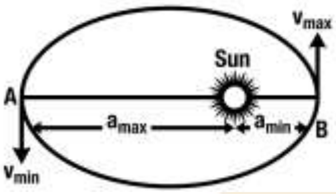
PHYSICS

1. **(b)** The laws of conservation have a deep connection with symmetries of nature. This is first theorised by Noether. Noether's first theorem states that any differentiable symmetry of the action of a physical system has a corresponding conservation law. This means that conservation laws are observed because of the symmetries of nature.

2. **(b)** Given, height of the tower, $h_1 = 200$ m
If R be the radius of the earth, then coverage range,
$d = \sqrt{2Rh} \Rightarrow d \propto \sqrt{h}$
$\Rightarrow \frac{d_1}{d_2} = \sqrt{\frac{h_1}{h_2}}$
Since, $d_2 = 3d_1$
$\therefore \frac{d_1}{3d_1} = \sqrt{\frac{h_1}{h_2}}$
$\Rightarrow \frac{1}{3} = \sqrt{\frac{200}{h_2}}$
$\Rightarrow \frac{1}{9} = \frac{200}{h_2}$
$\Rightarrow h_2 = 1800$ m
$\therefore$ Increase in the height of the tower
$= 1800\text{ m} - 200\text{ m} = 1600\text{ m}$

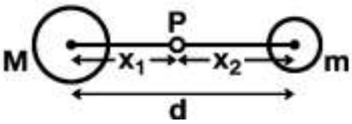
3. **(c)** The value of g at height h from the surface of the earth is given as
$g_h = \frac{g}{\left(1+\frac{h}{R_e}\right)^2}$
$= \frac{g}{\left(1+\frac{R_e/2}{R_e}\right)^2} \qquad \left(\because h = \frac{R_e}{2}\right)$
$= \frac{g}{\left(1+\frac{1}{2}\right)^2} = \frac{4g}{9}$

4. **(d)** The given situation is shown below



$a_{max} = 1.4 \times 10^{12}$ m
$a_{min} = 7 \times 10^{10}$ m
The velocity of the comet is maximum, when it is nearest to the sun and minimum when it is farthest from the sun.
$\therefore\ v_{max} = 6 \times 10^{14}$ m/s
Applying the law of conservation of angular momentum at points A and B, we get
$mv_{min}a_{max} = mv_{max}a_{min} \quad (\because L = mvr)$
$v_{min} = \frac{v_{max} \times a_{min}}{a_{max}}$
$= \frac{6 \times 10^{14} \times 7 \times 10^{10}}{1.4 \times 10^{12}}$
$= 3 \times 10^{13}\ ms^{-1}$

5. **(b)** The given situation is shown below



Let P be the location of centre of mass.

If x_1 and x_2 be the distances of the centre of mass P from the two bodies of mass M and $m (M > m)$ respectively, then

$Mx_1 = mx_2$

$\Rightarrow \frac{x_1}{x_2} = \frac{m}{M}$

Since, $M > m$

$\therefore \frac{x_1}{x_2} < 1$

$\Rightarrow x_1 < x_2$

Thus, position of centre of mass is closer to the heavier body.

6. **(b)** Volume of water in tank, $V = 30 \text{ m}^3$

Time taken to fill the tank,

$t = 15 \text{ min} = 15 \times 60 = 900 \text{ s}$

Height of the tank, $h = 40 \text{ m}$

$\therefore$ Mass of pumped water, $m =$ Volume × Density of water

$= 30 \times 10^3 = 3 \times 10^4 \text{ kg}$

Work done by the pump to fill the tank,

$W = mgh$

$= 3 \times 10^4 \times 10 \times 40$

$= 1.2 \times 10^7 \text{ J}$

$\therefore$ Output power of the pump,

$$P_o = \frac{W}{t} = \frac{1.2 \times 10^7}{900} = \frac{4}{3} \times 10^4 \text{ W}$$

As we know, efficiency $= \frac{\text{Output power } (P_o)}{\text{Input power } (P_i)}$

$$\Rightarrow 0.3 = \frac{\frac{4}{3} \times 10^4}{P_i}$$

$$\Rightarrow P_i = \frac{4 \times 10^4}{3 \times 0.3} = 4.4 \times 10^4 \text{ W} = 44 \text{ kW}$$

7. **(a)** Given, resistance of galvanometer, $R_g = 2.5\Omega$

Full scale deflection current,

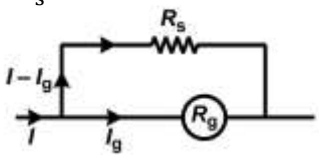
$I_g = 50 \text{ mA} = 50 \times 10^{-3} \text{ A}$

$= 0.05 \text{ A}$

Range of galvanometer is (0-5)A

$\therefore I = 5 \text{ A}$

If R_s be the shunt resistance, then



$(I - I_g)R_s = I_g R_g$

$$\Rightarrow R_s = \frac{I_g R_g}{I - I_g}$$

$$= \frac{0.05 \times 2.5}{5 - 0.05}$$

$= 0.025\Omega = 25 \times 10^{-2}\Omega$

8. **(a)** In a Zener diode, voltage across it remains constant even when current varies much or wide range. This property of the Zener diode is used to regulate the voltage supply.

9. **(a)** In an inelastic collision, momentum is conserved but kinetic energy is not conserved.

10. **(c)** Given, $\Delta I = 0.01$ A

$\Delta\phi = 2 \times 10^{-2}$ Wb

We know that, $\Delta\dot{\phi} = M\Delta I$ where, M is mutual inductance between the coils.

$\therefore\ M = \frac{\Delta\phi}{\Delta I}$

$= \frac{2 \times 10^{-2}}{0.01} = 2H$

11. **(c)** Given, velocity of aeroplane,

$v = 360 kmh^{-1}$

$= 360 \times \frac{5}{18}\ ms^{-1} = 100\ ms^{-1}$

Distance between the tips of wings,

$l = 50$ m

Vertical component of earth's magnetic field, $B_V = 4 \times 10^{-4} Wbm^{-2}$

$\therefore$ Induced emf, $e\ = B_V\ vl$

$= 4 \times 10^{-4} \times 100 \times 50 = 2$ V

12. **(a)** The inductance in a coil plays the same role as inertia plays in mechanics.

13. **(c)** The mutual inductance between the two coils depends upon the medium and the separation between them both.

14. **(d)** Given, magnetic flux

$\phi = 6t^2 - 5t + 1, R = 10\Omega$

Induced emf,

$e = -\frac{d\phi}{dt}$

$= -\frac{d}{dt}(6t^2 - 5t + 1)$

$e = -12t + 5$

At $t = 0.253$ s, $e = -12 \times 0.253 + 5$

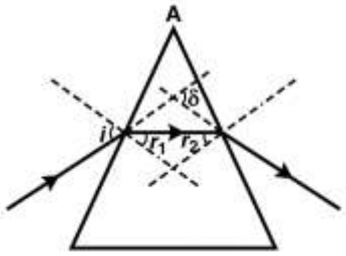
$= -3.036 + 5 = 1.964$ V

$\therefore$ Induced current, $I = \frac{e}{R}$

$= \frac{1.964}{10} = 0.1964\ A \simeq 0.2$ A

15. **(d)** In electromagnetic wave, both electric and magnetic fields vary with time.

16. **(b)** For refraction through prism,



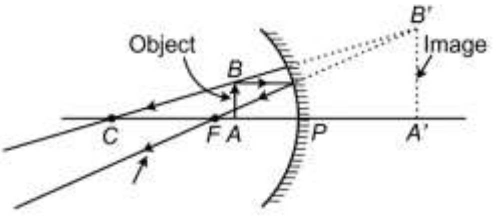
$A = r_1 + r_2$

For the case of minimum deviation,

$r_1 = r_2 = r$

$\therefore A = r + r = 2r$

17. **(b)** Image formed by a concave mirror is erect, enlarged and virtual only when object is placed between pole and focus as shown below,



18. **(b)** Number of images when an object is placed symmetrically between plane mirrors inclined at an angle θ, is given as

$$n = \frac{360}{\theta} - 1$$

Here,

$\theta = 90^\circ$

$$\therefore \; n = \frac{360}{90} - 1 = 4 - 1 = 3$$

19. **(b)** Given, $V_m = 300\text{ V}, \omega = 400\text{rads}^{-1}$

$R = 3\Omega, L = 20\text{mH} = 20 \times 10^{-3}\text{H}$

and

$C = 625\mu\text{ F} = 625 \times 10^{-5}\text{ F}$

Impedance of the circuit,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Here,

$X_L = \omega L = 400 \times 20 \times 10^{-3} = 8\Omega$

and

$$X_C = \frac{1}{\omega C} = \frac{1}{400 \times 625 \times 10^{-6}} = 4\Omega$$

$\because$

$$Z = \sqrt{3^2 + (8-4)^2} = \sqrt{25} = 5\Omega$$

$\therefore$ Peak current, $I_m = \frac{V_m}{Z} = \frac{300}{5} = 60\text{ A}$

20. **(b)** Given, angle of prism, $A = 60^\circ$

$\mu = \sqrt{2}$

We know that,

$$\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin A/2}$$

$$\Rightarrow \sqrt{2} = \frac{\sin\left(\frac{60^\circ + \delta_m}{2}\right)}{\sin\frac{60}{2}}$$

$$\Rightarrow \sin\left(\frac{60^\circ + \delta_m}{2}\right) = \sqrt{2}\sin 30^\circ$$

$$\Rightarrow \sin\left(\frac{60^\circ + \delta_m}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\left(\frac{60^\circ + \delta_m}{2}\right) = \sin 45^\circ$$

$$\Rightarrow \frac{60^\circ + \delta_m}{2} = 45^\circ$$

$\Rightarrow \delta_m = 30^\circ$

In the case of minimum deviation, angle of incidence,

$$i = \frac{A + \delta_m}{2} = \frac{60^\circ + 30^\circ}{2} = 45^\circ$$

21. **(b)** Given, wavelength of light used,

$\lambda = 5000\text{Å} = 5 \times 10^{-7}\text{ m}$

$D = 2\text{ m}$

$d = 5\text{ mm} = 5 \times 10^{-3}\text{ m}$

$$\therefore \text{ Fringe width, } \beta = \frac{D\lambda}{d}$$

$$= \frac{2 \times 5 \times 10^{-7}}{5 \times 10^{-3}} = 2 \times 10^{-4}\text{ m}$$

22. **(b)** Given, distance between source and screen, $D = 2$ m
Distance of first minimum from the central maximum, $y = 5$ mm $= 5 \times 10^{-3}$ m
Wavelength, $\lambda = 5 \times 10^{-7}$ m
We know that,
$y_n = \frac{n\lambda D}{d}$
For first minimum, n = 1
$\therefore y_1 = \frac{\lambda D}{d}$
$\Rightarrow d = \frac{\lambda D}{y_1} = \frac{5 \times 10^{-7} \times 2}{5 \times 10^{-3}} = 2 \times 10^{-4}$ m

23. **(b)** Given, wavelength of sodium line,
$\lambda = 589.0$ nm $= 5.89 \times 10^{-7}$ m
Observed wavelength,
$\lambda' = 589.6$ nm $= 5.896 \times 10^{-7}$ m
$\therefore$ Change in wavelength,
$\Delta\lambda = \lambda' - \lambda$
$= 5.896 \times 10^{-7} - 5.89 \times 10^{-7}$
$= 0.006 \times 10^{-7}$ m $= 6 \times 10^{-10}$ m
According to Doppler's shift equation, Speed of galaxy is given as
$v = \frac{\Delta\lambda}{\lambda} \times c$
$= \frac{6 \times 10^{-10}}{5.89 \times 10^{-7}} \times 3 \times 10^5$
$= 3.06 \times 10^5$ m/s
$= 306 \times 10^3$ m/s $= 306$ km/s

24. **(b)** In Newton's ring experiment, diameter of nth dark ring,
$D_n = 2\sqrt{n\lambda R}$
$\Rightarrow$ R
$= \frac{D_n^2}{4n\lambda}$ .....(i)
Given, $n = 10, D_{10} = 0.005$ m
$\lambda = 5000Å = 5 \times 10^{-7}$ m
$\therefore$ Putting these values in Eq. (i), we get
$R = \frac{(0.005)^2}{4 \times 10 \times 5 \times 10^{-7}} = 1.25$ m

25. **(b)** The amount of energy required to separate a hydrogen atom into a proton and an electron is equal to its ionisation energy which is equal to 13.6 eV .

