

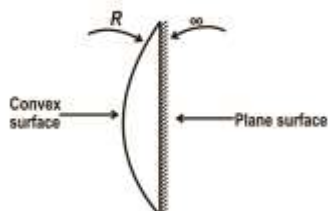
PHYSICS

1. (c) Resultant focal length (f) of the given combination is 0.2 m .

Refractive index of the material, $\mu = 1.5$

From the lens Maker formula, we get

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$



where, R_1 is the radius of curvature of convex surface
and R_2 is the radius of curvature of plane surface

($R_2 = \infty$).

Hence,

$$\frac{1}{f} = (1.5 - 1) \left(\frac{1}{R_1} - \frac{1}{\infty} \right)$$

$$\Rightarrow \frac{1}{0.2} = (0.5) \left(\frac{1}{R_1} \right)$$

$$\Rightarrow R_1 = 0.5 \times 0.2 = 0.1 \text{ m}$$

2. (d) Dimensional formula of Planck's constant = $[M^1 L^2 T^{-1}]$

Similarly,

Boltzmann constant is given

$$K = \frac{\text{Energy}}{\text{Temperature}}$$

$\therefore$ The dimension of K is

$$= \frac{[ML^2 T^{-2}]}{[K]} = [M^1 L^2 T^{-2} K^{-1}]$$

Dimensional formula of force is $[M^1 L^1 T^{-2}]$.

Dimensional formula of linear momentum is $[M^1 L^1 T^{-1}]$.

Dimensional formula of angular momentum is $[M^1 L^2 T^{-1}]$.

Hence, angular momentum and Planck's constant have same dimensional formula.

3. (b) Given, velocity of the balloon = 29 ms^{-1}

When the stone is dropped, then the velocity of the stone

$$u = 29 \text{ ms}^{-1}$$

Time taken to reach the ground, $t = 10 \text{ s}$

From the second equation of motion,

$$s = -ut + \frac{1}{2}gt^2$$

$$= -29 \times 10 + \frac{1}{2} \times 9.8 \times (10)^2$$

$$= -290 + 490 = 200 \text{ m}$$

4. (a) The potential energy due to surface tension is given by the formula

$$U = S \times A$$

where, S is the surface tension and A is the area of film. Since after puncturing, the film will try to minimise its potential energy and hence the surface area. Thus, the thread will become concave towards A.

5. (d) The velocity of sound in a gas at a fixed temperature is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

where, $\gamma = \left(\frac{C_p}{C_v}\right)$ is specific heat ratio and M is the molecular mass of the gas.

Let, velocity of sound in hydrogen, $v_1 = \sqrt{\frac{\gamma RT}{M_1}}$

Velocity of sound in oxygen, $v_2 = \sqrt{\frac{\gamma RT}{M_2}}$

Let, M_1 and M_2 be the molecular mass of hydrogen and oxygen respectively n_1 and n_2 be the moles of hydrogen and oxygen, respectively.

Molecular mass of the mixture,

$$M_{\text{mix}} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2}$$

According to the question, $n_1 = n_2$ at given NTP.

$$M_2 = 16M \quad (\text{given})$$

$$M_{\text{mix}} = \frac{M_1 + 16M_1}{2} = \frac{17M_1}{2}$$

Velocity of sound in mixture, $v_{\text{mix}} = \sqrt{\frac{\gamma RT}{M_{\text{mix}}}}$

$$\Rightarrow v_{\text{mix}} = \sqrt{\frac{2\gamma RT}{17M_1}}$$

$\therefore$ The ratio of velocity of sound in mixture to that of hydrogen

$$\frac{v_1}{v_{\text{mix}}} = \frac{\sqrt{\frac{\gamma RT}{M_1}}}{\sqrt{\frac{2\gamma RT}{17M_1}}}$$

$$\Rightarrow \frac{v_{\text{mix}}}{v_1} = \sqrt{\frac{2}{17}}$$

6. **(d)** Zero order principal maximum contains all incoming wavelets in diffraction pattern, Hence, zero order principal maximum will be white.
7. **(b)** H-polaroid is one of the most common type of polaroid used today. It is prepared by stretching a thin sheet of polyvinyl alcohol and then impregnating it with iodine. In this process, the molecules get oriented in the direction of the applied strain and the material becomes dichronic. Each polaroid sheet is mounted between thin glass plate to provide mechanical support.
8. **(d)** From Coulomb's law, we know

$$\Rightarrow F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

$$\Rightarrow N = \frac{1}{\epsilon_0} \frac{C^2}{m^2}$$

$$\Rightarrow \epsilon_0 = \frac{C^2}{m^2 N}$$

$$\Rightarrow \epsilon_0 = C^2 m^{-2} N^{-1}$$

9. **(c)** Given, capacitance of bigger drop = $1\mu F$

Let R be the radius of bigger drop and r be the radius of smaller drops.

According to the question, $\frac{4}{3}\pi R^3 = 8 \times \frac{4}{3}\pi r^3$

$$\Rightarrow R^3 = 8r^3$$

$$\Rightarrow R = 2r$$

The capacitance of bigger drop ,

$$C = 4\pi\epsilon_0 R$$

$$= 2 \times 4\pi\epsilon_0 r \quad (\because R = 2r)$$

Capacitance of smaller drop, $C' = 4\pi\epsilon_0 r$

$$\text{Thus, } C' = \frac{C}{2}$$

Hence, $C' = \frac{1}{2} \mu F$

10. (a) The magnitude of the resultant force can be given by parallelogram's law of vector addition i.e.,

$$\begin{aligned} \text{Resultant force} &= \sqrt{p^2 + p^2 + 2p \cdot p \cos \theta} \\ \Rightarrow F_{\text{net}} &= \sqrt{2p^2 + 2p^2 \times \cos(120^\circ)} \quad (\because \theta = 120^\circ) \\ &= \sqrt{2p^2 + 2p^2 \left(-\frac{1}{2}\right)} \\ &= \sqrt{p^2} \\ &= p \end{aligned}$$

11. (b) Given that, amplitude and frequency of two waves are same.

Hence, resultant amplitude due to superposition of two waves with phase difference ϕ is given by

$$\begin{aligned} A^2 &= A_1^2 + A_2^2 + 2A_1A_2\cos\phi \\ \Rightarrow A^2 &= A^2 + A^2 + 2A^2\cos\phi \quad (\because A_1 = A_2 = A) \\ \Rightarrow A^2 &= 2A^2 + 2A^2\cos\phi \\ \Rightarrow \cos\phi &= \frac{-1}{2} \\ \Rightarrow \phi &= \frac{2\pi}{3} \end{aligned}$$

12. (a) Given, time interval between echoes = 1 s

Speed of sound in air, $v = 340 \text{ ms}^{-1}$

Let the distance between man and cliff be x .

Time taken by sound to hit cliff and return = $\frac{2x}{v}$

$$\begin{aligned} 1 &= \frac{2x}{v} \\ \Rightarrow x &= \frac{340}{2} \text{ m} = 170 \text{ m} \end{aligned}$$

Thus, distance between cliffs

$$= 170 \times 2 = 340 \text{ m}$$

13. (d) Given, wavelength of light beam,

$$\lambda = 600 \text{ nm}$$

$$= 600 \times 10^{-9} \text{ m}$$

$$\text{Distance between slits, } d = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$\text{Distance between slits and screen, } D = 2 \text{ m}$$

Distance between first dark fringes from the central bright fringe, $2\beta = ?$

$$\text{As we know, fringe width, } \beta = \frac{\lambda D}{d}$$

$$= \frac{600 \times 10^{-9} \times 2}{10^{-3}}$$

$$= 1.2 \times 10^{-3} \text{ m}$$

$$\text{Distance between first dark fringes on either side of central bright fringe } (2\beta) = 2 \times \beta = 2 \times 1.2 \times 10^{-3} \text{ m}$$

$$\Rightarrow 2\beta = 2.4 \text{ mm}$$

14. (a) Given, specific rotation of sugar solution,

$$\alpha = 0.01 \text{ units}$$

$$\text{Length of polarimeter tube, } l = 0.25 \text{ m}$$

$$\text{Optical rotation, } \theta = 0.4 \text{ rad}$$

$$\text{Concentration of solution, } c = ?$$

As we know, specific rotation

$$= \frac{\text{Optical rotation}}{\text{Length of tube} \times \text{Concentration of sol.}}$$

$$\alpha = \frac{\theta}{l \times c}$$

$$c = \frac{\theta}{l \times \alpha}$$

$$= \frac{0.4}{0.01 \times 0.25}$$

$$= 160 \text{ kg/m}^3$$

Concentration of sugar solution = 200 kg/m^{-3} (given)

Percentage impurity of sugar in the sample

$$= \frac{160}{200} \times 100\%$$

$$= 80\%$$

15. (a) Given, potential difference, $V = 45.5 \text{ V}$

Velocity, $v = ?$

Energy acquired by the electron after getting accelerated through the potential difference V is eV, which is equal to its kinetic energy.

Hence, kinetic energy (KE) = $\frac{1}{2} m_e v^2 = eV$

[where, m_e = mass of electron]

$$\Rightarrow v = \sqrt{\frac{2eV}{m_e}}$$

Substituting the given values, we get

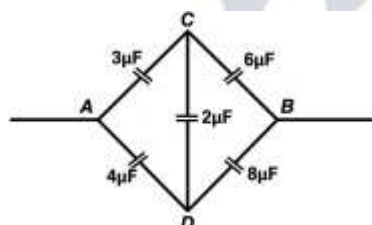
$$\Rightarrow v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 45.4}{9.11 \times 10^{-31}}}$$

$$= \sqrt{16 \times 10^{12}} = 4 \times 10^6 \text{ ms}^{-1}$$

16. (b) Earth is considered to be source of infinite charges having zero potential. When a body is connected to earth, it also acquires same potential as Earth i.e. zero. By this process excess charges flow through the body to the ground or the body acquires the opposite charges from the ground to make the body electrically neutral.

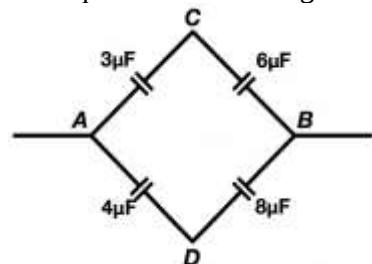
Thus, electrons flow from earth to the body to neutralise the positively charged body.

17. (d) The given combination is a balanced condition of Wheatstone bridge because potential at C and D points is same.



Hence, the $2 \mu \text{ F}$ capacitor can be neglected.

The equivalent circuit is given by the figure below



In the upper arm capacitors $3 \mu \text{ F}$ and $6 \mu \text{ F}$ are connected in series and similarly in lower arm capacitors $4 \mu \text{ F}$ and $8 \mu \text{ F}$ are connected in series. The equivalent capacitance is given by

$$\frac{1}{C_1} = \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6}$$

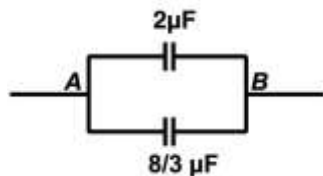
$$\Rightarrow C_1 = 2 \mu \text{ F}$$

and

$$\frac{1}{C_2} = \frac{1}{4} + \frac{1}{8} = \frac{2+1}{8} = \frac{3}{8}$$

$$\Rightarrow C_2 = \frac{8}{3} \mu \text{ F}$$

The reduced circuit with C_1 and C_2 capacitance is given as



Now, C_1 and C_2 are connected in parallel combination. Hence, equivalent capacitance is given by

$$C_{eq} = C_1 + C_2$$

$$C_{eq} = 2 + \frac{8}{3} = \frac{14}{3} \mu F$$

18. (b) Radius of n th-orbit in hydrogen like atom is given by

$$r = \frac{n^2 h^2}{4\pi^2 m k Z e^2}$$

$$\text{i.e., } r \propto \frac{n^2}{Z}$$

For hydrogen, $Z = 1, n = 1$ in ground state

$$\Rightarrow \frac{n^2}{Z} = \frac{1^2}{1} = 1$$

For Beryllium (Be^{+++}), $Z = 4$ orbital is same

$$\therefore \frac{n^2}{Z} = 1$$

$$\Rightarrow n^2 = 1 \times Z \Rightarrow n^2 = 4$$

$$\Rightarrow n = 2$$

Thus, the second level of triply ionised Be^{+++} has same radius as the ground state of hydrogen.

