Physics

1. **(c)** 1 parsec $= 3.26\text{ly} \cong 3.08 \times 10^{16}$ m; 1 nano meter $= 1 \times 10^{-9}$ m
1 fermi $= 1 \times 10^{-15}$ m and 1 angstrom $= 1 \times 10^{-10}$ m Therefore the least unit for length among the given is fermi.

2. **(b)** Time of flight $t = \frac{2u\sin\theta}{g}$ or, $8 = \frac{2\times u\times 1}{10}$

$\Rightarrow u = 40$ m/s

The position of the stone after 6 s from the ground.

$$h = ut + \frac{1}{2}gt^2 = 40 \times 6 + \frac{1}{2}(-10) \times 6^2$$
$$= 240 - 180 = 60 \text{ m}$$

3. **(a)** Maximum height, $H = \frac{u^2\sin^2\theta}{2g}$ and horizontal range,

$$R = \frac{u^2\sin 2\theta}{g}$$

$$H_1 = \frac{u^2\sin^2\theta}{2g} \text{ and } H_2 = \frac{u^2\sin^2(90^\circ - \theta)}{2g} = \frac{u^2\cos^2\theta}{2g}$$

$$H_1H_2 = \frac{u^2\sin^2\theta}{2g} \times \frac{u^2\cos^2\theta}{2g} = \frac{(u^2\sin 2\theta)^2}{16g^2} = \frac{R^2}{16}$$

$$R = 4\sqrt{H_1H_2}$$

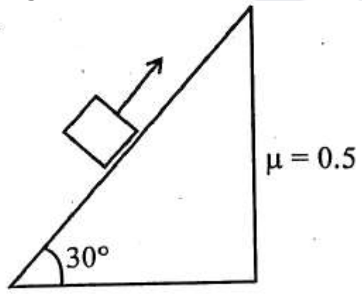
4. **(c)**

5. **(b)** Retarding force $F_{retard} = \frac{m\Delta v}{\Delta t} = \frac{1500\times(0-20)}{5}$

$$= \frac{1500 \times -20}{5} = -6000 \text{ N}$$

6. **(a)** Retardation, $a = g(\sin\theta + \mu\cos\theta)$

$= g[\sin 30^\circ + 0.5(\cos 30^\circ)]$



$$= g\left(\frac{1}{2} + \frac{1}{2} \times \frac{\sqrt{3}}{2}\right) = \left(\frac{1}{2} + \frac{\sqrt{3}}{4}\right)g = \left(\frac{2+\sqrt{3}}{4}\right)g$$

7. **(d)** Power of the machine gun,

$$P = \frac{\text{total K.E. of bullet fired}}{\text{time}}$$
$$= \frac{n \times \frac{1}{2}mv^2}{t} = \frac{n}{t} \times \frac{1}{2}mv^2$$
$$= \frac{300}{60} \times \frac{1}{2} \times \frac{4}{1000} \times (500)^2$$
$$= \frac{300}{60} \times \frac{2}{1000} \times 500 \times 500$$
$$P = \frac{300}{60} \times 500 = 2.5\text{KW}$$

8. **(c)** First height of rebound, $h_1 = e^2h_0$

$$36 = e^2(100) \Rightarrow e^2 = \frac{36}{100} \therefore e = \sqrt{\frac{36}{100}} = \frac{6}{10} = 0.6$$

9. **(b)** Torque, $\vec{\tau}=\frac{d\vec{L}}{dt}$ and $\vec{\tau}=0$ when $\vec{L}=$ constant

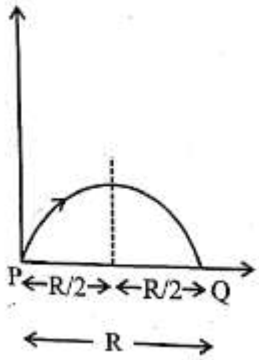
$\vec{\tau}=\vec{r}\times\vec{F}=0$ or, $(6\hat{i}+b\hat{j}+12\hat{k})\times(2\hat{i}+3\hat{j}+4\hat{k})=0$

or, $4b-36=0$, $b=\frac{36}{4}=9$

10. **(a)** Given, range of a projectile of weight $W=R$

The average torque on the projectile between the initial and final positions P and Q about the point of projection,

$T=\frac{WR}{2}$



11. **(d)** Spring constant, $K=\frac{F}{x}$

Given, $F_A=20$ N, $F_B=10$ N, $x_A=8$ cm and $x_B=16$ cm

$$\frac{K_A}{K_B}=\frac{\frac{F_A}{x_A}}{\frac{F_B}{x_B}}=\frac{F_A}{x_A}\times\frac{x_B}{F_B}=\frac{20}{8}\times\frac{16}{10}=4$$

$K_A:K_B=4:1$

12. **(a)** In damping harmonic oscillator, amplitude

$A=A_0e^{-bt}$

$A_1=A_0e^{-b\times2}\Rightarrow\frac{A_1}{A_0}=e^{-2b}$

And

$d\,A_2=A_0e^{-b\times4}\Rightarrow\frac{A_2}{A_0}=e^{-4b}=\left(e^{-2b}\right)^2=\left(\frac{A_1}{A_0}\right)^2$

or, $\frac{A_2}{A_0}=\left(\frac{A_1}{A_0}\right)^2=\frac{A_1^2}{A_0^2}$ or, $\frac{A_2}{A_1^2}=\frac{1}{A_0}\therefore A_1=\sqrt{A_0A_2}$

13. **(c)** Acceleration due to gravity ' g ' varies with height as

$g'=g\left(\frac{R}{R+h}\right)^2$

Given $R+h=5R$ and $g=10\ m/s^2$

$g'=10\left(\frac{R}{5R}\right)^2=\frac{10}{25}=0.4\ m/s^2$

14. **(b)** Elastic limit $=h\rho g$ Height, $h=\frac{\text{Elastic limit}}{\rho g}$

$h=\frac{30\times10^7}{3\times10^3\times10}=10{,}000\ m=10\ km$

15. **(a)** Piessure difference $\Delta p=\rho gh$ and force cxcrtcd

$F = \Delta P.A = \rho gh.2\pi r(r+h)$

$= 1.1\times10^{3}\times10\times1.65\times2\times\frac{22}{7}\times0.5\times10^{-3}$

$(0.5\times10^{-3}+h)$

$-18150\times2\times\frac{22}{7}\times0.5\times10^{-3}(0.5\times10^{-3}+10$

$10^{-3})$

$F \cong 0.57$ N.

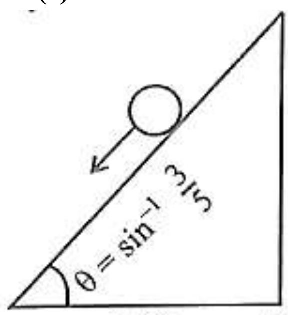
16. (c) Specific heat capacity of copper is 386.4 J kg^{-1} K^{-1} therefore specific heat capacity will remain same. It is independent of mass of the substance.

17. (d) Heat energy, Q at constant volume, $Q = nC_v\Delta T$ for monatomic gas $C_v = \frac{3}{2}R$ and for diatomic gas, $C_v = \frac{5}{2}R$

$Q = 2\times\frac{3}{2}R\times10$ and $Q' = 4\times\frac{5}{2}R\times5$

$\frac{Q'}{Q} = \frac{4\times\frac{5}{2}R\times5}{2\times\frac{3}{2}R\times10} = \frac{5}{3} \therefore Q' = \frac{5}{3}Q$

18. (c) Work done due to sliding down the block = heat gain by block of steel



mg cos θ (s) = mc ΔT

$\Delta T = \frac{g\cos\theta(s)}{C} = \frac{10\times\frac{4}{5}\times\frac{80}{100}}{420}$

$= \frac{10\times4\times80}{5\times100\times420} = \frac{16}{5\times210} = 0.0152^{\circ}C$

19. (c) Isothermal bulk modulus of a gas $B_{Isothermal}$

$= -\frac{\Delta P}{\frac{\Delta V}{V}} = P$

20. (b) Pressure, $P = \frac{2}{3}\frac{E}{V} = \frac{2}{3}\times\frac{4.5\times10^{5}}{10\times10^{-3}} = 30\times10^{6}Nm^{-2}$

21. (b) Given; $f = 100$ Hz, $v = 100$ cm $s^{-1} = 1$ ms^{-1},

$\Delta x = 2.75$ cm $= 2.75\times10^{-2}$ m, Phase difference $\Delta\phi = ?$

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\frac{v}{f}}\times\Delta x[v = f\lambda]$

or, $\Delta\phi = \frac{2\pi\times2.75\times10^{-2}}{1}\times100 = \frac{11}{2}\pi$

22. (d) Blue colour of the sky is due to scattering of light. Scattering $\propto \frac{1}{\lambda^4}$ and λ is least for blue colour among the fundamental colour red, green and blue. So blue colour of light scattered the most. Therefore sky appear blue.

23. (d) The property of light which cannot be explained by Huygen's construction of wave front is origin of spectra.

24. (c) The net electric flux through a square area $A = 2\times2 = 4$ m^2 parallel to $y - z$ plane only, as or.ly i component will cause any flux Flux $\phi = \vec{E}\cdot A = 3\times4 = 12$ NC^{-1} m^2.

25. (b) Let charge on inner shell after switch is closed be q then, $V_{inner} = V_{outer}$

then, $V_{inner} = V_{outer}$

$$\frac{Kq}{R}+\frac{K(Q-q)}{2R}=\frac{Kq}{2R}+\frac{K(Q-q)}{2R}$$

$q = 0$

26. **(c)** Capacitance, $C = \frac{(1-x.1)}{d} + \frac{K\epsilon_0 x}{d}$

$$= \frac{\epsilon_0}{d}(1-x+Kx)$$

or, $C = \frac{\epsilon_0}{d}[1+(K-1)x]$

$$\frac{dC}{dt} = \frac{\epsilon_0}{d}(K-1)V = \frac{8.85\times10^{-12}}{0.01}\times(11-1)\times0.001$$

$= 8.85\times10^{-12}$

$Q = CV$ or, $I = V.\frac{dC}{dt} = 500\times8.85\times10^{-12}$

$= 4.425$ A

27. **(d)** Current $I_1 = \frac{E}{R_1+r}$ and R_1 is replaced by R_2 then,

$$I_2 = \frac{E}{R_2+r}$$

$$\frac{I_1}{I_2} = \frac{R_2+r}{R_1+r} \Rightarrow I_1 r + I_1 R_1 = I_2 r + I_2 R_2$$

$$\Rightarrow r(I_1 - I_2) = I_2R_2 - I_1R_1 \Rightarrow r = \frac{I_2R_2 - I_1R_1}{(I_1-I_2)}$$

28. **(c)**

Given, area of cross-section $A = 0.3\ m^2$,

$n = 2\times10^{25}\ m^{-3}$ charge, $q = (3t^2+5t+2)C$,

$t = 2$ s, $V_d =$?

$$V_d = \frac{i}{neA} = \frac{dq/dt}{neA}\left[i = \frac{dq}{dt}\right]$$

$$i = \frac{dq}{dt} = \frac{d}{dt}(3t^2+5t+2) = 6t+5 = 6\times2+5$$

$= 17$

$$V_d = \frac{i}{neA} = \frac{17}{2\times10^{25}\times1.6\times10^{-19}\times0.3}$$

$$= \frac{17}{0.96\times10^6} = 1.77\times10^{-5}\ m/s$$

29. **(d)** Magnetic field intensity at the centre of a circular wire $B = \frac{\mu_0 i}{2r} = \frac{4\pi\times10^{-7}\times0.2}{2\times0.1} = 4\pi\times10^{-7}.T$

30. **(d)** Magnetic field at the centre of a circular coil $B = \frac{\mu_0 ni}{2r}$ where n = no. of turns and r = radius of circular coil.