26. **(c)** Given, $m = 1$ g $= 10^{-3}$ kg
According to Einstein's mass-energy equation,
$E = mc^2 = 10^{-3} \times (3 \times 10^8)^2 = 9 \times 10^{13}$ J

27. **(b)** As, radius of bigger drop, $R = n^{1/3}r = 2^{1/3}r$
$\Rightarrow R^2 = 2^{2/3}r^2 \Rightarrow \frac{r^2}{R^2}$ or $2^{-2/3}$
$\frac{\text{Initial surface energy}}{\text{Final surface energy}} = \frac{2(4\pi r^2 T)}{(4\pi R^2 T)} = 2\left(\frac{r^2}{R^2}\right)$
$= 2 \times 2^{-2/3} = 2^{1/3}$ or $2^{1/3} : 1$

28. **(c)** Given, mass is equivalent to 1 amu, i.e. m = 1 amu
$= 1.6605 \times 10^{-27}$ kg
According to Einstein's mass-energy equivalent equation,
$E = mc^2$
$= 1.6605 \times 10^{-27} \times (3 \times 10^5)^2$
$= 1.49 \times 10^{-10}$ J

$= \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} \text{eV}$

$= 9315 \times 10^{6} \text{eV}$

$= 9315 \text{MeV}$

29. (b) Voltage gain of first amplifier,

$A_1 = 10$

Voltage gain of second amplifier,

$A_2 = 20$

Input signal voltage,

$V_i = 0.01 \text{ V}$

Total voltage gain, when two amplifiers are connected in series (cascaded), is given as

$A = A_1 A_2 = 10 \times 20 = 200$

We know that, voltage gain

$A = \frac{\text{output voltage } (V_o)}{\text{input voltage } (V_i)}$

$\Rightarrow \; 200 = \frac{V_o}{0.01}$

$\Rightarrow \; V_o = 200 \times 0.01 = 2 \text{ V}$

30. (a) A transistor works as an amplifier, when emitter-base junction is in forward biased and collector-base junction is in reverse biased.

31. (b) In common base circuit, change in collector base voltage,

$V_{CB} = 0.6 \text{ V}$

$\Delta I_C = 0.02 \text{ mA} = 2 \times 10^{-5} \text{ A}$

$\therefore$ Output resistance,

$R_{out} = \frac{\Delta V_{CE}}{\Delta I_C} = \frac{0.6}{2 \times 10^{-5}} = 3 \times 10^{4} \Omega$

32. (c) Repeater is used between a receiver and a transmitter to extend transmission of radiowaves in communication system, so that signal can cover longer distances or be received on the other side of an obstruction.

33. (b) The waves suitable for transmission of radio signals are radiowaves. Radiowaves are a type of electromagnetic radiation with wavelengths in the electromagnetic spectrum longer than infrared radiation.

34. (d) Given, change in length of wire, $l_1 = 2 \text{ mm}, l_2 = ?$, length of wire, $L_1 = 2 \text{ m}$ and length of another wire, $L_2 = 8 \text{ m}$

Change in length, $l = \frac{FL}{AY} = \frac{FL^2}{(A \cdot L)Y} = \frac{FL^2}{VY}$ $\quad [\because V = A \cdot L]$

where, Y is Young's modulus.

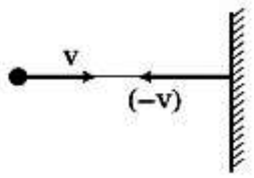
$\therefore \; l \propto L^2$ (as V, Y and F are constants)

$\frac{l_2}{l_1} = \left(\frac{L_2}{L_1}\right)^2 = \left(\frac{8}{2}\right)^2 = 16$

$\Rightarrow l_2 = 16 l_1 = 16 \times 2 \text{ mm}$

$= 32 \text{ mm} = 3.2 \text{ cm}$

35. (d) The given situation is shown below



Change in momentum,

$\Delta p = p_f - p_i = m(-v) - (mv) = -mv - mv = -2mv$

36. (a) Mass of the monkey, $m = 40 \text{ kg}$

Maximum tension that the rope can bear, $T_{max} = 600 \text{ N}$

In option (a), acceleration of monkey, $a = 6 \text{ m/s}^2$ (upward)

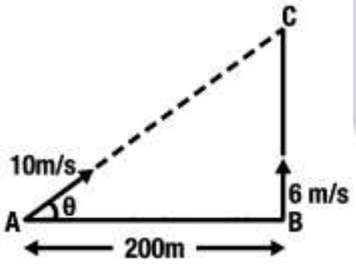
$\therefore$ By equation of motion,

$T - mg = ma$

$\Rightarrow T = m(g + a)$
$= 40(10 + 6) = 640$ N
Since, $T > T_{max}$, hence in this case, rope will break.
Therefore, option (a) is correct.
In option (b), acceleration of monkey, $a = 4\ m/s^2$ (downward)
$\therefore$ By equation of motion,
$T = m(g - a) = 40(10 - 4) = 240$ N
Since, $T < T_{max}$, then the rope will not break.
In option (c), monkey climbs up with uniform speed, $v = 5\ m/s$
$\therefore$ Acceleration, $a = 0\ m/s^2$
Hence, by equation of motion, $T = mg$
$= 40 \times 10 = 400$ N
Since, $T < T_{max}$, hence rope will not break.
In option (d), acceleration, $a = g$
$\therefore\ T = m(g - a) = m(g - g) = 0$ N.
Since, $T < T_{max}$, the rope will not break.

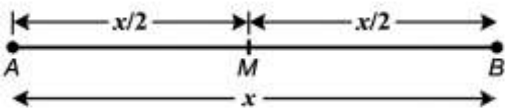
37. (b) Given, mass of shell, $m_x = 20\ g = 0.02$ kg
Mass of gun, $m_g = 100$ kg
Speed of shell, $v_s = 80\ m/s$
Let v_g be the speed of the recoil of gum, then according to law of conservation of linear momentum.
Total initial momentum = Total final momentum
$\Rightarrow 0 = m_s v_s + m_g v_g$
$\Rightarrow 0 = 0.02 \times 80 + 100 v_g$
$\Rightarrow 0 = 1.6 + 100 v_g$
$$\Rightarrow\ v_g = \frac{-1.6}{100}$$
$$= -1.6 \times \frac{10^{-2}\ m}{s} = -\frac{1.6\ cm}{s}$$
Negative sign indicates that gun moves in opposite direction to that of shell.
$\therefore$ Speed of recoil of gun $= 1.6\ cm/s$

38. (c) The given situation is shown below



$v_A = 10\ m/s$
$v_B = 6\ m/s$
The two boys meet at point C after a time t. Horizontal component of velocity of v_A –
$v_{AB} = v_A \cos\theta = 10\cos\theta$
$$= 10\left(\frac{\sqrt{10^2 - 6^2}}{10}\right)$$
$= 8\ m/s$
$$\therefore\ t = \frac{AB}{v_{AB}} = \frac{200}{8} = 25\ s$$

39. (a) The given situation is shown below



Let the distance between the two places is x km.
$$\therefore\ AM = MB = \frac{x}{2}\ km$$

Time taken by the car to cover first half (AM) with the speed of 40 km/h is given as

$t_1 = \frac{AM}{40} = \frac{x/2}{40} = \frac{x}{80}$ h

Similarly, time taken by the car to cover second half (MB) with the speed of 60 km/h is given as

$t_2 = \frac{MB}{60} = \frac{x/2}{60} = \frac{x}{120}$ h

∴ Average speed of car,

$v_{avg} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$

$= \frac{x}{\frac{x}{80} + \frac{x}{120}} = \frac{x}{\frac{5x}{240}}$

$= \frac{240}{5} = 48$ km/h

40. **(b)** According to Newton's law of gravitation,

$F = \frac{Gm_1m_2}{r^2}$

where, F = force between two objects of masses m_1 and m_2 and r = distance between m_1 and m_2.

$\therefore G = \frac{Fr^2}{m_1m_2}$

$\Rightarrow [G] = \frac{[F][r^2]}{[m_1][m_2]} = \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1} L^3 T^{-2}]$

41. **(c)** We know that,

Force = Mass × Acceleration

$\Rightarrow F = m \times a$

$\Rightarrow [F] = [m][a] = [M][LT^{-2}] = [MLT^{-2}]$

$\text{Pressure}(p) = \frac{\text{Force}(F)}{\text{Area}(A)}$

$\Rightarrow [p] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1} T^{-2}]$

Young's modulus, $Y = \frac{\text{Stress}}{\text{Strain}}$

$[Y] = \frac{[\text{ Stress }]}{[\text{ Strain }]} = \frac{[F/A]}{[\text{ Strain }]}$

$\Rightarrow = \frac{[ML^{-1} T^{-2}]}{[M^0 L^0 T^0]}$

$= [ML^{-1} T^{-2}]$

Energy = Work done = Force × Displacement

$\Rightarrow$ [Energy] = [Force] × [Displacement]

$= [MLT^{-2}][L] = [ML^2 T^{-2}]$

Hence, we see that dimensions of pressure and Young's modulus is same.

