PHYSICS

1. **(c)** The atomic mass or mass number of an atom $_ZX^A$ is given by
$A = Z + N$
where, A = mass number, Z = atomic number
and N = number of neutrons.
For, $_{92}U^{235}, A = 235$ and $Z = 92$
$N = A - Z = 235 - 92 = 143$

2. **(b)** With the rise of temperature, the density of minority charge carriers increases due to breaking of some covalent bonding in a semiconductor in reverse biased condition. So, the reverse saturation current increases.

3. **(d)** Given, $t_{1/2} = 10$ min, $N_0 = 64, t = 50$ min
The number of half-lives in 50 min ,
$$n = \frac{t}{t_{1/2}} = \frac{50}{10} = 5$$
In 5 half-lives, number of nuclei left undecayed,
$$N = N_0\left(\frac{1}{2}\right)^n = 64 \times \left(\frac{1}{2}\right)^5 = \frac{64}{32} = 2$$
$\therefore$ Number of nuclei decayed $= N_0 - N = 64 - 2 = 62$

4. **(b)** Photons of different energies carries the electromagnetic spectrum of all rays. They have no mass and travels at the speed of light.

5. **(a)** The NOT gate is also called invertor as it reverses the input in its output.

Input (A)	**Output ($Y = \overline{A}$)**
1	0
0	1

6. **(b)** LCD stands for Liquid Crystal Display. It is a special thin flat panel display or other electronically modulated optical device that uses the light modulating properties of liquid crystals combined with polarisers. Liquid crystal do not emit light directly, instead using a backlight or reflector to produce images in colour or monochromatic.

7. **(d)** Polarisation is the property of wave to oscillates in more than one orientation. The sound waves always oscillates in the direction of wave propagation, so polarisation is not possible in them. While light can be polarised.

8. **(b)** For an optically active solution, the angle through which a plane polarised light is rotated, is given as
$\theta = SLC$
where, S = correction factor, L = path length and C = concentration of solution.

9. **(c)** Given, $V_S = 50$ V, $V_R = 40$ V
For an L-R circuit,
$$V_S = \sqrt{V_R^2 + V_L^2}$$
$$\Rightarrow V_L = \sqrt{V_S^2 - V_R^2}$$
$$= \sqrt{50^2 - 40^2}$$
$$= \sqrt{2500 - 1600} = \sqrt{900} = 30 \text{ V}$$

10. **(a)** Common potential difference for a parallel combination of capacitors, is
$$V_C = \frac{V_1C_1 + V_2C_2}{C_1 + C_2}$$
Given, $V_2 = 0, V_1 = 10$ V,
$C_1 = C_2 = 10\mu\text{ F} = 10 \times 10^{-6}$ F
$$\therefore V_C = \frac{10 \times 10 \times 10^{-6}}{(10 + 10) \times 10^{-6}}$$

$= \frac{100}{20} = 5\text{ V}$

11. **(c)** In fusion of two lighter nuclei to form a relatively heavier nucleus, energy is released. So, the binding energy of the stable product nucleus must be greater than that of the reacting nuclei.

12. **(b)** Given, $Q_1 = 2\mu C = 2 \times 10^{-5} C$

$Q_2 = 1\mu C = 1 \times 10^{-6} C$

The electric field intensity at a point P due to a point charge is given by

$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$

$\Rightarrow E \propto \frac{Q}{r^2}$

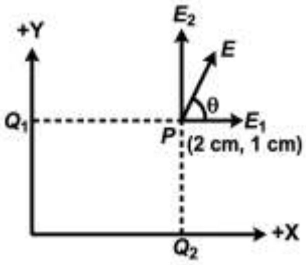
$\therefore$ Ratio of electric fields due to Q_1 and Q_2 at point P is

$\frac{E_1}{E_2} = \frac{Q_1}{r_1^2} \times \frac{r_2^2}{Q_2}$

$= \frac{2 \times 10^{-6}}{(2 \times 10^{-2})^2} \times \frac{(1 \times 10^{-2})^2}{1 \times 10^{-6}} = \frac{1}{2}$

Here, E_1 is along X-axis and E_2 is along Y-axis. So, the resultant field E have angle θ with X-axis as

$\tan\theta = \frac{E_2}{E_1} = 2$



13. **(d)** The direction of force on a current carrying conductor held perpendicular to an uniform magnetic field is given by Fleming's left hand rule.

It states that "If we hold the thumb, forefinger and middle finger of the left hand perpendicular to each other such that the fore finger points in the direction of magnetic field, the middle finger points in the direction of current, then the thumb shows the direction of force on the conductor."

14. **(a)** Given, $f_1 = F$ and $f_2 = -F$

The equivalent focal length of combination of two lenses is

or

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{F} + \frac{1}{-F} = 0$

or $f = \frac{1}{0} = \infty$ (infinity)

15. **(d)** Given, $x = at + bt^2$

Here, x is measured in metre(m) and t is in second(s).

So, for dimensional consistency of equation,

Dimensions of $a = [a] = \frac{[x]}{[t]} = \frac{[L]}{[T]} = [LT^{-1}]$

Similarly, for b. $[b] = \frac{[x]}{[t^2]} = \frac{[L]}{[T^2]} = [LT^{-2}]$

$\therefore$ Dimensions of $\left[\frac{b}{a}\right] = \frac{[LT^{-2}]}{[LT^{-1}]} = [T^{-1}]$

16. **(a)** Given, $u = 5\text{ ms}^{-1}$, Eastwards

$v = 5\text{ ms}^{-1}$, Northwards

and $\Delta t = 10\text{ s}$

Change in velocity $|\Delta v| = \sqrt{u^2 + v^2}$

$= \sqrt{5^2 + 5^2} = 5\sqrt{2}\text{ ms}^{-1}$

Direction of Δv,

$\tan\theta = \frac{v}{u} = \frac{-5}{5} = -1$

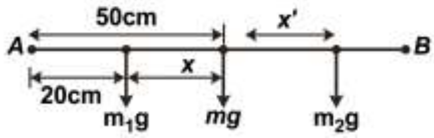
(taking eastward direction positive)

$\Rightarrow \theta = 135°$

$\therefore$ Average acceleration,

$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\ ms^{-2}$, towards North-West

17. (c) Given, $m = 0.1$ kg



Let x be the position of mass of 0.1 kg from its centre and x' be the position of second mass m_2 that must be suspended to the other end to prevent the rod from toppling. So,

$x = 50 - 20 = 30\ cm = 0.3\ m$

The rod will not topple if net torque on it is zero. So, balancing the moments about its centre, we get

$m_1 gx = m_2 gx'$

$\Rightarrow m_2 x = 0.1 \times 0.3 = 0.03\ kg - m$

From options, it is possible only in case of (c).

$m_2 = 0.15$ kg

$x' = \frac{0.03}{0.15} = 0.2\ m = 20\ cm$

$\therefore$ The second mass of 0.15 kg should be hanged at

$(50 + 20 =)70$ cm mark.

18. (b) Ink filler works on the principle of both surface tension and pump. Vacuum in the pen pulls the ink in the bottle to the pen. This ink travels in the pen tip through small openings due to surface tension.

19. (d) Ionosphere is the ionised part of Earth's atmosphere, which reflects the radiowaves due to total internal reflection towards earth, thus it helps in radio communication.

20. (b) The reaction of the floor on an object placed on the floor of an elevator is maximum, when the elevator accelerates upwards. In this case, normal reaction on it is

$N = m(g + a)$

where, $m =$ mass of object,

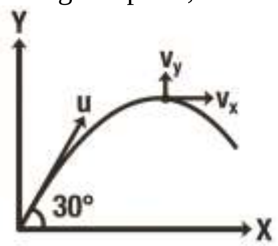
g = gravitational acceleration

and $a =$ acceleration of elevator.

21. (a) Let m be the mass of particle and u be its initial velocity, then

Initial momentum, $p = mu$ ....(i)

At highest point,



$v_x = u\cos\theta = u\cos 30°$

$= \frac{\sqrt{3}}{2}u$

$v_y = 0$

$\therefore$ Momentum at highest point is

$p' = mv_x = mu\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{2}p$ [using Eq.(i)]

22. **(a)** Since, the block is at rest, the upward frictional force is equal to the weight or gravitational pull of the Earth i.e.,

$F_S = w = mg = 0.1 \times 9.8 = 0.98\ \text{N}$

23. **(b)** The rotational kinetic energy of a ring is

$K_R = \frac{1}{2} I(\omega)^2$

For a ring, $I = MR^2$ and $\omega = \frac{v}{R}$

$\therefore\ K_R = \frac{1}{2}(MR^2) \times \left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2$

Also, translational kinetic energy, $K_T = \frac{1}{2}Mv^2$

$\therefore K_R = K_T$ or $K_R : K_T = 1:1$

24. **(d)** Given, work done, $W = 120\ \text{J}$

Time, $t = 2\ \text{min} = 2 \times 60 = 120\ \text{s}$

$\therefore\ \text{Power} = \frac{\text{Work done}}{\text{Time}} = \frac{W}{t} = \frac{120}{120} = 1\ \text{W}$

25. **(c)** As we know, escape velocity, $v_e = \sqrt{2gR}$

Let v_m be escape velocity from moon and v_e be the escape velocity from Earth. Then,

$$\frac{v_m}{v_e} = \sqrt{\frac{2 \times g_m \times R_m}{2 \times g_e \times R_e}}$$

$$= \sqrt{\frac{g_m \times D_m}{g_e \times D_e}} \quad (\because D = 2R)$$

$$= \sqrt{\frac{\frac{g_e}{6} \times \frac{D_e}{4}}{g_e D_e}} = \frac{1}{\sqrt{24}}$$

$$\therefore\ v_m = \frac{v_e}{\sqrt{24}} = \frac{11.2}{\sqrt{24}}\ \text{km/s}\ (\because v_e = 11.2\ \text{km/s})$$

26. **(d)** As work done in a cycle is equal to the area enclosed by the cycle.

