PHYSICS

1. **(a)** The time period of a pendulum is given by

$$T = \frac{1}{2\pi}\sqrt{\frac{l}{g}}$$

When a lift containing a simple pendulum is accelerated upwards with acceleration a, then its time period becomes

$$T' = \frac{1}{2\pi}\sqrt{\frac{l}{(g+a)}}$$

Clearly, $T' \angle T$

Hence, its time period decreases.

2. **(c)** The level of sound is given by

$$L = 10\log\left(\frac{I}{I_0}\right) \text{dB}$$

Let for 40 dB sound intensity be l and for 90 dB sound intensity be I_x, then

$$40 = 10\log\left(\frac{I}{I_0}\right)$$

$$\Rightarrow \log\left(\frac{I}{I_0}\right) = 4 \quad \ldots\text{(i)}$$

$$\text{and } 90 = 10\log\left(\frac{I_x}{I_0}\right)$$

$$\Rightarrow \log\left(\frac{I_x}{I_0}\right) = 9$$

Given, $\quad I_x = xI$

$$\Rightarrow \log\left(\frac{xI}{I_0}\right) = 9$$

$$\text{or } \log x + \log\left(\frac{I}{I_0}\right) = 9$$

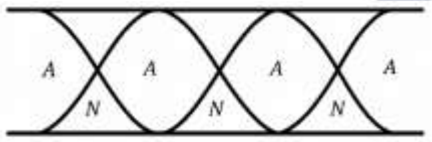
$$\Rightarrow \log x + 4 = 9 \quad [\text{from Eq.(i)}]$$

$$\Rightarrow \log x = 5 \Rightarrow x = 10^5$$

3. **(a)** The Doppler shift in wavelength of light from star is given by

$$d\lambda = \frac{\lambda v}{c}$$

4. **(b)** For third harmonic, n = 3. So, for an open pipe, the propagation of waves can be shown as



$\therefore$ Nodes = 3 and anti-nodes = 4

5. **(d)** In an adiabatic process, there is no transfer of heat takes place between a thermodynamic system and its surroundings.

6. **(a)** The efficiency of a Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Given, $T_1 = 627°C = 627 + 273 = 900$ K

and $T_2 = 27°C = 27 + 273 = 300$ K

$$\Rightarrow \eta = 1 - \frac{300}{900} = \frac{600}{900} = \frac{2}{3}$$

Also, $\eta = \frac{\text{Work done}}{\text{Heat taken}}$

$$\Rightarrow \text{Work done} = \eta \times \text{ Heat taken}$$

$$= \frac{2}{3} \times 300 = 200 \text{ J}$$

7. **(b)** The undecayed nuclei in a radioactive sample are given by

$$N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

Given, $N = N_0 - \frac{15}{16} N_0 = \frac{1}{16} N_0, t = 2$ h

$$\Rightarrow \frac{N_0}{16} = N_0 \left(\frac{1}{2}\right)^{\frac{2}{t_{1/2}}}$$

$$\Rightarrow \left(\frac{1}{2}\right)^{\frac{2}{t_{1/2}}} = \frac{1}{16} = \left(\frac{1}{2}\right)^4$$

$$\Rightarrow \frac{2}{t_{t/2}} = 4 \Rightarrow t_{1/2} = \frac{1}{2} \text{ h} = 30 \text{ min}$$

$\therefore$ Mean life, $\tau = 1.44 t_{1/2}$

$= 1.44 \times 30 = 43.2 \text{ min} \simeq 43 \text{ min}$

8. **(a)** MRI (magnetic resonance imaging) gives a detailed image of soft tissues like in brain.

9. **(d)** Of these leptons are truly elementary particles. While rest are composed of quarks, so they are composite particles.

10. **(d)** The output of NAND gate is as shown in the table below

Input		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Thus, it will give logic 0 only when both inputs are in logic 1 state.

11. **(c)** n-type semiconductor has electrons as majority charge carriers. So, they are obtained by doping with pentavalent atoms like phosphorous, arsenic, etc.

p-type semiconductor has holes as majority charge carriers. So, they are obtained by doping with trivalent atoms like aluminium, boron, indium, etc.

12. **(a)** Given, current gain, $\beta = 50$

Input resistance, $R_i = 500\Omega$

Output resistance, $R_L = 4k\Omega = 4000\Omega$

Voltage gain = Current gain $\times \frac{R_L}{R_i}$

$= 50 \times \frac{4000}{500} = 400$

Power gain = Current gain × Voltage gain

$= 50 \times 400 = 2 \times 10^4$

13. **(c)** In Balmer series, the electrons moves to $n = 2$ state

So, possible transitions are $4 \to 2, 3 \to 2$.

$\therefore$ Number of spectral lines are 2 .

14. **(b)** The IR region lies between microwave and visible region of electromagnetic spectrum.

15. **(b)** The de-Broglie wavelength of a particle is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

The momentum of proton,

$p_p = \sqrt{2mE} = \sqrt{2meV}$

Similarly, for α-particle,

$p_\alpha = \sqrt{2 \times 4m \times 2e \times V}$

$= \sqrt{16meV}$

$\therefore \frac{\lambda_p}{\lambda_\alpha} = \frac{p_\alpha}{p_p} = \frac{\sqrt{16meV}}{\sqrt{2meV}} = \sqrt{8}$

$= 2\sqrt{2}:1$

16. (a) Raman scattering is an inelastic scattering of a photon by molecules which are excited to higher vibrational or rotational energy level. Raman shift depends on the wavelength of incident radiation.

17. (c) $In\ {}_6C^{14}$, number of neutrons $= 14 - 6 = 8$

In ${}_7N^{15}$, number of neutrons $= 15 - 7 = 8$

These are the examples of isotones.

18. (c) Given, $\frac{I_{max}}{I_{min}} = \frac{9}{1}$

As we know,

$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{9}{1}$

$\Rightarrow \frac{a_1 + a_2}{a_1 - a_2} = \frac{3}{1}$

$\Rightarrow a_1 + a_2 = 3a_1 - 3a_2$

$\Rightarrow a_1 = 2a_2$

or $\frac{a_1}{a_2} = 2:1$

19. (a) For dark fringe, path difference,

$\Delta x = \left(n + \frac{1}{2}\right)\lambda$

Also,$\Delta x = \frac{dy}{D} \Rightarrow \frac{dy}{D} = \left(n + \frac{1}{2}\right)\lambda$

For 1st dark fringe at directly opposite point of slit

$n = 0, y = \frac{d}{2}$

$\Rightarrow \frac{d\left(\frac{d}{2}\right)}{D} = \frac{\lambda}{2} \Rightarrow \lambda = \frac{d^2}{D}$

20. (a) Newton's rings are fringes of equal thickness. These circular fringes were discovered by Newton, hence called Newton's rings. These rings are formed with equally spaced bright and dark band with central spot as dark.

21. (d) Diffraction pattern using light is difficult to observe in daily life because wavelength of light is very small.

22. (d) When a calcite crystal is placed over a dot on a piece of paper, one will observe two dots from above. If the crystal is rotated, then the dot produced by emergent ray will rotate around the stationary dot, which sometimes coincide with it.

23. (d) According to Brewster's law,

$\Rightarrow \mu = \tan i_p$

At critical angle, $\mu = \frac{1}{\sin i_C} = \frac{1}{\sin 45°}$

$\Rightarrow \mu = \sqrt{2}$

$\therefore \sqrt{2} = \tan i_p$

$\Rightarrow i_p = \tan^{-1}\sqrt{2} = 54.7°$

24. (b) For shunt resistance connected to galvanometer, current is given by

$I_g = \left(\frac{S}{S+G}\right)I$

$\frac{2}{100}I = \left(\frac{S}{S+G}\right)I$

$\Rightarrow S + G = 50S$

or $S = \frac{G}{49}$

25. (d) Magnetic moment of a coil of area A carrying current I is given by

$M = IA$

$\Rightarrow$ Current, $I = \frac{M}{A}$

26. (b) The magnetic field at the centre of a circular coil carrying current I is given by

$B = \frac{\mu_0 nI}{2R}$

where, n = number of turns.

For coil 1, $B_1 = \frac{\mu_0 \times 10 \times 0.2}{2 \times 0.2} = 5\mu_0$

For coil 2, $B_2 = \frac{\mu_0 \times 10 \times 0.3}{2 \times 0.4} = \frac{15}{4}\mu_0$

$\therefore$ Net magnetic field,

$B = B_1 - B_2 = 5\mu_0 - \frac{15}{4}\mu_0 = \frac{5}{4}\mu_0$

27. (a) Permanent magnet should have high retentivity to persist magnetism on removal of magnetic field. It should also have high coercivity, so that the magnetism is not lost by external magnetic field.

28. (c) The power factor is the cosine of phase angle between voltage and current. For $L - C - R$ circuit, it is given by

$\cos\phi = \frac{R}{Z}$

where, Z = impedance of circuit.

29. (c) Given, $L = 1H, V = 220\ V, f = 50\ Hz$

Peak value of current $I_0 = \frac{V_0}{X_L} = \frac{V_0}{\omega L}$

$\Rightarrow I_0 = \frac{\sqrt{2}V}{2\pi fL} = \frac{\sqrt{2} \times 220}{2 \times \pi \times 50 \times 1} \simeq 1\ A$

30. (c) Let I_0 be the intensity of light incident on polariser.

On passing through polariser, it gets reduced to $\frac{I_0}{2}$.