42. **(c)** Temperature of object in °C,

$T_C = 60°C$

Temperature of object in Fahrenheit scale, $T_F = ?$

We know that,

$\frac{T_C}{5} = \frac{T_F - 32}{9}$

$\Rightarrow \frac{60}{5} = \frac{T_F - 32}{9}$

$\Rightarrow 12 = \frac{T_F - 32}{9}$

$\Rightarrow T_F = 108 + 32 = 140°F$

43. **(d)** According to Newton's law of cooling,

$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_s\right)$ ....(i)

where, T_s is the temperature of surrounding.
For the first case, $T_1 = 94°C, T_2 = 86°C$
$t = 2$ min $= 120$ s, $T_s = 20°C$
$\therefore$ Putting these values in Eq. (i), we get

$$\frac{94-86}{120} = K\left(\frac{94+86}{2} - 20\right)$$

$$\Rightarrow K = \frac{8}{120 \times 70} \quad \text{....(ii)}$$

For the second case, $T_1 = 86°C, T_2 = 74°C$
$\therefore$ From Eq. (i), we get

$$\frac{86-74}{t} = K\left(\frac{86+74}{2} - 20\right)$$

$$\Rightarrow \frac{12}{t} = \frac{8}{120 \times 70}(60)$$

$$\Rightarrow t = 210 \text{ s}$$

44. **(b)** Kinetic energy of the system in SHM,

$$KE = \frac{1}{2}m\omega^2(A^2 - x^2)$$

Potential energy of the system in SHM, $PE = \frac{1}{2}m\omega^2x^2$
According to question, PE = KE

$$\frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$\Rightarrow x^2 = A^2 - x^2$$

$$\Rightarrow 2x^2 = A^2 \Rightarrow x = \frac{A}{\sqrt{2}}$$

45. **(c)** The universal gas law $\frac{pV}{T}$ = constant is applicable to both isothermal and adiabatic changes.

46. **(a)** Frequency of human heart,

$$f = \frac{72}{60} \text{ Hz} = \frac{6}{5} \text{ Hz} = 1.2 \text{ Hz}$$

Time period, $T = \frac{1}{f} = \frac{1}{1.2} = 0.83$ s

47. **(c)** Given, frequency, $f = 4.2\text{MHz} = 4.2 \times 10^6$ Hz
Velocity of sound, $v = 1.7\text{kms}^{-1} = 1.7 \times 10^3 \text{ ms}^{-1}$ The wavelength of sound in the tissue is given as

$$\lambda = \frac{v}{f} \quad (\because v = f\lambda)$$

$$= \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4 \times 10^{-4} \text{ m}$$

48. **(a)** Given, velocity of sound, $v = 330$ m/s
Let frequency of horn be f and speed of the car be $\mathbf{v_c}$. The frequency of the horn of the car heard by the policeman before it crosses him is given as

$$f' = f\left(\frac{v}{v - v_c}\right) \quad \text{....(i)}$$

and after it crosses him is given as

$$f'' = f\left(\frac{v}{v + v_c}\right) \quad \text{....(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{f''}{f'} = \frac{v - v_c}{v + v_c} = \frac{330 - v_c}{330 + v_c}$$

$$\Rightarrow \frac{f''}{f'} = \frac{330 - v_c}{330 + v_c} \quad \text{....(iii)}$$

Since, $f'' = f' - 15\%$ of $f' = 0.85f'$
$\therefore$ From Eq. (iii), we get

$$\frac{0.85f'}{f'} = \frac{330 - v_c}{330 + v_c}$$

$$\Rightarrow 0.85 = \frac{330 - v_c}{330 + v_c} \Rightarrow v_c = 26.7 \text{ ms}^{-1}$$

49. **(c)** When an electric dipole is placed in a non-uniform electric field, then it experiences both force and torque. Since, forces on the charges are not linear in non-uniform electric field, so the dipole will also experience a non-zero torque along with a net force.

50. **(b)** In a polar molecule, the centre of gravity of electrons and protons do not coincide.

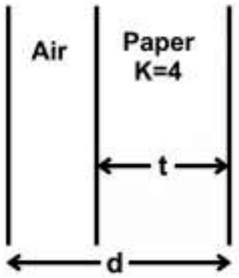
51. **(a)** Given, capacitance, $C = 10\text{mF} = 10^{-2}$ F

Potential, $V = 100$ V

When capacitor explodes, then its whole stored potential energy is given out in the form of heat and lightning.

$\therefore\ E_{out} = \frac{1}{2}CV^2 = \frac{1}{2} \times 10^{-2} \times (100)^2 = 50$ J

52. **(c)** The given situation is shown below



Thickness of paper, $t = 0.75$ mm

According to diagram, it is clear that given capacitor is equivalent to a two capacitors connected in series. In series combination, charge on each capacitor is same.

Hence, $Q_{air} = Q_{paper}$

$\Rightarrow\ C_{air} V_{air} = C_{paper} V_{paper}$

$$\frac{V_{air}}{V_{paper}} = \frac{C_{paper}}{C_{air}}$$

$$= \frac{\frac{\varepsilon_0 KA}{t}}{\frac{\varepsilon_0 A}{d-t}}$$

$$= \frac{K(d-t)}{t} = \frac{4(1-0.75)}{0.75} = \frac{4}{3}$$

53. **(b)** For first copper wire,

$l_1 = 1$ m

For second copper wire, $l_2 = 9$ m

We have to find, diameter ratio of two wires, i.e. $\frac{d_1}{d_2} = ?$

Since, both copper wires have same resistances.

i.e.

$R_1 = R_2$

$\Rightarrow \rho \cdot \frac{l_1}{A_1} = \rho \cdot \frac{l_2}{A_2} \qquad \left(\because R = \rho \cdot \frac{l}{A}\right)$

$$\Rightarrow \frac{l_1}{\left(\frac{\pi d_1^2}{4}\right)} = \frac{l_2}{\left(\frac{\pi d_2^2}{4}\right)}$$

$$\Rightarrow \frac{l_1}{d_1^2} = \frac{l_2}{d_2^2}$$

$$\Rightarrow \frac{d_1}{d_2} = \sqrt{\frac{l_1}{l_2}} = \sqrt{\frac{1}{9}} = \frac{1}{3} \text{ or } 1:3$$

54. **(c)** Heat produced in a conductor is given as

$H = i^2Rt$

$\Rightarrow\ H \propto i^2$

where, i = current, R = resistance and t = time.

55. **(c)** Radius of circular path of a charged particle in uniform magnetic field, when it enters perpendicular direction of magnetic field.

$r = \frac{mv}{Bq}$ ....(i)

where, m = mass, v = velocity and q = charge.

We know that, kinetic energy,

$K = \frac{1}{2}mv^2$

$\Rightarrow v = \sqrt{\frac{2K}{m}}$ .....(ii)

From Eqs. (i) and (ii), we get

$r = \frac{m}{Bq} \cdot \sqrt{\frac{2K}{m}}$

$r = \frac{\sqrt{2Km}}{Bq}$

For the same value of kinetic energy,

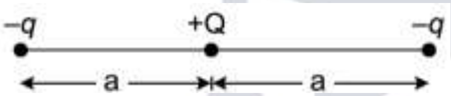
$r \propto \frac{\sqrt{m}}{q}$

$\Rightarrow \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha}} \cdot \frac{q_\alpha}{q_p} = \sqrt{\frac{m_p}{4m_p}} \cdot \left(\frac{2q_p}{q_p}\right) = \frac{1}{1}$ or $1:1$

56. (c) The magnetic properties of a magnet is lost at its Curie point because above this temperature. Magnetic domains to be disrupted permanently.

57. (b) Electromagnets are made of soft iron because soft iron has a very high value of susceptibility and low retentitivity.

58. (c) The given situation is shown below



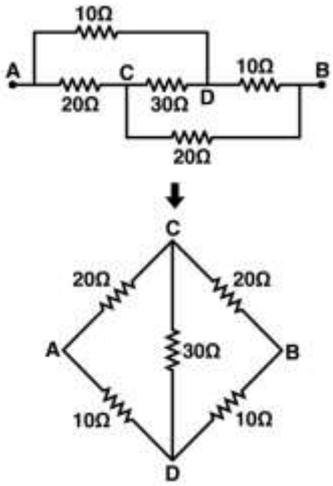
Total potential energy of the system = 0

$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left[\frac{(-q)Q}{a} + \frac{(+Q)(-q)}{a} + \frac{(-q)(-q)}{2a}\right] = 0$

$\Rightarrow -\frac{qQ}{a} - \frac{qQ}{a} + \frac{q^2}{2a} = 0 \Rightarrow \frac{2qQ}{a} = \frac{q^2}{2a}$

$\Rightarrow 2Q = \frac{q}{2} \Rightarrow \frac{Q}{q} = \frac{1}{4}$

59. (b) The given circuit diagram is redrawn as

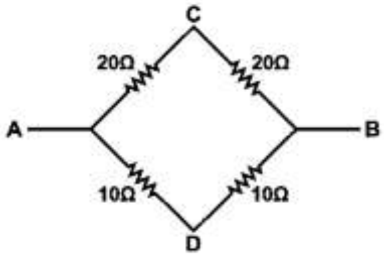


Since,

$\frac{R_{AC}}{R_{AD}} = \frac{R_{BC}}{R_{DB}}$

Given circuit is balanced Wheatstone bridge, hence 30Ω resistance is useless.

Therefore,



$\therefore\ R_{AB} = (20+20) \| (10+10)$

$= 40\|20 = \frac{40 \times 20}{40+20}$

$= \frac{800}{60} = \frac{40}{3}\Omega = 13.33\Omega$

60. **(c)** Bandwidth of each TV channel $= 3.7\text{MHz}$

$= 3.7 \times 10^{6}$ Hz

Total bandwidth available

$= 3700\text{GHz} = 3.7 \times 10^{12}$ Hz

Number of TV channels

$= \frac{\text{Total bandwidth available}}{\text{Bandwidth of one channel}}$

$= \frac{3.7 \times 10^{12}}{3.7 \times 10^{6}} = 10^{6}$

CHEMISTRY

61. **(d)** $d = 1.25$ g/mL, conc. of solution $= 3M$

$= 3$ moles in one litre of the solution.

Molar mass of NaCl $= 23 + 35.5 = 58.5\ \text{g mol}^{-1}$

Volume of solution $= 1\ \text{L} = 1000$ mL

Mass of solution $= d \times V$

$= 1.25\ \text{g mL}^{-1} \times 1000\ \text{mL} = 1250$ g

Mass of solute (NaCl) $= n \times$ molar mass

$= 3 \times 58.5 = 175.5$ g

Mass of solvent $=$ mass of solution $-$ mass of solute

$= 1250 - 175.5 = 10745$ g

62. **(c)** For 3p-orbital, $n = 3$ and $l = 1$.

63. **(a)** Power of the bulb $= 150\ \text{W} = 150\ \text{J s}^{-1}$

As only 8% of the energy is emitted as light so, the total energy emitted per second

$= \frac{150\ \text{J} \times 8}{100} = 12$ J

Energy of one photon, $E = h\nu = \frac{hc}{\lambda}$

$= \frac{(6.626 \times 10^{-34}\text{Js}) \times (3 \times 10^{8}\ \text{ms}^{-1})}{6600 \times 10^{-10}\ \text{m}}$

$= 3.0118 \times 10^{-19}$ J

$\therefore$ Number of photons emitted per second

$= \frac{12\ \text{J}}{3.0118 \times 10^{-19}\ \text{J}} = 3.98 \times 10^{19} = 4.0 \times 10^{19}$

64. **(d)** $O^{2-}(10e^{-}), F^{-}(10e^{-}), Na(11e^{-}), Mg^{2+}(10e^{-}), Al^{3+}(10e^{-})$and $Ne(10e^{-})$

Thus, Na is not isoelectronic with the rest of the species.

65. **(c)** The graph of volume vs temperature at constant pressure is called isobar.

