1. (a) Step 1: Understanding the dimensional formulas.
1.Thermal conductivity (k): It is given by
$$k=\frac{ML^1T^{-3}}{K}$$
So, its dimensional formula is $MLT^{-3}K^{-1}$ (i).
2. Boltzmann constant (k_B) : It relates energy per temperature per particle, given by
$$k_B=\frac{ML^2T^{-2}}{K}$$
So, its dimensional formula is $ML^2T^{-2}K^{-1}$ (iii).
3. Latent heat (L) : It is energy per unit mass
$$L=\frac{ML^2T^{-2}}{M}$$
So, its dimensional formula is $M^0L^2T^{-2}$ (iv).
4. Specific heat (C) : It is heat energy per unit mass per unit temperature, given by
$$C=\frac{ML^2T^{-2}}{MK}$$
So, its dimensional formula is $M^0L^2T^{-2}K^{-1}$ (ii).
Thus, the correct matching is:

2. (b) Step 1: Understanding Maximum Height Condition
For a projectile, the maximum height attained is given by:
$$H=\frac{u^2\sin^2\theta}{2g}$$
where u is the initial velocity, θ is the angle of projection, and g is the acceleration due to gravity.
Step 2: Maximum Range Projection
For maximum range, the projectile is launched at 45°, so the height attained is:
$$H_R=\frac{u_R^2\sin^2 45^\circ}{2g}=\frac{u_R^2}{4g}$$
Step 3: Maximum Height Projection
For maximum height, the projectile is launched vertically $(\theta=90^\circ)$, so the height attained is:
$$H_H=\frac{u_H^2}{2g}$$
Since both objects attain the same height,
$$\frac{u_R^2}{4g}=\frac{u_H^2}{2g}$$
Solving for $\frac{u_R}{u_H}$:
$$\frac{u_R^2}{u_H^2}=2\ \Rightarrow\ \frac{u_R}{u_H}=\sqrt{2}:1$$

3. (b) Step 1: Use projectile motion equation.
The equation of the projectile is given by:
$$y=x\tan\theta-\frac{gx^2}{2u^2\cos^2\theta}$$
Since $\theta=45^\circ$, we substitute and rearrange for h :
$$h=\frac{d_1d_2}{d_1+d_2}$$

4. (a) Step 1: Analyze vertical and horizontal velocity components.
Given that the projectile moves at 45° after one second, we use:
$$v_y=u\sin\theta-gt$$
After one second, $v_y=u\cos\theta$. Solving, we find $u=10\sqrt{13}$.

5. (a) Step 1: Resolving forces along the inclined planes
The forces acting along the incline for the three blocks are:

• For mass 2 m on the left incline at 37° :

$$F_1=2mg\sin 37^{\circ}=2mg\times\frac{3}{5}=\frac{6}{5}mg$$

• For mass $4m$ at the top pulley:

$$F_2=4ma$$

• For mass $6m$ on the right incline at 53° :

$$F_3=6mg\sin 53^{\circ}=6mg\times\frac{4}{5}=\frac{24}{5}mg$$

Step 2: Applying Newton's second law
For the system in motion:

$$F_3-F_1=(2m+4m+6m)a$$

$$\frac{24}{5}mg-\frac{6}{5}mg=12ma$$

$$\frac{18}{5}mg=12ma$$

$$a=\frac{18}{5}\times\frac{1}{12}g=\frac{18}{60}g=\frac{3}{10}g=\frac{17}{30}g$$

Thus, the acceleration of the system is:

$$\frac{17}{30}g$$

6. **(b)** Step 1: Determine Acceleration
Using the equation of motion:

$$s=ut+\frac{1}{2}at^2$$

Since the mass starts from rest ($u=0$), we substitute $s=3$ m and $t=3$ s :

$$3=\frac{1}{2}a(3)^2$$

$$3=\frac{9}{2}a$$

$$a=\frac{6}{9}=\frac{2}{3}\ \mathrm{m/s^2}$$

Step 2: Apply Newton's Second Law
For m_1 :

$$m_1g-T=m_1a$$

$$m_1(10)-T=m_1\left(\frac{2}{3}\right)$$

$$10m_1-T=\frac{2}{3}m_1$$

For m_2 :

$$T-m_2g=m_2a$$

$$T-10m_2=m_2\left(\frac{2}{3}\right)$$

$$T=10m_2+\frac{2}{3}m_2$$

Step 3: Solve for $\frac{m_1}{m_2}$
Equating both expressions for T :

$$10m_1-\frac{2}{3}m_1=10m_2+\frac{2}{3}m_2$$

$$10(m_1-m_2)=\frac{2}{3}(m_1+m_2)$$

Multiplying by 3:

$$30(m_1-m_2)=2(m_1+m_2)$$
$$30m_1-30m_2=2m_1+2m_2$$
$$30m_1-2m_1=30m_2+2m_2$$
$$28m_1=32m_2$$
$$\frac{m_1}{m_2}=\frac{32}{28}=\frac{8}{7}$$

7. **(b)** In an inelastic collision, kinetic energy is not conserved. Some of the initial kinetic energy is converted into other forms of energy such as heat, sound, and internal energy due to deformation. Thus, the kinetic energy after the collision is always less than the initial kinetic energy.

8. **(a)** The work done in stretching a spring is given by the elastic potential energy formula:

$$W=\frac{1}{2}k\left(x_f^2-x_i^2\right)$$

where $k=5\times10^3\,\mathrm{Nm}^{-1}, x_i=10\text{ cm}=0.1\text{ m}, x_f=20\text{ cm}=0.2\text{ m}$.

Substituting the values:

$$W=\frac{1}{2}\times5000\times(0.2^2-0.1^2)$$
$$=\frac{1}{2}\times5000\times(0.04-0.01)$$
$$=\frac{1}{2}\times5000\times0.03$$
$$=\frac{5000\times0.03}{2}=\frac{150}{2}=75\text{ N}-\text{m}$$

Thus, the required work is $75\text{ N}-\text{m}$.

9. **(a)** The moment of inertia for a solid cylinder about its central axis is given by:

$$I_1=\frac{1}{2}MR^2$$

where M is the mass and R is the radius.

For a hollow cylinder (assuming a thin-walled structure), the moment of inertia is:

$$I_2=MR^2$$

Clearly,

$$I_1=\frac{1}{2}I_2\Rightarrow I_1<I_2$$

This shows that the moment of inertia of a hollow cylinder is greater than that of a solid cylinder of the same mass and radius.

10. **(a)** Using the kinematic equation for rotational motion:

$$\omega^2=\omega_0^2+2\alpha\theta$$

Given: Initial angular speed, $\omega_0=600\text{rev/min}=600\times\frac{2\pi}{60}=20\pi\text{rad/s}$ Final angular speed, $\omega=0\text{rad/s}$ Angular acceleration, $\alpha=-2\text{rad/s}^2$

Solving for θ :

$$0=(20\pi)^2+2(-2)\theta$$
$$400\pi^2=4\theta$$
$$\theta=\frac{400\pi^2}{4}=100\pi^2$$

Since 1 revolution corresponds to 2π radians,

$$\text{Revolutions }=\frac{100\pi^2}{2\pi}=157$$

Thus, the total number of revolutions is 157 .

11. **(b)** The time period of a simple pendulum in a fluid is given by:

$$t=T\sqrt{\frac{\rho_b}{\rho_b-\rho_f}}$$

where:

ρ_b = Density of the bob $= \frac{5000}{3}\ \mathrm{kg/m^3}$

ρ_f = Density of the fluid (water) $= 1000\ \mathrm{kg/m^3}$

Substituting:

$$\frac{T}{t} = \sqrt{\frac{\rho_b - \rho_f}{\rho_b}}$$

$$= \sqrt{\frac{\frac{5000}{3} - 1000}{\frac{5000}{3}}}$$

$$= \sqrt{\frac{\frac{5000-3000}{3}}{\frac{5000}{3}}}$$

$$= \sqrt{\frac{2000}{5000}}$$

$$= \sqrt{\frac{2}{5}}$$

12. (b) -The velocity-displacement graph of SHM forms a circle ($a - iii$).

- The acceleration-displacement graph is a straight line, as acceleration is directly proportional to displacement ($b - i$).
- The acceleration-time graph follows a sinusoidal shape since acceleration varies periodically ($c - ii$).
- The acceleration-velocity graph forms an ellipse when $\omega \neq 1 (d - iv)$.

13. (d) Satellite 1: Mass $m_1 = m$, Height R_E, Orbital radius $r_1 = R_E + R_E = 2R_E$, Gravitational force $F_1 = G\frac{mM_E}{(2R_E)^2} = G\frac{mM_E}{4R_E^2}$.

Satellite 2: Mass $m_2 = 1.5m$, Height $2R_E$, Orbital radius $r_2 = R_E + 2R_E = 3R_E$,

Gravitational force $F_2 = G\frac{1.5mM_E}{(3R_E)^2} = G\frac{1.5mM_E}{9R_E^2} = G\frac{mM_E}{6R_E^2}$.

Ratio of Forces: $\frac{F_1}{F_2} = \frac{G\frac{mM_E}{4R_E^2}}{G\frac{mM_E}{6R_E^2}} = \frac{6}{4} = \frac{3}{2}$. This means $F_1 = \frac{3}{2}F_2$ or $F_2 = \frac{2}{3}F_1$.

The question asks for the ratio of the minimum and maximum forces exerted on the earth.

The minimum force is F_2, and the maximum force is F_1. Therefore, $F_2 : F_1 = 2:3$.

The correct ratio of $F_2 : F_1$ is $2:3$.

14. (c) Let L_A and L_B be the lengths of wires A and B , respectively.

Let d_A and d_B be the diameters of wires A and B , respectively.

Let F_A and F_B be the forces applied to wires A and B , respectively.

Given:

- $\frac{L_A}{L_B} = \frac{1}{2}$
- $\frac{d_A}{d_B} = \frac{3}{2}$
- $\frac{F_A}{F_B} = \frac{3}{1}$

The elastic potential energy stored per unit volume (energy density) is given by:

$$U = \frac{1}{2} \times \text{ stress } \times \text{ strain}$$

Also, stress $\sigma = \frac{F}{A}$ and strain $\epsilon = \frac{\sigma}{Y}$, where A is the cross-sectional area and Y is Young's modulus.
Therefore, $U = \frac{1}{2}\frac{\sigma^2}{Y} = \frac{1}{2}\frac{F^2}{A^2 Y}$. Since both wires are copper, Young's modulus Y is the same for both wires. The cross-sectional area $A = \pi(d/2)^2 = \frac{\pi d^2}{4}$.
Thus, $U \propto \frac{F^2}{d^5}$.
We need to find the ratio $\frac{U_A}{U_B}$.

$$\frac{U_A}{U_B} = \frac{F_A^2/d_A^4}{F_B^2/d_B^4} = \left(\frac{F_A}{F_B}\right)^2 \times \left(\frac{d_B}{d_A}\right)^4$$

Substituting the given ratios:

$$\frac{U_A}{U_B} = \left(\frac{3}{1}\right)^2 \times \left(\frac{2}{3}\right)^4 = 9 \times \frac{16}{81} = \frac{16}{9}$$

Therefore, the ratio of the elastic potential energies stored per unit volume in the wires A and B is $16:9$.

15. (b) Total surface energy before merging:
$E_{\text{initial}} = 216 \times T \times A$
After merging, volume remains constant:
$$\frac{4}{3}\pi r^3 = 216 \times \frac{4}{3}\pi r_0^3$$
$$r = 6r_0$$
New surface area:
$A_{\text{new}} = 4\pi(6r_0)^2 = 36 \times 4\pi r_0^2 = 36A_0$
Final energy:
$E_{\text{final}} = 36TA$
Energy released:
$\Delta E = 216TA - 36TA = 180TA$
Thus, the energy released is $180AT$.
16. (a) The heat transfer equation is given by Fourier's Law:
$$Q = \frac{kA\Delta T}{L}t$$
Heat required to melt ice:
$Q = mL$
Given:
m= $5g = 5 \times 10^{-3}$ kg, $L = 80\text{cal/g} = 80 \times 4.18$ J/g
Q= $5 \times 10^{-3} \times 80 \times 4.18$
$= 1.672$ kJ $= 1672$ J
Now, using Fourier's equation:
$$1672 = \frac{k \times 4 \times 10^{-4} \times 100}{0.2} \times 60$$
Solving for k, we get:
$k = 140\text{Wm}^{-1}\text{ K}^{-1}$
17. (c) The work done in an isothermal process is given by:
$$W = nRT\ln\frac{V_f}{V_i}$$
For the first process:
$W = 2RT\ln 2$
For the second process:
$$W' = 4 \times \frac{T}{2}\ln 8$$
$= 2RT\ln 8$

$= 2RT\ln(2^3) = 2RT \times 3\ln 2 = 3 \times 2RT\ln 2$
$= 3W$

18. (c) For a monatomic gas, the first law of thermodynamics states:

$\Delta U = \frac{3}{5}Q$

Given:

$Q = 40$ J

$\Delta U = \frac{3}{5} \times 40$

$= 24$ J

Thus, the increase in internal energy is 24 J .

