PHYSICS

1. **(d)** If frequency of source be f_s, then

$f_{approach} - f_{recession} = \frac{2}{100} \times f_o$

$\Rightarrow f_s\left(\frac{v+v_s}{v}\right) - f_s\left(\frac{v-v_s}{v}\right) = \frac{f_s}{50}$

$\Rightarrow \frac{2v_s}{v} = \frac{1}{50} \Rightarrow \frac{2v_s}{300} = \frac{1}{50}$

$\Rightarrow v_s = \frac{300}{2 \times 50} = 3\ ms^{-1}$

2. **(d)** The magnetic force between two current carrying wires for current in same direction is attractive.
Thus, force on B due to the magnetic field produced due to wire A is

$F_A = B_A I_B l = \frac{\mu_0 \times I_A}{2\pi d} \times I_B \times l$

$= \frac{\mu_0 \times 1 \times 2 \times l}{2\pi d} = \frac{2\mu_0 l}{2\pi d}$, towards A ….(i)

Similarly, force due to field of wire C,

$F_C = B_C I_B l = \frac{\mu_0 I_C I_B l}{2\pi d} = \frac{\mu_0 \times 3 \times 2 \times l}{2\pi d}$

$\Rightarrow F_C = \frac{6\mu_0 l}{2\pi d}$, towards C …..(ii)

From Eqs. (i) and (ii) we get

$F_C > F_A$

$\therefore$ The resultant force on B is directed towards C.

3. **(d)** Ferromagnetic materials like iron obeys Curie-Weiss law above its Curie temperature. According to Curie-Weiss law, magnetic susceptiblity (χ) of ferromagnetic material is related with temperature T as

$\chi = \frac{C}{T - T_C}$

where, C = Curie constant

and T_C = Curie temperature.

4. **(b)** As the magnet oscillates about its mean position performing SHM, the magnetic flux of the coil changes and emf is induced in it. So, the galvanometer shows deflection to the left and right for both in and out motion. But due to eddy currents, the velocity of magnet decreases So, the amplitude of deflection also decreases steadily.

5. **(a)** As we know, magnetic flux, $\phi = LI$

$\Rightarrow L = \frac{\phi}{I} = \frac{BA}{I}$

$\therefore$ Dimensions of $[L] = \frac{[B][A]}{[I]}$

$= \frac{[ML^0\ T^{-2}\ A^{-1}][L^2]}{[A]} = [ML^2\ T^{-2}\ A^{-2}]$

6. **(d)** Given, $G = 40\Omega, I_g = 10\ mA = 10 \times 10^{-3}\ A$

$V = 50\ V$

Let R be the resistance connected in series with the galvanometer to convert into voltmeter, then

$V = I_g(G + R)$

$\Rightarrow 50 = 10 \times 10^{-3}(40 + R)$

$\Rightarrow \frac{50}{10 \times 10^{-3}} = 40 + R$

$\Rightarrow R = 5000 - 40 = 4960\Omega$

7. **(d)** Chromosphere is the outer region of the sun just above the photosphere. The photosphere emits continuous spectrum that passes through the chromosphere after some absorption. These absorption lines are observed in solar spectrum.
During a total solar eclipse, when the light from photosphere is cut-off, the chromosphere is visible to us. Hence, the line absorption spectrum with continuous background is seen during a total solar eclipse.

8. **(a)** Heavy water (D_2O) contains heavy hydrogen or deuterium and oxygen.

9. **(d)** The nuclear reactor at Kaiga is a power reactor and named as Kaiga Atomic Power Station.
It is used for electricity and power generation.

10. **(a)** In a circular path, the centripetal force acts towards the centre of circle and the displacement is along the tangent to the circle. Hence, $\theta = 90°$

$\therefore$Work done $W = F.s = F s\cos\theta$

$= F s\cos 90° = 0$

11. **(d)** Given, $u_1 = 100\ ms^{-1}$

$\Rightarrow u_2 = 2 \times u_1 = 2 \times 100 = 200\ ms^{-1}$

Using equation of motion,

$v^2 = u^2 - 2as$ $\quad$ ($\because$ acceleration is negative)

As, final velocity, $v = 0$

$\Rightarrow u^2 = 2as$ or $s = \frac{u^2}{2a} \Rightarrow s \propto u^2$

$\therefore \frac{s_1}{s_2} = \frac{u_1^2}{u_2^2} = \frac{(100)^2}{(200)^2} = \frac{1}{4}$

$\Rightarrow s_2 = 4s_1 = 4 \times 2$ planks $= 8$ planks

12. **(a)** Given, $m_1 = 1$ kg, $m_2 = 2$ kg

The kinetic energy is related to momentum of body by the formula given as

$K = \frac{p^2}{2m}$

$\therefore \frac{K_1}{K_2} = \frac{m_2}{m_1} = \frac{2}{1}$ $\quad$ ($\because$ momentum is same)

13. **(a)** An open window is like a perfectly black body that absorbs completely the quantity being transferred, so its absorption coefficient is unity or 1 .

14. **(c)** The loudness depends on the amplitude of sound wave, thus it depends on its intensity.
The pitch of a sound describes how low or high a note is produced by the instrument. Thus, it depends on the frequency of sound source.

15. **(c)** In Melde's experiment, the change in frequency is produced when the tension in the string is increased. So, a resonance is required between tuning fork and the frequency of waves produced. For this, the frequencies of both should be same or their ratio be $1:1$.

16. **(c)** As we know, Kinetic energy of electron = Energy acquired due to potential (V_0)

$\Rightarrow \frac{1}{2} m_e v^2 = eV_0$

$\Rightarrow v = \sqrt{\frac{2eV_0}{m_e}}$

$= \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 45.5}{9.1 \times 10^{-31}}}$

$= \sqrt{16 \times 10^{12}} = 4 \times 10^6\ ms^{-1}$

17. **(a)** In first case, $P = R_1 + 10, Q = R_2, l = 50$ cm For balanced condition,

$\frac{P}{Q} = \frac{l}{100 - l}$

$\Rightarrow \frac{R_1 + 10}{R_2} = \frac{50}{50}$

$\Rightarrow R_1 + 10 = R_2$ $\quad$ ….(i)

Similarly, for second case,

$P = R_1, Q = R_2, l = 40$ cm

$\Rightarrow \frac{R_1}{R_2} = \frac{40}{100 - 40} = \frac{40}{60}$

$\Rightarrow R_2 = \frac{3}{2} R_1$ $\quad$ ….(ii)

Putting value of R_2 from Eq. (ii) in Eq (i), we get

$R_1 + 10 = \frac{3}{2}R_1$

$\Rightarrow R_1 = 2 \times 10 = 20\Omega$

18. (c) 2Ω and 3Ω resistors are in parallel, so their equivalent resistance,

$R_{eq} = \frac{2 \times 3}{2 + 3} = \frac{6}{5} = 1.2\Omega$

As capacitor blocks DC at steady state, so the resistor 4Ω is ineffective.

The R_{eq} and 28Ω are in series, so total resistance of circuit,

$R_T = 1.2 + 2.8 = 4\Omega$

Current in circuit, $I = \frac{V}{R_T} = \frac{6}{4} = 1.5$ A

Potential across the combination of 2Ω and 3Ω resistors $V' = 1.5 \times 1.2 = 1.8$ V

$\therefore$ The steady state current in 2Ω resistor is

$I' = \frac{V'}{2} = \frac{1.8}{2} = 0.9$ A

19. (d) Peak value of emf is induced, when longer side of rectangular coil moves perpendicular to the field.

Given,

$f = 50$cps,

$A = 25 \times 10 \times 10^{-4}$ m^2

$= 250 \times 10^{-4}$ m^2

$B = 4 \times 10^{-2}$ T, and $N = 300$

Peak value of induced emf, $e_0 = NAB\omega$

$= 300 \times 250 \times 10^{-4} \times 4 \times 10^{-2} \times 2\pi \times 50$

$= 30\pi$ V

20. (c) Given, $V_R = 40$ V, $V_L = 60$ V, $V_C = 30$ V

For an $L - C - R$ circuit, the supply voltage

$V = \sqrt{V_R^2 + (V_L - V_C)^2}$

$= \sqrt{40^2 + (60 - 30)^2}$

$= \sqrt{40^2 + 30^2}$

$= \sqrt{1600 + 900} = \sqrt{2500} = 50$ V

21. (a) Given, $R = 0.1$ m, $N = 10$, $B_H = 0.314 \times 10^{-4}$ T

For neutral point, the magnetic field at the centre of coil carrying current is equal to the horizontal component of earth's field.

i.e. $B_H = \frac{\mu_0 NI}{2R}$

$I = \frac{2B_H R}{\mu_0 N}$

$= \frac{2 \times 0.314 \times 10^{-4} \times 0.1}{4\pi \times 10^{-7} \times 10}$

$= \frac{0.314 \times 10^{-4}}{2 \times 3.14 \times 10^{-7} \times 100} = 0.5$ A

22. (a) When portion A is punctured, the soap solution comes out of the hole. So, due to surface tension, the solution in part B tries to pull it toward itself. Thus, the thread becomes concave towards A.

