PHYSICS

1. **(c)** The angle of incidence $= i$

Let r be the angle of refraction.

According to given situation, reflected rays and refracted rays are perpendicular to each other.

i.e. $i + r = 90^\circ$ ....(i)

$\therefore$ According to Snell's law,

$$\mu = \frac{\sin i}{\sin r}$$

$$= \frac{\sin i}{\sin(90 - i)}$$

$$= \frac{\sin i}{\cos i}$$

$\mu = \tan i$ ....(ii)

If i_c be the critical angle for the pair of media, then

$\mu = \frac{1}{\sin i_c}$ ....(iii)

From Eqs. (ii) and (iii), we get

$$\tan i = \frac{1}{\sin i_C}$$

$$\Rightarrow \sin i_C = \cot i$$

$$\Rightarrow i_C = \sin^{-1}(\cot i)$$

2. **(b)** Refractive index of water $= n$

Since, fish looks the bird from water (denser medium) to air (rarer medium).

Hence, normal shift $d = (n - 1)h$

But according to question, real depth $h = y$

$\therefore\ d = (a - 1)y$ ($\because$ height of bird $= y$) ....(i)

The bird's height from the surface of the water is equal to the summation of the normal shift and height of the bird from the surface.

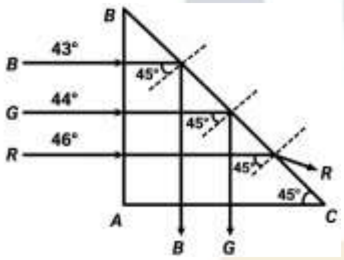
i.e. $d' = y + d = y + (n - 1)y$ [From Eq. (i)]

$d' = ny$ ....(ii)

Total actual distance between bird and fish

$= x + d' = x + ny$

3. **(a)** The given situation is shown below



Since, critical angles for blue colour and green colour are less than angle of incidence, hence total internal

4. **(a)** Let f_1 and f_2 are focal lengths of convex and concave lens respectively, when they are kept in contact co-axially, then focal length F of the combination is given as\

$$\Rightarrow \frac{1}{F} = \frac{1}{f_1} + \frac{1}{(-f_2)} \quad \left[\begin{array}{c}\text{Focal length of concave} \\ \text{lens is negative.}\end{array}\right]$$

$$\Rightarrow \frac{1}{F} = \frac{f_2 - f_1}{f_1 f_2}$$

$\Rightarrow F$

$= \frac{f_1 f_2}{f_2 - f_1}$ ....(i)

When both lenses are kept at a distance d, then their combined focal length F' is given as

$$\frac{1}{F'} = \frac{1}{f_1} + \frac{1}{(-f_2)} - \frac{d}{f_1(-f_2)}$$

$$\Rightarrow \frac{1}{F'} = \frac{1}{f_1} - \frac{1}{f_2} + \frac{d}{f_1 f_2}$$

$\Rightarrow \frac{1}{F'} = \frac{f_2 - f_1 + d}{f_1 f_2}$

$\Rightarrow F' = \frac{f_1 f_2}{f_2 - f_1 + d}$ ....(ii)

From Eqs. (i) and (ii), it is clear that $F' < F$

i.e. focal length of combination will decrease.

5. **(a)** Since, convex lens made of 3 layers of different materials have only one focal length. Thus, for a given object, there is only one image.

6. **(c)** Speed of light in terms of permittivity of free space ε_0 and permeability of free space μ_0 is given as

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

7. **(d)** When a third slit is made in between the double slits in YDSE, then contrast between bright and dark fringes is reduced. It is because in the case of double slits, light passes only two slits. Now, the same light is passing through the three slits which results in reducing the contrast of bright fringes as well as dark fringes.

8. **(b)** In double slit experiment, for possible interference maxima on the screen is given as

$d\sin\theta = n\lambda$

Given, slit width, $d = 4\lambda$

$\therefore 4\lambda\sin\theta = n\lambda$

$\Rightarrow 4\sin\theta = n$

Since, maximum value of $\sin\theta = 1$

$\therefore$ For maximum number of possible interference maxima,

$4 \cdot 1 = n$

$\Rightarrow n = 4$

i.e., $n = 0,1,2,3$

$\therefore$ Maximum number of possible maxima will be 7

i.e. $0, \pm 1, \pm 2 \pm 3$

9. **(c)** Given, wavelength of used light,

$\lambda = 400$ nm

For minima in Fraunhofer diffraction,

$d\sin\theta = n\lambda$

For first minima $n = 1$, where $\theta = 30°$

$\therefore d\sin 30° = 1 \times 400$

$\Rightarrow d = 800$ nm

For maxima in diffraction,

$d\sin\theta = (2n+1)\frac{\lambda}{2}$

For first maxima $n = 0$

$\therefore d\sin\theta = (0+1)\frac{\lambda}{2}$

$\Rightarrow d\sin\theta = \frac{\lambda}{2}$

$\Rightarrow 800\sin\theta = \frac{400}{2}$

$\Rightarrow \sin\theta = \frac{1}{4}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{4}\right)$

10. **(d)** Greater is the wavelength of wave higher will be its degree of diffraction, since radiowave has maximum wavelength, hence maximum diffraction takes place in a given slit for radio waves.

11. **(a)** When an unpolarised beam of intensity I_0 falls on a polaroid, then after polarisation, the intensity of the emergent beam of light becomes $\frac{I_0}{2}$.

12. **(b)** Materials which have different absorption for perpendicular incident plane for light are called dichoric material crystal. Tourmaline is one example of such type of crystal.

13. **(c)** Given, $Q_1 = 12\mu\ F = 12 \times 10^{-6}\ F$

$Q_2 = -8\mu\ F$

$= -8 \times 10^{-6}\ F$

Initial value of electrostatic force of attraction between both spheres,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{12 \times 10^{-6} \times 8 \times 10^{-6}}{r^2}$$

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{96\times10^{-12}}{r^2} \qquad \ldots\text{(i)}$$

When both charged spheres are kept in contact, then new charges on the spheres will be

$$Q_1' = Q_2' = \frac{Q_1 + Q_2}{2}$$

$$= \frac{12 + (-8)}{2} = 2\mu\ F = 2 \times 10^{-6}\ F$$

$\therefore$ Final value of electrostatic force between the spheres,

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1' \times Q_2'}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2 \times 10^{-6} \times 2 \times 10^{-6}}{r^2}$$

$$F_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4\times10^{-12}}{r^2} \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\therefore \frac{F_1}{F_2} = \frac{\frac{1}{4\pi\varepsilon_0}\cdot\frac{96 \times 10^{-12}}{r^2}}{\frac{1}{4\pi\varepsilon_0}\cdot\frac{4 \times 10^{-12}}{r^2}}$$

$$\Rightarrow \frac{F_1}{F_2} = \frac{24}{1}$$

$$\Rightarrow F_1 : F_2 = 24 : 1$$

14. **(a)** According to given situation, potential at the surface of small conducting sphere of radius r,

$$V_r = \frac{kq}{r} + \frac{kQ}{R}$$

Potential at the surface of bigger conducting sphere of radius R,

$$V_R = \frac{kq}{R} + \frac{kQ}{R}$$

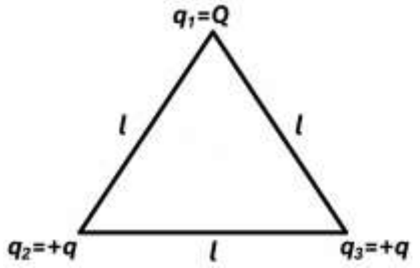
$\therefore$ Potential difference between the spheres is given as

$$V_r - V_R = \frac{kq}{r} + \frac{kQ}{R} - \frac{kq}{R} - \frac{kQ}{R}$$

$$= \frac{kq}{r} - \frac{kq}{R} = k\left(\frac{q}{r} - \frac{q}{R}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r} - \frac{q}{R}\right]$$

15. **(a)** The given situation is shown below



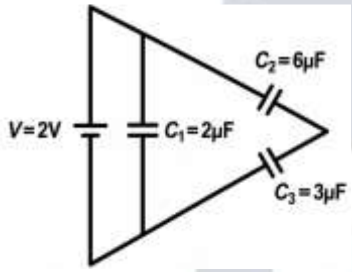
Net potential energy of the system = 0

i.e. $U_{net} = 0$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{l}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2q_3}{l}+\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_3q_1}{l}=0$$
$\Rightarrow q_1q_2+q_2q_3+q_3q_1=0$
$\Rightarrow\ Q\cdot q+q\cdot q+q\cdot Q=0$
$\Rightarrow 2Qq+q^2=0$
$\Rightarrow\ q(2Q+q)=0$
Since, q=0
$\therefore 2Q+q=0$
$\Rightarrow\ Q=\frac{-q}{2}$

16. **(d)** We know that, in series combination of capacitors, net potential difference is equal to sum of the individual potential difference of each capacitor
i.e., $V=V_1+V_2+V_3\ldots.[V_1=V_2=V_3=200\ \mathrm{V}]$
$\Rightarrow\ 600=n\times 200$
$\Rightarrow\ n=3$
So, there should be 3 capacitors in series to obtain the required potential difference. The equivalent capacitance of 3 capacitors in series is given as
$$\frac{1}{C_{eq}}=\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$$
$\Rightarrow C_{eq}\ =2\mu\ F$
Now, we require 18μ F capacitance for this, we need 9 number of such capacitors in parallel.
$\therefore$ Total number of capacitors (condensers)
$=9\times 3=27$

17. **(c)** The given circuit diagram is redrawn as

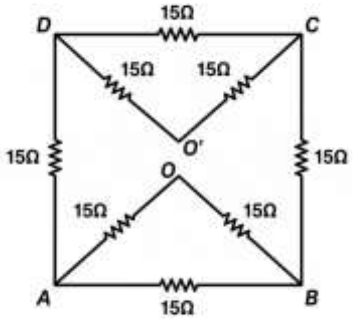


Equivalent capacitance is given as
$$C_{eq}=\frac{C_2C_3}{C_2+C_3}+C_1=\frac{6\times 3}{6+3}+2=4\mu\ F$$
$\therefore$ Total energy stored in the condenser system,
$$U=\frac{1}{2}C_{eq}\cdot V^2=\frac{1}{2}\times 4\times 10^{-6}\times 2^2$$
$=8\times 10^{-6}\ \mathrm{J}=8\mu\ \mathrm{J}$

18. **(d)** Drift velocity of the electron is given as
$$v_d=\frac{eE\tau}{m}$$
where, $e=$ charge on electron,
$E=$ electric field,
$\tau=$ relaxation time
and $m=$ mass of the electron.
Since,
$$E=\frac{V}{l}$$
$$\therefore\ v_d=\frac{eV\tau}{ml}$$
Since, e, V, m and l are constant.
$\therefore\ v_d\propto\tau$

On increasing the temperature of metal wire, relaxation time decreases, therefore drift velocity of electrons also decreases.
But on increasing temperature of metal wire, thermal velocity of electron increases.