19. (d) Packing fraction is equal to the ratio of mass defect to total number of nucleons.

$$\text{i.e. PF} = \frac{\text{Mass defect}}{\text{No. of nucleons}} = \frac{M - A}{A}$$

20. (a) Let N be the amount of substance left,

N_0 be the total amount of substance.

The amount of substance left after N half-life is given by

$$\frac{N}{N_0} = \frac{1}{2^n}$$

$$\text{Hence, } \frac{30}{240} = \frac{1}{2^n}$$

(Since, decay rate proportional to amount of substance)

$$\Rightarrow 2^n = 8 \Rightarrow n = 3$$

Number of half-life passed = 3

Time taken for the substance to decay from 240 count rate to 30 count rate = 1 h = 60 min

$$\text{Half-life} = \frac{\text{Total time}}{\text{Number of half - life}}$$

$$t_{1/2} = \frac{60}{3} = 20 \text{ min}$$

21. (a) Given, refractive index for blue light, $\mu_b = 1.67$

Refractive index for yellow light, $\mu_y = 1.65$

Refractive index for red light, $\mu_r = 1.63$

Dispersive power, $\omega = ?$

Dispersive power is the ratio of angular dispersion and mean deviation.

The formula of dispersive power is given as

$$\begin{aligned} \omega &= \frac{\mu_b - \mu_r}{\mu_y - 1} \\ &= \frac{1.67 - 1.63}{1.65 - 1} \\ &= \frac{0.04}{0.65} = 0.0615 \end{aligned}$$

22. (d) Given, amount of heat absorbed, $Q = 6 \times 10^4 \text{ J}$

Temperature of reservoir, $T_1 = 227^\circ\text{C} = 227 + 273 \text{ K}$
 $= 500 \text{ K}$

Temperature of sink, $T_2 = 127^\circ\text{C}$
 $= 127 + 273 \text{ K}$
 $= 400 \text{ K}$

Efficiency of Carnot engine, $\eta = 1 - \frac{T_2}{T_1}$

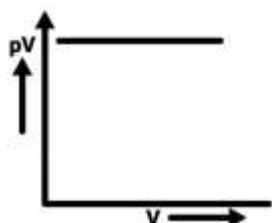
$$= 1 - \frac{400}{500} = \frac{1}{5}$$

Work done by Carnot engine (W)
 $= \text{Efficiency } (\eta) \times \text{heat absorbed } (Q)$

$$\left[\because \eta = \frac{W}{Q} \right]$$

$$W = \frac{1}{5} \times 6 \times 10^4 \text{ J} = 1.2 \times 10^4 \text{ J}$$

23. (b) According to Boyle's law, at constant temperature, the pV relation will be constant. The pV versus p or V graph will be a straight line parallel to X-axis. Hence, the pV versus V graph is given by



24. (b) Tiny water droplets suspended in air have greater density than air. The sunlight when changes medium, undergoes total internal reflection through these droplets thereby splitting in its component colours (since wavelength of each colour is different) and hence, results in rainbow formation. Thus, dispersion and total internal reflection together results in rainbow formation.

25. (c) The combined focal length of plano-convex lens is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

where, $f_1 = \infty$ for the plane surface and $f_2 = f$

$$\therefore \frac{1}{f} = \frac{1}{\infty} + \frac{1}{f}$$

$$\Rightarrow f = f$$

Now, when concave lens of same focal length is joined to first lens, then combined focal length

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2}$$

$$\Rightarrow \frac{1}{F} = \frac{1}{f} - \frac{1}{f} \quad (\because F_1 = f, F_2 = -f)$$

$$\Rightarrow \frac{1}{F} = 0 \Rightarrow F = \infty$$

Thus, the image can be focused on infinity or focus shifts to infinity.

26. (b) Ratio of diameters, $\frac{d_1}{d_2} = \frac{1}{2}$

$$\text{Ratio of lengths, } \frac{l_1}{l_2} = \frac{2}{1}$$

The rate of heat conduction is given by the formula,

$$H = \frac{dQ}{dt} = k \frac{A}{l} \Delta T$$

where, dQ is the amount of heat transferred in dt time, k is coefficient of heat conduction for a particular material.

Since, ΔT is same in both cases, the ratio is

$$\frac{H_1}{H_2} = \frac{A_1 l_2}{A_2 l_1}$$

$$\Rightarrow \frac{H_1}{H_2} = \frac{d_1^2 l_2}{d_2^2 l_1} = \frac{1}{4} \times \frac{1}{2} \Rightarrow R = 1:8$$

27. (c) The pressure inside our body is usually equal to the atmospheric pressure. Since, our mouth is open when blowing air, it will have the same pressure as the atmosphere, Hence, blowing air with open mouth is an example of isobaric process.
28. (d) Air is a completely elastic medium, i.e. it does not have modulus of rigidity, therefore sound wave in air are always longitudinal.
29. (d) White light consists innumerable wavelengths. If monochromatic light is replaced by white light in Young's double slit experiment, then each wavelength form their separate interference pattern. The resultant effect of all these patterns is obtained on the screen.
The waves of all colours reach at mid-point with same or zero phase difference. Therefore, central fringe is white. As fringe width i.e. wavelength increases in order of colours denoted by VIBGYOR, therefore on either side of it some coloured fringes are obtained in reverse order of VIBGYOR. After this the fringes of many colours overlap at each point of the screen and so the screen appears uniformly illuminated.

30. (b) Given, separation between two slits,

$$d = 0.9 \text{ mm} = 0.9 \times 10^{-3} \text{ m}$$

Distance between slits and screen, $D = 1 \text{ m}$

The distance between two consecutive dark or bright fringes is given as β (fringe width) and that between central fringe and first dark fringe on either side is $\frac{\beta}{2}$.

It is given that the spacing between second dark fringe and central fringe $= \beta + \frac{\beta}{2} = \frac{3\beta}{2}$

As we know, $\beta = \frac{\lambda D}{d}$

$$\text{Hence, } \frac{3}{2} \times \frac{\lambda D}{d} = 1 \times 10^{-3} \text{ m}$$

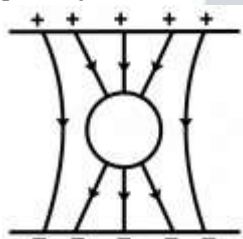
$$\Rightarrow \frac{3}{2} \times \lambda \times \frac{1}{0.9 \times 10^{-3}} = 1 \times 10^{-3} \text{ m}$$

$$\Rightarrow \lambda = \frac{2}{3} \times 10^{-3} \times \frac{0.9 \times 10^{-3}}{1} = 0.6 \times 10^{-6} \text{ m}$$

$\therefore$ Wavelength of monochromatic source of light,

$$\lambda = 600 \times 10^{-9} \text{ m} = 600 \text{ nm}$$

31. (c) When an uncharged sphere of metal is placed inside a charged parallel plate capacitor, charges of opposite polarity are induced on the surface of the sphere.



Negative charge would be induced on the upper surface of the sphere and similarly positive charge would be induced on lower surface of the sphere.

We know that, electric lines of force are always perpendicular to the surface and emerge from positive charge and enter into negative charge. Thus, the correct representation of the lines of force is given in the above figure.

32. (b) Given, resistance of the wire, $R = 6\Omega$

When it is cut in two equal parts, then resistance of the each part of wire is given by

$$r = \frac{R}{2} = 3\Omega \quad (\because R \propto l)$$

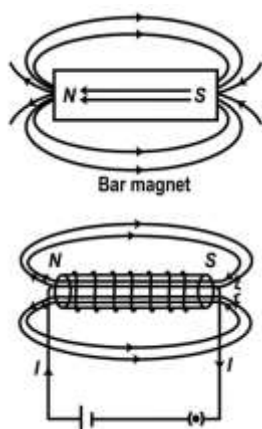
Now, these new resistances are connected in parallel combination. The equivalent resistance is given by

$$\frac{1}{R_{eq}} = \frac{1}{r} + \frac{1}{r}$$

$$\Rightarrow \frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$$\Rightarrow R_{eq} = 1.5\Omega$$

33. (a) The direction of magnetic field in a current carrying straight wire is given by right hand thumb rule. If we point our thumb in a direction of East to West, the direction of magnetic field is found to be towards North, which is given by the curled fingers of our hand.
34. (a) Magnetic field lines of a bar magnet is equivalent to the magnetic field lines due to a solenoid.



Hence, A bar magnet is equivalent to a solenoid carrying current.

35. (a) Excitation energy of hydrogen like ion in its first excitation state, $\Delta E = 40.8\text{eV}$

For the hydrogen like ion, energy in the ground state ($n = 1$),

$$E = \frac{-13.6Z^2}{n^2} \text{eV} = -13.6Z^2 \text{eV}$$

Energy in the first excited state, $E_2 = \frac{-13.6Z^2}{n^2}$

$$= \frac{-13.6Z^2}{4}$$

Excitation energy, $\Delta E = E_2 - E_1$

$$= \left(\frac{-13.6Z^2}{4} + 13.6Z^2 \right) \text{eV}$$

$$= (-3.4 + 13.6)Z^2 \text{eV}$$

$$\Rightarrow 40.8\text{eV} = 10.2Z^2 \text{eV}$$

$$\Rightarrow Z^2 = \frac{40.8}{10.2}$$

$$\Rightarrow Z^2 = 4$$

$$\Rightarrow Z = 2$$

Thus, energy of the electron in the ground state is given

$$\text{as } E_1 = -13.6Z^2 \text{eV}$$

$$\Rightarrow E_1 = -13.6(2)^2 \text{eV}$$

$$\Rightarrow E_1 = -54.4\text{eV}$$

Hence, the energy needed to remove the electron from the ion in ground state (ionisation energy)

$$= -(-54.4)\text{eV} = 54.4\text{eV}$$

36. (d) Threshold wavelength of the metal, $\lambda_0 = 5200\text{\AA}$

$$= 5200 \times 10^{-10} \text{m}$$

$$\text{Frequency of the radiation, } \nu_0 = \frac{c}{\lambda}$$

$$= \frac{3 \times 10^8}{5200 \times 10^{-10}}$$

$$= 5.7 \times 10^{14} \text{Hz}$$

$$= 5.7 \times 10^{14} \text{Hz}$$

$$= 5.7 \times 10^{14} \text{Hz}$$

The frequency of infrared radiation is lower than the threshold frequency of the metal. Hence, the surface of the metal is illuminated with monochromatic radiation which is greater than $5.7 \times 10^{14} \text{Hz}$.

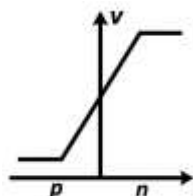
The 50 W UV-lamp is suitable for this purpose.

37. (d) The emitter is always forward biased to enable the majority charge carriers to cross the emitter-base junction and collector-base junction is in reverse biased to collect the charge carriers, so that current flows through the

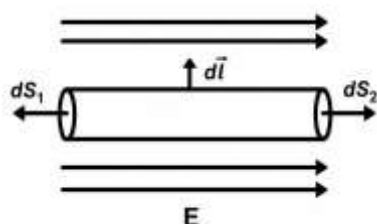
transistor.

Hence, the emitter-base junction of a transistor is forward biased, while collector-base junction is reverse biased.

38. (a) Potential across the p – n junction varies symmetrically linear, having p-side negative and n-side positive. In a forward biased p – n junction diode, the repulsion of holes and electrons takes place which decreases width of potential barrier by striking the combination of holes and electrons. The width of depletion region in forward biased is less than the reversed biased. Hence, the graph of the potential barrier in the depletion region is given by



39. (c) The given diagram represents a cylinder placed in a uniform electric field.