So, according to law of Malus,

$I = \frac{I_0}{2}\cos^2\theta$

Given, $I = \frac{75}{100} \times \frac{I_0}{2} = \frac{3I_0}{8}$

$\therefore \frac{3I_0}{8} = \frac{I_0}{2}\cos^2\theta \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30°$

31. (b) Potential due to a point charge at a distance r is given by

$V = \frac{1}{4\pi\varepsilon_0\varepsilon_r} \times \frac{q}{r}$

Given, $q = 10nC = 10 \times 10^{-9}C$,

$\varepsilon_r = 10, r = 0.1\ m$

$\Rightarrow V = \frac{9 \times 10^9 \times 10 \times 10^{-9}}{10 \times 0.1} = 90\ V$

32. (b) Permittivity of metal is very high, so its dielectric constant is infinite.

33. (d) Force between two point charges is given by

$F = \frac{kq_1q_2}{r^2}$

When $r' = r + 20\%$ of $r = 1.2r$, then

$F' = \frac{kq_1q_2}{(1.2r)^2} = \frac{kq_1q_2}{1.44r^2} = \frac{F}{1.44}$

Percentage decrease in force $= \frac{F-F'}{F} \times 100$

$= \frac{F - \frac{F}{1.44}}{F} \times 100 = \frac{0.44}{1.44} \times 100 = 31\%$

34. (a) Given $q_1 = q_2 = 10nC = 10 \times 10^{-9}C$

$r = 0.09\ m$

Potential energy of two point charges is given by

$U = \frac{1}{4\pi\varepsilon_0} \times \frac{q_1 q_2}{r}$

$= 9 \times 10^9 \times \frac{10 \times 10^{-9} \times 10 \times 10^{-9}}{0.09} = 10^{-5}$ J

$= 10\mu$ J

35. (b) Given, $t = \frac{d}{2}$

The capacitance is given by

$C = \frac{\varepsilon_0 A}{d}$

For dielectric medium between plates,

$C' = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$

For metal, K = infinity

$\Rightarrow C' = \frac{\varepsilon_0 A}{d - \frac{d}{2}} = \frac{2\varepsilon_0 A}{d} = 2C$

Hence, capacitance increases 2 times.

36. (d) The specific resistance is the property of material which does not depend on its geometry. It increases with increase in temperature.

37. (d) At 4.2 K , the resistance of mercury suddenly drops to zero and behaves as a super conductor at this temperature (called critical temperature).

38. (c) Given, $\alpha_r = 4 \times 10^{-3}/K, T_1 = 20°C$

Final resistance, $R_2 = R + 10\%R = 1.1R$

We know that, $\Delta R = \alpha_r R \Delta T$

$\Rightarrow 1.1R - R = 4 \times 10^{-3} \times R \times (T_2 - 20)$

$\Rightarrow 0.1 = 4 \times 10^{-3} \times (T_2 - 20)$

$\Rightarrow T_2 = 25 + 20 = 45°C$

39. (d) Here, 6Ω and 12Ω resistors are in parallel combination, so their equivalent resistance,

$R_{eq} = \frac{6 \times 12}{6 + 12} = \frac{72}{18} = 4\Omega$

Now, R_{eq} and 4Ω are in series, so total resistance,

$R_t = 4 + 4 = 8\Omega$

Emf of battery, E = 16 V

$\therefore$ Reading of voltmeter, $V' = \frac{4}{R_t} \times E = \frac{4 \times 16}{8} = 8$ V

40. (a) When a charged particle moves in a uniform magnetic field normally, then a magnetic force acts on it. Due to this force, it will move in a circular path.

At every point of circle, its velocity changes direction. So, its momentum also changes but the total energy will remain same.

41. (a) If n be the number of waves, then wavelength of light in glass, $\lambda_g = \frac{0.04}{n}$

and wavelength of light in water, $\lambda_w = \frac{0.05}{n}$

Speed of light in glass, $v_g = \lambda_g \times f = \frac{0.04f}{n}$

where, f = frequency of light.

Similarly, speed of light in water, $v_w = \lambda_w \times f = \frac{0.05f}{n}$

Relative refractive index

$= \frac{\mu_w}{\mu_g} = \frac{v_g}{v_w} = \frac{0.04f}{n} \times \frac{n}{0.05f} = \frac{4}{5}$

$\Rightarrow \mu_g = \frac{5}{4} \times \mu_w = \frac{5}{4} \times \frac{4}{3} = \frac{5}{3}$

42. (b) The critical angle between two media is given by

$i_c = \sin^{-1}\left(\frac{\mu_1}{\mu_2}\right)$

As we know that, $\mu_{air} = 1, \mu_{water} = 1.33, \mu_{glass} = 15$ and $\mu_{diamond} = 2.4$.

From given values, $\frac{\mu_1}{\mu_2}$ is maximum for water-glass interface. So, critical angle will also be maximum, when light travels from glass to water.

43. **(c)** Given, angle of prism, $A = 60°$

As, the ray of emergence is symmetrical to incident ray.

So, the angle of minimum deviation, $D_m = 60°$.

$\therefore$ Refractive index of prism, $\mu = \dfrac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$= \frac{\sin\left(\frac{60° + 60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin 60°}{\sin 30°}$$

$$= \frac{\sqrt{3}}{2} \times \frac{2}{1} = \sqrt{3} = 1.73$$

44. **(d)** In the spectrum of visible light produced by a prism, the violet light have minimum wavelength. So, the dispersion will be maximum towards violet.

45. **(b)** Given, $\mu_g = 15, \mu_w = \frac{4}{3}$

From lens Maker's formula,

$$\frac{1}{f} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \frac{1}{R_1} - \frac{1}{R_2} = \frac{1}{f(\mu_g - 1)} \quad \text{....(i)}$$

When lens is immersed in water, then

$$\frac{1}{f'} = \left(\frac{\mu_z}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \left(\frac{\mu_z}{\mu_w} - 1\right)\frac{1}{f(\mu_g - 1)} \quad \text{[using Eq. (i)]}$$

$$\frac{f}{f'} = \left(\frac{15}{4/3} - 1\right) \times \frac{1}{(15 - 1)}$$

$$= \left(\frac{9}{8} - 1\right) \times 2 = \frac{1}{4}$$

$\Rightarrow f' = 4f$ or $f' > f$

$\therefore$ The focal length of lens is greater than f in water.

46. **(a)** The focal length of combination of two lenses,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{(1/4)} + \frac{1}{(-1/2)} \quad \left(\because f = \frac{1}{P}\right)$$

$= 4 - 2 = 2$

$\Rightarrow f = 0.5$ m

47. **(b)** Eddy currents are produced in a material, when it is placed in a time varying magnetic field. It is based on the Faraday's law of electromagnetic induction.

48. **(d)** Given, $V_p = 220$ V, $\eta = 80\% = 0.8$

$P_0 = 8$ kW $= 8000$ W

Efficiency of transformer, $\eta = \frac{\text{Output power}}{\text{Input power}}$

$$\Rightarrow \eta = \frac{P_o}{P_i} \Rightarrow 0.8 = \frac{8000}{P_i}$$

$$\Rightarrow P_i = \frac{8000}{0.8} = 10000$$

$\Rightarrow V_p I_p = 10000$

or $I_p = \frac{10000}{220} = 45.5 \text{ A} \simeq 45 \text{ A}$

49. (b) Given, $f_0 = 600$ Hz, $Q_1 = 3, Q_2 = 2$

The bandwidth in L – C – R circuit,

$$\beta = \frac{f_0}{Q}$$

As, quality factor decreases, bandwidth increases. This increase in bandwidth is given by

$$\Delta\beta = \beta_2 - \beta_1 = \frac{f_0}{Q_2} - \frac{f_0}{Q_1} = f_0\left(\frac{1}{Q_2} - \frac{1}{Q_1}\right)$$

$$= 600\left(\frac{1}{2} - \frac{1}{3}\right) = 100 \text{ Hz}$$

50. (d) Given, $t = 4$ s, $u = 0, g = 10 \text{ ms}^{-2}$

Using equation of motion,

$$h = ut + \frac{1}{2}gt^2 = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times (4)^2 = 80 \text{ m}$$

51. (b) In liquid crystal phases, there are 4 types of phases as Nematic, Smectic, Cholesteric and Discotic. Of these, Smectic phase is more close to solid than to liquid.

52. (a) The acceleration due to gravity at earth's surface is given by

$$g = \frac{GM}{R^2}$$

As, mass remains constant.

$$\Rightarrow g \propto \frac{1}{R^2}$$

so, as the radius of earth decreases or the earth shrinks in size, the value of g increases.

53. (b) The thermal resistance of a rod is given by

$$R = \frac{l}{KA}$$

As, length and area of two rods are same, then in series combination,

$$R = R_1 + R_2$$

$$\Rightarrow \frac{2l}{K_s A} = \frac{l}{K_1 A} + \frac{l}{K_2 A}$$

$$\Rightarrow \frac{2}{K_s} = \frac{1}{K_1} + \frac{1}{K_2} = \frac{K_1 + K_2}{K_1 K_2}$$

$$\Rightarrow K_s = \frac{2K_1 K_2}{K_1 + K_2}$$

54. (c) Let F_1 and F_2 be the two forces acting at a point, then their resultant,

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

Given, $F_1 = F_2 = F$

$$F_R = \sqrt{2F^2 + 2F^2\cos\theta}$$

$$F_R^2 = 2F^2(1 + \cos\theta) \qquad \ldots\text{(i)}$$

As per question,

$$F_R^2 = 3F_1F_2 = 3F^2$$

$$\Rightarrow 2F^2(1 + \cos\theta) = 3F^2 \qquad \text{[using Eq.(i)]}$$

$$\Rightarrow 1 + \cos\theta = \frac{3}{2}$$

$$\Rightarrow \cos\theta = \frac{1}{2} \text{ or } \theta = 60^\circ$$

55. (d) The addition of impurities in a liquid may increase or decrease the surface tension of liquid depending on the nature of impurities.