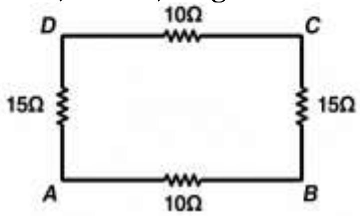
19. **(b)** Redrawing the given circuit network as



Equivalent resistance between points D and C,

$$R_{DC} = \frac{(15+15)\times 15}{(15+15)+15} = \frac{30\times 15}{45} = 10\Omega$$

Similarly, $R_{AB} = R_{DC} = 10\Omega$
Now, circuit, diagram reduces as



$\therefore$ Equivalent resistance between points A and B is given as

$$R_{AB} = (15+10+15)\|10 = 40\|10 = \frac{40\times 10}{40+10}$$

$$= \frac{400}{50} = 8\Omega$$

20. **(a)** Since, the terminals of battery is connected at one quarter distance. Hence, ratio of lengths between two parts will be

$$\frac{l_1}{l_2} = \frac{1}{3}$$

$\Rightarrow l_1 : l_2 = 1:3$
Since, $R \propto l$
$\therefore R_1 : R_2 = 1:3$

$$R_1 = \frac{24}{1+3} = 6\Omega$$

$$R_2 = 24 \times \frac{3}{1+3} = 18\Omega$$

Since, potential difference across R_1 and R_2 is same, so they are connected in parallel.
$\therefore$ Equivalent resistance of R_1 and R_2,

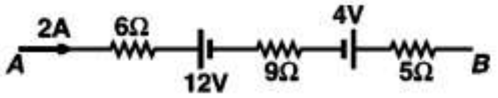
$$R' = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{6\times 18}{6+18} = 45\Omega$$

$\therefore$ Total resistance, $R_T = R' + r$
$= 4.5 + 1.5$ $\qquad [\because r = 1.5\Omega]$
$= 6\Omega$
Current, $I = \frac{V}{R_T} = \frac{18}{6} = 3\text{ A}$
According to current division rule, current through bigger arc (i.e. through R_2)

$$I_{R_2} = I \cdot \frac{R_1}{R_1 + R_2} = 3 \times \frac{6}{6+18} = 0.75 \text{ A}$$

21. (b)



Applying Kirchhoff's voltage law, we get

$-V_{AB} + 6 \times 2 + 12 + 9 \times 2 - 4 + 5 \times 2 = 0$

$\Rightarrow -V_{AB} + 48 = 0$

$\Rightarrow V_{AB} = 48$ V

22. (c)

Magnetic field at the centre of circular carrying conductor,

$B_c = \frac{\mu_0 I}{2r}$ ....(i)

Magnetic on the axis at a distance x from the centre is given as

$$B_{axis} = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}}$$

Here, $x = r$

$$B_{axis} = \frac{\mu_0 I r^2}{2(r^2 + r^2)^{3/2}}$$

$$= \frac{\mu_0 I r^2}{2(2r^2)^{3/2}} = \frac{\mu_0 I r^2}{2 \cdot 2\sqrt{2} \cdot r^3}$$

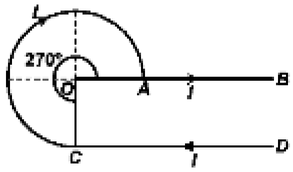
$$B_{axis} = \frac{\mu_0 I}{4\sqrt{2} r}$$

$$B_{axis} = B_a = \frac{\mu_0 I}{4\sqrt{2} r}$$

$$\therefore \frac{B_c}{B_a} = \frac{\mu_0 I/2r}{\frac{\mu_0 I}{4\sqrt{2} r}} = \frac{4\sqrt{2} r}{2r} = 2\sqrt{2}$$

$B_c : B_a = 2\sqrt{2} : 1$

23. (a) The given current carrying conductor is shown below



Magnetic field at the centre due to circular wire,

$$B_1 = \frac{\mu_0 I}{2r} \times \frac{270}{360} = \frac{\mu_0 I}{2r} \times \frac{3}{4}$$

$$B_1 = \frac{3\mu_0 I}{8r}$$

[in downward to the plane of paper]

Magnetic field at centre O due current carrying wire AB will be zero because point O lies along AB

$\therefore B_2 = 0$

Magnetic field at O due to current carrying wire CD

$B_3 = \frac{\mu_0 I}{4\pi r}$ (in downward to the plane of paper)

$\therefore$ Net magnetic field at the centre,

$$B_{net} = B_1 + B_2 + B_3 = \frac{3\mu_0 I}{8r} + 0 + \frac{\mu_0 I}{4\pi r}$$

$$= \frac{\mu_0 I}{r}\left[\frac{3}{8} + \frac{1}{4\pi}\right] = \frac{\mu_0 I}{4\pi r}\left[\frac{3\pi}{2} + 1\right]$$

24. (b) Given, $\theta_A = 60°, \theta_B = 30°$

We know that, deflection in galvanometer θ and current I are related as

$\tan\theta = KNI$
where, K = , constant
N = number of turns in the coil of galvanometer
and I = current.

$\therefore \frac{\tan\theta_A}{\tan\theta_B} = \frac{N_A}{N_B}$

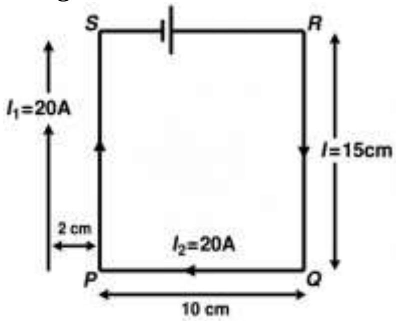
$\Rightarrow \frac{\tan 60°}{\tan 30°} = \frac{N_A}{N_B}$

$\Rightarrow \frac{\sqrt{3}}{1} = \frac{N_A}{N_B}$

$\Rightarrow \frac{\sqrt{3}}{1} = \frac{N_A}{N_B}$

$\Rightarrow N_A : N_B = 3 : 1$

25. (d) $I_1 = 20\text{ A}, I_2 = 20\text{ A}$
$r_1 = 2\text{ cm}$
$r_2 = (10 + 2)\text{cm} = 12\text{ cm}$
The given situation is shown in the figure.



The forces on armSR and PQ are equal in magnitude and opposite in direction and also having same line of action, hence they cancel to each other.
∴ Resultant force on the current loop PQRS,

$F = F_{PS} - F_{QR}$

$= \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2 \times l}{r_1} - \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2 l}{r_2}$

$= \frac{\mu_0}{2\pi} \cdot I_1 I_2 l \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$= 2 \times 10^{-7} \times 20 \times 20 \times 15 \times 10^{-2} \left[\frac{1}{2 \times 10^{-2}} - \frac{1}{12 \times 10^{-2}}\right]$

$= 5 \times 10^{-4}\text{ N}$

26. (b) Given, shunt resistance, $R_S = 1\Omega$
Total current, $I = 100\text{ A}$
Galvanometer current, $I_g = 1\text{ A}$
We know that,

$I_g = \frac{IR_5}{R_5 + G}$

$\Rightarrow 1 = \frac{100 \times 1}{1 + G}$

$\Rightarrow 1 + G = 100$

$\Rightarrow G = 99\Omega$

27. (a) On moving the circular loops further apart, flux linked with loops will be reduced. Therefore, according to Lenz's law, current will increase in both circular loops to increase the linked flux.

28. (c) The given circuit diagram is L-C-R. series circuit, where
Since,
$V_R = 80\text{ V}, V_C = 100\text{ V}$
$V_L = 40\text{ V}$

$V_C > V_L$

Value of emf E is given as

$E = \sqrt{V_R^2 + (V_C - V_L)^2}$

$= \sqrt{80^2 + (100 - 40)^2}$

$= \sqrt{6400 + 3600} = 100\ V$

29. **(c)** For a transformer,

$I_P = 5\ A, V_P = 220\ V$

$V_S = 2200\ V$

Power at primary side, $P_i = V_P I_P$

$= 220 \times 5 = 1100\ W$

Since, 50% power is lost, hence power at secondary side,

$P_0 = 50\%\ \text{of}\ P_i = \frac{50}{100} \times 1100$

$\Rightarrow P_0 = 550\ W$

$\Rightarrow V_s I_s = 550$

$\Rightarrow 2200 \times I_S = 550$

$\Rightarrow I_S = \frac{550}{2200} = \frac{1}{4} = 0.25\ A$

30. **(d)** In series $L - C - R$ circuit,

$X_L = X_C$

$\therefore Z = \sqrt{R^2 + (X_L - X_C)^2}$

$= \sqrt{R^2 + 0^2}$

$Z = R$

$\therefore$ Power factor $= \cos\phi$

$= \frac{R}{Z} = \frac{R}{R} = 1$

Hence, power factor is not zero.

Therefore option (d) is not correct.

In L-C-R series resonance circuit, peak energy stored by a capacitor is equal to peak energy stored by an inductor.

$P_{avg} = V_{rms} \times I_{rms} \times \cos 0°$

Average power, $= V_{rms} \times I_{rms}$

$=$ apparent power

L-C-R series resonance circuit is purely resistive because $X_L = X_C$, hence no wattless current exists in this circuit therefore wattless current is zero.

31. **(d)** Solar spectrum is a line absorption spectrum which is also called as Fraunhofer line of missing wavelengths.

32. **(a)** According to first situation wavelength of incident radiation,

$\lambda_1 = \lambda$

Stopping potential, $V_{01} = 3V_S$

According to Einstein's photoelectric equation,

$K_{max} = hv_1 - \phi$

$\Rightarrow eV_{01} = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_0}$ [where, $\lambda_0 =$ threshold wavelength.]

$\Rightarrow e \cdot 3V_s = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$

$\Rightarrow 3V_s e = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$ ....(i)

According to second situation, wavelength of incident radiation,

$\lambda_2 = 2\lambda$

Stopping potential, $V_{02} = V_S$

Again, Einstein's photelectric equation,

$K_{max} = hv_2 - \phi$

$\Rightarrow eV_{02} = hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_0}\right)$

$\Rightarrow eV_s = hc\left(\frac{1}{2\lambda} - \frac{1}{\lambda_0}\right)$ ....(ii)

Dividing Eq. (i) by Eq (ii), we get

$$3 = \frac{\frac{1}{\lambda} - \frac{1}{\lambda_0}}{\frac{1}{2\lambda} - \frac{1}{\lambda_0}}$$

$$\Rightarrow \frac{3}{2\lambda} - \frac{3}{\lambda_0} = \frac{1}{\lambda} - \frac{1}{\lambda_0}$$

$$\Rightarrow \lambda_0 = 4\lambda$$

33. **(c)** According to Einstein's photoelectric equation maximum kinetic energy,

$$K_{max} = eV_0 = hv - \phi_0$$

$$= h(v - v_0) = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

Hence, in photoelectric effect, maximum kinetic energy of emitted photoelectrons depends on work function of metal surface, frequency and wavelength of incident radiation but does not depend on intensity of incident radiation.

34. **(b)** We know that, wavelength of hydrogen spectrum is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

For minimum wavelength of Lyman series,

$n_1 = 1$ and $n_2 = \infty$

$$\therefore \frac{1}{(\lambda_L)_{min}} = R\left(\frac{1}{(1)^2} - \frac{1}{\infty}\right) = R$$

$$\Rightarrow (\lambda_2)_{min} = \frac{1}{R}$$

For minimum wavelength of Balmer series,

$n_1 = 2$ and $n_2 = \infty$

$$\frac{1}{(\lambda_B)_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$$

$$= R\left(\frac{1}{4} - 0\right) = \frac{R}{4}$$

$$\Rightarrow (\lambda_B)_{min}\frac{4}{R}$$

$$\therefore \frac{(\lambda_L)_{min}}{(\lambda_B)_{min}} = \frac{\frac{1}{R}}{\frac{4}{R}} = \frac{1}{4} = 0.25$$

35. **(d)** Hydrogen atom does not emit X-rays because its energy level are very close to each other. Hence, the transition of electron emits high energy radiations having short wavelength.

36. **(c)** Given,

$\lambda_e = \lambda_p$ .....(i)

Since, de-Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{2mK}}$$

where, K is kinetic energy.

$$\therefore \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}} \cdot \sqrt{\frac{K_p}{K_e}}$$

$\Rightarrow \frac{\lambda_e}{\lambda_e} = \sqrt{\frac{m_p}{m_e}} \cdot \sqrt{\frac{K_p}{K_e}}$ [From Eq. (i)]

$\Rightarrow 1=\sqrt{\frac{m_p}{m_e}}\cdot\sqrt{\frac{K_p}{K_e}}$

$\Rightarrow \sqrt{\frac{K_e}{K_p}}=\sqrt{\frac{m_p}{m_e}}$

Squaring both sides, we get

$\frac{K_e}{K_p}=\frac{m_p}{m_e}$

Since $m_p > m_e$

$\therefore K_e > K_p$

37. **(a)** Nuclear force between two protons at a separation of 40Å is much greater than electrostatic force between them.

i.e. $F_n \gg F_e$

38. **(b)** Blue colour of water in the sea is due to scattering of light by water molecules. As blue colour of light has smaller wavelength, therefore scattering of blue colour of light is exceptionally large.

Therefore, blue colour of sea water is due to scattering of sunlight by the water molecules.

39. **(c)** Nuclear radius and mass number A are related with following relation,

$R=R_0A^{\frac{1}{3}}$

$\therefore \frac{R_1}{R_2}=\left(\frac{A_1}{A_2}\right)^{1/3}$

Given, $A_1=216$ and $A_2=125$

$\therefore \frac{R_1}{R_2}=\left(\frac{216}{125}\right)^{1/3}$

$=\left[\left(\frac{6}{5}\right)^3\right]^{1/3}$

$=\frac{6}{5}$

$\therefore\ R_1:R_2=6:5$

40. **(b)** Given, output power, $P_g=1.6\text{MW}$

$=1.6\times10^6\text{ W}$

Energy released per fission,

$E_1=200\text{MeV}$

$=200\times10^6\times1.6\times10^{-19}\text{ J}$

$=320\times10^{-13}\text{ J}$

$=3.2\times10^{-11}\text{ J}$

The rate of fission R is given as

$R=\frac{P_0}{E_1}$

$=\frac{1.6\times10^6}{3.2\times10^{-11}}$

$=0.5\times10^{17}=5\times10^{16}/\text{s}$

41. **(a)** Let initial masses of two radioactive substances be N_0.