' B ' when the wire is bent in the form of a circular coil of 5 turns

' B ' when the wire is bent in the form of a circular coil of 10 turns

$$= \frac{\mu_0 ni'/2r}{\mu_{0'}n'/2r'} = \frac{n}{n'}\times\frac{r'}{r} = \frac{5}{10}\times\frac{R/10}{R/5} = \frac{25}{100} = \frac{1}{4}$$

31. **(d)** Given, $I = 4$ A, $n = 500\ m^{-1}, \mu_r = 400$

Magnetic intensity, $H = nI = 500\times4 = 2000Am^{-1}$

$\mu_r = 1+\chi$ and $\chi = (\mu_r - 1)$

Magnetising field, $M = \chi H = (\mu_r - 1)H$

$= (400-1)\times2000 = 399\times2000 = 7.98\times10^5 Am^{-1}$

32. **(a)** Flux, $\phi = B(NA)\cos\omega t$ and $\omega = 2\pi v$

emf, $\dot{e} = -\frac{d\phi}{dt} = \frac{-d}{dt}BNA\cos\omega t = -NBA\omega\sin(\omega t)$

$|e| = NBA(2\pi v)\sin(2\pi vt)$

33. **(b)** Given, capacitance, $C = 16\mu\ F = 16\times10^{-6}$ F

Inductance, $L = \frac{10}{\pi^2} mH = \frac{10}{\pi^2} \times 10^{-3} H$

$X_L = X_C$ or, $L\omega = \frac{1}{C\omega}$

or $L(2\pi f) = \frac{1}{C(2\pi f)}$

$f = \sqrt{\frac{1}{LC4\pi^2}}$ or, frequency, $f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

$f = \frac{1}{2\pi} \times \sqrt{\frac{1}{\frac{10}{\pi^2}\times10^{-3}\times16\times10^{-6}}} = \frac{1}{2\pi} \times \frac{\pi\times10^4}{4} = \frac{10}{8} kHz$

$f = 1.25 kHz$

34. (c) Maximum emf, $\varepsilon_0 = \sqrt{\frac{P}{2\pi R^2 E_0 c}}$

$$= \sqrt{\frac{1080}{2 \times 3.14 \times 3^2 \times 8.85 \times 10^{-12} \times 3 \times 10^8}}$$

$$= \sqrt{\frac{1080 \times 10^8}{54 \times 314 \times 885}} = 84.85$$

$$\varepsilon_{rms} = \frac{\varepsilon_0}{\sqrt{2}} = \frac{84.85}{1.414} \simeq 60 Vm^{-1}$$

35. (a) De-Broglie wavelength $\lambda = \frac{h}{p}$ or $\lambda \propto \frac{1}{mv}$

$$\frac{\lambda_A}{\lambda_p} = \frac{m_p v_p}{m_A v_A} \text{ or, } \frac{3}{2} = \frac{m_p v_p}{m_A 3v_p} \Rightarrow \frac{3}{2} = \frac{m_p}{3\, m_A}$$

$$m_A = \frac{2}{9} m_p$$

36. (d) Ultraviolet region i.e., Lyman series so photon whose wavelength lies in the ultraviolet region of the electromagnetic spectrum is $2 \rightarrow 1$.

37. (a) Half-life time $T_{1/2} = \frac{iog_e 2}{\lambda}$ and mean life-time

$T = \frac{1}{\lambda} \therefore T_{1/2} = T\log_e 2$

38. (c) Saturation current

$$I = \frac{v(\text{ reverse biased voltage })}{\text{diode resistance}}$$

$$= \frac{8}{4 \times 10^7} = 0.2\mu\ A$$

39. (a) In a p-n-p transistor emitter is heavily doped and collector is moderately doped

40. (c) The peak voltage for the modulating signal is

$m = \frac{A_m}{A_C}; A_m = mA_C = 0.60 \times 15 = 9\ V$

Chemistry

41. (a) $\lambda = \frac{c}{v} = \frac{3\times10^8\ ms^{-1}}{1.034\times10^8\ s^{-1}} \cong 2.90\ m$

42. (d)

43. (c) Al atom is smaller than Mg atom as Al lies to the right of Mg and it has greater effective nuclear charge. A cation is smaller than its parent atom so Al^{3+} is smaller than Al and therefore Al^{3+} is smallest in size.

Thus, statement I is correct.

Among lanthanides there is a gradual decrease in the atomic and ionic radii as we move from left to right due to lanthanoid contraction.

As per the latest data from rsc.org; the atomic radii of Sm, Eu and Gd are:-

$Sm = 236pm, 185pm$ (covalent)
$Eu = 235pm, 183pm$ (covalent)
$Gd = 234pm, 182pm$ (covalent)
Thus, the radii of lanthanides decreases gradually with europium following the same trend.
Thus, statement II is false.

44. **(c)** Fluorine (F) is the element with highest electronegativity, lying at the extreme right in the second period. Francium (Fr) is the most electropositive or least electronegative element, lying at the bottom left of the periodic table.

45. **(c)**

$$B.O. = \frac{N_b - N_a}{2}$$

$C_2 : \sigma 1\,s^2 \sigma^* 1\,s^2 \sigma 2\,s^2 \sigma^* 2\,s^2 \pi 2p_x^2 \pi 2p_y^2$

$$\Rightarrow B.O. = \frac{8-4}{2} = 2, \text{ so } x = 2$$

$B_2 : \sigma 1\,s^2 \sigma^* 1\,s^2 \sigma 2\,s^2 \sigma * 2\,s^2 \pi 2p_x^1 \pi 2p_y^1$

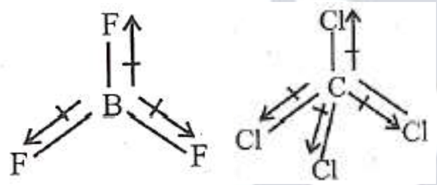
$$\Rightarrow B.O. = \frac{6-4}{2} = 1, \text{ so } B.O. = \frac{1}{2}x$$

$O_2 : \sigma 1\,s^2 \sigma^* 1\,s^2 \sigma 2\,s^2 \sigma^* 2\,s^2 \sigma 2p_z^2 \pi 2p_x^2 \pi 2p_y^2$

$\pi^* 2p^1 x \pi^* 2p_y{}^1$

$$\Rightarrow B.O. = \frac{10-6}{2} = 2, \text{ so } B.O. = x.$$

46. **(b)** BF_3 and CCl_4 are non-polar molecules due to symmetrical arrangement of the bond pairs that result in the cancellation of the individual bond dipoles.



$\mu = 0$

47. **(c)** $PV = nRT \Rightarrow P = \frac{nRT}{V} = CRT$

$$\Rightarrow C = \frac{P}{RT} = \frac{16.4}{0.082 \times 200} = 1.00M$$

48. **(c)** $n_1 = \frac{P_1 V}{RT} = \frac{4 V}{RT}$ and $n_2 = \frac{P_2 V}{RT} = \frac{3 V}{RT}$

$\Rightarrow$ Number of moles remaining =

$$\frac{4 V}{RT} - \frac{3 V}{RT} = \frac{1}{R}$$

49. **(a)** In the given reaction, Cl_2 is getting disproportionated into +1 state in ClO^- and -1 state in Cl^-. Cl_2 to ClO^- is oxidation while Cl_2 to Cl^- is reduction.

50. **(c)**

$$\Delta G^\circ = \Delta H^\circ - T\Delta s^\circ$$
$$= (-1860 \times 10^3) - 298(-550)$$
$$= -1696.1 \text{ kJMol}^{-1}$$

Thus, the reaction is Spontaneous as ΔG° is negative.
Now, Since the system loses the heat, the surrounding must gain it and therefore,

$$\Delta H^\circ \text{ Surrounding } = +1860 \times 10^3 \text{ J mol}^{-1}$$

$$\Rightarrow \Delta S^\circ_{\text{surrounding}} = \frac{\Delta H^\circ_{\text{surrounding}}}{T}$$

$= \dfrac{+1860 \times 10^3}{298}$

$= 6241.6 \text{ J}$

$\Rightarrow \Delta S^\circ_{\text{total}} = \Delta S^\circ_{\text{system}} + \Delta S^\circ_{\text{surrounding}}$

$= (-550) + (6241.6)$

$= +5692 \text{ J mol}^{-1} \text{ K}^{-1}$.

51. (a)

$\Delta H^\circ_f(H_2 \text{ gas}) = 0.0 \text{ kJ mol}^{-1}, \Delta H^\circ_f(Br_2 \text{ liquid})$

$= 0.0 \text{ kJ mol}^{-1}$

$\Delta H_r{}^\circ = -72.8$ kJ for 2 moles of HBr.

$\Rightarrow \Delta H^\circ_r = \left[2\Delta H^\circ_f(\text{HBr gas}) - \left(\Delta H^\circ_f(H_2 \text{ gas})\right)\right.$

$\left.+\left(\Delta H_f{}^\circ (Br_2 \text{ liq.})\right)\right]$

$-72.8 = 2\Delta H_f{}^\circ(\text{HBr gas}) - (0.0 + 0.0)$

$= 2\Delta H^\circ_f(\text{ HBr gas})$

$\Rightarrow \Delta H^\circ_f(\text{HBr gas}) = \dfrac{-72.8}{2} = -36.4 \text{ kJ mol}^{-1}$.

52. (c)

53. (a)

	$H_2O(g)$ +	$CO(g) \rightarrow$	$H_2(g)$ +	$CO_2(g)$
Moles Before	1.0	1.0	0.0	0.0
Moles At Equili.	1.0 – 0.4 = 0.6	1.0 – 0.4 =0.6	a = 0.4	a = 0.4

$\Rightarrow$ Number of moles of H_2O reacting

$= 40\%$ of 1 mole $= 0.4$ moles (degree of dissociation $= \alpha$)

$\Rightarrow$ 0.4 moles of CO and 0.4 moles of H_2O react.

Let C = initial concentrations of H_2O and CO.

$\Rightarrow$ At equilibrium;

$[H_2O] = \dfrac{0.6 \text{ mol}}{1 \text{ L}} = 0.6\text{M}$

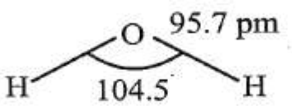
$[CO] = \dfrac{0.6 \text{ mol}}{1 \text{ L}} = 0.6\text{M}$

$[H_2] = [CO_2] = c\alpha = 1 \times 0.4 = 0.4\text{M}$

$\Rightarrow K_c = \dfrac{[H_2][CO_2]}{[H_2O][CO]} = \dfrac{(0.4)(0.4)}{(0.6)(0.6)}$

$= 0.444$

54. (a)



Water is a bent molecule in the gaseous phase with O – H bond length of 95.7 pm .

55. (d) Li and Mg being hard metals are used as an alloy in armour plates.

Cu – Be alloy is used in high-strength springs due to ductility of Cu .

Mg - Al alloy is used in aircraft construction as Al is a versatile metal.

56. (b) The atomic radii or metallic radii of elements increases down the group due to increasing shells with the exception of Gallium (Ga) in the 13th group.

This is due to the presence of d-subshell that offers poorer shielding of the outermost p-electrons. Thus, the order of metallic radius will be: $\text{Ga} < \text{Al} < \text{In} < \text{Tl}$

57. (b) GeO = Acidic oxide (Metalloid)

CO = Neutral oxide

SnO, PbO = Amphoteric oxides

58. (c)

59. (d) An aromatic species has planar structure, conjugated $C = C$ double bonds, cyclic structure with all ring carbon atoms with sp^2-hybridization and having a total of $(4n + 2)\pi$ electrons. Cycloheptatriene has a total of six π - electrons but one ring carbon is sp^3 - hybridized that restricts the conjugated movement of π-electrons. Thus, it is not aromatic.