$\therefore$ Work done, $W = \Delta p \times \Delta V$

$= (4 - 2) \times (6 - 1) = 10\ \text{J}$

27. **(b)** Speed of sound in a gas, $v = \sqrt{\frac{RT}{M}}$

Both, hydrogen and oxygen gas are diatomic. At same temperature,

$v \propto \frac{1}{\sqrt{M}}$

$$\therefore\ \frac{v_H}{v_O} = \sqrt{\frac{M_O}{M_H}} = \sqrt{\frac{32}{2}} = \sqrt{16} = 4 \text{ or } 4:1$$

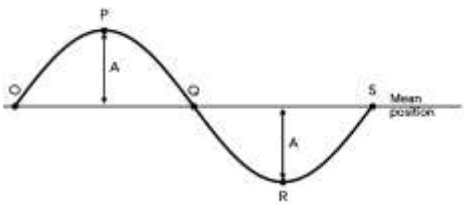
28. **(b)** According to Stefan's law,

Energy radiated $E \propto T^4$

$$\therefore\ \frac{E'}{E} = \frac{T'}{T} = \frac{\left(\frac{T}{2}\right)^4}{T^4} = \frac{1}{2^4} = \frac{1}{16}$$

or $E' = \frac{E}{16}$

29. **(d)** The motion of particle in one oscillation is shown below



In moving from $O \rightarrow P$ distance moved is A. Similarly,
for $P \rightarrow Q, Q \rightarrow R$ and $R \rightarrow S$, distance moved is A in each case.
$\therefore$ Total distance moved in one oscillation $= 4$ A

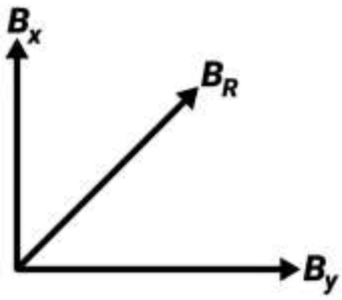
30. **(c)** Given, $C = 10\mu F = 10 \times 10^{-6}$ F
$V = 10$ V
Change, $q = C \times V = 10 \times 10^{-6} \times 10$
$= 10^{-4}C$
$= 100 \times 10^{6}C$
$= 100\mu C$

31. **(a)** When a wire of resistance R is cut into 10 equal parts, then resistance of each part,
$$R' = \frac{R}{10} = 0.1R$$
Equivalent resistance of parallel combination of resistors, is
$$R_{eq} = \frac{R'}{10} = \frac{0.1R}{10} = 0.01R$$

32. **(d)** The conductors have positive temperature coefficient, while semiconductors have negative temperature coefficient. So, when iron and silicon wires are cooled from 50°C to 30°C, the resistance of iron decreases, while that of silicon increases.

33. **(a)** Given, $P = 10\Omega, l = 0.2$ m
In balanced condition of meterbridge,
$$\frac{P}{Q} = \frac{l}{(1-l)}$$
$$\Rightarrow \frac{10}{Q} = \frac{0.2}{1-0.2} = \frac{0.2}{0.8}$$
$$\Rightarrow Q = 40\Omega$$

34. **(d)** As two coils are mutually perpendicular, so the field produced by them will also be perpendicular to each other as shown



The two coils are identical, so magnetic field,
$B \propto I$
or $\frac{B_x}{B_y} = \frac{I_x}{I_y} = \frac{1}{2}$
$\Rightarrow B_y = 2B_x$
$\therefore B_x = B$, then $B_y = 2B$
$\therefore$ Resultant field, $B_R = \sqrt{B_x^2 + B_y^2}$
$= \sqrt{B^2 + (2B)^2}$
$= \sqrt{5}B$

35. **(c)** AC or DC motor is an electrical device which is used to convert an electrical energy into mechanical energy.

36. **(c)** According to the Faraday's law of electromagnetic induction, whenever the magnetic flux linked with a coil changes continuously with time, an emf is induced in the coil.
Its magnitude is equal to the rate of change of magnetic flux i.e.,
$$e = N\frac{d\phi}{dt}$$
where, N is the number of turns in the coil.

37. **(a)** Given,
$I_1 = 1\text{ mA} = 1 \times 10^{-3}\text{ A}$
$I_2 = 5\text{ mA} = 5 \times 10^{-3}\text{ A}$
$t = 4\text{ ms} = 4 \times 10^{-3}\text{ s}$
$L = 10\text{mH} = 10 \times 10^{-3}\text{H}$
$\therefore$ Self-induced emf, $|e| = L\frac{dI}{dt}$
$$= 10 \times 10^{-3} \times \frac{(5-1) \times 10^{-3}}{4 \times 10^{-3}} = 10\text{mV}$$

38. **(a)** Work function of a metal,
$W = hv_0$
where, $h =$ Planck's constant
$v_0 =$ threshold frequency.
As, the threshold frequency of metal M is more than that of metal N, so work function of M is more than that of N, while their kinetic energies are equal.

39. **(d)** Electron wave is used in place of X-rays to study crystals of larger dimensions in electron crystallography. So, they can be diffracted.

40. **(c)** According to Bohr's atomic model, Kinetic energy K or $E_K = \frac{U}{2}$ where, U = potential energy. which is negative.
$\therefore\ 2E_K + U = 0$

41. **(b)** The magnetic field due to a circular coil carrying current at its axis is
$$B_x = \frac{\mu_0}{4\pi} \cdot \frac{2\pi N i r^2}{(x^2+r^2)^{3/2}} \quad \text{....(i)}$$
At centre of coil, $B_c = \frac{\mu_0}{4\pi}\frac{2\pi N i}{r}$ ....(ii)
From Eqs. (i) and (ii), we get
$$\frac{B_c}{B_x} = \frac{(x^2+r^2)^{3/2}}{r \times r^2}$$
At, $x = \frac{r}{2}, \frac{B_c}{B_x} = \frac{\left(\frac{r^2}{4}+r^2\right)^{3/2}}{r^3} = \frac{\left(\frac{5}{4}\right)^{3/2} r^3}{r^3} = \frac{5\sqrt{5}}{8}$
or $5\sqrt{5} : 8$

42. **(a)** Stokes lines are the radiation of certain wavelengths present in the line spectra, that is associated with the phenomena of fluorescence and the Raman effect. Anti-stokes lines are produced due to already excited atoms or molecules. These lines are symmetrical about the main lines and the wavelength of anti-stokes lines is less than the main line. So, if the difference between wavelengths of stokes line and corresponding anti-stokes line is 100Å, then the difference between anti-stokes line and main line is 50Å. Therefore, wavelength of incident radiation is 5050Å.

43. **(d)** Optical pumping is a process in which light is used to raise electrons from a lower energy state in an atom than the metastable state to a higher energy state than the metastable state.

44. **(b)** Let L_o be the length of air column in pipe. After immersion,
$$L_c = \frac{L_0}{2}$$
Fundamental frequency in open pipe, $f_o = \frac{v}{2L_o}$
Fundamental frequency in closed pipe, $f_e = \frac{v}{4L_e}$

$\therefore\ \frac{f_o}{f_e}=\frac{\frac{v}{2L_o}}{\frac{v}{4L_c}}=2\left(\frac{L_c}{L_o}\right)=2\left(\frac{L_o}{2L_o}\right)=1:1$

45. (a) Given, $\gamma=1.4=\frac{C_p}{C_V}$

$\Rightarrow\ C_p=1.4C_V$
Also, $R=C_p-C_V=8.3$(given)
$\Rightarrow\ 1.4C_V-C_V=83$
Or $C_V=\frac{8.3}{0.4}=20.75\text{Jmol}^{-1}\ \text{K}^{-1}$
$C_p=1.4C_V=1.4\times 20.75$
$=29.05\text{Jmol}^{-1}\ \text{K}^{-1}$

46. (c) Given, $\frac{I_A}{I_B}=100$

Using magnitude equation,
$m_B-m_A=-2.5\log\left(\frac{I_B}{I_A}\right)$
$=-2.5\log\left(\frac{1}{100}\right)=-2.5\times-2=5$

47. (d) When a ray of light enters a glass slab along the normal to the surface, it passes straight without deviation. So, angle of deviation is zero ($0°$).

48. (c) Given, $i=65°$ and $i_c=63°$
Since, $i>i_c$, so the ray undergo total internal reflection.
The angle of deviation, $d=\pi-2i$
$=180°-2(65°)=180°-130°=50°$

49. (a) Given, $i=49°30', A=60°$ (equilateral prism)
At minimum deviation position, angle of deviation is
$D=2i-A=2(49°30')-60°=99°-60°=39°$

50. (b) Here, $R=-0.1$ cm, $\mu_g=1.5$
$\therefore\ u=-0.1$ cm
From refraction formula,
$\frac{\mu_a-\mu_g}{R}=\frac{\mu_a}{v}-\frac{\mu_g}{u}$
$\Rightarrow\frac{1-15}{-0.1}=\frac{1}{v}-\frac{15}{(-0.1)}$ $\qquad(\because\ \mu_a=1)$
$\Rightarrow 5=\frac{1}{v}+15$
$\Rightarrow v=-\frac{1}{10}=-0.1$ cm
Here, negative sign shows that the image will be seen on the same size as that of object.
$\therefore$ The spot appears to be 0.1 cm below the top surface or exactly at the original position.