23. (b) Velocity of sound in a gas is given by

$v = \sqrt{\frac{\gamma RT}{M}}$

where, γ = specific heat ratio

and M = molecular mass of gas.

For hydrogen, $v_H = \sqrt{\frac{\gamma RT}{M_1}}$ ....(i)

For mixture of gases, molecular mass,

$M_{mix} = \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$
Here, $n_1 = n_2$ (for equal volumes)
And
$M_2 = 16M_1$ (given)
$M_{mix} = \frac{M_1 + 16M_1}{2} = \frac{17M_1}{2}$
∴ Velocity of sound, $v_{mix} = \sqrt{\frac{\gamma RT}{M_{mix}}}$ ….(ii)
Dividing Eq. (ii) by Eq. (i) we get
$\frac{v_{mix}}{v_H} = \sqrt{\frac{\gamma RT}{M_{mix}}} \times \sqrt{\frac{M_1}{\gamma RT}}$
$= \sqrt{\frac{M_1}{M_{mix}}} = \sqrt{\frac{2}{17}}$

24. (d) The resultant amplitude after super position of two wave differing in phase by ϕ is given by
$A_R^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi$
Here, $A_1 = A_2 = A_R = A$
$\Rightarrow A^2 = 2A^2 + 2A^2\cos\phi$
$\Rightarrow \cos\phi = -\frac{1}{2}$
$\Rightarrow \phi = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

25. (c) Let x be the distance between a cliff and man, then time taken by the echo,
$t = \frac{2x}{v} = 1$ (given)
$\Rightarrow x = \frac{v}{2} = \frac{340}{2} = 170 \text{ m}$
∴ Distance between two cliffs
$= 2x = 2 \times 170 = 340 \text{ m}$

26. (b) Given $\lambda = 600 \text{ nm} = 600 \times 10^{-9} \text{ m}$
$D = 2 \text{ m}, d = 1 \text{ mm} = 1 \times 10^{-3} \text{ m}$
The distance of first dark fringe from central bright fringe,
$y = \frac{D\lambda}{d}$
$= \frac{2 \times 600 \times 10^{-9}}{1 \times 10^{-3}} = 1.2 \text{ mm}$
∴ Distance between first dark fringe on either side of central bright fringe $= 2y = 1.2 \times 2 = 2.4$ mm

27. (c) Impulse = Force × Time
$= [MLT^{-2}][T] = [MLT^{-1}]$
Surface tension $= \frac{\text{Force}}{\text{Length}}$
$= \frac{[MLT^{-2}]}{[L]} = [ML^0 T^{-2}]$
Angular momentum
= Moment of intertia × Angular velocity
$= [ML^2][T^{-1}] = [ML^2 T^{-1}]$
Work = Force × Distance $= [MLT^{-2}][L] = [ML^2 T^{-2}]$
Torque = Force × Distance $= [ML^2 T^{-2}]$
Energy $= [ML^2 T^{-2}]$
Young's modulus=$\frac{\frac{\text{Force}}{\text{Area}}}{\frac{\Delta L}{L}}$

$= \frac{[MLT^{-2}] \times [L]}{[L^2] \times [L]} = [ML^{-1}\ T^{-2}]$

$\therefore$ Work and torque have same dimensions.

28. **(b)** Given, $t = \sqrt{x} + 3$

$\Rightarrow \sqrt{x} = t - 3$

Or $x = (t-3)^2$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(t-3)^2 = 2(t-3)$

When velocity of particle is zero, then

$2(t-3)= 0$

$t= 3$ s

At $t = 3$ s, $x = (3-3)^2 = 0$

29. **(c)** As per Boolean expressions,

$A \times (B \times C) = (A \cdot C)B - (A \cdot B)C = 0 - 0 = 0$

$\therefore$ $(B \times C)$ is parallel to A.

30. **(c)** A transformer does not work on DC supply because mutual induction does not take place due to constant value of current. So, $V_S = 0$ and $I_S = 0$.

31. **(d)** Here, work done = mgh

As, m and h is same, so work done will be same in both cases.

Power, $P = \frac{\text{Work done (W)}}{\text{Time taken (t)}}$

$\Rightarrow \frac{P_1}{P_2} = \frac{t_2}{t_1} = \frac{30}{60} = \frac{1}{2}$ or $1:2$

32. **(b)** Given $\theta_1 = 0°, \theta_2 = 90°$

The energy required to rotate the dipole from angle θ_1 to θ_2 is equal to work done.

i.e., $E = W = -pE(\cos\theta_2 - \cos\theta_1)$

$= -pE(\cos 90° - \cos 0°)$

$= -pE(0 - 1) = pE$

33. **(c)** Given, $r = 10$ m, $v = 10$ m/s

If θ be the angle made by the wire with the vertical, then

$\tan\theta = \frac{v^2}{rg}$

$= \frac{(10)^2}{(10)(10)} = 1$

$\Rightarrow \theta = 45°$ or $\frac{\pi}{4}$

34. **(b)** The escape velocity of earth,

$v_e = \sqrt{\frac{2GM_e}{R_e}} = R_e\sqrt{\frac{8}{3}\pi G\rho_e}$

Similarly for a planet,

$v_p = R_p\sqrt{\frac{8}{3}\pi G\rho_p}$

$\therefore \frac{v_e}{v_p} = \frac{R_e}{R_p}\sqrt{\frac{\rho_e}{\rho_p}}$

Here, $R_p = 2R_e, \rho_e = \rho_p \Rightarrow \frac{v_e}{v_p} = \frac{1}{2}$ or $v_p = 2v_e$

35. **(b)** Young's modulus, $Y = \frac{FL}{A\Delta L} = \frac{4FL}{\pi D^2 \Delta L}$

$\Rightarrow \Delta L = \frac{4FL}{\pi D^2 Y}$

The ratio of increase in length of steel and copper is

$$\frac{\Delta L_s}{\Delta L_c}=\frac{F_sL_s}{D_s^2Y_s}\times\frac{D_c^2Y_c}{F_cL_c}=\frac{F_s}{F_c}\times\frac{L_s}{L_c}\times\left(\frac{D_c}{D_s}\right)^2\times\frac{Y_c}{Y_s}$$

Force on steel wire, $F_s=(2m+5m)g=7mg$

Force on copper wire, $F_c=5mg$

$$\Rightarrow\frac{F_s}{F_c}=\frac{7}{5}$$

Also, $\frac{D_s}{D_c}=p,\frac{L_s}{L_c}=q,\frac{Y_s}{Y_c}=s$

$$\therefore\frac{\Delta L_s}{\Delta L_c}=\frac{7}{5}\times q\times\frac{1}{p^2}\times\frac{1}{s}=\frac{7q}{5sp^2}$$

36. (d) For adiabatic process,

$pV^\gamma=$ constant

For an ideal gas, $pV=nRT$ or $p=\frac{nRT}{V}$

$$\Rightarrow\frac{nRT}{V}V^\gamma=\text{ constant}$$

$\Rightarrow TV^{\gamma-1}=$ constant

Again, from ideal gas equation,

$$V=\frac{nRT}{p}$$

$$\Rightarrow\ p\left(\frac{nRT}{p}\right)^\gamma=\text{coantnst}$$

$\Rightarrow p^{1-\gamma}T^\gamma=$ constant

Hence, relation shown in option (d) is not for adiabatic process.

37. (a) The escape velocity from earth's surface is given by

$$v_e=\sqrt{\frac{2GM}{R+h}}$$

Given, $h=3R$

$$\Rightarrow\ v_e=\sqrt{\frac{2GM}{4R}}=\sqrt{\frac{GM}{2R}}$$

38. (d) H-polaroid is prepared by stretching a thin sheet of polyvinyl alcohol and then impregnating with iodine. In this process, the molecules are oriented in the direction of applied strain.

39. (b) From Coulomb's law, force between two charges,

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$\Rightarrow\ \varepsilon_0=\frac{q_1q_2}{4\pi r^2F}$$

$$\therefore\ \text{Unit of }\ \varepsilon_0=\frac{C\cdot C}{m^2N}=\frac{C^2}{m^2N}=C^2m^{-2}N^{-1}$$

40. (a) As, the volume remains constant, so

$$\frac{4}{3}\pi R^3=8\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow R=2r\text{ or }r=\frac{R}{2}$$

Capacitance of spherical capacitor,

$C=4\pi\varepsilon_0R$

$\Rightarrow\ C\propto R$

$$\therefore\frac{C_1}{C_2}=\frac{R}{r}=\frac{2r}{r}=2\Rightarrow C_2=\frac{C_1}{2}=\frac{1}{2}\mu F$$

41. (c) The magnitude of resultant of two forces is given by

$$R=\sqrt{A^2+B^2+2AB\cos\theta}$$

Given, $A = B = P$ and $\theta = 120^\circ$

$\therefore R = \sqrt{P^2 + P^2 + 2P^2 \cos 120^\circ}$

$= \sqrt{2P^2\left(1 - \frac{1}{2}\right)} = P$

42. (b) Given, $\lambda_0 = 5200Å = 5200 \times 10^{-10}$ m

$\therefore$ Threshold frequency, $f_0 = \frac{c}{\lambda_0} = \frac{3\times10^8}{5200\times10^{-10}}$

$= 5.76 \times 10^{14}$ Hz

It lies in the frequency range of UV-rays, so 50 W UV lamp is used for this emission.