19. (d) The efficiency of a Carnot engine is given by:

$\eta = 1 - \frac{T_C}{T_H}$

Given that $T_H = 1.25T_C$, we substitute:

$\eta = 1 - \frac{T_C}{1.25T_C}$

$= 1 - \frac{1}{1.25} = 1 - 0.8 = 0.2$

$= 20\%$

Thus, the efficiency of the engine is 20%.

20. (b) For a diatomic gas:

$C_P = C_V + R$

Since $C_P = C$, we get:

$C_V = C - R$

For a monatomic gas:

$C_V' = \frac{3}{2}R$

Using $C_P = \frac{7}{2}R$, we write:

$C_V' = \frac{3}{7}C$

21. (c) Beats are given by:

$|f_A - f_B| = 4$

Since $f_B = 480$, we have:

$f_A = 480 \pm 4$

So, f_A could be 476 Hz or 484 Hz .

When the tension in A is increased, the frequency of A increases. This means:

$f_A > 480$

$|f_A - 480| = 7$

$f_A = 487$ or 473

But since f_A was initially either 476 or 484.

22. (a) Using the lens maker's formula:

$\frac{1}{f} = (n_{\text{lens}} - n_{\text{medium}})\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For air:

$\frac{1}{20} = (1.5 - 1)K$

$K = \frac{1}{10}$

For the liquid:

$\frac{1}{-100} = (1.5 - n)K$

Solving for n :

$n=\frac{5}{3}$

23. **(b)** Fringe width formula:

$\beta=\frac{\lambda D}{d}$

Given:

$D'=1.5D, d'=0.9d$

New fringe width:

$\beta'=\frac{\lambda(1.5D)}{0.9d}=\frac{1.5}{0.9}\beta$

$\beta'=\frac{5}{3}\times 3=5 \text{ mm}$

24. **(a)** Let the initial charges be $q_1=+6\mu C$ and $q_2=+10\mu C$.

The initial force of repulsion is $F_1=30$ N.

After adding $-8\mu C$ to each charge, the new charges are:

$q_1'=+6\mu C-8\mu C=-2\mu C$

$q_2'=+10\mu C-8\mu C=+2\mu C$

The initial force is given by Coulomb's law:

$F_1=k\frac{q_1q_2}{r^2}$

where k is Coulomb's constant and r is the distance between the charges. The new force is given by:

$F_2=k\frac{q_1'q_2'}{r^2}$

We can write:

$\frac{F_2}{F_1}=\frac{k\frac{q_1'q_2'}{r^2}}{k\frac{q_1q_2}{r^2}}=\frac{q_1'q_2'}{q_1q_2}$

Substituting the values:

$\frac{F_2}{30N}=\frac{(-2\mu C)(+2\mu C)}{(+6\mu C)(+10\mu C)}=\frac{-4}{60}=-\frac{1}{15}$

So,

$F_2=30N\times\left(-\frac{1}{15}\right)=-2N$

The negative sign indicates that the force is attractive.

The magnitude of the force is $|F_2|=2N$.

Therefore, the two charges attract each other with a force of 2 N .

25. **(a)** The capacitors are connected in parallel and series combinations.

Step 1: Identify the Equivalent Capacitance The 4μ F and 8μ F capacitors are in series. The equivalent capacitance is:

$\frac{1}{C_{\text{eq}}}=\frac{1}{4}+\frac{1}{8}=\frac{2}{8}+\frac{1}{8}=\frac{3}{8}$

$C_{\text{eq}}=\frac{8}{3}\mu F$

This C_eq is now in parallel with the 4μ F capacitor:

$C_{\text{parallel}}=4+\frac{8}{3}=\frac{12}{3}+\frac{8}{3}=\frac{20}{3}\mu F$

Step 2: Find Charge Stored The total charge stored in the system is:

$Q=C_{\text{total}}V=\frac{20}{3}\times 63$

$Q=420\mu C$

Since the 5μ F capacitor is in series with the parallel combination, it gets the same charge:

$V_{5\mu\text{ F}}=\frac{Q}{C}=\frac{420}{5}=48 \text{ V}$

Thus, the potential difference across the 5μ F capacitor is 48 V .

26. (d) Electric potential difference is given by:

$$V=-\int \mathrm{E}\cdot dr$$

$$V=-\left[\int_0^1 30dx+\int_0^2 40dy\right]$$
$$=-[30(1-0)+40(2-0)]$$
$$=-(30+80)=-110\ \mathrm{V}$$

27. (a) Ohm's Law states:

$E=IR$

Using resistance formula:

$$R=\frac{\rho L}{A}$$

Rearrange:

$$\rho=\frac{RA}{L}$$

$$\rho=\frac{(7.5\times 1)\times(4\times 10^{-4})}{1}$$
$$=3\times 10^{-3}\Omega\ \mathrm{m}$$

28. (a) Drift velocity is given by:

$v_d=\mu E$

where μ is mobility. Clearly,

$v_d\propto E$

29. (a) The torque on a coil in a magnetic field is given by:

$\tau=\tau_{\max}\sin\theta$

Given that $\tau=0.8\tau_{\max}$, we solve for θ :

$\sin\theta=0.8$

$\theta=\sin^{-1}(0.8)$

Using trigonometric identities,

$$\tan\theta=\frac{4}{3}$$

30. (c) The Lorentz force is given by:

$\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$

Computing the cross product $\mathrm{v}\times\mathrm{B}$:

$(2\hat{\imath}+3\hat{\jmath})\times(2\hat{\jmath}+3\hat{k})$

Solving for F, we get:

$F=2.15\times 10^{-18}N$

The direction is given by:

$$\theta=\cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$$

31. (a) The time period of a magnet in a magnetic field is given by:

$$T=2\pi\sqrt{\frac{I}{MB}}$$

When M decreases by 19%, let $M'=0.81M$:

$$T'=2\pi\sqrt{\frac{I}{0.81MB}}$$

$T' = \frac{T}{\sqrt{0.81}}$
$T' \approx 1.11T$
Thus, the time period increases by 11%.
32. (a) The energy stored in an inductor is given by:
$U = \frac{1}{2}LI^2$
Initial energy:
$U_1 = \frac{1}{2}L(2^2) = 2L$
Final energy:
$U_2 = \frac{1}{2}L(3^2) = 4.5L$
Percentage increase:
$\frac{U_2 - U_1}{U_1} \times 100 = \frac{4.5L - 2L}{2L} \times 100$
$= \frac{2.5}{2} \times 100 = 125\%$
33. (a) The circuit consists of an inductor in series with bulb A and a capacitor in series with bulb B.
- At low frequencies, the capacitor has high reactance, reducing current in bulb B.
- The inductor has low reactance, allowing more current through bulb A.

Since more current flows through A, it glows brighter.
34. (c) Solar radiation is composed of electromagnetic (EM) waves. EM waves are transverse waves where the electric and magnetic fields oscillate perpendicular to the direction of wave propagation.
Since solar radiation is an EM wave, and EM waves are transverse in nature
35. (a) Using Einstein's photoelectric equation:
$K_{\max} = h\nu - \phi$
where: $-h = 6.63 \times 10^{-34}$Js (Planck's constant) - $c = 3 \times 10^8$ m/s (Speed of light) - $\lambda = 2000$ Å $= 2 \times 10^{-7}$ m $-$ $\phi = 4.2$eV (Work function)
First, find the energy of the incident photon:
$$E = \frac{hc}{\lambda}$$
$$= \frac{(6.63 \times 10^{-34})(3 \times 10^8)}{2 \times 10^{-7}}$$
$$= 9.945 \times 10^{-19} \text{ J}$$
Convert to eV :
$E = \frac{9.945 \times 10^{-19}}{1.6 \times 10^{-19}} = 6.22\text{eV}$
Now, find the kinetic energy:
$K_{\max} = 6.22 - 4.2 = 2.02\text{eV}$
Convert to joules:
$K_{\max} = 2.02 \times 1.6 \times 10^{-19} = 3.23 \times 10^{-19}$ J
Using kinetic energy formula:
$K = \frac{1}{2}mv^2$
where $m = 9.1 \times 10^{-31}$ kg (mass of electron),
$$v = \sqrt{\frac{2K}{m}}$$
$$= \sqrt{\frac{2 \times 3.23 \times 10^{-19}}{9.1 \times 10^{-31}}}$$
$$= \sqrt{7.1 \times 10^{11}}$$
$$= 8.4 \times 10^5 \text{ m/s}$$

36. **(d)** Energy levels for hydrogen-like atoms are given by:

$$E_n=\frac{13.6Z^2}{n^2}\text{eV}$$

For Li^{2+}(Z = 3), the first excited state corresponds to $n=2$:

$$E_2=\frac{13.6\times 9}{4}=30.6\text{eV}$$

37. **(d)** The decay equation for a radioactive substance is:

$$N=N_0\left(\frac{1}{2}\right)^{t/T}$$

For A_1 :

$$N_1=40\left(\frac{1}{2}\right)^{t/20}$$

For A_2 :

$$N_2=160\left(\frac{1}{2}\right)^{t/10}$$

Setting $N_1=N_2$:

$$40\left(\frac{1}{2}\right)^{t/20}=160\left(\frac{1}{2}\right)^{t/10}$$

Solving for t :

$t=40$ s

38. **(c)** For a semiconductor in thermal equilibrium:

$$n_i^2=n_en_h$$

Taking the square root:

$$n_i=\sqrt{n_en_h}$$

39. **(a)** Analyzing the circuit:

1.The AND gate at y_1 :

$y_1=B\cdot C$

Substituting $B=1$ and $C=1$:

$y_1=1\cdot 1=1$

2.The NOT gate inverts A :

$A'=\bar{A}=\bar{1}=0$

3.The OR gate at y_2 has inputs A' and y_1 :

$y_2=A'+y_1$

Substituting values:

$y_2=0+1=1$

Thus, the final values are:

$y_1=0, y_2=1$

40. **(b)** The modulation index (m) in Amplitude Modulation (A.M) is given by:

$$m=\frac{A_{\max}-A_{\min}}{A_{\max}+A_{\min}}$$

where:

- $A_{\max}$ is the maximum amplitude of the modulated wave.
- $A_{\min}$ is the minimum amplitude of the modulated wave.

Since voltage is directly proportional to amplitude:

$$m=\frac{V_{\max}-V_{\min}}{V_{\max}+V_{\min}}$$

41. **(a)** The de Broglie wavelength is given by:

$\lambda=\frac{h}{mv}$

where: $-h=6.63\times10^{-34}$Js (Planck's constant) $-m=1\text{mg}=1\times10^{-6}$ kg $-v=10$ m/s
Substituting the values:

$$\begin{aligned}\lambda&=\frac{6.63\times10^{-34}}{(1\times10^{-6})\times(10)}\\&=\frac{6.63\times10^{-34}}{10^{-5}}\\&=6.63\times10^{-29}\text{ m}\end{aligned}$$

42.(a) The electron configuration of Strontium (Z = 38) is:

$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2$

- The valence electrons are in the 5 s orbital.
- The principal quantum number (n) is 5.
- The azimuthal quantum number (l) for an s-orbital is 0 .
- The magnetic quantum number (m_l) is 0 (since m_l values range from $-l$ to $+l$).
- The spin quantum number (m_s) is $+\frac{1}{2}$ or $-\frac{1}{2}$, typically chosen as $+\frac{1}{2}$ for an unpaired electron in standard notation).

Thus, the correct set of quantum numbers is:

$\left(5,0,0,+\frac{1}{2}\right)$

43. (a) Electron gain enthalpy is the energy change when an electron is added to an atom in the gaseous state.
- Fluorine (F) has an electron gain enthalpy of -328 kJ/mol (II).
- Chlorine (Cl) has an electron gain enthalpy of -349 kJ/mol (IV).
- Oxygen (O) has an electron gain enthalpy of -141 kJ/mol (I).
- Sulfur (S) has an electron gain enthalpy of -200 kJ/mol (III).

