PHYSICS

1. **(a)** Frequency of incident light,
$v = 10^{15}$ Hz
Work function, $\phi_0 = 25\text{eV}$
Energy of incident photon,
$E = h\nu$
$= 6.62 \times 10^{-34} \times 10^{15}$
$= 6.62 \times 10^{-19}$ J
$= \frac{6.62 \times 10^{-19}}{1.6 \times 10^{-19}}$ eV
$= 4.1\text{eV}$
According to Einstein's photoelectric equation,
$K_{max} = h\nu - \phi_0$
$\Rightarrow eV_0 = h\nu - \phi_0$
$\Rightarrow eV_0 = 4.1\text{eV} - 25\text{eV}$
$\Rightarrow eV_0 = 1.6\text{eV}$
$\Rightarrow V_0 = 1.6$ V

2. **(c)** When a charged particle of charge q is accelerated through a potential V, then de-Broglie wavelength is given as
$$\lambda = \frac{h}{\sqrt{2mqV}}$$
$$\Rightarrow \lambda \propto \frac{1}{\sqrt{mq}}$$
$$\Rightarrow \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p}{m_\alpha}} \cdot \sqrt{\frac{q_p}{q_\alpha}}$$
$$\Rightarrow \frac{\lambda_\alpha}{\lambda} = \sqrt{\frac{m_p}{4m_p}}\sqrt{\frac{q_p}{2q_p}}$$
$$\left[\begin{array}{l}\because m_\alpha = 4m_p \\ \text{and } q_\alpha = 2q_p\end{array}\right]$$
$$\Rightarrow \frac{\lambda_\alpha}{\lambda} = \frac{1}{\sqrt{8}}$$
$$\Rightarrow \lambda_\alpha = \frac{\lambda}{2\sqrt{2}}$$

3. **(d)** Electric field due to thin spherical shell at a distance x outside from the centre is given as
$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{x^2}$ ....(i)
where, $Q = s \cdot A$
$= s \times 4\pi R^2$
$\therefore$ From Eq. (i), we get
$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{s \times 4\pi R^2}{x^2} = \frac{sR^2}{\varepsilon_0 x^2}$$
$$\therefore E \propto \frac{1}{x^2}$$

4. **(b)** For emission of energy, electron of H -atom must fall from higher energy state to lower energy state. Hence, options (a) and (c) are not possible.
In option (b), $n_1 = 2, n_2 = 1$
$\therefore$ Energy of emitted photon is given as
$$E' = -13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$
$$\Rightarrow h\nu' = -13.6Z^2\left(\frac{1}{2^2} - \frac{1}{1^2}\right)$$

$v' = \frac{13.6Z^2}{h} \times \frac{3}{4}$ ....(i)

In option (c), $n_1 = 2, n_2 = 6$, hence energy of emitted photon is given as

$E'' = -13.6Z^2\left(\frac{1}{6^2} - \frac{1}{2^2}\right)$

$\Rightarrow hv'' = 13.6Z^2 \times \frac{8}{36}$

$\Rightarrow v'' = \frac{13.6Z^2}{h} \times \frac{8}{36}$ ....(ii)

Hence, from Eqs. (i) and (ii), we get

$v' > v''$

5. **(c)** Nuclear force F_n are attractive in nature. As in the given case, the two protons are kept at a separation of 40Å (short range), hence in this range, nuclear force F_n is much greater than the electrostatic force F_e. i.e.

6. **(a)** Decays constants of radioactive materials X_1 and X_2

is given as $\lambda_1 = 5\lambda$ and $\lambda_2 = \lambda$

If N_0 be the initial number of nuclei in both

radioactive materials, then according to radioactive decay law,

$N_{X_1} = N_0 e^{-\lambda_1 t}$

and $N_{X_2} = N_0 e^{-\lambda_2 t}$

$\therefore \frac{N_{X_1}}{N_{X_2}} = \frac{N_0 e^{-\lambda_1 t}}{N_0 e^{-\lambda_2 t}}$

$\frac{1}{e} = e^{(-\lambda_1 + \lambda_2)t}$ $\left[\text{Given}, \frac{N_{X_1}}{N_{X_2}} = \frac{1}{e}\right]$

$\Rightarrow\ e^{-1} = e^{-(\lambda_1 - \lambda_2)t}$

$\Rightarrow\ (\lambda_1 - \lambda_2)t$

$\Rightarrow\ t = \frac{1}{\lambda_1 - \lambda_2}$

$= \frac{1}{5\lambda - \lambda} = \frac{1}{4\lambda}$

7. **(b)** If α and β be the longitudinal strain and lateral strain respectively, then

$\alpha = \frac{l_2 - l_1}{l_1} = 10^{-3} = 0.001$

$\Rightarrow \frac{l_2}{l_1} = 1.001$ ....(i)

Poisson's ratio, $\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$

$= \frac{\beta}{\alpha}$

$\Rightarrow \beta = \sigma\alpha = 0.1 \times 10^{-3} = 10^{-4}$

But, $\beta = \frac{r_1 - r_2}{r_1}$

$\therefore \frac{r_1 - r_2}{r_1} = 10^{-4} = 0.0001$

$\Rightarrow \frac{r_2}{r_1} = 0.9999$ ....(ii)

Hence, percentage increase in volume

$= \frac{V_2 - V_1}{V_1} \times 100$

$= \frac{\pi r_2^2 l_2 - \pi r_1^2 l_1}{\pi r_1^2 l_1} \times 100$

$= \left[\left(\frac{r_2}{r_1}\right)^2\left(\frac{l_2}{l_1}\right)^2 - 1\right] \times 100$