51. (d) Photoelectric effect and Raman effect can be explained on the basis of Planck's quantum theory of light. These verify the particle nature of light.

52. (b) Given, $\frac{I_{max}}{I_{min}}=9$

Since, intensity $\propto$ (amplitude) ^{2}

$\therefore\frac{A_{max}}{A_{min}}=\frac{a_1+a_2}{a_1-a_2}=\sqrt{\frac{9}{1}}=\frac{3}{1}$
$\Rightarrow a_1+a_2=3a_1-3a_2$
$\Rightarrow 2a_1=4a_2$
$\therefore a_1:a_2=2:1$

53. (c) Since, sound waves have higher wavelength, which are comparable to the dimension of interfering objects. So, diffraction effect can easily be detected in sound waves than light waves.

54. **(b)** According to Brewster's law,

$\mu = \tan\theta_p$

where, μ = refractive index $= \frac{c}{v}$

and θ_F = polarising angle $= \theta$

$\therefore\ \theta = \tan^{-1}\mu = \tan^{-1}\left(\frac{c}{v}\right)$

55. **(a)** For two polaroids placed at an axis making angle θ with each other, then according to law of malus, the intensity of transmitted light is

$I_t = I_0\cos^2\theta$

Here, $I_t = 0.75I_0$

$\Rightarrow 0.75I_0 = I_0\cos^2\theta$

$\Rightarrow \cos^2\theta = 0.75 = \frac{3}{4}$

$\Rightarrow \theta = \cos^{-1}\sqrt{\frac{3}{4}} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\therefore \theta = 30°$

56. **(d)** Let d be the distance in air between two point charges and d_m be the distance between them in medium.

According to question,

$F = F_m$

$\Rightarrow \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d^2} = \frac{1}{4\pi\varepsilon_0 k}\frac{q_1q_2}{d_m^2}$

$\Rightarrow d^2 = kd_m^2$

$\Rightarrow d_m = \frac{d}{\sqrt{k}}$

57. **(c)** The force on a charged particle in an electric field is

$F = qE$

Also, $F = ma$

$a = \frac{F}{m} = \frac{qE}{m}$

58. **(b)** Electric field intensity at a point due to a charge q is given by

$E = \frac{F}{q}$

As, the force between the charges is mutual, so the electric field intensity due to charge $-2q$ is

$E' = \frac{F}{|-2q|} = \frac{1}{2}\frac{F}{q} = \frac{E}{2}$

59. **(b)** As per Boolean postulates,

$1 \cdot A = A$

$\Rightarrow 1 \cdot 1 = 1$ and $1 \cdot 0 = 0$

p-type semiconductor is electrically neutral. The majority charge carriers in n-type semiconductor are electrons. A transistor can be used as a switch.

60. **(d)** J. J. Thomson gave the first ideal regarding structure of atom. According to him, "an atom is a solid sphere in which entire positive charge is distributed and negative charge or electrons are embedded like seeds in watermelon."

It is also called plum pudding model of an atom.

CHEMISTRY

61. **(b)** Cellulose is not a protein. It is a carbohydrate consisting of 3,000 or more glucose unit, while wool, hair and nails are proteins.

62. **(a)** Adsorption of a gas on a solid is exothermic process as during this process condensation of gas take place. According to Le-Chatelier principle, increase in temperature favors desorption.

63. **(d)** Peptisation is a method of preparation of colloids in which a suspension gets converted into a colloid on stirring with a peptising agent.

64. **(b)** Given, 39% by mass of organic solvent in benzene

∴ 39 g of organic solvent in 100 g solution.

or

39 g of organic solvent in $100-39=61$ g benzene.

Now, molar mass of organic solvent $=122$ g/mol

Molar mass of benzene $(C_6H_6)=12\times 6+6=78$ g/mol

∴ Moles of benzene $(n_s)=\frac{61}{78}$

Moles of organic solvent $(n_O)=\frac{39}{122}$

Mole fraction of benzene $=\frac{n_B}{n_B+n_o}=\frac{\frac{61}{78}}{\frac{61}{78}+\frac{39}{122}}=0.6$

65. **(b)** According to Nernst equation for a cell

$Zn(s)+Cl_2(1\text{ atm})\longrightarrow Zn^{2+}+2Cl^-;\ E^\circ=2\text{ V}$

$E=E^\circ-\frac{0.059}{2}\log\frac{[Zn^{2+}]}{[Cl^-]^2}$

∴ As$[Zn^{2+}]$ decreases E_{emf} of cell increases.

66. **(c)** $r^+_{A^+}=104\text{pm}$

$r^-_{B^-}=200\text{pm}$

Radius ratio $\frac{r^+_{A^+}}{r^-_{B^-}}=\frac{104}{200}=0.52$

∵ It lies between (0.414 to 0.732).

∴ Coordination number of A^+ is 6 .

67. **(d)** Given, molar mass of solute, $\mu_2=60$ g/mol

Mass of water $(W_1)=126$ g

Molar mass of water $(M_1)=18$ g/mol

$p_{Solution}\ (p_s)=0.99p^\circ$

Relative lowering of vapour pressure

$$\frac{p^\circ-p_s}{p^\circ}=\frac{W_2\,M_1}{M_2\,W_1}$$

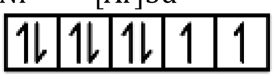
$$\frac{p^\circ-0.99p^\circ}{p^\circ}=\frac{W_2\times 18}{60\times 126}=\frac{0.1p^\circ}{p^\circ}=\frac{W_2\times 18}{60\times 126}$$

$$\Rightarrow\ W_2=\frac{60\times 126}{18\times 100}=4.2\text{ g}$$

68. **(c)** The pH of boiling water is 6.4 . This implies that boiling water is neutral. When water is boiled, then concentration of hydrogen ion and hydroxide ion increases with same extent.

69. **(b)** Electronic configuration of Ni $=[Ar]3d^84s^2$

$Ni^{2+}=[Ar]3d^5$



$3d^8$

∴ Number of unpaired electrons $=2$

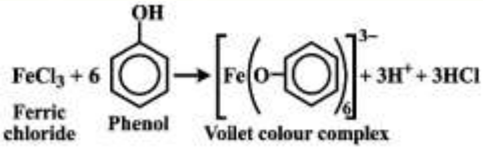
∴ Magnetic moment $(\mu)=\sqrt{n(n+2)}$

$=\sqrt{2(2+2)}=\sqrt{8}=2.84.$

70. **(c)** An organic compound containing halogen produces a bluish green flame when heated on a copper wire. This test is called Beilstein's test.

∴ Chlorobenzene will produce bluish green flame when heated on copper wire.

71. **(d)** Phenol can be distinguished from ethyl alcohol using $FeCl_3$ because phenol gives voilet colour compound with $FeCl_3$. While, alcohol do not give voilet colour.



72. (a) For zero order of reaction,

Rate, $\frac{-dx}{dt}=k$

$\therefore dt=\frac{-dx}{k}$

On integration,

$\int_0^T dt=-\int_a^0 \frac{dx}{k}$

$T=\frac{a}{k}$

73. (c) EAN of central atom in $[Co(NH_3)_6]Cl_3$

$=Z-(ON)+2\times(CN)$

where, $Z=$ atomic number

$ON=$ oxidation number

$CN=$ coordination number

ON of Co $\Rightarrow x+6\times0-3=0$

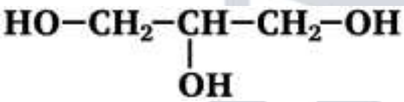
$\therefore\ x=3$

$\therefore\ CN=6$

$\therefore\ EAN=27-3+2\times6=36$

74. (d) The alcohol obtained during the hydrolysis of ground nut oil is propane 1, 2, 3-triol.

It's structure is



75. (c) Froth floatation method is used to concentrate sulphide ores.

Galena is sulphide ore with formula PbS .

$\therefore$ Galena is concentrated by froth floatation process.

76. (b) The energy for electron transition is

$E=-13.6Z^2\left[\frac{1}{n_2^2}-\frac{1}{n_1^2}\right]$

$\therefore E\propto\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)$

For $n=2$ to $n=1$ value of E is highest.

$E_2=13.6\times(0.75)=10.2$

For $n=6$ to $n=2$

$E=-13.6\left(\frac{1}{36}-\frac{1}{4}\right)$

$=13.6\times\frac{8}{36}=3.02$

It is less than E_2.

$\therefore$ For electron transition $n=2$ to $n=1$, energy released is maximum.

77. (c) Magnetic quantum number, $m=+3$

As $m=+l$ to $-l$

$\therefore l=3$ for f-orbital.

78. (d) The electronic configuration is

(a) $Z=16: 1s^2 2s^2 2p^6 3s^2 3p^4$

(b) $Z=14: 1s^2 2s^2 2p^6 3s^2 3p^2$

(c) $Z=13: 1s^2 2s^2 2p^6 3s^2 3p^1$

(d) $Z=15: 1s^2 2s^2 2p^6 3s^2 3p^3$

∵ For $Z = 15$, p-orbital has stable half-filled configuration.

∴ It will have highest ionisation energy.