If the impurities are highly soluble in liquid, then it will increase the surface tension. But if it is less soluble in liquid, then it will decrease the surface tension of liquid.

56. (c) The viscosity decreases with increase in temperature. Due to this reason, the hot water moves faster than cold water. Since in summer temperature is high, so more viscous oils are used in motorcars.

57. **(c)** Moment of momentum of an electron in nth orbit is given by

$L = \frac{nh}{2\pi}$

For an electron in second Bohr orbit, $n = 2$

$\therefore\ L = \frac{2h}{2\pi} = \frac{h}{\pi}$

58. **(a)** The excitation energy is the minimum energy required to excite an electron from ground state of atom to any of its excited state and ionisation energy is the amount energy needed to remove an electron from an atom.

So, lesser the excitation energy and more the ionisation energy, more the stability of atom.

59. **(c)** Given, work function, $\phi = 3eV$

The wavelength of incident radiation is given by

$\lambda = \frac{hc}{\phi} = \frac{1242}{\phi} = \frac{1242}{3} = 414\ nm$

60. **(d)** The equivalent capacitance of three capacitors connected in series is given by

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

Given, $C_1 = C_2 = C_3 = C$

$\frac{1}{C_s} = \frac{3}{C}$ or $C_s = \frac{C}{3}$

Similarly, for parallel combination,

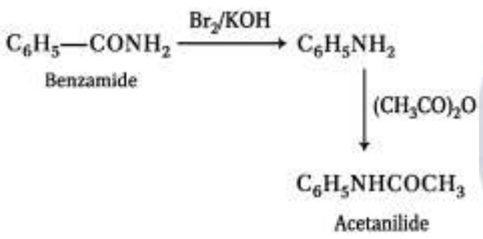
$C_p = C + C + C = 3C$

$\therefore$ Ratio of equivalent capacitance,

$\frac{C_s}{C_p} = \frac{C/3}{3C} = \frac{1}{9}$ or $1:9$

CHEMISTRY

61. **(d)** An organic compound gives oily liquid on heating with bromine and potassium hydroxide solution. On shaking with acetic anhydride, an antipyretic drug acetanilide obtained is benzamide.



62. **(d)** Silver salt of fatty acid on refluxing with an alkyl halide gives an ester.

$\underset{\text{Silver salt of fatty acid}}{CH_3COOAg} + \underset{\text{Allyl halide}}{C_2H_5Br} \longrightarrow \underset{\text{Ester}}{CH_3COOC_2H_5} + AgBr$

63. **(d)** Pepsin, ptyalin, lipase are all enzymes used for the breaking down of nutrients, while cellulose is a polysaccharide.

64. **(b)** Combination of H_2 and Br_2 to give HBr , is second order reaction.

$H_2 + Br_2 \longrightarrow 2HBr$

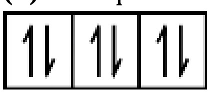
Rate law expression, $r = k[H_2][Br_2]$

$\therefore$ Order w.r.t. H_2 and Br_2 is one.

$\therefore$ Overall order $= 1 + 1 = 2$

65. **(c)** The diameter of colloidal particle ranges from 10^{-5} m to 10^{-9} m.

66. **(b)** The 2p-orbital has electronic configuration as follows



$\therefore$ Three electrons are with $-1/2$ spin quantum number and other three electrons are with $+1/2$ spin quantum number.

67. **(b)** The molecular formula of 2-methyl butane, 2,2-dimethyl propane and pentane is same i.e. (C_5H_{12}). They are structural isomers.

68. (a) $\underset{\substack{1\text{ mole} \\ (28\text{ g})}}{N_2} + \underset{\substack{3\text{ moles} \\ (6\text{ g})}}{3H_2} \longrightarrow \underset{\substack{2\text{ moles} \\ (34\text{ g})}}{2NH_3}$

$\therefore$ Mass of N_2 left $= 56 - 28 = 28$ g

$\therefore$ Moles of N_2 left $= \frac{28}{28} = 1$ mole

Mass of H_2 left $= 8 - 6 = 2$ g

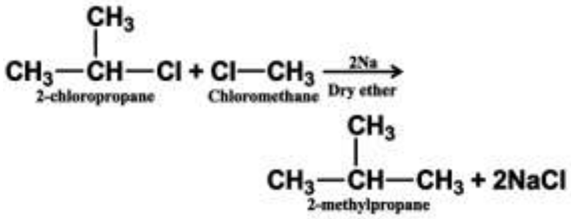
$\therefore$ Moles of H_2 left $= \frac{2}{2} = 1$ mole

Moles of ammonia $= \frac{34}{17} = 2$ moles

69. (a) For constant pressure and temperature process, ΔH and $\Delta U = 0$

70. (a) In a galvanic cell, the electron flow from anode to cathode through the external circuit.

71. (c) The mixture of 2-chloropropane and chloromethane on treating with sodium metal in dry ether gives 2 -methyl propane i.e.,



72. (c) Benzyl chloride is more reactive than alkyl halides as benzyl carbocation is stabilised by resonance. Hence, it easily gives nucleophilic substitution reaction.

73. (d) The main product obtained when sodium carbonate reacts with mercuric chloride is mercuric oxide.

$HgCl_2 + Na_2CO_3 \longrightarrow HgO + CO_2 + 2NaCl$ or $HgCO_3 + 2NaCl$

74. (b) In a electrothermal process, the compound displayed by silica from calcium phosphite is phosphorus pentoxide as follows:

$2Ca_3(PO_4)_2 + 6SiO_2 \longrightarrow \underset{(\text{ Slag })}{6CaSiO_3} + P_4O_{10}$

75. (b) $\underset{16\text{ g}}{CH_4} + \underset{64\text{ g}}{2O_2} \longrightarrow \underset{44\text{ g}}{CO_2} + \underset{36\text{ g}}{2H_2O}; \Delta H = 890$ kJ/mol

Heat liberated on combustion of 16 gCH_4 is 890 kJ .

$\therefore$ Heat liberation from 3.2 g methane

$= \frac{3.2}{16} \times 890 = \frac{890}{5}$

$= 178$ kJ

76. (a) According to gas equation,

$pV = nRT$

$\therefore\ V = \frac{nRT}{p}$

As both p and T are double, V remains constant i.e., $4dm^3$.

77. (d) Copper on reacting with conc. HNO_3 gives copper nitrate.

$Cu + HNO_3$ (conc.) $\longrightarrow \underset{\text{Copper nitrate}}{Ca(NO_3)_2} + 2H_2O + 2NO_2$

On strongly heating with copper nitrate, copper oxide is obtained.

$2Cu(NO_3)_2 \longrightarrow \underset{\substack{\text{Copper oxide} \\ (5g)}}{2CuO} + 4NO_2 + O_2$

$\therefore$ Equivalent weight of Cu

$=$ Weight of copper $\times$ Equivalent weight of oxygen

$= 4 \times 8 = 32$

78. (c) For the following reaction,

$N_2 + 3H_2 \rightleftharpoons 2NH_3 + 923$ kJ

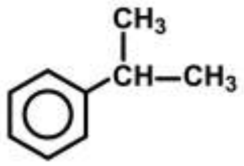
(i) Reducing temperature is favourable as it is an exothermic process and shift equilibrium towards right.

(ii) Also on removing NH_3 equilibrium shift towards right according to Le-Chatelier's principle.

(iii) Δn_g is negative for the process. Increase in pressure, equilibrium towards right.
For an exothermic reaction, increasing temperature shifts the reaction to reactant side.

79. **(d)** The chemical equilibrium of a reversible reaction is not influenced by catalyst as it only increase the rate of reaction to achieve the equilibrium faster. The increase in rate of forward direction is equal to increase in rate of backward direction at equilibrium.
$\therefore$ Equilibrium point is not shifted.

80. **(c)** The structure of cumene is



The IUPAC name is 1-methyl ethyl benzene.

81. **(d)** Moles of HCl $= \frac{1.2046\times10^{24}}{6.023\times10^{23}} = 2$

$\therefore$ Normality $= \dfrac{\text{moles of HCl}}{\text{volume of solution (L or dm}^3)}$

$= \dfrac{2}{1} = 2\ \text{N}$

82. **(c)** Nuclear theory of atom was put forward by Rutherford. He discovered α and β-rays and projected the law of radioactive decay.

83. **(c)** In acetylene or ethyne molecule, the two carbon atoms are linked by one σ and two π-bonds.

84. **(c)** $H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(g); \Delta H_1 = +ve$

$H_2O(g) \longrightarrow H_2O(l)\ ; \Delta H_3 = +ve$

$H_2(g) + \dfrac{1}{2}O_2(g) \longrightarrow H_2O(l)\ ; \Delta H_2 = \Delta H_1 + \Delta H_3$

$\therefore \Delta H_2 > \Delta H_1$

85. **(d)** For radioactive decay

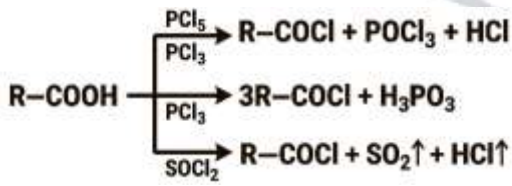
$t_{1/2} = \dfrac{0.693}{k}$

For concentration to reduce to $\frac{1}{16}$ of original amount

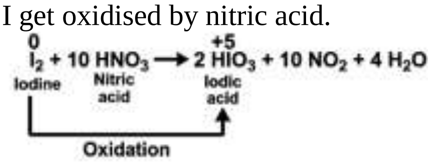
$t = t_{1/2} \times 4$

$\therefore t_{1/2} = \dfrac{192}{4} = 48\ \text{min}$

86. **(d)** The reagent, Cl_2 does not give acid chloride on heating with carboxylic acids, while other reagents give acid chloride.



