PHYSICS

1. **(d)** The dimension of Young's modulus (Y) is the same as stress, which is force per unit area.
•Force → [MLT-2]
•Area → [L^2]
•So, stress / Young's modulus → $[ML^{-1}T^{-2}]$

2. **(d)** Step 1: Let total distance = D
•Distance with $v_1: 0.4D$
•Distance with $v_2: 0.6D$
Step 2: Time taken for each part
$t_1 = \frac{0.4D}{v_1}, t_2 = \frac{0.6D}{v_2}$
Step 3: Total time
$T = t_1 + t_2 = \frac{0.4D}{v_1} + \frac{0.6D}{v_2} = D\left(\frac{0.4}{v_1} + \frac{0.6}{v_2}\right)$
Step 4: Average speed formula
$v_{avg} = \frac{\text{Total distance}}{\text{Total time}} = \frac{D}{D(0.4/v_1 + 0.6/v_2)} = \frac{1}{0.4/v_1 + 0.6/v_2}$
$v_{avg} = \frac{1}{\frac{2}{5v_1} + \frac{3}{5v_2}} = \frac{1}{\frac{2v_2 + 3v_1}{5v_1v_2}} = \frac{5v_1v_2}{3v_1 + 2v_2}$

3. **(b)** Let's solve this step by step.
We are asked: bullets are fired in all possible directions with initial speed $u = 10\ m/s$ and angle of projection $45°$. Gravity $g = 10\ m/s^2$. Find the area covered on the ground.
Step 1: Maximum range of a projectile
For a projectile with speed u and angle θ :
$R = \frac{u^2 \sin 2\theta}{g}$
Here, $u = 10\ m/s, \theta = 45° \to \sin 2\theta = \sin 90° = 1$
$R_{max} = \frac{10^2 \cdot 1}{10} = \frac{100}{10} = 10\ m$
Maximum horizontal distance in any direction is 10 m .
Step 2: Area covered on the ground
If bullets are fired in all directions (like a 3D "fountain"), the landing points form a circle of radius = maximum horizontal range:
Area $= \pi R^2$
Area $= \pi(10)^2 = 100\pi \approx 314\ m^2$

4. **(a)** Here's the reasoning step by step:
•Let the ball's speed be V_0 and the person's speed be $0.5V_0$.
•For the person to catch the ball, the horizontal component of the ball's velocity must be twice the person's speed:
$V_0 \cos\theta = 2 \times 0.5V_0 = V_0$
•So, $\cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$.

5. **(a)** Given:
•Mass of train: $m = 3 \times 10^6$ kg
•Speed: $v = 50\ m/s$
•Coefficient of friction: $\mu = 0.05$
• $g = 10\ m/s^2$
Step 1: Friction force
$F = \mu mg = 0.05 \times 3 \times 10^6 \times 10 = 1.5 \times 10^6$ N
Step 2: Power
$P = F \cdot v = 1.5 \times 10^6 \times 50 = 7.5 \times 10^7\ W = 75MW$

6. **(d)** Based on the standard version of this problem where the force F is applied at an angle of 60° to the vertical (or 30° to the horizontal):
(1).Vertical forces: The normal force $N = mg + F\cos(60°)$.

(2).Horizontal forces: To stay still, $F\sin(60^\circ) \leq \mu N$.

(3).Equation: $F\frac{\sqrt{3}}{2} = \frac{1}{2\sqrt{3}}\left(10\sqrt{3} + \frac{F}{2}\right)$.

(4).Solving: $3F = 10\sqrt{3} + \frac{F}{2} \rightarrow 2.5F = 10\sqrt{3} \rightarrow F \approx 20\text{ N}$.

7. **(b)** Given:

•Mass, $m = 8\text{ kg}$

•Initial momentum, $p = 24\text{ kg m/s} \rightarrow$ initial velocity $v_0 = \frac{p}{m} = \frac{24}{8} = 3\text{ m/s}$

•Force, $F = 24\text{ N}$

•Time, $t = 3\text{ s}$

Step 1: Find the impulse (change in momentum):

$\Delta p = F \cdot t = 24 \cdot 3 = 72\text{ kg m/s}$

Step 2: Find the final momentum:

$p_f = p + \Delta p = 24 + 72 = 96\text{ kg m/s}$

Step 3: Find the final velocity:

$$v_f = \frac{p_f}{m} = \frac{96}{8} = 12\text{ m/s}$$

Step 4: Increase in kinetic energy:

$$\triangle KE = \frac{1}{2}m\left(v_f^2 - v_0^2\right) = \frac{1}{2} \cdot 8 \cdot (12^2 - 3^2) = 4 \cdot (144 - 9) = 4 \cdot 135 = 540\text{ J}$$

8. **(b)** Given:

•Mass of ball, $m = 0.25\text{ kg}$

•Final speed, $v = 12\text{ m/s}$

•Distance moved by hand, $s = 0.9\text{ m}$

Step 1: Kinetic energy of the ball

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.25)(12^2) = 0.125 \times 144 = 18\text{ J}$$

Step 2: Work done = Force × distance

$W = F \cdot s$

Here, the net work done by the hand = kinetic energy gained. So:

$F \cdot 0.9 = 18$

Step 3: Solve for F

$$F = \frac{18}{0.9} = 20\text{ N}$$

9. **(a)** Given:

•Radius of gyration about an axis through center and perpendicular to plane: $k_\perp = 10\sqrt{2}$ cm

Step 1: Moment of inertia relation

For a thin circular ring:

$I_\perp = I_x + I_y$

where $I_\perp$ is about the axis perpendicular to plane (through center), and $I_x = I_y$ are about two perpendicular diameters.

$I_\perp = I_{\text{diameter}} + I_{\text{diameter}} = 2I_{\text{diameter}}$

Step 2: Relate radius of gyration

$I_\perp = Mk_\perp^2, I_{\text{diameter}} = Mk_{\text{diameter}}^2$

$$Mk_\perp^2 = 2Mk_{\text{diameter}}^2 \Rightarrow k_{\text{diameter}}^2 = \frac{k_\perp^2}{2}$$

$$k_{\text{diameter}}^2 = \frac{k_\perp}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10\text{ cm}$$

10. **(b)** We are given:

•Initial angular velocity, $\omega_0 = 0$ (starting from rest)

•Angular acceleration, $\alpha = \pi\text{rad/s}^2$

•Time, $t = 6\text{ s}$

We need number of rotations, N.

Step 1: Angular displacement formula

For constant angular acceleration:

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
Since $\omega_0 = 0$:
$\theta = \frac{1}{2}\alpha t^2$
Plug in values:
$\theta = \frac{1}{2} \cdot \pi \cdot 6^2 = \frac{\pi \cdot 36}{2} = 18\pi$ radians
Step 2: Convert radians to rotations
1 rotation $= 2\pi$ radians
$N = \frac{\theta}{2\pi} = \frac{18\pi}{2\pi} = 9$

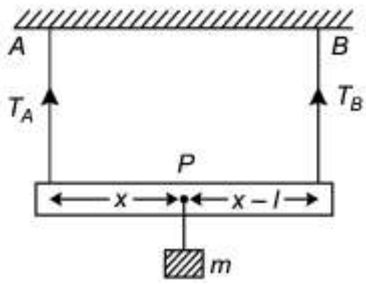
11. (d) We are given the SHM:
$y = 5\sin(3\pi t) + 5\sqrt{3}\cos(3\pi t)$
For SHM of the form $y = A\sin(\omega t) + B\cos(\omega t)$, the amplitude R is:
$R = \sqrt{A^2 + B^2}$
Here, $A = 5$ and $B = 5\sqrt{3}$.
$R = \sqrt{5^2 + (5\sqrt{3})^2} = \sqrt{25 + 75} = \sqrt{100} = 10$
So, the amplitude of the particle is 10 cm .

12. (b) For a mass-spring system, the angular frequency is:
$\omega = \sqrt{\frac{k}{m}}$
Here:
- $k = 2.5\ \mathrm{N/m}$
- $m = 0.1\ \mathrm{kg}$

$\omega = \sqrt{\frac{2.5}{0.1}} = \sqrt{25} = 5\mathrm{rad/s}$
$= 5\mathrm{rads}^{-1}$

13. (d) The gravitational potential at a point due to a mass m at distance r is:
$V = -\frac{Gm}{r}$
Here, masses are at $x = \pm 1, \pm 2, \pm 4, \pm 8, \ldots$ meters. Each mass is 1 kg .
Total potential at $x = 0$ is the sum of potentials from all masses:
$V = -G\left(\frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \cdots\right)$
•Combine pairs ($+x$ and $-x$):
$V = -G\left(2\left(1 + \frac{1}{2} + \frac{1}{4} + \cdots\right)\right)$
•The series is geometric: $1 + \frac{1}{2} + \frac{1}{4} + \cdots = 2$
$V = -G \cdot 2 \cdot 2 = -4G$

14. (a) Elongations in A and B are same ⇒



$\Delta l_A = \Delta l_B$

$$\frac{T_A l_A}{A_A \cdot Y_A} = \frac{T_B l_B}{A_B \cdot Y_B}$$
$$\Rightarrow \frac{T_A}{r_A^2 Y_A} = \frac{T_B}{r_B^2 Y_B}$$
$$\Rightarrow \frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^2 - \left(\frac{Y_A}{Y_B}\right)$$
Given, $\frac{r_A}{r_B} = \frac{2}{3}$ and $\frac{Y_A}{Y_B} = \frac{3}{2}$
$$\therefore \frac{T_A}{T_B} = \left(\frac{2}{3}\right)^2 \left(\frac{3}{2}\right) = \frac{2}{3}$$
For rotational equilibrium of bar; taking moments about point of suspension ' P ',
$T_A \cdot x = T_B(l - x)$
$$\Rightarrow \frac{l - x}{x} = \frac{T_A}{T_B} = \frac{2}{3}$$
$$\Rightarrow \frac{150 - x}{x} = \frac{2}{3}$$
$150 \times 3 = 5x$ [$\therefore l = 150$ cm]
Or
$x = 90$ cm(from P)

15. **(a)** We can determine the type of flow using the Reynolds number:
$$Re = \frac{\rho v D}{\eta}$$
Where:
• $\rho = 10^3\ kg/m^3$ (density of water)
• $v = 20$ cm/s $= 0.2$ m/s (velocity)
• $D = 2R = 2 \times 0.02 = 0.04$ m (diameter)
$\eta = 10^{-3}$ kg/(m
s) (viscosity)
Now calculate Re step by step:
$$Re = \frac{(10^3)(0.2)(0.04)}{10^{-3}}$$
Step by step:
(1). $10^3 \times 0.2 = 200$
(2). $200 \times 0.04 = 8$
(3). $8/10^{-3} = 8 \times 10^3 = 8000$
Interpretation:
•$Re < 2000 \rightarrow$ laminar (steady)
•$Re > 4000 \rightarrow$ turbulent
• $2000 < Re < 4000 \rightarrow$ transitional
Here, $Re = 8000 > 4000$, so the flow is turbulent.

16. **(c)** Power of kettle: $P = VI = 220 \times 4 = 880$ W
Heat needed: $Q = mc\Delta T = 1 \times 4200 \times (100 - 34) = 277200$ J
Time: $t = Q/P = 277200/880 \approx 315$ s ≈ 5.25 min

17. **(b)**

18. **(c)** The coefficient of performance (COP) of a refrigerator is:
$$COP = \frac{T_c}{T_h - T_c}$$
where temperatures are in Kelvin, T_c = freezer temperature, T_h = room temperature.
Given:
$COP = 5, T_c = -13°C = 260$ K
$$5 = \frac{260}{T_h - 260} \Longrightarrow T_h - 260 = 52 \Longrightarrow T_h = 312 \text{ K}$$
Convert to °C :
$T_h = 312 - 273 = 39°C$

19. **(a)**Step-by-Step Breakdown

(1).Identify Isochoric (Curve I): A vertical line on a $\mathbf{P}-\mathbf{V}$ diagram indicates that volume remains constant while pressure changes. This is an isochoric process (c).

(2).Identify Isobaric (Curve IV): A horizontal line indicates that pressure remains constant while volume changes. This is an isobaric process (b).

(3).Compare Adiabatic and Isothermal (Curves II & III): Both processes show an inverse relationship between pressure and volume, but their slopes differ.

(4).The adiabatic process (a) is steeper because its equation is $PV^{\gamma}=$ constant (where $\gamma>1$). Thus, Curve II is adiabatic.

(5).The isothermal process (d) follows $PV=$ constant and is less steep. Thus, Curve III is isothermal.