If T_1 and T_2 be the half-lives of two radioactive substances,

Then, $T_1=1\text{yr}$

$T_2=2\text{yrs}$

Total time, $t=6\text{yrs}$

Hence, remaining mass of the first subs

$t=6\text{yrs}$

$N_1=N_0\left(\frac{1}{2}\right)^{\frac{t}{T_1}}=N_0\left(\frac{1}{2}\right)^{6/1}$

$= \frac{N_0}{2^6}$

$\Rightarrow\ N_1 = \frac{N_0}{64}$

Similarly, remaining mass of the second substance after

$t = 6\text{yrs}$

$N_2 = N_0\left(\frac{1}{2}\right)^{\frac{t}{T_2}} = N_0\left(\frac{1}{2}\right)^{6/2} = \frac{N_0}{8}$

$\Rightarrow N_2 = \frac{N_0}{8}$

$\therefore \frac{N_1}{N_2} = \frac{\frac{N_0}{64}}{\frac{N_0}{8}} = \frac{1}{8} \Rightarrow \frac{N_1}{N_2} = \frac{1}{8} \qquad \ldots\text{(i)}$

Since, activity, $A = \lambda N = \frac{0.693}{T} N$

$\Rightarrow A \propto \frac{N}{T}$

$\therefore \frac{A_1}{A_2} = \frac{N_1 T_2}{N_2 T_1} = \frac{1}{8} \times \frac{2}{1} = \frac{1}{4}$

$\Rightarrow A_1 : A_2 = 1 : 4$

42. (c) When an α-particle emits from a radioactive nuclei, then its mass number is decreased by 4 unit and atomic number is decreased by 2 unit.

When a β-particle emits from a radioactive nuclei, then its mass number is unaffected, whereas atomic number is increased by one unit.

Here, initial nuclei = ${}_{92}U^{235}$

Final nuclei = ${}_{82}P^{203}$

Change in mass number $= 235 - 203 = 32$

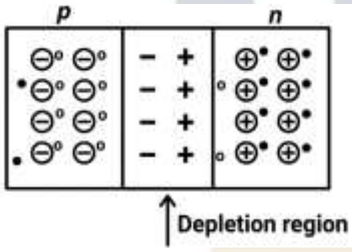
Number of α-particle emitted $= \frac{32}{4} = 8$

∴ Number of emitted β-particles

$= 8 \times 2 - (92 - 82) = 16 - 10 = 6$

43. (a) Neutron is the most stable particle in the baryon group.

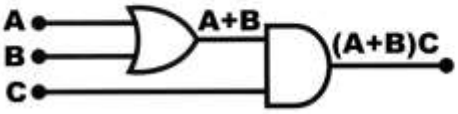
44. (b) An unbiased $p - n$ junction is shown below.



From the diagram, it is clear that in depletion region, n-side has immobile positive charge, whereas p-side has immobile negative charge.

Therefore, potential at p is less than that of n.

45. (c) The given logic diagram is shown as



Output of logic gate is given as

$Y = (A + B)C$

In option (a), $A = 0, B = 1, C = 0$

$\therefore\ Y = (0 + 1) \cdot 0 = 0$

In option (b), $A = 1, B = 0, C = 0$

$\therefore\ Y = (1 + 0) \cdot 0 = 0$

In option (c), $A = 1, B = 0, C = 1$

$\therefore\ Y = (1 + 0) \cdot 1 = 1$

In option (d), $A = 1, B = 1, C = 0$

$\therefore\ Y = (1 + 1) \cdot 0 = 0$

Hence, to get an output $Y = 1$

$A = 1, B = 0$ and $C = 1$

46. **(c)** According to Newton's law of gravitation,

$$F = \frac{Gm_1m_2}{r^2}$$

where, $F =$ gravitational force between two bodies of masses m_1 and m_2,

$r =$ distance between the two masses

and $G =$ universal gravitational constant.

$$\therefore\ G = \frac{Fr^2}{m_1m_2}$$

$$\therefore [G] = \frac{[F][r^2]}{[m_1][m_2]}$$

$$= \frac{[MLT^{-2}][L^2]}{[M][M]} = [M^{-1}\,L^3\,T^{-2}]$$

47. **(d)** Let u be the initial vertical velocity of the body by which it is projected vertically upward.

If t be the time taken by the body to reach a height h from the point of projection, then using equation of vertical motion,

$$h = ut - \frac{1}{2}gt^2$$

$$\Rightarrow gt^2 - 2ut + 2h = 0$$

$$\Rightarrow t = \frac{2u \pm \sqrt{4u^2 - 4g \times 2h}}{2g}$$

$$= \frac{u \pm \sqrt{u^2 - 2gh}}{g}$$

It means t has two values which is already given in question,

$$t_1 = \frac{u + \sqrt{u^2 - 2gh}}{g}$$

and $t_2 = \frac{u - \sqrt{u^2 - 2gh}}{g}$

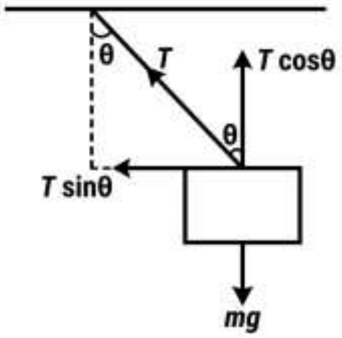
$$\therefore t_1 + t_2 = \frac{2u}{g}$$

$$u = \frac{g(t_1 + t_2)}{2}$$

48. **(a)** Given, mass suspended from spring balance, $m = 10$ kg

$\theta = 60°$

The given situation is shown below



From the diagram, reading of spring balance,

$F_s = T$ ....(i)

Since, $T\cos\theta = mg$

$T = \frac{mg}{\cos\theta}$ ....(ii)

From Eqs. (i) and (ii) we get

$F_s = \frac{mg}{\cos\theta} = \frac{mg}{\cos 60^\circ}$
$= 2mg$
$= 2 \times 10 \times g$
$= 20\ kg - wt$

49. (b) Weight of body in air,
$m_1 = 50\ g = 5 \times 10^{-2}\ kg$
Weight of body in water,
$m_2 = 40\ g = 4 \times 10^{-2}\ kg$
Buoyant force on the body,
$F' = m_1 g - m_2 g$
$= g(m_1 - m_2)$
$= g(5 \times 10^{-2} - 4 \times 10^{-2})$
$F' = 10^{-2} g\ N$ ….(i)
But buoyant force is given as
$F' = V\rho g$
$\Rightarrow 10^{-2}\ g = V\rho g$ [From Eq. (i)]
$\Rightarrow 10^{-2} = \rho V$
$10^{-2} = 10^3\ V$ $[\because \rho = 10^3\ kg/m^3]$
$\Rightarrow V = 10^{-5}\ m^3$
$\therefore$ Loss of weight in liquid $= \rho_l V g$
$W' = 1.5\rho \times 10^{-5} \times 10$ $[\because \rho_l = 1.5\rho]$
$= 1.5 \times 10^3 \times 10^{-5} \times 10$
$= 0.15\ N$
$\therefore$ Loss of weight (in kg) $= \frac{w'}{g} = \frac{0.15}{10} = 0.015\ kg = 15\ g$
$\therefore$ Weight of body in water $= 50 - 15 = 35\ g$

50. (c) We know that, displacement travelled by a moving body with constant acceleration a in nth second,
$s_n = u + \frac{1}{2}a(2n - 1)$
For first second, $n = 1$
$\therefore s_1 = u + \frac{1}{2}a(2 \times 1 - 1)$
$5 = u + \frac{a}{2}$ ... (i) $[\because s_1 = 5\ m]$
For third second, $n = 3$
$\therefore s_3 = u + \frac{a}{2}(2 \times 3 - 1)$
$2 = u + \frac{5a}{2}$ ... (ii) $[\because s_3 = 2\ m]$
Subtracting Eq. (i) from Eq. (ii), we get
$-3 = 2a$
$\Rightarrow a = \frac{-3}{2}\ m/s^2$
Force acting on the body,
$F = ma = 4 \times \left(\frac{-3}{2}\right) = -6\ N$
Hence, force acting on the body in opposite direction of its motion will be 6 N.

51. (b) Time-period of simple pendulum is given as
$T = 2\pi\sqrt{\frac{l}{g}}$ ….(i)
If a be the acceleration of the lift, so that time-period of simple pendulum is reduced to $T/2$
i.e. $T'T/2$.
$\Rightarrow 2\pi\sqrt{\frac{l}{g + a}} = \frac{1}{2} \times 2\pi\sqrt{\frac{l}{g}}$

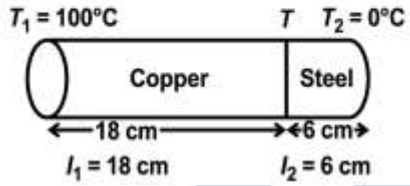
$\Rightarrow \frac{l}{g+a} = \frac{1}{4} \cdot \frac{l}{g}$
$\Rightarrow g + a = 4g$
$\Rightarrow a = 3g$

52. (c) According to Mayer's formula,
$C_p - C_v = R$
where, C_p = specific heat at constant pressure,
C_V = specific heat at constant volume
R = gas constant.
$\Rightarrow \frac{C_p}{C_V} - 1 = \frac{R}{C_V}$
$\Rightarrow \gamma - 1 = \frac{R}{C_V}$
$\Rightarrow C_V = \frac{R}{\gamma - 1}$

53. (b) Net work done in cyclic process ABCA,
W_{net} = Area enclosed by path ABCA
$= \frac{1}{2} \times (3V_1 - V_1)(4p_1 - p_1)$
$= \frac{1}{2} \times 2V_1 \times 3p_1 = 3p_1V_1$
Since, cyclic process is taken in anti-clockwise direction, hence net work done will be negative.
$W_{net} = -3p_1V_1$

54. (d) In isothermal process, temperature remains constant, hence internal energy of the system remains constant in isothermal process.

55. (a) The given situation is shown below



Thermal conductivity of copper
= 9 × thermal conductivity of steel
$K_c = 9K_s$
Since, both bar are connected in series, hence heat current along the bar will be same.
i.e $\frac{dQ}{dt} = \frac{K_cA(T_1 - T)}{l_1}$
$= \frac{K_sA(T - T_2)}{l_2}$
$\Rightarrow \frac{9K_sA(100 - T)}{18} = \frac{K_sA(T - 0)}{6}$
$\Rightarrow 3(100 - T) = T$
$300 - 3T = T$
$\Rightarrow 4T = 300$
$\Rightarrow T = 75°C$

56. (d) Equation of simple harmonic wave is given as
$y = 6\sin 2\pi(2t - 0.1x)$
Path difference, $\Delta x = 2$ mm
Comparing with standard equation of wave,
$y = A\sin(\omega t - kx)$
$= A\sin 2\pi\left(vt - \frac{k}{2\pi} \cdot x\right)$
$\frac{k}{2\pi} = 0.1$

$k = 0.2\pi$

Wavelength, $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = 10$ mm

$\therefore$ We know that, phase difference,

$\phi = \frac{2\pi}{\lambda} \times$ path difference (Δx)

$= \frac{2\pi}{10} \times 2 = \frac{2\pi}{5} = \frac{2 \times 180}{5} = 72^\circ$

57. (d) Given, sound source is stationary, $v_S = 0$

Apparent frequency, $f' = 2f$

where, f is actual frequency of sound.

By using Doppler's effect,

$f' = f\left(\frac{v + v_0}{v}\right)$

[where, v is velocity of sound A]

$\Rightarrow 2f = f\left(\frac{v + v_0}{v}\right)$

$\Rightarrow 2v = v + v_0$

$\Rightarrow v = v_0$

$\Rightarrow v_0 = v$

58. (a) Given, $T_1 = 27^\circ C = (273 + 27)K = 300$ K

$T_2 = 627^\circ C$

$= (627 + 273)K$

$= 900$ K

Energy emitted per second

$E_1 = 0.5$ J

$E_2 = ?$

According to Stefan's Boltzmann law,

Energy radiated per unit area per second is given by

$E = \sigma T^4$

$\Rightarrow \frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4$

$= \left(\frac{900}{300}\right)^4 = 81$

$\Rightarrow E_2 = 81E_1 = 81 \times 0.5$

$= 40.5$ J

59. (a) Given, $L_2 = 2L_1, f_1 = 200$ Hz, $f_2 = 300$ Hz

We know that, frequency of vibrating string is given as

$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

where, μ = mass per unit length of string

and L = length of the string.