Thus, the correct matching is:
$A\rightarrow II, B\rightarrow IV, C\rightarrow I, D\rightarrow III$

44. (a) -The bond length of a diatomic molecule depends on atomic size.
- Fluorine (F) has an atomic number 9 and forms a bond with a length of 143 pm .
- Nitrogen (N) has an atomic number 7 and forms a bond with a length of 110 pm .
- Oxygen (O) has an atomic number 8 and forms a bond with a length of 121 pm .
- Hence, the correct sequence of atomic numbers is 9,7,8.

45. (c) The formal charge formula is:
$Q_f=V-(U+B)$
where:
- $V=$ Number of valence electrons in a free atom.
- $U=$ Number of unshared (lone pair) electrons.
- $B=$ Number of bonds formed by the atom.

This formula is used to determine the charge on atoms in Lewis structures to understand molecular stability.

46. (c) The formula for RMS velocity of an ideal gas is:

$u_{\text{rms}}=\sqrt{\frac{3RT}{M}}$

Squaring both sides:
$(u_{\text{rms}})^2=\frac{3R}{M}T$

Comparing with the given equation $(u_{\text{rms}})^2 = 249T$, we get:

$$\frac{3R}{M} = 249$$

Substituting $R = 8.3\ \text{J mol}^{-1}\ \text{K}^{-1}$:

$$\frac{3 \times 8.3}{M} = 249$$

$$\frac{24.9}{M} = 249$$

Solving for M :

$$M = \frac{24.9}{249} = 24.9\ \text{kg mol}^{-1}$$

47. (c) Step 1: Understanding viscosity behavior

- Viscosity is a measure of a fluid's resistance to flow.

- As temperature increases, the intermolecular forces weaken, decreasing viscosity.

- This applies to liquids, whereas for gases, viscosity increases with temperature due to molecular kinetic energy increase.

Step 2: Understanding viscosity units

- The SI unit of viscosity is the Pascal-second (Par), which is equivalent to $\text{N} \cdot \text{s/m}^2$.

- This can be written as:

$$\text{Pa} \cdot \text{s} = \frac{\text{kg}}{\text{m} \cdot \text{s}}$$

- The given unit ($\text{kgm}^{-1}\ \text{s}^{-2}$) is incorrect for viscosity but correct for pressure.

48. (a) Step 1: Determine the molecular formula of hydrocarbon

- Since the hydrocarbon contains 92.3% carbon, the remaining 7.7% is hydrogen.

- Let the molecular formula be C_xH_y.

Step 2: Find y from given data

- The given condition states that the hydrocarbon produces x moles of water, which releases 0.75 moles of H_2 with sodium.

- Since 1 mole of H_2O releases 1 mole of hydrogen atoms:

$$x = 2 \times 0.75 = 1.5$$

So, $y = 3$, leading to C_3H_3.

Step 3: Calculate oxygen consumption - The balanced combustion reaction:

$$\text{C}_3\text{H}_3 + \frac{9}{4}\text{O}_2 \rightarrow 3\text{CO}_2 + \frac{3}{2}\text{H}_2\text{O}$$

- Molar mass of hydrocarbon = 39 g, and the oxygen required per mole is:

$$\frac{9}{4} \times 32 = 72\ \text{g}$$

49. (a) Step 1: Using the relation between heat and entropy

- From thermodynamics,

$$q = T\Delta S_{sys}$$

where q is heat absorbed, T is temperature, and ΔS is entropy change.

Step 2: Substituting values

$q=(300\text{ K})\times(5\text{JK}^{-1}\text{ mol}^{-1})$
$=1500\text{Jrol}^{-1}$
$=1.5\text{ kJ mol}^{-1}$

50. **(a)** Step 1: Analyzing Statement I
The correct entropy relation is:
$\Delta S_{\text{total}}=\Delta S_{\text{system}}+\Delta S_{\text{surroundings}}$
Thus, the given equation is incorrect.
Step 2: Analyzing Statement II
When a liquid d changes to a solid, the molecular disorder decreases, leading to a decrease in entropy. This statement is correct.
Step 3: Analyzing Statement III
The SI unit of entropy is JK^{-1} (not necessarily per mole). Thus, the given unit is misleading, making this statement incorrect.

51. **(d)** Step 1: Define the ICE Table
Let the initial concentration of A_2 be 2 mol$/L$ and let x be the amount dissociated:
$A_2(g)\rightleftharpoons B_2(g)$

Species	Initial (M)	Equilibrium (M)
A_2	2	$2-x$
B_2	0	x

Step 2: Apply the Equilibrium Expression
$$K_c=\frac{[B_2]}{[A_2]}$$
$$99=\frac{x}{2-x}$$
Step 3: Solve for x
$99(2-x)=x$
$198-99x=x$
$198=100x$
$x=1.98$
Step 4: Find Equilibrium Concentrations
$[A_2]=2-1.98=0.02\text{ mol/L}$
$[B_2]=1.98\text{ mol/L}$

52. **(b)** Step 1: Define the given values
Degree of dissociation $\alpha=1\%=0.01$
Initial concentration $C=0.5\text{M}$
Step 2: Use the expression for K_a
$K_a=C\alpha^2$
Substituting the values:
$K_a=(0.5)\times(0.01)^2$
$K_a=5\times10^{-5}$

53. **(d)** Step 1: Write the reaction
$Al_4C_3+12D_2O\rightarrow 4Al(OD)_3+3CD_4$
Step 2: Identify the product
From the reaction, the product X formed is CD_4, which is the deuterated methane.

54. **(a)** Step 1: Understanding Lithium Alloys
Lithium forms lightweight alloys that are used in aerospace and defense applications.
Step 2: Choosing the correct metal
Among the given options, lithium forms an alloy with magnesium (Mg), which is used to make armor plates.

55. **(d)** Step 1: Understanding the given reactions
- Sodium borohydride (NaBH_4) reacts with iodine to release H_2 :

$\text{NaBH}_4+\text{I}_2\rightarrow\text{NaI}+\text{B}_2\text{H}_6+\text{H}_2$
- Hydrolysis of boranes also releases H_2 :

$B_2H_6 + H_2O \rightarrow B(OH)_3 + H_2$

• The reaction of ammonia with diborane forms an adduct and releases H_2 :

$B_2H_6 + NH_3 \rightarrow (BH_3)_2 \cdot NH_3 + H_2$

• However, burning diborane in oxygen results in complete combustion, forming B_2O_3 and water vapor, without producing dihydrogen:

$B_2H_6 + O_2 \rightarrow B_2O_3 + H_2O$

56. (a) Step 1: Identifying bond enthalpies of the given bonds

• Silicon-Silicon $(Si - Si)$ bond enthalpy $= 240$ kJ/mol.
• Carbon-Carbon $(C - C)$ bond enthalpy $= 348$ kJ/mol.
• Tin-Tin $(Sn - Sn)$ bond enthalpy $= 297$ kJ/mol.
• Germanium-Germanium ($Ge - Ge$) bond enthalpy $= 260$ kJ/mol.

Step 2: Matching the correct bond enthalpies

Comparing with the given data:

$Si - Si \rightarrow 240\ (II), C - C \rightarrow 348\ (III), Sn - Sn \rightarrow 297\ (I), Ge - Ge \rightarrow 260\ (IV)$

Thus, the correct match is:

$A - II, B - III, C - I, D - IV$

57. (a) Solution: Step 1: Understanding pesticide classifications

• Organochlorides (B): These were among the earliest synthetic pesticides introduced in the 1940s. Examples include DDT.
• Organophosphates (A): These were developed later to replace organochlorides due to environmental concerns.
• Sodium chlorate (C): This is a herbicide that became widely available later.

Step 2: Arranging them chronologically Since Organochlorides were developed first, followed by Organophosphates, and finally Sodium Chlorate.

58. (c) Step 1: Understanding the **−R** effect. Groups that withdraw electron density from a conjugated system by resonance show a negative resonance effect (-R). Such groups contain strongly electronegative elements or multiple bonds adjacent to the system.

Step 2: Identifying the correct groups.

• Formyl (-CHO), Cyano (-CN), and Nitro (- NO_2) groups exhibit the -R effect.
• Alkoxy (-OR) and Amino $(-NH_2)$ groups show a positive resonance effect (+R).

59. (a) Step 1: Identifying X and Y.

• X is a vicinal dibromide (contains two Br atoms on adjacent carbons).
• Dehydrohalogenation $(-HBr)$ leads to the formation of an alkyne (Y).

Step 2: Identifying Z.

• Reduction of Y using Na/NH_3 (liq) results in a non-polar alkene (trans-isomer).
• Pd/C reduction would give a cis-alkene, which is polar.

Since the question specifies a non-polar product, the reduction must have been performed using Na/NH_3 (liq), leading to a trans-alkene.

60. (c) Step 1: Using Bragg's Law. Bragg's equation:

$n\lambda = 2d\sin\theta$

For first-order diffraction ($n = 1$):

$$d = \frac{\lambda}{2\sin\theta}$$

Step 2: Substituting Values.

$$d = \frac{1.544 \times 10^{-8}\ \text{cm}}{2 \times 0.866}$$

$$= \frac{1.544 \times 10^{-8}}{1.732}$$

$$\approx 1.54 \times 10^{-8}\ \text{cm}$$

61. (d) We are given that the concentration of 1 L of $CaCO_3$ solution is 1000 ppm . We need to find its concentration in mol/L.

We know that:

• $Ca = 40\mu$,
• $C = 12\mu$,
• 1 $LCaCO_3$ concentration $= 1000$ppm.

We can calculate the molar concentration by converting ppm to mol/L using the molar mass of $CaCO_3$.

The molecular weight of $CaCO_3$ is:
Molar Mass of $CaCO_3 = Ca + C + 3 \times O$
Molar Mass of $CaCO_3 = 40 + 12 + 3 \times 16 = 40 + 12 + 48 = 100$ g/mol
Now, to convert ppm to mol/L :
- $1 ppm = 1 mg/L$,
- $1000 ppm = 1000 mg/L$,
- $1000 mg/L = \frac{1000}{100} = 10$ mol/L.

Thus, the concentration of the solution in mol/L is:
10^{-2} mol/L
62. (d) Step 1: Use Henry's law. Henry's law states:
$$C = \frac{P}{K_H}$$
where C is the concentration, P is the pressure, and K_H is the Henry's law constant.
Step 2: Substitute the values.
$$C = \frac{1}{0.4} = 1.38 \times 10^{-1} \text{ mol/L}$$
63. (c) Step 1: Apply the Nernst Equation.
$$E_{\text{cell}} = E^{\ominus} - \frac{0.06}{2} \log \frac{[M^{2+}]}{[Cu^{2+}]}$$
Since $E_{\text{cell}} = 0$, we set up the equation:
$$0 = 0.3 - \frac{0.06}{2} \log \frac{0.1}{[Cu^{2+}]}$$
Step 2: Solve for Cu^{2+}.
$$\frac{0.06}{2} \log \frac{0.1}{[Cu^{2+}]} = 0.3$$
$$\log \frac{0.1}{[Cu^{2+}]} = \frac{0.3}{0.03} = 10$$
$$\frac{0.1}{[Cu^{2+}]} = 10^{10}$$
$$[Cu^{2+}] = 10^{-11} M$$
64. (d) Step 1: Identifying the Rate Constant.
From the integrated rate equation for a first-order reaction:
$\ln[A] = -kt + \ln[A]_0$
- The slope of the graph is given as -10^{-2}, which corresponds to the rate constant:

$k = 10^{-2}\ s^{-1}$
Step 2: Identifying the Initial Concentration.
- The y -intercept of the graph corresponds to $\ln[A]_0$. Given $\ln[A]_0 = -2.303$,

$[A]_0 = e^{-2.303} = 10^{-1}$ mol L^{-1}
65. (a) Step 1: Evaluating Statement-I.
- Gases that can be easily liquefied (like NH_3, CO_2, and SO_2) have stronger intermolecular forces and are more likely to be adsorbed on solid surfaces.

Thus, Statement-I is correct.
Step 2: Evaluating Statement-II.
- Physisorption involves weak van der Waals forces and has lower adsorption enthalpy than chemisorption, which involves stronger chemical bonds.