$= [(0.9999)^2 \times (1.0001)^2 - 1] \times 100$

$= 0.0008 \times 100 = 0.08\%$

8. **(d)** We know that, angular velocity,

$\omega = \frac{2\pi}{T}$

$\Rightarrow T = \frac{2\pi}{\omega}$

$\Rightarrow \frac{T_2}{T_1} = \frac{\omega_1}{\omega_2}$

$= \frac{\omega_1}{2\omega_1}$ $\quad [\because \omega_2 = 2\omega_1]$

$\Rightarrow \frac{T_2}{T_1} = \frac{1}{2}$

$\Rightarrow T_2 = \frac{T_1}{2}$ $\quad \ldots.(i)$

According to Kepler's law,

$T^2 \propto R^3$

$\Rightarrow R \propto T^{\frac{2}{3}}$

$\Rightarrow \frac{R_2}{R_1} = \left(\frac{T_2}{T_1}\right)^{\frac{2}{3}}$

$= \left(\frac{\frac{T_1}{2}}{T}\right)^{\frac{2}{3}}$

$= \left(\frac{1}{2}\right)^{\frac{2}{3}}$

$R_2 = \frac{R_1}{2^{\frac{2}{3}}}$

$= \frac{R}{4^{1/3}}$

9. **(b)** When a $p-n$ junction diode is connected in forward bias, then its potential barrier decreases because more number of charge carriers (electrons in n-side and holes in p-side) move through the junction. Due to increase of drifting of charge carriers, electric current is increased.

10. **(b)** Wavelength of ground waves have frequency greater than the wavelength of sky waves and space waves both.

11. **(b)** In the given situation, the weight of the ball is supported by the buoyant force of water and buoyant force due to air. As the density of air becomes less, the buoyant force due to the air becomes less. Therefore, to compensate a greater buoyant force due to water is required which is achieved by a greater immersion of ball into the water.

12. **(c)** Given, initial surface area, $A_1 = A$

Final surface area, $A_2 = A_1 - 50\%$ of A

$A_2 = A - 50\% \text{ of } A = A - \frac{A}{2} = \frac{A}{2}$

Initial surface energy, $E_1 = T \cdot 2A = 2TA \quad \ldots.(i)$

Final surface energy, $E_2 =$ Surface tension × Surface area

$\Rightarrow E_2 = T \times 2(A_1 - A_2)$

$= T \times 2\left(A - \frac{A}{2}\right)$

$= T \times 2 \times \frac{A}{2}$

$\Rightarrow E_2 = TA$

$\therefore$ Percentage change in the energy of soap film

$= \frac{E_1 - E_2}{E_1} \times 100$

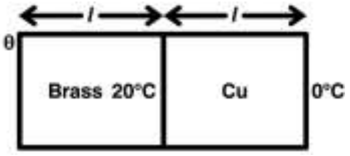
$= \frac{2TA - TA}{2TA} \times 100$

$= 50\%$

13. **(d)** Given, $K_{copper} : K_{brass} = 4:1$

$\Rightarrow \dfrac{K_{copper}}{K_{brass}} = \dfrac{4}{1}$

Let the temperature of the free surface of brass $= \theta$ Rate of flow of heat will be equal in both the plates as connected in series.



i.e. $H = K_{brass}\dfrac{A(\theta - 20)}{l} = K_{copper}\dfrac{A(20-0)}{l}$

$\Rightarrow \dfrac{\theta - 20}{20} = \dfrac{K_{copper}}{K_{brass}} = \dfrac{4}{1}$

$\Rightarrow \theta - 20 = 80$

$\theta = 100^\circ C$

14. **(a)** Given, $V_1 = 1\ mm^3$

$= 10^{-9}\ m^3$

$p_1 = 1\ atm$

$= 1 \times 10^5\ N/m^2$

$T_1 = 27 + 273$

$= 300\ K$

$T_2 = 627 + 273$

$= 900\ K$

Under adiabatic condition,

$p_1^{1-\gamma}T_1^{\gamma} = p_2^{1-\gamma}T_2^{\gamma}$

$\Rightarrow \left(\dfrac{p_2}{p_1}\right)^{1-\gamma} = \left(\dfrac{T_1}{T_2}\right)^{\gamma}$

$\Rightarrow \left(\dfrac{p_2}{p_1}\right)^{1-15} = \left(\dfrac{300}{900}\right)^{15}$

$\Rightarrow \left(\dfrac{p_2}{p_1}\right)^{-0.5} = \left(\dfrac{1}{3}\right)^{15}$

$\Rightarrow \left(\dfrac{p_1}{p_2}\right)^{0.5} = \left(\dfrac{1}{3}\right)^{15}$

$\Rightarrow \dfrac{p_1}{p_2} = \left(\dfrac{1}{3}\right)^{\frac{15}{05}} = \left(\dfrac{1}{3}\right)^{3} = \dfrac{1}{27}$

$\Rightarrow p_2 = 27p_1 = 27 \times 1 \times 10^5$

$= 27 \times \dfrac{10^5\ N}{m^2}$

15. **(b)** Given, temperature of sink,

$T_2 = -39^\circ C$

$= (273 - 39)K$

$= 234\ K$

Temperature of source,

$T_1 = 0^\circ C$

$= (273 + 0)K$

$= 273\ K$

$\therefore$ Efficiency,$\eta = 1 - \dfrac{T_2}{T_1}$

$= 1 - \dfrac{234}{273}$

$= \dfrac{39}{273}$

$= 0.142$

$= 14.2\%$

16. **(c)** Law of conservation of energy is valid across all domains of nature. In the universe, all kinds of violent phenomenon occur all the time. Yet the total energy of the universe is remains conserved.