$\Rightarrow f \propto \frac{\sqrt{T}}{L}$

$\Rightarrow \frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \cdot \frac{L_1}{L_2}$

$\Rightarrow \sqrt{\frac{T_2}{T_1}} = \frac{f_2 L_2}{f_1 L_1} = \frac{300}{200} \times \frac{2L_1}{L_1}$

$\sqrt{\frac{T_2}{T_1}} = 3$

$\Rightarrow \frac{T_2}{T_1} = 9$

$\therefore T_2 : T_1 = 9 : 1$

60. (a) Loudness of sound is given as

$L = 10\log_{10}\frac{I}{I_0}$

So, we get $L_2 - L_1 = 10\log_{10}\frac{I_2}{I_1}$

here $L_1 = 30$ dB, $L_2 = 60$ dB

$\therefore 60 - 30 = 10\log_{10}\frac{I_2}{I_1}$

$\Rightarrow 3 = \log_{10}\frac{I_2}{I_1}$

$\Rightarrow \frac{I_2}{I_1} = 10^3 \Rightarrow I_2 = 1000I_1$

CHEMISTRY

61. (c) Carnallite : $KCl \cdot MgCl_2 \cdot 6H_2O$

Dolomite: $MgCO_3 \cdot CaCO_3$

Calamine : $ZnCO_3$

Sea water : ore of magneisum

$\therefore$ Calamine is not an ore of magnesium.

62. (a) Electronic configuration of Ni, Cu and their ions are

$Ni = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^5 4s^2$

$Cu = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^1$

$Ni^{2+} = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^8 4s^0$

$Cu^+ = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^0$

Electronic configuration represents Cu^+ions.

63. (b) The element $[Ne]3s^2 3p^3$ have half-filled p-orbital.

As a result, it is more stable.

$\therefore$ The ionisation energy is highest for this element.

64. (a) The oxidation number of Br in Br_2 is zero.

The oxidation number of Br is BrO_3^-

$x + 3 \times (-2) = -1$

$x = -1 + 6$

$x = +5$

65. (d) Among the alkali metals, caesium is most reactive because the outermost electron is more loosely bound than the outermost electron of other alkali metals.

66. (a) Electronic configuration of nitrogen is

${}_7N = 1s^2 2s^2 2p^3$ i.e. 2,5

In outer most shell, it has five electrons.

$\therefore$ It share 3 electrons to complete its octet as follows :

67. (c) Hydrogen bond is strongest in $F - H - F$, as the electronegativity of fluorine is highest.

68. (a) Decomposition of $CaCO_3$ as follows:

$$\underset{100g}{CaCO_3} \xrightarrow{\Delta} CaO + \underset{44\,g}{CO_2}$$

$11.2 dm^3$ or(1/2 mole) 22 g or 1/2 mole

Neutralisation reaction of KOH and CO_2 as follows:

$2KOH + CO_2 \xrightarrow{\Delta} K_2CO_3 + H_2O$

2 moles 1 mole

1 mole $\frac{1}{2}$ mole

1 mole of CO_2 reacts with 2 moles of KOH

1/2 mole of CO_2 reacts with 1 mole of KOH

$\therefore$ Mass of KOH $= 1 \times 56.1 = 56.1\text{ g} = 56\text{ g}$

69. **(c)**

$$M = \frac{dRT}{p} = \frac{1.964 \times 0.0821 \times 273}{1}$$

$$\begin{bmatrix} \because p = 76\text{ cmHg} = 1\text{ atm}, \\ d = 1.964\text{ gdm}^{-3} \\ R = 0.0821\text{dm}^3\text{ atm K}^{-1}\text{ mol}^{-1} \end{bmatrix}$$

$M = 44\text{ g/mol}$

$\therefore$ So, gas is CO_2.

70. **(a)** 0.06 mole KNO_3 added to 100 cm^3 of water at 298 K .

$\Delta H = 35.8\text{ kJ/mol}$

$\Delta H = m \times C \times \Delta T$

$$\begin{bmatrix} \Delta H \text{ for } 0.06 \text{ mole } = 35.8 \times 0.06 = 2.148\text{ kJ} \\ \text{Mass} = 100\text{ cm}^3 = 100\text{ g} \end{bmatrix}$$

$\therefore\ 2148 = 100 \times 0.004184\text{ kJ} \times \Delta T$

$\Delta T = 5.133\text{ K}$

It is an endothermic process as heat absorbed.

So, temperature will decrease.

$\therefore$ Final temperature

$T = 298 - 5.133$

$T = 292867\text{ K} \simeq 293\text{ K}$

71. **(b)** Initially At equili.

$$\underset{\substack{4\text{ moles} \\ (4-2) \\ =2}}{2SO_2} + \underset{\substack{4\text{ moles} \\ (1-025\times4) \\ =3}}{O_2} \rightleftharpoons \underset{\substack{0 \\ 2}}{2SO_3}$$

$\therefore$ Total moles at equilibrium $= 2 + 3 + 2 = 7$ moles.

72. **(d)** An example of auto-catalysis is oxidation of oxalic acid by acidified $KMnO_4$ solution.

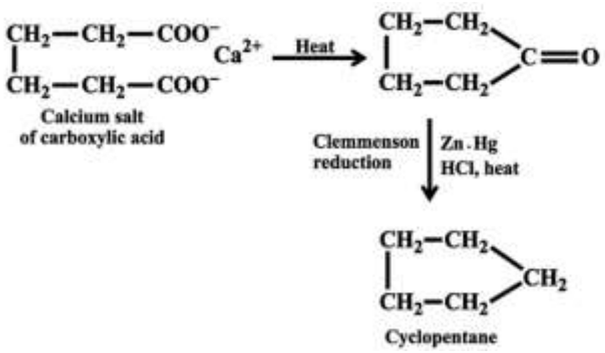
In this reaction, one of the product (Mn^{2+}) formed act as a catalyst and speed up the reaction.

$$\underset{\text{Permagnate ion}}{2MnO_4^-} + 6H^+ + \underset{\text{Oxalic acid}}{5C_2H_2O_4} \longrightarrow \underset{\text{Auto-catalyst}}{2Mn^{2+}} + 8H_2O + 10CO_2$$

73. **(c)** When a nitrogeneous organic compound is fused with sodium, the nitrogen present in the compound is converted into sodium cyanide.

This reaction is also called Lassaigne's test.

74. **(c)** The complete reaction is as follows :



75. **(c)** Williamson ether synthesis is an organic reaction, forming an ether from alkyl halide and deprotonated alcohol i.e.

$$\underset{\text{Sodium ethoxide}}{C_2H_5ONa} + \underset{\text{Ethane iodide}}{C_2H_5I} \longrightarrow \underset{\substack{\text{Ethoxyethane} \\ \text{(ester)}}}{C_2H_5OC_2H_5} + NaI$$

76. **(c)** For the reaction, $C + \frac{1}{2}O_2 \longrightarrow CO$,

ΔG° es T plot in the Ellingham's diagram slopes downwards.

As for this reaction ΔG° become more negative with increase in temperature.

77. **(a)** Decomposition of calcium carbonate to form calcium oxide and carbondioxide is endothermic reaction.

$CaCO_3 \longrightarrow CaO + CO_2$

78. **(a)** Liquor ammonia bottle is opened only after cooling because it is a mild explosive.

Note The concentrated ammonia solution in water results in liquor ammonia.

79. **(c)** The formation of $O_2^+[PtF_6]^-$is the basis for the formation of xenon fluorides because O_2 and Xe have comparable ionisation energies.

80. **(b)** Magnetic moment of the element / ion is highest for configuration $3d^5$,

1	1	1	1	1

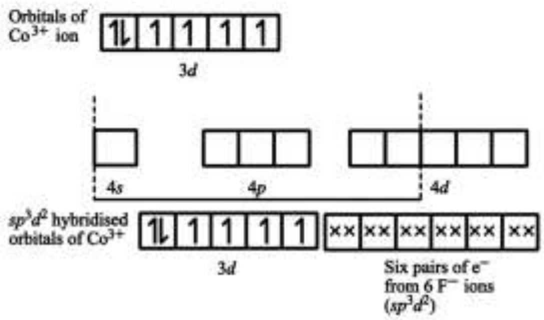
because number of unpaired electron is maximum i.e. five.

81. **(c)** If a transition metal ion exists in its highest oxidation state, then it behave as oxidising agent. The oxidation number of metal can only decrease from its highest oxidation number i.e. it will undergoes reduction and oxidise other element.

82. **(a)** In $[CoF_6]^{3-}$, $F^-$is a weak ligand. It cannot pair up d-electrons and form outer orbital octahedral complex while NH_3 and $CN^-$are strong ligand.

In $[CoF_6]^{3-}$, outer orbital 4d is used in hybridisation. Thus, it is called outer orbital or high spin or spin free complex.

$Co = [Ar]3d^7 4s^2$ and $Co^{3+} = [Ar]3d^6$



83. **(a)** Linkage isomerism can be shown by compound containing ambidentate ligand. In NO_2, NO_2 can bind either by N or by O .

e.g. $\left[\underset{\text{Yellow}}{Co(NH_3)_3(NO_2)}\right]Cl_2$ and $\left[\underset{\text{blue}}{Co(NH_3)\ _3ONO}\right]Cl_2$

84. **(a)** N_2 [14 electrons] =

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2$

Bond order $= \dfrac{N_b - N_a}{2}\left[\begin{array}{c}N_b \text{ and } N_a \text{ are bonding and} \\ \text{anti bonding molecular} \\ \text{orbital s - electron}\end{array}\right]$

$= \dfrac{10-4}{2} = 3$

$\therefore$ N_2 has highest bond order i.e. 3 .

While O_2, He_2 and H_2 have 2,0 and $1/2$ bond order respectively.

85. **(c)** $H_2^+$and He_2^+contains odd number of electrons

$\therefore$ They are paramagnetic.

Among Li_2 and O_2

$Li_2(6) = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2$

Li have no unpaired electron.

$\therefore$ It is diamagnetic

$O_2(18) = \sigma 1s^2, \sigma 1s^2, \sigma 2s^2, \sigma 2s^2 \pi 2p_x^2 = \pi 2p_y^2$

$\sigma 2p_z^2, \pi^* 2p_x^1 = \pi^* 2p_y^1$

It has two unpaired electrons. So, it is paramagnetic.

86. **(b)** For 1st order reaction

$k = \frac{2303}{t}\log\frac{[R_0]}{[R]} = \frac{2.303}{40}\log\left[\frac{0.1}{0.025}\right]$

$= \frac{2.303}{40} \times \log 4 = 0.0347 \text{ min}^{-1}$

Rate expression,

$r = k[A] = 0.0347 \times 0.01 = 3.47 \times 10^{-4} \text{M min}^{-1}$.

87. **(b)** According to Arrhenius equation

$k = Ae^{\frac{-E_u}{RT}}$

Here, k decreases with increase in E_a (activation energy).

Chemical reaction with very high E_a values are generally slow.

88. **(b)** Solid NaCl does not conduct electricity because the ions ($Na^+$and Cl^-) are well arranged in the crystal structure and not free to move to conduct electricity.

89. **(b)**According to second law of Faraday,

$\frac{w_1}{E_1} = \frac{w_2}{E_2}$

$\frac{0.16}{64} = \frac{w_2}{2}$

$w_2 = \frac{0.16 \times 2}{64} = \frac{0.01}{2} = 0.005 \text{ g}$

Volume of $H_2 = \frac{0.005}{2} \times 22.400 \text{ cm}^3 = \frac{22.4}{4} = 56 \text{ cm}^3$

90. **(c)** $K_{xp} = 10^{-8}M^2$

$K_{sp} = [A^+][B^-]$

$10^{-8} = (10^{-3})[B^-]$

$[B^-] = 10^{-5}M$

∴ Salt will precipitate, when concentration of $[B^-] > 10^{-5}$M.

91. **(d)** According to Nernst equation,

$E_{cell} = E^*_{cell} - \frac{0.0591}{n}\log\frac{[Cu^{+2}]}{[Ag^+]^2}$

∴ The voltage of cell will increase with increase in concentration of Ag^+ions.

92. **(c)** $pH = -\log[H^+]$

$[H^+] = 10^{-8} + 10^{-7} = 1.1 \times 10^{-7}$

$pH = -\log(1.1 \times 10^{-7})$

$pH = 7 - \log(1.1)$

∴ It is between 6 and 7.