Let ϕ_1 and ϕ_2 be the flux coming out of a circular cross-sectional surface. ϕ_3 is flux through the curved surface. Since, normal to a curved surface is always 90° to electric field E , so

$$\phi_3 = E d s_3 \cos \theta = E d s_3 \cos 90^\circ = 0$$

ϕ_1 and ϕ_2 are equal and opposite to each other, hence $\phi_1 + \phi_2 = 0$.

So, total flux through the surface of cylinder is zero.

40. (c) Given, power of bulb A, $P_A = 60 \text{ W}$ Power of bulb B, $P_B = 100 \text{ W}$

Since, both bulbs are connected to the same power source in parallel combination. Thus,

$$P_A = \frac{V^2}{R_A} = 60 \text{ W and } P_B = \frac{V^2}{R_B} = 100 \text{ W}$$

$$R_A = \frac{V^2}{60} \Omega \text{ and } R_B = \frac{V^2}{100} \Omega$$

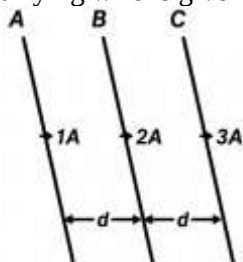
$$\text{Current in bulb A, } I_A = \frac{\text{Potential}}{\text{Resistance}}$$

$$\Rightarrow I_A = \frac{V}{\frac{V^2}{60}} \Rightarrow I_A = \frac{60}{V}$$

$$\text{Similarly, current in bulb B, } I_B = \frac{V}{\frac{V^2}{100}} = \frac{100}{V}$$

It is clear that $I_B > I_A$.

41. (b) Magnetic field at any point due to a straight current carrying wire is given by



$$B = \frac{\mu_0 I}{2\pi d}$$

where, d is the distance of the point from the wire and I_1 is current flowing in the wire.

Force due to this magnetic field on current carrying wire is given by

$$F = BIL$$

where, L is the length of the wire.

$\therefore$ Force on wire B due to wire A, $F_A = BIL$

$$\Rightarrow F_A = \frac{\mu_0 \times 1}{2\pi d} \times 2 \times L$$

Force on wire B due to wire C, $F_C = BIL$

$$\Rightarrow F_C = \frac{\mu_0 \times 3}{2\pi d} \times 2 \times L$$

It is clear that $F_C > F_A$. Hence, net force is towards wire C.

42. (b) Ferromagnetic substances like iron, nickel and cobalt obey Curie-Weiss law above Curie temperature.

43. (c) EMF induced in circular coil of self-inductance L is given by $\varepsilon = -L \frac{dI}{dt}$

EMF is equal to the potential difference dimensionally.

Hence,

$$\varepsilon = V$$

$$\text{We know, } V = \frac{W}{q}$$

Thus, dimensional formula of potential difference is given as

$$V = \frac{[M^1 L^2 T^{-2}]}{[AT]}$$

$$= [M^1 L^2 T^{-3} A^{-1}]$$

The self-inductance is given by, $L = V \times \frac{dt}{dI}$

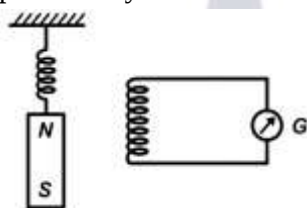
Dimensional formula of self-inductance is given by,

$$L = [M^1 L^2 T^{-3} A^{-1}] \times \frac{[T]}{[A]}$$

Dimensional formula of self-inductance,

$$L = [M^1 L^2 T^{-2} A^{-2}]$$

44. (d) The magnet is attached to the spring. This magnet will execute simple harmonic motion. When a coil is placed in such a way that magnet moves in and out of the coil, an emf is induced in the coil which opposes the movement of magnet in the coil. Due to this induced emf galvanometer will show deflections to the left and right periodically.



As the emf is induced in the coil eddy currents are also induced in the magnet, which lowers down the total magnetic strength of the magnet. Due to this reason, the magnetic flux linked with the coil decrease and hence amplitude of the deflections produced in the galvanometer also decreases.

45. (b) Given, resistance of the galvanometer, $G = 40\Omega$

Current in the galvanometer,

$$I_g = 10 \text{ mA} = 10 \times 10^{-3} \text{ A}$$

Maximum reading of voltmeter, $V = 50 \text{ V}$

The galvanometer can be converted into voltmeter by connecting a shunt resistance (S) in the series.

Voltage in the voltmeter is given by

$$V = I_g(G + R)$$

$$\therefore 50 = 10 \times 10^{-3}(40 + R)$$

$$\Rightarrow 5000 = 40 + R$$

$$\Rightarrow R = 4960\Omega$$

46. (c) R_1 is connected with 10Ω resistance. According to question, a balanced point is obtained at 50 .

$$\text{Thus, } \frac{R_1 + 10}{R_2} = \frac{50}{50}$$

$$\Rightarrow R_2 = R_1 + 10 \quad \dots(i)$$

When 10Ω is removed the balanced point shifts to 40 cm .

$$\text{Thus, } \frac{R_1}{R_2} = \frac{40}{60}$$

$$\Rightarrow R_2 = \frac{3}{2} R_1 \quad \dots(ii)$$

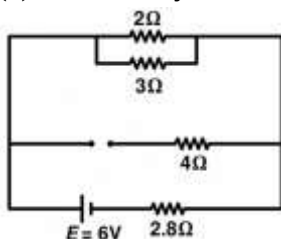
From Eqs. (i) and (ii), we get

$$\frac{3}{2} R_1 = R_1 + 10$$

$$\Rightarrow \frac{1}{2} R_1 = 10$$

$$\Rightarrow R_1 = 20\Omega$$

47. (a) In the steady state analysis capacitor acts as an open circuit. Therefore, the equivalent circuit is given as



Thus, current in the 4Ω resistor is zero. 2Ω and 3Ω resistors are parallel to each other. Thus, the resistance in the upper branch is given by

$$R = \frac{2 \times 3}{2 + 3} = \frac{6}{5} \Omega$$

The upper branch resistor are connected to 2.8Ω resistor in series. Therefore, equivalent resistance in the circuit is given by

$$R_{eq} = 2.8 + \frac{6}{5}$$

$$= \frac{14.0 + 6}{5} = 4\Omega$$

$$\text{Current, } I = \frac{\text{Voltage}}{\text{Resistance}} = \frac{6}{4} = 1.5 \text{ A}$$

Current in the 2Ω resistor is given by current division rule

$$\text{i.e. } I_{2\Omega} = I \times \frac{3\Omega}{2\Omega + 3\Omega}$$

$$\Rightarrow I_{2\Omega} = 1.5 \times \frac{3}{5} = 0.9 \text{ A}$$

48. (a) Given, area of rectangular coil, $A = 25 \text{ cm} \times 10 \text{ cm}$

$$\Rightarrow A = 250 \text{ cm}^2 = 0.025 \text{ m}^2$$

No. of turns in the coil, $N = 300$

Magnetic field strength, $B = 4 \times 10^{-2} \text{ T}$

Peak voltage will be induced when the longer side of the coil moves perpendicular to the magnetic field.

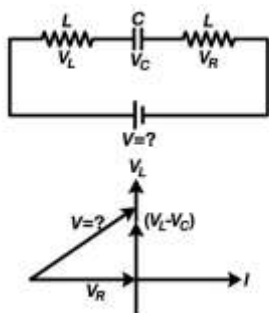
$$\omega = 2\pi\nu = 2\pi \times 50 = 100\pi$$

Induced emf is given by

$$\varepsilon = NBA\omega$$

$$= 300 \times 40 \times 10^{-2} \times 0.025 \times 100\pi = 3\pi \text{ V}$$

49. (a) The L-C-R circuit in series combination and phasor diagram is given below



According to the question,

Potential difference across resistor, $V_R = 40 \text{ V}$

Potential difference across inductor, $V_L = 60 \text{ V}$

Potential difference across capacitor, $V_C = 30 \text{ V}$

Magnitude of potential difference supplied is given by

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{40^2 + (60 - 30)^2}$$

$$= \sqrt{(40)^2 + (30)^2} = \sqrt{2500}$$

$$= 50 \text{ V}$$

50. (c) Given, radius of the coil = 0.1 m

No. of turns in the coil = 10

Horizontal component of magnetic field,

$$B_H = 0.314 \times 10^{-4} \text{ T}$$

The magnetic field at the centre of current carrying coil is given by the formula, $B = \frac{\mu_0 n I}{2R}$

According to question, magnetic field due to the coil obtains neutral point, in earth's magnetic field.

Hence, horizontal component of magnetic field,

$$B_H = \frac{\mu_0 n I}{2R}$$

$$\text{Thus, } I = \frac{2RB_H}{\mu_0 n}$$

$$\Rightarrow I = \frac{2(0.1)(0.314 \times 10^{-4})}{10 \times 4 \times (3.142) \times 10^{-7}} = 0.5 \text{ A}$$

51. (c) The spectrum obtained from the chromosphere of the sun at the time of total is line emission spectrum. The spectrum obtained is of $5876 \text{ \AA}$ wavelength from D_3 line in almost yellow colour.

52. (c) Heavy water is compound that contains heavy hydrogen or deuterium and oxygen. Heavy water is also called deuterium oxide D_2O .

53. (b) The nuclear reactor at Kaiga is a power reactor. The use of nuclear reactor for electricity and power generation classifies it as a power reactor.

54. (c) The work done by a force in moving a body is

$$W = F \cdot s$$

where, s is the displacement vector.

In circular path, the force is centripetal, thus always acting towards the centre along the radial direction.

However, the displacement is always along tangent to the circle at the instantaneous position of object.

Hence, $\theta = 90^\circ$

$$\text{Thus, } W = F \cdot s = F \cos 90^\circ = 0$$

55. (b) Given, velocity of the bullet $v = 100 \text{ ms}^{-1}$

Let, thickness of one plank, $t = s$

According to third equation of motion, we get

$$v^2 - u^2 = 2as$$

$$(0) - (100)^2 = 2 \times a \times (2s)$$

($\because$ two planks are penetrated)

$$\Rightarrow -1000 = 4as \quad \dots(i)$$

When velocity of bullet is doubled, $v = 200 \text{ ms}^{-1}$

Applying third equation of motion again, we get

$$\Rightarrow 0 - (200)^2 = 2 \times a \times (ns)$$

(where, n is number of planks)

$$-40000 = 2as \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{-40000}{-10000} = \frac{2as}{4as}$$

$$\Rightarrow n = 8$$

Hence, 8 planks can be penetrated by the same bullet if speed is doubled.

56. (c) Let m_1 be the mass of 1st body, $m_1 = 1 \text{ kg}$

m_2 be the mass of 2nd body, $m_2 = 2 \text{ kg}$

The relation, between kinetic energy and momentum is given by

$$K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

where, p is the momentum of the body. Since, momentum of both bodies are same.

$$\text{Hence, } \frac{k_1}{k_2} = \frac{p^2}{2m_1} \times \frac{2m_2}{p^2}$$

$$\frac{k_1}{k_2} = \frac{2}{1}$$

Thus, the ratio of their kinetic energies is 2: 1.

57. (a) Pitch is dependent on the frequency of the source of sound whereas loudness depends upon the amplitude of sound wave. Hence, the loudness and pitch of a sound note depends on intensity and frequency.
58. (c) Open windows and doors are considered to be perfect black body, hence absorption coefficient of an open window and door is 1 .
59. (a) In Melde's experiment, the variation in frequency is created if the tension is increased in the string. Melde's showed that mechanical waves also undergo phenomenon of interference. To do this a resonance in tuning fork and frequency of standing waves was developed.
- Hence, the frequency of the tuning fork and the frequency of the waves in the string are in the ratio 1: 1.