66. **(a)** Second ionisation enthalpy of Cr is highest because after the removal of 1st electron, Cr acquires a stable half-filled d^{5} configuration thus, removal of 2 nd electron is very difficult.

67. **(b)** $He_2^{+} = \sigma 1s^{2}, \sigma^{*}1s^{1}$

Bond order $= \frac{1}{2}(N_b - N_a)$

N_b is number of bonding electrons

N_d is number of anti-bonding electrons

$= \frac{1}{2}(2-1) = \frac{1}{2} = 05$

$C_2^{2-} = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, (\pi 2p_x^2 = \pi 2p_y^2), \sigma 2p_z^2$

$BO = \frac{1}{2}(10-4) = \frac{6}{2} = 3.0$

$O_2^+ = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, (\pi 2p_x^2 = \pi 2p_y^2)$

$\pi^* 2p_x^1$

$BO = \frac{1}{2}(10-5) = \frac{5}{2} = 2.5$

$O_2^- = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, (\pi 2p_x^2 = \pi 2p_y^2)$

$(\pi^* 2p_x^2 = \pi^* 2p_y^1)$

$BO = \frac{1}{2}(10-7) = \frac{3}{2} = 1.5$

Thus, the correct order of increasing bond order is

$He_2^+ < O_2^- < O_2^+ < C_2^{2-}$.

68. **(d)** O_2 and H_2 do not show bond dipole as they are homoatomic molecules, hence, they are non-polar.

69. **(d)** Given, $p_{total} = 25$ bar, $\chi_{O_2} = 0.18$

Using the relation; $\chi_{O_2} + \chi_{Ne} = 1$

$\chi_{Ne} = 1 - 0.18 = 0.82$

$p_{Ne} = \chi_{Ne} \times p_{tota} = 0.82 \times 25 = 20.5bar$

70. **(a)** $CaCO_3(s) \longrightarrow CaO(s) + CO_2(g)$

$\Delta_r H^\circ = \Delta_f H^\circ(CaO) + \Delta_f H^\circ(CO_2) - \Delta_f H^\circ(CaCO_3)$

$= -635.09 + (-393.5l) - (-1206.92)$

$= 178.3$ kJ mol^{-1}.

71. **(b)** $\Delta_r G^\circ = -2.303 RT \log K_p$

$= -2.303 \times 8.314 \times 298 \times \log(1 \times 10^{-29})$

$= 165469.6$ J $mol^{-1} = 165.47$ kJ mol^{-1}

72. **(a)** Equilibrium constant, $K_C = \frac{[\text{ Products }]}{[\text{ Reactants }]}$

Higher value of K_C, indicates higher concentration of products which means the reaction goes more towards the completion.

73. **(c)**

	$PCl_5 \rightleftharpoons$	PCl_3	$+ Cl_2$
Initial conc.	$\frac{1}{2} = 0.5$	0	0
Equili. conc.	$\frac{1-0.4}{2} = \frac{0.6}{2}$	$\frac{0.4}{2}$	$\frac{0.4}{2}$

$$K_C = \frac{[PCl_3][Cl_2]}{[PCl_5]} = \frac{\frac{0.4}{2} \times \frac{0.4}{2}}{\frac{0.6}{2}} = \frac{(0.4)^2}{2 \times 0.6} = 0.133$$

74. **(d)** The non-existence of PbI_4 and $PbBr_4$ is probably due to the strong oxidising power of Pb^{4+} ions and strong reducing power of I^- and Br^- ions.

75. **(b)** ClO_4^- does not show disproportionate because in this oxoanion chlorine is present in its highest oxidation state $(+7)$. The disproportionation reactions for the other three oxoanions of chlorine are as follows :

$3\overset{+1}{Cl}O^- \longrightarrow 2\overset{-1}{Cl}^- + \overset{+5}{Cl}O_3^-$

$6\overset{+3}{Cl}O_2^- \xrightarrow{hv} 4\overset{+5}{Cl}O_3^- + 2\overset{-1}{Cl}^-$

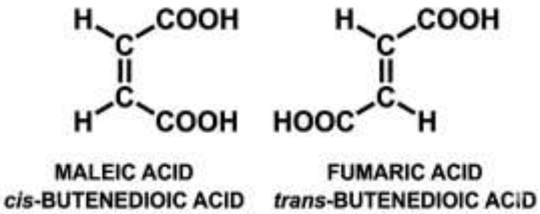
$4\overset{+5}{Cl}O_3^- \longrightarrow \overset{-1}{Cl}^- + 3\overset{+7}{Cl}O_4^-$

76. **(d)** $Na + (x+y)NH_3 \longrightarrow [Na(NH_3)_x]^+ + [e(NH_3)_y]^-$.

The deep blue colour of the solution is due to the ammoniated electron which absorbs energy in the visible region of light and thus, imparts blue colour to the solution.

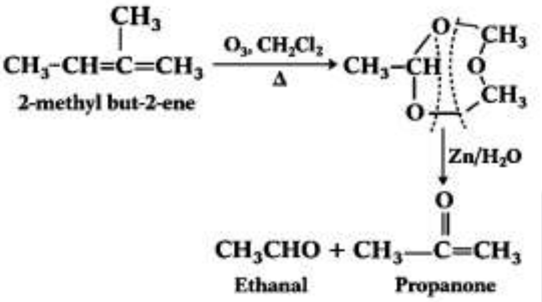
77. (b) In fullerene with formula C_{60}, all the carbon atoms are equal and they undergo sp^2 hybridisation.

78. (b) Maleic acid and fumaric acid both are geometrical isomers.Fumaric acid trans-butenedioic acid

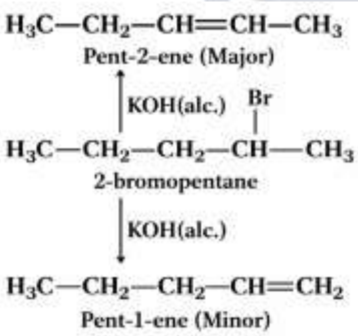


79. (a) IUPAC name of tertiary butyl chloride is 2-chloro-2-methylpropane.

80. (d) 2-methyl but-2-ene on ozonolysis produce mixture of ethanal and propanone.



81. (b) According to Saytzeff rule, in dehydrohalogenation reactions, the preferred product is that alkene which has the greater number of alkyl groups attached to the doubly bonded carbon atoms. Thus, 2-bromopentane gives pent-2-ene as the major product.



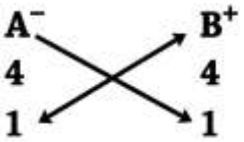
82. (b) Rain water is called acid rain when its pH falls below 5.6.

83. (c) The number of missing cations and anions are equal in Schottky defect to maintain electrical neutrality.

84. (d)

Number of A^{-} ions $= \underset{\text{(At corners)}}{\frac{1}{8} \times 8} + \underset{\text{(At face centres)}}{\frac{1}{2} \times 6} = 4$

Number of B^{+} ions $= \underset{\text{(At body centre)}}{1} + \underset{\text{(At edge centres)}}{12 \times \frac{1}{4}} = 4$



The simplest formula of the compound is AB.

85. (b) For fcc, $a = 2\sqrt{2}r$

$= 2 \times 1.414 \times 300\text{pm} = 848.5\text{pm}$

86. (b) When the concentration is expressed as the number of moles of solute per kilogram of solvent, it is known as molality.

87. (d) $NaCl(aq) \longrightarrow Na^{+}(aq) + Cl^{-}(aq)$

van't Hoff factor, $i = 2$

$BaCl_2(aq) \longrightarrow Ba^{2+}(aq) + 2Cl^{-}(aq), i = 3$

Glucose does not undergo dissociation or association hence, $i = 1$

88. **(a)** $\Delta T_b = iK_b m$

Two solutions having same molality and same van't Hoff factors will have same elevation in boiling points (ΔT_b) and thus, have same boiling points (T_b).

For NaCl, $i = 2$; $MgSO_4, i = 2$; $MgCl_2, i = 3$;

$Al_2(SO_4)_3, i = 5$ and $AlCl_3, i = 4$.

Thus, one molal solution of sodium chloride in water has the same boiling point as one molal solution of $MgSO_4$.

89. **(a)** $\frac{p^\circ - p_s}{p^\circ} = \frac{w_2 \times M_1}{w_1 \times M_2}$

$$\frac{17.535 - p_s}{17.535} = \frac{25 \times 18}{450 \times 180} = 5.55 \times 10^{-3}$$

$17.535 - p_s = 5.55 \times 10^{-3} \times 17.535$

$p_s = 17.535(1 - 0.00555)$

$p_s = 17.535 \times 0.99445$

$p_s = 17.437$ mmHg.

90. **(a)** In a electrochemical cell, the reaction will be feasible when, ΔG is - ve which is possible only when E is +ve as $\Delta G = -nFE$.

91. **(b)** For the cell, $Zn|Zn^{2+}||Ag^+|Ag$

$E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode}$

$1.56\ V = E^\circ_{Ag^+/Ag} - E^\circ_{Zn^{2+}/Zn}$

$1.56\ V = 0.8\ V - E^\circ_{Zn^{2+}/Zn}$

$E''_{Zn^{2+}/Zn} = -0.76\ V$

$\therefore\ E''_{Zn^{2+}/Zn^{2+}} = -E''_{Zn^{2+}/Zn}$

$= -(-0.76) = +0.76\ V$

92. **(a)** $\Lambda^\circ_m[Ba(OH)_2]$

$= \Lambda^\circ_m(BaCl_2) + 2\Lambda^\circ_m(NaOH) - 2\Lambda^\circ_m(NaCl)$

$= (2.8 + 2 \times 2.481 - 2 \times 1.265) \times 10^{-2}\ S\ m^2\ mol^{-1}$

$= (2.8 + 4.962 - 2.53) \times 10^{-2}\ S\ m^2\ mol^{-1}$

$= 5.232 \times 10^{-2}\ S\ m^2\ mol^{-1}$

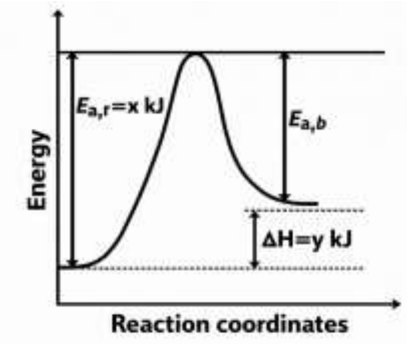
93. **(c)** For every 10°C rise in temperature, the rate of a chemical reaction doubles thus, for 60°C rise in temperature the rate of reaction increases by 2^n.

where, $n = \frac{\Delta T}{10} = \frac{60}{10} = 6$

$\therefore\ r_1 = 2^n r = 2^6 r \Rightarrow r_1 = 64r$

So, when the temperature increases by 60°C, then the rate of reaction increases by 64 times.

94. **(b)** Consider the following graph,



From the above graph

$x = y + E_{a,b}$

$E_{a,b} = x - y$ kJ

95. **(c)** For a reaction,

$aP + bQ \rightarrow$ Products

$r = k[P]^a[Q]^b$ ....(i)

$2r = k[2P]^a[Q]^b$ ....(ii)

$4r = k[P]^a[2Q]^b$ .....(iii)
On dividing Eqs. (ii) by (i), we get
$2 = 2^a \Rightarrow a = 1$
On dividing Eqs. (iii) by (i), we get
$4 = 2^b \Rightarrow b = 2$
$\therefore$ Overall order = a + b = 1 + 2 = 3.