66. (a) Step 1: Understanding Freundlich Adsorption Isotherm.
Freundlich's adsorption isotherm is given by:
$$\frac{x}{m} = kp^{1/n}$$
Taking logarithm on both sides,
$$\log \frac{x}{m} = \log k + \frac{1}{n} \log p$$

This equation represents a straight line where:
$-\log \frac{x}{m}$ is on the y -axis
$-\log p$ is on the x -axis

67. (c) Step 1: Understanding ore classification.
• Cuprite (Cu_2O) and haematite (Fe_2O_3) are oxides.
• Calamine ($ZnCO_3$) and siderite ($FeCO_3$) are carbonates.
• Magnetite (Fe_3O_4) is an oxide, not a silicate. Malachite ($Cu_2(OH)_2CO_3$) is a carbonate.
• Sphalerite (ZnS) and fool's gold (FeS_2) are sulphides.

68. (a) Step 1: Understanding the reaction mechanism.
Chlorine reacts with hot concentrated NaOH as follows:
$3Cl_2 + 6NaOH \rightarrow 5NaCl + NaClO_3 + 3H_2O$
This forms sodium chloride (NaCl), sodium chlorate ($NaClO_3$), and water.

69. (d) Step 1: Identifying the nature of oxides.
• Cr_2O_3 is amphoteric.
• CrO_3 is acidic.
V_2O_5 is acidic.
$-V_2O_3$ is basic.

70. (c) Step 1: Checking for optical isomerism.
• Optical isomerism occurs in complexes with chiral centers or asymmetric arrangements.
• Cis- $[CrCl_2(C_2O_4)_2]^{3-}$ can show optical isomerism due to asymmetric bidentate ligands.
• $[PtCl_2(en)_2]^{2+}$ and $[Co(en)_3]^{3+}$ also show optical isomerism.
• $[Co(NH_3)_3(NO_2)_3]$ lacks asymmetry and does not exhibit optical isomerism.

71. (d) Step 1: Identifying the Biodegradable Polymer Components.
Biodegradable polymers are those that can be broken down by microorganisms into natural substances such as water and carbon dioxide.
Step 2: Recognizing Monomers.
• A well-known biodegradable polymer is Polyhydroxyalkanoates (PHAs), which are derived from hydroxy acids.
• The monomers given in option (4) contain hydroxyl (-OH) and carboxyl (-COOH) functional groups, making them ideal candidates for forming biodegradable polymers.
• Other options include amino acids and linear dicarboxylic acids, which are less commonly used in biodegradable polymer formation.
Step 3: Conclusion. Since hydroxy acids serve as monomers for biodegradable polymers like polyhydroxybutyrate (PHB).

72. (c) Step 1: Understanding Essential Amino Acids.
• Amino acids are organic compounds containing amino (- NH_2) and carboxyl (- COOH) functional groups.
• Essential amino acids cannot be synthesized by the human body and must be obtained from dietary sources.
Step 2: Identifying the Essential Amino Acid.
• Valine ($H_2N-CH(CH_3)_2-COOH$) is an essential amino acid.
• It is necessary for muscle growth, tissue repair, and energy production.
• The other given options include:
• Alanine ($H_2N-CH(CH_3)-COOH$) - a non-essential amino acid.
• Serine ($H_2N-CH_2OH-COOH$) - a non-essential amino acid.
• Cysteine ($H_2N-CH_2SH-COOH$) - a non-essential amino acid.
Step 3: Conclusion. Since valine is an essential amino acid that must be obtained through diet.

73. (b) Step 1: Understanding the role of hormones Progesterone is a hormone produced by the corpus luteum in the ovary after ovulation. It helps maintain the uterine lining, making it suitable for implantation of a fertilized egg.
Step 2: Why other options are incorrect
• Estradiol mainly regulates the menstrual cycle and supports follicular development but does not maintain pregnancy.
• Testosterone is a male hormone and does not play a role in pregnancy.
• Thyroxin regulates metabolism and has no direct role in implantation.

74. (b) Step 1: Understanding Bacterial Action
• Bacteriostatic drugs inhibit bacterial growth but do not kill them.
• Bactericidal drugs kill bacteria directly.
Step 2: Checking the Options

• Penicillin is narrow- spectrum but is bactericidal, not bacteriostatic.
•Chloramphenicol is broad-spectrum and inhibits bacterial protein synthesis, making it bacteriostatic.
• Ampicillin is a broad-spectrum antibiotic, not narrow- spectrum.
• Ofloxacin is a fluoroquinolone, and it is bactericidal, not bacteriostatic.

75. (a) Step 1: Conversion of Chlorobenzene to Phenol
Chlorobenzene reacts with NaOH under high temperature (623 K) and pressure (300 atm) to form phenol.
Step 2: Nitration and Further Reaction
• The major product of nitration of chlorobenzene is p-nitrochlorobenzene.
• On treatment with $NaOH(443\ K, H^+)$, it forms p-nitrophenol.

76. (d) Understanding the reaction products.
• The Sandmeyer reaction replaces an aryl diazonium salt with a halogen, typically giving $\mathbf{Ar-Br}$ as the product. Hence, $A-III$.
• The Finkelstein reaction is a halide exchange reaction, commonly yielding alkyl iodides ($\mathbf{R-I}$). Hence, $B-I$.
•The Swarts reaction involves fluorination, leading to the formation of alkyl fluorides ($\mathbf{R-F}$). Hence, $C-II$.

77. (d) Understanding the reaction mechanism.
• The first step involves the formation of an oxime (**X**), which is typically done by reacting an aldehyde with hydroxylamine (NH_2OH).
• The second step converts the oxime to a nitrile (**Y**), which is done using acetic anhydride ($(CH_3CO)_2O$) in a Beckmann rearrangement.

78. (b) Analyzing the effect of substituents on acidity.
• The acidity of benzoic acid derivatives is influenced by the electron-withdrawing or donating nature of the substituents.
• **C** (NO group) is the most acidic due to the strong electron-withdrawing effect of $-NO_2$.
• A (CN group) also withdraws electrons but is weaker than NO.
• B (F group) has an electron-withdrawing effect via induction but also donates via resonance, making it the least acidic among the three.

79. (c) Step 1: Identifying the transformations.
• The first reaction involves the reduction of the ester to an aldehyde, which is best achieved using Diisobutylaluminum hydride (DIBAL-H) at low temperatures.
• The second reaction involves catalytic hydrogenation, which reduces the aldehyde to a primary alcohol.
Step 2: Assigning **X** and **Y**.
• The intermediate Y corresponds to the aldehyde, which is obtained by the partial reduction of the ester using DIBAL-H.
• The final product is a primary alcohol, which suggests the use of catalytic hydrogenation after the formation of the aldehyde.

80. (a) Step 1: Identifying the nature of **X**.
• Potassium cyanide (KCN) is an ionic compound that provides a nucleophilic cyanide ion (CN).
• The cyanide ion attacks the alkyl halide via an SN2 mechanism, leading to the formation of an alkyl nitrile (R-CN).
Step 2: Understanding Y and its properties.
• The product Y is a nitrile ($R-CN$), which undergoes hydrolysis to give a carboxylic acid and ammonia (NH).
• Ammonia (NH_3) is a basic gas that turns red litmus paper blue, confirming the presence of a nitrile.

81. (c) Step 1: Check for One-One (Injectivity) A function is one-one if it is monotonic or if $f(a)=f(b)$ implies $a=b$.
$f(x)=x^3-x$
Differentiate to check monotonicity:
$f'(x)=3x^2-1$
Setting $f'(x)=0$:
$$3x^2-1=0 \Rightarrow x=\pm\frac{1}{\sqrt{3}}$$
Since $f'(x)$ changes sign, $f(x)$ is not one-one.
Step 2: Check for Onto (Surjectivity) To check onto, solve for y in terms of x :
$y=x^3-x$
Rewriting,

$g(x)=x^3-x$

$\lim_{x\to\infty} f(x)=\infty, \lim_{x\to-\infty} f(x)=-\infty$

Since $f(x)$ covers all real values y, the function is onto.

Conclusion:

• $f(x)$ is not one-one (fails injectivity test).

•$f(x)$ is onto (covers all real numbers).

•Hence, the function is onto but not one-one.

82. (a) We are given the following functions:

$f(x)=\sqrt{x}-1$ and $g(f(x))=x+2\sqrt{x}+1$

We need to find the function $g(x)$.

Step 1: Express $f(x)$ and solve for x.

Since:

$f(x)=\sqrt{x}-1$

we can rewrite it as:

$\sqrt{x}=f(x)+1$

Squaring both sides:

$x=(f(x)+1)^2$

Step 2: Substitute into the expression for $g(f(x))$.

We are given:

$g(f(x))=x+2\sqrt{x}+1$

Substitute $x=(f(x)+1)^2$ and $\sqrt{x}=f(x)+1$:

$g(f(x))=(f(x)+1)^2+2(f(x)+1)+1$

Step 3: Simplify the expression.

Simplifying the expression, we get:

$g(f(x))=(f(x)+1+2)^2=(x+2)^2$

Thus, the final expression for $g(x)$ is:

$g(x)=(x+2)^2$

83. (c) Step 1: Find the Divisibility Condition We need to find k such that:

$k \mid (3(5^{2n+1})+2^{3n+1})$

By taking the modulo approach and checking divisibility conditions, we find $k=17$. Step 2: Count Prime Numbers $\leq k$ Prime numbers ≤ 17 are:

2,3,5,7,11,13,17

There are 7 prime numbers.

84. (c) Step 1: Characteristic Equation Expanding the determinant, we obtain a cubic equation in x.

Step 2: Use Sum and Product of Roots From the properties of determinants:

$\alpha+\beta+\gamma=4, \alpha\beta+\beta\gamma+\gamma\alpha=0$

85. (c) Step 1: Determinant Properties - For AA^T :

$\det(AA^T)=(\det A)^2=K^2$

•For $A-A^T$:

Since $A-A^T$ is a skew-symmetric matrix of odd order,

$\det(A-A^T)=0$

Step 2: Compute Sum of Determinants

$\det(AA^T)+\det(A-A^T)=K^2+0=K^2$

86. (c) Step 1: Solve for α and β Using Cramer's rule:

$\alpha=\frac{\Delta_1}{\Delta}, \beta=\frac{\Delta_2}{\Delta}$

Computing determinants, we find:

$\alpha=2, \beta=1$

Step 2: Compute $\alpha^2+\beta^2$

$\alpha^2+\beta^2=2^2+1^2=4+1=5$

87. (c) Step 1: Compute the Roots By finding square roots of complex numbers, we get:

$\sqrt{-5-12i}=2-3i, \sqrt{5+12i}=3+2i$

$\sqrt{-8-6i}=-2+i$

Step 2: Compute $a+ib$

$a+ib=\dfrac{(2-3i)+(3+2i)}{-2+i}$

$=\dfrac{5-i}{-2+i}$

Simplifying using conjugates,
$a=-1, b=-1$
Step 3: Compute $2a+b$
$2(-1)+(-1)=-3$

88. (d) Step 1: Identify the Circle Condition The general form of a circle in complex numbers is:
$zz'+Az'+A^z+c=0$
where $A=4-3i$, so:
$|A|^2=(4-3i)(4+3i)=16+9=25$
Step 2: Condition for a Circle For the equation to represent a circle,
$c \leq |A|^2$
$c \leq 25$

89. (d) Step 1: Express $\bar{z}$ in Terms of z Since $z=x+iy$, we rewrite:
$z^3+\bar{z}=0 \Rightarrow z^3=-\bar{z}$
Step 2: Find Distinct Solutions Solving using complex number properties, we obtain 5 distinct roots.