87. **(d)**Reducing property of halogens increases from F to I.
$\therefore$ I get oxidised by nitric acid.



88. **(a)** Lead form nitric oxide with dil. HNO_3. It does not form ammonium nitrate by reaction with dil. nitric acid.

$3Pb + 8HNO_3 \longrightarrow 3\ Pb(NO_3)_2 + \underset{\text{Nitric oxide}}{2NO} + 4H_2O$

89. **(d)** The elements with atomic number 9, 17, 35, 53 and 85 are all halogens, they have electronic configuration of ns^2np^5.

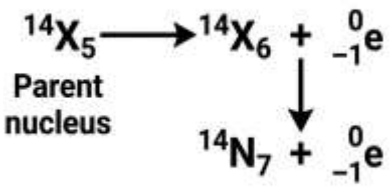
90. (b) In the electrolytic method of obtaining aluminium from purified bauxite, cryolite is added to the charge in order to lower the melting point of bauxite and brings conductivity.

91. (c) HNO_3 is not an amphoteric substance as, it behave only as acid. It can only donate H^+ions. While, substance like HCO_3^-, H_2O, and NH_3 can act as both donor and acceptor to H^+ions, so they are amphoteric in nature.

92. (b) Equivalent moles of H_2SO_4 and KOH undergoing neutralisation

$= 50 \times 0.2 \times 10^{-3} = 0.01$

93. (c)



$\therefore$ The number of neutrons in parent nucleus $= 14 - 5 = 9$

94. (d) Stainless steel does not rust because chromium forms an oxide layer and protects iron from rusting.

95. (b) Ethanol can be synthesised as follows:

$$CH_3MgI + \underset{\text{Formaldehyde}}{HCHO} \xrightarrow{H_2O} \underset{\text{Ethanol}}{CH_3 - CH_2OH} + Mg(OH)I$$

96. (b) The rate law expression is, $r = k[SO_2]^2[O_2]$

As, volume increase 2 times, concentration reduces by

$\frac{1}{2}$ times i.e., $[SO_2]_1 = [2SO_2]$ and $[O_2]_1 = [2O_2]$

$\therefore$ New rate, $r_1 = k[2SO_2]^2[2O_2]$

$\therefore r_1 = 8r$

$\therefore \frac{r_1}{r} = \frac{8}{1}$

97. (c) The pH of buffer solution is given as,

$pH_1 = pK_a + \log\frac{[CH_3COONa]}{[CH_3COOH]}$ ....(i)

Given, $[CH_3COONa]_2/[CH_3COOH]_2$

$= 10[CH_3COONa]/[CH_3COOH]$

$pH_2 = pK_a + \log 10[CH_3COONa]/[CH_3COOH]$

$pH_2 = pK_a + \log[CH_3COONa]/[CH_3COOH] + \log 10$

$\therefore pH_2 = pH_1 + 1$ [From Eq.(i)]

98. (b) When mixture of methane and oxygen is passed through heated molybdenum oxide, the main product formed is methanal.

$$\underset{\text{Methane}}{CH_4} + O_2 \xrightarrow{\text{Mo-oxide}} \underset{\text{Methanal}}{HCHO} + H_2O$$

99. (d) Benzene can be obtained by heating benzoic acid with soda lime and phenol with zine dust as follows:

$$\underset{\text{Benzoic acid}}{C_6H_5COOH} + \underset{\text{Soda lime (X)}}{2NaOH} \xrightarrow{\Delta} \underset{\text{Benzene}}{C_6H_6} + Na_2CO_3$$

$$\underset{\text{Phenol}}{C_6H_5 - OH} + \underset{\substack{\text{Zinc dust}\\ \text{(Y)}}}{Zn} \xrightarrow{\Delta} C_6H_6 + ZnO$$

100. (c) Ethyl benzoate is boiled with alc. potash to give ethyl alcohol and potassium benzoate.

$$\underset{\text{Ethyl benzoate}}{C_6H_5COOCH_2CH_3} \xrightarrow[H_2O]{\text{Alc.potash}} C_6H_5COOK + CH_3CH_2OH$$

Potassium benzoate is converted into benzoic acid, a white solid which separates out.

$C_6H_5COOK + HCl \longrightarrow C_6H_5COOH + KCl$

101. (a) On passing H_2S gas in presence of dil. HCl for qualitative analysis of second group due to common ion effect, lower concentration of sulphide ions is obtained. It is sufficient for the precipitation of second group cations in forms of their sulphides and hence their dissociation decreases.

102. (b) $E^\circ = E^\circ_{(Ag^+/Ag)} - E^\circ_{(Al^{3+}/Al)}$

$2.46 = 0.80 - E^{\circ}_{(Al^{3+}/Al)}$

$\therefore\ E^{\circ}_{(Al^{3+}/Al)} = -1.66\ V$

$\therefore$ Reduction potential $= -1.66$ V.

103. **(c)** The first fraction obtained during the fractionation of petroleum is hydrocarbon gases.

104. **(a)** Ethanol gives trichloromethane on distilling with bleaching powder.

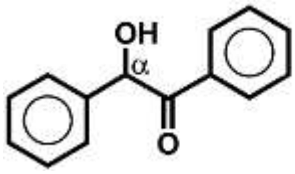
(I) $Cl_2 + H_2O \longrightarrow 2HCl + [O]$

$\underset{\text{Ethanol}}{CH_3CH_2OH} + [O] \longrightarrow CH_3CHO + H_2O$

(II) $CH_3CHO + 3Cl_2 \longrightarrow CCl_3CHO + 3HCl$

(III) $CCl_3CHO + Ca(OH)_2 \longrightarrow \underset{\text{Trichloro methane}}{2CHCl_3} + (HCOO)_2Ca$

105. **(b)** Benzoin is a α-hydroxy ketone.



106. **(b)** For every 10 K rise in temperature, rate of reaction double.

$\therefore$ From 298 K to 310 K , the reaction rate became $(2)^2$ times i.e., 4 times.

$\therefore k = 3.2 \times 10^{-3}\ s^{-1} \times 4 = 1.28 \times 10^{-2}\ s^{-1}$

107. **(c)** The acidic strength of an acid increase with ionisation.

The value of pK_q is inversely proportional to ionisation constant (K_a).

$\therefore$ The acid with pK_a value 1.0 is strongest.

108. **(a)** Unsaturated fatty acids contains one or more double bond in their carbon chains.

Their general formula is $C_nH_{2n-2x}O_2$.

$\therefore$ Oleic acid $(C_{18}H_{34}O_2)$ is unsaturated fatty acid with one double bond.

109. **(b)** Nylon is not a homopolymer. It is a polymer of hexamethylenediamine and adipic acid.

110. **(c)** The coaltar fraction which contains phenol is called middle oil.

111. **(b)**

	A	+ B	$\rightleftharpoons$	C	+ D
Initially	1	1		0	0
at equilibrium	$\frac{2}{3}$	$\frac{2}{3}$		$\frac{1}{3}$	$\frac{1}{3}$

$\therefore$ Equilibrium constant $(K) = \dfrac{[C][D]}{[A][B]}$

$$= \frac{\frac{1}{3}\cdot\frac{1}{3}}{\frac{2}{3}\cdot\frac{2}{3}} = \frac{1}{4} = 0.25$$

112. **(c)** The particles of ore float because their surface is not easily wetted by water.

113. **(d)** Amorphous solids are isotropic in nature i.e., they shows same properties in all the directions.

114. **(c)** Rate of diffusion $\propto \dfrac{1}{\sqrt{\text{Molar mass}}}$

$\therefore \dfrac{r_H}{r_A} = \sqrt{\dfrac{M_A}{M_H}}$

$\therefore r_H = 6r_A$

$\therefore 6 = \sqrt{\dfrac{M_A}{2}}$

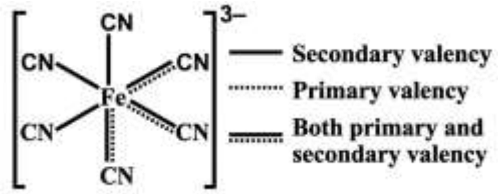
$\therefore M_A = 72$

115. **(b)** Dulong and Petit's law is valid only for solid elements. According to this law, the product of atomic mass and specific heat of a solid element is approximately equal to 6.4cal/mol.

116. **(d)** The gas which has greater critical temperature is readily adsorbed by activated charcoal.
The order of critical temperature is
$H_2 < N_2 < O_2 < SO_2$
$\therefore$ SO_2 gas is readily adsorbed by chorcoal.

117. **(b)** In fee unit cell of NaCl, Na^+ion is present at edge centre and Cl^-ion is present at comer.
$\therefore$ Edge length, $a = 2r^+ + 2r^-$
$\therefore$ $a = 2(r^+ + r^-)$
$= 2x \quad [\because x = r^+ + r^-]$

118. **(d)** In $K_3[Fe(CN)_6]$, the ligand has satisfied both primary and secondary valencies of ferric ion. The +3 charge of ferric ion is satisfied by negative charge on cyanide ion. The coordination sphere's secondary valency is satisfied by six cyanide ligands.



