	$\lvert x_i - \bar{x} \rvert$
5	4
6	3
7	2
8	1

6	3
9	0
13	4
12	3
15	6
$\sum x_i = 81$	$\sum\|x_i - \bar{x}\| = 26$

Mean $(\bar{x}) = \frac{\sum x_i}{n} = \frac{81}{9} = 9$

Mean deviation about mean $= \frac{\sum|x_i - \bar{x}|}{n} = \frac{26}{9} = 2.88$

115. **(a)** When 3 balls are picked then following events are possible:

Events	Sum of numbers
(a) All 3 even	Even
(b) All 3 odd	Odd
(c) 2 even and 1 odd	Odd
(d) 2 odd and 1 even	Even

$$\mathrm{P(\ odd\ )} = \frac{\text{Favourable cases i.e. b, c}}{\text{Total cases i.e. a, b, c, d}} = \frac{2}{4} = \frac{1}{2}$$

116. **(d)** n (Prize tickets) $= 10$

$\mathrm{n(\ No\ prize\ ticket\ )} = 35 - 10 = 25$

$$\mathrm{P(\ No\ prize\ )} = \frac{\mathrm{n(\ No\ prize\ ticket\ )}}{\mathrm{n(\ Total\ Tickets\ )}} = \frac{25}{35} = \frac{5}{7}$$

117. **(d)** Total probability $= \mathrm{P}(1^{\text{st}}\ \text{green}) \times \mathrm{P}(2^{\text{nd}}\ \text{green}) \times \mathrm{P}(3^{\text{rd}}\ \text{green})$

$$= \frac{7}{12} \times \frac{6}{11} \times \frac{5}{10} = \frac{7}{44}$$

118. **(d)** Product of 2 numbers will be odd only if both are odd. $\mathrm{P(xy\ is\ even\ )} = 1 - \mathrm{P(xy\ is\ odd\ )}$

$$\mathrm{P(xy\ is\ even\ )} = 1 - \mathrm{P(xy\ is\ )} = 1 - \frac{2}{4} \times \frac{2}{3} = \frac{2}{3}$$

119. **(b)** Here, $X \sim B(16, 0.25)$ with $n = 16$ and $P = 0.25$ and $Y \sim P(2)$ is a possion with $\lambda = 2$ variance of $x =$ $npq = 16 \times \frac{1}{4} \times \frac{3}{4} = 3$. Variance of $y = \lambda = 2$. So, required sum $= 2 + 3 = 5$

120. **(a)** $P(X = K) = e^{-\lambda} \cdot \frac{\lambda^K}{K!}$

Where $\lambda =$ mean and $\sum_{K=0}^{\infty} P(X = K) = 1$

$$P(X \geq 3) = 1 - [P(X = 0) + P(X = 1) + P(X = 2)]$$

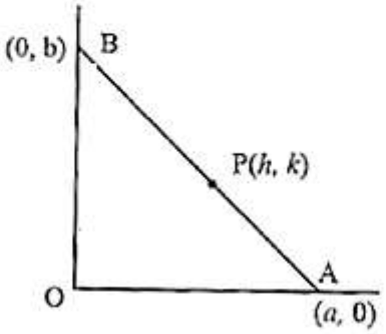
$$= 1 - \left[\frac{e^{-6}6^0}{0!} + \frac{e^{-6} \cdot 6^1}{1!} + \frac{e^{-6} \cdot 6^2}{2!}\right]$$

$$= 1 - \left[\frac{1}{e^6} + \frac{6}{e^6} + \frac{18}{e^6}\right] = 1 - \frac{25}{e^6}$$

121. **(d)** Let AB be the rod with

$(0, b)$ mid point at $P(h, k)$

$A(a, 0)$ and $B(0, b)$ are ends



$$h = \frac{a+0}{2} = \frac{a}{2}, k = \frac{0+b}{2} = \frac{b}{2}$$

$$AB = r = \sqrt{(a-0)^2 + (0-b)^2}$$

$$r^2 = a^2 + b^2$$

$$r^2 = 4h^2 + 4k^2, r^2 = 4(h^2 + k^2)$$

$$h^2 + k^2 = \frac{r^2}{4} = \left(\frac{r}{2}\right)^2$$

$$x^2 + y^2 = \left(\frac{r}{2}\right)^2$$

It represents a circle with radius $\frac{r}{2}$

length $=$ perimeter $= 2\pi(\text{ Radius }) = 2\pi \times \frac{r}{2} = \pi r$

122. **(d)** If point lies on $y = x + 3$ then let the point be $P(\alpha, \alpha + 3)$ distance between $P(\alpha, \alpha + 3)$ and $(0,3) = 2$

$$\sqrt{(\alpha - 0)^2 + (\alpha + 3 - 3)^2} = 2 \Rightarrow \sqrt{2\alpha^2} = 2$$

$$2\alpha^2 = 4 \Rightarrow \alpha^2 = 2 \Rightarrow \alpha = \pm\sqrt{2} = x$$

$$y = 3 \pm \sqrt{2} \therefore P(\alpha, \alpha + 3) \equiv (\pm\sqrt{2}, 3 \pm \sqrt{2})$$