93. **(b)** Lowering in vapour pressure is directly proportional to moles of solute particles

$(\Delta p)_{glucose} = (\Delta p)_{urea}$

$(\chi_B)_{glucuse} = (\chi_B)_{urea}$

i.e. $\frac{W_B}{50} \times \frac{18}{180} = \frac{1 \times 18}{50 \times 60}$

$W_B = 3$ g

94. **(c)** Osmotic pressure is a colligative property. It depends upon the number of solute particles. Benzoic acid undergoes dimerisation.

∴ Number of particles reduces ($i < 1$) Hence, osmotic pressure observed is less than the theoretical consideration.

95. **(a)** For a reaction to be spontaneous at all temperature ΔG should be negative. As we know $\Delta G = \Delta H - T\Delta S$

$= -ve - T(+ve) = -ve$

∴ ΔG and ΔH should be negative.

96. **(a)** Flocculation value $\propto \frac{1}{\text{Coagulating power}}$

Fe(OH) is a positively charged sol.

For coagulating $Fe(OH)_3$ negatively charged electrolyte is used and greater the value of -ve charge, coagulating power will be strong. Among given electrolyte, NaCl has lowest coagulating power. Hence, its flocculation value will be maximum.

97. **(b)** $\Delta H = -40\ \text{kJ} = -40 \times 10^3\ \text{J}$

$\Delta S = \Delta S_{\text{Product}} - \Delta S_{\text{Reactant}}$
$= 2(50) - 3(40) - 60$
$= 100 - 120 - 60 = -80$
$\Delta G = \Delta H - T\Delta S$
$0 = -40 \times 10^3 - T(-80)$
$T = \dfrac{40 \times 10^3}{80} = 500\ \text{K}.$

98. **(c)** NaCl forms fcc structure.

$\therefore (\text{ Diagonal length })^2 = (\sqrt{2}a)^2 = 2(r^+)^2 + 2(r^-)^2$
Where, a = edge length, r^+ and r^- are radii of Na^+ and Cl^- ions.
$a = 2[(r^+)^2 + (r^-)^2] = 2(95 + 181) = 2 \times 276 = 552\text{pm}$

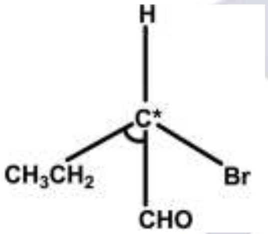
99. **(a)** Inductive effect involves the displacement of σ-electrons from less electronegative element to more electronegative element in a compound.

100. **(a)** The basicity of aniline is less than that of cyclohexylamine because of +R-effect of NH_2 group in aniline.

101. **(c)** Methyl bromide is converted into ethane by heating, it is ether medium with Na .

$$\underset{\text{Methyl bromide}}{2CH_3 - Br} + \underset{\text{Sodium}}{2Na} \longrightarrow \underset{\text{Ethane}}{2CH_3 - CH_3} + 2NaBr$$

102. **(c)** The compound having chiral centre, $CH_3CH_2CHBrCHO$ is expected to be optically active.



103. **(d)** Cyclohexane has the lowest heat of combustion per CH_2 group. As, stability increase upto 6 membered ring due to decrease in angular strain. Also heat of combustion is measure for chemical stability.

104. **(c)** The catalyst used in preparation of an alkyl chloride by action of dry HCl on alcohol is anhy. $ZnCl_2$. This process is also called Grove's process.

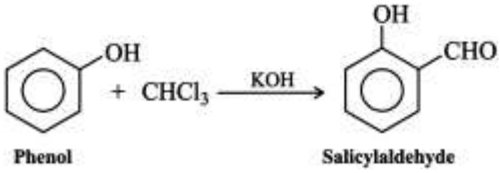
$$\underset{\text{Alcohhol}}{ROH} + HCl \xrightarrow{\text{Anky. } ZnCl_2} \underset{\text{Allyl chloride}}{RCl} + H_2O$$

105. **(c)** The complete reaction is as follows:

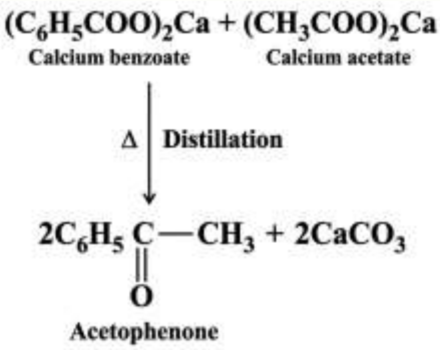
$$\underset{\text{Allkyl halide}}{R - X} \xrightarrow{\text{Alc. KCN}} \underset{\text{Allylcyanide}}{R - CN} \xrightarrow{\text{Dil. HCl}} R - \underset{\text{Carboxylic acid}}{COOH}$$

106. **(b)** Phenol does not decompose sodium bicarbonate ($NaHCO_3$) as it is a weaker acid than carbonic acid.
$\therefore CO_2$ gas is not evolved by phenol.

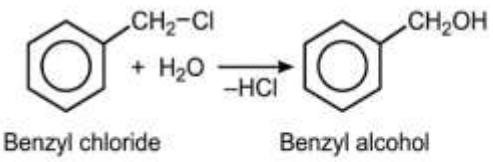
107. **(b)** Phenol on heating with chloroform in alkali get converted into salicylaldehyde.
This reaction is called Reimer-Tiemann reaction.



108. **(a)** When a mixture of calcium benzoate and calcium acetate is dry distilled, the resulting compound is acetophenone.



109. **(c)** Benzylchloride does not give benzoic acid on hydrolysis. It forms benzyl alcohol as major product.



110. **(c)** In Hofmann bromamide reaction, primary amide reacts with bromine and sodium hydroxide to form isocyanate, which is hydrolysed to give primary amine (with one carbon less) along with decarboxylation.

$$\underset{\text{Primary amide}}{RCONH_2} + Br_2 + NaOH \longrightarrow R-N=C=O \xrightarrow[-CO_2]{H_2O} \underset{\text{Primary arnine}}{RNH_2}$$

111. **(b)** Glucose contains in addition to aldehyde group on primary OH and four secondary OH groups.

The structure of glucose is

$CHO \longleftarrow$ **Aldehyde group**

$(CHOH)_4 \longleftarrow$ **Four secondary OH group**

$CH_2OH \longleftarrow$ **One primary OH group**

112. **(b)** Fats are triglycerides which are triesters of fatty acid with glycerol. So, the characteristic feature of fats is an ester group.

113. **(b)** At pH $= 4$, glycine reacts with H^+ion as

$$\underset{\text{Zwitter ion}}{H_3N^+ - CH_2 - COO^-} \xrightarrow{H^+} H_3N^+ - CH_2 - COOH$$

114. **(c)** Insulin regulate the metabolism of glucose (carbohydrates).

115. **(a)** The reaction between Mohr's salt and $KMnO_4$ is

$$2KMnO_4 + \underset{\text{Mohr's salt}}{10FeSO_4(NH_4)_2SO_4 \cdot 6H_2O} + 8H_2SO_4$$
$$\longrightarrow K_2SO_4 + 2MnSO_4 + 10(NH_4)_2SO_4$$
$$+2Fe_2(SO_4)_3 + 8H_2O$$

Oxidation sate of Fe changes from +2 to +3 .

$\therefore$ Equivalent weight of Mohr's salt

$$= \frac{\text{molecular weight}}{\text{n-factor}} = \frac{392}{1} = 392 \text{ g}$$

116. **(a)** The brown ring is appeared due to formation of a brown complex between nitric oxide and Fe^{2+} ion. This nitric oxide is formed as a result of reduction of nitrate. Brown ring test for nitrates depends on reduction of nitrate to nitric oxide.

117. **(c)** Acrolein test is positive for oils and fats when fat is heated in presence of $KHSO_4$, the glycerol portion of molecule is dehydrated and form unsaturated aldehyde. $CH_2 = CH - CHO$ (acrolein) a bad smelling compound.

118. **(a)** An organic compound which produces a bluish green flame, when heated in presence of copper is chlorobenzene. This is called Beilstein's test used to detect presence of halogen in an organic compound.

119. **(c)** Let rate expression, $r = k[A]^x[B]^y$ ….(i)

If, $[A_1] = [2A]$, then $r_1 = 2r$

$\therefore\ x = 1$

If, $[B_2] = [9B]$, then $r_2 = 3r$

$\therefore\ y=\frac{1}{2}$

Rate expression, $r=k[A][B]^{1/2}$ [By Eq. (i)]

$\therefore$ Order $=1+\frac{1}{2}=\frac{3}{2}$

120. **(a)** For, assume complete dissociation

For Na_2SO_4; $i=3, m=0.01$

$\Delta T_b = iK_b m$

$\Delta T_b = 3\times K_b\times 0.01 = 0.03K_b$

For, $0.01 M KNO_3$;

$i=2, m=0.01$

$\Delta T_b = 2\times K_b\times 0.01K_b = 0.02K_b$

While, urea and glucose are non-electrolyte

They do not dissociate into ions highest boiling point is shown by $0.01 M Na_2SO_4(aq)$ solution.

MATHEMATICS

121. **(a)** Given that, $u=a-b, v=a+b$

and

$|a|=|b|=2$

Now, $|u\times v| = |a-b\times a+b|$

$=2(a\times b)=2|a||b|\sin\theta$

$=2\times 2\times 2\sin\theta = 8\sin\theta = 8\sqrt{1-\cos^2\theta}$

$=8\sqrt{1-\left(\frac{a\cdot b}{|a||b|}\right)^2}$

$=8\sqrt{1-\left(\frac{a\cdot b}{4}\right)^2} = 8\sqrt{1-\left(\frac{a\cdot b}{16}\right)^2}$

$=\frac{8}{4}\sqrt{16-(a\cdot b)^2} = 2\sqrt{16-(a\cdot b)^2}$

122. **(a)** Let $A(1,1,1), B(2,1,3), C(3,2,2)$ and $D(3,3,4)$.

So, $AB=\hat{\imath}+2\hat{k}, AC=2\hat{\imath}+\hat{\jmath}+\hat{k}$

and $AD=2\hat{\imath}+2\hat{\jmath}+3\hat{k}$

Volume of tetrahedron $=\frac{1}{6}\begin{bmatrix}1 & 0 & 2\\ 2 & 1 & 1\\ 2 & 2 & 3\end{bmatrix}$

$=\frac{1}{6}|1(3-2)+0(6-2)+2(4-2)|$

$=\frac{1}{6}(1+0+4)=\frac{5}{6}$ cubic units

123. **(d)** Let $a=\hat{\imath}-2\hat{\jmath}+\hat{k}, b=\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $c=-\hat{\imath}+2\hat{\jmath}+\hat{k}$

Now, $b\times c=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & 1 & -2\\ -1 & 2 & 1\end{vmatrix} = 5\hat{\imath}+\hat{\jmath}+3\hat{k}$

Now, $a\times(b\times c)=\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 1 & -2 & 2\\ 5 & 1 & 3\end{vmatrix} = -8\hat{\imath}+7\hat{\jmath}+11\hat{k}$

Now, any vector perpendicular to a and lying in the plane containing $\hat{\imath}+\hat{\jmath}-2\hat{k}$ and $-\hat{\imath}+2\hat{\jmath}+\hat{k}$ is $\pm(a\times(b\times c))$.

$\therefore$ Required unit vector $=\pm\frac{(-8\hat{\imath}+7\hat{\jmath}+11\hat{k})}{\sqrt{(-8)^2+(7)^2+(11)^2}}$

$=\pm\frac{(-8\hat{\imath}+7\hat{\jmath}+11\hat{k})}{\sqrt{234}}$

124. **(d)** Let the inverse element of 10 be e, then

$a * e = 0$

Therefore, $10 * e = 10 + e + 10e = 0$
$\Rightarrow\ 10 + 11e = 0$
$\Rightarrow 11e = -10$
$\Rightarrow e = \frac{-10}{11}$

125. **(c)** Under Multiplication Modulo 7 , inverse of 2 is
$2 \times 4 = 1(\text{mod}7)$
Hence, $2^{-1} = 4$
Now, $2^{-1} \times 4 = 4 \times 4 = 16$
Now, $16 = 2 \times 7 + 2$
Hence dividing 16 by 7 gives a remainder 2,
Therefore, value of above multiplication mod is 2 .

126. **(d)** The group (z, +) has infinitely many subgroups.