79. (a) Moles of oxygen $= \frac{\text{volume }(\text{in}\,\text{cm}^3)}{22400\ \text{cm}^3} = \frac{560}{22400}$ mol

1 molecule of oxygen = 2 atoms of oxygen

∴ Number of atoms of oxygen

$= 6.022 \times 10^{23} \times \frac{560}{22400} \times 2$

$= \frac{1}{20} \times 6.022 \times 10^{23}$

80. (b) $\underset{\substack{1\ \text{mol} \\ =1.06\ \text{g}}}{Na_2CO_3} + \underset{(\text{dil.})}{2HCl(\text{ dil.})} \xrightarrow[\substack{1\ \text{mol} \\ =22400\ \text{cm}^3}]{} 2NaCl + CO_2 + H_2O$

$1.06\ \text{g}\,Na_2CO_3$ gives $= 22400\ \text{cm}^3$ of CO_2

∴ 1.06 g gives $= \frac{1.06 \times 22400}{106} = 224\ \text{cm}^3$ of CO_2

81. (c) General oxidation state of oxygen is (-2). But in peroxide, it's oxidation state is (-1).

∴ The oxidation number of oxygen is -1 in Na_2O_2.

$2 \times 1 + 2x = 0$

$x = -1$

82. (d) The heat of formation of $H_2O(g)$ will be less than the heat of formation of $H_2O(l)$, as heat of condensation is not included in formation of H_2O in gases state i.e.

$H_2(\text{ g}) + \frac{1}{2}O_2(\text{ g}) \xrightarrow{\Delta H_1} H_2O(\text{ g}) \xrightarrow{\Delta H_2} H_2O(l)$

∴ $\left|\Delta H_{f_{(H_2O)l}}\right| = |\Delta H_1 + \Delta H_2| = 286\ \text{kJ}$

But $\left|\Delta H_{f_{H_2O)g}}\right| = |\Delta H_1| < 286\ \text{kJ}$

∴ $\Delta H_{f_{(H_2O)g}} = -242.8\ \text{kJ}$

83. (a) According to first law of thermodynamics

$\Delta Q + \Delta W = \Delta U$

$\Delta W = 0$, then $\Delta Q = \Delta U$ {∵ Volume is constant }

$\Delta Q = 200\ \text{J}$

$\Delta U = 200\ \text{J}$

84. (b) Number of moles of $CaCO_3 = \frac{W}{M} = \frac{1}{100}$

$= 0.01$ mol

∴ Molar mass of $CaCO_3 = 100\ \text{g mol}^{-1}$

$100\ \text{mL of } \frac{N}{5}\,HCl = \frac{100}{1000} \times \frac{1}{5} = 0.02\ \text{mol}$

The reaction is

$\underset{1\ \text{mole}}{CaCO_3} + \underset{2\ \text{moles}}{2HCl} \longrightarrow CaCl_2 + CO_2 + H_2O$

∴ 0.01 mole of $CaCO_3$ reacts with 0.02 mole of HCl .

∴ Both are completely consumed in the given reaction.

85. (c) $v_{rms} = \sqrt{\frac{3RT}{M}}$

$v_{rms} \propto \frac{1}{\sqrt{M}}$, where M = molecular mass

$\Rightarrow v_{rms(CH_4)} : v_{rms(He)} : v_{rms(SO_2)}$

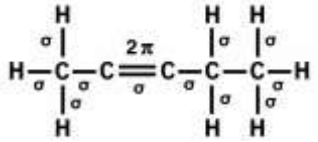
$\Rightarrow \frac{1}{\sqrt{16}} : \frac{1}{\sqrt{4}} : \frac{1}{\sqrt{64}} \Rightarrow \frac{1}{4} : \frac{1}{2} : \frac{1}{8}$

$\Rightarrow 2 : 4 : 1$

86. (d) A negative catalyst will increases the activation energy of the reaction.

87. (a)

Pent-2-yne is



$\therefore$ σ-bonds $= 12$ and π-bonds $= 2$

88. (b) The blue colouration is due to formation of ferric ferrocyanide, $Fe_4[Fe(CN)_6]_3$ in Lassaigne's test for nitrogen.

89. (c) The addition of HBr to propene in presence of peroxide follows Kharash effect.

$$\underset{\text{Propene}}{CH_3-CH=CH_2} + HBr \xrightarrow{\text{Peroxide}} \underset{\text{1-bromopropane}}{CH_3-CH_2-CH_2-Br}$$

90. (a) On passing H_2 S into a solution containing Zn^{2+} and Cu^{2+} ions, ZnS and CuS are formed.
The solubility product of CuS is less than that of ZnS .
$\therefore$ CuS get precipitated.

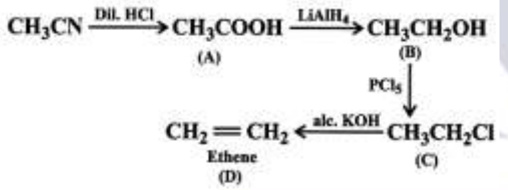
91. (d) $I = 0.5$ A
$t = 32.2 \times 60 = 19320$ s
$\therefore$ Mg obtained $= ZIt = \frac{12}{96500} \times 0.5 \times 1932.0$
$= 0.12012$ g
$Mg + 2HCl \longrightarrow MgCl_2 + H_2 \uparrow$
24 g of Mg gives $= 22400\ cm^3$ of H_2
0.12012 gMg gives $= \frac{22400}{24} \times 0.12012 = 112\ cm^3$

92. (a) According to Markownikoff's rule, addition of HBr to alkene take place such that a stable carbocation is formed.
It is applicable to propene.

93. (b) The reaction of ethene in the presence of $H^+$is an example of +E-effect as complete transfer of multiple bonded electron take place towards attacking reagent.

$$H_2C=CH_2 + H^+ \longrightarrow H_3C-\overset{+}{C}H_2$$

94. (c) The complete reaction is as follows

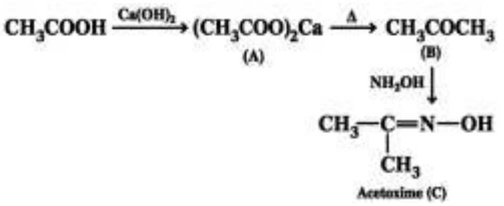


95. (d) An organic compound is optically active if, it is non-super imposable on its mirror image.

96. (c) $AlCl_3$ is an example of Lewis acid. It is an electron acceptor.
Hence, it is an example of electrophile.

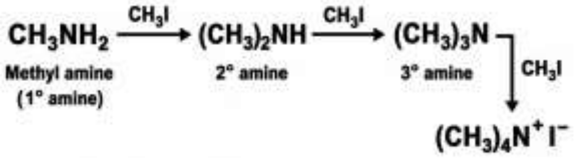
97. (b) Ionisation constant (K_a) of phenol is more than that of ethanol because phenol is more acidic than alcohol. On ionisation, phenol gives phenoxide ion which is stabilised due to resonance.

98. (d) The complete reaction is as follows



99. (a) Statement (a) is incorrect and can be corrected as, chloro acetic acid is stronger than acetic acid due to -I-effect of chloro group which leads to stabilisation of carboxylate ion.

100. (a) The reaction of methyl iodide and methyl amine is as follows



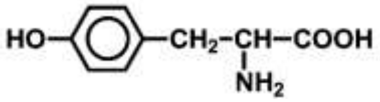
$\therefore$ 4 molecules of methyl iodide can react with one molecule of methyl amine.

101. **(c)** Only aliphatic primary amines react with nitrous acid to give alcohols with the evolution of N_2 gas.

$R-NH_2+HONO \xrightarrow{273-278\text{ K}} R-OH+N_2+H_2O$

$\because$ Dimethyl amine is a secondary amine, it will not give N_2 gas.

102. **(d)** Tyrosine is aromatic in nature. It's structure is



103. **(a)** One mole of triolein contain three double bonds. Hence, during hydrogenation, it adds three moles of hydrogen.

One mole of hydrogen $=2$g.

$\therefore$ Three moles of hydrogen $=6$ g.

104. **(d)** The reaction is as follows,

$CH_3-CH_2-OH \xrightarrow{PCl_5(A)} CH_3CH_2Cl$

$\therefore \xrightarrow{\text{alc. KOH(B)}} CH_2=CH_2$

$\therefore\ A=PCl_5$ and $B=$ alc. KOH

105. **(a)** Number of gram equivalent of HCl

$=N_1V_1=2\times\frac{500}{1000}=1$

No. of gram equivalent of NaOH

$=N_2V_2=4\times\frac{250}{1000}=1$

$\because$ Both NaOH and HCl are one gram equivalents.

$\therefore$ Heat evolved during reaction is equal to enthalpy of neutralisation i.e. x kJ.

106. **(b)**

	A	+	**B ⇌ C**		+	**D**
Initially	x		x	0		0
At equilibrium	y		y	2y		2y

Equilibrium constant

$k=\frac{[C][D]}{[A][B]}=\frac{2y\times 2y}{y\times y}=4$

107. **(c)** Dissociation of lime stone is an endothermic reaction. Hence, entropy increases in this reaction.

$CaCO_3(s) \xrightarrow{\Delta} CaO(s)+CO_2(g); \Delta H=+Q\text{ kJ}$

Lime stone

108. **(a)** Sulphuric acid (H_2SO_4) is hygroscopic in nature, it absorbs atmospheric moisture without changing physical state. It is not deliquescent in nature.

109. **(c)** van der Waals' force of attraction increase with increase in atomic mass. On going down the group atomic mass increases.

$\therefore$ Xe has the highest van der Waals' force of attraction.

110. **(c)** Atomic number of V is 23.

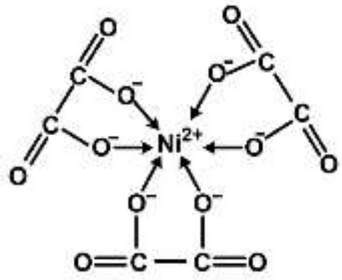
Electronic configuration of $V=[Ar]4s^2 3d^3$

$V^{4+}=[Ar]3d^1$

$\because$ It contains one unpaired electron, hence it is coloured. While Ti^+, Cu^+ and Zn^{2+} do not have unpaired electron.