Distance of P from origin $= \sqrt{(\alpha - 0)^2 + (\alpha + 3)^2}$

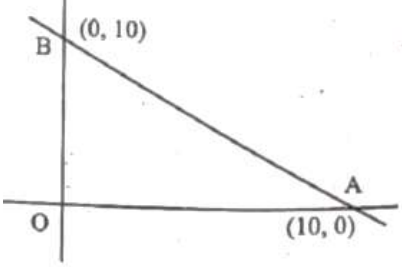
$= \sqrt{\alpha^2 + (\alpha + 3)^2} = \sqrt{2 + 2 + 9 + 6\alpha} = \sqrt{13 + 6\alpha}$

for least distance, $\alpha = -\sqrt{2}$,

Least distance $= \sqrt{13 - 6\sqrt{2}}$

123. **(b)**

124. **(a)** Let the line $x + y = 10$ meets the co-ordinate axes at A and B then for $\triangle$ OAB, if points lying inside then $x + y < 10\ \forall x, y \in \mathbb{N}$



x	y	Number of points
1	1,2,3,4,5,6,7,8	8
2	1,2,3,4,5,6,7	7
3	1,2,3,4,5,6	6
4	1,2,3,4,5	5
5	1,2,3,4	4

6	1,2,3	3
7	1,2	2
8	1	1

Total number of points $= 1 + 2 + \ldots. + 8 = \frac{8\times9}{2} = 36$

125. **(a)** $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ……(i)

the above represents pair of straight- line if

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ ….. (ii)

lines intersect at x axis $\Rightarrow y = 0$, so satisfy, it in eq. (i)

$ax^2 + 2gx + c = 0$

lines meet at x axis, then, Roots are equal

i.e., $D = 0 \Rightarrow (2g)^2 - 4(a)(c) = 0$

$4g^2 - 4ac = 0 \Rightarrow g^2 = ac$

Put $g^2 = ac$ in (ii)

$abc + 2fgh - af^2 - abc - ch^2 = 0 = 7$

$2fgh = af^2 + ch^2$

The incorrect option is $abc = 2fgh$

126. **(a)**

$$x\cos\theta - y = 0 \Rightarrow \frac{y}{x} = \cos\theta$$

$$m_1 = \cos\theta \ \left(\text{slope } = \frac{y}{x}\right)$$

$$(\cos\theta + \tan\alpha)x - (1 - \cos\theta\tan\alpha)y = 0$$

Similarly, $m_2 = \frac{\cos\theta+\tan\alpha}{1-\cos\theta\tan\alpha}$

let $\cos\theta = \tan\beta$

$$m_1 = \cos\theta = \tan\beta$$

$$m_2 = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \tan(\alpha+\beta)$$

If Angle between lines $= X$

$$\Rightarrow X = \tan^{-1}\left|\frac{m_2 - m_1}{1 + m_2m_1}\right|$$

$$= \tan^{-1}\left|\frac{\tan(\alpha+\beta) - \tan\beta}{1 - \tan(\alpha+\beta)\tan\beta}\right|.$$

$$= \tan^{-1}|\tan(\alpha+\beta-\beta)| = \alpha \ \left(\alpha \in \left[0, \frac{\pi}{2}\right]\right)$$

127. **(d)**

128. **(c)** If $\frac{x}{a} + \frac{y}{b} = 1$ is tangent to circle with centre (0,0) and radius 1 .

$\Rightarrow$ Radius of circle = Length of perpendicular from centre to the tangent

$$1 = \left|\frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}}\right| \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} = 1$$

comparing with $x^2 + y^2 = 1 \Rightarrow x = \frac{1}{a}, y = \frac{1}{b}$

$\left(\frac{1}{a}, \frac{1}{b}\right)$ lies on the circle.

129. **(d)**

130. **(a)** Let the pole be (x_1, y_1)

then the equation of polar with respect to circle is

$xx_1 + yy_1 = c^2$ …….(i)

The above equation represents the line

$\frac{x}{a} + \frac{y}{b} = 1$ ……. (ii)

From (i)

$x \cdot \frac{x_1}{c^2} + y \cdot \frac{y_1}{c^2} = 1$ .... (ii)

comparing coefficients of x and y from (ii) and (iii)

$\frac{1}{a} = \frac{x_1}{c^2}$ and $\frac{1}{b} = \frac{y_1}{c^2}, x_1 = \frac{c^2}{a}$ and $y_1 = \frac{c^2}{b}$

Pole $\equiv \left(\frac{c^2}{a}, \frac{c^2}{b}\right)$

131. **(a)** $S: x^2 + y^2 + 6x + 6y - 2 = 0$, $L: 5x - 2y + 6 = 0$

point Q lies on line $5x - 2y + 6 = 0$ which is on y axis i.e., $x = 0$

$5(0) - 2y + 6 = 0 \Rightarrow 7.2y = 6. y = 3$

Co-ordinates of Q are (0,3)

Now, PQ is length of tangent on S from Q

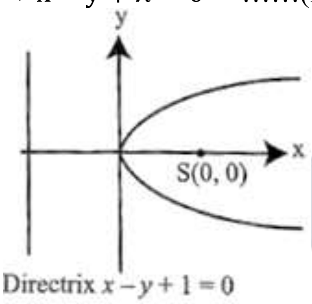
Length of tangent $= \sqrt{S_1} = S_1 = S_{(0,3)}$

$= \sqrt{(0)^2 + (3)^2 + 6(0) + 6(3) - 2} = \sqrt{25} = 5$

132. **(a)** Directrix is line parallel to $x - y + 1 = 0$

Equation of Directrix

$\Rightarrow x - y + \lambda = 0$ ......(i)