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Summary Table

Curve	Characteristic	Process	Match
(I)	Vertical line (Constant $\mathbf{V}$)	Isochoric	(C)
(II)	Steeper curve	Adiabatic	(A)
(III)	Less steep curve	Isothermal	(D)
(IV)	Horizontal line (Constant $\vec{P}$)	Isobaric	(B)

20. **(b)** We can solve this using the formula for mixing gases with negligible volume change:

$$T_{mix}=\frac{n_1T_1+n_2T_2}{n_1+n_2}$$

Here:

- $n_1=2, T_1=27+273=300\text{ K}$
- $n_2=4, T_2=327+273=600\text{ K}$

$$T_{mix}=\frac{2\cdot 300+4\cdot 600}{2+4}=\frac{600+2400}{6}=\frac{3000}{6}=500\text{ K}$$

Convert back to $^{\circ}C$:

$T_{mix}=500-273=227^{\circ}C$

21. **(d)** Here's a medium-short solution:

Frequencies: $f=v/\lambda$

$f_1=330/0.99\approx 333.33\text{ Hz}, f_2=330/1.00=330$

Beat frequency:

$f_{beat}=|f_1-f_2|=3.33\text{ Hz}$

Time for 10 beats:

$$t=\frac{10}{f_{beat}}=\frac{10}{3.33}\approx 3\text{ seconds}$$

22. **(d)** Given:

Far point $v=400\text{ cm}=4\text{ m}$

For distant objects, $u=\infty$

Lens formula:

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{v}-0=\frac{1}{v}$$

$f=4\text{ m}$

Power of lens P (in diopters) is:

$$P=\frac{100}{f(\text{ cm})}=\frac{100}{400}=0.25D$$

Since the person is short-sighted, we need a diverging lens, so the power is negative:

$-0.25D$

23. **(b)** In Young's double slit experiment, bright fringes satisfy:

$y=\frac{m\lambda L}{d}$

If the n^{th} red fringe coincides with $(n+1)^{th}$ blue fringe:

$n\lambda_{red}=(n+1)\lambda_{blue}$

Substitute values:

$n(7.5\times10^{-5})=(n+1)(5\times10^{-5})$

$7.5n=5n+5$

$2.5n=5\Longrightarrow n=2$

24. **(a)** We solve using series dielectrics formula:

$\frac{d}{\epsilon_{eq}}=\frac{d_1}{\kappa_1}+\frac{d_2}{\kappa_2}$

Given:

• $d=9\text{ cm}, d_1=6\text{ cm}, d_2=3\text{ cm}, \kappa_1=4, \kappa_2=9$

$\frac{9}{\epsilon_{eq}}=\frac{6}{4}+\frac{3}{9}=1.5+0.333=1.833$

$\epsilon_{eq}\approx4.91$

New force:

•$F=\frac{F_0}{\epsilon_{eq}}=\frac{98}{4.91}\approx20\text{ N}\approx18\text{ N}$ (closest)

25. **(c)** $U_{initial}=U_{12}+U_{13}+U_{23}$

$=\frac{kq^2}{2a}-\frac{kq^2}{a}-\frac{kq^2}{a}=\frac{kq^2}{2a}-\frac{2kq^2}{a}=\frac{-3kq^2}{2a}$

After exchanging the final charge with one of the negative charge, there are two q charges separated by a distance a and one +q charge separated by a distance ' a ' from one of the −q charges.

$\therefore U_{final}=U'_{12}+U'_{13}+U'_{23}$

$=\frac{kq^2}{a}-\frac{kq^2}{2a}-\frac{kq^2}{a}=\frac{-kq^2}{2a}$

$\therefore \Delta U=U_{final}-U_{initial}$

$=-\frac{kq^2}{2a}-\left(\frac{-3kq^2}{2a}\right)=\frac{2kq^2}{2a}$

$=\frac{kq^2}{a}=\frac{q^2}{4\pi\varepsilon_0 a}\left[\because k=\frac{1}{4\pi\varepsilon_0}\right]$

26. **(b)** •Initial energy: $U_0=\frac{1}{2}CV^2$

•Capacitor disconnected → charge constant, so $U=\frac{Q^2}{2C'}$ with $C'=KC$

•Energy decreases by 25%: $U=0.75U_0$

•Then: $\frac{U_0}{K}=0.75U_0\Rightarrow K=\frac{1}{0.75}=\frac{4}{3}$

27. **(a)** Reason: After stretching, $R=100\times1.2^2=144\Omega$. Rectangle sides: $3:2\rightarrow R_1=86.4\Omega, R_2=57.6\Omega$.

Diagonal: $R_d=\frac{R_1R_2}{R_1+R_2}\approx36\Omega$.

28. **(a)** Let the series length $l=160$ cm, and when reversed, it decreases by 75%, so new length $=0.25\times160=40$ cm .

For series aiding: $l\propto E_1+E_2\Rightarrow160\propto1.2+E_2$

For series opposing: $l\propto E_2-E_1\Rightarrow40\propto E_2-1.2$

Taking ratio: $\frac{160}{40}=\frac{1.2+E_2}{E_2-1.2}=4$

$\Rightarrow1.2+E_2=4(E_2-1.2)$

$\Rightarrow1.2+E_2=4E_2-4.8$

$\Rightarrow3E_2=6\Rightarrow E_2=2\text{ V}$

29. **(c)** Use formula: $B=\frac{\mu_0 I}{2\pi r}$

$\mu_0=4\pi\times10^{-7}, I=4\text{ A}, r=0.1\text{ m}$

$B = \dfrac{4\pi \times 10^{-7} \times 4}{2\pi \times 0.1} = \dfrac{16\pi \times 10^{-7}}{0.2\pi} = 8 \times 10^{-6}\ \mathrm{T} = 8\mu\ \mathrm{T}$

30. **(c)** For velocity selector: $v = \frac{E}{B} \Rightarrow B = \frac{E}{v}$

$E = 400\ \mathrm{V/m}, v = 6 \times 10^3\ \mathrm{m/s}$

$B = \dfrac{400}{6000} = \dfrac{1}{15}\ \mathrm{T}$

31. **(b)**Magnetic moment of a solenoid: $M = NIA$

Given: $M = 1.2\ \mathrm{J/T}, N = 1200, A = 5\ \mathrm{cm}^2 = 5 \times 10^{-4}\ \mathrm{m}^2$

$I = \dfrac{M}{NA} = \dfrac{1.2}{1200 \times 5 \times 10^{-4}} = \dfrac{1.2}{0.6} = 2\ \mathrm{A}$

32. **(b)**1. The standard formula for magnetic energy density is $u_B = \frac{B^2}{2\mu_0}$.

2.Using the relation for the speed of light, $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$, we can solve for the permeability constant: $\frac{1}{\mu_0} = \epsilon_0 c^2$.

3.Substituting $e_0 c^2$ for $\frac{1}{\mu_0}$ into the energy density formula gives:

$u_B = \dfrac{1}{2}(\epsilon_0 c^2)B^2$

$= \frac{\epsilon_0 c^2 B^2}{2}$.

33. **(c)** When a dielectric of constant $K = 16$ is inserted, the capacitance becomes $C' = KC = 16C$.

Resonant frequency:

$f = \dfrac{1}{2\pi\sqrt{LC'}} = \dfrac{1}{2\pi\sqrt{L \cdot 16C}} = \dfrac{f_0}{4}$

34. **(a)** The magnetic field is given as:

$\vec{B} = 3 \times 10^{-7} \sin(100\pi x + 10^{12} t)\hat{\imath}$

Compare with the general wave form:

$\sin(kx - \omega t)$ or $\sin(kx + \omega t)$

where $k = \frac{2\pi}{\lambda}$ is the wave number.

Here, $k = 100\pi \Rightarrow \frac{2\pi}{\lambda} = 100\pi$

$\lambda = \dfrac{2\pi}{100\pi} = 0.02\ \mathrm{m}$

35. **(c)** We know de Broglie wavelength:

$\lambda = \dfrac{h}{p}$

If momentum changes by p_0, then fractional change in wavelength:

$\dfrac{\Delta\lambda}{\lambda} = \dfrac{\lambda' - \lambda}{\lambda} \approx \dfrac{-\Delta p}{p} = \dfrac{-p_0}{p}$

Given $\frac{\Delta\lambda}{\lambda} = 0.25\% = 0.0025$,

$\dfrac{p_0}{p} = 0.0025 \Longrightarrow p = \dfrac{p_0}{0.0025} = 400p_0$

36. **(d)** Energy of photon:

$E = 5\mathrm{eV} = 5 \times 1.6 \times 10^{-19}\ \mathrm{J} = 8 \times 10^{-19}\ \mathrm{J}$

Wavelength:

$\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \cdot 3 \times 10^8}{8 \times 10^{-19}} = \dfrac{1.989 \times 10^{-25}}{8 \times 10^{-19}}$

$\approx 2.486 \times 10^{-7}\ \mathrm{m} = 248\ \mathrm{nm}$

37. **(a)** We are given:

•Half-life of X: $T_{1/2} = 1.4 \times 10^9 \mathrm{yr}$

•Current ratio: $X : Y = 1 : 7$

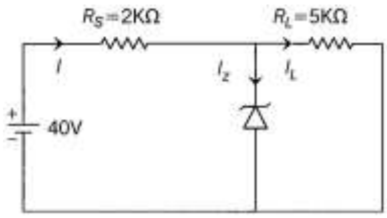
•Initially, $Y = 0$

Let the initial number of X atoms be N_0.

After time **t** :

$N_X = N_0 e^{-\lambda t}, N_Y = N_0 - N_X$
Given X: Y = 1: 7, so $N_X : N_Y = 1:7$
$\frac{N_X}{N_Y} = \frac{1}{7} \Rightarrow \frac{N_0 e^{-\lambda t}}{N_0 - N_0 e^{-\lambda t}} = \frac{1}{7}$
$\frac{e^{-\lambda t}}{1 - e^{-\lambda t}} = \frac{1}{7} \Rightarrow 7e^{-\lambda t} = 1 - e^{-\lambda t} \Rightarrow 8e^{-\lambda t} = 1$
$e^{-\lambda t} = \frac{1}{8}$
Decay constant: $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{1.4 \times 10^9}$
$t = \frac{\ln 8}{\lambda} = \frac{\ln 2^3}{\lambda} = \frac{3 \ln 2}{\lambda} = 3T_{1/2} = 3 \times 1.4 \times 10^9 = 4.2 \times 10^9 \text{yr}$

38. (c) $V_Z = 20$ V



$I = \frac{V - V_Z}{R_s} = \frac{40 - 20}{2 \times 10^3}$
$= \frac{20}{2 \times 10^3} = 10^{-2}$ A $= 10$ mA
$I_L = \frac{V_Z}{R_L} = \frac{20}{5 \times 10^3}$
$= 4 \times 10^{-3}$ A $= 4$ mA
$\therefore I_Z = I - I_L = 10 - 4 = 6$ mA

39. (c) We are given:
•Two amplifiers in series with voltage gains $A_1 = 8$ and $A_2 = 12.5$
•Input voltage $V_{in} = 200\mu$ V $= 200 \times 10^{-6}$ V
Step 1: Find total voltage gain
Since the amplifiers are in series, total voltage gain:
$A_{total} = A_1 \times A_2 = 8 \times 12.5 = 100$
Step 2: Find output voltage
$V_{out} = A_{total} \cdot V_{in} = 100 \times 200 \times 10^{-6} = 0.02$ V $= 20$mV

40. (a) For line-of-sight communication, the range R depends on the heights of the transmitting (h_1) and receiving (h_2) antennas:
$R \propto \sqrt{h_1} + \sqrt{h_2}$
We are given $h_1 + h_2 = h$. To maximize **R**, we maximize $\sqrt{h_1} + \sqrt{h_2}$ under the constraint $h_1 + h_2 = h$.
Step 1: Use substitution
Let $h_1 = h - h_2$, then
$f(h_2) = \sqrt{h - h_2} + \sqrt{h_2}$
Step 2: Maximize $f(h_2)$
Take derivative and set to zero:
$\frac{d}{dh_2}(\sqrt{h - h_2} + \sqrt{h_2}) = -\frac{1}{2\sqrt{h - h_2}} + \frac{1}{2\sqrt{h_2}} = 0$
$\sqrt{h_2} = \sqrt{h - h_2} \Rightarrow h_2 = h - h_2 \Rightarrow h_2 = \frac{h}{2}$

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41. (b) Use the photoelectric equation:
$K.E. = hf - hf_0$
Given:
$z = hx - hf_0, z/3 = hy - hf_0$

Subtract:

$z - z/3 = hx - hy \Rightarrow \frac{2z}{3} = h(x - y) \Rightarrow z = \frac{3}{2}h(x - y)$

Then:

$hf_0 = hx - z = hx - \frac{3}{2}h(x - y) = \frac{3y - x}{2}h$

42. (a) (I) is false - isotopes have the same chemical behavior.
(II) is true - Lyman series is in UV.
(III) is true - E and B fields ⊥ each other and to propagation.