111. **(c)** Oxalate $(C_2O_4^{2-})$ is an example of bidendate ligand.

Three oxalate ligands form six coordinate bond around Ni^{2+} ion. Hence, coordination number is 6 .



112. **(d)** Oxidation number of Cr in $[CrCl_2(NH_3)_4]NO_3$

$\Rightarrow x + 2 \times (-1) + 0 + (-1) = 0$

x= 3

IUPAC name of $[CrCl_2(NH_3)_4]NO_3$ is tetraamminedichlorochromium (III) nitrate.

113. **(b)** Unit of rate constant of nth order

$$(k) = \left(\frac{mol}{dm^3}\right)^{1-n} s^{-1}$$

for zero order $(n) = 0$

$\therefore\ k = moldm^{-3}\ s^{-1}$

∴ The order of reaction is 1 st.

114. **(b)** The concentration of gaseous $\propto \frac{1}{\text{volume}}$

∴ If volume of vessel is reduced of initial volume to half then concentration increases to two-times.

$\therefore [A'] = 2[A]$

$\therefore [B'] = 2[B]$

Initially, $r = k[A][B]$

After reduction of volume, $r_1 = k_2[A']2[B']$

$r_1 = 4k[A'][B']$ $\quad (k_2 = k)$

$r_1 = 4r$

∴ Rate increases to 4 times of original rate.

115. **(b)** The reduction potential of an electrode can be increased by increase in [M^{n+}] or decrease in temperature.

$M^{n+} + ne^- \longrightarrow M$ (Reduction)

$$E = E^\circ - \frac{RT}{nF}\ln\frac{[M]}{[M^{n+}]}$$

116. **(d)** According to Faraday second law,

$$\frac{W_{Ag}}{W_{O_2}} = \frac{E_{Ag}}{E_{O_2}}$$

$$\frac{0.054}{W_{O_2}} = \frac{108}{8}$$

$$W_{O_2} = \frac{0.054}{108} \times 8 = 4 \times 10^{-3}\ g$$

∴ 32 g of oxygen (O_2) at STP liberates 22400 cm^3.

∴ 4×10^{-3} g of oxygen will liberate

$$= \frac{4 \times 10^{-3} \times 22400}{32} = 2.8\ cm^3$$

117. **(a)** Millimoles of $H^+ = N_1V_1 + N_2V_2$

$10 \times 0.1 \times 2 + 5 \times 0.2 = 2 + 1 = 3$

Millimoles of $OH^- = N_3V_3$

$= 0.1 \times 5 \times 2 = 1$

∴ After neutralisation millimoles of H^+left

$= 3 - 1 = 2$

∴ Concentration of H^+

$$= \frac{\text{moles of } H^+}{\text{volumes of solution}}$$
$$= \frac{2}{10+5+5} = \frac{2}{20} = 0.1$$
$\therefore$ $pH = -\log[H^+] = -\log(10^{-1}) = 1$

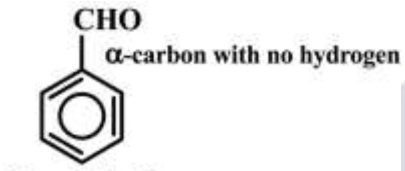
118. **(b)** Conjugate acid NH_3 is obtained by addition of H^+ ion to the base NH_2^- is

$$\underset{\text{Base}}{NH_2^-} + H^+ \longrightarrow \underset{\text{Conjugate acid}}{NH_3}$$

119. **(b)** Phenyl isocyanide is obtained from the carbylamine reaction as follows :

$$\underset{\text{Aniline}}{C_6H_5NH_2} + \underset{\text{Chloroform}}{CHCl_3} + \underset{\text{Potassium hydroxide}}{3\,KOH} \rightarrow \underset{\text{Phenyl isocyanide}}{C_6H_5NC} + 3\,KCl + 3H_2O$$

120. **(c)** Aldol condensation is a characteristic reaction of aldehyde having α-hydrogen atom :
Benzaldehyde do not contain any α-hydrogen atom.
$\therefore$ It do not undergoes aldol condensation reaction.



Benzaldehyde

MATHEMATICS

121. **(b)** We have,
$$7x^2 + 7y^2 - 7x + 14y + k = 0$$
$$x^2 + y^2 - x + 2y + \frac{k}{7} = 0$$
Here,
$$C \equiv \left(\frac{1}{2}, -1\right)$$
And
$$r = \sqrt{\frac{1}{4} + 1 - \frac{k}{7}}$$
$$r = \sqrt{\frac{5}{4} - \frac{k}{7}}$$
Given that, Area $= 12\pi$ sq. unit
$$\Rightarrow 12\pi = \pi\left(\frac{5}{4} - \frac{k}{7}\right)$$
$$\Rightarrow 12 = \frac{5}{4} - \frac{k}{7} \Rightarrow 12 = \frac{35-4k}{28}$$
$$\Rightarrow 336 = 35 - 4k$$
$$\Rightarrow 4k = -301 \Rightarrow k = -\frac{301}{4}$$

122. **(c)** Given that, $SP + S'P = 10$
$\Rightarrow 2a = 10 \Rightarrow a = 5$
and $\quad 2ae = 8$
$ae = 4$
$\because b^2 = a^2(1 - e^2)$
$= a^2 - (ae)^2$
$= 25 - 16 = 9$
Hence, required equation is

$$\frac{x^2}{25}+\frac{y^2}{9}=1$$

123. **(d)** We have,

$x^2+y^2-2x-4y+1=0$

and $x^2+y^2-12x-16y+91=0$

Here, $C_1 \equiv (1,2)$ and $r_1=\sqrt{1+4-1}=2$

and $C_2 \equiv (6,8)$ and $r_2=\sqrt{36+64-91}=3$

Now, $C_1C_2=\sqrt{25+36}=\sqrt{61}$

Here, $r_1+r_2=5<\sqrt{61}$

So, circle are far apart.

Hence, number of common tangents is 4 .

124. **(a)** Given, equation of circle is

$x^2+y^2+4x-6y-9=0$

Here, $T=S_1$

$x(0)+y(1)+2(x+0)-3(y+1)-9=0+1+0-6-9$

$\Rightarrow y+2x-3y-3-9=1-15$

$\Rightarrow 2x-2y-12=1-15$

$\Rightarrow 2x-2y+2=0$

$\Rightarrow x-y+1=0$

125. **(b)** Given, equation of hyperbola is $\frac{x^2}{25}-\frac{y^2}{16}=1$

Here,

$a^2=25 \Rightarrow a=5$

$b^2=16 \Rightarrow b=4$

Now, $\theta=2\tan^{-1}\left(\frac{b}{a}\right)=2\tan^{-1}\left(\frac{4}{5}\right)$

126. **(c)** Given that, $x=t^2+1 \Rightarrow x-1=t^2$

and

$y=2t+1 \Rightarrow y-1=2t$

Now,

$(y-1)^2=4t^2=4(x-1)$

$(y-1)^2=4(x-1)$

So,$x-1=-1$

$\Rightarrow x=0$

127. **(d)** We have, $|a\times b|=5$

and $|a\cdot b|=3$

$\because\ |a\times b|^2=|a|^2|b|^2-(a\cdot b)^2$

$\Rightarrow\ 25=|a|^2|b|^2-9$

$\Rightarrow |a|^2|b|^2=34$

128. **(a)** Let $a=2\hat{i}+\hat{j}-2\hat{k}$

So,$|a|=\sqrt{4+1+4}=\sqrt{9}=3$

So, direction cosines of vector are $\frac{2}{3},\frac{1}{3},-\frac{2}{3}$.

129. **(c)** We have,

$(3+3\omega^2+5\omega)^6-(2+6\omega^2+2\omega)^3$

$=[3(1+\omega^2)+5\omega]^6-[2(1+\omega)+6\omega^2]^3$

$=[3(-\omega)+5\omega]^6-[2(-\omega^2)+6\omega^2]^3$

$=(2\omega)^6-(4\omega^2)^3$

$=2^6\omega^6-64\omega^6$

$=64-64=0$

130. **(a)** We have,

$$\int_{\log 2}^{x}\frac{1}{\sqrt{e^y-1}}dy=\frac{\pi}{6}$$

Let $\sqrt{e^y-1}=t$

$\Rightarrow e^y - 1 = t^2$
$\Rightarrow e^y = 1 + t^2$
$\Rightarrow e^y dy = 2tdt$
$\Rightarrow (1 + t^2)dy = 2tdt$
$\Rightarrow dy = \frac{2t}{(1+t^2)}dt$
Now, $\int_1^{\sqrt{e^x-1}} \frac{2t}{(1+t^2)\times t}dt = \frac{\pi}{6}$
$\Rightarrow \int_1^{\sqrt{e^x-1}} \frac{dt}{1+t^2} = \frac{\pi}{12}$
$\Rightarrow [\tan^{-1}t]_1^{\sqrt{e^x-1}} = \frac{\pi}{12}$
$\Rightarrow \tan^{-1}\sqrt{e^x - 1} - \frac{\pi}{4} = \frac{\pi}{12}$
$\Rightarrow \tan^{-1}\sqrt{e^x - 1} = \frac{\pi}{3}$
$\Rightarrow \tan\left(\tan^{-1}\sqrt{e^x - 1}\right) = \tan\left(\frac{\pi}{3}\right)$
$\Rightarrow \sqrt{e^x - 1} = \sqrt{3}$
$\Rightarrow e^x - 1 = 3 \Rightarrow e^x = 4$
$\Rightarrow x = \log 4$