Also distance between focus and tangent at vertex is

$\left|\frac{0 - 0 + 1}{\sqrt{(1)^2 + (-1)^2}}\right| = \frac{1}{\sqrt{2}}$

Directrix $x - y + 1 = 0$ distance of directrix from focus $= 2 \times \frac{1}{\sqrt{2}}$

$\left|\frac{0 - 0 + \lambda}{\sqrt{(1)^2 + (-1)^2}}\right| = \sqrt{2}$

$\frac{\lambda}{\sqrt{2}} = \sqrt{2}, \Rightarrow \lambda = 2$

Put $\lambda = 2$ in (i)

Equation of directrix $= x - y + 2 = 0$

133. **(b)** $x^2 + 3y^2 = 6 \Rightarrow \frac{x^2}{6} + \frac{y^2}{2} = 1$

$a^2 = 6 \Rightarrow a = \sqrt{6}$
$b^2 = 2 \Rightarrow b = \sqrt{2}$ $(a > b)$

Let any point on ellipse be $(a\cos\theta, b\sin\theta)$

i.e., $P(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$

distance of $P(\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$ from centre (0,0)

$\Rightarrow \sqrt{(\sqrt{6}\cos\theta)^2 + (\sqrt{2}\sin\theta)^2} = 2$

$\Rightarrow \sqrt{4\cos^2\theta + 2} = 2 \Rightarrow 4\cos^2\theta + 2 = 4$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ $\therefore \theta = 45° = \frac{\pi}{4}$

134. **(b)** Foci$(\pm ae, 0) \equiv (\pm 3, 0)$.i.e., $ae = 3$

$\Rightarrow a\left(\frac{3}{2}\right) = 3$ $\left(e = \frac{3}{2}\right)$

$\Rightarrow a = 2$

also $e = \sqrt{1+\frac{b^2}{a^2}} \Rightarrow \frac{3}{2} = \sqrt{1+\frac{b^2}{4}}$

$\frac{5}{4} = \frac{b^2}{4} \Rightarrow b^2 = 5 \Rightarrow b = \sqrt{5}$

Equation of hyperbola is

$\frac{x^2}{4} - \frac{y^2}{5} = 1 \quad \ldots\ldots(i)$

given line is $2x - y - 1 = 0$. i.e., $y = 2x - 1$

Put $y = 2x - 1$ in (i)

$$\frac{x^2}{4} - \frac{(2x-1)^2}{5} = 1$$

$$5x^2 - 16x^2 - 4 + 16x = 20$$

$$11x^2 - 16x + 24 = 0$$

$$D = (-16)^2 - 4(11)(24) < 0$$

$$D < 0$$

No real values of x

the line dues not interseut hyperbula.

135. **(d)** $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \ldots.(i)$

Let the required point be (h, k)

Equation of chord of contact of (h, k) w.r.t. hyperbola is

$\frac{hx}{a^2} - \frac{ky}{b^2} = 1$

squaring (ii), $\left(\frac{hx}{a^2} - \frac{ky}{b^2}\right)^2 = 1^2$

$\frac{h^2x^2}{a^4} + \frac{k^2y^2}{b^4} - \frac{2hkxy}{a^2b^2} = 1 \quad \ldots\ldots(iii)$

from (i) and (iii)

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = \frac{h^2x^2}{a^4} + \frac{k^2y^2}{b^4} - \frac{2hkxy}{a^2b^2}$$

$$\frac{x^2}{a^2}\left(1 - \frac{h^2}{a^2}\right) - \frac{y^2}{b^2}\left(1 + \frac{k^2}{b^2}\right) + \frac{2hk}{a^2b^2}xy = 0$$

Now this equation will represent pair of perpendicular lines if

Coefficient of x^2 + coefficient of $y^2 = 0$

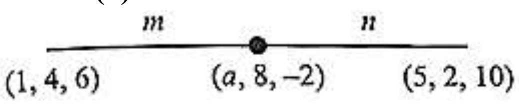
$$\frac{1}{a^2}\left(1 - \frac{h^2}{a^2}\right) - \frac{1}{b^2}\left(1 + \frac{k^2}{b^2}\right) = 0$$

$$\frac{h^2}{a^4} + \frac{k^2}{b^4} = \frac{1}{a^2} - \frac{1}{b^2}$$

replacing $(h, k) \to (x, y)$

$$\frac{x^2}{a^4} + \frac{y^2}{b^4} = \frac{1}{a^2} - \frac{1}{b^2}$$

136. **(b)**



Using section formula,

$$a = \frac{5m + n}{m + n} \quad \ldots\ldots(i)$$

$$8 = \frac{2m + 4n}{m + n} \quad \ldots\ldots(ii)$$

$$-2 = \frac{10m + 6n}{m + n} \quad \ldots\ldots(iii)$$

From (ii) and (iii)

$$8m + 8n = 2m + 4n \Rightarrow 3m = -2n$$

$$\frac{m}{n} = \frac{-2}{3}$$

From (i), $a = \frac{n\left(5\frac{m}{n}+1\right)}{n\left(\frac{m}{n}+1\right)}$

$$a = \frac{5\left(\frac{-2}{3}\right)+1}{\frac{-2}{3}+1} \Rightarrow a = -7. \left[\frac{m}{n} = \frac{-2}{3}\right]$$

$$a = -7$$

$$\frac{2m}{n} - \frac{a}{3} = 2\left(\frac{-2}{3}\right) + \frac{7}{3} = \frac{-4}{3} + \frac{7}{3} = 1$$