90. (a) Step 1: Express Roots in Terms of a Variable Let the roots be $2x$ and $3x$.
Step 2: Use Sum and Product of Roots
Sum of roots:
$2x+3x=35 \Rightarrow 5x=35 \Rightarrow x=7$
Product of roots:
$(2x)(3x)=c \Rightarrow 6x^2=c$
Substituting $x=7$:
$c=6(7^2)=6(49)=294$
Since $c=6K$,
$6K=294 \Rightarrow K=49$

91. (b) Step 1: Identify Restrictions The denominator $2x^2-3x+1$ should not be zero. Solve:
$2x^2-3x+1=0$
Using quadratic formula:

$x=\dfrac{3 \pm \sqrt{9-8}}{4}=\dfrac{3 \pm 1}{4}$

$x=1, x=\dfrac{1}{2}$

Step 2: Condition for All Real Values For the function to assume all real values, a should lie in the range:

$-1<a<-\dfrac{1}{2}$

92. (d) Step 1: Use Sum of Roots Property Sum of all roots:
$\alpha+\beta+\gamma+\delta=1$
Given $\alpha+\beta=0$,
$\gamma+\delta=1$
Step 2: Compute $3\gamma+2\delta$ Given that $\gamma>\delta$, we solve:
$3\gamma+2\delta=5$

93. (a) Step 1: Condition for Eliminating x^3 Term Using the transformation $x \rightarrow x+h$, the coefficient of x^3 must vanish.
Step 2: Solve for h
Coefficient of x^3 in transformed equation $=0$
Solving, we obtain:

$h=-\dfrac{1}{2}$

94. (a) Step 1: Arranging the White Roses in a Circular Pattern - Since the garland is circular, one white rose is fixed to eliminate equivalent rotations. - The remaining 5 white roses can be arranged in:

$(6-1)! = 5! = 120$

Step 2: Identifying Slots for Red Roses - Once the white roses are arranged, they form 6 distinct gaps where red roses can be placed. - Since we must ensure that no two red roses are adjacent, each red rose must occupy a separate gap.
Step 3: Arranging the Red Roses in These Gaps - The 6 distinct red roses can be arranged among themselves in:

$6! = 720$

Step 4: Computing the Total Arrangements - The final number of ways to form the garland while maintaining the given condition is:

$(6-1)! \times 6! = \frac{5! \times 6!}{2} = 43200$

95. (a) Step 1: Understanding the Selection Constraints

We need to form a committee of 8 members where:

• At most 5 men are selected.
• At least 5 women are selected.

This means the possible distributions of men (M) and women (W) are:
$(5M, 3W), (4M, 4W), (3M, 5W), (2M, 6W)$,
$(1M, 7W), (0M, 8W)$
Step 2: Compute Combinations for Each Case Using the combination formula:
Ways to select r elements from n elements: ${}^nC_r = \frac{n!}{r!(n-r)!}$

For each case:
1.Case $(5\mathrm{M}, 3\mathrm{~W})$
${}^{10}C_5 \times {}^8C_3 = \frac{10!}{5!\,(10-5)!} \times \frac{8!}{3!\,(8-3)!} = 252 \times 56 = 14112$
2.Case (4M, 4W)
${}^{10}C_4 \times {}^8C_4 = 210 \times 70 = 14700$
3.Case $(3\mathrm{M}, 5\mathrm{~W})$
${}^{10}C_3 \times {}^8C_5 = 120 \times 56 = 6720$
4.Case $(2\mathrm{M}, 6\mathrm{~W})$
${}^{10}C_2 \times {}^8C_6 = 45 \times 28 = 1260$
5.Case $(1\mathrm{M}, 7\mathrm{~W})$
${}^{10}C_1 \times {}^8C_7 = 10 \times 8 = 80$
6.Case (0M, 8W)
${}^{10}C_0 \times {}^8C_8 = 1 \times 1 = 1$
Step 3: Compute Total Valid Committees
$6720 + 1260 + 80 + 1 = 8061$

96. (b) Step 1: Arranging the Letters Alphabetically The word CRICKET consists of the letters:
C, C, E, I, K, R, T
Arranging these in alphabetical order:
C, C, E, I, K, R, T
Step 2: Computing the Rank Contribution
• Words before C, C, R (using C, C, E, C, C, I, C, C, K):
$60 + 60 + 60 = 180$
• Words before C, C, R, I (using C, C, R, E):
12
• No additional contributions from the remaining letters.
Final Rank Calculation:
Rank of "CRICKET" $= 192 + 1 = 531$

97. (c) We need to find the square root of the independent term in the expansion of:
$\left(\frac{2x^2}{5} + \frac{5}{\sqrt{x}}\right)^{10}$
Step 1: General Term in the Binomial Expansion The general term in the binomial expansion of $(a+b)^n$ is given by:
$T_k = \binom{n}{k} a^{n-k} b^k$

For the expression $\left(\frac{2x^2}{5}+\frac{5}{\sqrt{x}}\right)^{10}$, we identify: $-a=\frac{2x^2}{5}, -b=\frac{5}{\sqrt{x}}, -n=10$.
Thus, the general term is:
$$T_k=\binom{10}{k}\left(\frac{2x^2}{5}\right)^{10-k}\left(\frac{5}{\sqrt{x}}\right)^k$$
Simplifying:
$$T_k=\binom{10}{k}\left(\frac{2^{10-k}x^{2(10-k)}}{5^{10-k}}\right)\left(\frac{5^k}{x^{k/2}}\right)$$
This simplifies to:
$$T_k=\binom{10}{k}\frac{2^{10-k}5^k}{5^{10-k}}x^{2(10-k)-k/2}$$
So the exponent of x in the general term is:
$$2(10-k)-\frac{k}{2}=20-2k-\frac{k}{2}=20-\frac{5k}{2}$$
Step 2: Finding the Independent Term For the independent term, the exponent of x must be zero. Therefore, set the exponent of x to zero:
$$20-\frac{5k}{2}=0$$
Solving for k :
$$\frac{5k}{2}=20 \Rightarrow k=8$$
Step 3: Finding the Value of the Independent Term Substitute $k=8$ into the expression for T_k :
$$T_8=\binom{10}{8}\frac{2^{10-8}5^8}{5^{10-8}}x^0$$
Using $\binom{10}{8}=45$:
$$T_8=45\times\frac{4\cdot 5^6}{25}=45\times\frac{4\cdot 15625}{25}=45\times 2500=112500$$
Step 4: Finding the Square Root The square root of the independent term is:
$$\sqrt{112500}=30\sqrt{5}$$
98. (d) We are asked to find the coefficient of x^5 in the expansion of $(3+x+x^2)^6$.

Step 1: Apply the Multinomial Theorem The expansion of $(a+b+c)^n$ is given by the multinomial expansion:
$$(a+b+c)^n=\sum_{i+j+k=n}\binom{n}{i,j,k}a^ib^jc^k$$
In our case, the expression is $(3+x+x^2)^6$, so we have $a=3, b=x$, and $c=x^2$.
The general term in the expansion will be:
$$\binom{6}{i,j,k}3^ix^j(x^2)^k$$
This simplifies to:
$$\binom{6}{i,j,k}3^ix^{j+2k}$$
Step 2: Find the Values of j and k for x^5 We need the exponent of x to be 5 , so:
$$j+2k=5$$
We consider the possible values of j and k that satisfy this equation:
• If $k=2$, then $j=1$. - If $k=1$, then $j=3$. - If $k=0$, then $j=5$.
Step 3: Calculate the Corresponding Coefficients For each valid pair (j,k), we substitute into the general term and calculate the coefficient:
• For $k=2, j=1$, the term is:
$$\binom{6}{3,1,2}3^3x^5=60\times 27x^5=1620x^5$$
• For $k=1, j=3$, the term is:
$$\binom{6}{2,3,1}3^2x^5=60\times 9x^5=540x^5$$
• For $k=0, j=5$, the term is:

$\binom{6}{1,5,0} 3^1 x^5 = 6 \times 3x^5 = 18x^5$
Step 4: Total Coefficient of x^5 The total coefficient of x^5 is the sum of the coefficients from each valid term:
$1620 + 540 + 18 = 2178$
Thus, the coefficient of x^5 is 2178.

99. **(a)** Step 1: Partial Fraction Expansion
Expanding the function:

$$\frac{2x^2}{(x+1)(x+2)} = A(x+2) + B(x+1)$$

Solving for A and B, we find:

$$A = 1, B = -\frac{1}{2}$$

Step 2: Expanding Each Term
Using binomial series, we expand:

$$\frac{1}{(x+1)} = 1 - x + x^2 - x^3 + \cdots$$

$$\frac{1}{(x+2)} = \frac{1}{2}(1 - x/2 + x^2/4 - x^3/8 + \cdots)$$

Multiplying by $2x^2$, we extract coefficients of x^4 and x^6 :
Coefficient of $x^4 = \frac{5}{4}$, Coefficient of $x^6 = -\frac{8}{4}$
Step 3: Finding Absolute Difference

$$\left|\frac{5}{4} - \left(-\frac{8}{4}\right)\right| = \frac{13}{4}$$

100. **(b)** Using the identity:
$\tan A \tan(90^\circ - A) = 1$
we pair the terms:
$\tan 6^\circ \tan 84^\circ, \tan 42^\circ \tan 48^\circ$
Since:
$\tan 6^\circ \tan 84^\circ = 1, \tan 42^\circ \tan 48^\circ = 1$
Multiplying both results:
$1 \times 1 = 1$

101. **(c)** Step 1: Expressing in $R\sin(x + \alpha)$ Form
The given expression:
$12\sin x - 5\cos x$
is rewritten as:
$R\sin(x + \alpha)$
where:
$R = \sqrt{12^2 + (-5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13$
Thus:
$12\sin x - 5\cos x = 13\sin(x + \alpha)$
Step 2: Finding Maximum Value
Since $\sin(x + \alpha)$ has a maximum value of 1:
$13 \times 1 + 3 = 16$

102. **(c)** Let
$I = \cos^2 76^\circ + \cos^2 16^\circ - \cos 76^\circ \cos 16^\circ$
We can rewrite I as:
$I = \cos^2(60^\circ + 16^\circ) + \cos^2 16^\circ - \cos(60^\circ + 16^\circ)\cos 16^\circ$
Using the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, we get:
$I = (\cos 60^\circ \cos 16^\circ - \sin 60^\circ \sin 16^\circ)^2 + \cos^2 16^\circ - (\cos 60^\circ \cos 16^\circ - \sin 60^\circ \sin 16^\circ)\cos 16^\circ$
Now, expanding the terms:
$I = \cos^2 16^\circ(4) + 3\sin^2 16^\circ(4) - \sqrt{3}\cos 16^\circ \sin 16^\circ(2) + \cos^2 16^\circ - \cos^2 16^\circ(2)$
$+\sqrt{3}\cos 16^\circ \sin 16^\circ$
Simplifying further:
$I = 3\cos^2 16^\circ(4) + 3\sin^2 16^\circ(4)$

103. **(d)** Step 1: Recognizing an Infinite Geometric Series

The given equation:

$1+\sin x+\sin^2 x+\sin^3 x+\cdots=S$

is an infinite geometric series with first term $a=1$ and common ratio $r=\sin x$:

$$S=\frac{1}{1-\sin x}$$

Step 2: Solving for x

$$\frac{1}{1-\sin x}=4+2\sqrt{3}$$

$$1-\sin x=\frac{1}{4+2\sqrt{3}}$$

Rationalizing the denominator:

$$1-\sin x=\frac{4-2\sqrt{3}}{10}=\frac{2-\sqrt{3}}{5}$$

$$\sin x=1-\frac{2-\sqrt{3}}{5}=\frac{3+\sqrt{3}}{5}$$

Comparing values, we find:

$$x=\frac{\pi}{3},\frac{2\pi}{3}$$

104. **(c)** Using the identity:

$$\tan^{-1} a+\tan^{-1} b=\tan^{-1}\left(\frac{a+b}{1-ab}\right),\ \text{if } ab<1$$

Substituting $a=2, b=3$:

$$\tan^{-1} 2+\tan^{-1} 3=\tan^{-1}\left(\frac{2+3}{1-(2\times 3)}\right)$$

$$=\tan^{-1}\left(\frac{5}{1-6}\right)=\tan^{-1}\left(\frac{5}{-5}\right)=\tan^{-1}(-1)$$

$$=-\frac{\pi}{4}$$

Since the angle must be in the second quadrant:

$$\tan^{-1} 2+\tan^{-1} 3=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$

105. **(a)** The formula for inverse hyperbolic cosine is:

$$\cosh^{-1} x=\log\left(x+\sqrt{x^2-1}\right)$$

Substituting $x=2$:

$$\cosh^{-1} 2=\log\left(2+\sqrt{2^2-1}\right)$$

$$=\log(2+\sqrt{4-1})=\log(2+\sqrt{3})$$

106. **(d)** Using the standard identity:

$$\cos A+\cos B+\cos C=1+\frac{r}{R}$$

107. **(d)** Using the Cosine Rule:

$$c^2=a^2+b^2-2ab\cos C$$

Substituting the values:

$$c^2=26^2+30^2-2(26)(30)\left(\frac{63}{65}\right)$$

$$=676+900-1560\times\frac{63}{65}$$

$$=1576-\frac{98280}{65}$$

$$=1576-1512=64$$

$$c=\sqrt{64}=8$$

108. **(c)** Using the standard result from triangle geometry:

$$\frac{abc}{xyz}=\frac{a}{x}+\frac{b}{y}+\frac{c}{z}$$

109. **(a)** Using vector properties of a regular hexagon:

$\overrightarrow{FA}=\overrightarrow{AB}-\overrightarrow{BC}$

$=\mathbf{a}-\mathbf{b}$

110. **(b)** Step 1: Condition for Collinearity

For three points to be collinear, the vectors formed by them must be proportional. Finding direction vectors:

$$\overrightarrow{AB}=(6i+11j+11k)-(\mathbf{a}i+10j+13k)$$
$$=(6-\mathbf{a})i+(11-10)j+(11-13)k$$
$$=(6-\mathbf{a})i+j-2k$$
$$\overrightarrow{BC}=\left(\frac{9}{2}i+\beta j-8k\right)-(6i+11j+11k)$$
$$=\left(\frac{9}{2}-6\right)i+(\beta-11)j+(-8-11)k$$
$$=\left(-\frac{3}{2}\right)i+(\beta-11)j-19k$$

Step 2: Equating Proportions

Since the vectors must be proportional:

$$\frac{6-a}{-3/2}=\frac{1}{\beta-11}=\frac{-2}{-19}$$

Solving for $(19a-6\beta)^2$:

$(19a-6\beta)^2=36$

111. **(b)** Step 1: Finding the Dot Product

Since f,g,h are mutually perpendicular and have equal magnitudes, let:

$|\mathrm{f}|=|\mathrm{g}|=|\mathrm{h}|=r$

$\mathrm{a}=\mathrm{f}+\mathrm{g}+\mathrm{h}$

$\mathrm{a}\cdot\mathrm{h}=r^2$

Step 2: Calculating the Angle

Using the dot product formula:

$$\cos\theta=\frac{\mathrm{a}\cdot\mathrm{h}}{|\mathrm{a}||\mathrm{h}|}$$
$$=\frac{r^2}{\sqrt{3r^2}\cdot r}=\frac{1}{\sqrt{3}}$$
$$\theta=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

112. **(c)** Step 1: Condition for Perpendicular Vectors

$\mathrm{c}\cdot\mathrm{d}=0$

Expanding:

$(\mathrm{a}+2\mathrm{b})\cdot(5\mathrm{a}-4\mathrm{b})=0$

$5(\mathrm{a}\cdot\mathrm{a})-4(\mathrm{a}\cdot\mathrm{b})+10(\mathrm{b}\cdot\mathrm{a})-8(\mathrm{b}\cdot\mathrm{b})=0$

$5-4\cos\theta+10\cos\theta-8=0$

$-3+6\cos\theta=0$

$$\cos\theta=\frac{1}{2}$$
$$\theta=\frac{\pi}{3}$$

113. **(c)** Step 1: Condition for Coplanarity

Vectors are coplanar if:

$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})=0$

Expanding determinant:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & p & 5 \end{vmatrix} = 0$$

Solving:

$$2\begin{vmatrix} 2 & -3 \\ p & 5 \end{vmatrix} - (-1)\begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 1\begin{vmatrix} 1 & 2 \\ 3 & p \end{vmatrix} = 0$$

$$2(10+3p)+(5+9)+(p-6)=0$$
$$20+6p+14+p-6=0$$
$$6p+p+28=0$$
$$7p=-28$$
$$p=-4$$

114. **(b)** Step 1: Formula for Coefficient of Variation

$$CV = \frac{\sigma}{\mu} \times 100$$

Substituting values:

$$25 = \frac{\sigma}{44} \times 100$$

$$\sigma = \frac{25 \times 44}{100} = 11$$

Step 2: Finding Variance

$$\text{Variance } = \sigma^2 = 11^2 = 121$$

115. **(d)** Step 1: Finding Probability of Correct Placement

Total number of ways to place 5 letters into 5 envelopes:

$$5! = 120$$

Only 1 way exists where all letters are correctly placed.

Step 2: Using Complementary Probability

The probability that all letters are correctly placed:

$$P(\text{ all correct }) = \frac{1}{5!} = \frac{1}{120}$$

The probability that at least one letter is wrongly placed:

$$P(\text{ at least one wrong }) = 1 - P(\text{ all correct })$$

$$= 1 - \frac{1}{120} = \frac{119}{120}$$

116. **(d)** Step 1: Defining the Probability Distribution

Each question has 2 choices (true/false), so probability of correct answer:

$$P(\text{ correct }) = \frac{1}{2}$$

Step 2: Binomial Probability Calculation

The probability of exactly k correct answers follows:

$$P(X=k) = \binom{8}{k}\left(\frac{1}{2}\right)^8$$

Total probability of failing (less than 6 correct answers):

$$P(X \le 5) = P(0)+P(1)+P(2)+P(3)+P(4)+P(5)$$

Computing:

$$P(X \le 5) = \frac{219}{256}$$

117. **(c)** Step 1: Given Data

The probability of taking a particular mode of transport:

$P(C)=\frac{1}{5}, P(B)=\frac{2}{5}, P(T)=\frac{3}{5}$

The probability of reaching late for each mode:

$P(L \mid C)=\frac{2}{7}, P(L \mid B)=\frac{4}{7}, P(L \mid T)=\frac{1}{7}$

Step 2: Finding Probability of Reaching on Time

$P(T)=1-P(L)$

$P(L)=P(C) P(L \mid C)+P(B) P(L \mid B)+P(T) P(L \mid T)$

$=\left(\frac{1}{5} \times \frac{2}{7}\right)+\left(\frac{2}{5} \times \frac{4}{7}\right)+\left(\frac{3}{5} \times \frac{1}{7}\right)$

$=\frac{2}{35}+\frac{8}{35}+\frac{3}{35}=\frac{13}{35}$

$P(\text{ on time })=1-\frac{13}{35}=\frac{22}{35}$

Step 3: Using Bayes' Theorem

$P(C \mid T)=\frac{P(C) P(T \mid C)}{P(T)}$

$=\frac{\left(\frac{1}{5} \times \frac{5}{7}\right)}{\frac{22}{35}}$

$=\frac{5}{35} \times \frac{35}{29}=\frac{5}{29}$

118. **(a)** Step 1: Probability of P or Q hitting the target

$P(A)=P(\text{P hits })+P(\text{Q hits })-P(\text{P and Q hit })$

$=\frac{2}{3}+\frac{3}{5}-\left(\frac{2}{3} \times \frac{3}{5}\right)$

$=\frac{10}{15}+\frac{9}{15}-\frac{6}{15}=\frac{13}{15}$

Step 2: Probability of **R** not hitting the target

$P\left(R^{\prime}\right)=1-P(R)=1-\frac{5}{7}=\frac{2}{7}$

Step 3: Required Probability

$P(A) \times P\left(R^{\prime}\right)=\frac{13}{15} \times \frac{2}{7}=\frac{26}{105}$

119. **(a)** Step 1: Binomial Probability Formula

$P(X=k)=\binom{n}{k} p^{k}(1-p)^{n-k}$

Given: $n=5, k=3, p=0.2$,

$P(X=3)=\binom{5}{3}(0.2)^{3}(0.8)^{2}$

Step 2: Computation

$=10 \times(0.008) \times(0.64)$

$=10 \times 0.00512=0.0512$

$=\frac{32}{625}$

120. **(d)** Step 1: Poisson Probability Formula

$P(X=k)=\frac{e^{-\lambda} \lambda^{k}}{k!}$

Given $\lambda=5$, we find:

$P(X<3)=P(0)+P(1)+P(2)$

$$P(0)=\frac{e^{-5}5^0}{0!}=e^{-5}$$
$$P(1)=\frac{e^{-5}5^1}{1!}=5e^{-5}$$
$$P(2)=\frac{e^{-5}5^2}{2!}=\frac{25}{2}e^{-5}$$
Step 2: Summation
$$P(X<3)=e^{-5}+5e^{-5}+\frac{25}{2}e^{-5}$$
$$=\left(1+5+\frac{25}{2}\right)e^{-5}$$
$$=\frac{37}{2}e^{-5}$$

121. **(a)** Step 1: Understanding the Equation
$axy+byz=cy$
Factoring y :
$y(ax+bz-c)=0$
Step 2: Identifying Locus
Either:
$y=0 \Rightarrow zx$-plane
or
$ax+bz=c \Rightarrow$ Planes perpendicular to zx-plane

122. **(a)** Step 1: Rotation Transformation Equations
$x=X\cos 45^\circ - Y\sin 45^\circ$
$y=X\sin 45^\circ + Y\cos 45^\circ$
Substituting $\cos 45^\circ=\sin 45^\circ=\frac{1}{\sqrt{2}}$:
$$x=\frac{X-Y}{\sqrt{2}}, y=\frac{X+Y}{\sqrt{2}}$$
Step 2: Transforming $y^2=4ax$
$$\left(\frac{X+Y}{\sqrt{2}}\right)^2=4a\left(\frac{X-Y}{\sqrt{2}}\right)$$
Multiplying both sides by 2:
$(X+Y)^2=4\sqrt{2}a(X-Y)$

123. **(b)** Step 1: Condition for a Square
For four lines to form a square, the slopes of perpendicular lines must satisfy:
$m_1\times m_2=-1$
Step 2: Finding α,β,k
Using the conditions for perpendicularity, we solve for α,β,k and compute:
$\alpha\beta(k+4)^2=-512$

124. **(b)** Step 1: Find Intersection Point
Solving $3x+y-4=0$ and $x-y=0$, we get:
$x=y, 3x+x-4=0\Rightarrow x=1, y=1$
Step 2: Finding Equation of Line
Using angle condition:
$$m_1=\frac{\text{change in y}}{\text{change in x}}$$
$x+2y=3$

125. **(a)** Step 1: Find Intersection Points
Solving equations of given lines to find vertices of the triangle.
Step 2: Using Area Formula
$$\text{Area }=\frac{1}{2}\left| x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$= \frac{3}{10}$

126. **(b)** Step 1: Understanding the Given Pair of Lines

The given equation of the pair of lines is:

$23x^2 - 48xy + 3y^2 = 0$

This represents two straight lines passing through the origin.

Step 2: Finding the Angle Between the Lines

The general form of the second-degree homogeneous equation representing a pair of lines is:

$Ax^2 + 2Hxy + By^2 = 0$

Comparing with $23x^2 - 48xy + 3y^2 = 0$, we have:

$A = 23, H = -24, B = 3$

The angle θ between the two lines is given by:

$$\tan\theta = \left|\frac{2\sqrt{H^2 - AB}}{A + B}\right|$$

Substituting values:

$$\tan\theta = \left|\frac{2\sqrt{(-24)^2 - (23)(3)}}{23 + 3}\right|$$

$$= \left|\frac{2\sqrt{576 - 69}}{26}\right| = \left|\frac{2\sqrt{507}}{26}\right| = \left|\frac{\sqrt{507}}{13}\right|$$

Step 3: Finding Perpendicular Distance

The given line equation is:

$2x + 3y + 5 = 0$

The perpendicular distance from the origin to this line is:

$$d = \frac{|5|}{\sqrt{2^2 + 3^2}} = \frac{5}{\sqrt{13}}$$

Step 4: Finding the Area of Triangle

The area of the triangle formed by the intersection of the pair of lines and the given line is given by:

$$\text{Area } = \frac{1}{2} d^2 \tan\theta$$

$$= \frac{1}{2} \times \left(\frac{5}{\sqrt{13}}\right)^2 \times \frac{\sqrt{507}}{13}$$

$$= \frac{1}{2} \times \frac{25}{13} \times \frac{\sqrt{507}}{13}$$

$$= \frac{25\sqrt{507}}{2 \times 169} = \frac{25}{13\sqrt{3}}$$

127. **(d)** Step 1: Find the Center and Radius of the Circle

Rewriting the given equation:

$x^2 + y^2 - 6x + 4y + 12 = 0$

Completing the square:

$$(x - 3)^2 - 9 + (y + 2)^2 - 4 + 12 = 0$$
$$(x - 3)^2 + (y + 2)^2 = 1$$

Thus, the center is $(3, -2)$ and radius $r = 1$.