119. **(b)** 2-acetoxy benzoic acid is aspirin. It is used as an antipyretic to reduce body temperature.

120. **(a)** Nucleoside on hydrolysis gives an aldopentose and a heterocyclic base i.e. purine and pyrimidine.

MATHEMATICS

121. **(d)** Given that, $A = \{a, b, c\}, B = \{b, c, d\}$
and $c = \{a, d, c\}$
Now, $A - B = \{a, b, c\} - \{b, c, d\} = \{a\}$
and $B \cap C = \{b, c, d\} \cap \{a, d, c\} = \{c, d\}$
$(A - B) \times (B \cap C) = \{a\} \times \{c, d\} = \{(a, c), (a, d)\}$

122. **(b)** We have,
$f: X \rightarrow Y$
X : domain and Y : codomain
In one-one function,
$x_1 \neq x_2$
$f(x_1) \neq f(x_2)$
In onto function,
Range = domain
$f(x) = \sin x$
So, if $X = \left[0, \frac{\pi}{2}\right]$ and $Y = [-1,1]$
For each value of $x \in X$ and range $= [0,1]$, there exists a unique value for $y \in Y$
Hence, function is one-one but not onto.

123. **(a)** We have,
$\log_4 2 + \log_4 4 + \log_4 16 + \log_4 x = 6$
$\log_4(2 \times 4 \times x \times 16) = 6$
$\log_4(128x) = 6$
$\Rightarrow 128x = 4^6$
$\Rightarrow 128x = 64 \times 4^3$
$\Rightarrow 2x = 4^3$
$\Rightarrow x = \frac{64}{2} = 32$

124. **(c)** We have,
$$S_n = \frac{1}{6 \cdot 11} + \frac{1}{11 \cdot 16} + \frac{1}{16 \cdot 21} + \cdots + n \text{ terms}$$
$$= \frac{1}{5}\left(\frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \cdots \frac{1}{5n-1} - \frac{1}{5n+6}\right)$$

$=\frac{1}{5}\left(\frac{1}{6}-\frac{1}{5n+6}\right)$

$=\frac{1}{5}\left[\frac{5n+6-6}{6(5n+6)}\right]$

$=\frac{1}{5}\left[\frac{5n}{6(5n+6)}\right]$

So, $6S_n=\frac{n}{(5n+6)}$

125. **(b)** Here, terms greater than 5 !, i.e.

$(5!)^2,(6!)^2,\ldots\ldots,(100!)^2$ is divisible by 100.

∴ For terms

$(5!)^2,(6!)^2,\ldots(100!)^2$ remainder is 0.

Now, consider $(1!)^2+(2!)^2+(3!)^2+(4!)^2$

$=1+4+36+576$

$=617$

When, 617 is divided by 100 , its remainder is 17 .

So, required remainder is 17 .

126. **(d)** Given that, $(p\wedge\sim r)\rightarrow(\sim p\vee q)$ is false

$\Rightarrow$ $(p\wedge\sim r)$ is true and $(\sim p\vee q)$ is false

$\Rightarrow$ (p is true and $\sim$ r is true)

and ($\sim$ p is false and q is false)

$\Rightarrow$ p is true, r is false and q is false.

127. **(a)** Since, α,β and γ are the roots of the equation

$x^3-8x+8=0$, then

$\alpha+\beta+\gamma=0,\alpha\beta+\beta\gamma+\gamma\alpha=-8$

$\alpha\beta\gamma=-8$ ……(i)

Therefore, $(\alpha+\beta+\gamma)^2=0$

$\Rightarrow\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)=0$

$\Rightarrow\alpha^2+\beta^2+\gamma^2=-\chi(-8)=16$

$\Sigma\alpha^2=16$

and $\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\gamma\alpha}=\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}=\frac{0}{-8}=0$

$\Sigma\frac{1}{\alpha\beta}=0$

128. **(c)** $1080=2^3\times3^3\times5$ (prime factorization)

$675=3^3\times5^2$

So, Greatest Common divisor $=3^3\times5=27\times5=135$

129. **(d)** $b\equiv c(\mathrm{mod}\,a)$

So,

$\frac{b+c}{a}$ and $\frac{b-c}{a}=\frac{(b+c)(b-c)}{a^2}$

$=\frac{b^2-c^2}{a^2}$ or $\frac{a^2}{b^2-c^2}$

Here, $b^2\equiv c^2(\mathrm{mod}\,a^2)$

130. **(c)** By taking option (c),

$a|(b+c)\not\Rightarrow a|b$ and $a\mid c$

e.g. $6|18\not\Rightarrow6|1$ and $6\mid17$

131. **(c)** We have, $2A+3B=\begin{bmatrix}2&-1&4\\3&2&5\end{bmatrix}$ ….(i)

and $A+2B=\begin{bmatrix}5&0&3\\1&6&2\end{bmatrix}$ ….(ii)

Multiply Eq. (ii) by 2, we get

$2A+4B=\begin{bmatrix}10&0&6\\2&12&4\end{bmatrix}$ ….(iii)

Now, subtracting Eq. (i) from Eq. (iii), we get

$$B = \begin{bmatrix} 10 & 0 & 6 \\ 2 & 12 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$$

132. **(a)** Given that,

$O(A) = 2 \times 3, O(B) = 3 \times 2,$

$O(C) = 3 \times 3$

$\Rightarrow O(A') = 3 \times 2, O(B') = 2 \times 3$

(a)$C(A + B')$

Now, $O(A + B') = 2 \times 3$

and $O(C) = 3 \times 3$

So, matrix $C(A + B')$ cannot be determined.

(b)$C(A + B')'$

$O(A + B') = 2 \times 3$

$\Rightarrow O(A + B')' = 3 \times 2$ and $O(C) = 3 \times 3$

Therefore, matrix $C(A + B')'$ can be determined.

(c)$O(BA) = 3 \times 3$ and $O(C) = 3 \times 3$

Therefore, matrix BAC can be determined.

(d)$CB + A'$

Now, order of CB = (order of C) (order of B)

= (order of C is 3×3)(order of B is 3×2)

= order of CB is 3×2

Since, $O(A') = 3 \times 2$

Therefore, matrix $CB + A'$ can be determined.

133. **(a)** We have, $A = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$

Now,

$$A = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}\begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

$$= \begin{bmatrix} 1-6 & -3-3k \\ 2+2k & -6+k^2 \end{bmatrix} = \begin{bmatrix} -5 & -3-3k \\ 2+2k & k^2-6 \end{bmatrix}$$

Now, $A^2 - 4A + 101 = A$

$$\Rightarrow \begin{bmatrix} -5 & -3-3k \\ 2+2k & k^2-6 \end{bmatrix} - 4\begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix} + 10\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -5-4+10 & -3-3k+12+0 \\ 2+2k-8+0 & k^2-6-4k+10 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & -3k+9 \\ 2k-6 & k^2-4k+4 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ 2 & k \end{bmatrix}$$

On comparing both sides, we get

$k^2 - 4k + 4 = k$

$\Rightarrow k^2 - 5k + 4 = 0$

$\Rightarrow k^2 - 4k - k + 4 = 0$

$\Rightarrow k(k-4) - 1(k-4) = 0$

$\Rightarrow (k-4)(k-1) = 0$

$\Rightarrow k = 4,1$

134. **(a)** $\begin{vmatrix} x+y & y+z & z+x \\ x & y & z \\ x-y & y-z & z-x \end{vmatrix}$

$$= \begin{vmatrix} 2(x+y+z) & y+z & z+x \\ (x+y+z) & y & z \\ 0 & y-z & z-x \end{vmatrix}$$

$[\because C_1 \to C_1 + C_2 + C_3]$

$$= (x+y+z)\begin{vmatrix} 2 & y+z & z+x \\ 1 & y & z \\ 0 & y-z & z-x \end{vmatrix}$$

$$= (x+y+z)\begin{vmatrix} 2 & y+z & z+x \\ 0 & y-z & z-x \\ 0 & y-z & z-x \end{vmatrix}$$

$[\because R_2 \to 2R_2 - R_1]$

$= (x + y + z) \times 0 = 0$
$[\because R_2$ and R_3 are identical $]$

135. **(d)** If $a * b = b * a$, then the operation is commutative.
If $(a * b) * c = a * (b * c)$, then the operation is associative.
By option (d),
$a * b = a + b + ab$
$b * a = b + a + ba = a + b + ab$
Here, $a * b = b * a$, so the operation is commutative.
$(a * b) * c = (a + b + ab) * c$
$= (a + b + ab) + c + (a + b + ab)c$
$= a + b + ab + c + ac + bc + abc$
$= a + b + c + ab + ac + bc + abc$
$a * (b * c) = a * (b + c + bc)$
$= a + b + c + bc + a(b + c + bc)$
$= a + b + c + bc + ab + ac + abc$
Here, $(a * b) * c = a * (b * c)$.
So, the operation is associative.