43. (b) When the monochromatic light in Young's double slit experiment is replaced by white light, a white central bright fringe is observed as all wavelengths of light have zero phase difference at centre while at all other points, it is different for different colours. So different colour fringes are observed on both sides.

44. (d) The radius in Bohr's atomic model is given by

$r = 0.53\frac{n^2}{Z}Å$

For $Be^{3+}, Z_{Be} = 4, n_{Be} = ?$

For $H, Z_H = 1, n_H = 1$

As per question,

$r_{Be} = r_H$

$\Rightarrow \frac{n_{Be}^2}{Z_{Be}} = \frac{n_H^2}{Z_H}$

$\Rightarrow n_{Be} = \sqrt{n_H^2\left(\frac{Z_{Be}}{Z_H}\right)} = \sqrt{1 \times \frac{4}{1}} = 2$

45. (b) Packing fraction is the ratio of mass defect to the number of nucleons. Mass defect is the difference in the actual isotopic mass (M) and mass number (A).

$\therefore$ Packing fraction $= \frac{M-A}{A}$

where, A is mass number or number of nucleons.

46. (c) In radioactive decay, the decay rate is proportional to the amount of substance. So, for change of count rate from 240 per min to 30 per min, let N be the number of half-life passed in one hour. Then,

$\frac{240}{2^N} = 30$

$\Rightarrow 2^N = 8$

$\Rightarrow N = 3$

Half-life, $t_{1/2} = \frac{1\text{ h}}{3} = \frac{60}{3}$ min $= 20$ min

47. (d) The rate of heat conduction is given by

$H = \frac{d\theta}{dt} = \frac{kA}{l}\Delta T$

Here, ΔT is same for both conductors, so ratio of heat conduction,

$\frac{H_1}{H_2} = \frac{A_1}{l_1} \times \frac{l_2}{A_2}$

$= \frac{d_1^2}{l_1} \times \frac{l_2}{d_2^2} \quad \left(\because A = \frac{\pi d^2}{4}\right)$

$= \left(\frac{d_1}{d_2}\right)^2 \cdot \frac{l_2}{l_1}$

$= \left(\frac{1}{2}\right)^2 \times \frac{1}{2} = \frac{1}{8}$

48. (a) The air blown by mouth has same pressure as that of atmosphere, so it is an example of isobaric process.

49. (b) Longitudinal waves in air travels in the form as compression and rarefaction that changes its elastic properties. The air is a completely elastic medium, which does not have a modulus of rigidity. So, sound waves travel in it as a longitudinal wave.

50. (a) When an uncharged metal sphere is placed between two charged parallel plate capacitor, negative charge is induced on its upper side and on equal positive charge on its lower side. So, the lines of force enters the sphere normally as shown in figure of option (a).

51. (c) According to right hand rule, the thumb points in the direction of current, i.e. from East to West, then the curled fingers at a point above the conductor will point in the direction of magnetic field, i.e towards North.

52. (c) A bar magnet is equivalent to a solenoid carrying current as both produces similar magnetic field around them.

53. (c) The energy of nth state in hydrogen like atom,

$$E_n = -\frac{13.6Z^2}{n^2}\,\text{eV}$$

The energy of first excitation,

$$E_2 - E_1 = \frac{-13.6\times Z^2}{(2)^2} + \frac{13.6\times Z^2}{(1)^2} = 40.8 \ \text{(given)}$$

$$\Rightarrow\ Z^2 = \frac{40.8\times 4}{13.6\times 3} = 4$$

Or $\quad Z = 2$

∴ Energy needed to remove an electron from the ion in ground state, $E = -(E_1) = -\left[\frac{-13.6\times(2)^2}{(1)^2}\right] = 54.4\text{eV}$

54. (c) Given, $\mu_b = 1.67, \mu_y = 1.65, \mu_r = 1.63$

Dispersive power of material $= \dfrac{\mu_b - \mu_r}{\mu_y - 1}$

$$= \frac{1.67 - 1.63}{1.65 - 1} = \frac{0.04}{0.65} = 0.0615$$

55. (b) Given, $T_1 = 227^\circ\text{C} = 227 + 273 = 500\ \text{K}$

$T_2 = 127^\circ\text{C} = 127 + 273 = 400\ \text{K}$

Heat input, $H_i = 6\times 10^4\ \text{J}$

Efficiency of Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1}$$

$$= 1 - \frac{400}{500} = \frac{1}{5}$$

Also, efficiency,

$$\eta = \frac{\text{Work output}}{\text{Heat input}} = \frac{W}{6\times 10^4}$$

$$\Rightarrow\ W = \eta\times 6\times 10^4 = \frac{1}{5}\times 6\times 10^4$$

$$= 1.2\times 10^4\ \text{J}$$

56. (d) For an ideal gas, $pV = nRT$

At constant temperature, $pV = \text{constant}$.

So, pV does not vary with V and $pV - V$ graph is a straight line parallel to V - axis as shown in option (d).

57. (d) The rainbow is formed in the sky after rain shower, due to tiny droplets of water which act as optical medium. When sunlight falls on them, it is totally internally reflected inside them and when comes out it got dispersed in its constituent colours. Thus, a rainbow is formed due to dispersion and total internal reflection.

58. (a) The focal length of the combination is given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$f_1 =$ focal length of plano-convex lens $= f$

$f_2 =$ focal length of concave lens $= -f$

$$\therefore\ \frac{1}{F} = \frac{1}{f} + \frac{1}{-f} = 0$$

$$\Rightarrow\ F = \infty$$

Hence, focus shifts to infinity.

59. **(d)** Earth is a good conductor. So, when a body is earthed, electrons flow from earth to the body. It means, the body must be deficient of electrons i.e. it is positively charged.

60. **(b)** As it is a balanced Wheatstone bridge, so C and D are same potential and the capacitor 2μ F is ineffective.

$$C_1 = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\mu\text{ F}$$

The 3μ F and 6μ F in upper portion are in series, so equivalent capacitance,

$$C_1 = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\mu\text{ F}$$

Similarly, $C_2 = \frac{4\times8}{4+8} = \frac{32}{12} = \frac{8}{3}\mu\text{ F}$

C_1 and C_2 are in parallel, so their equivalent capacitance

$$C = C_1 + C_2 = 2 + \frac{8}{3} = \frac{14}{3}\mu\text{ F}$$

CHEMISTRY

61. **(c)** $MgSO_4$ is soluble in HCl and gives $MgCl_2$ and H_2SO_4.

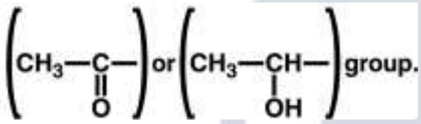
$$\underset{(x)}{MgSO_4(s)} + 2HCl(aq) \longrightarrow MgCl_2 + H_2SO_4(aq)$$

On adding NaOH to $MgCl_2$, it gives white ppt. of $Mg(OH)_2$.

$$MgCl_2(aq) + \underset{\text{Excess}}{2NaOH(aq)} \longrightarrow \underset{\text{White ppt.}}{Mg(OH)_2(s)} + NaCl$$

62. **(b)** Mn is used to make alloy steel for armour plates, safes and helmets as Mn increases harden-ability by lowering transformation points and causing transformation to be sluggish.

63. **(b)** Iodoform test is given by compounds having .



Thus, compound CH_3OH do not contain these groups. Hence, it do not give iodoform test.

64. **(d)** The gaseous carbon compound, methyl amine (CH_3NH_2) is soluble in dil. HCl and forms soluble hydrogen chloride salt solution.

This solution on treating with $NaNO_2$ gives nitrogen leaving behind a methanol solution with wood spirit smell.

65. **(d)** Benzyl chloride is more reactive than vinyl chloride as benzyl carbocation is stabilised by resonance. While, in vinyl chloride due to partial double bond character became of resonance, the $C-Cl$ bond strength increases. Hence, reactivity of vinyl chloride decreases.

66. **(a)** As enthalpy of formation of HF is more negative, than that of HCl and HF is more stable than HCl . The bond strength of hydrogen with halogen decrease down the group due to increases in size.

67. **(b)** Equivalent of NaOH neutralised by HCl

$= 100 \times 1 \times 10^{-3} = 0.1$ mol

$\therefore$ Heat released $= 57.3 \times 0.1 = 5.73$ kJ

68. **(c)** If [A] double's then, $r_1 = 2r$

If [B] increased 9 times then, $r_2 = 3r$

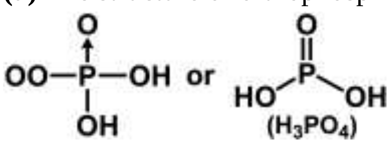
$\therefore$ Order w.r.t. A is 1 and order w.r.t B is $\frac{1}{2}$.

$\therefore$ Order of reaction $= 1 + \frac{1}{2} = 1\frac{1}{2}$.