60. (a) SiO_2 is a covalent network solid that forms a network of tetrahedral SiO_4^4 units.
MgO, CaF_2 and ZnS are ionic solids.

61. (b) $n_{HCl} = 0.1M \times 0.02\ L = 0.002$ mol
$n_{NaOH} = 0.1M \times 0.03\ L = 0.003$ mol
Number of moles of NaOH remaining after neutralization.
$= 0.003 - 0.002 = 0.001$ mol.
Total final volume $= 50\ mL + 50\ mL = 100\ mL = 0.1\ L$
$\Rightarrow$ Molarity $= \frac{0.001}{0.1} = 0.01M$

62. (b) $d = 1.2\ g\ mL^{-1}$, $m = 1\ mol\ kg^{-1}$.
Let's take 1 kg of the solution.
$\Rightarrow$ Volume of the solution $= \frac{\text{mass}}{\text{density}}$
$= \frac{1000\ g}{1.2\ g\ mL^{-1}} = 833.33\ mL$
$= 0.833\ L$
$\Rightarrow$ Number of moles $= 1.0$ mol
Therefore, molarity $= \frac{1.0\ mol}{0.833\ L}$
$= 1.20M \approx 1.01M$.

63. (a) Conductivity (k) $= 6 \times 10^{-3}\Omega^{-1}\ cm^{-1}$
$V = 200\ mL = 0.2\ L$
mass $= 2.08$ g
$\Rightarrow C = \frac{n}{V} = \frac{\left(\frac{\text{mass}}{\text{molar mass}}\right)}{V} = \frac{\left(\frac{2.08}{208}\right)}{0.2}$
$= 0.05M$
$\Rightarrow \Lambda_m = \frac{K \times 1000}{c} = \frac{(6 \times 10^{-3})(1000)}{0.05}$
$= 120 = 1.20 \times 10^2$
Thus, $x = 1.2$

64. (a) When initial concentration of A is kept constant and that of B is changed (tripled), the rate of the reaction increases by 9 times. Thus, the order of the reaction will be α with respect to B. When the concentration of B is kept constant and that of A is increased by 3 times, the rate of the reaction increases by 3 times too. Thus, the order of the reaction is 1 with respect to A.

65. (d) Zeolites are naturally found in the form of sedimentary rocks and volcanic products. Thus, statement III is correct. All other statements regarding zeolites are correct.

66. (d) $CaCl_2$ is hygroscopic in nature and it absorbs moisture rather than adsorbing it on its surface.

67. (b) Zone refining method is used for semiconductors that are B, In, Ge, Si and Ga .

68. (a) Ozone (O_3) is a bent molecule with a bond angle of about 117°.
Thus, (A) is incorrect.
The bond lengths of both O-O bonds are the same due to resonance.
It is less stable than oxygen and is electrophilic in nature. Thus, (D) is incorrect.
Therefore, only (B) and (C) are correct statement.

69. (b) Pr, Nd, Tb and Dy exhibit +4 state in their oxides as they acquire configurations closer to stable $4f^{\circ}$ and $4f^7$ configurations.

$Pr^{4+} = [Xe]4f^1, Nd^{4+} = [Xe]4f^2, Pm^{4+} = [Xe]4f^3,$
$Sm^{4+} = [Xe]4f^4, Eu^{4+} = [Xe]4f^5, Gd^{4+} = [Xe]4f^6,$
$Tb^{4+} = [Xe]4f^7, Dy^{4+} = [Xe]4f^8.$

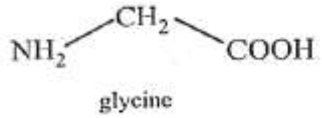
70. (a)

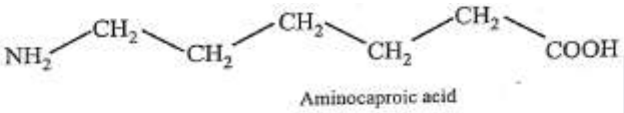
$E^\circ_{Cu^{2+}}/Cu = +0.34\ V, E_{I_2/I^-} = 0.54$

$2Cu^{2+} + 4I^- \rightarrow 2CuI + I_2$

Despite the above values of the standard reduction potentials, Cu^{2+} is able to oxidize iodide to I_2 due to all the species like Cu^{2+}, Cu^+, and CuI being present together in the solution.
The cumulative effect of this results in the $Cu^{2+} \rightarrow$ CuI reduction potential being +0.88 V that makes the reaction feasible.
Therefore, CuI or Cu_2I_2 exists instead of CuI_2 as Cu^{2+} oxidises $I^-$to I_2 and so (A) and (R) are true with (R) explaining (A).

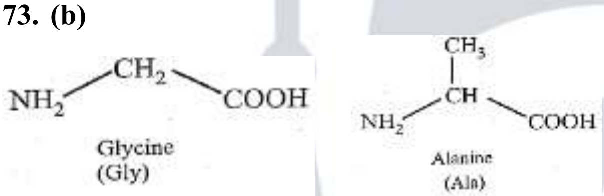
71. (c) Nylon-2- Nylon-6 is a polymer of glycine and aminocaproic acid.



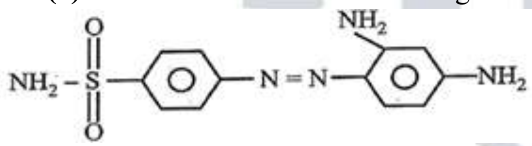


72. (d) The pairing of the two helices of DNA is such that A pairs with T and G pairs with C , through hydrogen bonding.
Thus, number of A = number of T
number of G = number of C
Therefore, statement in Assertion (A) is wrong and Reason (R) is correct.

73. (b)



74. (a) Prontosil is an antibacterial drug having the following structure:-

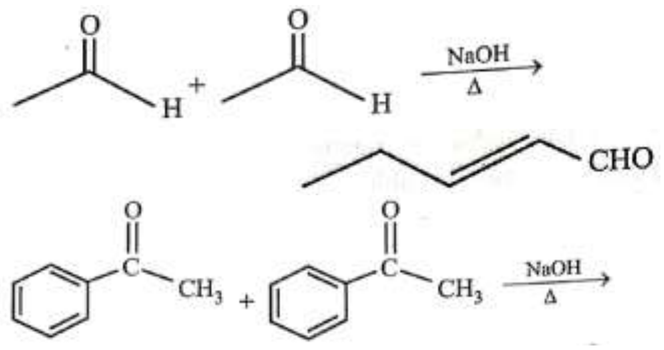


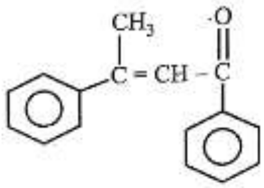
75. (b) $Br - CH_2 - CH_2 - CHO$
3- Bromopropanal is optically- inactive as there are no chiral centres.

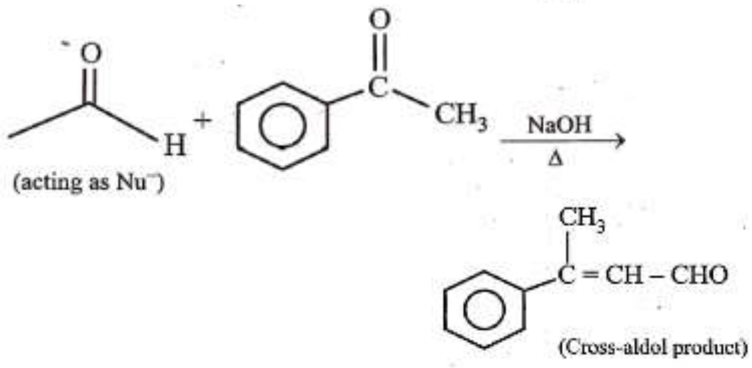
76. (a) $2CH_3CH_2Br \xrightarrow{2Na/\ dry\ ether} CH_3CH_2CH_2CH_3 + 2NaBr$

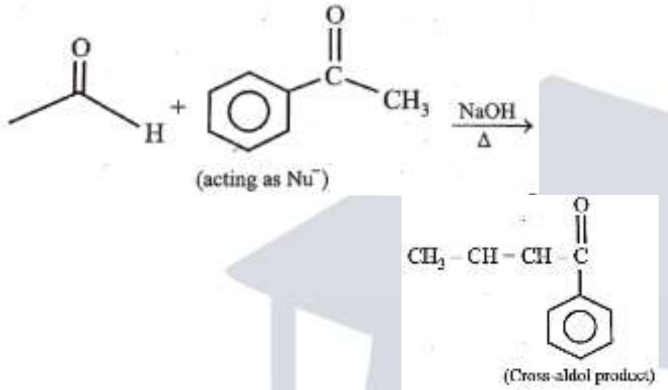
This reaction is known as wurtz reaction.

77. (d) Cross- aldol condensation :



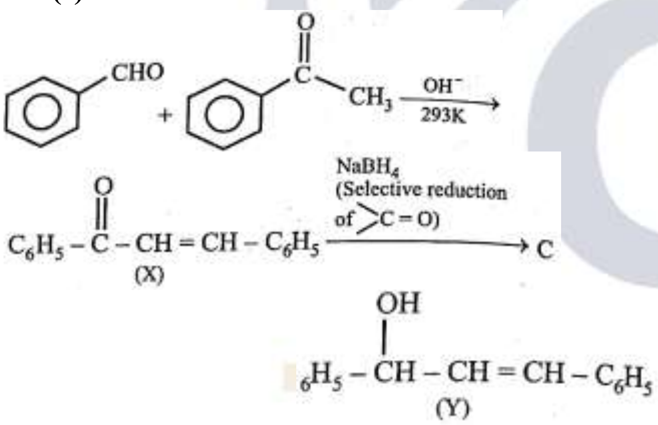






Thus, we get four different products.

78. (c)



79. (a) Esters $\left(R-\overset{\overset{\large O}{\|}}{C}-OR'\right)$ are derivatives of carboxylic acid and are characterized by fruity smell so they are used in perfumes.

80. (d) The boiling point of amines increases with increasing molecular mass and with decreasing number of alkyl groups to the amino N -atom because of H -bonding.