43. (b)(A) 112 → Copernicium → d-block
(B) 116 → Livermorium → p-block
(C) 88 → Radium → s-block
(D) 100 → Fermium → f-block

44. (c) (a) Salicylic acid -OH is ortho to -COOH. The -OH forms a hydrogen bond with the carbonyl of $-COOH$ → H -bond present.
(b) Salicylaldehyde - OH is ortho to - CHO. The - OH forms a hydrogen bond with the aldehyde oxygen → H - bond present.
(c) Quinol - If this is quinoline, the nitrogen in the ring is not positioned to form a stable intramolecular Hbond with any $-OH$ → H-bond absent.
(d) Catechol - Two - OH groups are ortho to each other; they can H -bond → H-bond present.
Quinol - intramolecular H-bonding is absent.

45. (d)

46. (b) Both statement-I and statement-II are not correct

47. (c) The van der Waals equation accounts for two factors that the ideal gas law ignores: the volume of gas molecules and the intermolecular forces (attractions) between them. At high temperature and low pressure, these factors become negligible, causing the van der Waals equation to reduce to the ideal gas equation ($PV = nRT$) for the following reasons:
•Low Pressure: At low pressure, the volume of the gas is very large compared to the actual volume of the molecules. Therefore, the volume correction constant b (representing the space occupied by molecules) becomes insignificant relative to the total volume.
•High Temperature: At high temperatures, gas molecules have high kinetic energy, which allows them to overcome intermolecular attractive forces. Consequently, the pressure correction term a (representing attractions) becomes negligible.

48. (d) To determine the number of significant figures, we apply standard scientific rules for each value:
(I) 0.0063 : This number has 2 significant figures (6 and 3).
•Rule: Leading zeros (zeros before the first non-zero digit) are never significant.
(II) 132.00: This number has 5 significant figures (1,3,2,0, and 0).
•Rule: Trailing zeros in a number with a decimal point are always significant as they indicate measurement precision.
(III) 1004: This number has 4 significant figures.
•Rule: All non-zero digits are significant, and zeros "sandwiched" between non-zero digits are also significant.
Summary of Significant Figures

Number	Significant Figures	Key Reason
0.0063	2	Leading zeros are placeholders only.
132.00	5	Trailing zeros with a decimal show precision.

1004	4	Zeros between non-zero digits are significant.

49. **(b)** Step 1: Formula

$$W_{max} = -nRT\ln\frac{P_2}{P_1}$$

Step 2: Number of moles

$$n = \frac{\text{mass}}{\text{molar mass}} = \frac{10}{2} = 5 \text{ mol}$$

Step 3: Plug values

$$W_{max} = -(5)(8.3)(273)\ln\frac{1}{10} = (5)(8.3)(273)\ln 10$$

$\ln 10 \approx 2.303$

$W_{max} = 5 \times 8.3 \times 273 \times 2.303 \approx 26077 \text{ J} \approx 26.1 \text{ kJ}$

Step 4: Sign

Since the gas expands, work done on surroundings is positive.

+26.09 kJ

50. **(b)** Step 1: Relation between $\mathbf{\Delta G^\circ}$ and **K**

$\Delta_r G^\circ = -RT\ln K$

$$\ln K = -\frac{\Delta_r G^\circ}{RT}$$

Given:

$\Delta_r G^\circ = 165.469 \text{ kJ/mol} = 165469 \text{ J/mol}$

$T = 298 \text{ K}, R = 8.3 \text{ J/mol}$

Step 2: Calculate In K

$$\ln K = -\frac{165469}{8.3 \times 298} = -\frac{165469}{2473.4} \approx -66.9$$

Step 3: Convert In K to K

$K = e^{-66.9} \approx 10^{-29}$ (since $\ln 10 \approx 2.303$)

$= 10^{-29}$

51. **(c)** Given:

- K_{sp} (AgBr) $= 4 \times 10^{-13}$
- $[Br^-]_{\text{from KBr}} = 0.1M$

Step 1: Write the solubility product expression

$AgBr(s) \leftrightarrow Ag^+ + Br^-$

$K_{sp} = [Ag^+][Br^-]$

Step 2: Include the common ion

Let the solubility of AgBr in 0.1 M KBr be **s**. Then:

$[Ag^+] = s, [Br^-] = 0.1 + s \approx 0.1$(since $s \ll 0.1$)

$K_{sp} = s \cdot 0.1$

$4 \times 10^{-13} = s \cdot 0.1$

Step 3: Solve for s

$$s = \frac{4 \times 10^{-13}}{0.1} = 4 \times 10^{-12} M$$

52. **(a)** Fraction of B atoms: $x = 0.2$

So B_2 fraction $= 1 - x = 0.8$

Kp for $B_2 \rightleftharpoons 2B$:

$$K_p = \frac{(P_B)^2}{P_{B_2}} = \frac{(0.2)^2}{0.8} = \frac{0.04}{0.8} = 0.05$$

53. **(c)** Given:

• $V_{KMnO_4} = 200\text{ mL} = 0.2\text{ L}, C = 0.4M$
• $10volH_2O_2 \rightarrow 1\text{ mL releases }10\text{ mL}O_2 \rightarrow 1\text{ mL}H_2O_2 \approx 0.294mmolO_2$
Moles $KMnO_4$: $0.2 \times 0.4 = 0.08$ mol
Moles H_2O_2 required: $\frac{5}{2} \times 0.08 = 0.2$ mol
Volume of $10volH_2O_2$:
$V = \frac{0.2\text{ mol}}{3.36\text{ mol/L}} \approx 0.224\text{ L} = 224\text{ mL}$

54. **(b)** Explanation: In alkaline earth metals, thermal stability of sulphates increases down the group. For example, $MgSO_4$ decomposes more easily than $BaSO_4$. So, it is wrong to say all are thermally stable.

55. **(c)** Here is the breakdown:
•Statement-I is correct: The electronegativity trend in Group 13 is unusual. It decreases from B(2.0) to AI(1.5), then increases slightly through In(1.7) to TI(1.8) due to poor shielding by d and f electrons. The specific order is B > TI > In > Ga > Al (or simply B > TI > Al > In as presented in many textbooks).
•Statement-II is incorrect: Boric acid (H_3BO_3) is a weak non-protic acid. It does not release H^+ions on its own; instead, it acts as a Lewis acid by accepting an OH^-ion from water, which then releases a proton from the water molecule.

56. **(a)** (a). $[SiCl_6]^{2-}$ - Silicon is small (period 3), high charge density; it cannot accommodate 6 bulky Cl^-ligands. So does not exist.
(b). $[GeCl_6]^{2-}$ - Germanium is larger, can form octahedral $GeCl_6{}^{2-}$. Exists.
(c). $[SiF_6]^{2-}$ - Fluoride is small; Si can accommodate 6 F^-ligands → exists.
(d). $[Sn(OH)_6]^{2-}$ – Tin(IV) forms $[Sn(OH)_6]^{2-}$ → exists.
$[SiCl_6]^{2-}$

57. **(c)**•Assertion (A) is correct: Carbon monoxide (CO) is highly poisonous because it reduces the blood's ability to carry oxygen.
•Reason (R) is incorrect: CO binds to hemoglobin to form carboxy haemoglobin, which is actually about 300 times more stable than the oxygen-haemoglobin complex (oxy haemoglobin). Because it is so stable, it prevents hemoglobin from binding with oxygen, leading to oxygen deprivation in the body.

58. **(c)** Vinylbenzene ($C_6H_5 - CH = CH_2$)
Step 1: $KMnO_4 + KOH$ → This is oxidative cleavage of the double bond under basic conditions.
•Vinyl group ($-CH = CH_2$) → – COOK (potassium benzoate) after oxidation.
Step 2: → Heating → decarboxylation (loss of CO_2) → Benzene (C_6H_6), which is X .
Step 3: NaOH + CaO → This is Kolbe's electrolysis (sodium salt of carboxylic acid +CaO → decarboxylation of salts to give alkane).
•Here, **X** (benzene) → **Y** → still benzene essentially.
Now, the question asks: ' Y ' can also be formed from?
•Kolbe's decarboxylation of sodium salt of carboxylic acid → gives alkanes.
•To get benzene, we need aromatisation of n-hexane ($C_6H_{14} \rightarrow C_6H_6$).
Aromatisation of n-hexane

59. **(a)** To determine the correct IUPAC name, we follow these steps:
Identify the longest chain → dec (10 carbons).
Number the chain from the end closest to the highest priority functional group → $-OH$ (alcohol) gets priority.
Locate substituents and double bond according to numbering.
Assign numbers to substituents: bromine (Br), methyl, ethyl.
From the options:
Alcohol at C-7, bromine at C-4, ethyl at C-6, methyl at C-9, double bond at C-5 → matches

60. **(b)** For an FCC lattice:
•Closest distance $x = \frac{a}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}Å$
•Density $y = \frac{4M}{a^3N_A} = \frac{4.197}{(4\times10^{-8})^3\cdot6\times10^{23}} 20.52\text{ g/cm}^3$
$= 2\sqrt{2}, 20.52$

61. **(c)** Step 1: Molar mass of ethylene glycol $C_2H_6O_2$
$M = 2(12) + 6(1) + 2(16) = 24 + 6 + 32 = 62\text{ g/mol}$
Step 2: Moles of ethylene glycol

$n = \frac{248}{62} = 4 \text{ mol}$

Step 3: Mass of solvent in kg

$m_{solvent} = 200 \text{ g} = 0.2 \text{ kg}$

Step 4: Molality

$$\text{Molality} = \frac{n}{m_{solvent\,(kg)}} = \frac{4}{0.2} = 20 \text{ m}$$

62. **(c)** Freezing point depression depends on molality:

$\Delta T_f = K_f \cdot m$

Since both solutions freeze at the same temperature, their molalities are equal:

$m_{urea} = m_X$

Step 1: Molality of urea

$$m_{urea} = \frac{\text{moles of urea}}{\text{kg of water}} = \frac{7.5/60}{1} = 0.125 \text{ mol/kg}$$

Step 2: Let molar mass of X be M_X

$$m_X = \frac{15}{M_X} \text{ mol/kg}$$

Equating molalities:

$$\frac{15}{M_X} = 0.125 \Rightarrow M_X = \frac{15}{0.125} = 120 \text{ g/mol}$$

63. **(d)** The cell is:

$Zn(s)|Zn^{2+}(0.01M)||Ni^{2+}(0.1M)|Ni(s)$

•Standard potentials: $E^\circ_{Zn^{2+}/Zn} = -0.76 \text{ V}, E^\circ_{Ni^{2+}/Ni} = -0.25 \text{ V}$

•Cathode: Ni, Anode: Zn

$E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode} = -0.25 - (-0.76) = 0.51 \text{ V}$

Step 2: Nernst equation

$$E_{cell} = E^\circ_{cell} - \frac{0.06}{n} \log \frac{[Zn^{2+}]}{[Ni^{2+}]}$$

• $n = 2$ electrons transferred

• $[Zn^{2+}] = 0.01, [Ni^{2+}] = 0.1$

$$E_{cell} = 0.51 - 0.06 \cdot \frac{1}{2} \log \frac{0.01}{0.1} = 0.51 - 0.03\log(0.1/1) = 0.51 - 0.03\log(0.1)$$

$\log(0.1) = -1$

$E_{cell} = 0.51 - 0.03 \cdot (-1) = 0.51 + 0.03 = 0.54 \text{ V}$

64. **(d)** For a first-order reaction, Rate $\propto$ [A]:

•$k = \text{Rate}/[A] = 0.2/0.02 = 10$

• $x = 0.4/10 = 0.04$

• $y = 1.0/10 = 0.10$

So, $x : y = 0.04 : 0.10 = 2 : 5$

65. **(d)** Let's analyze each statement:

(I). Sulphur sol → Many S_8 molecules dispersed → multimolecular colloid

(II). Starch sol → Large starch molecules dispersed → macromolecular colloid, not associated colloid

(III). Artificial rubber → Polymer molecules dispersed → macromolecular colloid

66. **(a)** Let's identify the enzymes:

(I). Reaction I:

Sucrose → Glucose + Fructose

•This is hydrolysis of sucrose.

Enzyme: Invertase

→ So, x = Invertase

(II). Reaction II:

Glucose → Ethanol + CO_2

•This is fermentation of glucose.

•Enzyme: Zymase

→ So, y = Zymase

Invertase, Zymase

67. **(c)** Let's analyze:

•Kaolinite → $Al_2Si_2O_5(OH)_4$ → ore of Aluminum (Al)
•Malachite → $Cu_2CO_3(OH)_2$ → ore of Copper (Cu)
So, $x = AI, y = Cu$

68. **(d)** Here is the breakdown:

(1). Identify Gas X: In Deacon's process, chlorine gas (Cl_2) is obtained by the oxidation of hydrogen chloride gas by atmospheric oxygen in the presence of a $CuCl_2$ catalyst. Thus, Gas X is Chlorine (Cl_2).
(2). The Reaction: When chlorine reacts with iodine (I_2) in the presence of water, it acts as an oxidizing agent.
(3). Chemical Equation:
$I_2 + 5Cl_2 + 6H_2O \rightarrow 2HIO_3 + 10HCl$
The product formed is iodic acid (HIO_3).