131. **(c)** Since, $(\sin^{93}x + x^{295})$ is a odd function.
So, $\int_{-8}^{8} (\sin^{93}x + x^{295})dx = 0$

132. **(b)** We have,
$\frac{x^2}{25} + \frac{y^2}{16} = 1$
Here,
$a^2 = 25 \Rightarrow a = 5$
$b^2 = 16 \Rightarrow b = 4$
So, required area $= \pi ab$
$= \pi(5)(4)$
$= 20\pi$ sq units

133. **(c)** Given, differential equation is
$$\left[1 + \left(\frac{dy}{dx}\right)^5\right]^{\frac{2}{3}} = \frac{d^3y}{dx^3}$$
Here, highest derivative is $\frac{d^3y}{dx^3}$.
So, order = 3

134. **(c)** Given, differential equation is
$\frac{dy}{dx} - 1 = e^{x-y}$
Put
$x - y = t$
$\Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx}$
So, given equation becomes,
$-\frac{dt}{dx} = e^t$
$-e^{-t}dt = dx$
Integrating both sides, we get
$e^{-t} = x + c$
$e^{-(x-y)} = x + c$

135. **(d)** We have,
$$\sin^{-1}\left(\frac{2p}{1+p^2}\right) - \cos^{-1}\left(\frac{1-q^2}{1+q^2}\right)$$

$= \tan^{-1}\left(\frac{2x}{1+x^2}\right)$
$\Rightarrow 2\tan^{-1}p - 2\tan^{-1}q = 2\tan^{-1}x$
$\Rightarrow 2(\tan^{-1}p - \tan^{-1}q) = 2\tan^{-1}x$
$\Rightarrow \tan^{-1}p - \tan^{-1}q = \tan^{-1}x$
$\Rightarrow \tan^{-1}\left(\frac{p-q}{1+pq}\right) = \tan^{-1}x$
$\Rightarrow x = \frac{p-q}{1+pq}$

136. **(a)** Given that, $a + 2b - c$

So, required unit vector $= \frac{a+2b-c}{\sqrt{1+4+1}} = \frac{a+2b-c}{\sqrt{6}}$

137. **(c)** It is a false option because every group of order 3 is abelian because it is closed as well.

138. **(b)** We have,

$y = \tan^{-1}\left(\frac{1}{1+x(x+1)}\right) +$
$\tan^{-1}\left(\frac{1}{1+(x+1)(x+2)}\right) +$
$\tan^{-1}\left(\frac{1}{1+(x+2)(x+3)}\right) + \cdots$
$+\tan^{-1}\left(\frac{1}{1+(x+n-1)(x+n)}\right)$

When $n = 1$, then

$y = \tan^{-1}\left(\frac{1}{1+x(x+1)}\right)$
$= \tan^{-1}\left(\frac{(x+1)-x}{1+x(x+1)}\right)$
$y = \tan^{-1}(x+1) - \tan^{-1}x$
$\frac{dy}{dx} = \frac{1}{1+(x+1)^2} - \frac{1}{1+x^2}$
$\left.\frac{dy}{dx}\right|_{x=0} = \frac{1}{1+(0+1)^2} - \frac{1}{1+(0)^2}$
$= \frac{1}{1+1} - \frac{1}{1} = \frac{1}{2} - 1 = -\frac{1}{2}$

139. **(c)** We have,

$\cot\alpha\cot\beta = 2$

Now, $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} = \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta}$

$= \frac{\cot\alpha\cot\beta - 1}{\cot\alpha\cot\beta + 1} = \frac{2-1}{2+1} = \frac{1}{3}$

140. **(b)** We have, $\begin{vmatrix} 1+\omega & \omega^2 & \omega \\ \omega^2+\omega & -\omega & \omega^2 \\ 1+\omega^2 & \omega & \omega^2 \end{vmatrix}$

$= \begin{vmatrix} 1+\omega+\omega^2 & \omega^2 & \omega \\ \omega^2 & -\omega & \omega^2 \\ 1+\omega+\omega^2 & \omega & \omega^2 \end{vmatrix} (C_1 \to C_1 + C_2)$

$= \begin{vmatrix} 0 & \omega^2 & \omega \\ \omega^2 & -\omega & \omega^2 \\ 0 & \omega & \omega^2 \end{vmatrix}$

$= 0 - \omega^2(\omega^4 - 0) + \omega(\omega^3 - 0)$
$= -\omega^6 + \omega^4$
$= -1 + \omega \quad [\because \omega^3 = 1]$
$= \omega - 1$

141. **(c)** Given, equation of curve is

$2y^3 = ax^2 + x^3$

$\Rightarrow 6y^2\dfrac{dy}{dx} = 2ax + 3x^2$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{2ax + 3x^2}{6y^2}$

$\Rightarrow \left.\dfrac{dy}{dx}\right|_{(a,a)} = \dfrac{2a^2 + 3a^2}{6a^2} = \dfrac{5a^2}{6a^2} = \dfrac{5}{6}$

So, x-intercept $= -\frac{a}{5} = \alpha$

and y-intercept $= \dfrac{a}{6} = \beta$

$\because \alpha^2 + \beta^2 = 61$

$\Rightarrow \dfrac{a^2}{25} + \dfrac{a^2}{36} = 61$

$\Rightarrow \dfrac{61a^2}{36 \times 25} = 61$

$\Rightarrow a^2 = 36 \times 25$

$\Rightarrow a = \pm 6 \times 5 = \pm 30$

142. **(d)** $y^2 = \frac{x}{2a+x}$

$2y\dfrac{dy}{dx} = \dfrac{2x}{2a + x} - \dfrac{x^2}{(2a + x)^2}$

At $(a, a), \left(\dfrac{dy}{dx}\right)_{(a,a)} = \dfrac{5}{18a}$

Length of subtangent at $(x_1, y_1) = \dfrac{y_1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}}$

$\therefore$ Length of subtangent at $(a, a) = \dfrac{a}{\left(\frac{dy}{dx}\right)_{(a,a)}} = \dfrac{18a^2}{5}$

143. **(a)** Given, $f(x) = 5 + 36x + 3x^2 - 2x^3$

$\Rightarrow f'(x) = 36 + 6x - 6x^2$

$= -6(x^2 - x - 6)$

$= -6(x^2 - 3x + 2x - 6)$

$= -6(x - 3)(x + 2)$

For increasing function, $f'(x) > 0$

$\Rightarrow (x - 3)(x + 2) < 0$

$\Rightarrow x < 3$ and $x > -2$

$\Rightarrow -2 < x < 3$

Hence, function f(x) is increasing in$(-2,3)$ interval.

144. **(b)** According to question,

$x + y = 20$

And $p = xy^3 = (20 - y)y^3$ [by Eq.(i)]

$p = 20y^3 - y^4$

$\Rightarrow \dfrac{dp}{dy} = 60y^2 - 4y^3$

$\Rightarrow \dfrac{d^2p}{dy^2} = 120y - 12y^2$

Now, $\dfrac{dp}{dy} = 0 \Rightarrow 60y^2 - 4y^3 = 0$

$\Rightarrow 60y^2 = 4y^3$

$\Rightarrow 4y = 60 \Rightarrow y = 15$

$x = 5$

Hence, two parts are (15,5).

145. **(c)** We have, $4896 = 2^5 \times 3^2 \times 17^1$

So, $T(a) = (1 + 5)(1 + 2)(1 + 1) = 36$

146. **(d)** We have, $583! \equiv 0(\bmod 10)$

and

$7^2 = -1(\bmod 10)$

So, $(7^2)^{145} \cdot 7^1 = (-1)^{145} \cdot 7^1(\bmod 10)$

$\Rightarrow 7^{291} = -7(\bmod 10) \equiv 3(\bmod 10)$

So, $583! + 7^{291} = 3(\bmod 10)$

Hence, last digit is 3 .

147. **(a)** We have, $I = \int x^x(1 + \log x)dx$

Let $x^x = t$

$\Rightarrow \frac{dt}{dx} = x^x \log x + x^x$

$\Rightarrow \frac{dt}{dx} = x^x(1 + \log x)$

$\Rightarrow dt = x^x(1 + \log x)dx$

Now, $\quad I = \int dt$

$= t + C = x^x + C$

148. **(c)** We have, $I = \int \frac{xe^x}{(1+x)^2}dx$

$$= \int \frac{(x+1-1)}{(x+1)^2}e^x dx$$

$$= \int \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right]e^x dx$$

$$= e^x \frac{1}{(1+x)} + C$$

On comparing, we get

$$f(x) = \frac{1}{1+x}$$

149. **(d)** We have,

$$I = \int_0^{\pi/2} \frac{\sin 2t}{\sin^4 t + \cos^4 t}dt$$

$$= \int_0^{\pi/2} \frac{2\sin t\cos t}{\sin^4 t + \cos^4 t}dt$$

$$= \int_0^{\pi/2} \frac{2\tan t\sec^2 t}{(\tan^2 t)^2 + 1}dt$$

Put $\tan^2 t = x$

$(2\tan t\sec^2 t)dt = dx$

$$\therefore I = \int_0^{\infty} \frac{1}{1+x^2}dx = [\tan^{-1}x]_0^{\infty} = \frac{\pi}{2}$$