69. **(c)** In Goldschmidt aluminothermic process, thermite contains 3 parts of Fe_2O_3 and 1 part of aluminium (Al).

$$Fe_2O_3 + 2Al \longrightarrow Al_2O_3 + 2Fe + \text{ Heat}$$

70. **(a)** The structure of orthophosphoric acid is



71. **(d)** At anode, oxidation occurs of H_2 (gas), while at cathode reduction of AgCl take place.

$\therefore$ The cell notation is

$$Pt|H_2(g), HCl(sol)||AgCl(s)|Ag$$

72. **(c)** If current is same then,

E_{wt} of Ag $= E_{wt}$ of H_2

$\frac{1.08}{108}=\frac{x}{1}$

$\therefore$ Mass of $H_2, x = 10^{-2}$ g

2 g of H_2 at STP occupy 22400 cm^3

$\therefore$ 10^{-2} g of H_2 at STP occupy

$=\frac{22400}{2}\times 10^{-2}\ cm^3 = 112\ cm^3$

73. **(d)** The flame colours of metal ions are due to metal excess defect. The free electrons can be exited to higher energy levels giving absorption spectra and as a consequence their compounds are coloured.

74. **(a)** The order of reaction of methyl halides increases from fluorine to iodine due to decrease in $C-X$ bond strength.

$\therefore$ The order of reactivities of methyl halides in formation of Grignard reagent is

$CH_3I > CH_3Br > CH_3Cl$

75. **(d)** Formaldehyde on reaction with ammonia gives urotropine which on nitration gives RDX.

$$\text{Formaldehyde} + NH_3 \longrightarrow \text{Urotropine} \xrightarrow{\text{Nitration}} \underset{\text{(RDX)}}{\text{Cyclo trimethylene-tri nitramine}}$$

76. **(d)** Number of carbon atoms

$=\frac{\text{mass}}{\text{molar mass}}\times 6.023\times 10^{23}$

$=\frac{10^{-3}}{12}\times 6.023\times 10^{23}$

$=5.02\times 10^{19}=0.502\times 10^{20}$

77. **(a)** Distance travelled by gas is directly proportional to rate of diffusion

$\therefore\ r_{NH_3}(l_1)\propto\frac{1}{\sqrt{M_{NH_3}}}\propto\frac{1}{\sqrt{17}}$

$r_{HCl}(l_2)\propto\frac{1}{\sqrt{M_{HCl}}}\propto\frac{1}{\sqrt{365}}$

$\because\ r_{NH_3} > r_{HCl}$

$\therefore$ Length travelled by NH_3 is greater than HCl .

Hence, white ring of NH_4Cl is formed nearer to HCl end.

78. **(d)** By the action of alc. KOH on ethyl iodide, ethylene gas is formed by elimination reaction. Ethylene decolourises alkaline $KMnO_4$.

$\underset{\text{Ethyl iodide}}{C_2H_5-I}\xrightarrow{\text{alc. KOH}}\underset{\text{Ethylene}}{C_2H_4}$

$\underset{\text{Ethylene}}{CH_2 = CH_2} + KMnO_4(\text{ alk. })\longrightarrow \underset{\text{Colourless}}{(CH_2OH)_2}$

79. **(c)** Chemisorption is highly specific and will not form if there is no possibility of chemical bonding between adsorbent and adsorbate.

$\therefore$ Chemisorption doesn't form multimolecular layer and hence, option (c) is incorrect.

80. **(d)** The concentration of electrolyte required to coagulate a given amount of As_2S_3 sol is minimum in case of aluminium nitrate.

As_2S_3 is negatively charge sol. In aluminium nitrate, the charge on Al cation is +3 which is maximum among given cations. Hence, its coagulating power is maximum.

81. **(d)** Benzaldehyde lacks $\alpha-H$ atom. On heating with strong NaOH solution, it undergoes Cannizzaro reaction to produce benzoic acid and benzyl alcohol. Benzoic acid forms sodium salt with NaOH .

$\underset{\text{Benzaldehyde}}{2C_6H_5CHO}\xrightarrow{\text{NaOH}}\underset{\text{Sodium salt of benzoic acid}}{C_6H_5COONa}+\underset{\text{Benzyl alcohol}}{C_6H_5CH_2OH}$

82. **(a)** The process by which synthesis of protein takes place based on the genetic information present in m-RNA is called translation.

83. **(b)** $\Delta H_{Reaction} = (\Delta H_f)_{Product} - (\Delta H_f)_{Reactant}$
$= -1596 - (-1134) = -462$ kJ

84. **(d)** For the reaction
$A + B \rightleftharpoons 2C + D + Q$
As the reaction is exothermic, low temperature favours formed direction according to Le-Chatelier's principle.
Also $\Delta n_{(g)} = (2+1) - (1+1) = 1$
$\because \Delta n_g = +ve$
(low pressure favours forward direction)
$\therefore$ Low pressure and temperature favours formed direction for the reaction.

85. **(a)** For every 10°C rise in temperature, rate increase by 2 times i.e.,
$r_1 = 2^n r$
where, $n = \frac{\Delta t}{10} = \frac{100-30}{10} = 7$
$\therefore r_1 = 2^7 r \Rightarrow r_1 = 128r$

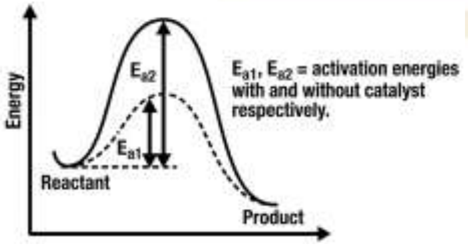
86. **(b)** Moles of HCl $= \frac{1}{2} \times V \quad \left\{\frac{N}{2} HCl\right\}$
For preparing $\frac{N}{10}$ solution,
$$\frac{1}{10} = \frac{\frac{1}{2} \times V}{500}$$
$$\therefore V = \frac{1000}{10} = 100 \text{ cm}^3$$

87. **(c)** Equivalent weight $= \frac{\text{molecular weight}}{\text{valence factor}}$
$\therefore$ Molecular weight $= 20 \times 3 = 60$
It's oxide is M_2O_3 and molecular weight
$= 2 \times 60 + 16 \times 3 = 120 + 48 = 168$

88. **(a)** The reaction that doesn't take place during the smelting process of extraction is
$2FeS + 3O_2 \longrightarrow 2FeO + 2SO_2 \uparrow$
This reaction occurs during roasting, which comes before smelting.

89. **(a)** In $K_4[Fe(CN)_6]$, the central metal atom obeys EAN rule.
EAN $= Z - X + Y = 26 - 2 + 12 = 36$
where,
Z = atomic number
X = number of electrons lost, while forming metal ion.
Y = number of electron donated by ligands.

90. **(c)** In a reversible reaction, the catalyst decreases the activation energy of both, forward and backward reaction.



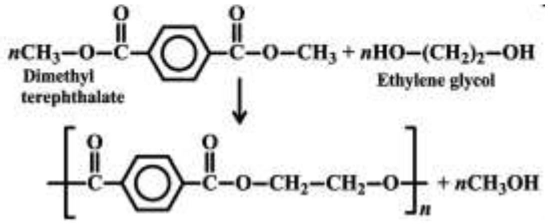
91. **(d)** $K_{sp} = [A][B]$
$10^{-8} = 10^{-3}[B]$
$[B] = 10^{-5}$M
$\therefore$ Salt will precipitate when $[B] > 10^{-5}$

92. **(b)** For reaction to be spontaneous, $E^\circ_{cell} > 0$
$E^\circ_{cell} = (E^\circ_{red})_{Cathode} - (E^\circ_{red})_{Anode}$
$= 0.80 - (-0.76) = 1.56$ V
$\therefore$ It will be spontaneous reaction

$\because E^{\circ}_{cell} > 0$.

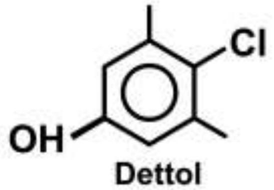
93. (b) When the radius ratio is between $0.732 - 1.000$, the coordination number is 8 and arrangement is body centered cubic (bec) packing.

94. (a) Dacron is obtained by the condensation polymerisation of ethylene glycol and terephthalic acid.



95. (d) 4-chloro-3, 5-dimethyl phenol is called dettol.

It is a well known antiseptic. It's structure is

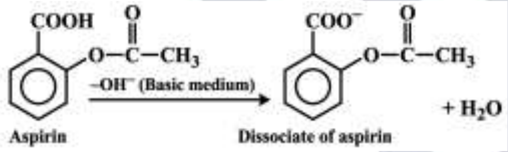


96. (a)

	Methane	Ethene	Ethyne
Hydrocarbon	CH_4	C_2H_4	C_2H_2
Hybridisation	sp^3	sp^2	sp
% s-character	25	33	50

97. (d) In the manufacture of sulphuric acid by contact process, Tyndall box is used to test the presence of dust particles.

98. (d) Aspirin is an acidic compound. It will not dissociate in acidic medium i.e., small intestine in order to react with base but will dissociate in basic medium.