96. **(c)** Semiconductor of very high purity are obtained by zone refining. Zone refining method is very useful for producing semiconductors and other metals of very high purity, e.g., Ge, Si, B, Ga and In.

97. **(b)** NaCN forms a soluble complex with ZnS thus, it selectively prevents ZnS from coming to the froth but allows PbS to come with the froth.

$$4NaCN + ZnS \longrightarrow \underset{\text{[Soluble complex]}}{Na_2[Zn(CN)_4]} + Na_2S$$

98. **(b)**

$$\underset{\substack{\text{white}\\\text{Phosphorus}\\\text{(A)}}}{P_4(s)} + 3NaOH(aq) + 3H_2O(l) \longrightarrow \underset{\substack{\text{Phosphine}\\\text{(B)}}}{PH_3(g)} + 3NaH_2PO_2(aq)$$

Phosphine has unpleasant odour like that of garlic or rotten fish. It burns in air to give clouds of P_4O_{10} which act as smoke screens.

99. **(b)** The reaction, $SO_2 + 2H_2S \xrightarrow{\text{Oxidation}} 3\,S\,(\text{sol.}) + 2H_2O$, represent the preparation of sulphur sol by oxidation.

100. **(d)** NeF_2 does not form inert gas compound due to high ionisation enthalpy because of small size.

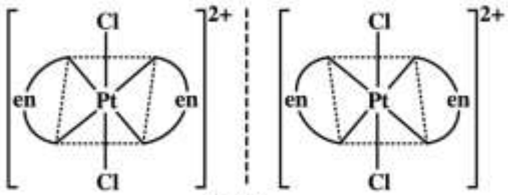
101. **(c)** Covalency of nitrogen is restricted to four due to absence of d-orbitals in its valence shell. Only four orbitals (one $2s$ and three $2p$-orbitals) are available in its valence shell.

102. **(d)** The photographic industry relies on the special light sensitive properties of AgBr.

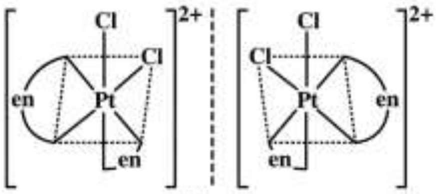
103. **(b)** In acidic medium, oxalic acid is oxidised to CO_2 (carbon dioxide) by $KMnO_4$.

$$5(COOH)_2 + 2KMnO_4 + 3H_2SO_4 \longrightarrow K_2SO_4 + 2MnSO_4 + 10CO_2 + 8H_2O$$

104. **(c)** $[PtCl_2(en)_2]^{2+}$ complex forms geometrical isomers (cis and trans). Trans-isomer does not show optical isomerism, since it is symmetrical while cis-isomer shows optical isomerism as it is unsymmetrical.



Mirror
trans-$[PtCl_2(en)_2]^{2+}$ isomer – optically inactive
(Superimposable mirror images)



Mirror
cis-$[PtCl_2(en)_2]^{2+}$ isomer – optically inactive
(Non-superimposable mirror images)

105. **(b, d)** Both options are same. The IUPAC name of complex $[Ag(NH_3)_2]^+[Ag(CN)_2]^-$ is diamminesilver (I) dicyanoargentate (I).

106. **(d)** Molar conductance of $[Co(NH_3)_3Cl_3]$ is zero as it does not ionise in solution.

107. **(b)** Alkyl halides are hydrolysed to corresponding alcohols by boiling with aqueous alkali solution (NaOH or KOH).

$$CH_3CH_2I + KOH \longrightarrow CH_3CH_2OH + KI$$

This is nucleophilic substitution reaction in which the attacking nucleophile is OH^-.

108. **(c)** S_N1 reaction is two steps reaction in which carbocation is formed as an intermediate in step I (rate determining step). Greater the stability of carbocation, greater will be its ease of formation from alkylhalide and faster will be the rate of reaction. Stability of carbocation follows the order :

Tertiary > Secondary > Primary
due to decreasing +I-effect.
Thus, the order of reactivity of haloalkanes towards S_N1 reaction is tertiary halide > secondary halide > primary halide.

109. **(a)** The given reaction takes place as follows:

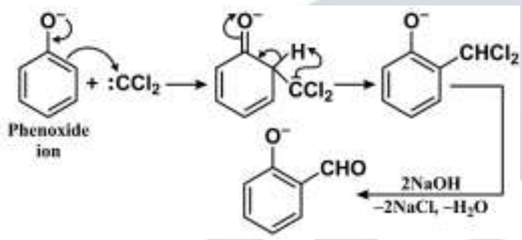
Chlorine is a o, p-directing and deactivating group. Chlorine is deactivating because of its -I-effect. As inductive effect is distance dependent so, electron density is lower at ortho position than para position. Thus, the nitration occurs at para position.

110. **(d)** The unstable intermediate[: CCl_2] is formed in Reimer-Tiemann reaction which is an electrophilic substitution reaction and occurs through the following steps.

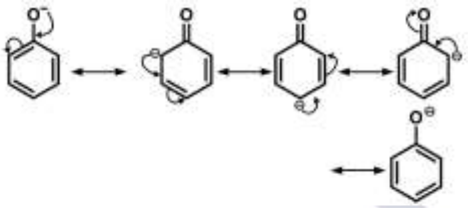
(i) Generation of electrophile

$$HO^- + CHCl_3 \rightleftharpoons H_2O + CCl_3^- \longrightarrow \underset{\substack{Dichorocarbene \\ (Electrophile)}}{:CCl_2} + Cl^-$$

(ii) Electrophilic substitution



111. **(d)** Phenols are highly acidic compared to alcohols due to formation of phenoxide ion which is stabilised by resonance.



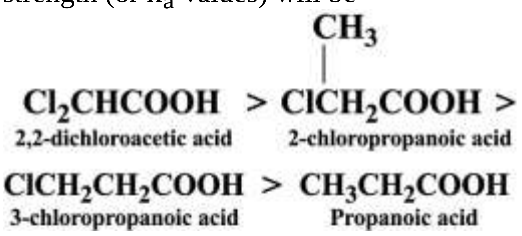
112. **(b)** Boiling point increases with increase in molecular mass so, 1-butanol has higher boiling point than 1-propanol.

Unlike alcohols, ethers do not form hydrogen bonds thus, they have lower boiling points than the corresponding alcohols.

Due to weak dipole-dipole interactions, the boiling points of lower ethers are only slightly higher than those of the n-alkanes having comparable molecular masses. Thus, the increasing order of boiling points is n-butane < ethoxyethane < 1-propanol < 1-butanol.

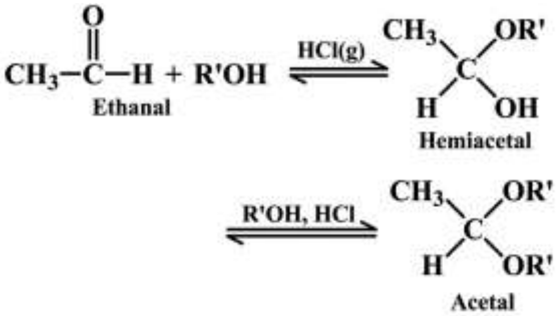
113. **(a)** Acidic strength $\propto k_a$ values

Due to $-$I-effect of electron withdrawing -Cl group, chloropropanoic acid is stronger acid than propanoic acid. Further, greater the number of electron withdrawing substitutents, greater would be the acidic strength. Inductive effect decreases rapidly with distance and so, is the acidic strength. Hence, the correct order of acidic strength (or k_a values) will be

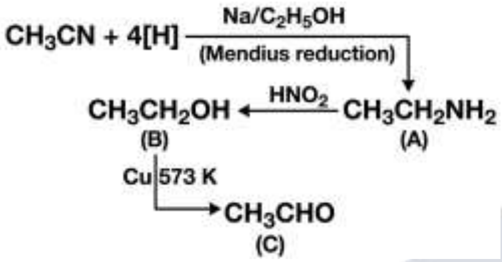


114. **(a)** Addition of Grignard reagents to carbonyl compounds is an example of nucleophilic addition reactions.

115. **(b)** The given reaction is known as acetal formation reaction.



116. **(b)** The complete reaction takes place as follows:



117. **(a)** In nucleic acids the nucleotides are joined together by phosphodiester linkage between $5'$ and $3'$ carbon atoms of the pentose sugar.

118. **(d)** Amino acids exists as Zwitter ions at isoelectrie pH.

119. **(c, d)** Neoprene and teflon are formed by addition polymerisation, while terylene and nylon-6,6 are formed by condensation polymerisation.

120. **(b)** Sucrose is natural sweetener while aspartame, sucralose and saccharin are artificial sweeteners.

MATHEMATICS

121. **(b)** In option (b), $x^2 + 3 = 0$

$\Rightarrow\ x = \sqrt{-3} \in C$

$\Rightarrow$ x is not a real number.

$\Rightarrow$ It is an empty set.

122. **(b)** If a set having n elements then its number of subsets $= 2^n$

$\therefore$ Numbers of proper susbets of a set having $(n+1)$ elements $= 2^{n+1} - 1$.

123. **(c)** Given, function is

$y = \sqrt{x-2} + \sqrt{1-x}$

Since, $x - 2 \geq 0$ and $1 - x \geq 0$

$\Rightarrow\ x \geq 2 \qquad \ldots.(i)$

$\Rightarrow\ x \leq 1 \qquad \ldots.(ii)$

$\therefore$ From Eqs. (i) and (ii), $x = \phi$

124. **(b)** Given, $A = [1,2,3,4]$

Let R be a reflexive relation on A, then for each $x \in A, (a, a) \in R$

125. **(d)** $f(x) = \frac{2x-3}{3x+4}$

Let $f(x) = y = \frac{2x-3}{3x+4}$

On cross multiplication, we get

$3xy + 4y = 2x - 3$

$\Rightarrow x(3y - 2) = -3 - 4y$

$$\Rightarrow\ x = \frac{-3-4y}{3y-2} \Rightarrow x = f^{-1}(y) = \frac{-3-4y}{3y-2}$$

Put $y = -\frac{4}{3}$, we get

$$f^{-1}\left(-\frac{4}{3}\right)=\frac{-3-4\times\left(-\frac{4}{3}\right)}{3\left(-\frac{4}{3}\right)-2}$$

$$=\frac{-3+\frac{16}{3}}{-4-2}=\frac{7}{3\times(-6)}=-\frac{7}{18}$$

126. **(a)** Let $f(x)=y=x+\frac{1}{x}$

or $xy=x^2+1$ or $x^2-xy+1=0$

Since, $x\in[0,\infty)$

$\therefore\ D\geq 0\Rightarrow y^2-4\geq 0\Rightarrow y\in[2,\infty)$

$$\therefore\ x=\frac{y\pm\sqrt{y^2-4}}{2}$$

i.e. $x=\frac{y-\sqrt{y^2-4}}{2}$ or $x=\frac{y-\sqrt{y^2-4}}{2}$

$$\Rightarrow f^{-1}(y)=\frac{y+\sqrt{y^2-4}}{2}$$

or $f^{-1}(y)=\frac{y-\sqrt{y^2-4}}{2}$

Replace by x,

$$f^{-1}(x)=\frac{x+\sqrt{x^2-4}}{2}$$

or $f^{-1}(x)=\frac{x-\sqrt{x^2-4}}{2}$

127. **(b)** We have, $\frac{\tan330^\circ\sec420^\circ\sin300^\circ}{\tan135^\circ\sin210^\circ\sec315^\circ}$

$$=\frac{\tan(360-30)^\circ\sec(360+60)^\circ\sin(360-60)^\circ}{\tan(180-45)^\circ\sin(180+30)^\circ\sec(360-45)^\circ}$$

$$=\frac{-\tan30^\circ\times\sec60^\circ\times(-\sin60^\circ)}{(-\tan45^\circ)(-\sin30^\circ)\sec45^\circ}$$

[$\because\ \tan(2\pi-\theta)=-\tan\theta,\sec(2\pi\pm\theta)=\sec\theta,$
$\sin(2\pi-\theta)=-\sin\theta,\sin(\pi+\theta)=-\sin\theta,$
$\tan(\pi-\theta)=-\tan\theta$]

$$=\frac{1\times2\times\sqrt{3}\times2}{\sqrt{3}\times2\times1\times\sqrt{2}}=\sqrt{2}$$

128. **(b)** Given, equation is

$\sin(120-A)=\sin(120-B)$

Since, sine is positive in II quadrant.