136. **(c)** Given that, $G = \{1,5,7,11\}$ is a group under multiplication module 12 .
$\because 7^{-1} = 7 \quad [\because 7^{-1} \otimes_{12} 7 = 1]$
Now, $7^{-1} \otimes_{12} (x \otimes_{12} 11) = 5$
$\Rightarrow 7 \otimes_{12} (11 \otimes_{12} x) = 5$
$\Rightarrow (7 \otimes_{12} 11) \otimes_{12} x = 5$
$\{7 \otimes_{12} 11 =$ remainder after dividing 77 from 12$\}$
$\Rightarrow 5 \otimes_{12} x = 5$
$\Rightarrow \; x = 1$

137. **(b)** In additive subgroup ' o ' is the identity.
So, required set $(N, +)$ which is not a subgroup.

138. **(d)** We have, $p = \hat{i} + \hat{j}, q = 4\hat{k} - \hat{j}$ and $r = \hat{i} + \hat{k}$
So, $3p + q - 2r = 3\hat{i} + 3\hat{j} + 4\hat{k} - \hat{j} - 2\hat{i} - = \hat{i} + 2\hat{j} + 2\hat{k}$
Now, required unit vector $= \dfrac{\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1 + 4 + 4}}$
$= \dfrac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$

139. **(a)** We have, $|a| = 3\sqrt{3}, |b| = 4$ and $|a + b| = \sqrt{7}$
Now, $|a + b|^2 = |a|^2 + |b|^2 + 2|a||b|\cos\theta$
$\Rightarrow (\sqrt{7})^2 = (3\sqrt{3})^2 + 16 + 2(3\sqrt{3})(4)\cos\theta$
$\Rightarrow 7 = 27 + 16 + 24\sqrt{3}\cos\theta$
$\Rightarrow 24\sqrt{3}\cos\theta = -36$
$\Rightarrow \; \cos\theta = -\dfrac{36}{24\sqrt{3}}$
$\Rightarrow \; \cos\theta = -\dfrac{\sqrt{3}}{2}$
$\Rightarrow \; \theta = 150^\circ$

140. **(b)** Given, a is perpendicular to b and c.
Thus, a is perpendicular to the plane of b and c.
Now, cross product of b and c will give a vector perpendicular to plane of b and c. This vector will be parallel to a.
Now, cross product of two parallel is zero vector.
Thus, $a \times (b \times c) = 0$

141. **(c)** We know that, if a and b are two adjacent sides of a parallelogram, then
Area $= |a \times b| = 15$ (given) $\qquad$ ….(i)
If the sides are $(3a + 2b)$ and $(a + 3b)$

Then,area of parallelogram

$= 3|(3a + 2b) \times (a + 3b)|$

$= |3a \times a + 9a \times b + 2b \times a + 6b \times a|$

$= |0 + 9a \times b - 2a \times b + 0|$

$= |7(a \times b)|$

$= 7|a \times b|$

$= 7 \times 15 = 105$ sq units

142. **(c)** The locus of the point which such that the ratio of its distance from two fixed point in the plane is always a constant $k(k < 1)$ is an ellipse.

143. **(d)** Given lines are concurrent, then there coefficient determinant is zero.

So, $\begin{vmatrix} 1 & 3 & -9 \\ 4 & b & -2 \\ 2 & -1 & -4 \end{vmatrix} = 0$

$\Rightarrow 1(-4b - 2) - 3(-16 + 4) - 9(-4 - 2b) = 0$

$\Rightarrow -4b - 2 + 36 + 36 + 18b = 0$

$\Rightarrow 14b + 70 = 0$

$\Rightarrow b = -5$

144. **(c)** We know that the angle between the two lines $ax^2 + 2hxy + by^2 = 0$ is given by

$\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$

$\therefore a = 90°$

$\therefore a + b = 0$

145. **(b)** Since, the equation of tangents $x - y - 2 = 0$ and $x - y + 2 = 0$ are parallel.

Therefore, distance between them = Diameter of the circle

$= \left|\frac{2 - (-2)}{\sqrt{1^2 + 1^2}}\right|$

$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$

Radius $= \frac{1}{2}(2\sqrt{2}) = \sqrt{2}$

Now, required equation of circle is

$(x - 0)^2 + (y - 0)^2 = (\sqrt{2})^2$

$x^2 + y^2 = 2$

146. **(b)** Let point $P(x_1, y_1)$ be any point on the circle, therefore it satisfy the circle

$(x_1 - 3)^2 + (y_1 + 2)^2 = 5r^2$ ....(i)

The length of the tangent drawn from point $p(x_1, y_1)$ to the circle $(x - 3)^2 + (y + 2)^2 = r^2$ is

$\sqrt{(x_1 - 3)^2 + (y_1 + 2)^2 - r^2} = \sqrt{5r^2 - r^2}$

$= \sqrt{4r^2} = 2r$

$\Rightarrow 2r = 16$

$\Rightarrow r = 8$

So, area between two circles

$= \pi(5r^2) - \pi r^2$

$= 5\pi r^2 - \pi r^2$

$= 4\pi r^2 = 4\pi \times 64$

$= 256\pi$ sq units

147. **(d)** Given equation of circles are

$ax^2 + ay^2 + 2g_1x + 2f_1y + c_1 = 0$

and $bx^2 + by^2 + 2g_2x + 2f_2y + c_2 = 0$

It can be rewritten as,

$x^2 + y^2 + \frac{2g_1}{a}x + \frac{2f_1}{a}y + \frac{c_1}{a} = 0$

and $x^2+y^2+\frac{2g_2}{b}x+\frac{2f_2}{b}y+\frac{c_2}{b}=0$

So, centres of circle $\left(-\frac{g_1}{a},-\frac{f_1}{a}\right)$ and $\left(-\frac{g_2}{b},-\frac{f_2}{b}\right)$ respectively.

We know that, if two circles cut orthogonally, then

$2(g_1g_2+f_1f_2)=c_1+c_2$

$\Rightarrow 2\left(\frac{g_1g_2}{ab}+\frac{f_1f_2}{ab}\right)=\frac{c_1}{a}+\frac{c_2}{b}$

$2(g_1g_2+f_1f_2)=bc_1+ac_2$

148. **(d)** Let $S_1\equiv x^2+y^2-6x-12y+37=0$

and $S_2\equiv x^2+y^2-6y+7=0$

The equation of common tangent of the two circles is

$S_1-S_2=0$

$\Rightarrow x^2+y^2-6x-12y+37-(x^2+y^2-6y+7)=0$

$\Rightarrow -6x+6y+30=0$

$\Rightarrow x-y-5=0$

149. **(d)** Given, vertex of the parabola $(h,k)=(-1,1)$ and its focus is

$(a+h,k)=(2,1)$ or $a+h=2\Rightarrow a=3$

We know that as the y-coordinate of vertex and focus are same,therefore axis of parabola is parallel to X-axis.

Thus, equation of parabola is

$(y-k)^2=4a(x-h)$

$(y-1)^2=4\times 3(x+1)$

$(y-1)^2=12x+12$

$y^2+1-2y=12x+12$

$y^2-12x-2y-11=0$

150. **(a)** Let the equation of line by $y=mx+C$

Since, this is the tangent to the circle $x^2+y^2=5$

So,$C=\pm a\sqrt{1+m^2}$

$=\pm\sqrt{5}\sqrt{1+m^2}$ ….(i)

Also, the above line is tangent to the parabola

$y^2=40x$

So,$c=\frac{a}{m}=\frac{10}{m}$ ….(ii)

By Eqs. (i) and (ii), we get

$\frac{10}{m}=\pm\sqrt{5}\sqrt{1+m^2}$

$m^4+m^2-20=0$

$(m^2+5)(m^2-4)=0$

$m^2=4(m^2\neq -5)$

Here, $m=\pm 2\Rightarrow c=\pm 5$

$y=\pm 2x\pm 5$

151. **(a)** Given, $x=4(1+\cos\theta)$

and $y=3(1+\sin\theta)$

$\Rightarrow \cos\theta=\frac{x}{4}-1=\frac{x-4}{4}$

and $\sin\theta=\frac{y}{3}-1=\frac{y-3}{3}$

$\because \sin^2\theta+\cos^2\theta=1$

$\because \frac{(x-4)^2}{16}+\frac{(y-3)^2}{9}=1$

152. **(d)** Equation of hyperbola is

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Distance between foci $=2ae$

and distance between directrices $=\frac{2a}{e}$

According to question, we have

$$\frac{2ae}{2a/e}=\frac{3}{2}$$
$$\Rightarrow e^2=\frac{3}{2}$$
$$\because b^2=a^2(e^2-1)$$
$$\Rightarrow \frac{b^2}{a^2}=\frac{3}{2}-1=\frac{1}{2}$$
$$\Rightarrow \frac{b}{a}=\frac{1}{\sqrt{2}}$$
$$\Rightarrow a:b=\sqrt{2}:1$$

153. **(a)** Equation of ellipse is

$$\frac{x^2}{25}+\frac{y^2}{16}=1, a>b$$

and equation of hyperbola is

$$\frac{x^2}{25}-\frac{y^2}{16}=1, a>b$$

Let e and e′ be the eccentricities of the ellipse and hyperbola.

So,

$$e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{25-16}{25}}=\frac{3}{5}$$

and $e'=\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{25+16}{25}}=\frac{\sqrt{41}}{5}$

(i) Centre of ellipse $(0,0)$ and centre of hyperbola is $(0,0)$

(ii) Foci of ellipse are $(\pm ae,0)$ or $(\pm 3,0)$. Foci of hyperbola are $(\pm ae', o)$ or $(\pm\sqrt{41},0)$.