99. (d) ${}^{232}_{90}Th \longrightarrow {}^{208}_{82}Pb + n\alpha + m\beta$

Number of α-particles, $n = \frac{232-208}{4} = 6$

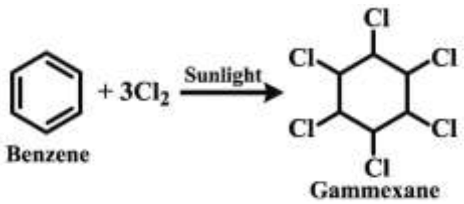
Now, for charge balanced

$90 = 82 + 2n - m$

$m = 4$

Number of β-particles, $m = 4$

100. (c) When chlorine is passed through warm benzene in presence of sunlight, the product obtained is gammexane.



101. (b) Ethyl benzoate reacts with PCl_5 to give C_2H_5Cl, C_6H_5COCl and $POCl_3$.

$$\underset{\text{Ethyl benxinate}}{C_6H_5COOCH_2CH_3} + PCl_5 \longrightarrow \underset{\text{Ethyl chloride}}{C_2H_5Cl} + \underset{\text{Benznate chloride}}{C_6H_5COCl} + \underset{\text{Phomphorus oxychloride}}{POCl_3}$$

102. **(a)** The statement that, sodium aluminium silicate is used in softening of hard water is not relevant with colloids.

It is a substitution reaction of calcium salts with zeolite.

103. **(c)** $\frac{dN}{dt} = \lambda N$

$\frac{dN}{dt} = \frac{1}{T}\frac{6.023\times10^{23}}{12} \quad \left\{\because \lambda = \frac{1}{T}\right\}$

$T = \frac{6.023\times10^{23}}{12\times1.18\times10^{13}} \quad \left\{\frac{dN}{dt} = 1.18 \times 10^{13}\right\}$

$= 42530 \times 10^6$ min

$= \frac{4253 \times 10^6}{60 \times 24 \times 365}$ years

$= 8092$ years

$= 8100$ years

104. **(a)** $K_p = K_C(RT)^{\Delta n}$

If Δn is positive than $K_p > K_C$ also, gaseous moles of product > moles of reactant.

∴ Forward reaction is favoured by low pressure, according to Le-Chatelier's principle.

105. **(d)** In a lime kiln, to get higher yield of CO_2, the measure that can be taken is to pump out CO_2. This will reduce the pressure of CO_2 and reaction will shift in forward direction.

$CaCO_3 \xrightarrow{\text{Heat}} CaO + CO_2$

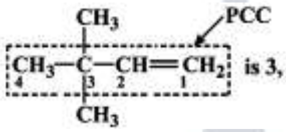
106. **(a)** 20 volume of H_2O_2 means that, 1 L of H_2O_2 on decomposition gives 20 L of oxygen.

∴ 5 L of oxygen will be obtained from

$= \frac{5}{20} \times 1 = 0.250$ LLof$_2O_2$

or = $250\ cm^3$ of 20 volume H_2O_2.

107. **(d)**



is 3, 3-dimethyl-1-butene.

108. **(b)** The process of hardening of iron rods of railway wagon axles embedded in charcoal powder is known as case-hardening. In this process surface of iron rods get hardened by depositing a layer of steel on its surface.

109. **(b)** Thomas slag is tetracalcium phosphorus nonaoxide. It's molecular formula is $Ca_3(PO_4)_2$.

110. **(d)** Urea is preferred to ammonium sulphate as a nitrogenous fertiliser because it does not cause acidity in the soil.

111. **(c)** Pressure in the cylinder is directly proportional to the number of moles of gases.

g of$_{H_2}$ has $= \frac{44}{2} = 22$ moles of H_2

g of CO_2 has $= \frac{44}{44} = 1$ mole of H_2

∴ Pressure of $H_2(p_{H_2}) = \frac{22}{1} \times 1 = 22$ atm.

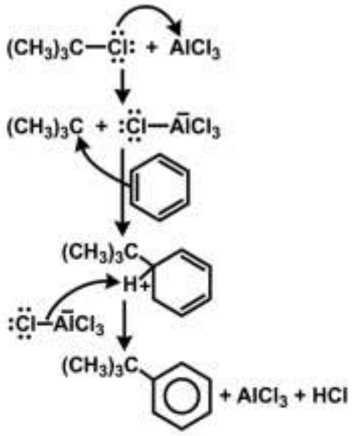
112. **(a)** According to Bohr's model, angular momentum of nth orbit of hydrogen atom is mvr_n or $\frac{nh}{2\pi}$.

113. **(a)** Electronic configuration of atomic number 36

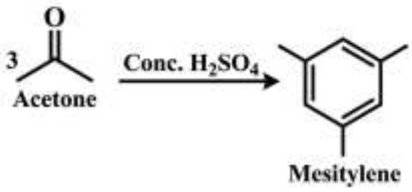
$= 1s^2, 2s^2, 2p^6, 3s^2 3p^6 3d^{10} 4s^2 4p^6$

As the last electrons is filled in p-orbital, so it belongs to p-block in periodic table.

114. **(d)** The function of $AlCl_3$ in Friedel-Craft's reaction is to produce electrophile.



115. **(a)** An important reaction of acetone is auto-condensation in presence of concentrated sulphuric acid to give the aromatic compound mesitylene.



116. **(a)**Kinetic energy $=\frac{3}{2}RT$

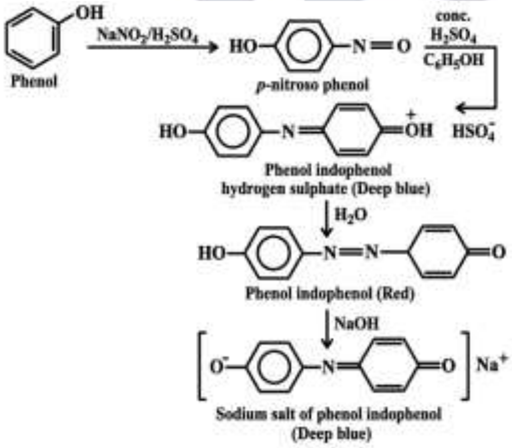
$=\frac{3}{2}\times 8314\times 300$

$= 3741.3\ \mathrm{J} = 3.74\ \mathrm{kJ}$

117. **(a)** The tripeptide hormone present in most living cell is glutathione.

The three amino acids present are glutamic acid, cysteine and glycine.

118. **(a)** The given reaction is Liebermann's reaction. It is test for phenol.



119. **(a)** Energy is stored in our body in the form of ATP (Adeniotriphosphate). The stored energy is released by the breakdown of ATP. The released energy is used by muscles and other parts of body.

120. **(d)** Molisch's test is used to detect all kinds of carbohydrates. The Benedict test identifies reducing sugars (mono saccharides and some disaccharides) which have free ketone or aldehyde functional group. Scliwanoff's test is used for detection of fructose or keto group in sugar solution. Therefore, give negative test for maltose. Hence, organic compound is maltose.

MATHEMATICS

121. **(d)**

p	q	~ p	~ p ∧ q	p → q	q → p	~ (p → q)	~ (q → p)
T	T	F	F	T	T	E	F

T	F	F	F	F	T	T	F
F	T	T	T	T	F	F	T
F	F	T	F	T	T	F	F

From table,

$\sim p \wedge q \cong \sim (q \rightarrow p)$

122. **(b)** Let p: A number is a prime.

q : A number is odd.

The given proposition is $p \rightarrow q$

Now, inverse of $(p \rightarrow q)$ is $\sim p \rightarrow \sim q$

Therefore, the inverse of the given proposition is 'If a number is not a prime, then it is not odd.'

123. **(a)** We have,

$$2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

124. **(c)** We have, $\begin{vmatrix} 1 & 1 & 1 \\ bc & ca & ab \\ b+c & c+a & a+b \end{vmatrix}$

On applying $C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3$, we get

$$\begin{vmatrix} 0 & 0 & 1 \\ -c(a-b) & -a(b-c) & ab \\ -(a-b) & -(b-c) & a+b \end{vmatrix}$$

Taking common $(a-b), (b-c)$ from C_1, C_2 respectively, we get

$$(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ -c & -a & ab \\ -1 & -1 & a+b \end{vmatrix}$$

On expanding along R_1, we get

$(a-b)(b-c) \cdot 1(c-a)$

$= (a-b)(b-c)(c-a)$

125. **(b)** We have,

$$\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix}$$

On applying, $C_1 \rightarrow C_1 - C_2, C_2 \rightarrow C_2 - C_3$, we get

$$\begin{vmatrix} -1 & -1 & 443 \\ -1 & -1 & 447 \\ -1 & -1 & 451 \end{vmatrix} = 0 \ [\because C_1 \text{ and } C_2 \text{ are identical }]$$

126. **(a)** Let $a = x\hat{i} + y\hat{j} + z\hat{k}$, then

$(a \cdot \hat{i}) + (a \cdot \hat{j})\hat{j} + (a \cdot \hat{k})\hat{k}$

$= [(x\hat{i} + y\hat{j} + z\hat{k} \cdot \hat{i}] \cdot \hat{i} + [(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{j}]\hat{j} +$