150. **(a)** We have,

$4^{\log_9 3} + 9^{\log_2 4} = 10^{\log_x 83}$

$\Rightarrow 4^{\frac{1}{2}\log_3 3} + 9^{2\log_2 2} = 10^{\log_x 83}$

$\Rightarrow 2 + 81 = 10^{\log_x 83}$

$\Rightarrow 83 = 10^{\log_x 83}$

$\Rightarrow \log_{10} 83 = \log_x 83$

$\Rightarrow x = 10$

151. **(b)** We have,

$$p = 3^{\frac{1}{3}} \cdot 3^{\frac{2}{9}} \cdot 3^{\frac{3}{27}}$$

$$= 3^{\frac{1}{3}\left[1+\frac{2}{3}+\frac{3}{9}+\cdots\infty\right]}$$

$$= 3^{\frac{1}{3}\left[\frac{a}{1-x}+\frac{ar}{(1-r)^2}\right]}$$

$= 3^{\frac{1}{3}}\left[\frac{1}{1-\frac{1}{3}}+\frac{1\cdot\frac{1}{3}}{\left(1-\frac{1}{3}\right)^2}\right]$

$= 3^{\frac{1}{3}\left[\frac{3}{2}+\frac{3}{4}\right]}$

$= 3^{\frac{1}{3}\left[\frac{12+6}{8}\right]} = 3^{\frac{1}{3}\left(\frac{18}{8}\right)} = 3^{\frac{3}{4}}$

Now, $p^{4/3} = \left(3^{\frac{3}{4}}\right)^{\frac{4}{3}}$

$= 3^{\frac{3}{4}\times\frac{4}{3}} = 3^1 = 3$

152. **(c)** We have,

$x^3 - 3x^2 + 2x - 1$

So,

$\alpha+\beta+\gamma = 3$

$\alpha\beta+\beta\gamma+\gamma\alpha = 2$

$\alpha\beta\gamma = 1$

Now, $(1-\alpha)(1-\beta)(1-\gamma)$

$= 1-(\alpha+\beta+\gamma)+(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma$

$= 1-3+2-1 = -1$

153. **(d)** The middle term in the expansion of $(1+x)^{2n}$ is $\left(\frac{2n}{2}+1\right)$ th term i.e. $(n+1)$ th term.

$= \frac{(2n)!}{n!\,n!}x^n$

$= \frac{2n(2n-1)(2n-2)\ldots3\cdot2\cdot1}{\{n(n-1)(n-2)\ldots3\cdot2\cdot1\}^2}x^n$

$= \frac{(2n(2n-2)(2n-4)\ldots4\cdot2\}\{(2n-1)(2n-3)\ldots(3\cdot1)\}}{\{n(n-1)(n-2)\ldots3\cdot2\cdot1\}^2}x^n$

$= \frac{2^n\{n(n-1)(n-2)\ldots3\cdot2\cdot1\}\{((2n-1)(2n-3)\ldots3\cdot1\}}{\{n(n-1)(n-2)\ldots3\cdot2\cdot1\}^2}x^n$

$= \frac{2^n\{(2n-1)(2n-3)\ldots3\cdot1\}}{\{n(n-1)(n-2)\ldots3\cdot2\cdot1\}}x^n$

$= \frac{(2n-1)(2n-3)\ldots3\cdot1}{n!}x^n2^n$

154. **(a)** Given that, $p \to (\sim q \vee r)$ is false.

$\Rightarrow (\sim q \vee r)$ is false and p is true.

$\Rightarrow$ q is true, r is false and p is true.

155. **(c)** We have,

$\frac{2}{9!}+\frac{2}{3!\,7!}+\frac{1}{5!\,5!}=\frac{2^a}{b!}$

$\Rightarrow \frac{1}{9!}\left[2+\frac{2\cdot8\cdot9}{3!}+\frac{6\cdot7\cdot8\cdot9}{5!}\right]=\frac{2^a}{b!}$

$\Rightarrow \frac{1}{9!}\left[2+24+\frac{126}{5}\right]=\frac{2^a}{b!}$

$\Rightarrow \frac{1}{9!}\left[26+\frac{126}{5}\right]=\frac{2^a}{b!}$

$\Rightarrow \frac{1}{9!}\left[\frac{256}{5}\right]=\frac{2^a}{b!}$

$\Rightarrow \frac{1}{9!}\frac{256\times2}{5\times2}=\frac{512}{10!}=\frac{2^9}{10!}=\frac{2^a}{b!} \Rightarrow a=9$ and $b=10$

156. **(d)** $\tan10^\circ\tan20^\circ\tan30^\circ\tan40^\circ\tan50$ $\tan60^\circ\tan70^\circ\tan80^\circ$

$= \tan10^\circ\tan20^\circ\left(\frac{1}{\sqrt{3}}\right)\tan40^\circ\tan(90^\circ-40^\circ)$

$(\sqrt{3})\tan(90^\circ-20^\circ)\tan(90^\circ-10^\circ)$

$= \tan 10^\circ \tan 20^\circ \tan 40^\circ \cot 40^\circ \cot 20^\circ \cot 10^\circ$

$= 1$

157. **(a)** We have, $\tan\theta = \frac{m}{n}$

Now, $n\cos 2\theta + m\sin 2\theta$

$$= n\left[\frac{1-\tan^2\theta}{1+\tan^2\theta}\right] + m\left[\frac{2\tan\theta}{1+\tan^2\theta}\right]$$

$$= n\left[\frac{1-\frac{m^2}{n^2}}{1+\frac{m^2}{n^2}}\right] + m\left[\frac{2\frac{m}{n}}{1+\frac{m^2}{n^2}}\right]$$

$$= n\left[\frac{n^2-m^2}{n^2+m^2}\right] + \left[\frac{\frac{2m^2}{n}\times n^2}{n^2+m^2}\right]$$

$$= n\left(\frac{n^2-m^2}{m^2+n^2}\right) + \frac{2m^2n}{n^2+m^2} = \frac{n(n^2-m^2)+2m^2n}{(m^2+n^2)}$$

$$= \frac{n[n^2-m^2+2m^2]}{(m^2+n^2)} = \frac{n(n^2+m^2)}{(m^2+n^2)} = n$$

158. **(b)** Let $\frac{b+c}{11} = \frac{c+a}{12} = \frac{a+b}{13} = k$

$\Rightarrow b+c = 11k, c+a = 12k, a+b = 13k$

Now, $2(a+b+c) = 36k$

$a+b+c = 18k$

So,

$a = 7k$

$b = 6k$

$c = 5k$

and Now, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$$= \frac{36k^2+25k^2-49k^2}{2(30k^2)} = \frac{12k^2}{60k^2} = \frac{1}{5}$$

159. **(d)** We have, $a = \cos 2\alpha + i\sin 2\alpha$ and $b = \cos 2\beta + i\sin 2\beta$

$$\frac{a}{b} = \frac{\text{Cis}2\alpha}{\text{Cis}2\beta} = \text{Cis}(2\alpha - 2\beta)$$

So, $\sqrt{\frac{a}{b}} = \text{Cis}(\alpha-\beta) = \cos(\alpha-\beta) + i\sin(\alpha-\beta)$

and $\sqrt{\frac{b}{a}} = \cos(\alpha-\beta) - i\sin(\alpha-\beta)$

Now, $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \cos(\alpha-\beta) + i\sin(\alpha-\beta)$

$+\cos(\alpha-\beta) - i\sin(\alpha-\beta) = 2\cos(\alpha-\beta)$

160. **(a)** We have,

$$y = \log\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$$

So, $\frac{dy}{dx} = \frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)} \cdot \sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right) \cdot \frac{1}{2}$

$$= \frac{1}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

$$= \frac{1}{\sin\left[2\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]} = \frac{1}{\cos x} = \sec x$$

161. **(b)** We have, $y = \sin^2\left[\tan^{-1}\sqrt{\frac{1-x^2}{1+x^2}}\right]$

Put $x^2 = \cos\theta$

So,

$$y = \sin^2\left[\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right]$$

$$= \sin^2\left[\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right]$$

$$= \sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2} = \frac{1-x^2}{2}$$

$$\frac{dy}{dx} = \frac{-2x}{2} = -x$$

162. **(c)** We have,

$$\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = \sqrt{a}$$

$$\left(\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}}\right)^2 = a$$

$$\frac{x}{y} + \frac{y}{x} + 2 = a$$

Differentiating both sides w.r.t. x,

$$\frac{y - x\frac{dy}{dx}}{y^2} + \frac{x\frac{dy}{dx} - y}{x^2} = 0$$

$$\Rightarrow \frac{y - x\frac{dy}{dx}}{y^2} = -\frac{\left(x\frac{dy}{dx} - y\right)}{x^2}$$

$$\Rightarrow xy^2\frac{dy}{dx} - y^3 = -x^2y + x^3\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(xy^2 - x^3) = -x^2y + y^3$$

$$\Rightarrow \frac{dy}{dx} = \frac{y(y^2 - x^2)}{x(y^2 - x^2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

Now, $y \cdot \frac{dx}{dy} = y \cdot \frac{x}{y} = x$

163. **(d)** We have, $x = \frac{1-t}{1+t}$

$$\frac{dx}{dt} = \frac{-(1+t)-(1-t)}{(1+t)^2} = \frac{-2}{1+t^2} \text{ and } y = \frac{2t}{1+t}$$

$$\frac{dy}{dt} = \frac{(1+t)2 - 2t}{(1+t)^2} = \frac{2}{(1+t)^2}$$

Now, $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt} = -1$

and $\dfrac{d^2y}{dx^2} = 0$

164. **(a)** Clearly, $11^{-1} = 11$ $\quad [\because 11 \otimes_{12} 11 = 1]$

So, $7 \otimes_{12} 11^{-1} = 7 \otimes_{12} 11 = 5$

165. **(b)** Option (c) cannot be subgroup as identity ' 1 ' is not present.