(iii) Direction of ellipse are

$$x=\pm\frac{a}{e}\Rightarrow x=\pm\frac{25}{3}$$

and directrices of hyperbola are $x=\pm\frac{a}{e}$

$$\Rightarrow\ x=\pm\frac{25}{\sqrt{41}}$$

(iv) Vertices of ellipse are $(\pm a,0)$ or $(\pm 5,0)$. Vertices of hyperbola are $(\pm a,0)$ or $(\pm 5,0)$.

From the above discussions, their are common is centre and vertices.

154. **(d)** We have, $\sec\theta=m$ and $\tan\theta=n$

Now, $\frac{1}{m}\left[(m+n)+\frac{1}{(m+n)}\right]$

$$=\frac{1}{\sec\theta}\left[(\sec\theta+\tan\theta)+\frac{1}{(\sec\theta+\tan\theta)}\right]$$
$$=\frac{1}{\sec\theta}\left[\frac{(\sec\theta+\tan\theta)^2+1}{(\sec\theta+\tan\theta)}\right]$$
$$=\frac{1}{\sec\theta}\left[\frac{\sec^2\theta+\tan^2\theta+2\sec\theta\tan\theta+1}{(\sec^2\theta+\tan\theta)}\right]$$
$$=\frac{1}{\sec\theta}\left[\frac{\sec^2\theta+\sec^2\theta-1+2\sec\theta+\tan\theta+1}{\sec\theta+\tan\theta}\right]$$
$$=\frac{1}{\sec\theta}\left[\frac{2\sec^2\theta+2\sec\theta\tan\theta}{\sec\theta+\tan\theta}\right]$$
$$=\frac{1}{\sec\theta}\left[\frac{2\sec\theta(\sec\theta+\tan\theta)}{(\sec\theta+\tan\theta)}\right]$$
$$=\frac{1}{\sec\theta}(2\sec\theta)=2$$

155. **(b)** $\frac{\sin 85^\circ-\sin 35^\circ}{\cos 65^\circ}$

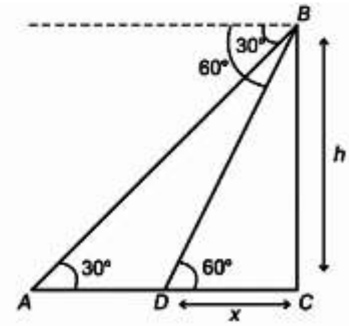
$$= \frac{2\cos\left(\frac{85^\circ+35^\circ}{2}\right)\sin\left(\frac{85^\circ-35^\circ}{2}\right)}{\cos 65^\circ}$$

$$= \frac{2\cos 60^\circ \sin 25^\circ}{\cos 65^\circ}$$

$$= \frac{2\times\frac{1}{2}\times\sin 25^\circ}{\cos(90^\circ-25^\circ)}$$

$$= \frac{\sin 25^\circ}{\sin 25^\circ} = 1$$

156. **(c)** Let the distance of two consecutive stones are ($x, x+1$).



In $\triangle$ BCD, we have

$$\tan 60^\circ = \frac{h}{x}$$

$\Rightarrow x = \frac{h}{\sqrt{3}}$ ….(i)

In $\triangle$ ABC, we have

$$\tan 30^\circ = \frac{h}{x+1}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x+1}$$

$\Rightarrow \frac{h}{\sqrt{3}} + 1 = \sqrt{3}h$ [by Eq.(i)]

$$\Rightarrow h\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right) = 1$$

$$\Rightarrow h\cdot\left(\frac{2}{\sqrt{3}}\right) = 1 \Rightarrow h = \frac{\sqrt{3}}{2}$$

157. **(d)** Let the angles of triangle are $3\theta, 4\theta, 5\theta$.

As we know that,

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow 3\theta + 4\theta + 5\theta = 180^\circ$

$\Rightarrow 12\theta = 180^\circ \Rightarrow \theta = 15^\circ$

So, angle are $45^\circ, 60^\circ, 75^\circ$.

Now, $\sin A = \sin 45^\circ = \frac{1}{\sqrt{2}}$

$$\sin B = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\sin C = \sin 75^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

So, $a : b : c = \sin A : \sin B : \sin C$

$$= \frac{1}{\sqrt{2}} : \frac{\sqrt{3}}{2} : \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$= 2 : \sqrt{6} : \sqrt{3}+1$

158. **(b)** Given that, $\cos^{-1}x = \alpha, (0 < x < 1)$ ....(i)

$\Rightarrow x = \cos\alpha$

Thus, $\sin^{-1}\left(2x\sqrt{1-x^2}\right) + \sec^{-1}\left(\frac{1}{2x^2-1}\right) = \frac{2\pi}{3}$

$\Rightarrow \sin^{-1}\left(2\cos\alpha\sqrt{1-\cos^2\alpha}\right)$

$+\sec^{-1}\left(\frac{1}{2\cos^2\alpha-1}\right)=\frac{2\pi}{3}$

$\Rightarrow \sin^{-1}(\sin 2\alpha)+\sec^{-1}(\sec 2\alpha)=\frac{2\pi}{3}$

$\Rightarrow 2\alpha+2\alpha=\frac{2\pi}{3}\Rightarrow \alpha=\frac{\pi}{6}$

Now, $x=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}\Rightarrow 2x=\sqrt{3}$

Therefore, $\tan^{-1}(2x)=\tan^{-1}\sqrt{3}=\frac{\pi}{3}$

159. **(a)** We have,

$\tan^{-1}\left(\frac{a}{b}\right)+\tan^{-1}\left(\frac{a+b}{a-b}\right)$

$=\tan^{-1}\left[\frac{\frac{a}{b}+\frac{a+b}{a-b}}{1-\left(\frac{a}{b}\right)\left(\frac{a+b}{a-b}\right)}\right]$

$=\tan^{-1}\left[\frac{\frac{a^2-ab+ab+b^2}{b(a-b)}}{\frac{b(a-b)-a(a+b)}{b(a-b)}}\right]$

$=\tan^{-1}\left[\frac{a^2+b^2}{ab-b^2-a^2-ab}\right]$

$=\tan^{-1}\left[\frac{a^2+b^2}{-(a^2+b^2)}\right]=\tan^{-1}(-1)$

It does not depends neither a nor b.

160. **(c)** By option (c),

$\operatorname{cosec}\theta\cdot\sec\theta=1$

$\sin\theta\cos\theta=1$

$2\sin\theta\cos\theta=2$

$\sin 2\theta=2$

As we know that range of $\sin x$ is $[-1,1]$.

Hence, this equation has no solution.

161. **(b)**Let $z=\frac{(-\sqrt{3}+3i)(1-i)}{(3+\sqrt{3}i)(i)(\sqrt{3}+\sqrt{3}i)}$

$=\frac{\sqrt{3}(-1+\sqrt{3}i)(1-i)}{(\sqrt{3})^2(\sqrt{3}+i)(1+i)(i)}$

$=\frac{(-1+\sqrt{3}i)(1-i)}{\sqrt{3}(\sqrt{3}i+i^2)(1+i)}=\frac{(-1+\sqrt{3}i)(1-i)}{\sqrt{3}(-1+\sqrt{3}i)(1+i)}$

$=\frac{1-i}{(\sqrt{3})1+i}\times\frac{(1-i)}{(1-i)}=\frac{(1-i)^2}{(\sqrt{3})(1+1)}$

$=\frac{1+i^2-2i}{2\sqrt{3}}=\frac{-2i}{2\sqrt{3}}=-\frac{i}{\sqrt{3}}$

$=$ Purely imaginary

162. **(a)** We have,

$2x=-1+\sqrt{3}i\Rightarrow x=\frac{-1+\sqrt{3}i}{2}=\omega$

So, $(1-x^2+x)^6-(1-x+x^2)^6$

$=(1-\omega^2+\omega)^6-(1-\omega-\omega^2)^6$

$=(-2\omega^2)^6-(-\omega-\omega)^6=2^6\omega^{12}-2^6\omega^6$

$=2^6-2^6=0 \qquad [\because \omega^3=1]$

163. **(c)** Let $z=(1+i\sqrt{3})^8$

$$=\left(2\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)\right)^8$$

$$=[2(\cos 60^\circ+i\sin 60^\circ)]^8=\left(2^8e^{i\pi/3}\right)^8$$

$$=2^8\cdot e^{\frac{8\pi}{3}i}=2^8\cdot e^{\left(2\pi+\frac{2\pi}{3}\right)i}=2^8\cdot e^{\frac{2\pi}{3}i}$$

So, modulus $=2^8=256$

and amplitude $=\frac{2\pi}{3}$

164. **(d)** $\lim_{x\to 0}\frac{5^x-5^{-x}}{2x}=\lim_{x\to 0}\frac{5^x\log 5+5^{-x}\log 5}{2}$

(Applying L' Hospital rule)

$$=\frac{\log 5+\log 5}{2}=\log 5$$

165. **(c)** If a function f(x is continuous at $x=a$).

Then, it may or may not be differentiable at $x=a$.