69. **(d)** Brass, 'Silver' UK coin

• $X(Cu + Zn) \rightarrow$ Brass
• $Y(Cu + Ni) \rightarrow$ 'Silver' used in UK coins (Cu-Ni alloy)

70. **(c)** Geometrical isomerism occurs in $[Co(\text{ en })(NH_3)_2Cl_2]Cl_1[Co(NH_3)_4Cl_2]Cl, [Co(\text{ en })_2Cl_2]Br$.
$[Co(en)_3]_5Cl_3$ does not show geometrical isomerism (only optical).

71. **(d)**High density polythene

•Ziegler-Natta catalyst → used for HDPE and stereoregular polymers.
•LDPE is made by free radical polymerization, not Ziegler-Natta.

72. **(c)** It is a highly branched polymer of α-D-(+)-glucose units

•Amylose is linear (not highly branched), with $\alpha - 1$,4-glycosidic linkages.
•Amylopectin is the branched component of starch.

73. **(a) X** → Adrenal Cortex: Its dysfunction → Addison's disease (deficiency of cortisol/aldosterone).
Y → Estradiol: Responsible for secondary female sexual characteristics.

74. **(d)**Phenelzine → MAO inhibitor (antidepressant), not an antacid.
Cimetidine, Ranitidine, $NaHCO_3$ → antacids/acid reducers.

75. **(c)**

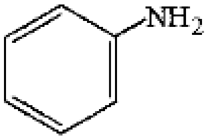
76. **(c)**

77. **(c)** (a) z gives yellow precipitate of CHI_3 with $NaOH + I_2$ solution: Correct. Acetone contains a methyl ketone group ($CH_3CO -$), which gives a positive iodoform test.
(b) z gives isopropyl alcohol on reduction with H_2 in the presence of Pd catalyst: Correct. Acetone (CH_3COCH_3) reduced with H_2/Pd yields isopropyl alcohol $(CH_3CH(OH)CH_3)$.
(c) z on reaction with CH_3MgBr followed by hydrolysis gives 2° alcohol: Incorrect. Acetone reacts with CH_3MgBr (Grignard reagent) followed by hydrolysis to give a tertiary (3°) alcohol, specifically tert-butyl alcohol $((CH_3)_3COH)$.
(d) z does not give positive test with Fehling's reagent: Correct. Acetone is a ketone and does not reduce Fehling's solution (only aldehydes do).
z on reaction with CH_3MgBr followed by hydrolysis gives 2° alcohol

78. **(d)**

79. **(c)**

80. **(b)**



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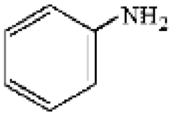

MATHEMATICS

81. **(d)**For $f(x) = \cos x - 3$:

•Domain: cosx is defined for all real x ⇒ Domain = ℝ
•Range:
$\cos x \in [-1,1]$
So, $\cos x - 3 \in [-1 - 3, 1 - 3] = [-4, -2]$
ℝ and $[-4, -2]$

82. **(a)** $(gof)(x) = g(f(x)) = g(2x-3) = 5(2x-3)^2 - 2$

Since $(2x-3)^2 \geq 0$, its minimum value is 0 (when $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$).

So, Minimum of $(gof)(x) = 5(0) - 2 = -2$

83. **(a)** To find the correct value for x such that $1^3 + 2^3 + 3^3 + \cdots + n^3 > x$ for all $n \in \mathbb{N}$, we first use the standard formula for the sum of the cubes of the first n natural numbers.

(I). Identify the Sum Formula

The sum of the cubes of the first n natural numbers is given by:

$$S_n = 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Expanding this formula gives:

$$S_n = \frac{n^2(n+1)^2}{4}$$

(II). Compare the Sum with the Given Options

We need to determine which of the following options is always strictly less than S_n for all $n \in \mathbb{N}$.

(a) $\frac{n^2}{4}$: Since $n \geq 1$, we have $(n+1)^2 > 1$. Therefore, $\frac{n^2(n+1)^2}{4} > \frac{n^2(1)}{4}$, which means $S_n > \frac{n^2}{4}$ is always true.

(b) n^2 : For $n = 1, S_1 = 1^3 = 1$. The option is $1^2 = 1$. Since $1 \not> 1$, this is false for $n = 1$.

(c) n^4 : For $n = 2, S_2 = 1^3 + 2^3 = 9$. The option is $2^4 = 16$. Since $9 \not> 16$, this is false.

(d) $\frac{n^2(n+1)^2}{4}$: This is exactly equal to S_n. Since the condition requires $S_n > x$, and S_n is not strictly greater than itself, this is false.

84. **(b)** (i) False: For matrices, $AB = 0$ does not imply $A = 0$ or $B = 0$ (non-zero matrices can multiply to zero).

(ii) True: If $AB = I_3$, then A is invertible and $A^{-1} = B$.

(iii) False:

$(A-B)^2 = A^2 - AB - BA + B^2$, not $A^2 - 2AB + B^2$ (since generally $AB \neq BA$).

85. **(b)** $AA^T - A - A^T =$

$$\begin{bmatrix} 4 & -8 & 12 \\ -8 & 16 & -28 \\ 12 & -28 & 47 \end{bmatrix}$$

86. **(a)** Trace $A = x + y - 1 = 0 \Rightarrow x + y = 1$

Determinant:

$$\begin{vmatrix} x & 2 & 1 \\ -2 & y & 0 \\ 2 & 0 & -1 \end{vmatrix} = x(-y) - 2(2) + 1(-2y) = -xy - 4 - 2y$$

Given $= -6$:

$-xy - 4 - 2y = -6 \Rightarrow xy + 2y = 2 \Rightarrow y(x+2) = 2$

Using $x + y = 1 \Rightarrow x = 1 - y$:

$y(3-y) = 2 \Rightarrow y^2 - 3y + 2 = 0 \Rightarrow y = 1$ or 2

Non-zero $\Rightarrow$ both valid, but $x = 1 - y$ gives:

•If $y = 1, x = 0$ (not allowed)

•So $y = 2, x = -1$

Minor of element 1 (position (1,3)):

$$\begin{vmatrix} -2 & y \\ 2 & 0 \end{vmatrix} = (-2)(0) - (2y) = -2y = -4$$

87. **(b)** Using periodicity i^n (cycle of 4):

First sum:

$$\sum_{n=2}^{30} i^n$$

Since cycle length $= 4$, compute from i^2 to i^{30}.

Total terms $= 29$.

Full cycles = 28 terms $\Rightarrow$ sum $= 0$

Remaining term $= i^{30} = i^2 = -1$

So,

$$\sum_{n=2}^{30} i^n = -1$$

Second sum:

$$\sum_{n=30}^{65} i^{n+3} = \sum_{k=33}^{68} i^k$$

Total terms = 36 (multiple of 4) $\Longrightarrow$ sum = 0

88. **(a)** Let z_1, z_2 be two n^{th} roots of unity.

Roots of unity lie on the unit circle at angles:

$$\theta = \frac{2\pi r}{n}$$

If the line segment joining z_1 and z_2 subtends a right angle at the origin, then the angle between their position vectors is:

$$\frac{\pi}{2}$$

So,

$$\left|\frac{2\pi(r_1 - r_2)}{n}\right| = \frac{\pi}{2}$$

$$\frac{2\pi m}{n} = \frac{\pi}{2}\ (m \in \mathbb{Z})$$

$$\Rightarrow \frac{2m}{n} = \frac{1}{2} \Rightarrow n = 4m$$

89. **(c)** Let

$$z = \sqrt{\sqrt{2}+1} + i\sqrt{\sqrt{2}-1}$$

First find modulus:

$$|z| = \sqrt{(\sqrt{\sqrt{2}+1})^2 + (\sqrt{\sqrt{2}-1})^2} = \sqrt{(\sqrt{2}+1) + (\sqrt{2}-1)} = \sqrt{2\sqrt{2}} = 2^{3/4}$$

Now argument:

$$\tan\theta = \frac{\sqrt{\sqrt{2}-1}}{\sqrt{\sqrt{2}+1}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \sqrt{(\sqrt{2}-1)^2} = \sqrt{2}-1$$

We know:

$$\tan\frac{\pi}{8} = \sqrt{2}-1 \Rightarrow \theta = \frac{\pi}{8}$$

So,

$$z = 2^{3/4}\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$$

Now,

$$z^8 = \left(2^{3/4}\right)^8(\cos\pi + i\sin\pi) = 2^6(-1) = -64$$

90. **(c)** Let roots be α, β.

Harmonic mean:

$$HM = \frac{2\alpha\beta}{\alpha+\beta}$$

Given quadratic:

$$\sqrt{2}x^2 - bx + (8 - 2\sqrt{5}) = 0$$

So,

$$\alpha+\beta = \frac{b}{\sqrt{2}},\ \alpha\beta = \frac{8-2\sqrt{5}}{\sqrt{2}}$$

Using HM = 4 :

$$\frac{2\alpha\beta}{\alpha+\beta} = 4$$

Substitute:

$$\frac{2\cdot\frac{8-2\sqrt{5}}{\sqrt{2}}}{\frac{b}{\sqrt{2}}}=4$$

Simplify:

$$\frac{2(8-2\sqrt{5})}{b}=4$$

$$\frac{16-4\sqrt{5}}{b}=4$$

$$b=\frac{16-4\sqrt{5}}{4}=4-\sqrt{5}$$

91. (a) We have,

$2kx^2-(4k+1)x+2<0$

$\because 2kx^2-4kx-x+2<0$

$2kx(x-2)-1(x-2)<0$

$(x-2)(2kx-1)<0$

$2<x<\frac{1}{2k}$

Expression is negative for exactly 3 integral values of x which is $x=3,4,5$

$5<x\leq 6$

$\because \frac{1}{2k}\leq 6\Rightarrow k\geq\frac{1}{12}$

$\therefore \frac{1}{2k}>5\Rightarrow k<\frac{1}{10}$

$\therefore k\in\left[\frac{1}{12},\frac{1}{10}\right)$

92. (c) Factor the equation:

$4x^4+4x^3-23x^2-12x+36=(x+2)^2(2x-3)^2$

So the multiple roots are:

$\beta=-2,\alpha=\frac{3}{2}(\text{ since }\alpha>\beta)$

Now,

$2\alpha-\beta=2\left(\frac{3}{2}\right)-(-2)=3+2=5$

93. (a) Let

$\beta=a+1,\gamma=a-1\ (\because \beta-\gamma=2)$

Then

$\beta+\gamma=2a$

Using sum of roots:

$\alpha+\beta+\gamma=13\Rightarrow\alpha+2a=13\Rightarrow\alpha=13-2a$

Using product:

$\alpha\beta\gamma=-189$

$(13-2a)(a^2-1)=-189$

Solving:

$(13-2a)(a^2-1)=-189\Rightarrow 2a^3-13a^2-2a-176=0$

Trial gives $a=8$

So,

$\beta+\gamma=16,\alpha=13-16=-3$

Now,

$k=\alpha\beta+\beta\gamma+\gamma\alpha$

$=2a\alpha+(a^2-1)=16(-3)+(64-1)=-48+63=15$

Thus,

$\beta+\gamma:k+\alpha=16:(15-3)=16:12=4:3$

94. (d) Prime factorization: $30=2^1\cdot 3^1\cdot 5^1$

For each prime, distribute exponent among x,y,z :

Number of non-negative solutions of $a+b+c=1=\binom{3}{2}=3$

So total solutions:

$3\times3\times3=27$

95. **(d)**Here is the quick breakdown:

(I). Letter counts: $N(4), E(3), I(2), C(2), O(1), V(1)$. Total unique letters $=6$.

(II). Total 5-letter words: Calculated by summing all possible combinations ($4+1,3+2,3+1+1,2+2+1,2+1+1+1$, and $1+1+1+1+1$) which equals 3925 .

(III). Words with all distinct letters: $\binom{6}{5}\times5!=6\times120=720$.

(IV). Words with at least one repetition: Total words - Distinct words = 3925-720 = 3205.

96. **(c)**Vowels in PERFECTION: E, E, I, O (4 vowels)

Consonants: 6 distinct

Condition ⇒ pattern must be:

v ___ V ___ V ___ V (exactly 2 consonants between vowels)

So:

•Arrange 6 consonants in 6 places → 6 !