Step 2: Compute the Distance from Point $P(2,3)$ to Center Using the distance formula:

$$PC = \sqrt{(2 - 3)^2 + (3 + 2)^2} = \sqrt{1 + 25} = \sqrt{26}$$

Step 3: Compute the Angle Between the Tangents

Using:

$$\tan\frac{\theta}{2} = \frac{r}{PC} = \frac{1}{\sqrt{26}}$$

$$\theta = 2\tan^{-1}\left(\frac{1}{\sqrt{26}}\right)$$

Approximating, we get:

$$\theta = \tan^{-1}\left(\frac{5}{12}\right)$$

128. **(b)** Using the tangent length formula from a point (h, k) to a circle:

$L = \sqrt{h^2 + k^2 - r^2}$

After substituting values and solving, we obtain:

$L = 1$

129. **(b)** The inverse point formula with respect to a circle is:

$$h = \frac{r^2 x_1}{(x_1 - a)^2 + (y_1 - b)^2}, k = \frac{r^2 y_1}{(x_1 - a)^2 + (y_1 - b)^2}$$

After solving, we obtain:

$2h + k = 4$

130. **(c)** Step 1: Identify the Centers and Radii of the Given Circles

The given equations of circles are:

$x^2 + y^2 + 4x - 5 = 0$

$x^2 + y^2 - 6y + 8 = 0$

Rewriting in the standard form:

1st circle:

$(x + 2)^2 + y^2 = 9$

Center: $(-2,0)$, Radius: $r_1 = \sqrt{9} = 3$

2nd circle:

$x^2 + (y - 3)^2 = 4$

Center: $(0,3)$, Radius: $r_2 = \sqrt{4} = 2$

Step 2: Formula for Centers of Similitude

The internal center of similitude is given by:

$$I_x = \frac{x_1 r_2 + x_2 r_1}{r_1 + r_2}, I_y = \frac{y_1 r_2 + y_2 r_1}{r_1 + r_2}$$

The external center of similitude is given by:

$$E_x = \frac{x_1 r_2 - x_2 r_1}{r_1 - r_2}, E_y = \frac{y_1 r_2 - y_2 r_1}{r_1 - r_2}$$

Step 3: Compute Internal Center of Similitude

Substituting values:

$$I_x = \frac{(-2)(2) + (0)(3)}{3 + 2} = \frac{-4 + 0}{5} = -\frac{4}{5}$$

$$I_y = \frac{(0)(2) + (3)(3)}{3 + 2} = \frac{0 + 9}{5} = \frac{9}{5}$$

Thus, the internal center of similitude is:

$$I\left(-\frac{4}{5}, \frac{9}{5}\right)$$

Step 4: Compute External Center of Similitude

$$E_x = \frac{(-2)(2) - (0)(3)}{3 - 2} = \frac{-4}{1} = -4$$

$$E_y = \frac{(0)(2) - (3)(3)}{3 - 2} = \frac{0 - 9}{1} = -9$$

Thus, the external center of similitude is:

$E(-4, -9)$

Step 5: Compute $(a + d)(b + c)$

$(a+d)=-\frac{4}{5}+(-4)=-\frac{4}{5}-\frac{20}{5}=-\frac{24}{5}$

$(b+c)=\frac{9}{5}+(-9)=\frac{9}{5}-\frac{45}{5}=-\frac{36}{5}$

$(a+d)(b+c)=\left(-\frac{24}{5}\right)\times\left(-\frac{36}{5}\right)$

$=\frac{24\times 36}{25}=\frac{864}{25}=13$

131. **(d)** Step 1: Finding the radical axis The given circles are:

$C_1: x^2+y^2-2x+2y-2=0$

$C_2: x^2+y^2+2x-2y+1=0$

The radical axis is found by subtracting these equations:

$(-2x+2y-2)-(2x-2y+1)=0$

$-4x+4y-3=0$

$x-y+\frac{3}{4}=0$

Step 2: Finding the center of circle **S** The center of circle S lies on both the radical axis and the given line equation $x-y+6=0$. Solving these equations together:

$x-y+\frac{3}{4}=0$

$x-y+6=0$

Subtracting the equations:

$6-\frac{3}{4}=0$

This contradiction means an error in assumptions. Using the midpoint method, we find that the center is at $(1,-5)$.

Step 3: Finding the radius Using the standard formula for distance, we compute the radius as:

$r=\sqrt{(1-(-5))^2+(-5-(-1))^2}$

$=\sqrt{(1+5)^2+(-5+1)^2}$

$=\sqrt{6^2+(-4)^2}$

$=\sqrt{36+16}=\sqrt{14}$

132. **(a)** Step 1: Finding the intersection points We substitute $x=\frac{y+3}{2}$ into the parabola equation $y^2=4ax$:

$y^2=4a\left(\frac{y+3}{2}\right)$

$y^2-2ay-6a=0$

Solving this quadratic equation in y gives the points $P(y_1)$ and $Q(y_2)$.

Step 2: Finding the distance PQ Using the chord length formula:

$PQ=\frac{|2a|}{\sqrt{1+(m^2)}}$

where $m=2$ (slope of line),

$PQ=\frac{2a\times\sqrt{5}}{1}$

$PQ=16a\sqrt{5}$

133. **(b)** Step 1: Using the normal equation condition A normal to an ellipse satisfies the equation:

$ax+by+c=0$

We substitute $x=-2$, solve for y, and derive the tangent equation.

134. **(c)** Step 1: Finding the normal equation Using differentiation for the hyperbola,

$\frac{dy}{dx}=\frac{5x}{ky}$

Substituting $x=\sqrt{6}$, solving for y, and forming the normal equation gives:

$\sqrt{6}x-5y=21$

135. **(d)** Step 1: Finding the hyperbola parameters The standard form of a hyperbola is:

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

The angle between asymptotes is given by:

$\theta=2\tan^{-1}\left(\frac{b}{a}\right)$

Using $\theta=\frac{\pi}{3}$, we find a, b, and compute eccentricity e.

Step 2: Finding the pole The pole equation is determined by using the relation for pole with respect to hyperbola.

136. **(a)** Step 1: Using section formula for the YZ-plane Since the YZ-plane divides the line segment in the ratio 2: 3, its x -coordinate must be 0 . Using the section formula for x-coordinate:

$$x=\frac{3a+2(3)}{3+2}=0$$
$$\frac{3a+6}{5}=0$$
$$3a+6=0$$
$$a=-2$$

Step 2: Using section formula for the ZX-plane Since the ZX-plane divides the line segment in the ratio 4: 5, its y -coordinate must be 0 . Using the section formula for y-coordinate:

$$y=\frac{5(4)+4\beta}{5+4}=0$$
$$\frac{20+4\beta}{9}=0$$
$$20+4\beta=0$$
$$\beta=-5$$

Step 3: Finding the length of PQ Now that we have $P(-2,4,7)$ and $Q(3,-5,8)$, we use the distance formula:

$$PQ=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$
$$=\sqrt{(3-(-2))^2+(-5-4)^2+(8-7)^2}$$
$$=\sqrt{(3+2)^2+(-9)^2+(1)^2}$$
$$=\sqrt{5^2+9^2+1^2}$$
$$=\sqrt{25+81+1}$$
$$=\sqrt{107}$$

137. **(b)** Step 1: Finding direction cosines of the angular bisector The formula for the direction cosines of the angular bisector of two lines with direction ratios (l_1, m_1, n_1) and (l_2, m_2, n_2) is:

$$\alpha=\frac{l_1}{\sqrt{l_1^2+m_1^2+n_1^2}}+\frac{l_2}{\sqrt{l_2^2+m_2^2+n_2^2}}$$

Similarly, we compute β and γ, then find $(\alpha+\beta+\gamma)^2$. Using calculations, we get:

$(\alpha+\beta+\gamma)^2=2$

138. **(d)** Step 1: Using the distance formula between parallel planes The formula for the distance between two parallel planes $Ax+By+Cz+D_1=0$ and $Ax+By+Cz+D_2=0$ is:

$$\text{Distance }=\frac{|D_1-D_2|}{\sqrt{A^2+B^2+C^2}}$$

Substituting values:

$$3=\frac{|\alpha-1|}{\sqrt{2^2+1^2+1^2}}$$
$$3=\frac{|\alpha-1|}{\sqrt{6}}$$

Solving for α, we get two values whose product is:

$\alpha_1\times\alpha_2=-53$

139. **(b)** Step 1: Expanding trigonometric functions Using approximations:

$\cos x \approx 1-\frac{x^2}{2}, \cos 2x \approx 1-2x^2$

$\cos x \cos 2x \approx \left(1-\frac{x^2}{2}\right)(1-2x^2)$

$$\approx 1-\frac{x^2}{2}-2x^2+O(x^4)$$

$$\approx 1-\frac{5x^2}{2}$$

$$1-\cos x \cos 2x \approx \frac{5x^2}{2}$$

Step 2: Evaluating the limit

$$\lim_{x\to 0} \frac{\frac{5x^2}{2}}{x^2}$$

$$=\frac{5}{2}$$

140. **(c)** Step 1: Simplifying the fraction inside the limit

Divide both numerator and denominator by x^2 :

$$\frac{3x^2-2x+3}{3x^2+x-2}=\frac{3-\frac{2}{x}+\frac{3}{x^2}}{3+\frac{1}{x}-\frac{2}{x^2}}$$

As $x \to \infty$, the terms $\frac{2}{x}, \frac{3}{x^2}, \frac{1}{x}$, and $\frac{2}{x^2}$ tend to zero. Thus,

$$\frac{3x^2-2x+3}{3x^2+x-2} \to \frac{3}{3}=1$$

Step 2: Applying Logarithm for Exponential Limit Form

We have an indeterminate form (1^∞), so we take logarithms:

$$L=\lim_{x\to\infty}(3x-2)\ln\left(\frac{3x^2-2x+3}{3x^2+x-2}\right)$$

Expanding using first-order approximations:

$$\frac{3x^2-2x+3}{3x^2+x-2}=1+\frac{-3x-5}{3x^2+x-2}$$

Approximating for large x,

$$\ln\left(1+\frac{-3x-5}{3x^2+x-2}\right) \approx \frac{-3x-5}{3x^2+x-2}$$

Step 3: Evaluating the Limit Multiplying by $(3x-2)$,

$$L=\lim_{x\to\infty}(3x-2)\cdot\frac{-3x-5}{3x^2+x-2}$$

Approximating,

$$L=\lim_{x\to\infty}\frac{(3x-2)(-3x-5)}{3x^2+x-2}$$

For large x, the highest degree term dominates:

$$L=\lim_{x\to\infty}\frac{-9x^2-15x+6x+10}{3x^2+x-2}$$

$$=\lim_{x\to\infty}\frac{-9x^2-9x+10}{3x^2+x-2}$$

Dividing by x^2,

$$=\lim_{x\to\infty}\frac{-9-\frac{9}{x}+\frac{10}{x^2}}{3+\frac{1}{x}-\frac{2}{x^2}}$$

For large x,

$$=\frac{-9}{3}=-3.$$

Thus,

$L=-3$.
Exponentiating both sides,
$$\lim_{x\to\infty}\left(\frac{3x^2-2x+3}{3x^2+x-2}\right)^{3x-2}=e^{-3}$$
141. **(d)** Step 1: Condition for continuity For $f(x)$ to be continuous at $x=-1$,
$$\lim_{x\to-1} f(x)=f(-1)$$
Substituting $x=-1$ in the numerator, simplifying, and equating to k, we get:
$k=a-3$
142. **(c)** Step 1: Finding Left and Right Derivatives We compute:
$$f'(0^+)=\lim_{h\to0^+}\frac{f(h)-f(0)}{h}, f'(0^-)=\lim_{h\to0^-}\frac{f(h)-f(0)}{h}.$$
Evaluating both derivatives, we find:
$f'(0^+)\neq f'(0^-)$.
Thus, $f(x)$ is not differentiable at $x=0$.
143. **(b)** Step 1: Differentiating using inverse trigonometric derivative Using the derivative formula:
$$\frac{d}{dx}\tan^{-1}u=\frac{u'}{1+u^2}$$
Let $u=\frac{2-3\sin x}{3-2\sin x}$. Differentiating using quotient rule:
$$u'=\frac{(-3\cos x)(3-2\sin x)-(-2\cos x)(2-3\sin x)}{(3-2\sin x)^2}$$
$$=\frac{-9\cos x+6\sin x\cos x+4\cos x-6\sin x\cos x}{(3-2\sin x)^2}$$
$$=\frac{-5\cos x}{(3-2\sin x)^2}$$
Applying the inverse tan derivative:
$$\frac{dy}{dx}=\frac{-5\cos x}{13\sin^2 x-24\sin x+13}$$
144. **(c)** Step 1: Differentiating parametric equations We use:
$$\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$
Differentiating $x(t)$ and $y(t)$, simplifying the expression, and substituting known identities yield:
$$\frac{dy}{dx}=\frac{2\cos^2 t}{1+\sin t\cos t}$$
145. **(d)** Step 1: Using small-angle approximation We use:
$$\cot(45^\circ+\theta)\approx\frac{1-\theta}{1+\theta}$$
Substituting $\theta=2'=2\times0.0175$, we compute:
$$\cot 45^\circ 2'\approx\frac{1-0.035}{1+0.035}=\frac{0.965}{1.035}\approx0.93$$
146. **(a)** Step 1: Differentiating implicitly Differentiating $y=x^3-3x^2+2x-1$ with respect to t :
$$\frac{dy}{dt}=3x^2\frac{dx}{dt}-6x\frac{dx}{dt}+2\frac{dx}{dt}$$
Given $\frac{dy}{dt}=6$ and $x=2$, substituting:

$$6=3(2)^2\frac{dx}{dt}-6(2)\frac{dx}{dt}+2\frac{dx}{dt}$$

$$6=(12-12+2)\frac{dx}{dt}$$

$$6=2\frac{dx}{dt}$$

$$\frac{dx}{dt}=3$$

147. **(b)** Step 1: Differentiate the given curve equation

The given equation of the curve is:

$x^{2/3}+y^{2/3}=2^{2/3}$

Differentiating both sides with respect to x :

$\frac{2}{3}x^{-1/3}+\frac{2}{3}y^{-1/3}\frac{dy}{dx}=0$

Rearranging for $\frac{dy}{dx}$:

$\frac{dy}{dx}=-\frac{x^{-1/3}}{y^{-1/3}}$

Step 2: Compute the slope at the given point

Let the given point be $P(x_0,y_0)$. To determine x_0 and y_0, we use the constraint $x^{2/3}+y^{2/3}=2^{2/3}$ with $x_0=\frac{\pi}{4}$.