⇒ Butanamine	> N-Ethylethanamine	> N, N – Dimethyl Ethanamine
(1°)	(2°)	(3°)

Mathematics

81. (c)

$$g\left(\frac{\pi}{12}\right) = \sin^2\left(\frac{\pi}{12}\right) = \frac{1}{2}$$

$$\Rightarrow f\left(g\left(\frac{\pi}{12}\right)\right) = f\left(\frac{1}{2}\right) = \frac{1}{8} - \frac{1}{2} = -\frac{3}{8}$$

82. (b)

Since $\log_{10}\left(\frac{3-x}{x}\right) \geq 0$ | also Since, $\frac{3-x}{x} > 0$

$$\Rightarrow \frac{3-x}{x} \geq 1 \quad \Rightarrow x \in (0,3)$$

$$= x \quad \in \left(0, \frac{3}{2}\right]$$

Hence domain of $f(x) = \left(0\,\frac{3}{2}\right] \cap (0,3)$

$$= \left(0, \frac{3}{2}\right]$$

83. (b)

$$X^T B^T = A^T$$

$$\Rightarrow (X^T B^T) = A^T \Rightarrow BX = A$$

$$|B| = \begin{vmatrix} 3 & 5 & -7 \\ 0 & -1 & 8 \\ 6 & -1 & 0 \end{vmatrix} = 222, B^{-1} = \frac{1}{222}\begin{vmatrix} 8 & 7 & 33 \\ 48 & 42 & -24 \\ 6 & 33 & -3 \end{vmatrix}$$

$$X = B^{-1}A = \frac{1}{222}\begin{bmatrix} 8 & 7 & 33 \\ 48 & 42 & -24 \\ 6 & 33 & -3 \end{bmatrix}\begin{bmatrix} 0 \\ -6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{222}\begin{bmatrix} 222 \\ -444 \\ -222 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = D = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

Now, $D^{T_A}A = [1 \quad -2 \quad -1]\begin{bmatrix} 0 \\ -6 \\ 8 \end{bmatrix} = [0 + 12 - 8] = 4$

84. (a) $S^{-1} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ (obtainded from matrix ' S ')

Consider $SA = \frac{1}{2}\begin{bmatrix} 0 & 2a & 2a \\ 2\,b & 0 & 2\,b \\ 2c & 2c & 0 \end{bmatrix}$

Hence $SAS^{-1} = (SA)S^{-1}$

$$= \frac{1}{4}\begin{bmatrix} 4a & 0 & 0 \\ 0 & 4b & 0 \\ 0 & 0 & 4c \end{bmatrix} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

85. (a)

$$A^2 = A \cdot A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, B^2 = B \cdot B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$C^2 = C \cdot C = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Hence,

$$A^2 + B^2 + C^2 = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

and $3\,A^2\,B^2C^2 = 3[(A^2\,B^2)C^2]$

$$= 3\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$A^2 + B^2 + C^2 = 3\,A^2\,B^2C^2$$

86. (a) Given $z_1 = 2 - i,\ z_2 = 6 + 3i$

$\operatorname{amp}\left(\frac{z_1-z_2}{z_1+z_2}\right)=\operatorname{amp}\left(\frac{-4-4i}{8+2i}\right)$
$=\operatorname{amp}(-4-4i)-\operatorname{amp}(8+2i)$
$=[\tan^{-1}(1)-\pi]-\left[\tan^{-1}\left(\frac{2}{8}\right)\right]$
$=\left(\frac{\pi}{4}-\pi\right)-\tan^{-1}\left(\frac{1}{4}\right)=-\frac{3\pi}{4}-\tan^{-1}\left(\frac{1}{4}\right)$

87. (a) Given that $z^3+\bar{z}=0\Rightarrow z^3=-\bar{z}$
$\Rightarrow |z|^3=|-\bar{z}|=|z|\Rightarrow |z|(|z|^2-1)=0$
$|z|=0$ or $|z|^2=1\Rightarrow z\bar{z}=1\Rightarrow \bar{z}=\frac{1}{z}$
$z^3+\bar{z}=0\Rightarrow z^3+\frac{1}{z}=0$
$\Rightarrow z^4+1=0\rightarrow 4$ solution
Total 5 solution.

88. (d)
$[\sqrt{2}(\cos 56°15'+i\sin 56°15')]^8=(\sqrt{2})^8(i^f6°15^1)^8$
$=2^4e^{i(450°)}=2^4e^{i(360+90)}\ \{\because e^{i\theta}=\cos\theta+i\sin\theta\}$
$=$
$=2^4\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=16i$

89. (b)
$(1+i)^{2024}+(1+i)^{2024}=[(1+i)^2]^{1012}+[(1-i)^2]^{1012}$
$=[(2i)^{1012}]+[(-2i)]^{1012}$
$=(2)^{1012}(i)^{1012}+(-2)^{1012}(i)^{1012}$
$=2\cdot 2^{1012}=2^{1013}$

90. (c) $x^2+2(k+2)x+6k+7=0$ has equal roots
Hence
$b^2-4ac=0$
$\Rightarrow [2(k+2)]^2-4(1)(6k+7)]=0$
$\Rightarrow k=3,1$
Hence $k_1=3,k_2=1$
$k_1^2+k_2^2=10$

91. (d) Since $(3+2\sqrt{2})\cdot(3-2\sqrt{2})=9-8=1$
$\Rightarrow (3-2\sqrt{2})=\frac{1}{(3+2\sqrt{2})}$
Now $(3+2\sqrt{2})^{x^2-4}+(3-2\sqrt{2})^{x^2-4}=6$
$\Rightarrow (3+2\sqrt{2})^{x^2-4}+\frac{1}{(3+2\sqrt{2})^{x^2-4}}=6$
Let $y=(3+2\sqrt{2})^{x^2-4}$
$\Rightarrow y+\frac{1}{y}=6\Rightarrow y^2-6y+1=0$
$\Rightarrow y=3+2\sqrt{2}$ or $y=3-2\sqrt{2}$
$\Rightarrow (3+2\sqrt{2})^{x^2-4}=(3+2\sqrt{2})$ or
$(3+2\sqrt{2})^{x^2-4}=\frac{1}{(3+2\sqrt{2})}$
$\Rightarrow x^2-4=1$ or $x^2-4=-1$
$\Rightarrow x=\sqrt{5}$ or $x=3i$
Since $x=3i$ is imaginary
Hence $x=\sqrt{5}$
Therefore $x^4+x^2+5=35$

92. (d) Let $\alpha, \alpha, \alpha, \beta$ be the roots of given equation
$x^4 + ax^3 + bx^2 + cx + d = 0$
Hence $3\alpha + \beta = -a,\ 3\alpha(\alpha + \beta) = b$
$\alpha^2(\alpha + 3\beta) = -c,\ \alpha^3\beta = d$
Now consider $6c - ab = \alpha(3\alpha^2 - 6\alpha\beta + 3\beta^2)$ .....(i)
also consider $3a^2 - 8b = (3\alpha^2 - 6\alpha\beta + 3\beta^2)$ ......(ii)
dividing equation (i) by equation (2),

$$\Rightarrow \alpha = \frac{6c - ab}{3a^2 - 8b}$$

93. (b) Since $x = 1$ is a twice repeated root of $f(x) = ax^3 + ax^2 + bx + c = 0$. Hence $f(-1) = 0$ and $f'(-1) = 0$
Now $f(-1) = 0$
$\Rightarrow -a + a + b + c = 0 \Rightarrow c = b$
Now $f'(x) = 3ax^2 + 2ax + b$
$\Rightarrow f'(-1) = 3a - 2a + b$
$\Rightarrow a + b = 0$
$\Rightarrow a = -b$
$a : b : c = (-b) : (b) : (b)$
$= -1 : 1 : 1$

94. (d) General term for given polinomial

$$\frac{r \cdot {}^nC_r}{{}^nC_{r-1}} = \frac{r \cdot \frac{n!}{r!\,(n-r)!}}{\frac{n!}{(r-1)!\,(n-r+1)!}} = (n - r + 1)$$

Hence

$$\frac{C_1}{C_0} + \frac{2C_2}{C_1} + \cdots + \frac{nC_n}{C_{n-1}} = n + (n-1) + (n-2) + \cdots$$
$$= n + (n-1) + (n-2) + \cdots + 3 + 2 + 1 \qquad + [(n) - (n-1)]$$
$$= \sum_{k=1}^{n} k$$

95. (b) We know that, Number of diagonals of a regular polygon of n sides $= \frac{(n-3)n}{2}$

$$\Rightarrow 104 = \frac{(n-3)n}{2}$$

$\Rightarrow n = 16$ and $n = -13$ (Rejected)
$\Rightarrow n = 16$

96. (d)

$$(2 - 5x)^{-1/5} = (2)^{-1/5}\left(1 - \frac{5}{2}x\right)^{-1/5}$$
$$= (2)^{-1/5}\left[1 + \frac{1}{2}x + \frac{3}{4}x^2 + \cdots\right]$$
$$= (2)^{-1/5} + \frac{1}{2} \cdot (2)^{-1/5}x + \frac{3}{4}(2)^{-1/5}x^2 + \cdots$$

Hence $a_1 = \frac{1}{2}(2)^{-1/5}$ and $a_2 = \frac{3}{4} \cdot (2)^{-1/5} \Rightarrow \frac{a_1}{a_2} = \frac{2}{3}$

97. (d) Natural numbers less than 10000 which are devisible by 5 , will be the numbers upto 4 digits which ends with zero or 5 .
Number of all -natural numbers up to 4 digits which ends with zero when no digits are repeated
$= (9)(8)(7)(1) + (9)(8)(1) + (9)(1) = 585$
(Note that zero will not be counted as $0 \notin \mathbb{N}$)
Number of all- Natural number 5 up to 4 digits which ends with 5 when no digits are repeated.
$= (8)(8)(7)(1) + (8)(8)(1) + (8)(1) + (1) = 521$
Therefore, required number of Natural number

$= 585 + 521 = 1106$

98. **(c)** Since 2 particular students are always being selected Hence number of vacant place in 5 member team $= 5 - 2 = 3$ Hence Now total available students $= 12 - 2 = 10$ Therefore

Required No. of ways = ways for selecting 3 students from available 10 students

$= {}^{10}C_3 = 120$

99. **(b)** $\sin 21^\circ \cos 9^\circ - \cos 84^\circ \cos 6^\circ$

$= \frac{1}{2}[2\sin 21^\circ \cos 9^\circ - 2\cos(90^\circ - 6^\circ) \cdot \cos 6^\circ]$

$= \frac{1}{2}[\sin 30^\circ + \sin 12^\circ - 2\sin 6^\circ \cos 6^\circ]$

$= \frac{1}{2}\left[\frac{1}{2} + \sin 12^\circ - \sin 12^\circ\right] = \frac{1}{4}$

100. **(a)** Given $(1 + \sqrt{1+a}) = (1 + \sqrt{1-a})\cot\alpha$

$\Rightarrow \quad (1 + \sqrt{1+a})\sin\alpha = (1 + \sqrt{1-a})\cos\alpha$

$\Rightarrow \quad (\sin\alpha - \cos\alpha) = (\sqrt{1-a}\cos\alpha - \sqrt{1+a}\sin\alpha)$

Squaring both side, we get

$\Rightarrow \; 1 - \sin 2\alpha = 1 + a(\sin^2\alpha - \cos^2\alpha) - \sqrt{1-a^2}\sin 2\alpha$

$\Rightarrow \; [(a\cos 2\alpha) - \sin 2\alpha]^2 = \left[-\sqrt{1-a^2}\sin 2\alpha\right]^2$

$\Rightarrow a^2\cos^2 2\alpha + \sin^2 2\alpha - 2a\sin 2\alpha\cos 2\alpha$

$= (1-a^2)\sin^2 2\alpha$

$\Rightarrow \; a^2 = a\sin 4\alpha$

$\Rightarrow \sin 4\alpha = a$

101. **(a)**

Let $I = \dfrac{\cos A + \cos 3A + \cos 5A + \cos 7A}{\sin A + \sin 3A + \sin 5A + \sin 7A}$

$\Rightarrow I = \dfrac{(\cos A + \cos 7A) + (\cos 5A + \cos 3A)}{(\sin A + \sin 7A) + (\sin 5A + \sin 3A)}$

$\Rightarrow I = \dfrac{\cos 4A[2\cos 3A + 2\cos A]}{\sin 4A[2\cos 3A + 2\cos A]}$

$\Rightarrow I = \cot 4A = \cot\frac{\pi}{6} = \sqrt{3}$

102. **(d)** Since $\sec(\theta+\alpha), \sec\theta, \sec(\theta-\alpha)$ are in A.P.