166. **(a)** Given, $y=\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\cdots\infty$

This is an infinite GP with $a=1$

and $r=\frac{1}{x}$

$$S_\infty=\frac{a}{1-r}$$

$$\because y=\frac{1}{1-\frac{1}{x}}$$

$$y=\frac{x}{x-1}$$

Now, $\frac{dy}{dx}=\frac{(x-1)-x}{(x-1)^2}=\frac{x-1-x}{(x-1)^2}=\frac{-1}{(x-1)^2}$

$$=-\frac{1}{\left(\frac{x^2}{y^2}\right)}=-\frac{y^2}{x^2}$$

167. **(c)** $f\{g(x)\}=x^3-\frac{1}{x^3}$

$$f\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+1\right)$$

$$=\left(x-\frac{1}{x}\right)\left\{\left(x-\frac{1}{x}\right)^2+2+1\right\}$$

$$f\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)\left\{\left(x-\frac{1}{x}\right)^2+3\right\}$$

Replace $x-\frac{1}{x}$ by x,

$f(x)=(x)(x^2+3)=x^3+3x$

$f'(x)=3x^2+3$

168. **(b)** Let $u=a^{\sec x}\Rightarrow v=a^{\tan x}$

$\log u=\sec x\log a$

$\frac{du}{dx}=u\log a\sec x\tan x$

$=a^{\tan x}\log a\sec x\tan x$

And

$\log v=\tan x\log a$

$\frac{dv}{dx}=v\log a\sec^2x$

$=a^{\tan x}\log a\sec^2 x$

$$\frac{du}{dv}=\frac{\left(\frac{du}{dx}\right)}{\frac{dv}{dx}}=\frac{a^{\sec x}\log a\sec x\tan x}{a^{\tan x}\log a\sec^2 x}$$

$$=\frac{a^{\sec x}\sin x}{a^{\tan x}}=\sin x a^{\sec x-\tan x}$$

169. **(c)** Given that, $\sin(x+y)+\cos(x+y)=\log(x+y)$

On differentiating w.r.t. x,

$$\cos(x+y)\cdot\left(1+\frac{dy}{dx}\right)-\sin(x+y)\left(1+\frac{dy}{dx}\right)$$

$$=\frac{1}{(x+y)}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow\left(1+\frac{dy}{dx}\right)\left\{\cos(x+y)-\sin(x+y)-\frac{1}{(x+y)}\right\}=0$$

$$\Rightarrow\ 1+\frac{dy}{dx}=0$$

Again differentiating w.r.t. ' x '

$$0+\frac{d^2y}{dx^2}=0\Rightarrow\frac{d^2y}{dx^2}=0$$

170. **(a)** We have,

$g(x)=[f(x)]^2+[f'(x)]^2$

Differentiate the function $g(x)$

$g'(x)=2f(x)f'(x)+2f'(x)f''(x)$

Use chain rule,

$2f'(x)[f(x)+f''(x)]=2f'(x)(0)=0$

Hence, $g(x)$ is a constant function

$\Rightarrow\ g(x)=c$, constant

But, $g(3)=8$, so $g(x)=8$

For all real x.

Hence, $g(8)=8$

171. **(c)** $y=2x^3+ax^2+bx+c$ ....(i)

Since, it passes through $(0,0)$,

$0=2(0)+a(0)+b(0)+c$

$c=0$ ....(ii)

$\frac{dy}{dx}=6x^2+2ax+b$

Since, tangents at $x=-1$ and $x=2$ are parallel to X-axis.

$\therefore\frac{dy}{dx}=0$

At $x=-1$

$6(-1)^2+2a(-1)+b=0$

$6-2a+b=0$ ....(iii)

At $x=-2$

So, $6(2)^2+2a(2)+b=0$

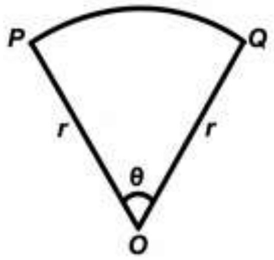
$24+4a+b=0$ ....(iv)

By solving Eqs. (iii) and (iv), we get

$a=-3, b=-12$

Hence, $a=-3, b=-12, c=0$

172. **(b)**



Perimeter of sector $= 2r + r\theta$

$\Rightarrow 60 = 2r + r\theta$ (given)

$\Rightarrow \theta = \dfrac{60 - 2r}{r}$

Now, area of sector,

$(A) = \dfrac{\pi r^2\theta}{360°} = \dfrac{\pi r^2(60 - 2r)}{360r}$

$= \dfrac{\pi r}{180}(30 - r)$

$\dfrac{dA}{dr} = \dfrac{\pi}{180}(30 - 2r)$

For maximum area $\frac{dA}{dr} = 0$

$\Rightarrow 30 - 2r = 0$

$\Rightarrow 2r = 30 \Rightarrow r = 15$

and $\dfrac{d^2A}{dr^2} = \dfrac{\pi}{180°}(-2) = -\dfrac{\pi}{90} < 0$

Hence, it is maximum at $r = 15$ m

173. **(a)** Given,equation of curve is

$y = x^2 - x + 4$

Slope of tangent at $P(1,4)$ is

$\dfrac{dy}{dx} = 2x - 1 \Rightarrow \left(\dfrac{dy}{dx}\right)_{(1,4)} = 2 - 1 = 1$

So, equation of tangent is

$y - 4 = 1(x\text{-}1)$

$\Rightarrow y - x{=}3$ ....(i)

and equation of normal at point $P(1,4)$ is

$y - 4 = -1(x - 1)$

$x + y = 5$ ....(ii)

Since, the tangent cuts X-axis at A.

Therefore, coordinates of A are $(-3,0)$ and the normal cuts X-axis at B and coordinates of B are (5,0).

Therefore, area of $\triangle$ PAB

$= \dfrac{1}{2}\begin{vmatrix} 1 & 4 & 1 \\ -3 & 0 & 1 \\ 5 & 0 & 1 \end{vmatrix}$

$= \dfrac{1}{2}[1(0) - 4(-3 - 5) + 1(0)]$

$= \dfrac{1}{2}|32| = 16$ sq. units

174. **(d)** $\int \frac{x^3+3x^2+3x+1}{(x+1)^5}dx$

$= \displaystyle\int \frac{(x+1)^3}{(x+1)^5}dx = \int \frac{dx}{(x+1)^2}$

$= -\dfrac{1}{x+1} + c$

175. **(c)** Let $I = \int \frac{\operatorname{cosec} x}{\cos^2\left(1+\log\tan\frac{x}{2}\right)}dx$

Put $1 + \log\tan\frac{x}{2} = t$

$$\Rightarrow \frac{1}{\tan\frac{x}{2}}\cdot\sec^2\frac{x}{2}\cdot\frac{dx}{2} = dt$$

$$\Rightarrow \frac{dx}{2\sin\frac{x}{2}\cos\frac{x}{2}} = dt \Rightarrow \frac{dx}{\sin x} = dt$$

$\Rightarrow \operatorname{cosec}x dx = dt$

So, $I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t dt = \tan t + c$

$= \tan\left(1 + \log\tan\frac{x}{2}\right) + c$

176. **(b)** Let $I = \int \frac{dx}{x\sqrt{(x^3)^2-16}}$

Put $x^3 = t \Rightarrow 3x^2dx = dt$

So, $I = \frac{1}{3}\int \frac{dt}{x^3\sqrt{(x^3)^2-16}}$

$$= \frac{1}{3}\int \frac{dt}{t(\sqrt{t^2-16}}$$

$$= \frac{1}{3\times 4}\sec^{-1}\left(\frac{t}{4}\right) + c$$

$$= \frac{1}{12}\sec^{-1}\left(\frac{x^3}{4}\right) + c$$

177. **(d)** $I_1 = \int_0^{\pi/2} x\sin x dx$

$$= \left[-x\cos x + \int \cos x\right]_0^{\pi/2}$$

$= [-x\cos x + \sin x]_0^{\pi/2}$

$$= 0 + \sin\frac{\pi}{2} - 0 = 1$$

Similarly, $I_2 = \int_0^{\pi/2} x\cos x dx$

$$= \left[x\sin x - \int \sin x\right]_0^{\pi/2}$$

$= [x\sin x + \cos x]_0^{\pi/2}$

$$= \frac{\pi}{2}\sin\frac{\pi}{2} - 1 = \frac{\pi}{2} - 1$$

Hence, $I_1 + I_2 = 1 + \frac{\pi}{2} - 1 = \frac{\pi}{2}$

178. **(b)** Given that, $f(x) = 4x^2 - 3x + 1$

$$g(x) = \frac{f(-x) - f(x)}{x^2+3}$$

Therefore,

$$g(x) = \frac{(4x^2+3x+1) - (4x^2-3x+1)}{x^2+3} = \frac{6x}{x^2+3}$$

Now, $g(-x) = -\frac{6x}{x^2+3} = g(x)$

which is an odd function.

Thus, $\int_{-2}^{2} g(x)dx = 0$

179. **(a)** Required area

$$= \int_0^2 [(x+2) - (x^2 - x + 2)]dx$$

$$= \int_0^2 [x + 2 - x^2 + x - 2]dx = \int_0^2 (-x^2 + 2x)dx$$

$$= \left[-\frac{x^3}{3} + x^2\right]_0^2$$

$= -\frac{8}{3} + 4 - (0) = \frac{-8+12}{3} = \frac{4}{3}$

180. **(d)** Given, differential equation is

$e^{-x}(y+1)dy + (\cos^2 x - \sin 2x)ydx = 0$

$\Rightarrow \left(\frac{y+1}{y}\right)dy + e^x(\cos^2 x - \sin 2x)dx = 0$

$\Rightarrow \left(1 + \frac{1}{y}\right)dy + (e^x\cos^2 x - e^x \sin 2x)dx = 0$

$\Rightarrow d(y + \log y) + d(e^x \cos^2 x) = 0$

On integrating above equation, we get

$y + \log y + e^x \cos^2 x = c$

At $y(0) = 1$

$\Rightarrow 1 + 0 + e^0 \cdot 1 = c \Rightarrow c = 2$

Hence, required solution is

$y + \log y + e^x \cos^2 x = 2$