$[(x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{k}]\hat{k}$

$= (x)\hat{i} + (y)\hat{j} + (z)\hat{k} = x\hat{i} + y\hat{i} + \hat{k} = a$

127. **(d)** Let $A = \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}$

Now, $|A| = \begin{vmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{vmatrix}$

$= \cos^2 2\theta - (-\sin^2 2\theta)$

$= \cos^2 2\theta + \sin^2 2\theta = 1$

Again, $A^{-1} = \frac{1}{|A|}\begin{vmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{vmatrix}$

$\left[\because \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right]$

$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

128. **(b)** Since, $(a+\lambda b) \perp (a-\lambda b)$, then

$(a+\lambda b)\cdot(a-\lambda b) = 0$

$\Rightarrow |a|^2 - \lambda^2|b|^2 = 0 \Rightarrow |a^2| = \lambda^2|b|^2$

$\Rightarrow \lambda \frac{|a|}{|b|} = \frac{3}{4}$

129. **(b)** We have,

$a = 2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}, b = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

Now,

$a \cdot b = (2\hat{\imath} + 3\hat{\jmath} - 2\hat{k}) \cdot (\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$

$= 2 + 6 - 6 = 2$

And

$|b| = \sqrt{(1)^2 + (2)^2 + (3)^2}$

$= \sqrt{1+4+9} = \sqrt{14}$

$\therefore$ Projection of a on b $= \frac{a\cdot b}{|b|} = \frac{2}{\sqrt{14}}$

130. **(c)** We have,

$2 \times 4 = 1(\text{mod}7)$

Hence, $2^{-1} = 4(\text{mod}7)$

Now, $2^{-1} \times 4 = 4 \times 4 = 16$

Now, $16 = (2 \times 7) + 2$

$\therefore 16 = 2(\text{mod}7)$

$\Rightarrow 2^{-1} \times 4 = 2(\text{mod}7)$

131. **(d)** We know that,

$a\sin x + b\cos x \le \sqrt{a^2 + b^2}$

$\therefore 3\cos x - 4\sin x \le \sqrt{(-4)^2 + (3)^2} = \sqrt{16+9} = 5$

$\therefore$ Maximum value is 5 .

132. **(c)** We have,

$S = t^3 - 3t^2$

$\Rightarrow \frac{dS}{dt} = 3t^2 - 6t$ and $\frac{d^2S}{dt^2} = 6t - 6$

Now, acceleration, $\frac{d^2S}{dt^2} = 0$

$\Rightarrow 6t - 6 = 0$

$\Rightarrow t = 1$

$\therefore$ Velocity $= \left.\frac{dS}{dt}\right|_{t=1}$

$= 3(1)^2 - 6(1) = 3 - 6 = -3$ m/sec

133. **(b)** We have,

$y^n = a^{n-1}x$

$\Rightarrow n \cdot y^{n-1}\frac{dy}{dx} = a^{n-1}$

$\Rightarrow y^{n-1}\frac{dy}{dx} = \frac{1}{n}a^{n-1}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{n}a^{n-1}y^{1-n}$

$\therefore$ Length of the subnormal $= \left|y\frac{dy}{dx}\right|$

$= \left|y \times \frac{1}{n}a^{n-1}y^{1-n}\right|$

$= \left|\frac{1}{n}a^{n-1}y^{2-n}\right|$

Since, length of subnormal is constant.

$\therefore 2 - n = 0$

$\Rightarrow n = 2$

134. **(a)** We have,

$x = A\cos 4t + B\sin 4t$

$\Rightarrow \frac{dx}{dt} = A(-\sin 4t \cdot 4) + B(\cos 4t \cdot 4)$

$= -4A\sin 4t + 4B\cos 4t$

Again,

$\frac{d^2x}{dt^2} = -4A(\cos 4t \cdot 4) + 4B(-\sin 4t \cdot 4)$

$= -16A\cos 4t - 16B\sin 4t$

$= -16[A\cos 4t + B\sin 4t] = -16x$

135. **(c)** We have,

$x = at^2, y = 2at$

Now,

$\frac{dx}{dt} = 2at, \frac{dy}{dt} = 2a$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2a}{2at} = \frac{1}{t}$

Since, tangent is perpendicular to X-axis, then

$\frac{dy}{dx} = \frac{1}{0}$

$\Rightarrow \frac{1}{t} = \frac{1}{0} \Rightarrow t = 0$

$\therefore x = at^2 = 0$ and $y = 2at = 0$

So, required point is (0,0).

136. **(b)** We have,

$\frac{dy}{dx} + \frac{1 + \cos 2y}{1 - \cos 2x} = 0$

$\Rightarrow \frac{dy}{dx} = -\left(\frac{1 + \cos 2y}{1 - \cos 2x}\right)$

$\Rightarrow \frac{dy}{dx} = -\frac{2\cos^2 y}{2\sin^2 x}$

$\Rightarrow \sec^2 y dy = -\mathrm{cosec}^2 x dx$

$\Rightarrow \int \sec^2 y dy + \int \mathrm{cosec}^2 x dx = 0$

$\Rightarrow \tan y - \cot x = c$

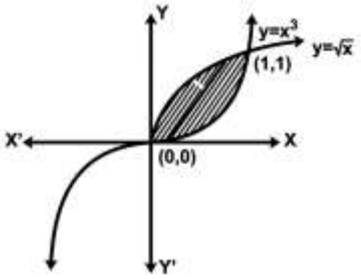
137. **(b)** We have,

$\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/4} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$

$\Rightarrow \left(1 + \left(\frac{dy}{dx}\right)^2\right)^9 = \left(\frac{d^2y}{dx^3}\right)^4$

$\therefore$ Degree $= 4$

138. **(c)**



Required area $= \int_0^1 (\sqrt{x} - x^3)dx$

$= \left[\frac{2x^{3/2}}{3} - \frac{x^4}{4}\right]_0^1$

$= \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

139. **(d)** Let $I = \int_0^{\pi/5} \cos^3 4\theta d\theta = \int_0^{\pi/8} \frac{\cos 12\theta + 3\cos 4\theta}{4} d\theta$

$= \frac{1}{4}\left[\frac{\sin 12\theta}{12} + \frac{3\sin 4\theta}{4}\right]_0^{\pi/8} = \frac{1}{4}\left[\frac{1}{12}\sin\frac{3\pi}{2} + \frac{3}{4}\sin\frac{\pi}{2}\right]$

$= \frac{1}{4}\left[-\frac{1}{12} + \frac{3}{4}\right] = \frac{1}{4}\left(\frac{-1+9}{12}\right) = \frac{8}{4 \times 12} = \frac{1}{6}$

140. **(a)**Let $I = \int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \cos x \sin x} dx$ ....(i)

$= \int_0^{\pi/2} \frac{\cos\left(\frac{\pi}{2} - x\right) - \sin\left(\frac{\pi}{2} - x\right)}{1 + \cos\left(\frac{\pi}{2} - x\right)\sin\left(\frac{\pi}{2} - x\right)} dx$

$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$

$\therefore I = \int_0^{\pi/2} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx$ ....(ii)

On adding Eqs. (i) and (ii), we get

$2I = 0 \Rightarrow I = 0$

141. **(b)** $ax^2 - y^2 + 4x - y = 0$ represents a pair of straight line, if

$\begin{vmatrix} a & 0 & 2 \\ 0 & -1 & -\frac{1}{2} \\ 2 & -\frac{1}{2} & 0 \end{vmatrix} = 0$

$\Rightarrow a\left(0 - \frac{1}{4}\right) + 2(0 + 2) = 0$

$\Rightarrow -\frac{a}{4} + 4 = 0$

$\Rightarrow a = 16$

142. **(b)** Let the required point be $P(x, y)$. Then,

$4|y| = \left(\sqrt{(x-0)^2 + (y-0)^2}\right)^2$

$\Rightarrow 4|y| = x^2 + y^2$

$\Rightarrow x^2 + y^2 - 4|y| = 0$

143. **(c)** Let the equation of line be

$x + y = a$ $[\because a = b]$

Now, given line passes through (2,4), then

$\Rightarrow 2 + 4 = a$

$\Rightarrow a = 6$

$\therefore$ Equation of line is $x + y - 6 = 0$

144. **(c)** We have,

$$\frac{1}{2}\begin{vmatrix} x & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 2 & 1 \end{vmatrix} = \pm 4$$
$\Rightarrow x(1-2)+1(2-0) = \pm 8$
$\Rightarrow -x+2 = \pm 8$
$\Rightarrow x = 2 \pm 8 \Rightarrow x = 10, -6$

145. **(c)** We have,

$$\lim_{\theta\to\frac{\pi}{2}} \frac{\frac{\pi}{2}-\theta}{\cot\theta} = \lim_{\theta\to\frac{\pi}{2}} \frac{-1}{-\operatorname{cosec}^2\theta} \quad \text{[using L' Hospital Rule]}$$
$$\Rightarrow \lim_{\theta\to\frac{\pi}{2}} \sin^2\theta = \sin^2\frac{\pi}{2} = 1$$

146. **(a)** The equation $x^2+y^2+2gx+c=0$ represents co-axial system of circle having centre at $(-g, 0)$ and radius $= \sqrt{g^2-c}$.