137. **(b)** Direction ratio of line joining points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is $(x_2 - x_1, y_2 - y_1, z_2 - z_1)$

For $(4,3,-5)$ and $(-2,1,-8)$

Direction Ratio $= (-2-4, 1-3, -8-(-5))$

$= (-6,-2,-3)$

or $(6,2,3)$

$a = 6, b = 2, c = 3$

$P(a, 3b, 2c) \equiv P(6,6,6)$

$a + 3b - 2(2c) = 0$

correct option is $x + y - 2z = 0$

138. **(a)** Let equation of plane π is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

where a, b, c are intercepts on X, Y, Z axes

plane passes through $(1,1,1)$ and have x intercept $\frac{5}{2}$ then

$$\frac{1}{\left(\frac{5}{2}\right)} + \frac{1}{b} + \frac{1}{c} = 1$$

$$\frac{1}{b} + \frac{1}{c} = 1 - \frac{2}{5} - \frac{3}{5} \quad \ldots\ldots (i)$$

also, perpendicular distance (d) from origin (0,0,0)

$$\Rightarrow d = \frac{-1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}}} \Rightarrow \frac{5}{7} = \frac{-1}{\sqrt{\frac{1}{\left(\frac{5}{2}\right)^2} + \frac{1}{b^2} + \frac{1}{c^2}}}$$

$$\frac{49}{25} = \frac{4}{25} + \frac{1}{b^2} + \frac{1}{c^2} \Rightarrow \frac{1}{b^2} + \frac{1}{c^2} = \frac{49}{25} - \frac{4}{25} = \frac{45}{25}$$

$$\frac{1}{b^2} + \frac{1}{c^2} = \frac{9}{5} \quad \ldots\ldots (ii)$$

squaring equation (i),

$$\left(\frac{1}{b} + \frac{1}{c}\right)^2 = \left(\frac{3}{5}\right)^2 \Rightarrow \frac{2}{bc} = \frac{9}{25} - \left(\frac{1}{b^2} + \frac{1}{c^2}\right)$$

$$= \frac{9}{25} - \frac{9}{5} = \frac{-36}{25} \left(\text{from(ii)}\right) \frac{2}{bc} = \frac{-36}{25}$$

$$\left(\frac{1}{b} - \frac{1}{c}\right)^2 = \frac{1}{b^2} + \frac{1}{c^2} - \frac{2}{bc} = \frac{9}{5} + \frac{36}{25} = \frac{81}{25}$$

$$\frac{1}{b} - \frac{1}{c} = \pm\frac{9}{5} \quad \ldots\ldots (iii)$$

b is negative and c is positive

$\frac{1}{b} - \frac{1}{c} = -\frac{9}{5}$ .... (iv)

Equation (i) + (iv)

$$\frac{2}{b} = \frac{-6}{5} \therefore b = \frac{10}{-6} \; b = \frac{-5}{3}$$

y intercept is $\frac{-5}{3}$

139. **(c)** $f(x) - x = \lambda$, Put $x = 0 \Rightarrow f(0) = \lambda$ ....(i)

$f(x) = x + \lambda, f(y) = y + \lambda$
$x \cdot f(y) = x(y + \lambda) = xy + \lambda x$
$f(xf(y)) = f(xy + \lambda x) = f(xy) + x$
Put $x = 1, y = 0 \Rightarrow f(\lambda) = f(0) + 1, f(\lambda) = \lambda + 1$
$(f(0) = \lambda)$
$f(x) = x + 1$

$$\lim_{x\to 0} \frac{(f(x))^{1/3} - 1}{(f(x))^{1/2} - 1} = \lim_{x\to 0} \frac{(x+1)^{1/3} - 1}{(x+1)^{1/2} - 1} \quad \because \text{ It is } \frac{0}{0} \text{ form}$$

using L-Hospital Rule

$$\lim_{x\to 0} \frac{\frac{1}{3}(x+1)^{-2/3}}{\frac{1}{2}(x+1)^{-1/2}} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

140. **(a)** $\lim_{x\to 0} \frac{|x|}{\sqrt{x^4+4x^2+5}} = \frac{0}{\sqrt{0+0+5}} = 0$

$K = 0$

$$\lim_{x\to 0} x^4 \sin\left(\frac{1}{3\sqrt{x}}\right)$$

RHL: $\lim_{x\to 0^+} x^4 \sin\left(\frac{1}{3\sqrt{x}}\right)$

$$\lim_{h\to 0} (0+h)^4 \sin\left(\frac{1}{3\sqrt{0+h}}\right) = \lim_{h\to 0} h^4 \sin\left(\frac{1}{3\sqrt{h}}\right)$$

$-1 \le \sin x \le 1$

$\sin\left(\frac{1}{3\sqrt{h}}\right)$ is a finite number between -1 and 1 .

$\lim_{h\to 0} h^4 \sin\left(\frac{1}{3\sqrt{h}}\right) = 0 \times$ finite number $= 0$

$\ell = 0, K + \ell = 0 + 0 = 0$

141. **(a)** $\log_e x$ is a finite quantity $\Rightarrow \lim_{n\to\infty} x^n = 0$

This is possible if $-1 < x < 1$
base of log can't be 0 or negative
Considering $0 < x < 1$,
$\log_x 12$ will be always negative.