Option (d) cannot be a subgroup of $2 \otimes_7 3 = 6 \notin \{1,2,3\}$.

Option (a) cannot be a subgroup as $2 \otimes_7 6 = 5 \notin \{2,6,1\}$.

So, option (b) is a subgroup.

166. **(c)** We have,

$A=\begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$

Now,

$|A|=\begin{vmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{vmatrix}$

$=3(-3+4)+3(2-0)+4(-2-0$

$=3+6-8=1$

Now, $\frac{|C|}{|\text{adj}B|}=\frac{|5A|}{|B|^{3-1}}=\frac{5^3|A|}{|B|^2}=\frac{5^3|A|}{|\text{adj}A|^2}$

$=\frac{5^3|A|}{(|A|^2)^2}=\frac{5^3|A|}{|A|^4}=\frac{5^3}{|A|^3}$

$=\frac{5^3}{1^3}=5^3$

167. **(d)** We have, $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 3\\ 1 & 3 & 4\end{bmatrix}, B=\begin{bmatrix}7\\ 16\\ 22\end{bmatrix}$

$X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$

Now, $AX=B$

$x+y+z=7$ …..(i)

$x+2y+3z=16$ …….(ii)

$x+3y+4z=22$ ……(iii)

Subtracting Eq. (i) from Eq. (ii), we get

$y+2z=9$ ……..(iv)

Subtracting Eq. (iii) from Eq. (ii), we get

$y+z=6$ ……..(v)

Subtracting Eq. (v) from Eq. (iv), we get

$z=3$

168. **(a)** We have, $A=\begin{bmatrix}1 & \log_b a\\ \log_a b & 1\end{bmatrix}$

Now,

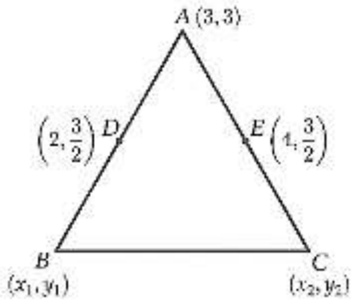
$|A|=\begin{vmatrix}1 & \log_b a\\ \log_a b & 1\end{vmatrix}$

$=1-\log_a b\log_b a$

$=1-\log_a b\times\frac{1}{\log_a b}\left\{\because \log_b a=\frac{1}{\log_a b}\right\}$

$=1-1=0$

169. **(c)** Let the vertices of $\triangle ABC$ are $A(3,3), B(x_1,y_1)$ and $C(x_2,y_2)$.



Now, $\frac{x_1+3}{2}=2$ and $\frac{y_1+3}{2}=\frac{3}{2}$

$x_1+3=4$ and $y_1+3=3$

$x_1 = 1$ and $y_1 = 0$

So, $B \equiv (1,0)$
and $\frac{x_2+3}{2} = 4$ and $\frac{y_2+3}{2} = \frac{3}{2}$

$x_2 + 3 = 8$ and $y_2 + 3 = 3$

$x_2 = 5$ and $y_2 = 0$
So,$C \equiv (5,0)$

Hence, centroid $= \left(\frac{3+1+5}{3}, \frac{3+0+0}{3}\right)$

$= \left(\frac{9}{3}, \frac{3}{3}\right) = (3,1)$

170. **(c)** Let the image of point $(2,4) = (h,k)$, then

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1+by_1+c)}{a^2+b^2}$$
$$\frac{h-2}{1} = \frac{k-4}{1} = \frac{-2(2+4-10)}{1^2+1^2}$$
$$\frac{h-2}{1} = \frac{k-4}{1} = \frac{-2(-4)}{2}$$
$$\frac{h-2}{1} = \frac{k-4}{1} = 4$$

$h - 2 = 4$ and $k - 4 = 4$
$h = 6$ and $k = 8$
So, $(h,k) = (6,8)$

171. **(d)** Given, equation of line is $x^2 - 4pxy + 8y^2 = 0$
On comparing with $ax^2 + 2hxy + by^2 = 0$
$\Rightarrow\ a = 1, b = 8, 2h = -4p$
Given that, $m_1 + m_2 = 3m_1m_2$

$$\Rightarrow -\frac{2h}{b} = \frac{3a}{b}$$
$\Rightarrow -2h = 3a$
$\Rightarrow 4p = 3a$
$$\Rightarrow\ p = \frac{3a}{4}$$
$\Rightarrow\ p = \frac{3}{4}$ $\qquad [\because a = 1]$

172. **(a)** $\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}}$

$$= \lim_{x\to 0}\frac{(1+5x^2)^{\frac{1}{5x^2}\cdot 5}}{(1+3x^2)^{\frac{1}{3x^2}\cdot 3}} = \frac{e^5}{e^3} = e^{5-3} = e^2$$

173. **(b)** $f(x)$ is continuous at $x = 0$
So, $\lim_{x\to 0} f(x) = f(0)$
$$\Rightarrow \lim_{x\to 0}\frac{e^{3x-1}}{4x} = \frac{k+0}{4}$$
$$\Rightarrow \frac{1}{4}\lim_{x\to 0} 3\left(\frac{e^{3x}-1}{3x}\right) = \frac{k}{4}$$
$$\Rightarrow 3\lim_{x\to 0}\frac{e^{3x}-1}{3x} = k \Rightarrow 3\times 1 = k \Rightarrow k = 3$$

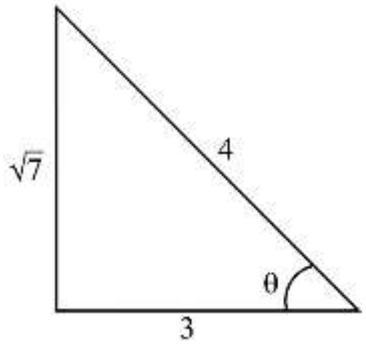
174. **(d)** Non-adjacent vertex are not connected through edge.
$\therefore V_1V_3$ is correct option.

175. **(a)** $\sin\left[2\cos^{-1}\left\{\cot\left(\tan^{-1}\frac{1}{1-\frac{1}{4}}\right)\right\}\right]$

$= \sin\left[2\cos^{-1}\left\{\cot\left(\tan^{-1}\frac{4}{3}\right)\right\}\right]$

$= \sin\left[2\cos^{-1}\frac{3}{4}\right] = \sin 2\theta$

$\left[\text{where, } \cos\theta = \frac{3}{4} \text{ and } \theta = \cos^{-1}\frac{3}{4}\right]$



$= 2\sin\theta\cos\theta = 2\frac{\sqrt{7}}{4}\cdot\frac{3}{4} = \frac{3\sqrt{7}}{8}$

176. **(b)** $\frac{3+4i}{4-5i}$

Now, multiplicative inverse is

$\frac{4-5i}{3+4i}\times\frac{3-4i}{3-4i}$

$= \frac{(4-5i)(3-4i)}{9+16}$

$= \frac{12-16i-15i+20i^2}{25} = \frac{-8-31i}{25} = \left(-\frac{8}{25}, -\frac{31}{25}\right)$

177. **(a)** $\tan x - \sin x = 1 - \tan x\sin x$

$\Rightarrow \tan x + \tan x\sin x = 1 + \sin x$

$\tan x(1+\sin x) - (1+\sin x) = 0$

$(1+\sin x)(\tan x - 1) = 0$

$\Rightarrow \tan x - 1 = 0$ or $1 + \sin x = 0$

$\tan x = 1 = \tan\frac{\pi}{4}$ or $\sin x = -1 = \sin\left(-\frac{\pi}{2}\right)$

$x = n\pi + \frac{\pi}{4}$ or $x = n\pi + (-1)^n\left(-\frac{\pi}{2}\right)$

178. **(d)** Let θ be the angle of intersection of two given circles having centres $(-g,-f) = (-2,-1)$ and $(-g_1,-f_1) = (1,-3)$ respectively, then

$\cos\theta = \left|\frac{-2gg_1 - 2ff_1 + c + c_1}{2\sqrt{g^2+f^2-c}\sqrt{g_1^2+f_1^2-c_1}}\right|$

$= \left|\frac{-2(2)(-1) - 2(1)(3) + 1 - 6}{2\sqrt{4+1-1}\sqrt{1+9+6}}\right|$

$= \left|\frac{4-6-5}{2(2)(4)}\right| = \left(\frac{7}{6}\right)$

$\theta = \cos^{-1}\left(\frac{7}{16}\right)$

179. **(b)** We have, $|\mathrm{a}| = 2, |\mathrm{b}| = 7$

and $\mathrm{a}\times\mathrm{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$

So, $|a\times b| = \sqrt{9+4+36} = \sqrt{49} = 7$

Since, $|a\times b| = |a||b|\sin\theta|\hat{n}|$

$\Rightarrow 7 = (2)(7)\sin\theta + 1[\because |\hat{n}| = 1]$

$\Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \sin\theta = \sin\frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{6}$

180. **(b)** We have, $f(x) = \log(1-x) + \sqrt{x^2-1}$

$\log(1-x)$ is defined if $1 - x > 0 \Rightarrow x < 1$

$\Rightarrow x \in (-\infty, -1)$ ....(i)

and $\sqrt{x^2-1}$ is defined if $x^2 - 1 \geq 0$

$\Rightarrow x^2 \geq 1$

$\Rightarrow x \geq 1$ or $x \leq -1$ ......(ii)

By Eqs. (i) and (ii), required domain is $(-\infty, -1]$.