127. **(c)** Given, $3x \equiv 5(\text{mod}7)$
$\Rightarrow 3x - 5 \equiv 0(\text{mod}7) = \frac{7}{3x-5}$
$\Rightarrow 3x - 5 = 7P$, where $P \in Z$
$\Rightarrow x \equiv 4(\text{mod}7)$

128. **(c)** Let $z = \sin\left(\frac{6\pi}{5}\right) + i\cos\left(1 + \cos\frac{6\pi}{5}\right)$
Putting $\sin\frac{6\pi}{5} = r\cos\alpha$ ....(i)
and $1 + \cos\frac{6\pi}{5} = r\sin\alpha$ ....(ii)
On dividing Eq. (ii) by Eq. (i), we get
$\tan\alpha = \frac{1+\cos\frac{6\pi}{5}}{\sin\left(\frac{6\pi}{5}\right)} = \frac{2\cos^2\frac{3\pi}{5}}{2\sin\frac{3\pi}{5}\cos\frac{3\pi}{5}}$
$= \cot\frac{3\pi}{5} = \tan\left(\frac{\pi}{2} - \frac{3\pi}{5}\right)$
$\tan\alpha = \tan\left(-\frac{\pi}{10}\right)$
$\therefore$ Argument to z is $\alpha = -\frac{\pi}{10}$

129. **(c)** We have,
$$\sum_{n=1}^{n} k^k = i + i^2 + i^3 + i^4 + i^5 + i^6 + i^7 + i^8 + i^9 + \cdots$$
Now as, $i^4 = i^5 = i^{12} = \cdots i^{4n} = 1$
and $i + i^2 + i^3 + i^4 = 0$
$$\text{So, } \sum_{n=1}^{n} i^k = i + i^2 + i^3 + i^4 + i^5 + i^6 + \cdots i^7 + i^8 + i^9 + \cdots$$
$= i + i^2 + i^3 + i^4 + (i + i^2 + i^3 + i^4) + (i + i^2 + i^3 + i^4) + \cdots$
$= 0 + 0 + 0 + \cdots$
When we take $n = 99$ after 96th term, we will be left with $0 + i + i^2 + i^3 = -1$
So, for $n = 99$
$$\Rightarrow\ 1 + \sum_{k=1}^{n} k^k = 0$$

130. **(c)** We have,
$(-1 + \sqrt{-3})^{62} + (-1 - \sqrt{-3})^{62}$
$= 2^{62}\left[\left(\frac{-1+\sqrt{3}i}{2}\right)^{62} + \left(\frac{-1-\sqrt{3}i}{2}\right)^{62}\right]$
$= 2^{62}[\omega^{62} + \omega^{124}]$
$= 2^{62}[\omega + \omega^2]$

$= 2^{62}(-1) = -2^{62}$

131. **(b)** We have,

$$\left|\frac{z-6i}{z+6i}\right| = 1$$

or $|z-6i| = |z+6i|$

Let $z = x+iy$

or $|x+iy-6i| = |x+iy+6i|$

$|x+(y-6)i| = |x+(y+6)i|$

$\Rightarrow \sqrt{x^2+(y-6)^2} = \sqrt{x^2+(y+6)^2}$

$\Rightarrow x^2+(y-6)^2 = x^2+(y+6)^2$

$\Rightarrow y^2+36-12y = y^2+36+12y$

$\Rightarrow 24y = 0$

$\Rightarrow y = 0$

Hence, it is lie on the real axis.

132. **(a)** We have,

$\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]$

Now,

$$\cos(\tan^{-1}x) = \cos\left[\cos^{-1}\frac{1}{\sqrt{x^2+1}}\right]$$

$$= \frac{1}{\sqrt{x^2+1}}$$

$[\because \cos(\cos^{-1}x) = x, \forall x \in [-1,1]]$

So, $\cot^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right) = \sin^{-1}\left(\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}\right)$

Thus, $\sin\left[\cot^{-1}\left(\frac{1}{\sqrt{x^2+1}}\right)\right] = \sin\left[\sin^{-1}\left(\frac{\sqrt{x^2+1}}{\sqrt{x^2+2}}\right)\right]$

$$= \frac{\sqrt{(x^2+1)}}{\sqrt{x^2+2}} \quad [\because \sin(\sin^{-1}x) = x, \forall x \in [-1,1]$$

133. **(c)**Let $y = 1+\alpha x$ $\quad [f(x) = y]$

$$\Rightarrow x = \frac{y-1}{\alpha}$$

$$\Rightarrow f^{-1}(x) = \frac{x-1}{\alpha}$$

It is given that, $f(x) = f^{-1}(x)$

$$\Rightarrow 1+\alpha x = \frac{x-1}{\alpha}$$

$\Rightarrow \alpha + \alpha^2 x = x-1$

On compare the coefficients, we get

$\alpha^2 = 1$ and $\alpha = -1$

$\Rightarrow \alpha = -1$

134. **(c)** Let x^r occurs in the expansion of $\left(x+\frac{1}{x}\right)^n$

$$T_{r+1} = {}^nC_p x^{n-p}\left(\frac{1}{x}\right)^p$$

$= {}^nC_p x^{n-p}x^{-p} = {}^nC_p x^{n-2p}$

let $n-2p = r$

$$\Rightarrow n-r = 2p \Rightarrow p = \frac{n-r}{2}$$

So, coefficient of $x^r = {}^nC_p = \frac{n!}{p!(n-p)!}$

$$= \frac{n!}{\left(\frac{n-r}{2}\right)!\left(n-\frac{n-r}{2}\right)!}$$

$$= \frac{n!}{\left(\frac{n-r}{2}\right)!\left(\frac{n+r}{2}\right)!}$$

135. **(c)** We have,

$\tan A - \tan B = x$

and $\cot B - \cot A = y$

$\Rightarrow \frac{1}{\tan B} - \frac{1}{\tan A} = y$

$\Rightarrow \frac{\tan A - \tan B}{\tan A \tan B} = y$

$\Rightarrow \tan A \tan B = \frac{x}{y}$

$\Rightarrow \cot A \cot B = \frac{y}{x}$

Now, $\cot(A-B) = \frac{1 + \cot A \cot B}{\cot B - \cot A}$

$= \frac{1 + \frac{y}{x}}{y} = \frac{x+y}{xy} = \frac{1}{y} + \frac{1}{x}$

136. **(a)** $\cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \cos^2\frac{5\pi}{12}$

$= \cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{12}\right)$

$= \cos^2\frac{\pi}{12} + \cos^2\frac{\pi}{4} + \sin^2\frac{\pi}{12}$

$= 1 + \cos^2\frac{\pi}{4} = 1 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1 + \frac{1}{2} = \frac{3}{2}$

137. **(a)** $\because \sin\theta, \cos\theta$ and $\tan\theta$ are in GP.

So, $\cos^2\theta = \sin\theta \cdot \tan\theta$

$\Rightarrow \cos^3\theta = \sin^2\theta$

$\Rightarrow \cot^3\theta = \frac{1}{\sin\theta} = \operatorname{cosec}\theta$

Squaring both sides, we get

$\cot^6\theta = \operatorname{cosec}^2\theta$

$\Rightarrow \cot^6\theta = 1 + \cot^2\theta$

$\Rightarrow \cot^6\theta - \cot^2\theta = 1$

138. **(d)** We have,

$\frac{3x^2 - 2x + 4}{(x-1)^6} = \frac{A_1}{x+1} + \frac{A_2}{(x+1)^2} + \frac{A_3}{(x+1)^3}$

$+ \frac{A_4}{(x+1)^4} + \frac{A_5}{(x+1)^5} + \frac{A_6}{(x+1)^6}$ ....(i)

Now, put $x = 0$ in Eq. (i), we get

$A_1 + A_2 + A_3 + A_4 + A_5 + A_6 = 4$ ....(ii)

If $A_1 + A_3 + A_5 = -8$ and $A_2 + A_4 + A_6 = 12$

satisfies the equation.

Hence, only option (i.e. $-8, 12$) satisfies the solution.

139. **(d)** We have,

$\log_2(2^{x-1} + 6) + \log_2(4^{x-1}) = 5$

$\Rightarrow 2^{x-1} + 6)(2^{2x-2}) = 2^5$

Let $y = 2^{x-1}$

$\Rightarrow (y+6)y^2 = 2^5$

$\Rightarrow y^3 + 6y^2 - 32 = 0$

$\Rightarrow (y-2)(y^2 + 8y + 16) = 0$

So, $y = 2$

$2^{x-1} = 2^1$

On comparing both sides, we get

$x - 1 = 1$

$x = 2$

140. **(b)** As, $(a+b+c+d)^2 = a^2 + b^2 + c^2 + d^2$

$+2(ab+bc+ad+bc+bd+cd)$
$\Rightarrow a^2+b^2+c^2+d^2=(a+b+c+d)^2$
$-2[ab+bc+ac+ad+bd+cd]$
Now, as a, b, c, d are the roots of the equation.
We have,
$$a+b+c+d=\frac{-2}{1}=-2$$
and $ab+ac+ad+bc+bd+cd=\frac{3}{1}=3$
We have, $1+a^2+b^2+c^2+d^2$
$=1+(-2)^2-2(3)$
$=1+4-6=-1$

141. **(b)** We have,
$$c_1x+c_3x^3+c_5x^5+\cdots=\frac{1}{2}\{(1+x)^n-(1-x)^n\}$$
$$\Rightarrow \int_0^1 (c_1x+c_3x^3+c_5x^5+\cdots)dx$$
$$=\frac{1}{2}\int_0^1 (1+x)^n-(1-x)^n dx$$
$$\Rightarrow \frac{c_1}{2}+\frac{c_3}{4}+\frac{c_5}{6}+\cdots=\frac{2^n-1}{n+1}$$
$$\log_5\left(\frac{1}{4}+\frac{1}{4\cdot 2}+\frac{1}{4}\left(\frac{1}{2}\right)^2+\cdots\infty\right)$$

142. **(a)** $(0.2)^{\log_{\sqrt{5}}\frac{1/4}{1-\frac{1}{2}}}$
$$=(0.2)^{\log_{\sqrt{5}}\left(\frac{1}{2}\right)}$$
$$=\left(\frac{1}{5}\right)^{\log_5\left(\frac{1}{2}\right)}=\left(\frac{1}{2}\right)^{2\log_5\frac{1}{5}}$$
$$=\frac{1^{-2\log_6 5}}{2}=\left(\frac{1}{2}\right)^{-2}$$
$$=(2^{-1})^{-2}=2^2=4$$

143. **(c)** If set A and B has m elements, then the number of bijections from A to B is the total number of arrangements of n distinct items taken all at a time, i.e. m !.

144. **(c)**Let,$y=\sin^{-1}\left[x\sqrt{1-x}-\sqrt{x}\sqrt{1-x^2}\right]$
$$=\sin^{-1}\left[x\sqrt{1-(\sqrt{x})^2}-\sqrt{x}\sqrt{1-x^2}\right]$$
$$=\sin^{-1}x-\sin^{-1}\sqrt{x}$$
$$\left[\text{Using } \sin^{-1}x-\sin^{-1}y=\sin^{-1}\left[x\sqrt{1-y^2}-y\sqrt{1-x^2}\right]\right]$$

145. **(b)** We have,
$\tan\theta+\tan 4\theta+\tan 7\theta=\tan\theta\tan 4\theta\tan 7\theta$
$\tan\theta+\tan 4\theta=\tan\theta\tan 4\theta\tan 7\theta-\tan 7\theta$
$\tan\theta+\tan 4\theta=-\tan 7\theta(1-\tan\theta\tan 4\theta)$
$$\frac{\tan\theta+\tan 4\theta}{1-\tan\theta\tan 4\theta}=-\tan 7\theta$$
$\tan(\theta+4\theta)=-\tan 7\theta$
$\tan 5\theta=\tan(-7\theta)$
$\therefore\ 5\theta=-7\theta$
$12\theta=0$ or $12\theta=n\pi$
So, $\theta=\frac{n\pi}{12}, \forall n\in I$

146. **(b)** The normal to the tangent will pass through the centre. Also, the intersection of the normal and tangent will give the point of contact.

Hence, cosider the equation of tangent

$y = -\frac{n}{2} - 45$

Here, $m = \frac{-1}{2}$

Therefore, slope of the normal is 2 .

Since, slope of normal and tangent are mutually perpendicular.

Therefore, equation of the normal at $(1, -1)$ is

$\frac{y-1}{x+1} = 2$ or $y - 1 = 2x + 2$

$\Rightarrow 2x - y = -3$

Therefore, we have two equations $2x - y = -3$ and $x + 2y + 9 = 0$

Solving these two equations give us $(x, y) = (-3, -3)$

Hence, point of contact is $(-3, -3)$.