•Arrange vowels $(E, E, I, O)\rightarrow\frac{4!}{2!}$

Total ways $=6!\times\frac{4!}{2!}=6!\times12=8640$

$=2!\cdot3!\cdot6!$

97. **(d)** Given $C_r=\binom{n}{r}$

Expression:

$C_0+(C_0+C_1)+(C_0+C_1+C_2)+\cdots+(C_0+\cdots+C_n)$

Rewrite by counting contributions:

• C_0 appears $n+1$ times

• C_1 appears n times

• C_2 appears $n-1$ times

• C_n appears 1 time

So sum =

$$\sum_{r=0}^{n}(n+1-r)\binom{n}{r}$$

Split:

$$=(n+1)\sum\binom{n}{r}-\sum r\binom{n}{r}$$

Use identities:

$$\sum\binom{n}{r}=2^n,\sum r\binom{n}{r}=n2^{n-1}$$

So:

$=(n+1)2^n-n2^{n-1}=2^{n-1}(2n+2-n)=2^{n-1}(n+2)$

$(n+2)2^{n-1}$

98. **(b)**$\left(\frac{3x-5}{4x^2+3}\right)^{-4/5}=\left(\frac{4x^2+3}{3x-5}\right)^{4/5}$

Factor highest powers:

$$=\left(\frac{4x^2\left(1+\frac{3}{4x^2}\right)}{3x\left(1-\frac{3}{3x}\right)}\right)^{4/5}=\left(\frac{4x}{3}\cdot\frac{1+\frac{3}{4x^2}}{1-\frac{5}{3x}}\right)^{4/5}$$

Now expand:

$$\frac{1}{1-\frac{5}{3x}}\approx1+\frac{5}{3x}+\frac{25}{9x^2}$$

So,

$$\frac{1+\frac{3}{4x^2}}{1-\frac{5}{3x}}\approx\left(1+\frac{3}{4x^2}\right)\left(1+\frac{5}{3x}+\frac{25}{9x^2}\right)$$

Multiply:
$$\approx 1+\frac{5}{3x}+\left(\frac{25}{9}+\frac{3}{4}\right)\frac{1}{x^2}=1+\frac{5}{3x}+\frac{127}{36x^2}$$
Now raise to power 4/5:
$$(1+u)^{4/5}\approx 1+\frac{4}{5}u-\frac{2}{25}u^2$$
Take $u=\frac{5}{3x}+\frac{12n^2}{36x^2}$
First term:
$$\frac{4}{5}\cdot\frac{5}{3x}=\frac{4}{3x}$$
Second-order terms:
$$\frac{4}{5}\cdot\frac{127}{36x^2}=\frac{127}{45x^2},-\frac{2}{25}\cdot\frac{25}{9x^2}=-\frac{2}{9x^2}$$
So total:
$$\frac{127}{45}-\frac{2}{9}=\frac{127-10}{45}=\frac{117}{45}=\frac{13}{5}$$
Thus:
$$\approx\left(\frac{4x}{3}\right)^{4/5}\left(1+\frac{4}{3x}+\frac{13}{5x^2}\right)$$

99. **(c)** We have,
$$\frac{H(x)}{(x-1)(x+1)(x-2)}=f(x)+\frac{g(x)}{(x-1)(x+1)(x-2)}$$
$\Rightarrow H(x)=(x-1)(x+1)(x-2)f(x)+g(x)$
$\Rightarrow H(-1)=0+g(-1)=g(-1)$
$H(2)=0+g(2)=g(2)$
$H(1)=0+g(1)=g(1)$
$\therefore H(-1)+2H(2)-3H(1)$
$=g(-1)+2g(2)-3g(1)$

100. **(a)** Reduce the angle:
$630^\circ<\theta<810^\circ\Rightarrow -90^\circ<\theta-720^\circ<90^\circ$
Soθ is in 4th quadrant $\Rightarrow \cos\theta>0,\sin\theta<0.$
Given $\tan\theta=-\frac{7}{24}\Rightarrow$ triangle gives
$$\cos\theta=\frac{24}{25}$$
Now,
$$\cos\frac{\theta}{2}=\sqrt{\frac{1+\cos\theta}{2}}=\sqrt{\frac{1+\frac{24}{25}}{2}}=\sqrt{\frac{49}{50}}=\frac{7}{5\sqrt{2}}$$
Then,
$$\cos\frac{\theta}{4}=\sqrt{\frac{1+\cos(\theta/2)}{2}}=\sqrt{\frac{1+\frac{7}{5\sqrt{2}}}{2}}=\sqrt{\frac{7+5\sqrt{2}}{10\sqrt{2}}}$$
$=-\sqrt{\frac{7+5\sqrt{2}}{10\sqrt{2}}}$

101. **(b)** $2\cos\theta+\sin\theta=1$
Let $\cos\theta=c,\sin\theta=s$
Then $s=1-2c$
Using identity:
$c^2+s^2=1\Rightarrow c^2+(1-2c)^2=1\Rightarrow 5c^2-4c=0\Rightarrow c(5c-4)=0$
So,
- $c=0\Rightarrow s=1\Rightarrow k=7(0)+6(1)=6$
- $c=\frac{4}{5}\Rightarrow s=-\frac{3}{5}\Rightarrow k=\frac{28}{5}-\frac{18}{5}=2$

102. **(d)** $\sum_{k=0}^{12} \frac{1}{\sin\left((k+1)\frac{\pi}{6}+\frac{\pi}{4}\right)\sin\left(k\frac{\pi}{6}+\frac{\pi}{4}\right)}$

Use identity:

$$\frac{1}{\sin A \sin B} = \frac{\cot B - \cot A}{\sin(A-B)}$$

Here,

$$A = (k+1)\frac{\pi}{6} + \frac{\pi}{4}, B = k\frac{\pi}{6} + \frac{\pi}{4}$$

So,

$$A - B = \frac{\pi}{6}, \sin(A-B) = \frac{1}{2}$$

Thus term becomes:

$2(\cot B - \cot A)$

Sum telescopes:

$$\sum = 2\left[\cot\left(\frac{\pi}{4}\right) - \cot\left(13\frac{\pi}{6} + \frac{\pi}{4}\right)\right]$$

$$13\frac{\pi}{6} + \frac{\pi}{4} = \frac{29\pi}{12}$$

$$\cot\left(\frac{\pi}{4}\right) = 1, \cot\left(\frac{29\pi}{12}\right) = \cot\left(\frac{5\pi}{12}\right) = 2 - \sqrt{3}$$

So,

$$= 2[1 - (2 - \sqrt{3})] = 2(\sqrt{3} - 1)$$

103. **(b)** Given

$2\sin^2\theta - 3\cos^2\theta = \sin\theta\cos\theta$

Divide by $\cos^2\theta$ (check separately $\cos\theta = 0$):

$2\tan^2\theta - 3 = \tan\theta$

$2t^2 - t - 3 = 0$ $(t = \tan\theta)$

$(2t + 3)(t - 1) = 0$

$$t = 1 \text{ or } t = -\frac{3}{2}$$

Now count solutions in $(-\pi, \pi)$:

• $\tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}, -\frac{3\pi}{4} \to 2$ solutions

• $\tan\theta = -\frac{3}{2} \to 2$ solutions in one period $\to 2$ solutions

Also check $\cos\theta = 0 \to$ no solution.

Total $= 2 + 2 = 4$

104. **(a)** $\sqrt{8 - 2\sqrt{15}} = \sqrt{(\sqrt{5} - \sqrt{3})^2} = \sqrt{5} - \sqrt{3}$

So expression becomes:

$$\tan^{-1}\left(\frac{\sqrt{5} - \sqrt{3}}{\sqrt{15} + 1}\right) + \tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

Now use identity:

$$\tan^{-1}a + \tan^{-1}b = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$$

After simplification, this evaluates to:

$= \frac{\pi}{6} v$

105. **(a)** Given

$\cos\alpha = \operatorname{sech}\beta = 1/\cosh\beta$

$\Rightarrow \cosh\beta = \sec\alpha$

Now,

$\beta = \cosh^{-1}(\sec\alpha)$

Using identity:

$\cosh^{-1}x = \log\left(x + (x^2 - 1)\right)$

$\Rightarrow \beta = \log\left(\sec\alpha + (\sec^2\alpha - 1)\right)$

$\Rightarrow \beta = \log(\sec\alpha + \tan\alpha)$

106. **(a)** Given:

$a+b=x, ab=y, x^2-c^2=y$

Now,

$x^2=(a+b)^2=a^2+b^2+2ab$

So,

$x^2-c^2=a^2+b^2+2ab-c^2$

Given $x^2-c^2=y=ab$, hence:

$a^2+b^2+2ab-c^2=ab$

$a^2+b^2+ab=c^2$

Using identity:

$$a^2+b^2+ab=\frac{(a+b)^2+(a^2+b^2-ab)}{?}\text{(skip)}$$

Instead use cosine rule:

$c^2=a^2+b^2-2ab\cos C$

Compare with:

$c^2=a^2+b^2+ab$

So,

$$-2ab\cos C=ab\Rightarrow\cos C=-\frac{1}{2}\Rightarrow C=120^\circ$$

Circumradius:

$$R=\frac{c}{2\sin C}=\frac{c}{2\sin 120^\circ}=\frac{c}{2\cdot\frac{\sqrt{3}}{2}}=\frac{c}{\sqrt{3}}$$

107. **(c)** Given $\tan\frac{B}{2}=\frac{\sqrt{5}}{4}$

$$\sin B=\frac{2t}{1+t^2}=\frac{2(\sqrt{5}/4)}{1+5/16}=\frac{8\sqrt{5}}{21},\cos B=\frac{1-t^2}{1+t^2}=\frac{11}{21}$$

Area:

$$\Delta=\frac{1}{2}ac\sin B=4\sqrt{5}$$

$$\frac{1}{2}ac\cdot\frac{8\sqrt{5}}{21}=4\sqrt{5}\Rightarrow ac=21$$

Using cosine rule ($b=6$):

$36=a^2+c^2-2ac\cos B$

$$36=a^2+c^2-2(21)\cdot\frac{11}{21}\Rightarrow a^2+c^2=58$$

$(a+c)^2=a^2+c^2+2ac=58+42=100\Rightarrow a+c=10$

So,

$t^2-10t+21=0\Rightarrow t=3,7$

Sides are 3, 6, 7 .

Smallest side = 3 units

108. **(d)** Using exradii:

$$r_1=\frac{\Delta}{s-a},r_2=\frac{\Delta}{s-b},r_3=\frac{\Delta}{s-c}$$

Given $r_1=2r_2=3r_3$:

(1). $r_1=2r_2$

$$\frac{1}{s-a}=\frac{2}{s-b}\Rightarrow s-b=2(s-a)\Rightarrow 2a-b=s$$

(2). $r_1=3r_3$

$$\frac{1}{s-a}=\frac{3}{s-c}\Rightarrow s-c=3(s-a)\Rightarrow 3a-c=2s$$

Now use $s=\frac{a+b+c}{2}$:

From $2a-b=s$:

$$2a-b=\frac{a+b+c}{2}\Rightarrow 3a=3b+c$$

From $3a - c = 2s = a + b + c$:
$3a - c = a + b + c \Rightarrow 2a = b + 2c$
Solve (1) & (2):
$c = 3a - 3b$
$2a = b + 2(3a - 3b) = 6a - 5b \Rightarrow 4a = 5b$
$a : b = 5 : 4$

109. **(b)** Given position vectors:
$\vec{A} = (2, -1, -1), \vec{B} = (5, 1, -2), \vec{C} = (-13, -11, 4)$
$\overrightarrow{AB} = \vec{B} - \vec{A} = (3, 2, -1)$
$\overrightarrow{BC} = \vec{C} - \vec{B} = (-18, -12, 6)$
Given $\overrightarrow{AB} = \lambda\overrightarrow{BC}$:
$$(3, 2, -1) = \lambda(-18, -12, 6) \Rightarrow \lambda = -\frac{1}{6}$$
Now,
$\overrightarrow{AC} = \vec{C} - \vec{A} = (-15, -10, 5)$
$\overrightarrow{CB} = \vec{B} - \vec{C} = (18, 12, -6)$
Given $\overrightarrow{AC} = \mu\overrightarrow{CB}$:
$$(-15, -10, 5) = \mu(18, 12, -6) \Rightarrow \mu = -\frac{5}{6}$$
$$\lambda + \mu = -\frac{1}{6} - \frac{5}{6} = -1$$

110. **(b)** Given $\vec{A} = \vec{a}, \vec{B} = \vec{b}$
$\overrightarrow{AB} = \vec{b} - \vec{a}$
For point C:
$\overrightarrow{AC} = 3\overrightarrow{AB} = 3(\vec{b} - \vec{a})$
$\vec{C} = \vec{A} + \overrightarrow{AC} = \vec{a} + 3(\vec{b} - \vec{a}) = 3\vec{b} - 2\vec{a}$
For point D:
$\overrightarrow{BA} = \vec{a} - \vec{b}$
$\overrightarrow{BD} = 2\overrightarrow{BA} = 2(\vec{a} - \vec{b})$
$\vec{D} = \vec{B} + \overrightarrow{BD} = \vec{b} + 2(\vec{a} - \vec{b}) = 2\vec{a} - \vec{b}$
Now,
$\overrightarrow{CD} = \vec{D} - \vec{C} = (2\vec{a} - \vec{b}) - (3\vec{b} - 2\vec{a})$
$= 2\vec{a} - \vec{b} - 3\vec{b} + 2\vec{a} = 4\vec{a} - 4\vec{b}$
$4\vec{a} - 4\vec{b}$