As $c<0$, radius is always greater than 0 . So, it has imaginary limiting points which show that circles are intersecting.

147. **(d)** We have, diameters of circle are
$x+y=6$ and $x+2y=4$
On solving above equations, we get
$x=8$ and $y=-2$
∴ Centre of circle is (8-2).
Since, circle passes through (6, 2).
∴ Radius $= \sqrt{(8-6)^2+(-2-2)^2} = \sqrt{4+16} = \sqrt{20}$

148. **(a)** The vertex and focus of the parabola are $(0,6)$ and $(0,3)$ respectively.
Clearly, axis is Y-axis.
∴ Latusrectum = 4(Distance between vertex and focus)
$= 4\sqrt{(0-0)^2+(6-3)^2} = 12$
Also, the vertex is on the right hand side of the focus, so the parabola opens downward.
Hence, equation of the parabola is
$(x-0)^2 = -12(y-6)$
$\Rightarrow x^2 = -12y+72$
$\Rightarrow x^2+12y = 72$

149. **(c)** We have,
$25x^2+9y^2-150x-90y+225=0$
$\Rightarrow 25(x^2-6x)+9(y^2-10y)+225=0$
$\Rightarrow 25[(x-3)^2-9]+9[(y-5)^2-25]+225=0$
$\Rightarrow 25(x-3)^2-225+9(y-5)^2-225+225=0$
$\Rightarrow 25(x-3)^2+9(y-5)^2=225$
$$\Rightarrow \frac{(x-3)^2}{9}+\frac{(y-5)^2}{25}=1$$
On comparing with $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, we have
$a=3, b=5$
$$\therefore \text{ Eccentricity,e} = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

150. **(b)** The equation of the ellipse is $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and equation of the hyperbola is $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ or $\frac{x^2}{\left(\frac{12}{5}\right)^2}-\frac{y}{\left(\frac{9}{5}\right)^2}=$

1

∴ Eccentricity of ellipse, $e = \sqrt{1-\frac{b^2}{16}}$

and eccentricity of hyperbola,

$$e' = \sqrt{1 + \frac{\left(\frac{9}{5}\right)^2}{\left(\frac{12}{5}\right)^2}} = \sqrt{1 + \frac{81}{144}} = \frac{15}{12}$$

Since, foci of ellipse and hyperbola coincide.

$\therefore 4e = \frac{12}{5}e'$

$\Rightarrow e = \frac{3}{5}e'$

$\Rightarrow \sqrt{1 - \frac{b^2}{16}} = \frac{3}{5} \times \frac{15}{12}$

$\Rightarrow 1 - \frac{b^2}{16} = \left(\frac{3}{4}\right)^2 \Rightarrow b^2 = 7$

151. **(b)** We have,

$f(x) = \log x$

$\therefore f(\sin x) = \log \sin x$

On differentiating both sides w.r.t.x, we get

$\frac{d}{dx}f(\sin x) = \frac{1}{\sin x} \cdot \cos x = \cot x$

152. **(a)** We have,

$$f(x) = \begin{cases} \frac{1 - \cos x}{x}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

Since, f(x) is continuous at $x = 0$

$\therefore f(0) = \lim_{x \to 0} f(x)$

$\Rightarrow k = \lim_{x \to 0} \frac{1 - \cos x}{x}$

$\Rightarrow k = \lim_{x \to 0} \frac{2\sin^2(x/2)}{x}$

$\Rightarrow k = \lim_{x \to 0} \frac{\sin(x/2)}{(x/2)} \times \sin(x/2)$

$\Rightarrow k = 1 \times 0$

$\Rightarrow k = 0$

153. **(c)** We have,

$(3 + \omega + 3\omega^2)^4$

$= (3 + 3\omega + 3\omega^2 - 2\omega)^4$

$= [3(1 + \omega + \omega^2) - 2\omega]^4$

$= (3 \times 0 - 2\omega)^4$

$[\because 1 + \omega + \omega^2 = 0 \text{ or } \omega^3 = 1]$

$= (-2\omega)^4 = 16\omega^4$

$= 16\omega \ [\because \omega^4 = \omega^3 \cdot \omega = \omega]$

154. **(d)** We have,

$y = \tan^{-1}(\sec x - \tan x)$

$y = \tan^{-1}\left[\frac{1 - \sin x}{\cos x}\right]$

$= \tan^{-1}\left[\frac{1 - \cos\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right)}\right]$

$= \tan^{-1}\left[\frac{2\sin^2\left(\frac{\pi}{4} - \frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\cos\left(\frac{\pi}{4} - \frac{x}{2}\right)}\right]$

$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right\} = \frac{\pi}{4}-\frac{x}{2}$

$\therefore \frac{dy}{dx} = -\frac{1}{2}$

155. **(d)** We have,

$x+\frac{1}{x} = 2\cos\alpha$

$\Rightarrow x^2+1 = 2x\cos\alpha$

$\Rightarrow x^2-(2\cos\alpha)x+1 = 0$

$\Rightarrow x = \frac{2\cos\alpha \pm \sqrt{4\cos^2\alpha-4}}{2}$

$= \frac{2\cos\alpha \pm \sqrt{-4\sin^2\alpha}}{2}$

$= \frac{2\cos\alpha \pm 2(\sin\alpha)i}{2} = \cos\alpha \pm (\sin\alpha)i$

Now, $x^n = (\cos\alpha \pm (\sin\alpha)i)^n = \cos n\alpha \pm i\sin n\alpha$

and $\frac{1}{x^n} = (\cos\alpha \pm (\sin\alpha)i)^{-n} = \cos n\alpha \mp i\sin n\alpha$

$\therefore x^n+\frac{1}{x^n} = (\cos n\alpha \pm i\sin n\alpha)+(\cos n\alpha \mp i\sin n\alpha)$

$= 2\cos n\alpha$

156. **(c)** Let $I = \int_{-1}^{1} |1-x|dx$

$= \int_{-1}^{1}(1-x)dx \left[\because |x| = \begin{cases}-x, & x<0\\ x, & x\geq 0\end{cases}\right]$

$= \left[x-\frac{x^2}{2}\right]_{-1}^{1} = \left(1-\frac{1}{2}\right)-\left(-1-\frac{1}{2}\right) = \frac{1}{2}+\frac{3}{2} = 2$

157. **(b)**Let $I = \int \frac{1}{x(x^7+1)}dx$

$= \int \frac{x^6}{x^7(x^7+1)}dx$

Put $x^7 = t \Rightarrow 7x^6dx = dt$

$\therefore I = \frac{1}{7}\int \frac{d}{t(t+1)} = \frac{1}{7}\int \left(\frac{1}{t}-\frac{1}{t+1}\right)dt$

$= \frac{1}{7}[\log t - \log(t+1)]+c$

$= \frac{1}{7}\log\left(\frac{t}{t+1}\right)+c = \frac{1}{7}\log\left(\frac{x^7}{x^7+1}\right)+c$

158. **(b)** Let $I = \int \sqrt{x}e^{\sqrt{x}}dx$

Put $\sqrt{x} = t \Rightarrow x = t^2 \Rightarrow dx = 2tdt$

$\therefore I = 2\int t_1^2 e_{11}^t dt$

$= 2\left[t^2e^t - \int e^t\cdot 2tdt\right]$

$= 2\left[t^2e^t - 2\int te^tdt\right]$

$= 2\left[t^2e^t - 2\left(te^t - \int e^t\cdot 1dt\right)\right]$

$= 2[t^2e^t - 2te^t + 2e^t]+c$

$= (2t^2-4t+4)e^t+c$

$= (2x-4\sqrt{x}+4)e^{\sqrt{x}}+c$

159. **(d)** Let $I = \int \frac{dx}{x^2+2x+2}$

$= \int \frac{dx}{(x+1)^2+1} = \tan^{-1}(x+1)+c$

160. **(a)** Let $P(x_1, y_1)$ be the required point. The given curve is

$y = 6x - x^2$

$\Rightarrow \frac{dy}{dx} = 6 - 2x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 6 - 2x_1$

Since, the tangent at (x_1, y_1) is parallel to the line

$4x - 2y - 1 = 0$.

$\therefore$ Slope of the tangent at (x_1, y_1) = Slope of the line

$4x - 2y - 1 = 0$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(x_1,y_1)} = 2$

$\Rightarrow 6 - 2x_1 = 2 \Rightarrow x_1 = 2$

$\therefore\ y_1 = 6x_1 - x_1^2 = 6 \times 2 - 2^2 = 8$

Thus, required point is (2,8).

161. **(b)** Let $x = 0.5737373 ....$

$\Rightarrow 10x = 5.737373 ...$

$\Rightarrow 1000x = 573.7373 ...$

$\Rightarrow 1000x - 10x = 573 - 5$

$\Rightarrow 990x = 568$

$\Rightarrow x = \frac{568}{990}$

$\Rightarrow x = \frac{284}{495}$

162. **(a)** We have,

$x^2 - 5|x| + 6 = 0$

$|x|^2 - 5|x| + 6 = 0\ [\because x^2 = |x|^2]$

$(|x| - 3)(|x| - 2) = 0$

$|x| = 3,2$

$\Rightarrow\ x = \pm 3, \pm 2$

$\therefore$ Number of solution is 4 .