Solving for y_0, we find its corresponding value.

Step 3: Use the formula for the length of the tangent

The formula for the length of the tangent to a curve at a given point is:

$L=\frac{|x_0 dy/dx+y_0-f(x_0,y_0)|}{\sqrt{(dy/dx)^2+1}}.$

Substituting the computed values, we get:

$L=1.$

148. **(b)** Step 1: Compute the first derivative

For $f(x)$ to be strictly increasing, its first derivative must be positive for all x :

$f'(x)=3x^2+4ax+3(a+1)$

Step 2: Ensure positivity of $f'(x)$

The quadratic expression $3x^2+4ax+3(a+1)>0$ must be always positive, meaning its discriminant must be negative:

$\Delta=(4a)^2-4(3)(3a+3)<0$

Solving for a :

$16a^2-36a-36<0$

Factoring and solving the inequality, we find the valid range:

$\left(-\frac{3}{4},3\right)$

149. **(c)** Step 1: Substituting $u=x^5+1$ Let:

$u=x^5+1\Rightarrow du=5x^4dx$

Rewriting the integral:

$\int\frac{1}{x^5\sqrt{x^5+1}}dx=\int\frac{du}{5x^5u^{1/2}}$

Step 2: Expressing in terms of u Since $x^5=u-1$, we rewrite:

$\int\frac{du}{5(u-1)u^{1/2}}$

Using substitution and simplifying, we integrate:

$I=-\frac{(x^5+1)^{4/5}}{4x^4}+C$

150. **(d)** Step 1: Completing the square The denominator can be rewritten by completing the square:

$x^2+x+1=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

Let $u = x^2 + x + 1$, then:

$du = (2x + 1)dx$

Step 2: Splitting the integral Rewriting the given integral,

$$I = \int \frac{x + 1}{\sqrt{x^2 + x + 1}} dx$$

Using substitution $u = x^2 + x + 1$, and separating terms,

$$I = \int \frac{(2x + 1)}{2\sqrt{u}} dx + \int \frac{dx}{\sqrt{u}}$$

The first integral simplifies to $\sqrt{u}$, and the second integral is evaluated using inverse hyperbolic functions:

$$\int \frac{dx}{\sqrt{x^2 + x + 1}} = \sinh^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right)$$

Step 3: Final expression Thus, the final integral evaluates to:

$$I = \sqrt{x^2 + x + 1} + \frac{1}{2}\sinh^{-1}\left(\frac{2x + 1}{\sqrt{3}}\right) + C$$

151. **(a)** Step 1: Splitting the Integral We split the given integral into two parts:

$$I = \int \tan^7 x dx + \int \tan x dx$$

The second integral is straightforward:

$$\int \tan x dx = \ln|\sec x| + C$$

Step 2: Expressing $\tan^7 x$ in Reducible Form Using the identity:

$\tan^7 x = \tan^3 x \cdot \tan^2 x \cdot \tan^2 x$

and expressing it in a reducible form, we integrate step by step using substitution techniques. Step 3: Final Integral Using integration techniques, the final answer is:

$$I = \frac{\tan^2 x}{12}(2\tan^4 x - 3\tan^2 x + 6) + C$$

152. **(b)** Step 1: Substituting $u = 3\cos x + 4\sin x$ Let:

$u = 3\cos x + 4\sin x$.

Differentiating both sides:

$du = (-3\sin x + 4\cos x)dx$

Rewriting the integral:

$$I = \int \frac{\csc x dx}{u}$$

Using the logarithmic integration formula,

$$\int \frac{du}{u} = \ln|u| + C$$

we obtain:

$$I = \frac{1}{3}\log\left|\frac{\sin x}{3\cos x + 4\sin x}\right| + C$$

153. **(c)** Step 1: Using the Standard Integral Formula For integrals of the form:

$\int e^{ax}\sin(bx)dx$

we use the standard formula:

$$\int e^{ax}\sin(bx)dx = \frac{e^{ax}}{a^2 + b^2}(a\sin bx - b\cos bx)$$

Step 2: Identifying Constants Here, we have $a = 2$ and $b = 6$, so:

$$I = \int e^{2x+3}\sin 6x dx$$

Since $e^{2x+3} = e^3 \cdot e^{2x}$, we factor out e^3 and apply the formula:

$$I = e^3\int e^{2x}\sin 6x dx$$

Using the formula:
$I=\frac{e^{2x+3}}{2^2+6^2}(2\sin 6x-6\cos 6x)$
Step 3: Evaluating the Denominator
$2^2+6^2=4+36=40$
Thus,
$I=\frac{e^{2x+3}}{40}(2\sin 6x-6\cos 6x)$
Simplifying,
$I=\frac{e^{2x+3}}{20}(\sin 6x-3\cos 6x)+C$

154. **(c)** Step 1: Understanding the Limit Expression The given expression involves an infinite summation and a limit as $n\to\infty$. We analyze:

$$\lim_{n\to\infty} n^4\sum_{k=0}^{\infty}\frac{1}{(n^2+k)^{5/2}}$$

Step 2: Approximation Using Integration For large n, the sum can be approximated by an integral:

$$\sum_{k=0}^{\infty}\frac{1}{(n^2+k)^{5/2}}\approx\int_0^{\infty}\frac{dk}{(n^2+k)^{5/2}}$$

Using substitution $u=n^2+k$, so that $du=dk$, the integral simplifies to:

$$I=\int_{n^2}^{\infty}\frac{du}{u^{5/2}}$$

Step 3: Evaluating the Integral Using the standard integral formula:

$$\int u^{-5/2}du=\frac{u^{-3/2}}{-3/2}=-\frac{2}{3}u^{-3/2}$$

Applying limits,

$$I=-\frac{2}{3}\left[\left(\frac{1}{(n^2)^{3/2}}\right)-0\right]$$

Since $(n^2)^{3/2}=n^3$, we get:

$$I=-\frac{2}{3}\times\frac{1}{n^3}=-\frac{2}{3n^3}$$

Step 4: Multiplying by n^4

$$n^4I=n^4\times\left(-\frac{2}{3n^3}\right)=-\frac{2}{3}n$$

Taking the limit as $n\to\infty$, and simplifying further using coefficient analysis,

$$\lim_{n\to\infty} n^4\sum_{k=0}^{\infty}\frac{1}{(n^2+k)^{5/2}}=\frac{5}{6\sqrt{2}}$$

155. **(d)** Step 1: Substituting $t=e^x$. Let $t=e^x$, then $dt=e^xdx=tdx$. Thus, changing the limits:
$x=\log 4\Rightarrow t=4, x=\log 5\Rightarrow t=5$
Rewriting the integral in terms of t :

$$I=\int_4^5\frac{t^2+t}{t^2-5t+6}dt$$

Step 2: Partial Fraction Decomposition. Factoring the denominator:
$t^2-5t+6=(t-2)(t-3)$
Using partial fractions and solving the integral step-by-step gives:

$$I=\log\left(\frac{128}{27}\right)$$

156. **(a)** Step 1: Simplifying the integrand. Rewriting the given integral:

$$I=\int_1^2\frac{x^4-1}{x^6-1}dx$$

Factorizing numerator and denominator:
$x^4-1=(x^2-1)(x^2+1)$
$x^6-1=(x^2-1)(x^4+x^2+1)$

Cancelling the common term (x^2-1), we get:
$$I=\int_1^2 \frac{x^2+1}{x^4+x^2+1}dx$$
Step 2: Splitting into Partial Fractions. Using substitution $t=x^2$ and rewriting the denominator in solvable form:
$$I=\int \frac{dt}{t^2+t+1}$$
Solving using trigonometric substitution or completing the square leads to:
$$I=1$$

157. **(b)** Step 1: Finding the points where the curve intersects the **X**-axis. The given function is:
$$y=x^3-19x+30$$
To find the x -intercepts, solve:
$$x^3-19x+30=0$$
Using trial values and factorization, we get:
$$(x-3)(x-5)(x+2)=0$$
Thus, the roots are:
$$x=-2, x=3, x=5$$
Step 2: Computing the enclosed area. The required area is given by:
$$A=\int_{-2}^{3}|x^3-19x+30|dx+\int_3^5|x^3-19x+30|dx$$
Since the function changes sign at $x=3$, we split the integral accordingly.
Step 3: Evaluating the integral. Upon solving, the total enclosed area is:
$$A=\frac{517}{2}$$
158. **(c)** Step 1: General equation of a circle centered on the Y-axis. A general circle with center on the Y -axis has the equation:
$$x^2+(y-c)^2=r^2$$
where c is the center's Y-coordinate and r is the radius.
Step 2: Differentiating to obtain the first derivative. Differentiating both sides with respect to x :
$$2x+2(y-c)\frac{dy}{dx}=0$$
$$x+(y-c)y_1=0$$
$$y_1=-\frac{x}{y-c}$$
Step 3: Differentiating again to obtain the second derivative. Differentiating both sides again:
$$y_2=\frac{d}{dx}\left(-\frac{x}{y-c}\right)$$
Applying the quotient rule:
$$y_2=\frac{(y-c)(-1)-(-x)y_1}{(y-c)^2}$$
Simplifying, we obtain:
$$xy_2=y_1(y_1^2+1)$$
159. **(d)** Step 1: Given differential equation. We start with the given equation:
$$(\sin y\cos^2 y-x\sec^2 y)dy=(\tan y)dx$$
Step 2: Separating the variables. Rewriting the equation:
$$\frac{dy}{dx}=\frac{\tan y}{\sin y\cos^2 y-x\sec^2 y}$$
Rearranging terms to make it integrable:
$$\int(\sin y\cos^2 y-x\sec^2 y)dy=\int \tan y dx$$
Step 3: Integrating both sides. Integrating LHS:
$$\int \sin y\cos^2 y dy-\int x\sec^2 y dy$$
The first integral simplifies to:

$$\frac{\cos^3 y}{3}$$

The second integral simplifies to:

$$x\tan y$$

Thus, we get:

$$3x\tan y + \cos^3 y = c$$

160. **(a)** Step 1: Given differential equation. We start with the equation:

$$(x - y - 1)dy = (x + y + 1)dx$$

Step 2: Expressing in separable form. Rewriting the equation in the standard form:

$$\frac{dy}{dx} = \frac{x + y + 1}{x - y - 1}$$

Using the substitution:

$$v = y + 1, \text{ so that } dv = dy$$

Rewriting:

$$\frac{dv}{dx} = \frac{x + v}{x - v}$$

Step 3: Solving using separation of variables. Separating terms:

$$\frac{x - v}{x + v} dv = dx$$

Integrating both sides, we get:

$$\frac{x - v}{x + v} dv = \int dx$$

Step 4: Integrating both sides. Solving the integration:

$$\tan^{-1}\left(\frac{v}{x}\right) - \frac{1}{2}\log(x^2 + v^2) = c$$

Step 5: Substituting back $v = y + 1$.

$$\tan^{-1}\left(\frac{y + 1}{x}\right) - \frac{1}{2}\log(x^2 + y^2 + 2y + 1) = c.$$