17. **(d)** Given, centripetal force, $F = -\frac{A}{R^2}$

$\therefore$ Potential energy, $U = -\frac{dF}{dR}$

$= -\frac{d}{dR}\left(-\frac{A}{R^2}\right)$

$= -\frac{A}{R}$

In circular motion, centripetal force is responsible for the motion of object.

$\therefore \frac{mv^2}{R} = \frac{A}{R^2}$

$\Rightarrow mv^2 = \frac{A}{R}$

$\Rightarrow \frac{1}{2}mv^2 = \frac{A}{2R}$

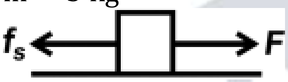
$\therefore$ Kinetic energy, $K = \frac{1}{2}mv^2 = \frac{A}{2R}$

$\therefore$ Total energy $= U + K$

$= -\frac{A}{R} + \frac{A}{2R} = -\frac{A}{2R}$

18. **(b)** Given, force $F = 20$ N

$m = 5$ kg



Net force on the body,

$F_{net} = F - f_s$ [where, f_s = friction]

$\therefore$ Work done by displacing 2 m is given as

$W = F_{net} \times$ displacement

$= (F - f_s)2$

$= (20 - f_s)2$

$\Rightarrow W = 40 - 2f_s$

According to work-energy theorem,

Work done = gain in kinetic energy

$\Rightarrow 40 - 2f_s = 10$

$\Rightarrow 2f_s = 40 - 10 = 30$

$\Rightarrow f_s = \frac{30}{2}$

$= 15$ N

19. **(c)** Given, $F = 6\hat{i} - 8\hat{j} + 10\hat{k}$

$a = 1\ m/s^2$

$|F| = \sqrt{6^2 + (-8)^2 + 10^2}$

$= \sqrt{200}$

$\Rightarrow |F| = 10\sqrt{2}$ N

$\Rightarrow ma = 10\sqrt{2}$ $[\because F = ma]$

$\Rightarrow m = \frac{10\sqrt{2}}{a}$

$= \frac{10\sqrt{2}}{1}$

$= 10\sqrt{2}$ kg

20. **(a)** Mass of the body, $m = 1000$ kg

$v = 50$ m/s

After adding 250 kg , new mass of the body,

$m' = m + 250$
$= 1000 + 250$
$= 1250$ kg
If v' be the final velocity of the body, then by conservation of linear momentum,
$mv = m'v'$
$\Rightarrow 1000 \times 50 = 1250 \times v'$
$$\Rightarrow v' = \frac{1000 \times 50}{1250}$$
$= 40$ m/s

21. **(d)** Densities ratio of two gases,
$\rho_1 : \rho_2 = 1:16$
$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{1}{16}$$
Since, pressure of both gases are equal.
$\therefore\ p_1 = p_2$
$$\frac{1}{3}\rho_1 c_1^2 = \frac{1}{2}\rho_2 c_2^2$$
$$\Rightarrow \frac{c_1^2}{c_2^2} = \frac{\rho_2}{\rho_1} = \frac{16}{1}$$
$$= \left(\frac{4}{1}\right)^2$$
$$\Rightarrow \frac{c_1}{c_2} = \frac{4}{1}$$
$\Rightarrow c_1 : c_2 = 4:1$

22. **(d)** Mass of helium, $m_1 = 16$ g
Mass of oxygen, $m_2 = 16$ g
$\therefore$ Number of moles of He gas,
$$n_1 = \frac{m_1}{4} = \frac{16}{4} = 4$$
Number of moles of O_2 gas,
$$n_2 = \frac{m_2}{32} = \frac{16}{32} = \frac{1}{2}$$
We know that for the mixture,
$$C_V = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}, \text{ where } C_V = \frac{fR}{2}$$
and $C_p = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 + n_2}$, where $C_p = \left(\frac{f}{2} + 1\right) R$
For He gas, $f = 3, n_1 = 4$
For O_2 gas, $f = 5, n_2 = \frac{1}{2}$
$$\therefore \frac{C_p}{C_V} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{V_1} + n_2 C_{V_2}}$$
$$= \frac{4 \times \left(\frac{3}{2} + 1\right) R + \frac{1}{2}\left(\frac{5}{2} + 1\right) R}{4 \times \frac{3}{2} R + \frac{1}{2} \times \frac{5}{2} R}$$
$$= \frac{\left(4 \times \frac{5}{2} R\right) + \left(\frac{1}{2} \times \frac{7}{2} R\right)}{\left(4 \times \frac{3}{2} R\right) + \left(\frac{1}{2} \times \frac{5}{2} R\right)} = \frac{47}{29}$$
$= 1.62$

23. **(a)** Amplitude of the particle is SHM,
$A = 4$ cm
At mean position velocity,
$v_{max} = 10$ cm/s

$\Rightarrow\ A\omega = 10$

$\Rightarrow\ \omega = \frac{10}{A}$

$= \frac{10}{4} = \frac{2.5\text{rad}}{\text{s}}$

Again when v = 5 cm/s, then displacement, x = ?