60. (b) Given, speed of sound in air, $c = 300 \text{ ms}^{-1}$

Difference between apparent frequency during approach and apparent frequency during recession = 2%

$$\text{Thus, frequency during approach - frequency during recession} = \frac{2}{100} \times f_0$$

$$\Rightarrow f_0 \left(\frac{c+v}{c} \right) - f_0 \left(\frac{c-v}{c} \right) = \frac{2}{100} f_0$$

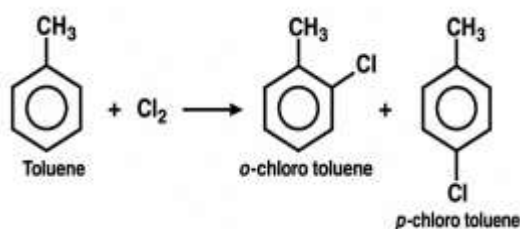
where, c is speed of sound in air and v is the velocity of source.

$$\Rightarrow \frac{2v}{300} = \frac{2}{100}$$

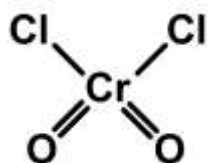
$$\Rightarrow v = 3 \text{ ms}^{-1}$$

CHEMISTRY

61. (a) Ozone in stratosphere is depleted by number of pollutants like CFC 's (CF_2Cl_2), nitrogen oxide, CCl_4 , Cl_2 etc.
62. (c) Linolenic acid is an unsaturated fatty acid. It is a carboxylic acid with 18 -carbon chain having three double bond.
63. (c) When chlorine is passed through boiling toluene, mixture of o and p-chloro toluene is formed.

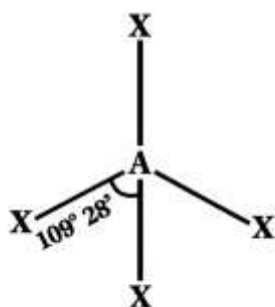


64. (b) The standard temperature used in thermo chemical calculation is 25°C or 298 K
 $K = T + 273 = 25 + 273 = 298 \text{ K}$
65. (c) Density is an intensive property. It is independent of the quantity of substance. While, enthalpy, entropy and mass are extensive properties.
66. (c) Schiff's reagent contains rosaniline ($\text{C}_{19}\text{H}_{18} \text{N}_3\text{Cl}$). It is used to check the presence of aldehyde in the given compound by giving purple or magenta colour.
67. (d) Chromyl chloride is inorganic compound with formula CrO_2Cl_2 . It is reddish compound that is volatile at room temperature. It's structure is



68. (c) Horn silver is a halide ore. The formula for horn silver is AgCl .

69. (b) Tetrahedral structure is formed by sp^3 hybridisation. Here, four sp^3 -hybridised orbitals are arranged in tetrahedral structure.



Tetrahedral geometry

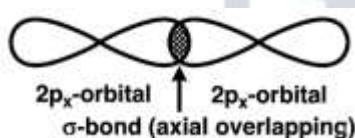
$$\Rightarrow f_0 \left(\frac{c+v}{c} \right) - f_0 \left(\frac{c-v}{c} \right) = \frac{2}{100} f_0$$

where, c is speed of sound in air and v is the velocity of source.

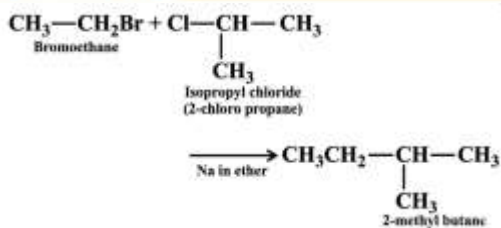
$$\Rightarrow \frac{2v}{300} = \frac{2}{100}$$

$$\Rightarrow v = 3 \text{ ms}^{-1}$$

70. (c) NO^+ ligand is called nitrosonium ligand. It is positive type of ligand, which donate pair of electrons to central metal atom and forms coordinate bond with them.
71. (a) A compound in which the complex ion carries positive charge is called cationic complex.
For hexaamino platinum chloride, chemical formula is $[\text{Pt}(\text{NH}_3)_6]\text{Cl}_4$.
 $\therefore$ The complex ion $[\text{Pt}(\text{NH}_3)_6]^{4+}$ is positively charge.
72. (b) $2p_x$ atomic orbital undergoes linear combination with $2p_x$ -orbital. For linear combination, the combining orbitals should have same energy and same orientation of overlapping of p_x -orbital.

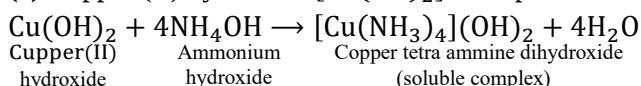


73. (b) $[A_0] = 0.1$
 $[A_t] = 0.01$
 $t = 100 \text{ s}$
1st order reaction,
 $k = \frac{2303}{t} \log \frac{[A_0]}{[A_t]}$
 $= \frac{2303}{100} \log \frac{0.1}{0.01} = 0.02303 \text{ s}^{-1}$
74. (d) When Δn_g i.e. difference in gaseous moles of product and reactant is zero, then pressure has no effect and it is equilibrium point for reaction.
 $\text{N}_2(\text{g}) + \text{O}_2(\text{g}) \rightleftharpoons 2\text{NO}(\text{g})$
 $\Delta n_g = 2 - (1 + 1) = 0$
 $\therefore$ Pressure has no effect for this reaction.
75. (d) Conductivity of a solution is not affected by addition of ethanol as it is a covalent compound and it do not dissociate into ions to change the number of ions in the solution.
76. (d) Lowering in vapour pressure is a colligative property so, the number of particle (i.e. ion) increase, which results in decrease of vapour pressure. The lowering in vapour pressure is maximum for $0.1\text{MK}_4[\text{Fe}(\text{CN})_6]$, as it dissociate into free ions i.e. four K^+ and one $[\text{Fe}(\text{CN})_6]^-$.
77. (b) The given reaction takes place as follows :

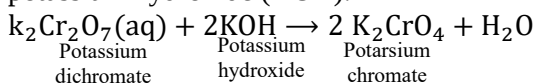


78. (c) NH_3 (ammonia) gas is dried by using sodalime, because NH_3 itself is basic in nature and only basic compound can be used to dry NH_3 , while conc. H_2SO_4 , anhy. CaCl_2 and P_2O_5 can not react with ammonia.

79. (c) Copper (II) hydroxide $[\text{Cu}(\text{OH})_2]$ is amphoteric in nature and is soluble in excess of ammonium hydroxide.



80. (a) Potassium dichromate ($\text{K}_2\text{Cr}_2\text{O}_7$) can be converted into potassium chromate (K_2CrO_4) by adding potassium hydroxide (KOH).



81. (b) Equivalent moles of NaOH = Equivalent moles of acid

$$\begin{aligned}
 N_1 V_1 &= \frac{\text{Weight}}{\text{Equivalent mass}} \\
 40 \times 10^{-3} \times 0.125 &= \frac{0.5}{\text{Equivalent mass}}
 \end{aligned}$$

$\therefore$ Equivalent mass of acid = 100

82. (d) $M_1 = 5$ L and $\text{pH} = 12$

$$\text{pOH} = 14 - \text{pH} = 14 - 12 = 2$$

$$[\text{OH}^-] = 10^{-\text{pOH}} = 10^{-2}$$

$$\therefore \text{Moles of NaOH} = M_1 V_1 = 5 \times 10^{-2} \text{ moles}$$

$$\therefore \text{Mass of NaOH} = \text{moles} \times \text{molar mass}$$

$$= 5 \times 10^{-2} \times 40 = 2 \text{ g}$$

83. (a) Equivalent moles of oxalic acid = Equivalent moles of KMnO_4

$$= N_1 V_1 = 25 \times 0.2 \times 10^{-3} = 5 \times 10^{-3} = 0.005$$

$$\frac{\text{Weight of oxalic acid}}{\text{Eq. weight of oxalic acid}} = 0.005$$

$$\text{Weight of oxalic acid} = 0.05 \times 63 = 0.315 \text{ g}$$

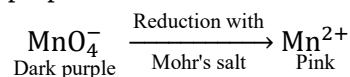
$$\therefore 0.315 \text{ g in 50 cc of oxalic acid.}$$

In 500 cc solution, mass of oxalic acid is

$$= 0.315 \times \frac{500}{50} = 3.15 \text{ g}$$

84. (a) Pure water is neutral as it has equal concentration of H^+ and OH^- ions. It has $\text{pH} = 7$.

85. (b) In the titration of Mohr's salt with KMnO_4 , the indicator used is KMnO_4 itself. Because KMnO_4 is very intense purple in colour. When, the solution changes to pink, end point is noted.



86. (d) The relationship between half-life of a reaction and order of reaction is

$$t_{1/2} \propto \frac{1}{a^{(n-1)}}$$

$$87. (a) \frac{p^\circ - p_s}{p^\circ} = \frac{w_2 M_1}{M_2 w_1}$$

Where, w_2 and M_2 are weight and molecular mass of solute.

M_1 and w_1 are molecular mass and weight of solvent.

$$\frac{\Delta p}{p^\circ} = \frac{6}{60} \times \frac{18}{90} = \frac{1}{50} = 0.02$$

88. (c) Molar mass of sucrose ($\text{C}_{12}\text{H}_{22}\text{O}_{11}$) = 342

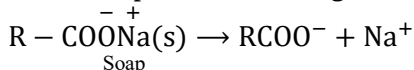
$$\text{Molality} = \frac{\text{Weight of sucrose}}{\frac{\text{Molar mass of sucrose}}{\text{Mass of solvent (g)}}} \times 1000$$

$$= \frac{6.84 \times 1000}{342 \times 200} = 0.1\text{M}$$

89. (a) NaCl on dissociation gives Na^+ and Cl^- ions as follows:



While, soap in the solution gives RCOO^- and Na^+ ions as follows:



$\therefore$ Solubility of soap decrease due to common ion (Na^+) present in the solution due to Le-Chatelier's principle, reaction shifts towards the reactant side.

90. (a) Concentrations of ions in 0.1 M aluminium sulphate $[\text{Al}_2(\text{SO}_4)_3]$ is

$$= 0.1[2 \times (\text{Al}^{3+}) \text{ ions} + 3 \times (\text{SO}_4^{2-}) \text{ ions}]$$

$$= 0.1[2 \times 1 + 3 \times 1] = 0.1 \times 5 = 0.5\text{M}$$

Concentrations of ions in 0.1 M potassium nitrate (KCl)

$$\text{is} = 0.1(1 \text{ mole of } \text{K}^+ + 1 \text{ mole of } \text{Cl}^-)$$

$$= 0.1 \times 2 = 0.2\text{M}$$

$$\text{Concentration of ions in 0.1 M magnesium chloride (MgCl}_2\text{) is} = 0.1(1 \text{ mole of } \text{Mg}^{2+} + 2 \text{ moles of } \text{Cl}^-)$$

$$= 0.1 \times 3 = 0.3\text{M}$$

$$\text{Concentration of ions in 0.1 M barium chloride (BaCl}_2\text{) is} = 0.1(1 \text{ mole of } \text{Ba}^{2+} + 2 \text{ moles of } \text{Cl}^-)$$

$$= 0.1 \times 3 = 0.3\text{M}$$

$\therefore$ Order of concentration is

$$0.1\text{MAl}_2(\text{SO}_4)_3 > 0.1\text{MKCl} > 0.1\text{MgCl}_2$$

$$= 0.1\text{MBaCl}_2$$

$\therefore$ Osmotic pressure is colligative property and, it depends on concentration of ions.

$\therefore$ Order of osmotic pressure is

$$0.1\text{M}(\text{Al}_2\text{SO}_4)_3 > 0.1\text{MKCl} > 0.1\text{MMgCl}_2 > 0.1\text{MBaCl}_2$$

91. (d) As reaction is completed 50% so, $t_{1/2} = 30 \text{ min}$.