142. **(b)**

$$f'(x) = \frac{d}{dx} \cot^{-1}\left(\frac{x^x + x^{-x}}{2}\right)$$

$$= \frac{-1}{1 + \left(\frac{x^x + x^{-x}}{2}\right)^2} \cdot \frac{d}{dx}\left(\frac{x^x + x^{-x}}{2}\right)$$

$$= \frac{-1}{2\left[1 + \left(\frac{x^x + x^{-x}}{2}\right)^2\right]} \times -[x^x(1 + \ln x)]^2$$

$$\left(\frac{d}{dx} x^x \quad = x^x(1 + \ln x)\right)$$

$$f'(1) \quad = \frac{+1}{2\left(1 + \left(\frac{2}{2}\right)^2\right)} \times [1(1 + \ln 1)]^2 = \frac{1}{4}$$

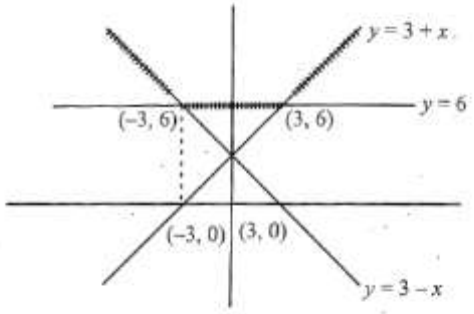
Note: If in question it is given

$f(x) = \cot^{-1}\left(\frac{x^x - x^x}{2}\right)$ then

$$f(x) = \tan^{-1}\left(\frac{2}{x^x - \frac{1}{x^x}}\right) = -\tan^{-1}\left(\frac{2 \cdot x^x}{1 - (x^x)^2}\right)$$

If $x^x = \tan\theta = -\tan^{-1}(\tan 2\theta) = -2\theta = -2\tan^{-1} x^x$
Now, $f'(1) = -1$ (as per Answer key)

143. (c)



Clearly for max. value, $f(x)$ is defined as

$$f(x) = \begin{cases} 3 - x, & x < -3 \\ 6, & -3 \le x < 3 \\ 3 + x, & x \ge 3 \end{cases}$$

Checking differentiability at $x = -3$ and $x = 3$,
(i) At $x = -3$
LHD: $\lim_{\substack{x \to -3^- \\ h \to 0}} \frac{f(x-h) - f(x)}{-h}$
or $\lim_{x \to -3^-} f'(x) = \frac{d}{dx}(3 - x) = -1$
RHD: $\lim_{x \to -3^+} f'(x) = \frac{d}{dx} 6 = 0$
LHD $\neq$ RHD
$f(x)$ is not differentiable at $x = -3$
(ii) At $x = 3$
LHD: $\lim_{x \to -3^-} f'(x) = \frac{d}{dx} 6 = 0$
RHD: $\lim_{x \to 3^+} f'(x) = \frac{d}{dx}(3 + x) = 1$
$\text{LHD}_{x=3} \neq \text{RHD}|_{x=3}$
$f(x)$ is not differentiable at $x = 3$
$a = -3, b = 3$
$|a| + |b| = |-3| + |3| = 6$

144. (d) $x^3 - 2x^2y^2 + 5x + y - 5 = 0$

$x^3 + 5x - 5 = 2x^2y^2 - y$

Differentiating both sides w.r.t x

$$\frac{d}{dx}(x^3 + 5x - 5) = \frac{d}{dx}(2x^2y^2 - y)$$

$$3x^2 + 5 = \frac{d}{dx}(2x^2y^2) - \frac{dy}{dx}$$

$$3x^2 + 5 = 2\left[2yx^2\frac{d}{dx} + 2xy^2\right] - \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3x^2 - 4xy^2 + 5}{4x^2y - 1}$$

Differentiating again w.r.t. x

$$\frac{d^2y}{dx^2} = \frac{(4x^2y - 1)\frac{d}{dx}(3x^2 - 4xy^2 + 5) - (3x^2 - 4xy^2 + 5) \cdot \frac{d}{dx}(4x^2y - 1)}{(4x^2y - 1)^2}$$

$$= \frac{(4x^2y - 1)\left[6x - 4\left\{2xy\frac{dy}{dx} + y^2\right\}\right] - (3x^2 - 4xy^2 + 5)\left[4\left\{x^2\frac{dy}{dx} + 2xy\right\}\right]}{(4x^2y - 1)^2}$$

Now substituting $\frac{dy}{dx} = \frac{3x^2 - 4xy^2 + 5}{4x^2y - 1}$ and $x = 1, y = 1$

$$y''(1,1)=\frac{3\left[6-4\left(\frac{8}{3}+1\right)\right]-4\left[4\left(\frac{4}{3}+2\right)\right]}{9}$$

$$y''(1,1)=\frac{3\left(6-\frac{44}{3}\right)-4\left(\frac{40}{3}\right)}{9}=\frac{-\frac{78}{3}-\frac{160}{3}}{9}$$

$$=\frac{-238}{27}$$

145. **(a)** For common tangent point of contact, equating both coefficients

$x^3-3x^2-8x-4=3x^2+7x+4$

$x^3-6x^2-15x-8=0 \Rightarrow (x+1)^2(x-8)=0$

$x=-1,8$

$y=(-1)^3-3(-1)^2-8(-1)-4=-1-3+8-4=0$

Common point is $(-1,0)\equiv(x_1,y_1)$

$y=x^3-3x^2-8x-4$

$$\frac{dy}{dx}=3x^2-6x-8$$

$$\left.\frac{dy}{dx}\right|_{(-1,0)}=3(-1)^2-6(-1)-8=3+6-8=1=m$$

Equation of common tangent is $y-y_1=m(x-x_1)$

$y-0=1(x+1)$

$x-y+1=0$

146. **(c)** $ax+by=1 \Rightarrow by=-ax+1 \Rightarrow y=\frac{-a}{b}x+\frac{1}{b}$