147. **(b)** For given condition,

$\frac{2g}{2g'} = \frac{2f}{2f'}$

$\Rightarrow gf' = fg'$

148. **(b)** Given, circles are $x^2 + y^2 = 4$ and

$x^2 + y^2 - 4x + 2y - 4 = 0$

i.e. $x^2 + y^2 = 4$ ....(i)

and $(x-2)^2 - 4 + (y+1)^2 - 1 - 4 = 0$

$\Rightarrow x^2 + y^2 = 4$ and $(x-2)^2 + (y+1)^2 = 9$ ....(ii)

Center and radius of circles (i) is

$c_1 = (0,0), r_1 = 2$

and center and radii of circle (ii) is

$c_2 = (2-1), r_2 = 3$

So, difference between c_1 and $c_2 = \sqrt{5}$

Now here,

$1 = r_2 - r_1 < \sqrt{5} = c_1c_2 < 5 = r_1 + r_2$

$r_2 - r_1 < c_1c_2 < r_1 + r_2$

Hence, there will be two common tangents.

149. **(c)** Length of tangent from $x^2 + y^2 + 2gx + 2fy + c_1 = 0$

To the circle $x^2 + y^2 + 2gx + 2fy + c_2 = 0$ is $\sqrt{c_2 - c_1}$

Here, $c_2 = 4$ and $c_1 = 0$

So, required length $= \sqrt{4-0} = \sqrt{4} = 2$

150. **(c)** Eccentricity for $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $b^2 = a^2(1 - e^2)$ and eccentricity for

$\frac{x^2}{\frac{144}{25}} - \frac{y^2}{\frac{81}{25}} = 1$ is

$e_1 = \frac{a_1^2 + b_1^2}{a_1^2}$

So, $e_1 = \sqrt{1 + \frac{81}{144}} = \frac{15}{12}$

Again, foci $= a_1e_1 = \frac{12}{5} \times \frac{15}{12} = 3$

So, focus of hyperbola is $(3,0) = (ae, 0)$

and focus of ellipse is $(ae, 0) = (5a, 0)$

As, this foci are same, so

$5a = 3$

$\therefore e = \frac{3}{5}$

So, $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{b^2}{25}$

$\Rightarrow \frac{b^2}{25} = 1 - e^2 = 1 - \frac{9}{25}$

$\Rightarrow \frac{b^2}{25} = \frac{16}{25}$

$\Rightarrow b^2 = 16$

151. **(c)** Given that, $\frac{2b^2}{a} = \frac{2b}{2}$

$a = 2b$

Now, $b^2 = a^2(1 - e^2)$

$b^2 = 4b^2(1 - e^2)$

$\frac{1}{4} = 1 - e^2$

$\Rightarrow e^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\Rightarrow e = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

152. **(a)** Given, equation of parabola is

$x^2 + 10x - 16y + 25 = 0$

It can be rewritten as

$(x + 5)^2 = 16y$ .....(i)

Here $a = 4$ and vertex $\equiv (-5,0)$ and focus $\equiv (-5,4)$

So, y-coordinate of ends of latusrectum will be 4 .

Putting in equation, we have

$(x + 5)^2 = 16 \times 4 = 64$

$\Rightarrow x + 5 = \pm 8$

So, $x = 3, -13$

Substituting x in Eq. (i), we get $y = 4,4$

Hence, ends of latusrectum will be (3,4) and (−13,4).

153. **(d)** (a) We have,

$f(x) = \cos|x| + |x|$

$= \begin{cases} \cos(-x) + (-x), & x < 0 \\ \cos(x) + x, & x \geq 0 \end{cases}$

$= \begin{cases} \cos x - x, & x < 0 \\ \cos x + x, & x \geq 0 \end{cases}$

$\therefore f'(x) = \begin{cases} -\sin x - 1, & x < 0 \\ -\sin x + 1, & x \geq 0 \end{cases}$

$\therefore f'(0^-) = -1$ and $f'(0^+) = 1$

$\Rightarrow f'(0^-) \neq f'(0^+)$

So, it is not differentiable at $x = 0$.

(b) We have,

$f(x) = \cos|x| - |x|$

$= \begin{cases} \cos(-x) - (-x), & x < 0 \\ \cos(x) - x, & x \geq 0 \end{cases}$

$= \begin{cases} \cos x + x, & x < 0 \\ \cos x - x, & x \geq 0 \end{cases}$

$\therefore f'(x) = \begin{cases} -\sin x + 1, & x < 0 \\ -\sin x - 1, & x \geq 0 \end{cases}$

$\therefore f'(0^-) = 1$ and $f'(0^+) = -1$

$\Rightarrow f'(0^-) \neq f'(0^+)$

So, it is not differentiable at $x = 0$.

(c) We have,

$f(x) = \sin|x| + |x|$

$= \begin{cases} \sin(-x) + (-x), & x < 0 \\ \sin(+x) + (+x), & x \geq 0 \end{cases}$

$= \begin{cases} -\sin x - x, & x < 0 \\ \sin x + x, & x \geq 0 \end{cases}$

$\therefore\ f'(x) = \begin{cases} -\cos x - 1, & x < 0 \\ \cos x + 1, & x \geq 0 \end{cases}$

$\therefore f'(0^-) = -1 - 1 = -2$

and $f'(0^+) = 1 + 1 = 2 \Rightarrow f'(0^-) \neq f'(0^+)$

So, it is not differentiable at $x = 0$.

(d) We have,

$f(x) = \sin|x| - |x|$

$= \begin{cases} \sin(-x) - (-x), & x < 0 \\ \sin x - x, & x \geq 0 \end{cases}$

$= \begin{cases} -\sin x + x, & x < 0 \\ \sin x - x, & x \geq 0 \end{cases}$

$\therefore f'(x) = \begin{cases} -\cos x + 1, & x < 0 \\ \cos x - 1, & x \geq 0 \end{cases}$

$\therefore f'(0^-) = -1 + 1 = -0$

and $f^1(0^+) = 1 - 1 = 0$

$\Rightarrow f'(0^-) = f'(0^+)$

So, it is differentiable at $x = 0$.

154. **(d)** We have,

$x = a\left(\cos t + \log\tan\frac{t}{2}\right)$

Now, $\frac{dx}{dt} = a\left\{-\sin t + \frac{1}{\tan\frac{t}{2}} \cdot \sec^2\frac{t}{2} \cdot \frac{1}{2}\right\}$

$= a\left\{-\sin t + \frac{1}{\sin t}\right\}$

$= a\left(\frac{1 - \sin^2 t}{\sin t}\right) = \frac{a\cos^2 t}{\sin t}$

and $y = a\sin t$

$\frac{dy}{dt} = a\cos t$

So, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \frac{a(\cos t \sin t)}{a\cos^2 t} = \tan t$

155. **(b)** We have,

$\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \frac{1}{1 + \tan^2\theta}\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}$

$\left[\because \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right]$

Now, $\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\theta \\ -\tan\theta & 1 \end{bmatrix}^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix}\left(\frac{1}{1 + \tan^2\theta}\right)\begin{bmatrix} 1 & -\tan\theta \\ \tan\theta & 1 \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \frac{1}{1 + \tan^2\theta}\begin{bmatrix} 1 - \tan^2\theta & -2\tan\theta \\ 2\tan\theta & 1 - \tan \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$\therefore a = \cos 2\theta, b = \sin 2\theta$

156. **(c)** We have, $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+2 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix}$

$A^3 = A^2 \times A \begin{bmatrix} 1 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 2+4 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \times 3 \\ 0 & 1 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & 2n \\ 0 & 1 \end{bmatrix}$

157. **(b)** We have, α, β and γ are roots of equation

$x^3 + px + q = 0 \quad \ldots.(i)$

Here,

$\alpha + \beta + \gamma = 0$

$\alpha\beta + \beta\gamma + \gamma\alpha = p$

$\alpha\beta\gamma = -q$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$\begin{vmatrix} \alpha+\beta+\lambda & \beta & \gamma \\ \alpha+\beta+\lambda & \gamma & \alpha \\ \alpha+\beta+\lambda & \alpha & \beta \end{vmatrix} = (\alpha+\beta+\lambda) \begin{vmatrix} 1 & \beta & \gamma \\ 1 & \gamma & \alpha \\ 1 & \alpha & \beta \end{vmatrix}$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$, we get

$(\alpha+\beta+\lambda) \begin{vmatrix} 1 & \beta & \gamma \\ 0 & \gamma-\beta & \alpha-\gamma \\ 0 & \alpha-\beta & \beta-\gamma \end{vmatrix}$

$= (\alpha+\beta+\lambda)[(\gamma-\beta)(\beta-\gamma) - (\alpha-\gamma)(\alpha-\beta)]$

$= 0 \times [(\gamma-\beta)(\beta-\gamma) - (\alpha-\gamma)(\alpha-\beta)] = 0$

158. **(b)** $\begin{vmatrix} \sin x & \cos x & \cos x \\ \cos x & \sin x & \cos x \\ \cos x & \cos x & \sin x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$

$\begin{vmatrix} 2\cos x + \sin x & \cos x & \cos x \\ 2\cos x + \sin x & \sin x & \cos x \\ 2\cos x + \sin x & \cos x & \sin x \end{vmatrix} = 0$

$\Rightarrow (2\cos x + \sin x) \begin{vmatrix} 1 & \cos x & \cos x \\ 1 & \sin x & \cos x \\ 1 & \cos x & \sin x \end{vmatrix} = 0$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$(2\cos x + \sin x) \begin{vmatrix} 1 & \cos x & \cos x \\ 0 & \sin x - \cos x & 0 \\ 0 & 0 & \sin x - \cos x \end{vmatrix} = 0$

$\Rightarrow (2\cos x + \sin x)[1 \cdot (\sin - \cos x)^2] = 0$

$\Rightarrow (2\cos x + \sin x)(\sin x - \cos x)^2 = 0$

$\Rightarrow 2\cos x = -\sin x$ or $\sin x = \cos x$

$\Rightarrow \tan x = -2$ which is not possible as for

$-\frac{\pi}{4} \le x \le \frac{\pi}{4}$, we get $-1 \le \tan x \le 1$

Or $\tan x = 1$

So, $x = \frac{\pi}{4}$. Hence, only one real root exist.

159. **(c)** Prime factors of 450 is $2 \times 3 \times 3 \times 5 \times 5$.

Now, non-prime divisors will be combinations of these.

When two are multiplied,

$2 \times 3 = 6, 2 \times 5 = 10$

$3 \times 3 = 9, 3 \times 5 = 15, 5 \times 5 = 25$

When three are multiplied,

$2 \times 3 \times 3 = 18, 2 \times 3 \times 5 = 30, 2 \times 5 \times 5 \times = 50$

$3 \times 3 \times 5 = 45, 3 \times 5 \times 5 = 75$

When four are multiplied,

$2 \times 3 \times 3 \times 5 = 90, 2 \times 3 \times 5 \times 5 = 150$

$3 \times 3 \times 5 \times 5 = 225$
When five are multiplied,
$2 \times 3 \times 3 \times 5 \times 5 = 450$
Also, 1 is also a non-prime divisor.
So, sum of divisors
$= 9 + 15 + 25 + 30 + 50 + 45 + 75 + 90 + 150 + 225 + 450 + 1 = 1199$

160. **(a)** $\sum_{\substack{1<p<100 \\ p \to \text{prime}}} P! - \sum_{n=1}^{50} (2n!) = (2! + 3! + 5! + 7! + \cdots)$
$-(2! + 4! + 6! + \cdots)$
After 5 ! the number will have 0 at the one's place.
So, they will no effect the last digit of the total sum.
Now last digit of $(2! + 3!) = 2 + 6 = 8$
Last digit of $(2! + 4!) = 2 + 4 = 6$
Last digit of $\sum_{\substack{1<p<100 \\ p \to \text{prime}}} p! - \sum_{n=1}^{50} (2m!) = 8 - 6 = 2$

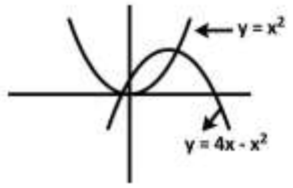
161. **(b)** Let $I = \int_0^1 \frac{1}{\sqrt{1+x^4}} dx$
We have, $0 \le x \le 1$
$\Rightarrow 1 \le 1 + x^4 \le 2$
$\Rightarrow 1 \le \sqrt{1 + x^4} \le \sqrt{2}$
$$\Rightarrow \frac{1}{\sqrt{2}} \le \frac{1}{\sqrt{1 + x^4}} \le 1$$
$$\Rightarrow \int_0^1 \frac{1}{\sqrt{2}} dx \int_0^1 \frac{1}{\sqrt{1 + x^4}} dx \le \int_0^1 1dx$$
$$\Rightarrow \frac{1}{\sqrt{2}} \le \int_0^1 \frac{1}{\sqrt{1 + x^4}} dx \le 1$$
$$\therefore I \in \left[\frac{1}{\sqrt{2}}, 1\right]$$

162. **(b)** Let $I = \int_0^{\pi/2} \log(\tan x)dx$
$$\Rightarrow I = \int_0^{\pi/2} \log\tan\left(\frac{\pi}{2} - x\right) dx$$
$$\Rightarrow I = \int_0^{\pi/2} \log\cot x dx$$
$$\Rightarrow I = \int_0^{\pi/2} \log\frac{1}{\tan x} dx$$
$$\Rightarrow I = -\int_0^{\pi/2} \log\tan x dx$$
$\Rightarrow I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$

163. **(b)** Let $I = \int_{-2}^{2} (ax^3 + bx + c)dx$ ….(i)
$$= \int_{-2}^{2} (a(-2 + 2 - x)^3 + b(-2 + 2)x + c)dx$$
$$\left[\because \int_a^b f(x)dx = \int_a^b f(a + b - x)dx\right]$$
$\Rightarrow I = \int_{-2}^{2} (-ax^3 - bx + c)dx$ ….(ii)
On adding Eqs. (i) and (ii), we get
$$2I = \int_{-2}^{2} 2cdx$$
$I = 4c$
So, value of I depends on the value of c.