We know that,

$v = \omega\sqrt{A^2 - x^2}$

$\Rightarrow 5 = 25\sqrt{4^2 - x^2}$

$\Rightarrow 2 = \sqrt{A^2 - x^2}$

$\Rightarrow 4 = A^2 - x^2$

$\Rightarrow 4 = 4^2 - x^2$

$\Rightarrow x^2 = 16 - 4 = 12$

$\therefore x = \sqrt{12} = 2\sqrt{3}$ cm

24. **(a)** Given, equation of progressive wave,

$y = 15\sin(660\pi t - 0.02\pi x)$

Comparing this equation with standard equation of progressive wave as $y = A\sin\left(\frac{2\pi}{T}\cdot t - \frac{2\pi}{\lambda}\cdot x\right)$

We get,

$\frac{2\pi}{T} = 660\pi$

$\Rightarrow\ T = \frac{2\pi}{660\pi}$

$\Rightarrow\ T = \frac{1}{330}$

$\therefore$ Frequency of the wave,

$v = \frac{1}{T}$

$= \frac{1}{1/330} = 330$ Hz

25. **(b)** Let the frequency of the sound is f.

In first case, $f = 100 \pm 5$

$= 105$ Hz or 95 Hz

The frequency of second harmonic = 2f

According to second case,

$2f = 200 \pm 10$

$= 210$ Hz or 190 Hz

If 5 beats are produced with frequency 105 Hz , then frequency of second harmonic will be 210 Hz .

i.e. $2f = 210$

$\Rightarrow\ f = 105$ Hz

$= 105\ s^{-1}$

26. **(a)** Given,$Q_1 + Q_2 = 0.8C$

$r = 30\text{ cm} = 0.3\text{ m}$

$\therefore$ According to Coulomb's law

$F = k\frac{Q_1Q_2}{r^2}$

$\Rightarrow F = k\frac{Q_1(0.8 - Q_1)}{r^2}$

The force on Q_1 will be maximum if

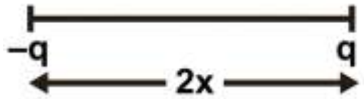
$\frac{dF}{dQ_1} = 0$

$\Rightarrow \frac{d}{dQ}k\frac{Q_1(0.8 - Q_1)}{r^2} = 0$

$0.8 - 2Q_1 = 0$

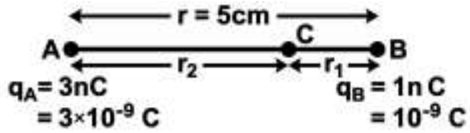
$\Rightarrow Q_1 = 0.4C$
$\therefore Q_2 = 0.8 - Q_1$
$= 0.8 - 0.4 = 0.4C$

27. (b) The axis of the electric dipole is directed along the direction of dipole moment and the direction of electric dipole moment is taken from negative charge to positive charge.



28. (c) The orientation of the dipole for the stable equilibrium, $\theta = 0°$.

29. (b) The given situation is shown below



$r_1 = 1$ cm
$= 1 \times 10^{-2}$ m
$r_2 = 5 - 1$
$= 4$ cm
$= 4 \times 10^{-2}$ m
Required work done = Changes in potential energy
$= kq_Aq_B\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$
$= 9 \times 10^9 \times 3 \times 10^{-9} \times 10^{-9}$
$\left[\frac{1}{4 \times 10^{-2}} - \frac{1}{1 \times 10^{-2}}\right]$
$= 1.35 \times 10^{-7}$ J

30. (c) We know that, potential energy of electric dipole,
$U = -p \cdot E = -pE\cos\theta$
When, $90 < \theta < 180$, then $\cos\theta$ is negative.
As θ increases, $\cos\theta$ decreases.
$\therefore\ \theta_{(iv)} > \theta_{(iii)}$
When $0° < \theta \leq 90°$, then $\cos\theta$ is positive.
Also, when θ increases, then $\cos\theta$ decreases.
$\therefore$ Hence, $\theta_{(i)} > \theta_{(ii)}$
Therefore, $\theta_{(iv)} > \theta_{(iii)} > \theta_{(i)} > \theta_{(ii)}$.

31. (d) Refractive index, $\mu = A + \frac{B}{\lambda^2}$
According to principle of homogeneity,
Dimension of $\frac{B}{\lambda^2}$ = Dimension of μ
$\Rightarrow\ [B] = [\mu][\lambda^2]$
$= [M^0L^0T^0][L^2]$
$= [L^2]$
= Dimension of area

32. (d) Voltage, $V = (200 \pm 8)V$
Current, $I = (20 \pm 0.5)A$
$\therefore$ Resistance, $R = \frac{V}{I} = \frac{200}{20} = 10\Omega$
Taking maximum error condition,
$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = \frac{8}{200} + \frac{05}{20} = \frac{13}{200}$
$\Rightarrow \frac{\Delta R}{R} \times 100 = \frac{13}{200} \times 100$
$= \frac{13}{2}\% = 65\%$

33. **(b)** Let u be the initial velocity of vertical projection and t be the time taken by the body to reach at height h from the ground.

$\therefore$ Using equation, $h = ut + \frac{1}{2}at^2$

Here, $u = u, a = -g$

$\therefore\ h = ut - \frac{1}{2}gt^2$

$\Rightarrow\ gt^2 - 2ut + 2h = 0$

$\therefore t = \dfrac{2u \pm \sqrt{4u^2 - 4g \times 2h}}{2g}$

$= \dfrac{u \pm \sqrt{u^2 - 2gh}}{g}$

It means t has two values, t_1 and t_2. Where, t_1 is time taken by the body in reaching maximum height and t_2 be the time taken by the body in reaching ground from maximum height.