111. **(c)** Given unit vectors $\Rightarrow |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$
$|\vec{a} - \vec{b}|^2 = 2 - 2\vec{a} \cdot \vec{b}$
Similarly for others,
$$\Rightarrow \sum |\vec{a} - \vec{b}|^2 = 6 - 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a})$$
Given = 15:
$$6 - 2S = 15 \Rightarrow S = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = -\frac{9}{2}$$
Now,
$|\vec{a} - \vec{b} - \vec{c}|^2 = 3 - 2\vec{a} \cdot \vec{b} - 2\vec{a} \cdot \vec{c} + 2\vec{b} \cdot \vec{c}$
Required:
$|\vec{a} - \vec{b} - \vec{c}|^2 - 4(\vec{b} \cdot \vec{c})$
$= 3 - 2(\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{b} \cdot \vec{c}) = 3 - 2S$
$$= 3 - 2\left(-\frac{9}{2}\right) = 3 + 9 = 12$$

112. **(b)** $\vec{a} = (1, p, -3), \vec{b} = (p, -3, 1), \vec{c} = (-3, 1, 2)$
Given:
$|\vec{a} \times \vec{b}| = |\vec{a} \times \vec{c}| \Rightarrow |\vec{a} \times \vec{b}|^2 = |\vec{a} \times \vec{c}|^2$

Using:
$|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2$
Compute:
$|\vec{a}|^2 = p^2 + 10, |\vec{b}|^2 = p^2 + 10, |\vec{c}|^2 = 14$
$\vec{a}\cdot\vec{b} = -2p - 3, \vec{a}\cdot\vec{c} = p - 9$
So, $(p^2 + 10)^2 - (2p + 3)^2 = (p^2 + 10)\cdot 14 - (p - 9)^2$
Simplifying:
$p^4 + 3p^2 - 30p + 32 = 0$
Testing options $\Rightarrow p = 2$ satisfies.

113. **(b)** Use identity:
$(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d}) = [\vec{a}\cdot(\vec{c}\times\vec{d})]\vec{b} - [\vec{b}\cdot(\vec{c}\times\vec{d})]\vec{a}$
Compute:
$$\vec{c}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 1 & 1 \end{vmatrix} = (-3\hat{i} - \hat{j} + 4\hat{k})$$
Now,
$\vec{a}\cdot(\vec{c}\times\vec{d}) = (2,-3,4)\cdot(-3,-1,4) = -6 + 3 + 16 = 13$
$\vec{b}\cdot(\vec{c}\times\vec{d}) = (1,2,-1)\cdot(-3,-1,4) = -3 - 2 - 4 = -9$
So, $= 13\vec{b} - (-9)\vec{a} = 13\vec{b} + 9\vec{a}$
$= 13(1,2,-1) + 9(2,-3,4) = (13 + 18, 26 - 27, -13 + 36) = (31,-1,23)$
$31\vec{i} - \vec{j} + 23\vec{k}$

114. **(c)** Mean $= (2 + 12 + 3 + 11 + 5 + 10 + 6 + 7)/8 = 56/8 = 7$
Squared deviations:
$(-5)^2 + 5^2 + (-4)^2 + 4^2 + (-2)^2 + 3^2 + (-1)^2 + 0^2$
$= 25 + 25 + 16 + 16 + 4 + 9 + 1 + 0 = 96$
Variance $= 96/8 = 12$

115. **(b)** Given:
$P(A\cup B) = 3/4, P(A\cap B) = 1/4, P(\bar{A}) = 2/3$
First, $P(A) = 1 - P(\bar{A}) = 1 - 2/3 = 1/3$
Now use:
$P(A\cup B) = P(A) + P(B) - P(A\cap B)$
$3/4 = 1/3 + P(B) - 1/4$
$\Rightarrow P(B) = 3/4 - 1/3 + 1/4$
$\Rightarrow P(B) = (9/12 - 4/12 + 3/12) = 8/12 = 2/3$
Now,
$P(\bar{A}\cap B) = P(B) - P(A\cap B)$
$= 2/3 - 1/4$
$= (8/12 - 3/12) = 5/12$

116. **(d)** Given both cards are black $\rightarrow$ sample space $= \binom{26}{2} = 325$
Black face cards $= 6 \Rightarrow$ non-face black cards $= 20$
No face card cases $= \binom{20}{2} = 190$
Favourable $= 325 - 190 = 135$
Probability $= \frac{135}{325} = \frac{27}{65}$

117. **(a)** Given:
A man is known to speak truth three out of four times.
He throws a die and observe that it is a 6 .
Probability that the man speaks the truth is $P(A) = \frac{3}{4}$
The probability that the man lies is $P(B) = 1 - P(A) = 1 - \frac{3}{4}$
$= \frac{1}{4}$

probability of getting 6 is $\frac{1}{6}$

probability of not getting 6 $= 1 - \frac{1}{6} = \frac{5}{6}$

$$P\left(\frac{A}{B}\right) = \frac{P\left(\frac{B}{A}\right) \cdot P(A)}{P(B)}$$

Applying Bayes' theorem, we get the required probability,

$$= \frac{\frac{1}{6} \times \frac{3}{4}}{\frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}} = \frac{3}{8}$$

Hence, the probability that it is actually a 6 $= \frac{3}{8}$

118. **(c)** Given:

- $P(\text{man}) = 0.7, P(\text{ woman }) = 0.3$
- $P(TA \mid \text{man }) = 0.3, P(TA \mid \text{woman }) = 0.15$
- We need P (man | TA).

Step 1: Use Bayes' theorem

$$P(\text{man} \mid TA) = \frac{P(TA \mid \text{man})P(\text{man})}{P(TA)}$$

where

$P(TA) = P(TA \mid \text{ man })P(\text{ man }) + P(TA \mid \text{ woman })P(\text{ woman })$

Step 2: Compute P(TA)

$P(TA) = (0.3)(0.7) + (0.15)(0.3) = 0.21 + 0.045 = 0.255$

Step 3: Compute P (man | TA)

$$P(\text{man} \mid TA) = \frac{0.3 \cdot 0.7}{0.255} = \frac{0.21}{0.255}$$

Divide numerator and denominator by 0.015 :

$$\frac{0.21}{0.255} = \frac{14}{17}$$

119. **(c)** We use the normalization condition:

$$\sum_{x=0}^{\infty} P(X = x) = 1$$

Given:

$$P(X = x) = k\frac{2^{2x+1}}{(2x + 1)!}$$

So,

$$k\sum_{x=0}^{\infty} \frac{2^{2x+1}}{(2x + 1)!} = 1$$

Now recall the expansion:

$$\sinh 2 = \sum_{n=0}^{\infty} \frac{2^{2n+1}}{(2n + 1)!}$$

Thus,

$$k \cdot \sinh 2 = 1 \Rightarrow k = \frac{1}{\sinh 2} = \operatorname{cosech} 2$$

120. **(b)** $np - npq = 1$

$\Rightarrow np^2 = 1$

$2^nC_2p^2q^{n-2} = 3^nC_1pq^{n-1}$

$\Rightarrow np - p = 3q$

$(\therefore q = 1 - p)$

$\Rightarrow p = \frac{1}{2}$

Hence $n = 4$

$P(x > 1) = 1 - (p(x = 0) + p(x = 1)$

$= 1 - \left({}^4C_0 \left(\frac{1}{2}\right)^4 + {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^3 \right)$
$= \frac{11}{16}$

121. **(a)** Let $P(x, y)$.
Distance from $\mathbf{A}(2, -2)$:
$\sqrt{(x-2)^2 + (y+2)^2}$
Distance from Y-axis $= |x|$
Given:
$\sqrt{(x-2)^2 + (y+2)^2} = 2|x|$
Square:
$(x-2)^2 + (y+2)^2 = 4x^2$
Expand:
$x^2 - 4x + 4 + y^2 + 4y + 4 = 4x^2$
$x^2 + y^2 - 4x + 4y + 8 = 4x^2$
$3x^2 - y^2 + 4x - 4y - 8 = 0$

122. **(c)** Translate origin to (α, β) so linear terms vanish:
$\frac{\partial f}{\partial x} = 4x + 3y - 17 = 0, \frac{\partial f}{\partial y} = 3x - 4y + 6 = 0$
At (α, β) :
$4\alpha + 3\beta = 17, 3\alpha - 4\beta = -6$
Solving:
$\alpha = 2, \beta = 3$
Now constant term:
$c = f(2,3) = 0$
So,
$3\alpha + c = 3(2) + 0 = 6 = 2\beta$

123. **(a)** Line: $x - y - 2 = 0 \Rightarrow y = x - 2$
Direction vector of line $= (1,1)$, unit vector:
$\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
Points at distance 4 from $P(6,4)$ on both sides:
$A = \left(6 + \frac{4}{\sqrt{2}}, 4 + \frac{4}{\sqrt{2}}\right) = (6 + 2\sqrt{2}, 4 + 2\sqrt{2})$
$B = \left(6 - \frac{4}{\sqrt{2}}, 4 - \frac{4}{\sqrt{2}}\right) = (6 - 2\sqrt{2}, 4 - 2\sqrt{2})$
Now:
$\alpha^2 + \beta^2 = (6 + 2\sqrt{2})^2 + (4 + 2\sqrt{2})^2$
$= (36 + 24\sqrt{2} + 8) + (16 + 16\sqrt{2} + 8) = 68 + 40\sqrt{2}$
$\gamma^2 + \delta^2 = (6 - 2\sqrt{2})^2 + (4 - 2\sqrt{2})^2 = 68 - 40\sqrt{2}$
Adding:
$= 136$

124. **(c)** Given angle bisector:
$\frac{2x + 3y + 1}{\sqrt{13}} = \pm \frac{3x + 2y + 4}{\sqrt{13}}$
Take negative sign:
$2x + 3y + 1 = -(3x + 2y + 4) \Rightarrow 5x + 5y + 5 = 0 \Rightarrow x + y + 1 = 0$
Pair of lines formed:
$(2x + 3y + 1)^2 = (3x + 2y + 4)^2$
Simplifying gives two lines:
One is $3x + 2y + 4 = 0$, other comes out as:
$9x - 23y - 26 = 0$

125. **(b)** Slopes satisfy: $6m^2 + 2hm + 1 = 0$

$m_1+m_2=-\frac{h}{3}, m_1m_2=\frac{1}{6}$

$\tan\theta=\frac{|m_1-m_2|}{1+m_1m_2}=\frac{1}{7}\Rightarrow\frac{\sqrt{\frac{h^2}{9}-\frac{2}{3}}}{\frac{7}{6}}=\frac{1}{7}$

$\sqrt{\frac{h^2}{9}-\frac{2}{3}}=\frac{1}{6}\Rightarrow h^2=\frac{25}{4}, h=-\frac{5}{2}$

Equation: $x^2-5xy+6y^2=0=(x-2y)(x-3y)$

Distances from $(1,0)$:

$d_1=\frac{1}{\sqrt{5}}, d_2=\frac{1}{\sqrt{10}}$

Product:

$d_1d_2=\frac{1}{5\sqrt{2}}$

126. **(c)**The angle between the positive coordinate axes is bisected by the line $\mathbf{y}=\mathbf{x}$.

Substitute $y=x$ into

$ax^2+2hxy+by^2=0$

$ax^2+2hx^2+bx^2=0$

$(a+2h+b)x^2=0$

For a non-trivial solution:

$a+2h+b=0$

127. **(d)** Let $P(x_1,y_1)$ lie on $x^2+y^2=4$.

Circle:

$x^2+y^2-6x-6y+14=0\Rightarrow(x-3)^2+(y-3)^2=4$

So center $C(3,3)$, radius 2 .

Chord of contact AB from P is:

$xx_1+yy_1-3(x+x_1)-3(y+y_1)+14=0$

Circle through P, A, B has center = midpoint of P and C (property of tangents).

So centre $(h,k)=\left(\frac{x_1+3}{2},\frac{y_1+3}{2}\right)$

Now since P lies on $x^2+y^2=4$:

$x_1^2+y_1^2=4$

Put $x_1=2h-3, y_1=2k-3$:

$(2h-3)^2+(2k-3)^2=4$

$4(h^2+k^2)-12(h+k)+18=4$

$h^2+k^2-3h-3k+\frac{7}{2}=0$

Multiply by 2:

$2h^2+2k^2-6h-6k+7=0$

$2x^2+2y^2-6x-6y+7=0$

128. **(c)** Let endpoints be (x,y) and $(-x,-y)$ on $x^2+y^2=4$.