164. **(b)** Given equation of curves are $y = x^2$ and
$y = 4x - x^2$

$y - 4 = -4 + 4x - x^2$
$y - 4 = -(x^2 - 4x + 4) = -(x - 2)^2$



Now, required area $= \int_0^2 \{(4x - x^2) - x^2\}dx$

$= \int_0^2 \{(4x - 2x^2)dx$

$= \left[2x^2 - \frac{2}{3}x^3\right]_0^2$

$= \left[\left(8 - \frac{16}{3}\right) - 0\right]$

$= \frac{24 - 16}{3} = \frac{8}{3}$ sq. units

165. **(b)** We have,

$\frac{y}{x}\frac{dy}{dx} = \frac{1 + y^2}{1 + x^2}$

$\frac{ydy}{1 + y^2} = \frac{xdx}{1 + x^2}$

$\frac{2ydy}{1 + y^2} = \frac{2xdx}{1 + x^2}$

Integrating both sides, we get

$\int \frac{2ydy}{1 + y^2} = \int \frac{2xdx}{1 + x^2}$

$\ln(1 + y^2) = \ln(1 + x^2) + \ln c$

$1 + y^2 = c(1 + x^2)$

Now, $y = 2$ at $x = 1$

So,

$5 = c(2)$

$2c = 5$

Therefore, $2(1 + y^2) = 5(1 + x^2)$

166. **(d)** We have,

$\frac{dy}{dx} = (x + y)^2$

Let $x + y = z$

$\Rightarrow \frac{dy}{dx} + 1 = \frac{dz}{dx} \Rightarrow \frac{dy}{dx} = \frac{dz}{dx} - 1$

Now, given equation becomes

$\frac{dz}{dx} - 1 = z^2 \Rightarrow \frac{dz}{dx} = 1 + z^2$

$dx = \frac{dz}{1 + z^2}$

On integrating both sides, we get

$\int dx = \int \frac{dz}{1 + z^2}$

$\Rightarrow c + x = \tan^{-1}(z)$

$\Rightarrow \tan^{-1}z = x + c$

$\Rightarrow \tan^{-1}(x + y) = x + c$

167. **(a)**Let $f(x) = \left(\frac{1}{x}\right)^{2x^2} = x^{-2x^2}$

$\therefore \log f(x) = -2x^2 \log x$

$\Rightarrow \frac{1}{f(x)}f'(x) = -4x\log x - \frac{2x^2}{x}$

$\Rightarrow f'(x) = x^{-2x^2}[-4x\log x - 2x]$

For maximum,

$f'(x) = 0$

$\Rightarrow x^{-2x^2}[-4x\log x - 2x] = 0$

$\Rightarrow -4x\log x - 2x = 0$

$\Rightarrow \log x = -\frac{1}{2}$

$\Rightarrow x = e^{-\frac{1}{2}}$

Again, $f''(x) = x^{-2x^2}[-4x\log x - 2x]^2$

$+x^{-2x^2}[-4\log x - 4 - 2]$

$\therefore f''\left(e^{-\frac{1}{2}}\right) < 0$

So, f(x) has maximum value at $x = e^{-\frac{1}{2}}$

$$\therefore f_{max} = f\left(e^{-\frac{1}{2}}\right) = \left(\frac{1}{\frac{1}{\sqrt{e}}}\right)^{2\left(\frac{1}{\sqrt{e}}\right)^2} = (\sqrt{e})^2\left(\frac{1}{e}\right) = e^{1/e}$$

168. **(a)** Let the required numbers be x. Then,

$f(x) = x - x^2$

$\Rightarrow f'(x) = 1 - 2x$

For a local maxima or local minima, we must have

$f'(x) = 0$

$\Rightarrow 1 - 2x = 0$

$\Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

Now, $f''(x) = -2 < 0$

So, $x = \frac{1}{2}$ is point of local maxima.

Hence, required numbers is $\frac{1}{2}$.

169. **(c)** Given, curve is $y = x\sin x$ ....(i)

At $x = \frac{\pi}{2}$, we have $y = \frac{\pi}{2}$

Slope of the tangent at $x = \frac{\pi}{2}$

From Eq. (i), we get

$\frac{dy}{dx} = \sin x + x\cos x$

$\Rightarrow \frac{dy}{dx} = 1$ at $\frac{\pi}{2}$

Now, equation of tangent will be

$x - \frac{\pi}{2} = y - \frac{\pi}{2}$

$\Rightarrow x = y$

This will intersect X-axis at (0,0).

So, the subtangent will be $= \frac{\pi}{2} - 0 = \frac{\pi}{2}$

170. **(a)** Let $I = \int \frac{10^{x/2}}{\sqrt{10^{-x} - 10^x}}dx$

$= \int \frac{10^{x/2}}{\sqrt{\frac{1}{10^x} - 10^x}}dx = \int \frac{10^{x/2} \cdot 10^{x/2}}{\sqrt{1 - (10^x)^2}}dx$

$= \int \frac{10^x}{\sqrt{1 - (10^x)^2}}dx$

Put $10^x = t \Rightarrow 10^x \log 10 dx = dt$

So, $I = \frac{1}{\log 10}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{\log 10}\sin^{-1}(t) + c = \frac{1}{\log 10}\sin^{-1}(10^x) + c$

171. **(c)** We have,

$\int e^x\left\{\frac{1+\sin x\cos x}{\cos^2 x}\right\}dx$

$= \int e^x\left\{\frac{1}{\cos^2 x} + \frac{\sin x\cos x}{\cos^2 x}\right\}dx$

$= \int e^x(\sec^2 + \tan x)dx = \int e^x\{f'(x) + f(x)\}dx$

[where $f(x) = \tan x$ and $f'(x) = \sec^2 x$]

$= e^x f(x) + c = e^x \tan x + c$

172. **(d)** We have, $\int \frac{x^2+1}{x^4+1}dx$

Divide numerator and denominator by x^2, we get

$\int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}-2+2}dx$

$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}dx$

Let $x - \frac{1}{x} = t$

$\left(1+\frac{1}{x^2}\right)dx = dt = \int \frac{dx}{t^2+2}$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{t}{\sqrt{2}}\right) + c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$

173. **(a)** Given, line segment is $x\cos\alpha + y\sin\alpha = p$

Therefore, it cuts the X-axis and Y-axis at points $(p\sec\alpha, 0)$ and $(0, p\operatorname{cosec}\alpha)$.

Hence, the coordinate of the mid point are

$\left(\frac{p\sec\alpha}{2}, \frac{p\operatorname{cosec}\alpha}{2}\right)$

Therefore, $x = \frac{p\sec\alpha}{2}$ and $y = \frac{p\operatorname{cosec}\alpha}{2}$

or $\cos\alpha = \frac{p}{2x}$ and $\sin\alpha = \frac{p}{2y}$

$\because \sin^2\alpha + \cos^2\alpha = 1$

$\left(\frac{p}{2x}\right)^2 + \left(\frac{p}{2y}\right)^2 = 1$

$\Rightarrow \frac{p^2}{4x^2} + \frac{p^2}{4y^2} = 1 \Rightarrow \frac{1}{4x^2} + \frac{1}{4y^2} = \frac{1}{p^2}$

$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2} \Rightarrow x^{-2} + y^{-2} = 4p^{-2}$

174. **(c)** If the line is inclined at angle 45° with the positive direction of the X-axis i.e. the points are situated at angle $\theta = 45°$ from X-axis at a distance $r = 3\sqrt{2}$ from point $(4, -5)$.

The two points can be given by

$(x_0 \pm r\cos\theta, y \pm r\sin\theta)$

The points are

$\left(4+3\sqrt{2}\cos 45°, -5+3\sqrt{2}\sin 45°\right)$

$= \left(4+3\sqrt{2}\frac{1}{\sqrt{2}}, -5+3\sqrt{2}\times\frac{1}{\sqrt{2}}\right) = (7,-2)$

and $\left(4-3\sqrt{2}\cos 45°, -5-3\sqrt{2}\sin 45°\right)$

$= \left(4-3\sqrt{2}\times\frac{1}{\sqrt{2}}, -5-3\sqrt{2}\times\frac{1}{\sqrt{2}}\right) = (1,-8)$

Thus, the points are $(1,-8)$ and $(7,-2)$.

175. **(c)** The equation $ax^2+2hxy+by^2=0$ can be rewritten as

$a+\frac{2hy}{x}+b\left(\frac{y}{x}\right)^2=0$ ....(i)

Consider,

$px+qy=0$

$px=-qy \Rightarrow \frac{y}{x}=\frac{-p}{q}$

Substituting in Eq. (i), we get

$a+2h\left(-\frac{p}{q}\right)+b\left(\frac{-p}{q}\right)^2=0 \Rightarrow a-\frac{2hp}{q}+b\frac{p^2}{q^2}=0$

$\Rightarrow aq^2-2hpq+bp^2=0$

176. **(b)** We have, $f(x)=\frac{\log_e(1+ax)-\log_e(1-bx)}{x}$

$=\lim\limits_{x\to 0}\frac{\log_e(1+ax)-\log_e(1-bx)}{x}$ $\left[\frac{0}{0}\text{ form}\right]$

Applying L' Hospital rule,

$\lim\limits_{x\to 0}\frac{\frac{a}{1+ax}+\frac{b}{1-bx}}{1}=\frac{\frac{a}{1}+\frac{b}{1}}{1}=a+b$

177. **(b)** We have,

$\lim\limits_{n\to\infty}\frac{(1^2+2^2+\cdots+n^2)\sqrt[n]{n}}{(n+1)(n+10)(n+100)}$

$=\lim\limits_{n\to\infty}\frac{n(n+1)(2n+n)}{6(n+1)(n+10)(n+100)}\left(\lim\limits_{n\to\infty}\sqrt[n]{n}\right)$

$=\lim\limits_{n\to\infty}\frac{2n^2+n}{6(n+10)(n+100)}$ ....(i)

$=\lim\limits_{n\to\infty}\frac{2+\frac{1}{n}}{6\left(1+\frac{10}{n}\right)\left(1+\frac{100}{n}\right)}$

$=\frac{2+0}{6(1+0)(1+0)}$

$=\frac{2}{6}=\frac{1}{3}$

178. **(c)** Consider that there are m collinear points out of total n points in a plane. To construct a triangle we require 3 non-collinear points.

Hence, the number of triangles will be ${}^nC_3-{}^mC_3$.

Since, all the point in the above question are non-collinear, hence $n=10$ and $m=0$

Therefore, the total number of triangles are $= {}^{10}C_3$

$=\frac{10\times 9\times 8}{3!}=\frac{90\times 8}{6}=15\times 8=120$

179. **(a)** Angle traced by the hour hand in 12 h $=360°$

Anlge traced by it in 4 h 25 min , i.e.

$4\frac{5}{12}\text{ h}=\frac{53}{12}\text{ h}$

$=\frac{360}{12}\times\frac{53}{12}=\frac{265}{2}=132\frac{1°}{2}$

Angle traced by it in 25 min

$= \frac{360^\circ}{60^\circ} \times 25 = 150^\circ$

Now, required angle $= \left(150^\circ - 132\frac{1}{2}^\circ\right) = 17\frac{1}{2}^\circ$

180. **(a)** $x^2 + y^2 - 8x = 0$

$\Rightarrow 5^2 + (-7)^2 - 8 \times 5$

$\Rightarrow 25 + 49 - 40 = 34$

$\Rightarrow 34 > 0$

Here, $34 > 0$, so the points $(5, -7)$ lies outside the circle.