$\therefore$ Either $120-A=120-B$

$\Rightarrow A=B$

or $120-A=180-(120-B)$

$\Rightarrow 120-A=60+B\Rightarrow A+B=\frac{\pi}{3}$

129. **(a)** Given, equation is

$$\cot^{-1}\frac{1}{5}+\cot^{-1}\frac{1}{3}-\cot^{-1}\frac{4}{7}=\cot^{-1}x$$

$$\Rightarrow\tan^{-1}5+\left(\tan^{-1}3-\tan^{-1}\frac{7}{4}\right)=\tan^{-1}\frac{1}{x}$$

$$\Rightarrow\tan^{-1}5+\tan^{-1}\left(\frac{3-\frac{7}{4}}{1+3\times\frac{7}{4}}\right)=\tan^{-1}\frac{1}{x}$$

$$\Rightarrow\tan^{-1}5+\tan^{-1}\left(\frac{1}{5}\right)=\tan^{-1}\frac{1}{x}$$

$\Rightarrow \tan^{-1}5 + \cot^{-1}5 = \tan^{-1}\frac{1}{x}$

$\Rightarrow \tan^{-1}\frac{1}{x} = \frac{\pi}{2}$ $\quad \left[\because \tan^{-1}\theta + \cot^{-1}\theta = \frac{\pi}{2}\right]$

$\Rightarrow \frac{1}{x} = \tan\frac{\pi}{2} = \infty \Rightarrow x = 0$

130. **(c)** Given, equation is

$1 + i = (x + iy)(u + iv)$

$\Rightarrow 1 + i = (xu - yv) + i(xv + yu)$

Comparing real and imaginary parts, we get

$xu - yv = 1 \quad \ldots.(i)$

$xv + yu = 1 \quad \ldots.(ii)$

Multiply Eq. (i) by u and Eq. (ii) by v and then adding, we get

$x(u^2 + v^2) = u + v \Rightarrow x = \frac{u + v}{u^2 + v^2}$

From Eq. (i)

$y = \frac{xu - 1}{v} = \frac{u - v}{u^2 + v^2}$

(Substituting the value of x)

Now, $\tan^{-1}(y/x) + \cot^{-1}(u/v)$

$\tan^{-1}\left(\frac{u - v}{u + v}\right) + \cot^{-1}(u/v)$

$= \tan^{-1}\left(\frac{1 - \frac{v}{u}}{1 + \frac{v}{u}}\right) + \cot^{-1}(u/v)$

$= \tan^{-1}(1) - \tan^{-1}\left(\frac{v}{u}\right) + \tan^{-1}\left(\frac{v}{u}\right)$

$= n\pi + \pi/4, n \in I$

131. **(a)** $1^3 + 2^3 + \cdots. + n^3$

$= \left(\frac{n(n + 1)}{2}\right)^2 = (1 + 2 + \cdots. + n)^2$

132. **(b)** Let $z = a + ib$

Then, in third quadrant $a < 0, b < 0$.

Its conjugate $\bar{z} = \overline{a + ib} = a - ib = a + (-ib)$

$= a + ik$, where $k = -b$

$\Rightarrow a < 0, k > 0$

$\Rightarrow \bar{z}$ lies in II quadrant.

133. **(b)** Since, β satisfy the inequation

$x^2 - x - 6 > 0$

$\Rightarrow \beta^2 - \beta - 6 > 0$

$\Rightarrow (\beta - 3)(\beta + 2) > 0$

$\Rightarrow \beta > 3, \beta < -2$

134. **(a)** Given, ${}^{56}P_{r+6} : {}^{54}P_{r+3} = 30800 : 1$

$\Rightarrow \frac{{}^{56}P_{r+6}}{{}^{54}P_{r+3}} = \frac{30800}{1}$

$\Rightarrow \frac{56! \times (51 - r)!}{(50 - r)! \times 54!} = \frac{30800}{1}$

$\Rightarrow 56 \times 55 \times (51 - r) = 30800$

$\Rightarrow 51 - r = 10 \Rightarrow r = 41$

135. **(c)** Given, expansion is $\left(2 + \frac{x}{3}\right)^n$

Let t_{r+1} be general term.

Then, $t_{r+1} = {}^nC_r 2^{n-r}\left(\frac{x}{3}\right)^r = {}^nC_r 2^{n-r} \cdot 3^{-r} x^r$

Since, coefficient of x^5 and x^6 are equal.

$\therefore\ {}^nC_6 2^{n-6}3^{-6} = {}^nC_5 2^{n-5}3^{-5}$

$\Rightarrow \frac{{}^nC_6}{{}^nC_5} = 2 \times 3 \Rightarrow \frac{n! \times 5! \times (n-5)!}{(n-6)! \times 6! \times n!} = 6$

$\Rightarrow \frac{n-5}{6} = 6 \Rightarrow n - 5 = 36 \Rightarrow n = 41$

136. **(d)** Given, expansion is $\left[\sqrt{\left(\frac{x}{3}\right)} + \sqrt{\left(\frac{3}{2x^2}\right)}\right]^{10}$

Let t_{r+1} be general term, then

$$t_{r+1} = {}^{10}C_r \left(\sqrt{\frac{x}{3}}\right)^{10-r} \left(\sqrt{\frac{3}{2x^2}}\right)^r$$

$$= {}^{10}C_r \frac{x^{\frac{10-r}{2}}}{3^{\frac{10-r}{2}}} \cdot \frac{3^{r/2}}{2^{r/2} \cdot x^r} = {}^{10}C_r x^{5-\frac{r}{2}-r} \cdot \frac{3^{r-5}}{2^{r/2}}$$

For the term independent of x.

Puts $5 - \frac{r}{2} - r = 0$

$\Rightarrow\ 5 - \frac{3}{2}r = 0 \Rightarrow \frac{3}{2}r = 5 \Rightarrow r = \frac{10}{3}$

Fractional value of r is not possible. So, no term is independent of x.

137. **(c)** Let

$$S = \frac{1^2}{1 \cdot 2} + \frac{1^2 + 2^2}{2 \cdot 3} + \frac{1^2 + 2^2 + 3^2}{3 \cdot 4} + \cdots$$

upto 20 terms

Let t_n be nth terms of series.

$$t_n = \frac{1^2 + 2^2 + 3^2 + \cdots + n^2}{n \cdot (n+1)}$$

Then, $= \frac{\Sigma n^2}{n(n+1)} = \frac{n(n+1)(2n+1)}{6 \cdot n(n+1)} = \frac{2n+1}{6}$

Taking summation on both sides

$$\sum_{n=1}^{20} t_n = \frac{2}{6}\sum_{n=1}^{20} n + \frac{1}{6}\sum_{n=1}^{20} 1$$

$$= \frac{1}{3} \times \left(\frac{20(20+1)}{2}\right) + \frac{1}{6} \times 20$$

$$= \frac{1}{3}\left(\frac{20 \times 21}{2}\right) + \left(\frac{10}{3}\right) = \frac{210}{3} + \frac{10}{3} = \frac{220}{3}$$

138. **(a)** Given, equations of lines are

$\sqrt{3}x + y = 1$ ....(i)

and $x + \sqrt{3}y = 1$ ....(ii)

Let m_1 and m_2 be slopes of Eqs. (i) and (ii), then

$m_1 = -\sqrt{3}, m_2 = -\frac{1}{\sqrt{3}}$

Let θ be angle between them, then

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-\sqrt{3} + \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \cdot \frac{1}{\sqrt{3}}}\right| = \left|\frac{-3+1}{2\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow \theta = \frac{\pi}{6}$

139. **(d)** Since, equation of family of lines is $\frac{x}{a} + \frac{y}{b} = 1$ .... (i)

Sum of intercepts $= a + b = 7$ (given)

$\Rightarrow\ b = 7 - a$

Substitute $b = 7 - a$ in Eq. (i), we get

$$\frac{x}{a} + \frac{y}{7-a} = 1$$

$\Rightarrow (7-a)x + ay = a(7-a) \Rightarrow ay = (7-a)(a-x)$

140. **(b)** Let equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Since it passes through the points $(-3,1)$ and $(2,-2)$

$\therefore \frac{(-3)^2}{a^2} + \frac{(1)^2}{b^2} = 1$ and $\frac{(2)^2}{a^2} + \frac{(-2)^2}{b^2} = 1$

$\Rightarrow \frac{9}{a^2} + \frac{1}{b^2} = 1$ ....(i)

$\Rightarrow \frac{4}{a^2} + \frac{4}{b^2} = 1$ ....(ii)

Multiply by 4 in Eq. (i) and subtracting Eq. (i) from Eq. (ii),

$3a^2 = 32 \Rightarrow a^2 = \frac{32}{3}$

Substituting in Eq. (i), gives

$$\frac{9 \times 3}{32} + \frac{1}{b^2} = 1$$

$\Rightarrow \frac{1}{b^2} = 1 - \frac{27}{32} = \frac{5}{32} \Rightarrow b^2 = \frac{32}{5}$

$\therefore$ Required equation is

$\frac{3x^2}{32} + \frac{5y^2}{32} = 1$ or $3x^2 + 5y^2 = 32$

141. **(b)** Given, equation of parabola is $y^2 = 6x$

Substitute $x = 24$

$y^2 = 6 \times 24$

$= \pm\sqrt{6 \times 24} = \pm\sqrt{6 \times 6 \times 4} = \pm(6 \times 2) = \pm 12$

$\therefore$ Point on the parabola is $(24,12), (24,-12)$ vertex of given parabola is $(0,0)$.