Hence $\sec\theta = \frac{1}{2}[\sec(\theta+\alpha) + \sec(\theta-\alpha)]$

$\Rightarrow \sec\theta = \frac{1}{2}\left[\dfrac{2\cos\theta \cdot \cos\alpha}{\cos(\theta+\alpha)\cdot\cos(\theta-\alpha)}\right]$

$\Rightarrow \cos(\theta+\alpha)\cdot\cos(\theta-\alpha) = \cos^2\theta \cdot \cos\alpha$

$\Rightarrow \cos^2\theta - \sin^2\alpha = \cos^2\theta \cdot \cos\alpha$

$\Rightarrow \cos^2\theta = \dfrac{\sin^2\alpha}{1-\cos\alpha} = 2\cos^2\frac{\alpha}{2}$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta = 1 - 2\cos^2\frac{\alpha}{2}$

$\Rightarrow \sin^2\theta = -\cos\alpha$

103. **(d)** $\cos\alpha + \cos\beta = a \Rightarrow a = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

and $\sin\alpha + \sin\beta = b \Rightarrow b = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)\frac{b}{a} = \tan\left(\frac{\alpha+\beta}{2}\right)$

Now $\tan(\alpha+\beta) = \dfrac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)} = \dfrac{2ab}{a^2-b^2}$ [Hence (I) → (a)]

[Hence (IV) → (c)]

Since $\tan(\alpha+\beta) = \frac{2ab}{a^2-b^2}$

$\sin(\alpha+\beta)=\frac{2ab}{a^2+b^2}$

and [Hence III → b]

$\cos(\alpha+\beta)=\frac{a^2-b^2}{a^2+b^2}$

[Hence II → d]

104. (d)

Let $I=\frac{\cos h\alpha}{1-\tan h\alpha}+\frac{\sin h\alpha}{1-\cot h\alpha}$

$\Rightarrow I=\frac{\cos h^2\alpha}{(\cos h\alpha-\sin\alpha)}+\frac{\sin h^2\alpha}{(\sin h\alpha-\cos h\alpha)}$

$\Rightarrow I=\frac{\cos h^2\alpha-\sin h^2\alpha}{(\cos h-\sin h\alpha)}=\cos h\alpha+\sin h\alpha$

$\Rightarrow I=e^{\alpha\{\because e^{\alpha}=\cos h\alpha+\sin h\alpha\}}$

$\Rightarrow I=e^{\log_e(2+\sqrt{3})}=2+\sqrt{3}$

105. (a)

$\cot\frac{A}{2}:\cot\frac{B}{2}:\cot\frac{C}{2}=3:7:9$

$\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}:\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}:\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$=3:7:9$

$\Delta\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}:\Delta\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}:\Delta\sqrt{\frac{s(s-c)}{(s-a)(s-b)}}$

$=3:7:9$

$\Rightarrow s(s-a):s(s-b):s(s-c)=3:7:9$

$\Rightarrow (s-a):(s-b):(s-c)=3:7:9$

So, we have:

$s-a=3k\Rightarrow\frac{a+b+c-a}{2}=3k\Rightarrow b+c-a=6k\ \ldots..(i)$

and $s-b=7k\Rightarrow a+c-b=14k\ \ldots\ldots(ii)$

$s-c=9k\Rightarrow a+b-c=18k\ \ldots\ldots(iii)$

Solving equation (i), (ii) and (iii), we get:

$a=16k,\ b=12k,\ c=10k$

$a:b:c=16:12:10=8:6:5$

106. (d) $\frac{1}{r_1},\frac{1}{r_2},\frac{1}{r_3}$ are in arithmetic progression

Then, $\frac{2}{r_2}=\frac{1}{r_1}+\frac{1}{r_3}$

$\Rightarrow\frac{2(s-b)}{\Delta}=\frac{s-a}{\Delta}+\frac{s-c}{\Delta}$

$\Rightarrow 2(s-b)=2s-(a+c)$

$\Rightarrow 2s-2b=2s-(a+c)\Rightarrow 2b=a+c$

Now, $\frac{r_2}{r_1}=\frac{\frac{\Delta}{s-b}}{\frac{\Delta}{s}}=\frac{s}{s-b}=\frac{2s}{2s-2b}=\frac{a+b+c}{a+b+c-2b}$

$=\frac{2b+b}{a+c-b}=\frac{3b}{2b-b}=\frac{3b}{b}\Rightarrow\frac{r_2}{r}=\frac{3}{1}$

107. (a) We know that:

$$P_1=\frac{2\Delta}{a},P_2=\frac{2\Delta}{b},P_3=\frac{2\Delta}{c}$$

$$\text{Now, }\frac{\cos A}{P_1}+\frac{\cos B}{P_2}+\frac{\cos C}{P_3}$$

$$=\frac{a\cos A}{2\Delta}+\frac{b\cos B}{2\Delta}+\frac{c\cos C}{2\Delta}$$

$$=\frac{2R\sin A\cos A+2R\sin B\cos B+2R\sin C\cos C}{2\Delta}$$

$$=\frac{R(\sin 2A+\sin 2B+\sin 2C)}{2\Delta}$$

$$=\frac{2R(\sin(A+B)\cos(A-B)+\sin C\cos C)}{2\Delta}$$

$$=\frac{2R(\sin C\cos(A-B)-\sin C\cos(A+B))}{2\Delta}$$

$$=\frac{4R\sin C[\sin B\sin C]}{2\Delta}$$

$$=\frac{4R\left(\frac{a}{2R}\right)\left(\frac{b}{2R}\right)\left(\frac{c}{2R}\right)}{2\Delta}=\frac{1}{R^2}\frac{abc}{4\Delta}$$

$$=\frac{1}{R^2}\times R=\frac{1}{R}$$

108. **(c)**

$$\text{Given }\frac{2x^2+5x+6}{(x+2)^3}=\frac{a}{(x+2)}+\frac{b}{(x+2)^2}+\frac{c}{(x+3)^3}$$

$$\Rightarrow 2x^2+5x+6=a(x+2)^2+b(x+2)+c$$

$$\Rightarrow 2x^2+5x+6=x^2(a)+x(4a+b)+(4a+2b+c)$$

Comparing we get,

$a=2, b=-3, c=4$

Hence

$a\cdot b+b\cdot c+c\cdot a=-10$

109. **(d)** $\bar{p}=2\bar{q}$

$$\Rightarrow (x+2y+3)\bar{a}+(5x-y+2)\bar{b}$$

$$(4x+6y+10)\vec{a}+(2x-10y-9)\vec{b}$$

Comparing both sides, we get

$3x+4y+7=0$ ….. (i)

and $3x+9y+6=0$ ….. (ii)

Solving (1) & (2) we get,

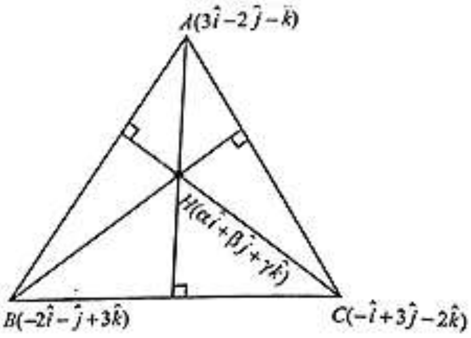
$x=-\frac{13}{5},\ y=\frac{1}{5}$

Hence $x-2y=-3$

110. **(b)** Let $H(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k})$ be the ortho centre of the $\triangle$ ABC.

Since $\overrightarrow{AH}\cdot\overrightarrow{BC}=0$ $\{\because \overrightarrow{AH}\perp\overrightarrow{BC}\}$

$\Rightarrow \alpha+4\beta-5\gamma=0$ ….(i)



Since $BH \perp \overrightarrow{AC} \Rightarrow \overrightarrow{BH} \cdot \overrightarrow{AC} = 0$
$\Rightarrow 5\alpha - \beta - 4\gamma = 0$ ……(ii)
Since $\overrightarrow{CH} \perp \overrightarrow{AB}$ Hence $\overrightarrow{CH} \cdot \overrightarrow{AB} = 0$
$\Rightarrow 5\alpha - \beta - 4\gamma = 0$ …..(iii)
Solving equation (i), (ii) & (iii) we get
$\alpha = \beta = \gamma = 0$
Hence orthocentre is $H(0,0,0)$

Hence $\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} = (3\hat{\imath} - 2\hat{\jmath} - \hat{k}) + (-2\hat{\imath} - \hat{\jmath} + 3\hat{k})$

$+(-\hat{\imath} + 3\hat{\jmath} - 2\hat{k})$
$= \vec{0}$

111. **(b)** Let $\vec{A}_1 = \hat{\imath} + 2\hat{\jmath}$ and $\vec{A}_2 = 3\hat{\jmath} - 2\hat{k}$ be the vectors related to π_1 and $\vec{B}_1 = \hat{\jmath} - 2\hat{k}, \vec{B}_2 = 3\hat{k} - 2\hat{\imath}$ be the vectors related to π_2

Vector perpendicular to plane $\pi_1 = \vec{n}_1 = \vec{A}_1 \times \vec{A}_2$
$\vec{n}_1 = -4\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$
Vector perpendicular to plane $\pi_2 = \vec{n}_2 = \vec{B}_1 \times \vec{B}_2$
$\vec{n}_2 = 3\hat{\imath} + 4\hat{\jmath} + 2\hat{k}$
Hence
angle between π_1 and π_2 = angle between normals to the plane $\vec{n}_1$ and $\vec{n}_2$

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{-12 - 8 + 6}{\sqrt{29}\sqrt{29}}$$
$$\cos\theta = \frac{-14}{29}$$

112. **(d)** Vector perpendicular to $\vec{a}$ and $(\vec{b} \times \vec{c})$ will be
$$\vec{a} \times (\vec{b} + \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$
$$= (10)(3\hat{\jmath} + 4\hat{k}) - (9)(5\hat{\imath} + 4\hat{k})$$
$$= (-45\hat{\imath} + 30\hat{\jmath} + 4\hat{k})$$

113. **(a)** Position vector of centroid $= \overrightarrow{OG} = \frac{\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}}{3}$
$\Rightarrow \overrightarrow{OG} = -2^{\wedge}$
Now $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = -4\hat{\imath} + 5\hat{\jmath} + 5\hat{k}$
$\overrightarrow{CA} = \overrightarrow{OA} - \overrightarrow{OC} = -5\hat{\imath} - 4\hat{\jmath} - \hat{k}$
$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = -\hat{\imath} - \hat{\jmath} - 4\hat{k}$
Therefore
$$BC^2 + CA^2 + AB^2 + 9(OG)^2 = \overrightarrow{BC} \cdot \overrightarrow{BC} + \overrightarrow{CA} \cdot \overrightarrow{CA} + \overrightarrow{AB} \cdot \overrightarrow{AB}$$

$$+9(\overrightarrow{OG} \cdot \overrightarrow{OG})$$
$$= 66 + 42 + 18 + 9(4)$$
$$= 162$$

114. **(c)**
Variance $= 5$ and $n = 20$

$$\text{Variance } = 5 = \frac{1}{n}\Sigma(xi - \bar{x})^2$$
$\Rightarrow \Sigma(xi - \bar{x})^2 = 100$ … … (i)
Let after multiplication, the new observations be $y_1, y_2 \ldots y_{20}$
Where $y_i = 2(xi)$ for some i …….(ii)
New variance $= \frac{1}{2}\Sigma(y_i - y)^2$ where $\bar{y} = \frac{1}{n}\Sigma y_i$

$$\Rightarrow \bar{y}=\frac{1}{20}\Sigma 2xi=2\left(\frac{1}{20}\Sigma xi\right)=2\bar{x} \quad \ldots\ldots \text{(iii)}$$

$\Sigma(xi-\bar{x})^2=100$ {From (1)}

$$\Rightarrow \Sigma\left(\frac{1}{2}y_i-\frac{1}{2}\bar{y}\right)^2=100$$

$$\Rightarrow \left(\frac{1}{2}\right)^2\Sigma(y_i-y)^2=100 \Rightarrow \Sigma(y_i-y)^2=400$$

New variance $=\frac{1}{n}\Sigma(yi-\bar{y})^2=\frac{1}{20}\times 400=20$

115. **(a)** Given that S contains n elements and two sets A and B are selected.