For first order reaction

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{30} = 0.0231 \text{ min}^{-1}$$

92. (b) The ebullioscopic constant (K_b) is the elevation in boiling point produced by 1 molal solution i.e.

$$\Delta T_b = K_b m$$

When, $m = 1$

$$K_b = \Delta T_b$$

93. (c) Molecular mass of glucose = 180 g/mol

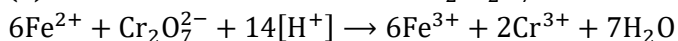
$$\text{Mass of water} = 50 \text{ g} = 0.05 \text{ kg}$$

$$\text{Molality (m)} = \frac{\text{Mass}}{\text{Molar mass} \times \text{mass of solvent (in kg)}}$$

$$0.3 = \frac{\text{Mass}}{160 \times 0.05}$$

$$\text{Mass} = 2.7 \text{ g}$$

94. (b) The reaction of Mohr's salt and $\text{K}_2\text{Cr}_2\text{O}_7$ is as follow:



n-factor of $\text{Cr}_2\text{O}_7^{2-}$ is 6 and n-factor of Fe^{2+} is 1.

As n-factor for Fe is 1 so,

$$\text{Molarity} = \text{Normality} \text{ Concentration of Mohr's salt}$$

$$= 0.08\text{M}$$

Moles in 25 mL of 0.08 M Mohr's salt

$$= 0.08 \times \frac{25}{1000} = 0.002$$

$\therefore$ Moles in 500 cc of Mohr's salt

$$= 0.002 \times \frac{500}{25} = 0.04$$

Hence, mass of Mohr's salt in 500 cc

$$= 0.04 \times 392 = 15.68 \text{ g}$$

95. (a) For a reaction to be spontaneous change in Gibbs free energy should be negative.

i.e.

$$\Delta G = \Delta H - T\Delta S < 0$$

$$= (-ve) - (+ve) = -ve - ve = -2ve$$

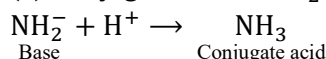
$$\Delta H < 0 \text{ and } \Delta S > 0$$

So, $\Delta H = -ve$ and $\Delta S = +ve$

96. (c) Sodium chloride is in simple closed cubic packing.

Its coordination number is 6.

97. (a) Conjugate acid of NH_2^- is NH_3 .



98. (a) Molar conductivity increases with decrease in concentration as the total volume, (V) of a solution containing one mole of electrolyte also increases.

$\therefore$ 0.005MNaCl have highest molar conductivity.

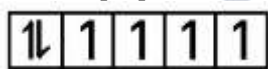
99. (d) In the detection of III group, basic radicals NH_4OH is added after NH_4Cl to decrease the ionisation of NH_4OH due to common ion effect.

100. (a) Just before attaining chemical equilibrium rate of forward reaction decrease and rate of backward reaction increases.

As the concentration of product is slightly more than the equilibrium concentration.

101. (a) Atomic number of Fe(26) = $[\text{Ar}]3d^64s^2$

$$\text{Fe}^{2+} = [\text{Ar}]3d^64s^0$$



$3d^6$

Number of unpaired electrons = 4

$$\text{For } \text{Co}^{2+} = [\text{Ar}]3d^7$$

Number of unpaired electrons = 3

It shows highest magnetic moment. As magnetic moment depends on number of unpaired electrons.

$$\text{For } \text{Cr}^{3+} = [\text{Ar}]3d^3$$

$\therefore$ Number of unpaired electrons = 3

$$\text{For } \text{Ni}^+ = [\text{Ar}]3d^8$$

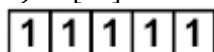
$\therefore$ Number of unpaired electrons = 2

102. (c) In 3d series as we move from Sc to Zn paramagnetism first increases as number of unpaired electron increases. After the elements get half-filled the number of unpaired electron decreases as pairing take place.

$\therefore$ The number of unpaired electron, decreases thus, paramagnetism also decreases.

103. (d) $\text{Fe}(26) = [\text{Ar}]3d^64s^2$

$$\text{Fe}^{3+}(23) = [\text{Ar}]3d^54s^0$$



$$\text{Fe}^{3+} = 3d^5$$

$\therefore$ Number of unpaired electrons = 5

104. (c) In $\text{K}_4[\text{Fe}(\text{CN})_6]$,

$$\text{Oxidation number of Fe} \Rightarrow 4 + x + 6 \times (-1) = 0$$

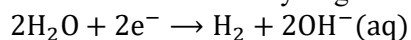
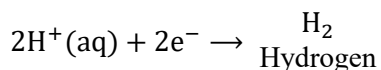
$$x = +2$$

It's IUPAC name is potassium hexacyanoferrate (II).

105. (d) Adsorption of an inert gases on activated charcoal increases with decrease of temperature as it is an exothermic process. The adsorption of gas on solid surface is similar to condensation process, which is exothermic in nature.

106. (b) Brine is a solution of sodium chloride and water. On electrolysis, it gives chlorine (Cl_2), hydrogen (H_2) and sodium hydroxide (NaOH).

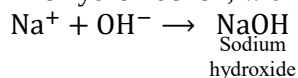
At cathode



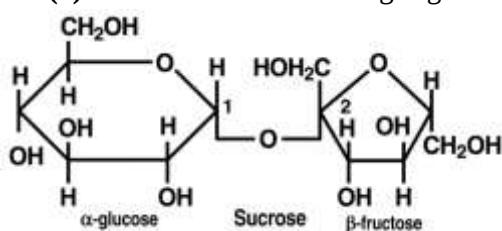
At anode



The hydroxide ion, with the unchanged sodium ion, forms NaOH .



107. (a) Sucrose is a non-reducing sugar due to 1, 2-glycosidic linkage between anomeric carbon atoms.

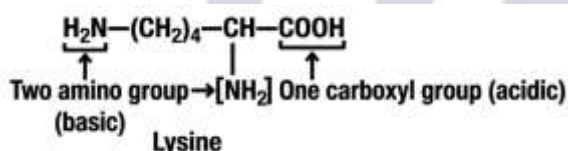


Therefore, there is no free aldehyde or ketone group (reducing of group) as they are involved in glycosidic linkage.

108. (d) Sulphur containing amino acid is cysteine. Its structure is

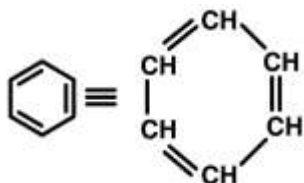


109. (c) Lysine is a basic amino acid as it contains more number of amino groups than carboxyl groups. Its structure is



110. (b) In Molisch's reagent substance used is α -naphthol dissolved in ethanol. It gives positive test with carbohydrates, nucleic acids and glycoproteins by forming purple-coloured product.

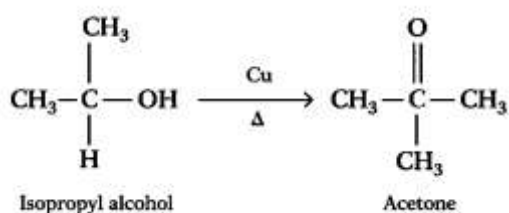
111. (b) In benzene, each carbon have sp^2 hybridisation. The structure of benzene is



Each carbon have three σ -bonds.

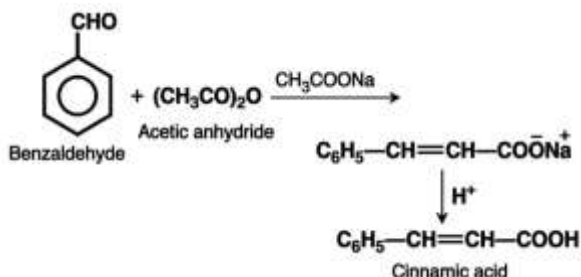
$\therefore$ Hybridisation = sp^2

112. (d) Alcohols are oxidised by passing their vapours over heated copper. 2° alcohol (isopropyl alcohol) is oxidised to acetone i.e. it undergoes dehydrogenation as follows :

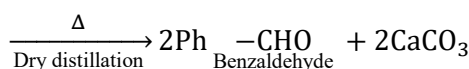
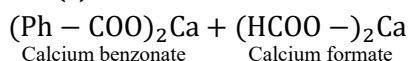


113. (c) The IUPAC name of $\text{CH}_3 - \overset{\text{CH}_3}{\underset{|}{\text{N}}} - \text{CH}_3$ is N,N-dimethyl methanamine

114. (b) The reaction of benzaldehyde with acetic anhydride in presence of fused sodium acetate gives cinnamic acid. This reaction is called Perkin condensation.



115. (c) When a mixture of calcium benzoate and calcium formate is dry distilled, we get benzaldehyde.

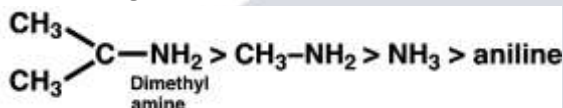


116. (a) Basic strength of amine increase with increases in strength of electron donating group. In dimethyl amine there are two methyl group with +1 -effect and is most basic.

While, methyl amine has only one methyl group and ammonia has no methyl group.

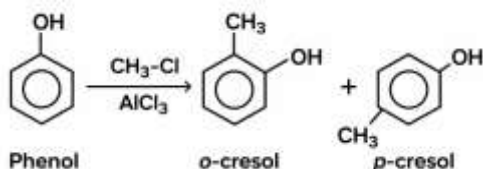
In aniline due to resonance the lone pair of amines are delocalised and is least basic.

Basic strength order

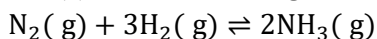


117. (c) Benzoic acid is bi-functional compound as it has two functional groups, carboxylic acid and aromatic ring. Cinnamic acid is tri-functional compound as it has three functional groups, i.e., -COOH, C = C and aromatic ring. Formic acid and acetic acid are mono-functional as they have one functional group i.e., (- COOH group).

118. (d) When phenol is treated with chloromethane in presence of AlCl₃ we get mixture of o and p-cresol. It is an example of Friedel-Craft's alkylation reaction.



119. (c) For the following reaction,



Δn = number of moles of product in gaseous state - number of moles of reactant in gaseous state

$$= 2 - (3 + 1) = -2$$

$$\therefore K_p = K_c(\text{RT})^{\Delta n}$$

$$\therefore K_p = K_c(\text{RT})^{-2}$$

120. (a) According to Faraday's second law,

$$\frac{w_1}{E_1} = \frac{w_2}{E_2}$$

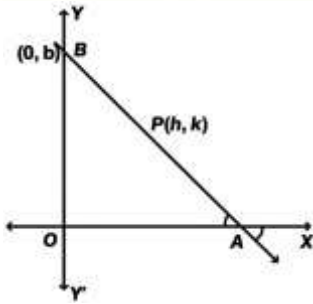
where, w and E are mass and equivalent weight respectively.

$$\frac{0.4}{\frac{63.5}{2}} = \frac{w_2}{108}$$

$$\therefore w_2 = 1.35 \text{ g}$$

MATHEMATICS

121. (d)



We have, equation of line $\frac{x}{a} + \frac{y}{b} = 1$.

Let $P(h, k)$ be the required point.

Now, coordinates A and B are $(a, 0)$ and $(0, b)$ respectively. Since, P is mid-point of AB.

Therefore,

$$(h, k) = \left(\frac{a + 0}{2}, \frac{0 + b}{2} \right)$$

$$\Rightarrow h = \frac{a}{2} \text{ and } k = \frac{b}{2}$$

$$\Rightarrow a = 2h \text{ and } b = 2k$$

Now, it is given that

$$a + b = 4$$

$$\Rightarrow 2h + 2k = 4$$

$$\Rightarrow h + k = 2$$

So, locus of P is $x + y = 2$

122. (a) Let $S_1: x^2 + y^2 - 8x = 0$

$$S_2: x^2 + y^2 - 5x + 7y = 0$$

$$S_3: x^2 + y^2 - 5x + 7y - 1 = 0$$

$$S_4: x^2 + y^2 - 8x + 7y - 2 = 0$$

Now, at $(5, -7)$

$$S_1 = 25 + 49 - 40 = 34 > 0$$

$$S_2 = 25 + 49 - 25 - 49 = 0$$

$$S_3 = 25 + 49 - 25 - 49 - 1 = -1 < 0$$

$$S_4 = 25 + 49 - 40 - 49 - 2 = -17 < 0$$

$\therefore (5, -7)$ lies outside the circle S_1 .