$\therefore$ Equation of lines passing through $(0,0)$ and $(24,12)$, $(24,-12)$ is

$y = \frac{12}{24}x, y = \frac{-12}{24}x$

$\Rightarrow 2y - x = 0, 2y + x = 0$

$\Rightarrow 2y \pm x = 0$ are required equations of lines.

142. **(b)** $P(a,b,c)$ and PA and PB are perpendicular to YZ and ZX planes. Hence, coordinate of A and B are $(0,b,c)$ and $(a,0,c)$ respectively.

Equation of plane passing through $(0,0,0), (0,b,c)$ and

$(a,0,c)$ is $\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$

$\Rightarrow x(bc - 0) - y(0 - ac) + z(0 - ab) = 0$

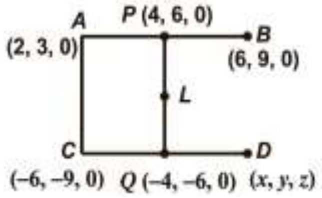
$\Rightarrow bcx + acy - abz = 0$

143. **(a)** Perpendicular distance of point $(1,2,3)$ from plane

$3y + 4z + 4 = 0$

$= \left|\frac{3(2) + 4(3) + 4}{\sqrt{(3)^2 + (4)^2}}\right| = \left|\frac{6+16}{5}\right| = \left|\frac{22}{5}\right| = 4.4$

144. **(c)**



Let coordinate of D is (x,y,z).

Using mid-point formula,

$Q = \left(\frac{x-6}{2}, \frac{y-9}{2}, \frac{z+0}{2}\right)$

Also,

$P = \left(\frac{2+6}{2}, \frac{3+9}{2}, \frac{0+0}{2}\right) = (4,6,0)$

Since, AC||PQ

$\therefore$ D.R'.s of line AC = D.R' s of line PQ

$\Rightarrow (-8,-12,0) = \left(\frac{x-14}{2}, \frac{y-21}{2}, \frac{z}{2}\right)$

$\Rightarrow x = -2, y = -3, z = 0$

$\Rightarrow D(-2,-3,0) \Rightarrow Q(-4,-6,0)$

If L is mid-point of PQ, then

$L\left(\frac{4-4}{2}, \frac{6-6}{2}, 0\right) = (0,0,0)$

$\therefore$ Perpendicular distance of L(0,0,0) from the plane $3x + 4z + 25 = 0$ is

$\left|\frac{3(0)+4(0)+25}{\sqrt{(3)^2+(4)^2}}\right| = \left|\frac{25}{\sqrt{25}}\right| = \sqrt{25} = 5$

145. **(b)** Equation of plane through the intersection of planes $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$ is

$(x + 2y + 3z - 4) + k(2x + y - z + 5) = 0$

or$(1 + 2k)x + (2 + k)y + (3 - k)z + (5k - 4) = 0$ ….(i)

D.R.' s of normal of plane (i) are

$= \langle(1 + 2k), (2 + k), (3 - k)\rangle$

Given, $5x + 3y + 6z + 8 = 0$ ….(ii)

D.R.'s of plane (ii) are $\langle 5,3,6\rangle$.

Since Eq. (i) is Perpendicular to the plane (ii),

$\therefore 5(1 + 2k) + (2 + k)3 + 6(3 - k) = 0$

$\Rightarrow 5 + 10k + 6 + 3k + 18 - 6k = 0$

$\Rightarrow 7k + 29 = 0 \Rightarrow k = \frac{-29}{7}$

$\therefore$ Required equation of plane is

$(x - 2y + 3z - 4) + \left(-\frac{29}{7}\right)(2x + y - z + 5) = 0$

$\Rightarrow 7x + 14y + 21z - 28 - 58x - 29y + 29z - 145 = 0$

$\Rightarrow -51x - 15y + 50z - 173 = 0$

$\Rightarrow 51x + 15y - 50z + 173 = 0$

146. **(c)** $\lim_{x\to 0} \frac{xa^x - x}{1 - \cos x} = \lim_{x\to 0} \frac{x(a^x - 1)}{2\sin^2\frac{x}{2}}$

$[\because \cos 2\theta = 1 - 2\sin^2\theta]$

$= \lim_{x\to 0} \frac{a^x - 1}{\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2} \times \frac{2}{x}$

$= \frac{\lim_{x\to 0} 2\left(\frac{a^x - 1}{x}\right)}{\left[\lim_{x\to 0} \frac{\sin\frac{x}{2}}{\frac{x}{2}}\right]^2} = 2\log a$

147. **(b)** Given, $y = \tan x$

Differentiating w.r.t. x both sides,

$\frac{dy}{dx} = \sec^2 x$

Taking again derivative w.r.t. x,

$\frac{d^2y}{dx^2} = 2\sec x \cdot \sec x \tan x$

$= 2\sec^2 x \tan x = 2\tan x(1 + \tan^2 x)$

$= 2y(1+y^2)$

148. **(c)** Given, $y = \tan^{-1}\left(\frac{a-x}{1+ax}\right)$

$\Rightarrow\ y = \tan^{-1}a - \tan^{-1}x$

Taking derivative w.r.t. x on both sides, we get

$\frac{dy}{dx} = -\frac{1}{1+x^2}$

149. **(c)** $f(x) = \begin{cases} [x]+[-x], & x \neq 2 \\ K, & x = 2 \end{cases}$

Since, f(x) is continuous at $x = 2$

$\therefore \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = f(2)$

Now, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}[x]+[-x]$

$= 1 + (-2) = -1$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}[x]+[-x] = 2 - 3 = -1$

$\Rightarrow\ K = -1$

150. **(b)** Given, equation of curve is

$x^2 - xy + y^2 = 27 \qquad \ldots.(i)$

Taking derivative w.r.t. x on both sides

$2x - \frac{xdy}{dx} - y + 2y\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx}(2y - x) = y - 2x \Rightarrow \frac{dy}{dx} = \frac{y-2x}{2y-x}$

Since, curve has tangent parallel to X-axis.

$\therefore$ Slope of tangent $= 0$

$\Rightarrow \frac{dy}{dx} = 0 \Rightarrow \frac{y-2x}{2y-x} = 0$

$\Rightarrow y = 2x \qquad \ldots.(ii)$

Now, solving Eqs. (i) and (ii), we get

$x^2 - 2x^2 + 4x^2 = 27 \Rightarrow 3x^2 = 27 \Rightarrow x = \pm 3$

For $x = 3, y = 6$ and $x = -3, y = -6$

$\therefore$ Points are $(3,6)$ and $(-3,-6)$.

151. **(b)** Given, equation of circle is

$x^2 + y^2 = 2 \qquad \ldots.(i)$

Taking derivative w.r.t. ' t ' on both sides.

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

If abscissa and ordinate increase at the same rate, we have

$\frac{dx}{dt} = \frac{dy}{dt}$

$x\frac{dx}{dt} + y\frac{dx}{dt} = 0 \Rightarrow \frac{dx}{dt}(x+y) = 0$

$\frac{dx}{dt} \neq 0$

Since,

$\Rightarrow\ x + y = 0 \Rightarrow x = -y \qquad \ldots.(ii)$

Solving Eqs. (i) and (ii), we get

$x^2 + (-x)^2 = 2 \Rightarrow x = \pm 1$

For $x = 1, y = -1$ and $x = -1, y = 1$

Required point are $(1,-1)$ and $(-1,1)$.

152. **(a)** Given, $\frac{dV}{dt} = 35$

where, V is volume of spherical balloon.

Also, $V = \frac{4}{3}\pi r^3$

$\Rightarrow\ \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right) = 35 \Rightarrow \frac{4}{3}\pi \times 3r^2\frac{dr}{dt} = 35$

$\Rightarrow \frac{dr}{dt} = \frac{35 \times 3}{4\pi \times 3r^2}$

Let S be surface area of sphere, then $S = 4\pi r^2$

Taking derivative w.r.t. r

$\frac{dS}{dt} = 8\pi \times r \frac{dr}{dt} = 8\pi \times r \times \frac{35 \times 3}{4\pi \times 3r^2}$

Substitute, $r = 7$

$\frac{dS}{dt} = \frac{2 \times 35 \times 3}{3 \times 7} = 10\ cm^2/min$

153. **(a)** Let $I = \int e^x(\cos x - \sin x)dx$

$= \int e^x \cos x dx - \int e^x \sin x dx$

$= \int e^x \cos x dx - \left[e^x(-\cos x) - \int e^x(-\cos x)dx\right]$

$= \int e^x \cos x dx + e^x \cos x - \int e^x \cos x dx$

$= e^x \cos x + C$

154. **(a)** Given, differential equation is

$(2x - 10y^3)\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = \frac{-y}{2x - 10y^3}$

$\Rightarrow \frac{dx}{dy} = \frac{2x - 10y^3}{-y} = \frac{-2x}{y} + 10y^2$

$\Rightarrow \frac{dx}{dy} + \frac{2}{y}x = 10y^2$

Compare with linear differential equation

$\frac{dx}{dy} + Px = Q \Rightarrow IF = e^{\int \frac{2}{y}dy} = e^{\log y^2} = y^2$

$\therefore$ Required solution is $x \cdot y^2 = \int 10y^2 \cdot y^2 dy + C$

i.e. $xy^2 = 2y^5 + C$

155. **(a)** Given, differential equation is

$\Rightarrow (xy)^{-1}\left(x\frac{dy}{dx} + y\right) = (x + y)^{-2}\left(1 + \frac{dy}{dx}\right)$

$\Rightarrow \int (xy)^{-1}\left(x\frac{dy}{dx} + y\right)dx = \int (x + y)^{-2}\left(1 + \frac{dy}{dx}\right)dx \ldots (i)$

Using integral,

$\int (f(x))^n f'(x)dx = \frac{(f(x))^{n+1})}{n + 1}$

and

$\int \frac{f'(x)}{f(x)}dx = \log(f(x)) + C$

From Eq. (i) $\log(xy) = \frac{(x+y)^{-1}}{-1} + C$

$\Rightarrow \log(xy) = \frac{-1}{x + y} + C$

156. **(a)** Let $a = 3\hat{i} + \hat{j} + 2\hat{k}$ and $b = \hat{i} + \hat{j} + 2\hat{k}$

$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 1 & 1 & 2 \end{vmatrix}$

$= \hat{i}(2 - 2) - \hat{j}(6 - 2) + \hat{k}(3 - 1) = -4\hat{j} + 2\hat{k}$

$|a \times b| = \sqrt{(-4)^2 + (2)^2} = \sqrt{16 + 4} = \sqrt{20}$

Since, $|a \times b| = |a||b|\sin\theta$, If θ is angle between a and b.

$\sqrt{20} = \sqrt{9 + 1 + 4}\sqrt{1 + 1 + 4}\sin\theta$

$\Rightarrow \sin\theta = \frac{\sqrt{20}}{\sqrt{6}\sqrt{14}}$

$\Rightarrow \sin\theta = \sqrt{\frac{20}{6 \times 14}} = \sqrt{\frac{5}{21}}$

157. **(a)** Let θ be angle between a and b then $\theta = 60°$ (given)

$|a + b|^2 = |a|^2 + |b|^2 + 2a \cdot b$

$= 4 + 4 + (2 \times 2 \times 2 \times \cos 60°)$

Since, $= 8 + 8\cos 60° = 8 + 4 = 12$

$\Rightarrow \; |a + b| = \sqrt{12} = 2\sqrt{3}$

Now, $a \cdot (a + b) = |a||a + b|\cos x$

where x is angle between a and a + b.