Compare with $y=mx+c \Rightarrow m=\frac{-a}{b}, c=\frac{1}{b}$

Equation of parabola: $y^2=4ax$

Compare with $y^2=4ax \Rightarrow a=p$

Now condition for normal line is

$$c=-2am-am^3, \frac{1}{b}=\frac{2pa}{b}+\frac{pa^3}{b^3}$$

$$\frac{pa^3}{b^3}=\frac{1}{b}-\frac{2pa}{b} \Rightarrow pa^3=b^2(1-2pa)$$

$pa^3=b^2-2pab^2$

147. **(c)** $y=\frac{x}{1+4x+x^2} \Rightarrow y'=\frac{(x^2+4x+1)-x(2x+4)}{(x^2+4x+1)^2}$

$$y'=\frac{-x^2+1}{(x^2+4x+1)^2} \quad \ldots(i)$$

$$y''=\frac{(x^2+4x+1)^2\cdot(-2x)-(1-x^2)\cdot 2(x^2+4x+1)(2x+4)}{(x^2+4x+1)^4}$$

$$y''=\frac{-2x(x^2+4x+1)^2-2(2x+4)(1-x^2)(x^2+4x+1)}{(x^2+4x+1)^4}$$

For maxima or minima, $y'=0$

$$\frac{-x^2+1}{(x^2+4x+1)^2}=0 \text{ i.e., } 1-x^2=1 \Rightarrow x=\pm1$$

$$y''(x=1)=\frac{-2(36)-2(6)(0)(6)}{6^4}=\frac{-72}{6^4}<0$$

at $x=1, f(x)$ will be maximum

maximum value of $f(x)=f(1)$

and $f(1)=\frac{1}{1+4(1)+(1)^2}=\frac{1}{6}$

148. **(d)** $f(x)=x+\frac{4}{x+2} \Rightarrow f'(x)=1-\frac{4}{(x+2)^2}$

$$f''(x)=-4\frac{(-2)}{(x+2)^3}=\frac{8}{(x+2)^3}$$

For maximum or minimum value, $f'(x)=0$

$$1-\frac{4}{(x+2)^2}=0$$
$$(x+2)^2=4 \Rightarrow x+2=\pm 2$$
$$x=0 \text{ or } -4$$
$$f''(0)=\frac{8}{(0+2)^3}=1>0, f'(0)>0$$
$f(x)$ is minimum at $x=0$

Minimum of $f(x)=0+\frac{4}{0+2}=2$

149. (c)

$$f(x)=ax^3+bx^2+cx+d$$
$$f(x)=3ax^2+2bx+c$$
If $f(x)$ has no extreme values then $f'(x)$ should have no turning points (i.e., monotonic function) which is possible if $f'(x)$ have no real roots

For $3ax^2+2bx+c, D<0$
$$(2b)^2-4(3a)(c)<0$$
$$b^2-3ac<0 \Rightarrow b^2<3ac$$

150. (c)

$$I_n=\int \cot^n x dx=\int \cot^{n-2} x \cdot \cot^2 x dx$$
$$I_n=\int \cot^{n-2} x \cdot \operatorname{cosec}^2 x dx-\int \cot^{n-2} x dx$$
$$I_n=\int \cot^{n-2} x \cdot \operatorname{cosec}^2 x dx-I_{n-2}$$
$$I_n+I_{n-2}=\int \cot^{n-2} x \cdot \operatorname{cosec}^2 x dx;\ \text{Put } n=6$$
$$I_6+I_4=\int \cot^4 x \cdot \operatorname{cosec}^2 x dx$$
Let $\cot x=t \Rightarrow -\operatorname{cosec}^2 x dx=dt$
$$I_6+I_4=-\int t^4 dt \Rightarrow I_6+I_4=-\frac{t^5}{5}=-\frac{\cot^5 x}{5}$$
Assertion is correct.
$$I_n+I_{n-2}=-\frac{\cot^{n-1} x}{n-1}$$

151. (b)

$$I_n=\int \tan^n x dx=\int \tan^{n-2} x \cdot \tan^2 x dx$$
$$=\int \tan^{n-2} x(\sec^2 x-1) dx$$
$$I_n=\int \tan^{n-2} x \sec^2 x dx-\int \tan^{n-2} x dx$$
$$I_n=\int \tan^{n-2} x \sec^2 x dx-I_{n-2}$$
$$I_n+I_{n-2}=\int \tan^{n-2} x \cdot \sec^2 x dx$$
Let $\tan x=t \Rightarrow \sec^2 x dx=dt$
$$I_n+I_{n-2}=\int t^{n-2} \cdot dt \Rightarrow I_n+I_{n-2}=\frac{t^{n-1}}{n-1}\ \forall n \geq 2$$
Put $n=2,3,4,5,6$ and add all $I_2+I_0=\frac{t^1}{1}$
$$I_3+I_1=\frac{t^2}{2}, I_4+I_2=\frac{t^3}{3}, I_5+I_3=\frac{t^4}{4}, I_6+I_4=\frac{t^5}{5}$$
Adding all, we get

$$I_0 + I_1 + 2I_2 + 2I_3 + 2I_4 + I_5 + I_6$$
$$= \frac{t^1}{1} + \frac{t^2}{2} + \frac{t^3}{3} + \cdots \ldots \ldots + \frac{t^5}{5} = \sum_{r=1}^{5} \frac{t^r}{r}$$
$$= \sum_{K=1}^{5} \frac{\tan^K x}{K} \Rightarrow n = 5 \quad (t = \tan x)$$