123. (c) Let $S_1: x^2 + y^2 = 9$

and $S_2: x^2 + y^2 + 2\alpha x + 2y + 1 = 0$

Now, centre and radius of S_1 are

$$\text{Centre} = C_1(0, 0)$$

$$\text{Radius} = 3$$

Centre and radius of S_2 are

$$\text{Centre} = C_2(-\alpha, -1)$$

$$\text{Radius} = \sqrt{(-\alpha)^2 + (-1)^2 - 1} = \alpha$$

Since, S_1 and S_2 touch each other internally, then

$$C_1C_2 = |r_2 - r_1|$$

$$\Rightarrow \sqrt{(-\alpha - 0)^2 + (-1 - 0)^2} = |\alpha - 3|$$

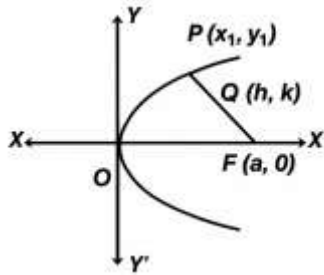
$$\Rightarrow \sqrt{\alpha^2 + 1} = |\alpha - 3|$$

$$\Rightarrow \alpha^2 + 1 = (\alpha - 3)^2$$

$$\Rightarrow \alpha^2 + 1 = \alpha^2 - 6\alpha + 9$$

$$\Rightarrow 6\alpha = 8 \Rightarrow \alpha = \frac{4}{3}$$

124. (c) Let the mid-point of any point $P(x_1, y_1)$ on parabola and focus $F(a, 0)$ be $Q(h, k)$.



Then, $(h, k) = \left(\frac{x_1+a}{2}, \frac{y_1+0}{2}\right)$

$$\Rightarrow h = \frac{x_1+a}{2} \text{ and } k = \frac{y_1}{2}$$

$$\Rightarrow x_1 = 2h - a \text{ and } y_1 = 2k$$

Since, $P(x_1, y_1)$ lies on parabola $y^2 = 4ax$, then

$$(2k)^2 = 4a(2h - a)$$

$$4k^2 = 4a(2h - a)$$

$$k^2 = 2a\left(h - \frac{a}{2}\right)$$

$$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$$

Equation of its directrix is

$$x - \frac{a}{2} = \frac{-a}{2}$$

$$\Rightarrow x = 0$$

125. (b) The tangent at the extremities of a focal chord of a parabola intersect at right angles on the directrix.

Now, equation of parabola is $y^2 = 16x$.

So, its directrix is $x = -4$ or $x + 4$.

So, tangent intersect on the line $x + 4 = 0$.

126. (d) We have,

$$a * 3 = a + b - 5$$

$$\text{Now, } x * 3 = x + 3 - 5 = x - 2$$

$$\text{Again, } 2 * (x * 3) = 5$$

$$\Rightarrow 2 * (x - 2) = 5$$

$$\Rightarrow 2 + x - 2 - 5 = 5$$

$$\Rightarrow x = 10$$

127. (c) (a) $a + b = b + a, \forall a, b \in \mathbb{N}$.

So, addition is commutative in $\mathbb{N}$.

$$(b) (a \times b) \times c = a \times (b \times c), \forall a, b, c \in \mathbb{N}.$$

So, multiplication is associative in $\mathbb{N}$.

$$(c) a * b = a^b \text{ and } b * a = b^a$$

$$\therefore a * b \neq b * a; \forall a, b \in \mathbb{N}$$

So, $*$ is commutative in $\mathbb{N}$ is false.

$$(d) (a + b) + c = a + (b + c), \forall a, b \text{ and } c \in \mathbb{N}.$$

So, addition is associative in $\mathbb{N}$.

128. (c) Let $a = x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{Now, } a \cdot \hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \hat{i} = x$$

$$a \cdot (\hat{i} + \hat{j}) = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j}) = x + y$$

$$\text{and } a \cdot (\hat{i} + \hat{j} + \hat{k}) = (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = x + y + z$$

Now, according to the question,

$$a \cdot \hat{i} = a(\hat{i} + \hat{j}) = a \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

$$\Rightarrow x = x + y = x + y + z = 1$$

$$\Rightarrow x = 1, y = 0, z = 0$$

$$\therefore a = \hat{i}$$

129. (d) We have

$$|a| = |b| = 1, |a + b| = 1$$

We know that,

$$\begin{aligned} |a + b|^2 + |a - b|^2 &= 2|a|^2 + 2|b|^2 \\ \Rightarrow (1)^2 + |a - b|^2 &= 2((1)^2 + (1)^2) \\ \Rightarrow 1 + |a - b|^2 &= 4 \\ \Rightarrow |a - b| &= \sqrt{3} \end{aligned}$$

130. (c) Projection of a on b = $\frac{a \cdot b}{|b|}$

$$\begin{aligned} &= \frac{(3\hat{i} - \hat{j} + 5\hat{k}) \cdot (2\hat{i} + 3\hat{j} + \hat{k})}{|2\hat{i} + 3\hat{j} + \hat{k}|} \\ &= \frac{6 - 3 + 5}{\sqrt{4 + 9 + 1}} = \frac{8}{\sqrt{14}} \end{aligned}$$

131. (a) Let $f^{-1}(8) = x$

$$\begin{aligned} \Rightarrow f(x) &= 8 \\ \Rightarrow x^3 &= 8 \\ \Rightarrow x &= 2 \quad [\because x \in \mathbb{R}] \\ \therefore f^{-1}(8) &= \{2\} \end{aligned}$$

132. (d) (a) When $x = -4$, then

$$4(-4) + 3y = 20 \Rightarrow y = 12$$

But $x = -4 \notin \mathbb{N}$

$$\therefore (-4, 12) \notin R$$

(b) When $x = 5$, then

$$4(5) + 3y = 20 \Rightarrow y = 0$$

$$\therefore (5, 0) \notin R$$

(c) When $x = 3$, then

$$4(3) + 3y = 20 \Rightarrow y = \frac{8}{3}$$

$$\therefore (3, 4) \notin R$$

(d) When $x = 2$, then

$$4(2) + 3y = 20 \Rightarrow y = 4$$

$$\therefore (2, 4) \in R$$

133. (b) We have,

$$\log_{10} 7 = 0.8451$$

$$\text{Let } x = 7^{-20}$$

$$\Rightarrow \log x = -20 \log 7 = -20 \times 0.8451 = -16.902$$

$$\Rightarrow x = 10^{-16.902}$$

Now, -16.902 lies between -16 and -17

So, position of first significant figure of 7^{-20} is 17.

134. (c) We have,

$$\begin{aligned} &= \frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \dots + \frac{1}{[(2 + (n-1)3)][5 + (n-1)3]} \\ &= \frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \dots + \frac{1}{(3n-1)(3n+2)} \\ &= \frac{1}{3} \left[\frac{3}{2 \cdot 5} + \frac{3}{5 \cdot 8} + \frac{3}{8 \cdot 11} + \dots + \frac{3}{(3n-1)(3n+2)} \right] \\ &= \frac{1}{3} \left[\frac{5-2}{2 \cdot 5} + \frac{8-5}{5 \cdot 8} + \frac{11-8}{8 \cdot 11} + \dots + \frac{(3n+2)-(3n-1)}{(3n-1)(3n+2)} \right] \\ &= \frac{1}{3} \left[\frac{1}{2} - \frac{1}{5} + \frac{1}{5} - \frac{1}{8} + \frac{1}{8} - \frac{1}{11} + \dots + \frac{1}{3n-1} - \frac{1}{3n+2} \right] \\ &= \frac{1}{3} \left[\frac{1}{2} - \frac{1}{3n+2} \right] = \frac{1}{3} \left[\frac{3n+2-2}{2(3n+2)} \right] = \frac{n}{6n+4} \end{aligned}$$

135. (b) We have,

$$1! = 1, 4! = 24, 7! = 5040, 10! = 3628800$$

$$11! = 39916800$$

So, last two digit from $10!$ is always 00.

Now, $1! + 4! + 7! = 1 + 24 + 5040 = 5065$

So, ten's digit in given addition is 6 which is divisible by 3 !.

136. (b) We have,

$$\frac{x^2}{2-\lambda} - \frac{y^2}{\lambda-5} - 1 = 0$$

$$\Rightarrow \frac{x^2}{2-\lambda} + \frac{y^2}{5-\lambda} = 1$$

It will represent an ellipse, when

$$2-\lambda > 0 \text{ and } 5-\lambda > 0$$

$$\Rightarrow \lambda < 2 \text{ and } \lambda < 5$$

$$\Rightarrow \lambda < 2$$

137. (d) We have,

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Since, $(-4,0)$ is vertex of the hyperbola, so equation of normal at $(-4,0)$ is $y = 0$.

138. (d) We have,

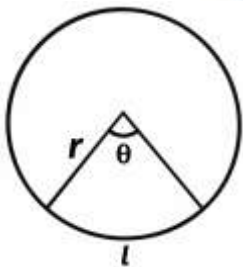
$$p \rightarrow \sim q$$

The contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$ and converse of $p \rightarrow q$ is $q \rightarrow p$.

$\therefore$ Contrapositive of $p \rightarrow \sim q$ is $\sim(\sim q) \rightarrow \sim p \equiv q \rightarrow \sim p$.

Converse of $q \rightarrow \sim p$ is $\sim p \rightarrow q$.

139. (a) It is given that, Perimeter of sector = Length of arc of the semi-circles



$$\Rightarrow 2r + l = \pi r$$

$$\Rightarrow 2r + r\theta = \pi r \quad [\because l = r\theta]$$

$$\Rightarrow r(2 + \theta) = \pi r$$

$$\Rightarrow \theta = \pi - 2$$

140. (c) We have,

$$\tan 67\frac{1}{2}^\circ + \cot 67\frac{1}{2}^\circ$$

$$= \frac{\sin 67\frac{1}{2}^\circ}{\cos 67\frac{1}{2}^\circ} + \frac{\cos 67\frac{1}{2}^\circ}{\sin 67\frac{1}{2}^\circ}$$

$$= \frac{\sin^2 67\frac{1}{2}^\circ + \cos^2 67\frac{1}{2}^\circ}{\cos 67\frac{1}{2}^\circ \sin 67\frac{1}{2}^\circ}$$

$$= \frac{1}{\frac{\cos 67\frac{1}{2}^\circ \sin 67\frac{1}{2}^\circ}{2}}$$

$$= \frac{2}{\sin 135^\circ}$$

$$= \frac{2}{\sin(90^\circ + 45^\circ)}$$

$$= \frac{2}{\cos 45^\circ} = 2 \times \sqrt{2} = 2\sqrt{2}$$

141. (b) We have, equation of hyperbola

$$3x^2 - 2y^2 = 25$$

$$\Rightarrow \frac{x^2}{\left(\frac{5}{\sqrt{3}}\right)^2} - \frac{y^2}{\left(\frac{5}{\sqrt{2}}\right)^2} = 1$$

Equation of conjugate hyperbola is

$$\frac{y^2}{\left(\frac{5}{\sqrt{2}}\right)^2} - \frac{x^2}{\left(\frac{5}{\sqrt{3}}\right)^2} = 1$$

Now, eccentricity of hyperbola,

$$e_1 = \sqrt{1 + \frac{\left(\frac{5}{\sqrt{2}}\right)^2}{\left(\frac{5}{\sqrt{3}}\right)^2}}$$

$$= \sqrt{1 + \frac{3}{2}} = \sqrt{\frac{5}{2}}$$

and eccentricity of conjugate hyperbola,

$$e_2 = \sqrt{1 + \frac{\left(\frac{5}{\sqrt{3}}\right)^2}{\left(\frac{5}{\sqrt{2}}\right)^2}} = \sqrt{1 + \frac{2}{3}} = \sqrt{\frac{5}{3}}$$

$$\text{Now, } e_1^2 + e_2^2 = \frac{5}{2} + \frac{5}{3} = \frac{25}{6} \approx 4$$