$\therefore\ t_1 = \dfrac{u + \sqrt{u^2 - 2gh}}{g}$

and $t_2 = \dfrac{u - \sqrt{u^2 - 2gh}}{g}$

$\therefore\ t_1 + t_2 = \dfrac{2u}{g}$ or $u = \dfrac{g(t_1 + t_2)}{2}$

34. **(d)** Slope of distance-time graph gives the velocity of the particle. Since at the point C, slope of the curve is maximum, hence maximum instantaneous velocity of particle occurs at point C.

35. **(c)** According to the equation of continuity,

$Av =$ constant

Therefore, velocity of the fluid is minimum at the widest part and maximum at the narrowest part of the horizontal pipe.

Again, according to Bernoulli's principle for horizontal pipe,

$p + \frac{1}{2}\rho v^2 =$ constant

Hence, p is more, v is less and vice-versa.

Therefore, when a fluid is flowing through the horizontal pipe of variable area of cross-section, then velocity is maximum at the narrowest part of the pipe and pressure is maximum at the widest part of the pipe.

36. **(b)** Given, $V_1 = 120\ V, V_2 = 200\ V$

Since, potential on each capacitor is zero when they are connected in parallel. Hence, charge on each capacitor must be equal with unlike polarities together.

i.e. $q_1 = q_2$

$\Rightarrow C_1V_1 = C_2V_2$

$\Rightarrow C_1 \times 120 = C_2 \times 200$

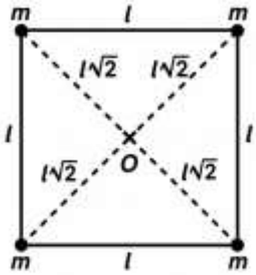
$\Rightarrow 3C_1 = 5C_2$

37. **(d)** According to Kirchhoff's junction rule, $\Sigma I = 0$

i.e. sum of total incoming currents = sum of total outgoing currents.

Hence, it is based on the law of conservation of charges. Hence, all the statements are correct.

38. **(a)** The given situation is shown in figure.



Moment of inertia of the systems about an axis perpendicular to the square and passing through the centre is given as

$$I = m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$= 4\frac{ml^2}{2} = 2ml^2$ ....(i)

If k be the radius of gyration, then

$I = 4mk^2$ ....(ii)

$\therefore$ From Eqs. (i) and (ii), we have

$2ml^2 = 4mk^2$

$\Rightarrow\ k = \frac{l}{\sqrt{2}}$

39. **(a)** Given, for steel wire,

Area, $A_1 = 3 \times 10^{-5}\ m^2$

Length, $L_1 = 4.7$ m

For copper wire,

$A_2 = 4 \times 10^{-5}\ m^2, L_2 = 35$ m

If Y_1 and Y_2 be the Young's modulus for the material steel wire and copper wire respectively, then

$$\frac{Y_1}{Y_2} = \frac{\frac{F_1 L_1}{A_1 \Delta l}}{\frac{F_2 L_2}{A_2 \Delta l}}$$

$\Rightarrow \frac{Y_1}{Y_2} = \frac{L_1 A_2}{L_2 A_1}$ $[\because F_1 = F_2]$

$= \frac{4.7 \times 4 \times 10^{-5}}{3.5 \times 3 \times 10^{-5}} = 1.8$

$\Rightarrow\ Y_1 : Y_2 = 1.8 : 1$

40. **(b)** Given, $m_1 = 200\ g = 0.2$ kg

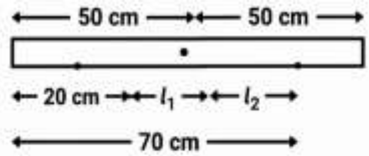
$m_2 = 300\ g = 0.3$ kg

$l_1 = 50 - 20 = 30$ cm

$\Rightarrow\ l_1 = 0.3$ m

$l_2 = 70 - 50 = 20\ cm = 0.2$ m

$\therefore$ Moment of inertia of the system about an axis passing through 50 cm mark (i.e. mid-point)

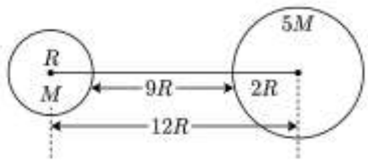


$I = m_1 l_1^2 + m_2 l_2^2$

$= 0.2 \times (0.3)^2 + 0.3 \times (0.2)^2$

$= 0.036$ kg m^2

41. **(c)** The given situation is shown below.



Let the smaller body travels a distance x before collision due to gravitation attraction.

Hence, $Mx = 5M(9R - x)$

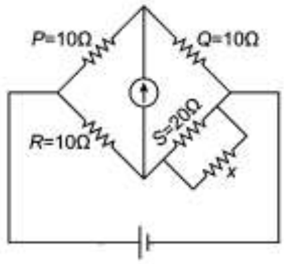
$\Rightarrow x = 5(9R - x)$

$\Rightarrow x = 45R - 5x$

$\Rightarrow 6x = 45R$

$\Rightarrow\ x = \frac{45}{6}R = \frac{15}{2}R = 75R$

42. **(c)** The given Wheatstone bridge circuit is shown below.



Wheatstone is balanced only when

$$\frac{P}{Q}=\frac{R}{S}$$

Thus, the value of resistance S should be 10Ω. But it is given to 20Ω.

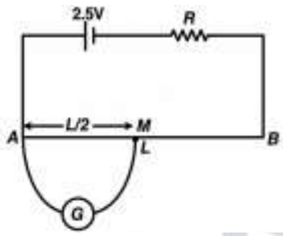
Hence, to decrease its value upto 10Ω, suppose xΩ resistance is connected parallel to it.

$$\therefore \frac{1}{10}=\frac{1}{20}+\frac{1}{x}$$

$$\Rightarrow \frac{1}{x}=\frac{1}{10}-\frac{1}{20}=\frac{1}{20}$$

$$\Rightarrow x=20\Omega$$

43. **(d)** The given situation is shown below.