152. **(b)** $\int e^{\cot x}\left(2\log \operatorname{cosec}^{2x} + \frac{2\tan x}{1+\tan^2 x}\right) \cdot \operatorname{cosec}^2 x dx$

Let $\cot x = t \Rightarrow \operatorname{cosec}^2 x dx = -dt$ and $\tan x = \frac{1}{t}$

$\operatorname{cosec}^2 x = 1 + \cot^2 x = 1 + t^2$.

$$= -\int e^t \left[\log(1+t^2) + \frac{2 \times \frac{1}{t}}{1 + \frac{1}{t^2}}\right] dt$$
$$\frac{d}{dt}\log(1+t^2) = \frac{2t}{1+t^2}$$

Integral is of the form

$$= -\int e^t[f(t) + f'(t)]dt = -e^t f(t) + C \; (t = \cot x)$$
$$= -e^{\cot x} \cdot \log(1 + \cot^2 x) + C = -e^{\cot x} \cdot (\log \operatorname{cosec}^2 x) + C$$
$$= -2e^{\cot x} \cdot (\log \operatorname{cosec} x) + C$$

153. **(a)**

$$x = \frac{t^3}{t^2 - 1} \Rightarrow \frac{dx}{dt} = \frac{d}{dt}\left(\frac{t^3}{t^2 - 1}\right)$$
$$= \frac{t^2(3t^2 - 3 - 2t^2)}{(t^2 - 1)^2} = \frac{t^2(t^2 - 3)}{(t^2 - 1)^2}$$
$$\frac{dx}{dt} = \frac{t^2(t^2 - 3)}{(t^2 - 1)^2} \Rightarrow dx = \left(\frac{t^2(t^2 - 3)}{(t^2 - 1)^2}\right) dt$$
$$\text{and } x - 3y = \frac{t^3}{t^2 - 1} - \frac{3t}{t^2 - 1} = \frac{t^3 - 3t}{t^2 - 1} = \frac{t(t^2 - 3)}{(t^2 - 1)}$$
$$\int \frac{dx}{x - 3y} = \int t^2 \left[\frac{t^2 - 3}{(t^2 - 1)^2}\right]\left(\frac{(t^2 - 1)}{t(t^2 - 3)}\right) dt = \int \frac{t}{t^2 - 1} dt$$
$$\text{Let } t^2 - 1 = u \Rightarrow t dt = \frac{1}{2} du$$
$$= \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + c = \frac{1}{2}\ln|t^2 - 1| + c$$
$$= \frac{1}{2}\log(t^2 - 1) + c$$

154. **(a)** For converting integral into limit of sum considering option (a),

$$\lim_{n\to\infty} \frac{a^k(1 + 2^k + 3^k + - - - + n^k)}{n^{k+1}} \left(\lim_{n\to\infty} a^k \cdot \sum_{r=1}^{n} \frac{rk}{n^k \cdot n}\right)$$
$$\lim_{n\to\infty} \left(\frac{a^k}{n}\right) \cdot \sum_{r=1}^{n} \left(\frac{r}{n}\right)^k = a^k, \lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{n}\left(\frac{r}{n}\right)^k$$

we know that

$$\lim_{n\to\infty}\sum_{r=\eta}^{r_2}\frac{1}{n}f\left(\frac{r}{n}\right)=\int_{\lim_{n\to\infty}\frac{n}{n}}^{\lim_{n\to\infty}\frac{r_2}{n}}f(x)dx$$

$$a^k\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{r}{n}\right)^k=a^k\int_{\lim_{n\to\infty}\frac{1}{n}}^{\lim_{n\to\infty}\frac{n}{n}}x^k dx=a^k\int_0^1 x^k dx.$$

$$=\int_0^1 a^k, x^k dx$$

155. (d)

$$18x^2-9\pi x+\pi^2=0\Rightarrow 6x(3x-\pi)-\pi(3x-\pi)=0$$

$$(3x-\pi)(6x-\pi)=0\Rightarrow x=\frac{\pi}{6},\frac{\pi}{3}$$

$$\alpha=\frac{\pi}{6},\beta=\frac{\pi}{3}$$

$$g\circ f(x)=g\{f(x)\}=g(x^2)=\cos x^2$$

$$\int_\alpha^\beta x(gof(x))dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}x\cdot\cos x^2 dx$$

$$\text{let } x^2=t, x=\frac{\pi}{6}\Rightarrow t=\frac{\pi^2}{36}, 2x\,dx=dt$$

$$xdx=\frac{1}{2}dt\; x=\frac{\pi}{3}\Rightarrow t=\frac{\pi^2}{9}$$

$$\int_{\frac{\pi^2}{36}}^{\frac{\pi^2}{9}}\frac{1}{2}\cos t dt=\frac{1}{2}[\sin t]_{\frac{\pi^2}{36}}^{\frac{\pi^2}{9}}=\frac{1}{2}\left[\sin\frac{\pi^2}{9}-\sin\frac{\pi^2}{36}\right]$$