Two set A and B can be selected in 2^n ways.

The no. of ways of selecting two sets such that their union is S and intersection is 2^n.

Therefore the probability $=\frac{2^n}{2^n.2^n}=\frac{1}{2^n}$

116. **(a)** No. of events in sample space when a pair of dice is rolled 24 times $=(6\times 6)^{24}=(36)^{24}$.

No. of event of not getting on both of the dice $=36$

The required probability is $=\frac{(35)^{24}}{(36)^{24}}$

117. **(d)** Let $P(B_1), P(B_2)$ and $P(W)$ are the probabilty of selecting bag 1, bag 2 and white bag respectively.

$$P(B_1)=\frac{1}{2}=P(B_2)$$

$$P\left(\frac{W}{B_1}\right)=\frac{4}{6}=\frac{2}{3}$$

$$P\left(\frac{W}{B_2}\right)=\frac{3}{7}$$

Now, from the total probability theorem:

$$P(W)=P\left(\frac{W}{B_1}\right)P(B_1)+P\left(\frac{W}{B_2}\right)P(B_2)$$

$$=\frac{1}{2}\times\frac{2}{3}+\frac{1}{2}\times\frac{3}{7}=\frac{23}{42}$$

118. **(a)** Since there are 6 total samples. Hence the probability of getting number 1 on face $=\frac{1}{6}$

Probability of not getting number 1 on face of a dice

$$=1-\frac{1}{6}=\frac{5}{6}$$

Since 4 dice are thrown simultaneously. Hence probability of not getting number 1 on face of any of the dice

$$=\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right)=\frac{625}{1296}$$

119. **(b)**

$P(X=0)+P(X=1)+P(X=2)=1$

$\Rightarrow 3c^3+4c-10c^2+5c-1=1$

$\Rightarrow 3c^3+4c-10c^2+5c-2=0$

$\Rightarrow (c-1)(3c-1)(c-2)=0$

$\Rightarrow c=\frac{1}{3},1,2$

When $c=2$, Then $P(X=2)=10-1=9>1$

$c\neq 2$

When $c=1$, Then $P(X=2)=5-1=4>1$

- $c\neq 1$

So, $c=\frac{1}{3}$.

120. **(c)** Since 8 coins are tossed simultaneously.

Hence total samples $2^8=256=n(S)$

Number of ways of getting at least 6 heads

$= {}^8c_6 + {}^8c_7 + {}^8c_8 = 37 = n(E)$
Therefore,
Required probability $= \frac{n(E)}{n(S)} = \frac{37}{256}$

121. **(b)** Let $P(h, k)$ be the point whose locus is to be find.
given $\sqrt{(h-5)+(k-0)^2} = \sqrt{13}$
$\Rightarrow\ h^2 = -k^2 + 10h - 12$ ....(i)
also given, $\frac{2h-3k+4}{\sqrt{4+9}} = 2$
Squaring both sides, we get-

$$\Rightarrow 4h^2 + 9k^2 + 16 - 12hk - 24k + 16h = 52$$

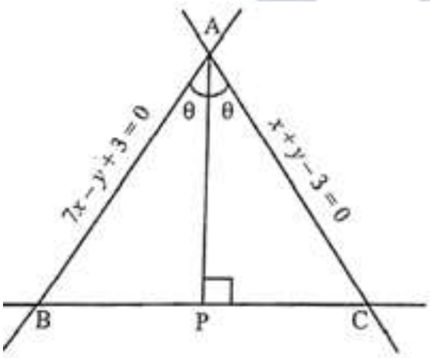
Putting h^2 value in above equation we get

$$\Rightarrow 12hk - 5k^2 - 56h + 24k + 84 = 0$$

for getting locus replacing h, k by x, y respectively
$\Rightarrow 12xy - 5y^2 - 56x + 24y + 84 =$

122. **(a)** All the lines passing from pt(3,4) is $(y-4)$
$= m(x-3)$ ..... (i)
where $m =$ slope
Slope of given line $x + y + 1 = 0$ is $\frac{3\pi}{4}$
Hence $m_1 = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$ and $m_2 = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$ will be the slopes of lines which make $\frac{\pi}{4}$ angle with $x + y + 1 = 0$
When $m = m_1 = \pi$ then equation of line is $y - 4 = 0(x-3) \Rightarrow (y-4) = 0$
When $m = m_2 = \frac{\pi}{2}$ then equation of line is $y - 4 = \frac{1}{0}(x-3)$
$\Rightarrow (x-3) = 0$
Hence combined equation is $\rightarrow (x-3)(y-4) = 0$
$\Rightarrow xy - 4x - 3y + 12 = 0$

123. **(a)** Let $\triangle$ ABC be the isosceles triangle where AB = AC Hence Bisector of Angle $\angle BAC$ will also be the perpendicular online BC.



Let m_1 and m be the slopes of bisector lines AP and line BC respectively.
Equation of Bisector lines (AP) are

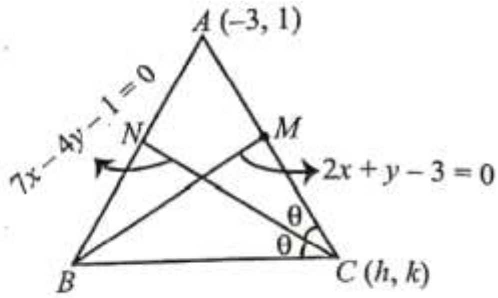
$$\frac{7x - y + 3}{\sqrt{49+1}} = \pm\frac{(x+y-3)}{\sqrt{(1)+(1)}}$$

$\Rightarrow x - 3y + 6 = 0$ and $3x + y - 12 = 0$

Here $m_1 = \frac{1}{3}$	Here $m_1 = -3$
Since AP ⊥ BC	Since AP ⊥ BC
$m_1 \cdot m = -1$	$m_1 \cdot m = -1$
$m = -3$	$\Rightarrow m = \left(\frac{1}{3}\right)$

Since m is integer (given)
Hence $m = -3$

124. **(b)**



$M = \text{Mid point of } AC = \left(\frac{-3+h}{2}, \frac{1+k}{2}\right)$ ....(i)

(h, k) lies on $7x - 4y - 1 = 0$.

$\Rightarrow 7h - 4k - 1 = 0 \Rightarrow h = \frac{4k+1}{7}$ ....(ii)

$$M = \left(\frac{-3 + \frac{4k+1}{7}}{2}, \frac{1+k}{2}\right) = \left(-10 + 2k, \frac{1+k}{2}\right)$$

M lies on BM.

$$\Rightarrow 2(-10 + 2k) + \left(\frac{1+k}{2}\right) - 3 = 0 \Rightarrow k = 5$$

from equation (2) $\Rightarrow h = \frac{4\times5+1}{7} = 3$

$(h, k) = (3,5)$

Slope of line AC $= m_1 = \frac{5-1}{3+3} = \frac{4}{6} = \frac{2}{3}$

Slope of line CN $= m_2 = \frac{7}{4}$.

Let slope of BC is m.

Since $\angle BCN = \angle NCA$.

$$\Rightarrow \left|\frac{m - \frac{7}{4}}{1 + \frac{7}{4}m}\right| = \left|\frac{\frac{7}{4} - \frac{2}{3}}{1 + \frac{7}{4} \times \frac{2}{3}}\right| = \frac{1}{2}$$

$$\Rightarrow \frac{m - \frac{7}{4}}{1 + \frac{7}{4}m} = \frac{1}{2} \Rightarrow m = 18$$

Equation of BC is

$$(y - 5) = m(x - 3) \Rightarrow (y - 5) = 18(x - 3)$$
$$\Rightarrow 18x - y = 49$$

125. **(c)** Given pair of lines

$$(x^2 + y^2)\sin^2\alpha = (x\cos\alpha - y\sin\alpha)^2$$
$$\Rightarrow x^2\cos 2\alpha + y^2(0) - 2xy\sin\alpha\cos\alpha = 0$$

Since lines are perpendicular.

Coefficient of x^2 + coefficient of $y^2 = 0$

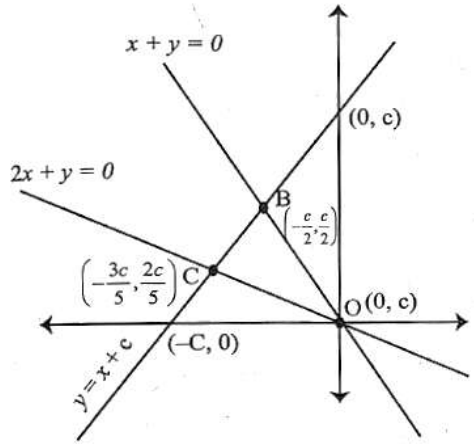
$\Rightarrow \cos 2\alpha = 0$

$$\Rightarrow 1 - 2\sin^2\alpha = 0$$
$$\Rightarrow \sin^2\alpha = \frac{1}{2}$$
$$\text{and } \cos^2\alpha = 1 - \frac{1}{2} = \frac{1}{2}$$
$$\tan^2\alpha = \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)} = 1$$
$$\sin^2\alpha + \tan^2\alpha = 1 + \frac{1}{2} = \frac{3}{2}$$

126. **(a)**

$$2x^2 + 5xy + 3y^2 = 0$$
$$(x + y)(2x + 3y) = 0$$

Let $\triangle$ ABC be the region formed by



$$y = x + c$$
$$x + y = 0$$
$$2x + 3y = 0$$

Now, area of $\triangle ABC = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ -\frac{c}{2} & \frac{c}{2} & 1 \\ -\frac{3c}{5} & \frac{2c}{5} & 1 \end{vmatrix}$

$$\Rightarrow \frac{1}{20} = 0 + 0 + 1\left[\left(-\frac{c}{2}\right)\left(\frac{2c}{5}\right) - \left(\frac{c}{2}\right)\left(-\frac{3c}{5}\right)\right]$$

$\Rightarrow c^2 = 1 \Rightarrow c = \pm 1$

127. **(b)** Equation of line through $(5, 1, a)$ and $(3, b, 1)$

is $\frac{x-5}{(5-3)} = \frac{y-1}{1-b} = \frac{z-a}{a-1} = \lambda$ (say)

Since this line passess through $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