Resistance of potentiometer wire, $R_P=10\Omega$ Emf, $E=25$ V

Total resistance, $R'=R_P+R=(10+R)\Omega$

Current through potentiometer wire,

$$I=\frac{E}{R_P}=\frac{25}{10+R}$$

Since, potential difference across length $AL=\frac{L}{2}$,

$$V=I\left(\frac{R_P}{2}\right)$$

$$\Rightarrow 1=\frac{25}{10+R}\times\frac{10}{2}$$

$$\Rightarrow 10+R=125$$

$$\Rightarrow R=25\Omega$$

44. **(c)** Given,

$$v=2\times10^5\hat{\imath}\ m/s$$

$$B=(\hat{\imath}-4\hat{\jmath}-3\hat{k})T$$

Charge on electron, $q=1.6\times10^{-19}C$

$\therefore$ Force, $F=q(v\times B)$

$$=1.6\times10^{-19}\left[(2\times10^5\hat{\imath})\times(\hat{\imath}-4\hat{\jmath}-3\hat{k})\right]$$

$$=1.6\times10^{-19}\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ 2\times10^5 & 0 & 0\\ 1 & -4 & -3\end{vmatrix}$$

$$F=1.6\times10^{-19}\left(6\times10^5\hat{\jmath}-8\times10^5\hat{k}\right)$$

$$=9.6\times10^{-14}\hat{\jmath}-128\times10^{-14}\hat{k}$$

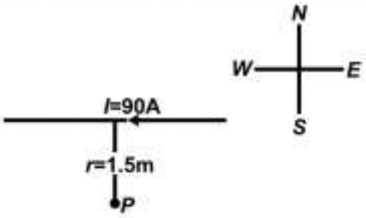
$$\Rightarrow |F|=\sqrt{(9.6\times10^{-14})^2+(-12.8\times10^{-14})^2}$$

$$=10^{-14}\sqrt{(9.6)^2+(128)^2}$$

$$=16\times10^{-14}\ N$$

$$=1.6\times10^{-13}\ N$$

45. **(a)** The given situation is shown below



Magnetic field at point P,

$B=\frac{\mu_0}{2\pi}\cdot\frac{I}{r}$

$=2\times10^{-7}\times\frac{90}{15}$

$=1.2\times10^{-5}$ T (South)

46. **(b)** Given, number of turns in moving coil galvanometer, $N_i=28$

Initial sensitivity, $S_i=S$

Final sensitivity, $S_f=S_i+25\%$ of S_i

$=S+25\%$ of S

$=S+\frac{S}{4}=\frac{5S}{4}$

Since, sensitivity of moving coil galvanometer is given as

$S=\frac{NAB}{K}$

$\Rightarrow S\propto N$ [A,B and K are constant]

$\Rightarrow \frac{S_f}{S_i}=\frac{N_f}{N_i}$

$\Rightarrow \frac{5S}{4}=\frac{N_f}{28}$

$\Rightarrow N_f=\frac{28\times5}{4}=35$

47. **(a)** The intensity of magnetic field due to isolated pole of strength m_p at a point distance r from it is given as

$B=\frac{\mu_0}{4\pi}\cdot\frac{m_p}{r^2}=m_p/r^2$

48. **(c)** Light particle (like electron) cannot be accelerated by a cyclotron because due its small mass, a small increase in its energy makes the electrons move with a very high speed. As a result of it, the electrons go quickly out of step with oscillation electric field.

49. **(d)** The angle which the total magnetic field of earth makes with the surface of the earth is called inclination or angle of dip.

50. **(a)** Given, $B_H=B_V$

If δ be the angle of dip, then

$\tan\delta=\frac{B_V}{B_H}=\frac{B_V}{B_V}=1$

$=\tan45^\circ$

$\Rightarrow\ \delta=45^\circ$

51. **(b)** Given, number of turns in the first coil, $N_1=100$

Its self-inductance, $L_1=15\text{mH}=1.5\times10^{-2}\text{H}$

Number of turns in the second coil,

$N_2=500$

$\therefore L_2=?$

We know that, self inductance of the coil is given by

$L=\frac{\mu_0\pi N^2R}{2}$

$\Rightarrow\ L\propto N^2$

$\therefore \frac{L_2}{L_1}=\left(\frac{N_2}{N_1}\right)^2=\left(\frac{500}{100}\right)^2=25$

$\Rightarrow L_2=25L_1$

$= 25 \times 15 \times 10^{-2}$
$= 375 \times 10^{-2} \mathrm{H}$
$= 375 \times 10^{-3} \mathrm{H} = 375 \mathrm{mH}$

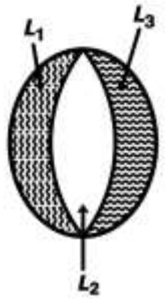
52. (d) Given, number of turns, $N = 100$
Current, $I = 5 \text{ mA} = 5 \times 10^{-3} \text{ A}$
Magnetic flux, $\phi = 10^{-5}$ Wb
Inductance, $L = ?$
We know that,
$N\phi = LI$
$\Rightarrow L = \frac{N\phi}{I} = \frac{100 \times 10^{-5}}{5 \times 10^{-3}}$
$= 20 \times 10^{-2} \mathrm{H} = 0.2 \mathrm{H}$

53. (a) In step-up transformer, number of turns in secondary coil is greater than the number of turns in the primary coil.
i.e. $N_S > N_P$

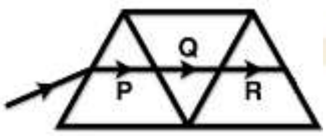
54. (b) In $L - C - R$ series resonance circuit,
$X_L = X_C$
$\therefore$ Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + 0^2}$
$\Rightarrow Z = R$
$\therefore$ Power factor, $\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$
Which is not zero.
Since, circuit behaves like a resistive circuit. So, power loss occurs always in this circuits.
i.e. wattless current is zero.