$\Rightarrow a \cdot a + a \cdot b = 4\sqrt{3}\cos x$

$\Rightarrow 4 + 2 \times 2\cos 60° = 4\sqrt{3}\cos x$

$\Rightarrow 6 = 4\sqrt{3}\cos x$

$\Rightarrow \cos x = \frac{\sqrt{3}}{2} = \cos\frac{\pi}{6} \Rightarrow x = 30°$

158. **(d)** $\begin{bmatrix} -2 & 5 \\ 3 & -1 \end{bmatrix}_{2\times2} \begin{bmatrix} x \\ y \end{bmatrix}_{2\times1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2} \begin{bmatrix} 3 \\ -1 \end{bmatrix}_{2\times1}$

$\Rightarrow \begin{bmatrix} -2x + 5y \\ 3x - y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$

$\Rightarrow -2x + 5y = 1, 3x - y = 5$

or $y = 3x - 5$ ....(i)

$\Rightarrow \; -2x + 5(3x - 5) = 1$

$\Rightarrow \; -2x + 15x - 25 = 1$

$\Rightarrow \; 13x = 26 \Rightarrow x = 2$

Substituting $x = 2$ in Eq. (i), we get

$y = 6 - 5 = 1$

Hence, $(x, y) = (2,1)$

159. **(d)** $(BA)^{-1} = C$ (given)

or $A^{-1}B^{-1} = C$

or $A^{-1}\begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$

Multiply by B on both sides, we get

$A^{-1}(B^{-1}B) = \begin{bmatrix} -1 & 0 & 1 \\ 1 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & 6 & 4 \\ 1 & 0 & 1 \\ -1 & 1 & -1 \end{bmatrix}$

or $A^{-1} = \begin{bmatrix} -3 & -5 & -5 \\ 0 & 9 & 2 \\ 2 & 14 & 6 \end{bmatrix}_{3\times3}$

160. **(c)** Given, $AB = B$

Multiply by A on both sides

$ABA = BA$ ....(i)

Also, $BA = A$

Multiply by B on both sides

$BAB = AB$ ....(ii)

Adding Eqs. (i) and (ii), we get

$ABA + BAB = BA + AB$

$\Rightarrow A(BA) + B(AB) = BA + AB$

$\Rightarrow AA + BB = A + B$

$\Rightarrow A^2 + B^2 = A + B$

161. **(d)** Given, equation is $x^3 + a^2x + b = 0$

Since, α, β, γ are its roots.

$\therefore$ Sum of roots $= \alpha + \beta + \gamma = 0$ ....(i)

Now, $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$

Using operation $C_1 \rightarrow C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} (\alpha+\beta+\gamma) & \beta & \gamma \\ (\alpha+\beta+\gamma) & \gamma & \alpha \\ (\alpha+\beta+\gamma) & \alpha & \beta \end{vmatrix} = \begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix} = 0 \qquad \text{[Using Eq. (i)]}$$

162. **(a)** Given determinant is

$$\begin{vmatrix} 4 & 4 & 4 \\ (a+a^{-1})^2 & (b+b^{-1})^2 & (c+c^{-1})^2 \\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2 \end{vmatrix}$$

Using operation $R_2 \rightarrow R_2 - R_3$, we get

$$\begin{vmatrix} 4 & 4 & 4 \\ 4aa^{-1} & 4bb^{-1} & 4cc^{-1} \\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 4 & 4 \\ 4 & 4 & 4 \\ (a-a^{-1})^2 & (b-b^{-1})^2 & (c-c^{-1})^2 \end{vmatrix} = 0$$

[$\because R_1$ and R_2 are identical rows]

163. **(a)** Given, $P(B \mid A) = 0.6$

$\Rightarrow \dfrac{P(B \cap A)}{P(A)} = 0.6$

$\Rightarrow P(B \cap A) = 0.6 \times 0.1 = 0.06$

Also, $P(B \mid A^C) = 0.3$

$\Rightarrow \dfrac{P(B \cap A^C)}{P(A^C)} = 0.3$

$\Rightarrow \dfrac{P(B) - P(B \cap A)}{1 - P(A)} = 0.3$

$\Rightarrow \dfrac{P(B) - 0.06}{0.9} = 0.3$

$\Rightarrow P(B) = 0.33$

Now, $P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.06}{0.33} = \frac{2}{11}$

164. **(c)** Let A be the event that a card is queen and B be the event that it is a spade.

Now, $P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{1/52}{4/52} = \frac{1}{4}$

165. **(a)** $P(A^C \cap B^C) = P\left((A \cup B)^C\right) = 1 - P(A \cup B)$

Since, probability of occurrence of an event A implies the occurrence of event B,

$\therefore A \subset B \Rightarrow A \cup B = B$

$\Rightarrow P(A^C \cap B^C) = 1 - P(B) = P(B^C)$

166. **(d)** Let events T_1, T_2, T_3 be the following

T_1 : The item is manufactured by factory F_1.

T_2 : The item is manufactured by factory F_2.

T_3 : : The item is manufactured by factory F_3.

Clearly, T_1, T_2, T_3 are mutually exclusive and exhaustive events.

$P(T_1) = 30\% = 0.3, P(T_2) = 20\% = 0.2$

$P(T_3) = 50\% = 0.5$

Let E be the event that item is defective.

Now,

$P(E \mid T_1) = 2\% = 0.02$

$P(E \mid T_2) = 3\% = 0.03$

$P(E \mid T_3) = 4\% = 0.04$

Hence, by Bayes' theorem, we have

$$P(T_1 \mid E) = \frac{P(T_1)P(E \mid T_1)}{P(T_1)P(E \mid T_1) + P(T_2)(P(E \mid T_2) + P(T_3)P(E \mid T_3)}$$

$$= \frac{0.3 \times 0.02}{0.3 \times 0.02 + 0.2 \times 0.03 + 0.5 \times 0.04}$$

$$= \frac{0.006}{0.006 + 0.006 + 0.20} = \frac{0.006}{0.032} = \frac{6}{32} = \frac{3}{16}$$

167. **(a)** To get an average of atleast 55 marks, We have,

$$\frac{60 + 85 + x}{3} \geq 55$$

$\Rightarrow 145 + x \geq 165$

$\Rightarrow x \geq 165 - 145$

$\Rightarrow x \geq 20$

168. **(b)** Coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

For first distribution, $60 = \frac{21}{\bar{x}} \times 100$

$$\bar{x} = \frac{21 \times 100}{60} = 35$$

For second distribution, $70 = \frac{16}{\bar{x}} \times 100$

$$\bar{x} = \frac{16 \times 100}{70} = 22.85$$

Hence, required means are 35,22.85.

169. **(c)** Required probability $= \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$

170. **(d)**Two non-empty sets have always non-empty intersection $\rightarrow$ logical

The real number n is less 2 $\rightarrow$ logical

Two individuals are always related $\rightarrow$ logical

171. **(d)** Objective function is not linear.

172. **(b)** Minimise $5x + 10y$

Subject to

$x \geq 6$

$y \leq 2$

$x, y \geq 0$

173. **(b)** Given,

$f(x) = \log\left(x + \sqrt{x^2 + 1}\right)$

$\therefore f(x) + f(-x) = \log\left(x + \sqrt{x^2 + 1}\right)$

$+\log\left(-x + \sqrt{x^2 + 1}\right) = \log(1) = 0$

$\Rightarrow f(-x) = -f(x)$

Hence, f(x) is an odd function.

174. **(b)** Given, $x^4 + \sqrt{x^4} + 20 = 22$

Add both sides 20 , we get

$x^4 + 20 + \sqrt{x^4 + 20} = 22 + 20$

Let $\sqrt{x^4 + 20} = y$

$\therefore y^2 + y - 42 = 0$

$\Rightarrow (y - 6)(y + 7) = 0$

$\Rightarrow y=6$ $\qquad [\because y \neq -7]$

$\Rightarrow \sqrt{x^4 + 20} = 6 \Rightarrow x^4 + 20 = 36$

$\Rightarrow x^4 = 16 \Rightarrow x = \pm 2$

Hence, the number of real roots of the equation is 2 .

175. **(b)**Since, $\Sigma n = \frac{1}{78}\Sigma n^3$

$$\Rightarrow \frac{n(n + 1)}{2} = \frac{1}{78} \times \frac{n^2(n + 1)^2}{4}$$

$\Rightarrow n^2+n-156=0$
$\Rightarrow (n+13)(n-12)=0$
$\Rightarrow n=12 \quad [\because n \neq -13]$

176. **(a)** A polygon of n sides has number of diagonals

$=\frac{n(n-3)}{2}=275 \quad$ [given]
$\Rightarrow n^2-3n-550=0$
$\Rightarrow (n-25)(n+22)=0$
$\therefore n=25 \quad [\because n \neq -22]$

177. **(c)** Now, $7^9=(8-1)^9=-1(1-8)^9$

$=-1+{}^9C_1 8-{}^9C_2 8^2+\cdots+{}^9C_9 8^9$
And $9^7=(1+8)^7=1+{}^7C_1 8+{}^7C_2 8^2$
$+{}^7C_3 8^3+\cdots.+{}^7C_7 8^7$
$\therefore 7^9+9^7=8({}^9C_1+{}^7C_1)+8^2({}^7C_2-{}^9C_2)+\cdots$
$=8(9+7)+8^2(21-36)+\cdots$
$=64\times 2+64(-15)+\cdots$
Hence, it is divisible by 64 .

178. **(a)** Let $\Delta=\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2-bc & b^2-ca & c^2-ab\end{vmatrix}$

On applying $C_3\rightarrow C_3-C_2$ and $C_2\rightarrow C_2-C_1$, we get

$$\Delta=\begin{vmatrix}1 & 0 & 0\\ a & b-a & c-b\\ a^2-bc & \begin{matrix}(b^2-a^2)\\+c(b-a)\end{matrix} & \begin{matrix}(c^2-b^2)\\+a(c-b)\end{matrix}\end{vmatrix}$$

$=1[(b-a)(c-b)(c+b+a)$
$-(c-b)(b-a)(b+a+c)]=0$

179. **(d)** Given, $3\cos 2x-10\cos x+7=0$

$\Rightarrow 6\cos^2 x-10\cos x+4=0$
$[\because \cos 2x=2\cos^2 x-1]$
$\Rightarrow 2(3\cos x-2)(\cos x-1)=0$
$\Rightarrow \cos x=1$ or $\cos x=\frac{2}{3}$
Hence, cosx is positive in Ist and IVth quadrants. Hence, the total number of solutions is 4 .

180. **(b)** $\lim\limits_{x\rightarrow-\infty}\frac{2x-1}{\sqrt{x^2+2x+1}}$

$=\lim\limits_{y\rightarrow\infty}\frac{-2-\frac{1}{y}}{\sqrt{1-\frac{2}{y}+\frac{1}{y^2}}}$ [put $x=-y, x\rightarrow-\infty$]

$=-\frac{2}{1}=-2$