Distance to line $x+y+1=0$:

$d=\frac{|x+y+1|}{\sqrt{2}}, d'=\frac{|-x-y+1|}{\sqrt{2}}$

Product:

$P=\frac{|(x+y+1)(1-x-y)|}{2}=\frac{|1-(x+y)^2|}{2}$

Maximize $\Rightarrow$ maximize $|1-(x+y)^2|$.

Since $x^2+y^2=4\Rightarrow(x+y)^2\le 8$,

Max occurs at $(x+y)^2=8\Rightarrow x+y=\pm2\sqrt{2}$.

Solving with $x^2+y^2=4\Rightarrow x=y=\sqrt{2}$ or $-\sqrt{2}$.

$(\sqrt{2},\sqrt{2}),(-\sqrt{2},-\sqrt{2})$

129. **(d)** Let centre be (h, k) and radius r.
Intercepts:
•On x-axis: $2\sqrt{r^2 - k^2} = 8 \Rightarrow r^2 - k^2 = 16$
•On y-axis: $2\sqrt{r^2 - h^2} = 6 \Rightarrow r^2 - h^2 = 9$
Subtract:
$(r^2 - k^2) - (r^2 - h^2) = 16 - 9 \Rightarrow h^2 - k^2 = 7$
$x^2 - y^2 - 7 = 0$

130. **(c)** Tangent from $(3,4)$ with slope m :
$y - 4 = m(x - 3)$
Distance from centre $(0,0)$ to line = radius 3 :
$$\frac{|-3m + 4|}{\sqrt{m^2 + 1}} = 3$$
Squaring:
$$(-3m + 4)^2 = 9(m^2 + 1) \Rightarrow 9m^2 - 24m + 16 = 9m^2 + 9 \Rightarrow -24m + 7 = 0 \Rightarrow m = \frac{7}{24}$$

131. **(a)** For circles
$S: x^2 + y^2 + 2kx + 4y - 3 = 0$ and $S_1: x^2 + y^2 - 4x + 2ky + 9 = 0$
Angle formula:
$$\cos\theta = \frac{2(g_1g_2 + f_1f_2) - (c_1 + c_2)}{2\sqrt{(g_1^2 + f_1^2 - c_1)(g_2^2 + f_2^2 - c_2)}}$$
Substitute:
$g_1 = k, f_1 = 2, c_1 = -3;\ g_2 = -2, f_2 = k, c_2 = 9$
Solving with $\cos\theta = \frac{3}{8}$ gives:
$k = -3$ (centre $(2, -k) = (2,3)$ in first quadrant)
Radical axis $= S - S_1 = 0$:
$(2k + 4)x + (4 - 2k)y - 12 = 0$
Put $k = -3$:
$-2x + 10y - 12 = 0 \Rightarrow x - 5y + 6 = 0$

132. **(d)** Parabola: $y^2 = 8x \Rightarrow a = 2$
Point at $t = \frac{1}{\sqrt{2}}$:
$(x, y) = (2t^2, 4t) = (1, 2\sqrt{2})$
Slope of normal $= -t = -\frac{1}{\sqrt{2}}$
Equation of normal:
$$y - 2\sqrt{2} = -\frac{1}{\sqrt{2}}(x - 1) \Rightarrow x + \sqrt{2}y - 5 = 0$$
Focus: $(2,0)$
Foot of perpendicular from $(2,0)$ to $x + \sqrt{2}y - 5 = 0$:
$= (3, \sqrt{2})$

133. **(d)** Tangent to ellipse $x^2 + 2y^2 = 2$ at (x_1, y_1) is:
$xx_1 + 2yy_1 = 2$
Intercepts:
•On x-axis $(y = 0)$: $x = \frac{2}{x_1}$
•On y-axis $(x = 0)$: $y = \frac{1}{y_1}$
Midpoint (h, k) :
$$h = \frac{1}{x_1}, k = \frac{1}{2y_1}$$
So:
$$x_1 = \frac{1}{h}, y_1 = \frac{1}{2k}$$
Substitute into ellipse:

$$\frac{1}{h^2}+2\left(\frac{1}{4k^2}\right)=2\Rightarrow\frac{1}{h^2}+\frac{1}{2k^2}=2$$
Divide by 2 :
$$\frac{1}{2h^2}+\frac{1}{4k^2}=1$$
$$\frac{1}{2x^2}+\frac{1}{4y^2}=1$$

134. **(a)** $m=\frac{b^2}{a^2e}=\frac{3}{2},b^2=36\Rightarrow\frac{36}{a^2e}=\frac{3}{2}\Rightarrow a^2e=24$
$$e^2=1+\frac{36}{a^2}\Rightarrow\left(\frac{24}{a^2}\right)^2=1+\frac{36}{a^2}\Rightarrow a^2=12,e=2$$
$$\sqrt{a^2+e^2}=\sqrt{12+4}=4$$

135. **(c)** Foci: (8,3) and (0,3) $\Rightarrow$ centre $(\alpha,\beta)=(4,3)$, so $\alpha=4,\beta=3$
$$c=4,e=\frac{4}{3}\Rightarrow a=\frac{c}{e}=\frac{4}{4/3}=3\Rightarrow a^2=9$$
$$c^2=a^2+b^2\Rightarrow 16=9+b^2\Rightarrow b^2=7$$
Thus $p=a^2=9,q=b^2=7$
$$p+q=16=\alpha^2$$

136. **(d)** Using centroid formula:
$$G=\frac{A+B+C}{3}\Rightarrow C=3G-A-B$$
$$3G=(3,0,3),A+B=(4,-3,2)$$
$$C=(3,0,3)-(4,-3,2)=(-1,3,1)$$
Now,
$$AG=G-A=(0,4,-1)\Rightarrow AG^2=17$$
$$CG=G-C=(2,-3,0)\Rightarrow CG^2=13$$
So,
$$AG^2+CG^2=17+13=30$$
$$BG=G-B=(-2,-1,1)\Rightarrow BG^2=6$$
Thus,
$$30=5\times 6=5BG^2$$

137. **(b)** Distance from axis:

•From X-axis: $\sqrt{y^2+z^2}=\sqrt{16+\alpha^2}$
•From Y-axis: $\sqrt{x^2+z^2}=\sqrt{9+\alpha^2}$
•From Z-axis: $\sqrt{x^2+y^2}=5$

Sum:
$$S=\sqrt{16+\alpha^2}+\sqrt{9+\alpha^2}+5$$
To minimize, differentiate:
$$\frac{dS}{d\alpha}=\frac{\alpha}{\sqrt{16+\alpha^2}}+\frac{\alpha}{\sqrt{9+\alpha^2}}=0$$
$$\alpha\left(\frac{1}{\sqrt{16+\alpha^2}}+\frac{1}{\sqrt{9+\alpha^2}}\right)=0\Rightarrow\alpha=0$$
So,
$$\sec\alpha=\sec 0=1$$

138. **(c)** Normals of given planes:
(3, −2,1) and (1,1,1)
Required plane ⊥ both ⇒ its normal | | cross product:
$$\vec{n}=(3,-2,1)\times(1,1,1)=(-3,-2,5)$$
So, let plane be:
$$lx+my+nz=1$$
with
$$(l,m,n)=k(-3,-2,5)$$
Use point (2, -1, 3):

$2l - m + 3n = 1$
Substitute:
$2(-3k) - (-2k) + 3(5k) = 1$
$-6k + 2k + 15k = 1$
$11k = 1 \Rightarrow k = \frac{1}{11}$
Thus:
$l = -\frac{3}{11}, m = -\frac{2}{11}, n = \frac{5}{11}$
Now:
$4m + 2n - 3l = 4\left(-\frac{2}{11}\right) + 2\left(\frac{5}{11}\right) - 3\left(-\frac{3}{11}\right)$
$= \frac{-8 + 10 + 9}{11} = \frac{11}{11} = 1$

139. **(a)** Using series expansion:

- $\cos x \approx 1 - \frac{x^2}{2}$

Numerator:

$$\sqrt{2} - \sqrt{1 + \cos x} \approx \sqrt{2} - \sqrt{2 - \frac{x^2}{2}} \approx \sqrt{2} - \left(\sqrt{2} - \frac{\sqrt{2}x^2}{8}\right) = \frac{\sqrt{2}x^2}{8}$$

Denominator:
$\sqrt{15 + \cos 2x} - 4$
$\cos 2x \approx 1 - 2x^2 \Rightarrow 15 + \cos 2x \approx 16 - 2x^2$
$\sqrt{16 - 2x^2} \approx 4 - \frac{x^2}{4}$
$\Rightarrow$ Denominator $\approx -\frac{x^2}{4}$
Limit:
$$\frac{\frac{\sqrt{2}x^2}{8}}{-\frac{x^2}{4}} = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$$

140. **(a)** Continuity at $x = -3$:
Numerator at $x = -3$ must be 0
$9 + (a + 3)(-3) + (a + 1) = 0$
$9 - 3a - 9 + a + 1 = 0 \Rightarrow -2a + 1 = 0 \Rightarrow a = \frac{1}{2}$
Now,
$\lim_{x \to a}(x^2 + x + 1) = a^2 + a + 1$
$= \frac{1}{4} + \frac{1}{2} + 1 = \frac{7}{4}$

141. **(c)** $\lim_{x \to 0} \frac{x\tan 2x - 2x\tan x}{(1 - \cos 3x)(\operatorname{cosec}x - \cot x)^2}$
Use expansions:
$\tan x \approx x, \tan 2x \approx 2x$
Numerator:
$x(2x) - 2x(x) = 2x^2 - 2x^2 = 0$(take next term)
Use better expansion:
$\tan x = x + \frac{x^3}{3}, \tan 2x = 2x + \frac{8x^3}{3}$
$x\tan 2x = 2x^2 + \frac{8x^4}{3}, 2x\tan x = 2x^2 + \frac{2x^4}{3}$
Numerator $= \frac{6x^4}{3} = 2x^4$

Denominator:

$1-\cos 3x \approx \frac{(3x)^2}{2}=\frac{9x^2}{2}$

$\operatorname{cosec}x-\cot x=\frac{1-\cos x}{\sin x} \approx \frac{x^2/2}{x}=\frac{x}{2}$

$(\operatorname{cosec}x-\cot x)^2 \approx \frac{x^2}{4}$

Denominator $=\frac{9x^2}{2}\cdot\frac{x^2}{4}=\frac{9x^4}{8}$

$\frac{2x^4}{9x^4/8}=\frac{16}{9}$

142. **(c)** Check quickly:

(A) $x|x| \rightarrow$ differentiable in $(-1,1) \Rightarrow$ III

(B) $\sqrt{|x|} \rightarrow$ continuous but not differentiable at $0 \Rightarrow$ II

(C) $x+[x] \rightarrow$ strictly increasing but not differentiable (jump at integers) $\Rightarrow$ V

(D) $|x-1|+|x+1|+|x| \rightarrow$ differentiable in $(-1,0)\cup(0,1) \Rightarrow$ IV

143. **(d)** Let $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right)$, and $t=\sqrt{1-x^2}$.

We need $\frac{dy}{dt}=\frac{dy/dx}{dt/dx}$.