55. (c) Both magnet and electric fields have zero average value in a plane electromagnetic wave.

56. (c) From the given diagram, we see that lenses L_1 and L_3 are identical but contribute opposite sense, hence their combined effect of making image is ineffective.
Hence, only lens L_2 forms the image.



57. (c) The complete ray diagram for the given combination of prisms is shown below.



Since, ray of light incident on prism P suffers minimum deviation, hence it becomes parallel to the base of prism P. As prism Q and R are of identical shape and of same material as that of prism P, therefore, the ray continues to be parallel to base of prisms Q and R. Hence, the ray will now suffer same deviation as before.

58. (c) Phase difference between the two identical waves $= \delta$
For identical wave,
$a_1 = a_2 = a$
After superposition, intensity of resultant wave
$I \propto (\text{ Amplitude })^2$
$\propto A^2$
$\propto (a_1^2 + a_2^2 + 2a_1a_2\cos\delta)$
$\propto (a^2 + a^2 + 2a^2\cos\delta)$
$\propto 2a^2(1 + \cos\delta)$

$\propto 2a^2 \cdot \left(2\cos^2\frac{\delta}{2}\right)$

$\propto 4a^2\cos^2\left(\frac{\delta}{2}\right)$

$\Rightarrow\ I \propto \cos^2\left(\frac{\delta}{2}\right)$

59. **(d)** Given, wavelength of monochromatic light,

$\lambda = 5867\text{Å}$

Refractive index, $\mu = 1.6$

Let t be the thickness of the plastic.

Since, the centre of the screen appears dark when plastic sheet is introduced, hence path difference,

when, $\mu t = \frac{3\lambda}{2}$

$\Rightarrow\ t = \frac{3\lambda}{2\mu} = \frac{3 \times 5867}{2 \times 1.6} = 5500\text{Å}$

60. **(a)** Position vector of starting point (2, 10, 1) of particle is given as

$r_1 = 2\hat{i} + 10\hat{j} + \hat{k}$

If final co-ordinate of the particle be (x, y, z), then its position vector

$r_2 = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore$ Displacement, $\Delta r = r_2 - r_1$

$8\hat{i} - 2\hat{j} + \hat{k} = x\hat{i} + y\hat{j} + z\hat{k} - (2\hat{i} + 10\hat{j} + \hat{k})$

$\Rightarrow 8\hat{i} - 2\hat{j} + \hat{k} = (x-2)\hat{i} + (y-10)\hat{j} + (z-1)\hat{k}$

Comparing the coefficients of $\hat{i}, \hat{j}$ and $\hat{k}$, we get

$x - 2 = 8, y - 10 = -2, z - 1 = 1$

$\Rightarrow\ x = 10, y = 8, z = 2$

$\therefore$ Final co-ordinate $= (10,8,2)$

CHEMISTRY

61. **(a)** During the extraction of gold, following reactions take place

$$\underset{\text{Impure gold}}{4Au(s)} + 8CN^-(aq) + 2H_2O(aq) \xrightarrow{O_2} \underset{(X)}{4[Au(CN)_2]^-} + 4OH^-(aq)$$

$$2\underset{(X)}{[Au(CN)_2]^-} + Zn(s) \longrightarrow \underset{(y)}{[Zn(CN)_4]^{2-}}(aq) + \underset{\text{Pure gold}}{2Au(s)}$$

One chlorine molecule contains 2 chlorine atoms and one HCl molecule contains 1 chlorine atom.

62. **(b)** Number of gram molecules of chlorine in HCl

$= \frac{\text{number of molecules of HCl}}{N_A \times 2}$

$= \frac{6.02 \times 10^{25}}{6.02 \times 10^{23} \times 2}$

$= \frac{100}{2} = 50$

63. **(b)** Graphite is a soft solid lubricant extremely difficult to melt. The reason for this anomalous behaviour is that graphite has carbon atoms arranged in large plates of rings of strongly bound carbon atoms with weak interplate bonds.

64. **(d)** Paracetamol is an analgesic and antipyretic drug. It is used in medication for treatment of fever, mild to moderate pain.

65. **(a)** 2 g of water has maximum number of molecules as its molecular mass is minimum among the given compounds.