156. (b) $I=\int_0^\pi x[\sin^2(\sin x)+\cos^2(\cos x)]dx$ .....(i)

using property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$$I=\int_0^\pi(\pi-x)[\sin^2(\sin(\pi-x))+\cos^2(\cos(\pi-x))]dx$$

$$I=\int_0^\pi(\pi-x)[\sin^2\sin x+\cos^2\cos x]dx \quad \text{... ... (ii)}$$

(i) + (ii)

$$2I=\int_0^\pi\pi(\sin^2\sin x+\cos^2\cos x)dx$$

f(x) is even function

$$\int_0^{2a}f(x)dx=2\int_0^a f(x)dx$$

$2I=2\pi\int_0^{\frac{\pi}{2}}[\sin^2(\sin x)+\cos^2(\cos x)]dx$ .....(iii)

again using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$2I=2\pi\int_0^{\frac{\pi}{2}}[\sin^2(\cos x)+\cos^2(\sin x)]dx$ ......(iv)

(iii) + (iv)

$$4I = 2\pi \int_0^{\frac{\pi}{2}} \begin{bmatrix} \sin^2(\sin x) + \cos^2(\sin x) \\ +\cos^2(\cos x) + \sin^2(\cos x) \end{bmatrix} dx$$

$$4I = 2\pi \int_0^{\frac{\pi}{2}} (1+1)dx\ [\sin^2\theta + \cos^2\theta = 1]$$

$$4I = 4\pi \int_0^{\frac{\pi}{2}} dx = 4\pi[x]_0^{\frac{\pi}{2}} = 4\pi\left(\frac{\pi}{2}\right)$$

$$4I = \frac{4\pi^2}{2} \quad \therefore I = \frac{\pi^2}{2}$$

157. (d)

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{r^4}{r^5 + n^5} = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{r^4}{n^5\left(\frac{r^5}{n^5} + 1\right)}$$

$$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{\left(\frac{r}{n}\right)^4 \left(\frac{1}{n}\right)}{\left\{\left(\frac{r}{n}\right)^5 + 1\right\}} = \lim_{n\to\infty} \sum_{r=1}^{n} f\left(\frac{r}{n}\right) \cdot \frac{1}{n}$$

$$= \lim_{n\to\infty} \frac{n}{n}$$

$$= \int_{n\to\infty} f(x) \cdot dx \quad \text{[By Leibnitz Formula]}$$

$$= \frac{r}{n} \to x, \frac{1}{n} \to dx$$

$$\int_0^1 \frac{x^4}{1+x^5} dx$$

Let $1 + x^5 = t \Rightarrow x = 0 \Rightarrow t = 1$

$5x^4 dx = dt \Rightarrow x = 1 \Rightarrow t = 2$

$$\frac{1}{5}\int_1^2 \frac{dt}{t} = \frac{1}{5}[\ln t]^2 = \frac{1}{5}\log 2 = \log \sqrt[5]{2}$$

158. (d) $\frac{dy}{dx} = y\log_e \frac{1}{2}, \frac{dy}{y} = -\log_\varepsilon 2dx$

$$\int \frac{dy}{y} = -\int \log_e 2dx$$

$\ln y = -x\log_e 2 + c$

at $x = 0, y = 1, \ln 1 = 0 + c, c = 0$

$\log_e y = -x\log_e 2$

$\log_e y = \log_e 2^{-x}$

$y = 2^{-x}, \because \lim_{x\to\infty} 2^{-x} = k$

$$\lim_{x\to\infty} \frac{1}{2^x} = k\dot{k} = 0$$

159. (d)

Length of subnormal $= y\frac{dy}{dx}$

$$y\frac{dy}{dx} = x - 1 \Rightarrow ydy = (x-1)dx$$

$$\int ydy = \int (x-1)dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} - x + c$$

curve passes through (1,2)

$$\frac{2^2}{2} = \frac{1^2}{2} - 1 + c \Rightarrow c = \frac{5}{2}$$

i.c. $\frac{y^2}{2}=\frac{x^2}{2}-x+\frac{5}{2} \Rightarrow y^2=x^2-2x+5$

For vertex, $x=0, y^2=5 \Rightarrow y=\pm\sqrt{5}$

A vertex of the conic is $(0, \sqrt{5})$

160. **(d)** $y=Ae^x+Be^{-2x}$

Differentiating, both sides w.r.t. x,

$\frac{dy}{dx}=Ae^x-2\,Be^{-2x} \quad \ldots\ldots(i)$

differentiating again w.r.t. x.

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\,Ae^x-\frac{d}{dx}2\,Be^{-2x}$$

$$\frac{d^2y}{dx^2}=Ae^x+4\,Be^{-2x} \quad \ldots\ldots(ii)$$

(i) + (ii)

$$\frac{d^2y}{dx^2}+\frac{dy}{dx}=2\,Ae^x+2\,Be^{-2x}$$

$$=2(Ae^x+Be^{-2x})=2y, \frac{d^2y}{dx^2}+\frac{dy}{dx}-2y=0$$