142. (d) We have,

$$p^2 - 2q^2 = 1$$

$$\text{Let } p = 3, q = 2$$

$$\text{Then, } p^2 - 2q^2$$

$$= (3)^2 - 2(2)^2 = 9 - 8 = 1$$

$$\therefore p^2 - 2q^2 = 1$$

$$\text{Now, } p^2 + 2q^2 = (3)^2 + 2(2)^2$$

$$= 9 + 8 = 17$$

143. (b) We have,

$$A(\text{adj}A) = 51$$

$$\Rightarrow |A(\text{adj}A)| = |51|$$

$$\Rightarrow |A||\text{adj}A| = 5^3|1| \quad [\because |KB| = K^n|B|]$$

$$\Rightarrow |A||A|^{3-1} = 125 \times 1 \quad [\because |\text{adj}A| = |A|^{n-1}]$$

$$\Rightarrow |A|^3 = 125$$

$$\Rightarrow |A| = 5$$

$$\text{Now, } |\text{adj}A| = |A|^{n-1}$$

$$= |A|^2 \quad [\because n = 3]$$

$$= (5)^2 = 25$$

144. (a) We have,

$$\sin 2x + \cos 4x = 2$$

$$\Rightarrow \sin 2x + 1 - 2\sin^2 2x = 2$$

$$\Rightarrow 2\sin^2 2x - \sin 2x + 1 = 0$$

$$\text{Now, Discriminant} = b^2 - 4ac$$

$$= (-1)^2 - 4(2)(1)$$

$$= 1 - 8 = -7$$

$$< 0$$

So, there is no real root of the given equation.

145. (c) Let $I = \int e^x \cdot x^5 dx$

$$= x^5 \cdot e^x - \int e^x \cdot (5x^4) dx$$

$$\begin{aligned}
 &= x^5 e^x - 5 \int e^x \cdot x^4 dx \\
 &= x^5 e^x - 5 \left[x^4 \cdot e^x - \int e^x \cdot 4x^3 dx \right] \\
 &= x^5 e^x - 5x^4 e^x + 20 \int e^x \cdot x^3 dx \\
 &= x^5 e^x - 5x^4 e^x + 20 \left[x^3 \cdot e^x - \int e^x \cdot 3x^2 dx \right] \\
 &= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60 \int e^x \cdot x^2 dx \\
 &= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60 \left[x^2 \cdot e^x - \int e^x \cdot 2x dx \right] \\
 &= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120 \int e^x \cdot x dx \\
 &= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120 \left[x e^x - \int e^x dx \right] \\
 &= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120x e^x - 120e^x + C \\
 &= e^x [x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120] + C
 \end{aligned}$$

146. (b) We have, $f(x)$ is an even function.

$$\begin{aligned}
 \therefore f(-x) &= f(x) \\
 \Rightarrow f'(-x)(-1) &= f'(x) \\
 \Rightarrow f'(x) + f'(-x) &= 0 \\
 \Rightarrow f'(e) + f'(-e) &= 0
 \end{aligned}$$

147. (a) We have, $\alpha^2 + \alpha + 1 = 0$

$$\begin{aligned}
 \Rightarrow \alpha &= \omega \text{ or } \omega^2 \\
 \text{Now, if } \alpha &= \omega, \text{ then} \\
 \alpha^{31} &= \omega^{31} = (\omega^3)^{10} \cdot \omega = \omega = \alpha \\
 \text{and if } \alpha &= \omega^2, \text{ then} \\
 \alpha^{31} &= (\omega^2)^{31} = \omega^{62} = (\omega^3)^{20} \cdot \omega^2 = \omega^2 = \alpha \\
 \therefore \alpha^{31} &= \alpha
 \end{aligned}$$

148. (c) Let $u = \sin(x^3)$

$$\text{and } v = \cos(x^3)$$

$$\text{Then, } \frac{du}{dx} = \cos(x^3) \cdot (3x^2)$$

$$\text{and } \frac{dv}{dx} = -\sin(x^3)(3x^2)$$

$$\text{Now, } \frac{du}{dv} = \frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)}$$

$$= \frac{3x^2 \cos(x^3)}{-3x^2 \sin(x^3)} = -\cot(x^3)$$

149. (d) Let $a = \hat{i} + \hat{j}$ and $b = \hat{j} + \hat{k}$

Then, unit vector perpendicular to both a and b is $\frac{a \times b}{|a \times b|}$

$$\text{Now, } a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \hat{i} - \hat{j} + \hat{k}$$

$$\text{and } |a \times b| = \sqrt{(1)^2 + (-1)^2 + (1)^2} = \sqrt{3}$$

$$\text{Thus, required vector} = \frac{\hat{i} - \hat{j} + \hat{k}}{\sqrt{3}}$$

150. (b) We have,

$$A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \quad [\because |A| = |A^T|]$$

$$= (-) \begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \quad [R_1 \leftrightarrow R_2]$$

$$= (-)(-)\begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_2 \end{vmatrix} = B[R_1 \leftrightarrow R_3]$$

151. (c) The locus of a point which moves such that the sum of its distances from the two fixed points is a constant is an ellipse.

152. (a) Centroid of $\triangle ABC = \left(\frac{2+8+5}{3}, \frac{3+10+5}{3}\right) = (5, 6)$

153. (d) If $3x^2 + xy - y^2 - 3x + 6y + k = 0$ represents a pair of lines, then

$$\begin{vmatrix} 3 & \frac{1}{2} & -\frac{3}{2} \\ \frac{1}{2} & -1 & 3 \\ -\frac{3}{2} & 3 & k \end{vmatrix} = 0$$

$$\Rightarrow 3(-k-9) - \frac{1}{2}\left(\frac{1}{2}k + \frac{9}{2}\right) - \frac{3}{2}\left(\frac{3}{2} - \frac{3}{2}\right) = 0$$

$$\Rightarrow -3k - 27 - \frac{1}{4}k - \frac{9}{4} = 0$$

$$\Rightarrow -\frac{13}{4}k - \frac{117}{4} = 0$$

$$k = -9$$

154. (b) (a) Put $x = 2, y = 2$ we have

$$(2)^2 + (2)^2 + 5 \times 2 + 5 \times 2 + 12 = 4 + 4 + 10 + 10 + 12 = 40 \neq 0$$

(b) Put $x = 2, y = 2$, we have

$$(2)^2 + (2)^2 - 5 \times 2 - 5 \times 2 + 12 = 4 + 4 - 10 - 10 + 12 = 0$$

Put $x = 3, y = 3$, we get

$$(3)^2 + (3)^2 - 5 \times 3 - 5 \times 3 + 12 = 9 + 9 - 15 - 15 + 12 = 30 - 30 = 0$$

So, both points satisfies the given equation.

(c) Put $x = 2, y = 2$, we get

$$(2)^2 + (2)^2 + 5 \times 2 - 5 \times 2 + 12 = 4 + 4 + 10 - 10 + 12 = 20 \neq 0$$

(d) Put $x = 2, y = 2$, we get

$$(2)^2 + (2)^2 - 5 \times 2 + 5 \times 2 - 12 = 4 + 4 - 10 + 10 - 12 = -4 \neq 0$$

155. (a) We have,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

which is a lower triangular matrix.

Since, characteristic roots of a lower triangular matrix are its diagonal elements.

So, characteristics roots of A are 1, 3, 6.

156. (a) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then

$$A^{-1} = \frac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

So, if $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, then

$$A^{-1} = \frac{1}{4-6} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= -\frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

157. (d) We have, $A = \{-1, 0, 1\}$

Now, let e be the identity element.

Then, $a \times e = a \Rightarrow e = 1 \in A$

Again, a^{-1} be the inverse element.

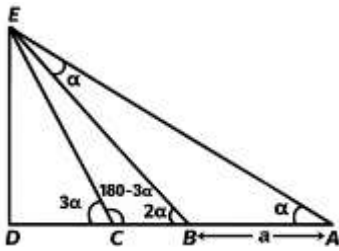
Then, $a \times a^{-1} = e$

$$\Rightarrow a \times a^{-1} = 1 \Rightarrow a^{-1} = \frac{1}{a}$$

Now, for $a = 0$, $a^{-1} = \frac{1}{0} = \infty \notin A$

So, inverse law fails.

158. (c)



Let DE be the height of the tower.

Now, in $\triangle ABE$,

$$\angle AEB = \angle EBC - \angle EAB$$

$$= 2\alpha - \alpha = \alpha$$

$$\therefore \angle AEB = \angle BAE$$

$$\Rightarrow BE = AB = a$$

$$\text{Now, } \angle ECB = 180^\circ - \angle DCE = 180 - 3\alpha$$

$$\text{Now, in } \triangle BEC, \frac{BE}{\sin(180-3\alpha)} = \frac{CE}{\sin 2\alpha}$$

[using sine rule]

$$\Rightarrow \frac{a}{\sin 3\alpha} = \frac{CE}{\sin 2\alpha}$$

$$\Rightarrow CE = \frac{a \sin 2\alpha}{\sin 3\alpha}$$

In $\triangle EDC$,

$$\sin 3\alpha = \frac{DE}{CE}$$

$$\Rightarrow DE = CE \sin 3\alpha$$

$$= \frac{a \sin 2\alpha}{\sin 3\alpha} \times \sin 3\alpha = a \sin 2\alpha$$

Hence, height of tower is $a \sin 2\alpha$.

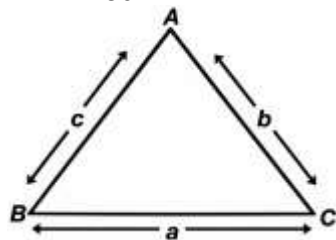
159. (c) It is given that A, B, C are in AP.

$$\therefore 2B = A + C$$

$$\text{Now, } A + B + C = 180^\circ$$

$$\Rightarrow B + 2B = 180^\circ$$

$$\Rightarrow B = 60^\circ$$



Now, using sine rule, we have

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \sin C = \frac{c}{b} \sin B$$

$$= \frac{\sqrt{2}}{\sqrt{3}} \times \sin 60^\circ \quad [\because b:c = \sqrt{3}:\sqrt{2}]$$

$$= \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}}$$

$$\therefore c = 45^\circ$$

$$\text{Now, } A + B + C = 180^\circ$$

$$\Rightarrow A + 60^\circ + 45^\circ = 180^\circ$$

$$\Rightarrow A = 75^\circ$$

160. (a) We have,

$$\sin \left(2 \sin^{-1} \sqrt{\frac{63}{65}} \right)$$

$$= \sin \left\{ \sin^{-1} \left(2 \sqrt{\frac{63}{65}} \sqrt{1 - \left(\sqrt{\frac{63}{65}} \right)^2} \right) \right\}$$

$$[\because 2 \sin^{-1} x = \sin^{-1} 2x \sqrt{1 - x^2}]$$

$$= \sin \left\{ \sin^{-1} \left(2 \sqrt{\frac{63}{65}} \sqrt{1 - \frac{63}{65}} \right) \right\}$$

$$= \sin \left\{ \sin^{-1} \left(2 \sqrt{\frac{63}{65}} \times \sqrt{\frac{2}{65}} \right) \right\} = \sin \left\{ \sin^{-1} \frac{2\sqrt{126}}{65} \right\}$$

$$= \frac{2\sqrt{126}}{65} \quad [\because \sin(\sin^{-1} x) = x]$$

161. (c) We have, $|\sin x| = \cos x$

$$\Rightarrow \sin x = \pm \cos x$$

$$\Rightarrow \tan x = \pm 1$$

$$\Rightarrow \tan x = \tan \left(\pm \frac{\pi}{4} \right)$$

$$\Rightarrow x = n\pi \pm \frac{\pi}{4}$$

162. (d) We have,

$$x^3 - 6x + 9 = 0$$

$$\Rightarrow (x+3)(x^2 - 3x + 3) = 0$$

$$\Rightarrow x+3 = 0 \quad [\because x^2 - 3x + 3 \neq 0, x \in \mathbb{R}]$$

$$\Rightarrow x = -3$$

163. (d) We know that,

$$5^1 = 5$$

$$5^2 = 25$$

$$5^3 = 125 \text{ and so on.}$$

So, unit place is always 5.