At $x=\frac{1}{2}$:

- $2x^2-1=-\frac{1}{2} \Rightarrow u=\frac{1}{2x^2-1}=-2$
- $\frac{dy}{dx}=\frac{u'}{|u|\sqrt{u^2-1}}=\frac{-8}{2\sqrt{3}}=-\frac{4}{\sqrt{3}}$
- $\frac{dt}{dx}=-\frac{x}{\sqrt{1-x^2}}=-\frac{1}{\sqrt{3}}$

So, $\frac{dy}{dt}=\frac{-4/\sqrt{3}}{-1/\sqrt{3}}=4$

144. **(b)** Given:

$5f(x)+3f(1/x)=x+2$

Differentiate w.r.t **x** :

$5f'(x)+3f'(1/x)(-1/x^2)=1$

$\Rightarrow 5f'(x)-(3/x^2)f'(1/x)=1$

Put $x=1$:

$5f'(1)-3f'(1)=1$

$\Rightarrow 2f'(1)=1$

$\Rightarrow f'(1)=1/2$

Now find $f(1)$:

$5f(1)+3f(1)=3$

$\Rightarrow 8f(1)=3$

$\Rightarrow f(1)=3/8$

Given $y=xf(x)$

$\Rightarrow dy/dx=f(x)+xf'(x)$

At $x=1$:

$dy/dx=f(1)+f'(1)=3/8+1/2=7/8$

145. **(b)** Curve: $x^3+y^3=2xy$

Differentiate:

$3x^2+3y^2\frac{dy}{dx}=2y+2x\frac{dy}{dx}$

At (1,1):

$3+3\frac{dy}{dx}=2+2\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = -1$
Tangent: slope = -1
$y - 1 = -1(x - 1) \Rightarrow y = -x + 2$
Normal: slope = 1
$y - 1 = 1(x - 1) \Rightarrow y = x$
Intersections with x-axis $(y = 0)$:
•Tangent: $0 = -x + 2 \Rightarrow x = 2 \Rightarrow (2,0)$
•Normal: $0 = x \Rightarrow x = 0 \Rightarrow (0,0)$
Triangle vertices: $(0,0), (2,0), (1,1)$
Area $= \frac{1}{2} \times 2 \times 1 = 1$

146. **(d)** Given $f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$
Check Rolle's theorem: $f(0) = 0, f(\pi) = 0 \Rightarrow$ applicable
Differentiate:
$f'(x) = 2\cos x + 2\cos 2x$
Set $f'(c) = 0$:
$2(\cos x + \cos 2x) = 0$
$\Rightarrow \cos x + \cos 2x = 0$
Use identity: $\cos 2x = 2\cos^2 x - 1$
$\Rightarrow \cos x + 2\cos^2 x - 1 = 0$
Let $t = \cos x$:
$2t^2 + t - 1 = 0$
$\Rightarrow (2t - 1)(t + 1) = 0$
So $t = 1/2$ or $t = -1$
In $(0, \pi)$:
$\cos x = 1/2 \Rightarrow x = \pi/3$

147. **(d)** $y = 3x^4 - 5x^3 - 12x^2 + 18x + 3$
Slope function:
$g(x) = \frac{dy}{dx} = 12x^3 - 15x^2 - 24x + 18$
For $g(x)$ to be strictly increasing:
$g'(x) > 0$
$g'(x) = 36x^2 - 30x - 24 = 6(6x^2 - 5x - 4)$
Factor:
$6x^2 - 5x - 4 = (3x - 4)(2x + 1)$
Roots:
$x = \frac{4}{3}, x = -\frac{1}{2}$
Since parabola opens upward:
$g'(x) > 0$ for $x \in \left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{4}{3}, \infty\right)$
So domain:
$\mathbb{R} - \left[-\frac{1}{2}, \frac{4}{3}\right]$

148. **(a)** Check each:
(I)At $x = \frac{1}{2}$: LHS $= \frac{1}{2} - \frac{1}{2} = 0$, RHS $= 0^2 = 0 \Rightarrow$ continuous
Derivative:
LHS $= -1$, RHS $= 2\left(\frac{1}{2} - x\right)(-1) = 0$ at $x = \frac{1}{2} \Rightarrow$ not equal $\Rightarrow$ not differentiable
(II) $f(x) = |3x - 1|$
Not differentiable at $x = \frac{1}{3} \in (0,1)$
(III) $f(x) = x|x|$
On $[0,1], = x^2 \Rightarrow$ continuous & differentiable
(IV) $f(x) = |x|$

On [0,1], = x ⇒ continuous & differentiable

149. **(d)** Given:

$$\int \frac{e^{\sin x}(\sin 2x - 8\cos x)}{2(\sin x - 3)^2} dx$$

Simplify numerator:

$\sin 2x = 2\sin x\cos x$

$\sin 2x - 8\cos x = 2\cos x(\sin x - 4)$

So integral becomes:

$$\int \frac{e^{\sin x} \cdot 2\cos x(\sin x - 4)}{2(\sin x - 3)^2} dx = \int \frac{e^{\sin x}\cos x(\sin x - 4)}{(\sin x - 3)^2} dx$$

Now observe:

$$\frac{d}{dx}\left(\frac{e^{\sin x}}{\sin x - 3}\right) = \frac{e^{\sin x}\cos x(\sin x - 4)}{(\sin x - 3)^2}$$

Hence,

$$= \frac{e^{\sin x}}{\sin x - 3} + C$$

150. **(c)** Differentiate RHS:

$$\frac{d}{dt}[f(t)\sin(1/t)] = f'(t)\sin(1/t) - \frac{f(t)}{t^2}\cos(1/t)$$

Match with integrand:

$3t^2\sin(1/t) - t\cos(1/t)$

Comparing coefficients:

- $f'(t) = 3t^2$
- $\frac{f(t)}{t^2} = t \Rightarrow f(t) = t^3$

So,

$f(2) = 2^3 = 8$

151. **(a)**Use standard reduction:

$$\int x^n(\log x)^m dx = \frac{x^{n+1}}{n+1}\left[(\log x)^m - \frac{m}{n+1}(\log x)^{m-1} + \frac{m(m-1)}{(n+1)^2}(\log x)^{m-2} - \cdots\right]$$

Here $n = 4, m = 3$:

$$\int x^4(\log x)^3 dx = \frac{x^5}{5}\left[(\log x)^3 - \frac{3}{5}(\log x)^2 + \frac{6}{25}\log x - \frac{6}{125}\right]$$

$$= x^5\left[\frac{1}{5}(\log x)^3 - \frac{3}{25}(\log x)^2 + \frac{6}{125}\log x - \frac{6}{625}\right] + C$$

152. **(d)** $I = \int \frac{\sin 2x}{\sin^2 x + 3\cos x - 3} dx$

Use $\sin 2x = 2\sin x\cos x$ and $\sin^2 x = 1 - \cos^2 x$:

$$I = \int \frac{2\sin x\cos x}{1 - \cos^2 x + 3\cos x - 3} dx$$

$$= \int \frac{2\sin x\cos x}{-(\cos^2 x - 3\cos x + 2)} dx = \int \frac{-2\sin x\cos x}{(\cos x - 1)(\cos x - 2)} dx$$

Let $t = \cos x \Rightarrow dt = -\sin x dx$

$$I = \int \frac{2t}{(t-1)(t-2)} dt$$

Partial fraction:

$$\frac{2t}{(t-1)(t-2)} = \frac{-2}{t-1} + \frac{4}{t-2}$$

$$I = \int \left(\frac{-2}{t-1} + \frac{4}{t-2}\right) dt = -2\ln|t-1| + 4\ln|t-2| + C$$

Back substitute $t = \cos x$:

$$I = 2\ln\left|\frac{(\cos x - 2)^2}{\cos x - 1}\right| + C = \ln\left(\frac{(\cos x - 2)^4}{(\cos x - 1)^2}\right) + C$$

153. **(b)** Use $t = \sin x - \cos x$

Then,
$\sin^3 x + \cos^3 x = (\sin x + \cos x)(1 - \sin x \cos x)$
After substitution:
$$I = 2\int \frac{dt}{(2 - t^2)(1 + t^2)}$$
Split:
$$= \int \frac{dt}{2 - t^2} + \int \frac{dt}{1 + t^2}$$
So,
$$A = \frac{1}{2\sqrt{2}}, B = 1 \Rightarrow \frac{B}{A} = 2\sqrt{2}$$
$(2\sqrt{2}, \sin x - \cos x)$

154. **(a)** $\int_{\pi/4}^{\pi/3} \frac{\cos x - \sin x}{\sin 2x} dx$
$$\sin 2x = 2\sin x \cos x \Rightarrow \frac{\cos x - \sin x}{\sin 2x} = \frac{1}{2}(\csc x - \sec x)$$
$$= \frac{1}{2}\left(\int \csc x dx - \int \sec x dx\right)$$
$$= \frac{1}{2}[\ln|\csc x - \cot x| - \ln|\sec x + \tan x|]$$
Now substitute limits:
At $x = \frac{\pi}{3}$:
$$\csc x - \cot x = \frac{1}{\sqrt{3}}, \sec x + \tan x = 2 + \sqrt{3}$$
At $x = \frac{\pi}{4}$:
$$\csc x - \cot x = \sqrt{2} - 1, \sec x + \tan x = \sqrt{2} + 1$$
$$\text{Integral } = \frac{1}{2}\ln\left[\frac{(3 + 2\sqrt{2})(2 - \sqrt{3})}{\sqrt{3}}\right]$$

155. **(b)** We evaluate:
$$I = \int_0^{\pi/2} \frac{\sin x}{1 + \cos x + \sin x} dx$$
Step 1: Divide numerator and denominator by $\sin x$:
$$I = \int_0^{\pi/2} \frac{1}{\csc x + \cot x + 1} dx$$
Step 2: Use the substitution $x = \pi/2 - t \Rightarrow dx = -dt$, then $\sin x = \cos t, \cos x = \sin t$:
$$I = \int_0^{\pi/2} \frac{\cos t}{1 + \sin t + \cos t} dt$$
Step 3: Add original and transformed integrals:
$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{1 + \sin x + \cos x} dx = \int_0^{\pi/2} \left(1 - \frac{1}{1 + \sin x + \cos x}\right) dx$$
Step 4: Simplify $1 + \sin x + \cos x = 1 + \sqrt{2}\sin(x + \pi/4)$
$$2I = \int_0^{\pi/2} 1 dx - \int_0^{\pi/2} \frac{dx}{1 + \sqrt{2}\sin(x + \pi/4)} = \frac{\pi}{2} - \int_0^{\pi/2} \frac{dx}{1 + \sqrt{2}\sin(x + \pi/4)}$$
Step 5: Standard integral formula
$$\int \frac{dx}{a + b\sin x} = \frac{2}{\sqrt{a^2 - b^2}} \arctan\left(\frac{(a - b)\tan(x/2) + b}{\sqrt{a^2 - b^2}}\right) \text{ or ln form}$$
After calculation, we get:
$$I = \frac{\pi}{4} - \frac{1}{2}\ln 2$$
$= \pi/4 - \frac{1}{2}\log 2$

156. **(c)** We have the limit:

$$\lim_{n\to\infty}\sum_{k=1}^{2n}\frac{n+k}{n^2+k^2}.$$

Divide numerator and denominator by n :

$$\frac{n+k}{n^2+k^2}=\frac{1+\frac{k}{n}}{n\left(1+\left(\frac{k}{n}\right)^2\right)}$$

Set $x=\frac{k}{n}$, then $\Delta x=\frac{1}{n}$, and sum becomes a Riemann sum:

$$\sum_{k=1}^{2n}\frac{1+x}{1+x^2}\Delta x\to\int_0^2\frac{1+x}{1+x^2}dx$$

Split:

$$\int_0^2\frac{1+x}{1+x^2}dx=\int_0^2\frac{dx}{1+x^2}+\int_0^2\frac{x}{1+x^2}dx=[\tan^{-1}x]_0^2+\frac{1}{2}[\ln(1+x^2)]_0^2$$

Evaluate:

$$\tan^{-1}2-\tan^{-1}0+\frac{1}{2}\ln(5/1)=\tan^{-1}2+\frac{1}{2}\ln 5.$$

157. **(a)** We have:

$$I=\int_0^{\pi}\frac{x\sin x}{1+\cos^2x}dx$$

Use the property $\int_0^a f(x)dx=\int_0^a f(a-x)dx$:

$$I=\int_0^{\pi}\frac{(\pi-x)\sin x}{1+\cos^2x}dx$$

Add the two forms:

$$2I=\int_0^{\pi}\frac{[x+(\pi-x)]\sin x}{1+\cos^2x}dx=\pi\int_0^{\pi}\frac{\sin x}{1+\cos^2x}dx$$

Substitute $u=\cos x, du=-\sin x dx$:

$$\int_0^{\pi}\frac{\sin x}{1+\cos^2x}dx=\int_{-1}^{1}\frac{du}{1+u^2}=\arctan 1-\arctan(-1)=\frac{\pi}{2}$$

So:

$$I=\frac{\pi}{2}\cdot\frac{\pi}{2}=\frac{\pi^2}{4}$$

158. **(b)** The family of parabolas with axis along $x=1$ can be written as:

$$y=a(x-1)^2+b$$

(1). First derivative:

$$\frac{dy}{dx}=2a(x-1)$$

(2). Second derivative:

$$\frac{d^2y}{dx^2}=2a$$

Eliminate a from these:

$$\frac{d^2y}{dx^2}=\frac{dy/dx}{x-1}\Longrightarrow(x-1)\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$$

159. **(c)** The given differential equation is:

$$\frac{dy}{dx}+\frac{1}{x}y=\frac{1}{x}e^x$$

This is a linear differential equation of the form:

$$\frac{dy}{dx}+P(x)y=Q(x)$$

Here, $P(x)=\frac{1}{x}$ and $Q(x)=\frac{e^x}{x}$.

The integrating factor is:

$\mu(x) = e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

Multiply both sides by x :

$x\frac{dy}{dx} + y = e^x$

Left-hand side is $\frac{d}{dx}(xy) = e^x$

Integrate:

$xy = \int e^x dx = e^x + C$

$y = \frac{e^x + C}{x}$

160. **(d)** Use substitution $v = y/x \Rightarrow y = vx, dy = vdx + xdv$

$x\sin(v)(vdx + xdv) = (vx\sin(v) - x)dx$

Simplify:

$x^2\sin(v)dv = -xdx \Rightarrow \sin(v)dv = -\frac{dx}{x}$

Integrate:

$-\cos(v) = -\ln x + C \Rightarrow \ln x - \cos(y/x) = C$

$\log x - \cos(y/x) = c$