Hence $\frac{0-5}{5-3} = \lambda \Rightarrow \lambda = -\frac{5}{2}$

and $\dfrac{\frac{17}{2} - 1}{1 - b} = \lambda = \dfrac{-5}{2}$

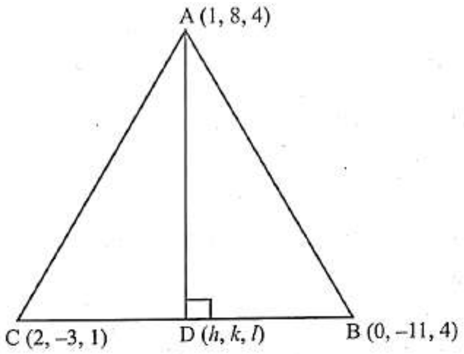
$\Rightarrow b = 4$

and $\dfrac{-\frac{13}{2} - a}{a - 1} = \lambda = -\dfrac{5}{2}$

$\Rightarrow a = 6$

Therefore $a + b = 6 + 4 = 10$

128. **(d)** Let $D(h, k, l)$ be foot of Perpendicular AD on BC . Equation of line BC is



$$\frac{x-2}{2} = \frac{y+3}{8} = \frac{z-1}{-3} = \lambda(\text{say})$$

Since equation passes through $D(h,k,l)$

Hence $\frac{h-2}{2}=\lambda, \frac{k+3}{8}=\lambda, \frac{l-1}{-3}=\lambda$

$\Rightarrow h=2\lambda+2,\ k=8\lambda-3,\ l=-3\lambda+1$

Since $AD \perp BC$ Hence

$\Rightarrow [(2\lambda+2)-1]\cdot(2-0)+[(8\lambda-3)-8]\cdot(-3+11)$

$+[(-3\lambda+1)-4]\cdot(1-4)=0$

$\Rightarrow \lambda=1$

Hence $h=4,\ k=5,\ l=-2$

$D(h,k,l)=D(4,5,-2)$

129. **(c)** Given equation are

$2x-y+2z+3=0$ ......(i)

and $4x-2y+4z+\lambda=0$

$\Rightarrow\ 2x-y+2z+\frac{\lambda}{2}=0$ ......(ii)

and $2x-y+2z+\mu=0$ ......(iii)

Since distance between equation (i) and (ii) is $\frac{1}{3}$. Hence,

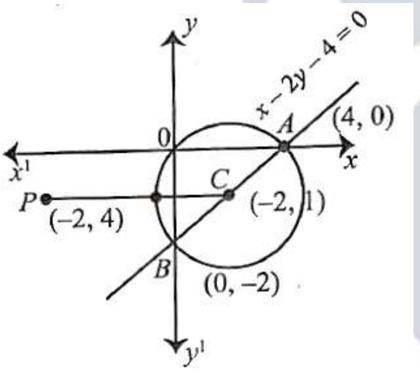
$$\Rightarrow \frac{\left|\frac{\lambda}{2}-3\right|}{\sqrt{4+1+4}}=\frac{1}{3}\Rightarrow \lambda=8$$

Since distance between equation (i) and (iii) is $\frac{2}{3}$. Hence,

$$\Rightarrow \frac{|\mu-3|}{\sqrt{4+1+4}}=\frac{2}{3}\Rightarrow \mu=5$$

Therefore $\lambda+\mu=8+5=13$

130. **(a)** Line $x-2y-4=0$ intersect the axes at $A(4,0)$ and $B(0,-2)$



Centre of circle $=\left(\frac{4+0}{2},\frac{-2+0}{2}\right)=(2,-1)$

radius $=\sqrt{(2-0)^2+(0+1)^2}=\sqrt{5}$

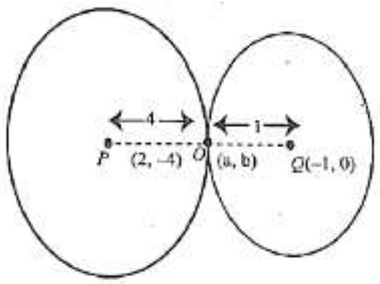
$CP=\sqrt{(2+2)^2+(-1+4)^2}=5$

Distance of PQ is least when points P, Q and C are collinear.

$PQ=CP-CQ=5-\sqrt{5}$

131. **(b)** Circle $x^2+y^2-4x+8y+4=0$

have centre $P(2,-4)$ and radius 4 unit and circle $x^2+y^2+2x=0$ have centre $Q(-1,0)$ and radius 1 unit



Let $O(a, b)$ be the point of contact.

$$O(a, b) = \frac{4(-1)+1(2)}{4+1}, \frac{4(0)+1(-4)}{4+1}$$

$$= \left(-\frac{2}{5}, -\frac{4}{5}\right)^{4+1}$$

$a + 2b = -2$

132. **(d)**

133. **(c)** Equation of pair of tangents from point (1,1) on circle

$S = x^2 + y^2 + 2x + 2y + 1$ can be given as,

$SS_1 = T^2$

$\Rightarrow (x^2 + y^2 + 2x + 2y + 1)(1^2 + 1^2 + 2 + 2 + 1)$

$= [x(1) + y(1) + (x + 1) + (y + 1) + 1]^2$

$\Rightarrow 7x^2 + 7y^2 + 14x + 14y + 7 = (2x + 2y + 3)^2$

$\Rightarrow 3x^2 + 3y^2 + 2x + 2y - 2 - 8xy = 0$

134. **(d)** In circle $S = x^2 + y^2 + 2gx + 2fy + c = 0$ ……(i)

centre $= (-g, -f), g_{\hat{1}} = g, f_1 = f, c_1 = c$

In circle $x^2 + y^2 - 2x + 2y - 2 = 0$ …… (ii)

$g_2 = -1, f_2 = 1, c_2 = -2$

In circle $x^2 + y^2 + 4x - 6y + 9 = 0$ …….. (iii)

$g_3 = 2, f_3 = -3, c_3 = 9$

Since eq (i) & (ii) are orthogonal hence

$\Rightarrow 2g_1g_2 + 2f_1f_2 = c_1 + c_2$

$\Rightarrow -2g + 2f = c - 2$ …. (iv)

Since eq (i) & (iii) are also orthogonal

Hence $4g - 6f = c + 9$ ……(v)

Since center $(-g, -f)$ lies on $2x + 3y - 2 = 0$ Hence $\Rightarrow -2g - 3f - 2 = 0$

Solving eq (4), (5) and (6), we get

$g = \frac{1}{2}, f = -1, c = -1$

Therefore $2g + f = 0 = c - f$

135. **(d)** $x^2 - 4x - 4y + 16 = 0$

$\Rightarrow 2x - 4 - 4\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{4-2x}{-4} = -1 + \frac{1}{2}x$

$2x - y - 5 = 0$ is tangent on the parabola.

$m = 2$

$\Rightarrow -1 + \frac{1}{2}x = 2 \because m = \frac{dy}{dx}$

$\Rightarrow \frac{1}{2}x = 3 \Rightarrow x = 6$

So, $2 \times 6 - y - 5 = 0 \Rightarrow y = 7$.

$P \equiv (6,7)$

Now, equation of normal at $P(6,7)$ is

$\Rightarrow (y - 7) = -\frac{1}{2}(x - 6)$

$\Rightarrow 2y - 14 = -x + 6 \Rightarrow x + 2y - 20 = 0$

$\Rightarrow \frac{1}{2}x + y - 10 = 0$ …. . (i)

The given equation normal is $\Rightarrow ax + y + c = 0$ …..(ii)

from (1) and (2) $\Rightarrow a = \frac{1}{2}$ and $c = -10$

$ac = \frac{1}{2} \times (-10) = -5$

136. **(b)** Equation of director circle ellipse is

$x^2 + y^2 = a^2 + b^2 \Rightarrow a^2 + b^2 = 20$ …….(i)

Given that lotus rectum $= \frac{2b^2}{a} = 2 \Rightarrow b^2 = a$ ……(ii)
from (i) and (ii) we get $a = 4$
We have $c^2 = a^2 - b^2 = 16 - 4 = 12$
$\Rightarrow c = 2\sqrt{3}$
Distance between foci $= 2c = 4\sqrt{3}$.

137. **(c)** Given equation of ellipse $\frac{x^2}{6} - \frac{y^2}{5} = 1$

Let equation of tangent $y = mx + c$
It passes through (2,3) i.e $c = 3 - 2m$ ….. (i)
By condition of tangency.

$$c^2 = a^2m^2 - b^2 \Rightarrow c^2 = 6m^2 - 5$$
$$\Rightarrow m^2 + 6m - 7 = 0 \ \text{(from (i))}$$
$$\Rightarrow m = 1, -7$$

Now, $\tan\theta = \left|\frac{1+7}{1-7}\right| = \frac{8}{6} = \frac{4}{3}$.

138. **(d)** We have difference between foci $= 2a$

$2a = 6 \Rightarrow a = 3$,

End of focal chord is $\left(c, \pm\frac{b^2}{a}\right) \Rightarrow c = \sqrt{13}$

$$c^2 = a^2 + b^2 \Rightarrow 13 = 9 + b^2 \Rightarrow b^2 = 4$$
$$k = \pm\frac{b^2}{a} = \pm\frac{4}{3}.$$

139. **(a)** $f(x) = \begin{cases} 1 + 6x - 3x^2, & x \le 1 \\ x + \log_2(b^2 + 7), & x > 1, \end{cases}$

Since f(x) is continuous at $x = 1$,
Hence

$$\Rightarrow \lim_{h\to 0}(1 + h) = f(1)$$
$$\Rightarrow \lim_{h\to 0}[(1 + h) + \log_2(b^2 + 7)] = 1 + 6 - 3$$
$$\Rightarrow \log_2(b^2 + 7) = 3 \Rightarrow b = \pm 1$$

140. **(d)** Since $f'(x)$ is a constant

$f'(x) = a$ (say) ….. (i)
$\Rightarrow f(x) = ax + b$ where b is arbitrary constant. ….. (ii)
Since $f(0) = 2 \Rightarrow b = 2$
Since $f'(0) = 1 \Rightarrow a = 1$
$f(x) = (x + 2)$
Which is continuous on $(-\infty, \infty)$ i.e R

141. **(a)** Since f(x) is also continuous on $\mathbb{R}$

$$\text{Hence } f(0) = \lim_{h\to 0} f(0 - h) = \lim_{h\to 0}\frac{\sin(-h) - \sin\left(-\frac{h}{2}\right)}{(-h)}$$
$$\Rightarrow f(0) = \lim_{h\to 0}\left[\frac{\sin h}{h} - \frac{1}{2}\cdot\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right] = 1 - \frac{1}{2}$$
$$\Rightarrow f(0) = \frac{1}{2}$$

142. **(a)** Since $f(3) \ne f(2)$

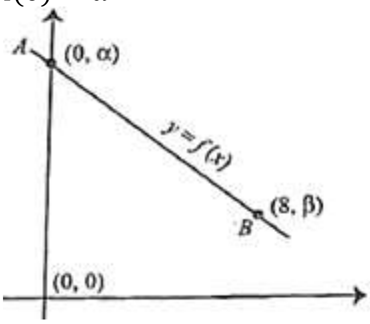
Using Mean value theorem

$$f'(x) = \frac{f(3) - f(2)}{3 - 2} = \frac{15 - 4}{3 - 2}$$
$\Rightarrow f'(x) = 11$
$\Rightarrow f'(x) \geq 5$
$\Rightarrow \frac{f(6) - f(2)}{6 - 2} \geq 5$
$\Rightarrow f(6) - 4 \geq 20$
$\Rightarrow f(6) \geq 24$
$\Rightarrow$ Possible values of $f(6) = 24$.

143. **(d)** $f(x) = \sqrt{x + \sin x}$ (given) ….. (i)
Since $f'(x) = 0$
$\Rightarrow \frac{(1 + \cos x)}{2\sqrt{x + \sin x}} = 0$
$\Rightarrow \cos x = -1$
$\Rightarrow x = (2n + 1)\pi, n \in \mathbb{Z}$
at $x = (2n + 1)\pi$
$f[(2n + 1)\pi] = \sqrt{(2n + 1)\pi + \sin(2n + 1)\pi}$
$\Rightarrow f[(2n + 1)\pi] = \sqrt{(2n + 1)\pi}\{\sin(2n + 1)\pi = 0 n \in \mathbb{Z}\}$
$\Rightarrow f(t) = \sqrt{t}$
$\Rightarrow$ Which represents a Hyperbola.