163. **(c)** Required number of 6 -digits number

$= \frac{6!}{2!\,2!\,2!} = \frac{720}{8} = 90$

164. **(c)** We have,

$7^1 = 7$

$7^2 = 49$

$7^3 = 343$

$7^4 = 2401$

and so on.

Thus, the last digit repeat itself in multiple of 4 .

Since, 300 is divisible by 4 , the units digit of 7^{300} would be 1 .

165. **(b)** Let $\frac{\log x}{a-b} = \frac{\log y}{b-c} = \frac{\log z}{c-a} = k$

$\Rightarrow \log x = k(a - b)$,

$\log y = k(b - c)$

and $\log z = k(c - a)$

Now, $\log x + \log y + \log z =$

$k(a - b + b - c + c - a)$

$\Rightarrow \log(xyz) = 0$

$\Rightarrow xyz = 1$

166. **(b)** We have,

$(1 + i)^{2n} = (1 - i)^{2n}$

$\Rightarrow \left(\frac{1 + i}{1 - i}\right)^{2n} = 1$

$\Rightarrow \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right)^{2n} = 1$

$\Rightarrow \left(\frac{1+i^2+2i}{1-i^2}\right)^{2n} = 1$

$\Rightarrow \left(\frac{1-1+2i}{1+1}\right)^{2n} = 1$

$\Rightarrow i^{2n} = 1$

$\Rightarrow 2n = 4 \qquad [\because i^4 = 1]$

$\Rightarrow n = 2$

167. **(b)** We have,

$= \cos^{-1}p + \cos^{-1}q + \cos^{-1}r = \pi$

$\Rightarrow \cos^{-1}p + \cos^{-1}q = \pi - \cos^{-1}r$

$\Rightarrow \cos^{-1}\left(pq - \sqrt{1-p^2}\sqrt{1-q^2}\right) = \pi - \cos^{-1}r$

$\Rightarrow pq - \sqrt{1-p^2}\sqrt{1-q^2} = \cos(\pi - \cos^{-1}r)$

$\Rightarrow pq - \sqrt{1-p^2}\sqrt{1-q^2} = -\cos(\cos^{-1}r)$

$\Rightarrow pq - \sqrt{1-p^2}\sqrt{1-q^2} = -r$

$\Rightarrow pq + r = \sqrt{1-p^2}\sqrt{1-q^2}$

$\Rightarrow (pq+r)^2 = (1-p^2)(1-q^2)$

$\Rightarrow p^2q^2 + r^2 + 2pqr = 1 - p^2 - q^2 + p^2q^2$

$\Rightarrow p^2 + q^2 + r^2 + 2pqr = 1$

168. **(c)** We have,

$\Rightarrow \sin^{-1}\frac{x}{5} + \mathrm{cosec}^{-1}\frac{5}{4} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{x}{5} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{x}{5} + \cos^{-1}\frac{3}{5} = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\frac{x}{5} = \frac{\pi}{2} - \cos^{-1}\frac{3}{5}$

$\Rightarrow \sin^{-1}\frac{x}{5} = \sin^{-1}\frac{3}{5}$

$\Rightarrow \frac{x}{5} = \frac{3}{5}$

$\Rightarrow x = 3$

169. **(a)** We have,

$81^{\sin^2 x} + 81^{\cos^2 x} = 30$

$\Rightarrow 81^{\sin^2 x} + 81^{\cos^2 x} = 3 + 27$

$= 81^{\sin^2 x} + 81^{\cos^2 x}$

$= (81)^{\frac{1}{4}} + (81)^{\frac{3}{4}}$

$\Rightarrow \sin^2 x = \frac{1}{4}$ and $\cos^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \frac{\pm 1}{2}$ and $\cos x = \frac{\pm\sqrt{3}}{2}$

Since, $0 < x < \pi$

$\Rightarrow \sin x = \frac{1}{2}$ and $\cos x = \frac{\sqrt{3}}{2}$

$\Rightarrow x = \frac{\pi}{6}$

170. **(d)** Equation of director circle of hyperbola,

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $x^2 + y^2 = a^2 - b^2$

Hence, director circle of $\frac{x^2}{16}-\frac{y^2}{4}=1$ is

$x^2+y^2=16-4 \Rightarrow x^2+y^2=12$

171. **(b)** Let e be the identity element, then

$a * e = a$

$\Rightarrow\ a + e - ae = a$

$\Rightarrow e(1-a)\ = 0$

$\Rightarrow e = 0 \quad [a \neq 1]$

172. **(b)** We have, $x^2+y^2-8x+4y+4=0$

$\Rightarrow (x^2-8x+16)+(y^2+4y+4)=16$

$\Rightarrow (x-4)^2+(y+2)^2=4^2$

$\therefore$ Centre is $(4,-2)$ and radius is 4 .

Hence, given circle touches Y-axis.

173. **(a)** (a) Set of all fourth roots of unity is $a=\{1,-1,i,-i\}$

Now, $1\times(-1)=-1\in G$

$1\times i=i\in G$

$1\times(-i)=-i\in G$

$(-1)\times i=-i\in G$

$(-1)\times(-i)=i\in G$

$i\times(-i)=-i^2=1\in G$

So, it is true.

(b) Set of all cube root of unit is

$H=\{1,w,w^2\}$

Now, $1+w=-w^2\notin H$

So, it is false.

(c) It is not true, as this is possible only, when a is an abelian group, i.e. $ab=ba$

(d) We have,

$(ab)^2=a^2b^2$

$\Rightarrow (ab)(ab)\ = (aa)(bb)$

$\Rightarrow (b)(a)\ = (a)(b) \Rightarrow ba = ab$

$\Rightarrow$ a is an abelian group.

So, it is false.

174. **(d)** Let H consist of the integral multiple of 5, i.e.

$H=\{5x: x\in I\}$

This H is the non-empty subset of I.

Now, let $a=5r$ and $b=5s$ any two elements of H, where r and s are some integers.

We have, $a+b=5r+5s=5(r+s)\in H$. $(r+s)$ comes out to be the some integer.

$\therefore a\in H, b\in H \Rightarrow (a+b)\in H$

Hence, H is a subgroup of set of all integers under addition.

175. **(c)** Given, circles can be written as

$x^2+y^2+kx+4y+2=0$

and $x^2+y^2-2x-\frac{3}{2}y+\frac{k}{2}=0$

Since, both circles are orthogonal, then

$2g_1g_2+2f_1f_2=c_1+c_2$

$\Rightarrow -k-3=2+\frac{k}{2}$

$\Rightarrow\ \frac{3k}{2}=-5$

$\Rightarrow\ k=-\frac{10}{3}$

176. **(b)** $\lim_{x\to\infty}\left(1-\frac{4}{x-1}\right)^{3x-1}$ $[1^\infty \text{ form}]$

$= e^{\lim_{x\to\infty}}\left(\frac{-4}{x-1}\right)(3x-1)$

$= e^{\lim_{x\to\infty} -\frac{12x-4}{x-1}} = e^{-12}$

177. **(b)** We have,

$A + B + C = \pi$

$\Rightarrow A + B = \pi - C$

$\Rightarrow \frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

$\Rightarrow \tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$

$\Rightarrow \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \cot\frac{C}{2}$

$\Rightarrow \frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \frac{1}{\tan\frac{C}{2}}$

$\Rightarrow \tan\frac{C}{2}\tan\frac{A}{2} + \tan\frac{B}{2}\tan\frac{C}{2}$

$= 1 - \tan\frac{A}{2}\tan\frac{B}{2}$

$\Rightarrow \tan\frac{A}{2}\tan\frac{B}{2} + \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} = 1$

$\Rightarrow \Sigma\tan\frac{A}{2}\tan\frac{B}{2} = 1$

178. **(c)** We have,

$\frac{b}{\sin B} = 2R$

$\Rightarrow R = \frac{B}{2\sin B} = \frac{2}{2 \times \sin 30°} = 2$

$\therefore$ Area of circumcircle $= \pi R^2$

$= \pi(2)^2 = 4\pi$

179. **(a)** We have,

$\sin x + \sin^2 x = 1$

$\Rightarrow \sin x = 1 - \sin^2 x$

$\Rightarrow \sin x = \cos^2 x$

Now, $\cos^{12}x + 3\cos^{10}x + 3\cos^8 x + \cos^6 x$

$= \sin^6 x + 3\sin^5 x + 3\sin^4 x + \sin^3 x$

$= \sin^3 x[\sin^3 x + 3\sin^2 x + 3\sin x + 1]$

$= \sin^3 x(\sin x + 1)^3$

$= (\sin^2 x + \sin x)^3$

$= (1)^3 = 1$

180. **(d)** We have,

$f(x) = |x|$

Now, $f(1) = |1| = 1$

and $f(-1) = |-1| = 1$

$\therefore f(1) = f(-1)$ but $1 \neq -1$

So, f(x) is not one-one.

Again, $|x| \geq 0, \forall x \in R$

So, range of $f(x) = [0, \infty]$

$\therefore$ Range $\neq$ Co-domain

So, f(x) is not onto.