So, unit's place of 5^{834} is 5.

164. (d) We have,

$$3^2 = 4(\text{mod } 5)$$

$$(3^2)^2 = 16(\text{mod } 5) = 1(\text{mod } 5)$$

$$(3^4)^{25} = (1)^{25}(\text{mod } 5)$$

$$\Rightarrow 3^{100} = 1(\text{mod } 5)$$

$$\text{Again, } 2^2 = 4(\text{mod } 5)$$

$$(2^2)^2 = 16(\text{mod } 5) = 1(\text{mod } 5)$$

$$(2^4)^{12} = (1)^{12}(\text{mod } 5) = 1(\text{mod } 5)$$

$$2^{18} \cdot 2^2 = 4(\text{mod}5)$$

$$\therefore 3^{100} \times 2^{50} = 1 \times 4(\text{mod}5) = 4(\text{mod}5)$$

165. (a) Let $I = \int \frac{\sin x \cos x}{\sqrt{1 - \sin^4 x}} dx$

Put $\sin^2 x = t \Rightarrow 2 \sin x \cos x dx = dt$

$$\therefore I = \frac{1}{2} \int \frac{dt}{\sqrt{1 - t^2}}$$

$$= \frac{1}{2} \sin^{-1} t + C = \frac{1}{2} \sin^{-1}(\sin^2 x) + C$$

166. (b) Let $I = \int_{-2}^2 (ax^3 + bx + c) dx \dots(i)$

$$\Rightarrow I = \int_{-2}^2 [(a(-2 + 2 - x)^3 + b(-2 + 2 - x) + c)] dx$$

$$\left[\because \int_a^b f(x) dx = \int_a^b f(a + b - x) dx \right]$$

$$= \int_{-2}^2 (-ax^3 - bx + c) dx \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-2}^2 2c dx$$

$$I = c(x^2)_{-2}^2 = 4c$$

So, I depends on value of c.

167. (b) We have,

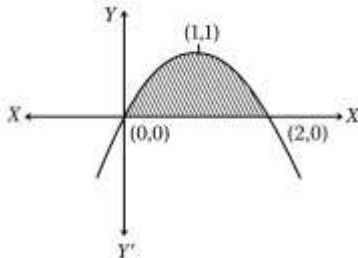
$$y = 2x - x^2$$

$$\Rightarrow y = -(x^2 - 2x)$$

$$\Rightarrow y = -[(x - 1)^2 - 1]$$

$$\Rightarrow y = -(x - 1)^2 + 1$$

$$\Rightarrow (x - 1)^2 = -(y - 1)$$



$$\text{Required area} = \int_0^2 y dx$$

$$= \int_0^2 (2x - x^2) dx$$

$$= \left[x^2 - \frac{x^3}{3} \right]_0^2 = 4 - \frac{8}{3}$$

$$= \frac{4}{3} \text{ sq unit}$$

168. (d) We have,

$$y \frac{dy}{dx} + x = c$$

$$\Rightarrow y \frac{dy}{dx} = (c - x)$$

$$\Rightarrow y dy = (c - x) dx$$

$$\Rightarrow y dy = -(x - c) dx$$

$$\Rightarrow \int y dy = - \int (x - c) dx$$

$$\Rightarrow \frac{y^2}{2} = - \frac{(x - c)^2}{2} + K$$

$$\Rightarrow (x - c)^2 + y^2 = 2K$$

which is a circle with centre $(c, 0)$ on the X-axis.

169. (a) We have,

$$f(x^5) = 5x^3$$

$$\Rightarrow f(x) = 5x^{3/5}$$

$$\Rightarrow f'(x) = 5 \times \frac{3}{5} x^{\frac{3}{5}-1} = 3x^{-\frac{2}{5}}$$

$$= \frac{3}{\sqrt[5]{x^2}}$$

170. (d) We have,

$$f(x) = \begin{cases} 2a - x, & -a < x < a \\ 3x - 2a, & x \geq a \end{cases}$$

At $x = a$

$$\text{LHL} = \lim_{x \rightarrow a^-} (2a - x) = 2a - a = a$$

$$\text{RHL} = \lim_{x \rightarrow a^+} (3x - 2a) = 3a - 2a = a$$

$$f(a) = 3a - 2a = a$$

$$\therefore \text{LHL} = \text{RHL} = f(a)$$

So, $f(x)$ is continuous at $x = a$.

Now, when $x < a$, $f(x) = 2a - x$,

which is continuous for all $x < a$.

Again, when $x > a$, $f(x) = 3x - 2a$, which is continuous for all $x > a$.

So, $f(x)$ is continuous for all x .

Now, at $x = a$

$$\text{LHD} = \lim_{x \rightarrow a^-} \frac{(2a - x) - a}{x - a}$$

$$= \lim_{x \rightarrow a^-} \frac{-(x - a)}{(x - a)} = -1$$

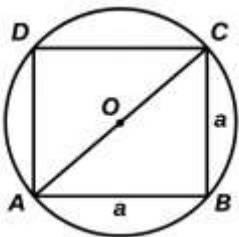
$$\text{and RHD} = \lim_{x \rightarrow a^+} \frac{(3x - 2a) - a}{x - a}$$

$$= \lim_{x \rightarrow a^+} \frac{3(x - a)}{x - a} = 3$$

$$\therefore \text{LHD} \neq \text{RHD}$$

So, $f(x)$ is not differentiable at $x = a$.

171. (c) The area of a rectangle that can be inscribed in a circle will have maximum area when it is a square.



Let a be the side of square ABCD.

$$\text{Then, } AB^2 + BC^2 = AC^2$$

$$\Rightarrow a^2 + a^2 = (4)^2$$

$$\Rightarrow 2a^2 = 16$$

$$\Rightarrow a^2 = 8$$

$\therefore$ Area of square is 8 sq units.

172. (b) Let $Z = x + iy$

$$\text{Then, } \bar{Z} = x - iy$$

Now, it is given that

$$Z = -\bar{Z}$$

$$\Rightarrow x + iy = -(x - iy)$$

$$\Rightarrow x + iy = -x + iy$$

$$\Rightarrow 2x = 0$$

$$\Rightarrow x = 0$$

$\therefore Z$ is purely imaginary.

173. (a) We have,

$$\begin{aligned} & \sum_{K=1}^6 \left(\sin \frac{2K\pi}{7} - i \frac{\cos 2K\pi}{7} \right) \\ &= -i \sum_{K=1}^6 \left(\cos \frac{2K\pi}{7} + i \sin \frac{2K\pi}{7} \right) \\ &= -i \sum_{K=1}^6 e^{\frac{2K\pi}{7}} \\ &= -i \left[e^{\frac{2\pi}{7}} + e^{\frac{4\pi}{7}} + e^{\frac{6\pi}{7}} + e^{\frac{8\pi}{7}} + e^{\frac{10\pi}{7}} + e^{\frac{12\pi}{7}} \right] \\ &= -i \left[1 + e^{\frac{2\pi}{7}} + e^{\frac{4\pi}{7}} + e^{\frac{6\pi}{7}} + e^{\frac{8\pi}{7}} + e^{\frac{10\pi}{7}} + e^{\frac{12\pi}{7}} - 1 \right] \\ &= -i[0 - 1] \quad [\because \text{Sum of all } n\text{th roots of unit is zero}] \\ &= i \end{aligned}$$

174. (c) We have,

$$\lim_{x \rightarrow \infty} x \sin \left(\frac{2}{x} \right) = \lim_{x \rightarrow \infty} \frac{\sin \left(\frac{2}{x} \right)}{\left(\frac{2}{x} \right)} \times 2$$

$$\text{Put } \frac{2}{x} = y$$

$$\text{Then } x \rightarrow \infty$$

$$\Rightarrow \frac{2}{x} \rightarrow 0 \Rightarrow y \rightarrow 0$$

$$\therefore \lim_{x \rightarrow \infty} \frac{\sin \left(\frac{2}{x} \right)}{\left(\frac{2}{x} \right)} \times 2$$

$$= \lim_{y \rightarrow 0} \frac{\sin y}{y} \times 2 = 1 \times 2 = 2$$

175. (b) We have,

$$x = 80t - 16t^2$$

$$\Rightarrow \frac{dx}{dt} = 80 - 32t$$

For maximum value of x ,

$$\frac{dx}{dt} = 0$$

$$\Rightarrow 80 - 32t = 0 \Rightarrow t = \frac{80}{32} = 2.5$$

$$\text{Again, } \frac{d^2x}{dt^2} = -32 < 0$$

So, at $t = 2.5$ sec x is maximum.

176. (d) We have,

$$f(x) = \frac{\log x}{x}$$

$$\Rightarrow f'(x) = \frac{\frac{1}{x} \times x - \log x \cdot 1}{x^2} = \frac{1 - \log x}{x^2}$$

For maxima or minima,

$$f'(x) = 0$$

$$\Rightarrow \frac{1 - \log x}{x^2} = 0$$

$$\Rightarrow \log x = 1$$

$$\Rightarrow x = e$$

$$\text{Now, } f''(x) = \frac{-\frac{1}{x}x^2 - (1 - \log x) \cdot 2x}{x^4}$$

$$= \frac{-x - 2x + 2x \log x}{x^4}$$

$$= \frac{-3x + 2x \log x}{x^4} = \frac{-3 + 2 \log x}{x^3}$$

$$\therefore f''(e) = \frac{-3 + 2}{e^3} = \frac{-1}{e^3} < 0$$

Hence, $f(x)$ attains a local maxima at $x = e$.

$$\therefore f_{\max} = f(e) = \frac{\log e}{e} = \frac{1}{e}$$

177. (c) We have,

$$f(x) = be^{ax} + ae^{bx}$$

$$\Rightarrow f'(x) = abe^{ax} + abe^{bx}$$

$$\Rightarrow f''(x) = a^2be^{ax} + ab^2e^{bx}$$

$$\therefore f''(0) = a^2b + ab^2 = ab(a + b)$$

178. (c) We have,

$$\sqrt{\frac{1 + \cos A}{1 - \cos A}} = \frac{x}{y}$$

$$\Rightarrow \sqrt{\frac{2\cos^2 \frac{A}{2}}{2\sin^2 \frac{A}{2}}} = \frac{x}{y}$$

$$\Rightarrow \cot \frac{A}{2} = \frac{x}{y} \Rightarrow \tan \frac{A}{2} = \frac{y}{x}$$

$$\text{Now, } \tan A = \frac{2\tan \frac{A}{2}}{1 - \tan^2 \frac{A}{2}} = \frac{2\left(\frac{y}{x}\right)}{1 - \left(\frac{y}{x}\right)^2} = \frac{2xy}{x^2 - y^2}$$

179. (a) Let $I = \int \frac{\sec x}{\sec x + \tan x} dx$

$$= \int \frac{1}{1 + \sin x} dx = \int \frac{1 - \sin x}{1 - \sin^2 x} dx$$

$$= \int \frac{1 - \sin x}{\cos^2 x} dx$$

$$= \int (\sec^2 x - \sec x \tan x) dx$$

$$= \tan x - \sec x + C$$

180. (b) We have,

$$f(x)dx = g(x)$$

$$\Rightarrow f(x) = g'(x)$$

Now, let

$$I = \int f(x) \cdot g(x) dx = \int g'(x)g(x) dx$$

$$\text{Put } g(x) = t$$

$$\Rightarrow g'(x)dx = dt$$

$$\therefore I = \int t dt = \frac{t^2}{2} + C = \frac{1}{2}(g(x))^2 + C$$