144. **(a)**
$y = \frac{\log x}{x}$
$\Rightarrow \frac{dy}{dx} = \frac{1 - \log x}{x^2}$
and $\frac{d^2y}{dx^2} = \frac{-3x + 2x\log x}{x^4}$
Now Consider,
$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y$
$= x^2\left(\frac{-3x + 2x\log x}{x^4}\right) + 3x\left(\frac{1 - \log x}{x^2}\right) + \frac{\log x}{x} = 0 \ \forall x, y$

145. **(a)** Point $A(0, \alpha)$ is on $y = f(x)$
$f(0) = \alpha$



and also given $f(0) = 2$
Therefore $\alpha = 2$.
Since slope of chord AB = Slope of tangent on $y = f(x)$ at $x = 4$
$\Rightarrow \frac{\beta - \alpha}{8 - 0} = f'(4)$
$\Rightarrow \frac{\beta - 2}{8} = -\frac{3}{4}$
$\Rightarrow \beta = -4$

146. **(d)** Given $y = x^3 - ax^2 + x + 1$

$\Rightarrow \dfrac{dy}{dx} = 3x^2 - 2ax + 1$

Since θ is an acute angle. Hence $\tan\theta > 0$

$\Rightarrow 3x^2 - 2ax + 1 > 0$

Which is true off $A > 0$ and $D < 0$

Here $A = 3 > 0$

$\Rightarrow D < 0$

$\Rightarrow B^2 - 4AC < 0$

$\Rightarrow (2a)^2 - 4(3)(1) < 0$

$\Rightarrow (a - \sqrt{3})(a + \sqrt{3}) < 0$

$\Rightarrow a \in (-\sqrt{3}, \sqrt{3})$

147. **(d)** $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{4t+3}{(3t^2-8t-3)}$

Hence slope of normal $= -\frac{1}{\left(\frac{dy}{dx}\right)}$

$= \dfrac{(3t^2 - 8t - 3)}{(4t + 3)}$

Since slope of normal = Slope of x-axis

$\Rightarrow \dfrac{3t^2 - 8t - 3}{4t + 3} = 0$

$\Rightarrow 3t^2 - 8t - 3 = 0$

$\Rightarrow t = 3$ and $t = -\frac{1}{3}$

Hence required number of points = 2

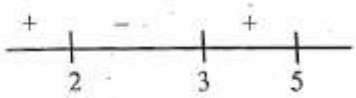
148. **(b)** Given $A = \{x: 9x \geq x^2 + 20\}$

$A = \{x: x \in [4,5]\} = [4,5]$

Given $f(x) = 2x^3 - 15x^2 + 36x - 48$

$f'(x) = 6(x^2 - 5x + 6) = 6(x - 3)(x - 2)$

For maxima or minima $f'(x) = 0$



$\Rightarrow x = 3,2$

but $A = [4,5]$

Hence $f(x)$ is increasing function in $A = [4,5]$

Therefore maximum value of $f(x)$ will be obtained at $x = 5$

$f(5) = 7$

149. **(b)** Let $I = \int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = \int \frac{4e^{2x}+6}{9e^{2x}-4}dx$

Let $4e^{2x} + 6 = A(9e^{2x} - 4) + B(18e^{2x})$

$4e^{2x} + 6 = (9A + 18B)e^{2x} - 4A$

Comparing both sides we get, $A = -\frac{3}{2}, B = \frac{35}{36}$

$$I = -\frac{3}{2}\int \frac{(9e^{2x} - 4)dx}{(9e^{2x} - 4)} + \frac{35}{36}\int \frac{18e^{2x}}{9e^{2x} - 4}dx$$

$$\Rightarrow I = -\frac{3}{2}\int dx + \frac{35}{36}\int \frac{18e^{2x}}{9e^{2x} - 4}dx$$

$$\Rightarrow I = -\frac{3}{2} + \frac{35}{36}\log|(9e^{2x} - 4)| + c$$

Hence $A = -\frac{3}{2}$, $B = \frac{35}{36}$

150. **(c)** $f(x) = \int \frac{dx}{x^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

Since at $x = \sqrt{2}, f(x) = 0$

$\Rightarrow 0 = \frac{1}{\sqrt{2}}\tan^{-1}(1) + c \Rightarrow c = -\frac{\pi}{4\sqrt{2}}$

Hence

$f(0) = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{0}{\sqrt{2}}\right) + \left(-\frac{\pi}{4\sqrt{2}}\right) = -\frac{\pi}{4\sqrt{2}}$

151. (b)

Let $I = \int e^x\left(\frac{2+\sin 2x}{1+\cos 2x}\right)dx$

$= \int e^x\left(\frac{2+\sin 2x}{2\cos^2 x}\right)dx$

$= \int e^x \sec^2 x dx + \int e^x \tan x dx$

$= \left[e^x \int \sec^2 dx - \int e^x \tan x dx\right] + \int e^x \tan x dx + c$

$= e^x \tan x + c$

152. (c)

$I = \int_{-a}^{a} (x^4 - 2x^2)dx$

$\frac{dI}{da} = \frac{d}{da}\left[\int_{-a}^{a} (x^4 - 2x^2)dx\right]$

$= 2a^4 - 4a^2$

and $\frac{d^2I}{da^2} = 8a^3 - 8a$

For minima, $\frac{dI}{d\alpha} = 0$

$\Rightarrow a = 0, \pm\sqrt{2}$

but when $c = \sqrt{2}$ then $\frac{d^2I}{da^2} = 8\sqrt{2} > 0$

Hence $a = \sqrt{2}$ is minima

153. (a) Let $\int_{\pi/6}^{\pi/3} \frac{(\sin x)^{\sqrt{2}}dx}{\sqrt{2}+(\cos x)^{\sqrt{2}}}$ …..(i)

Since $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Hence $I = \int_{\pi/6}^{\pi/3} \frac{(\cos x)^{\sqrt{2}} \cdot dx}{(\sin x)^{\sqrt{2}}+(\cos x)^{\sqrt{2}}}$ ……(ii)

Equation (i) +(ii),

$\Rightarrow I\int_{\pi/6}^{13} \frac{(\sin x)^{\sqrt{2}}(\cos x)^{\sqrt{2}}}{(\sin x)^{\sqrt{2}}} (\cos x)^{\sqrt{2}}$

$\Rightarrow Ix$

Hence Assertion ' A ' is correct.

Let $P = \int_{\pi/6}^{\pi/3} \frac{f(x)dx}{f(x)+f\left(\frac{\pi}{2}-x\right)}$ …..(iii)

$\Rightarrow P = \int_{\pi/6}^{\pi/3} \frac{f\left(\frac{\pi}{2}-x\right)dx}{f(x)+f\left(\frac{\pi}{2}-x\right)}$ …..(iv)

Equation (iii) + (iv),

$2P = \int_{\pi/6}^{\pi/3} dx \Rightarrow P = \frac{\pi}{12}$

Hence Reason R is also true

putting $f(x) = (\sin x)^{\sqrt{2}}$ in Reason ' R ' we get-

$\int_{\pi/6}^{\pi/3} \frac{(\sin x)^{\sqrt{2}}dx}{(\sin x)^{\sqrt{2}} + (\cos x)^{\sqrt{2}}} = \frac{\pi}{12}$

Hence R is correct explanation of ' A '

154. (b)

1.5 $x^2_{-0.5}[x]dx = \int_{-0.5}^{0} x^2(-1)dx + \int_{0}^{1} x^2(0)dx + \int_{1}^{1.5} x^2(1)dx$

$= \quad -\frac{1}{3}[x^3]^0_{-\frac{1}{2}} + \frac{1}{3}[x^3]_1^{3/2} = \frac{3}{4}$

155. **(b)**

Let $I = \int_0^{50\pi} \sqrt{1-\cos^2 x}dx$

$= \sqrt{2}\int_0^{50\pi} \sqrt{\sin^2 x}dx = \sqrt{2}\int_0^{50\pi} |\sin x|dx$

$= 50\sqrt{2}\int_0^{\pi} \sin xdx = -50\sqrt{2}[\cos x]_0^{\pi}$

$= 100\sqrt{2}$

156. **(a)** Let $I = \int_{-5}^{5} \frac{x^3+5}{\sqrt{12+x}}dx$ ......(i)

$\Rightarrow I = \int_{-5}^{5} \frac{(5-5-x)^3+5}{\sqrt{12+(5-5-x)}}dx$ (By property)

Adding equation (i) & (ii)

$2I = \int_{-5}^{5}\left(\frac{x^3+5}{\sqrt{12+x}} + \frac{-x^3+5}{\sqrt{12-x}}\right)dx$

Since $f(-x) = f(x)$

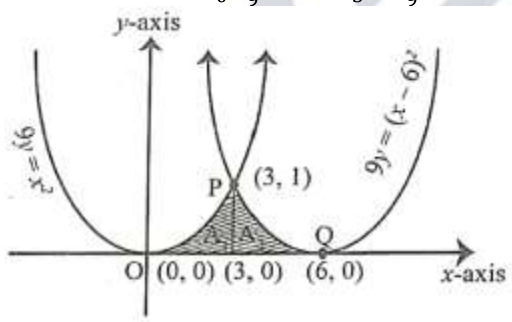
Hence $2I = 2\int_0^5 \left(\frac{x^3+5}{\sqrt{12+x}} + \frac{5-x^3}{\sqrt{12-x}}\right)dx$

$\Rightarrow \int_{-5}^{5} \frac{x^3+5}{\sqrt{12+x}}dx = \int_0^5 \left(\frac{x^3+5}{\sqrt{12+x}} + \frac{5-x^3}{\sqrt{12-x}}\right)dx$

By comparision, $g(x) = \frac{5-x^3}{\sqrt{12-x}}$.

157. **(c)** Here we have to the area of shaded region

Required Area $= \int_0^3 \frac{x^2}{9}dx + \int_3^6 \frac{(x-6)^2}{9}dx$



$= \frac{1}{27}[x^3]_0^3 + \frac{1}{27}[(x-6)^3]_3^6 = 2$

158. **(d)** x = vy converts those differential equation to variable separable form which we have Homogenous function $f(x,y) = \frac{dy}{dx}$ of order zero ie each term should have equal power.

Option (a) & (b) are not homogeneous. Hence they are incorrect option. Option (c) have homogeneus function of order 1 because $\frac{dy}{dx} = \frac{y^2}{x+\sqrt{xy}} = f(x,y)$. Hence option (c) is also not correct.

Option (d) is correct because it have homogenous function of order zero $\frac{dy}{dn} = f(x,y) = \frac{-\left(1+2e^{\frac{x}{y}}\right)}{2e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)}$

159. **(c)** Since $\frac{dy}{dx} = f(x,y)$ is said to be a homogeneous differential equation if $f(x,y)$ is a homogenous function of degree zero.

i.e. $\frac{dy}{dx} = f(x,y) = \phi(y/x)$

160. **(c)** Given, $y_3^{\frac{1}{2}} - 2(y_1)^{\frac{1}{4}} + xy = 0$

$\Rightarrow \left(y_3^{\frac{1}{2}} - xy\right)^4 = \left[2(y_1)^{\frac{1}{4}}\right]^4$

$\Rightarrow y_3^{\frac{1}{2}}[\,{}^4C_1(y_3)xy + {}^4C_3(xy)^3]$

$= [16y_1 - {}^4C_0(y_3)^2 - {}^4C_2(y_3)(xy)^2 - {}^4C_4(xy)^4]$

Squaring both side we have =

$y_3[\,{}^4C_1(y_3)xy + {}^4C_3(xy)^3]^2 = [16y_1 - {}^4C_0(y_3)^2 - {}^4C_2(y_3)$
$(xy)^2 - {}^4C_4(xy)^4]^2$ which is now free from fractions in the power of derivatives (y_1, y_2, y_3).

Clearly, order = 3

Since y_3 will be having power 4 in right side.

Hence degree = 4