Number of O -atoms in $H_2O = \frac{2}{18}N_A = \frac{1}{9}N_A$

Number of O -atoms in $SO_2 = \frac{2}{64}N_A = \frac{1}{32}N_A$

Number of O -atoms in $CO_2 = \frac{2}{44} \times 2N_A = \frac{1}{11}N_A$

Number of O -atoms in $CO = \frac{2}{28}N_A = \frac{1}{14}N_A$

66. **(b)** C_2H_5OH (alcohol) shows functional group isomerism. It's isomers are
$\underset{\text{Alcohol}}{C_2H_5OH}, \underset{\text{Ether}}{CH_3 - O - CH_3}$

67. **(c)** For the following reaction, $BiO_3^- + 6H^+ + xe^- \longrightarrow Bi^{3+} + 3H_2O$
On balancing charge on both sides
$-1 + 6 + x(-1) = 3 + 3 \times 0$
$\therefore\ x = 2$

68. **(d)** For orthophosphoric acid solution,
Molarity = 3 M, n-factor = 3
$\therefore$ Normality = n-factor $\times$ molarity
$= 3 \times 3$
$= 9$ N

69. **(c)** Acidified sodium fusion extract on addition of ferric chloride solution gives blood red colouration, which confirms the presence of N and S.
$Na + C + N + S \longrightarrow NaCNS$ (Sodium thiocyanate)
$Fe^{3+} + 3CNS^- \longrightarrow \underset{\text{Red ppt}}{Fe(CNS)_3}$

70. **(c)** $m = 10mg, v = 100\ ms^{-1}, \lambda = ?$
de-Broglie wavelength, $\lambda = \dfrac{h}{mv}$
$= \dfrac{6.63 \times 10^{-34}}{10 \times 10^{-3} \times 100}$
$= 6.63 \times 10^{-34}$ m

71. **(b)** Number of electrons in Mg = 12
Number of electrons in $Mg^{2+} = 10$
Number of electrons in $Na^+ = 10$
$\therefore Mg^{2+}$ is isoelectronic with Na^+.

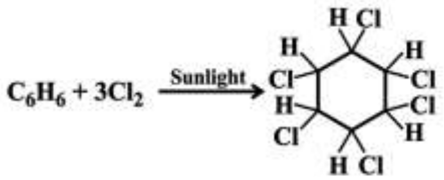
72. **(b)** Gram molecular volume of oxygen at STP is $22400\ cm^3$.
i.e. volume of 1 mole of oxygen molecule
$= \dfrac{22400}{1} = 22400\ cm^3$.

73. **(a)** Presence of halogen in organic compounds can be detected using Beilstien's test. In this test, an organic compound react with copper wire, when they are heated in a flame and produce a brilliant, long lasting green flame.

74. **(c)** Electronic configuration of $Cr = [Ar]3d^5 4s^1$
Electronic configuration of $Cr^{3+} = [Ar]3d^3 4s^0$

75. **(a)** Equivalent of metal = equivalent of oxygen
$\dfrac{w}{31.75} = \dfrac{8}{8}$
$\therefore w = 31.75$ g

76. **(c)** Benzene reacts with chlorine in sunlight to give a final product gammaxene.



$C_6H_6Cl_6$
(Gammaxene)

77. **(c)** In the periodic table metals usually used as catalyst belong to d-block as they have variable oxidation state.

78. **(a)** Dalton's law is applicable to mixture of gases, when they are non-reacting, so that the total pressure of all gases is equal to the sum of the partial pressure of each gas. CO and H_2 are non-reacting gases.
$\therefore\ p_T = p_{CO} + p_{H_2}$

79. **(b)** The general formula of cycloalkene is C_nH_{2n-2}.

80. **(c)** In acetylene, between carbon atoms, there are one σ and two π-bonds i.e.

Acetylene

81. (d) Denaturated alcohol is ethanol + methanol. Ethanol is made unfit for drinking by adding poisonous substance such as methanol. It is called denaturation.

82. (a) During the formation of a chemical bond energy of system decreases with the release of energy. This result in stability of bond formed.

83. (b) O_2 is diatomic and degree of freedom is 5 .

$\therefore$ Kinetic energy of $O_2=\frac{5}{2}\times k\times T$

$=\frac{5}{2}\times 273\times k$

SO_2 is triatomic and degree of freedom is 7 .

$\therefore$ Kinetic energy of $SO_2=\frac{7}{2}\times 546\times k$

$\therefore$ So, kinetic energy of SO_2 is more than that of O_2 i.e.

$SO_2>O_2$

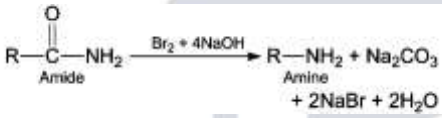
84. (d) + 1-effect is shown by $-CH_3$ group.

While, $-NO_2, -Br$ and -Cl shows $-$I-effect.

85. (b) Formation of coloured solution is possible, when metal ion in the compound contains unpaired electrons due to transition of electrons in excited state.

86. (d) Temperature, surface tension and viscosity are intensive properties. They are independent of the quantity of matter.

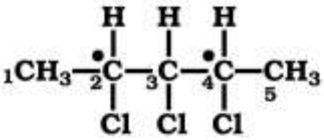
87. (b) Hofmann's bromamide reaction is used to convert amide to amine.



88. (c) IUPAC name of $Na_3[Co(NO_2)_6]$ is sodium hexanitro cobaltate (III).

89. (a) Thermodynamic standard condition of temperature and pressure is $273\ K(0^{\circ}C)$ and $1\ atm(760\ mmHg)$.

90. (b) The structure of 2, 3, 4-trichloropentane is



Carbon 2 and 4 are chiral carbons as all the groups are different.

$\therefore$ Two chiral carbons are present in 2,3-4-trichloropentane.

91. (b) The number of unidentate ligands in the complex ion is called coordination number. Coordination is the number of lone pair of electron shared by ligands with central metal atom.If all the ligand are unidentate, then number of ligand is equal to coordination number.

92. (b) The reaction, $2SO_2(g)+O_2(g)\overset{V_2O_5}{\rightleftharpoons}$ is an example of heterogeneous catalysis as the reactant and catalyst are not present in same phase.

93. (a) The amino acid which is not optically active is glycine.

$$\underset{\underset{NH_2}{|}}{CH_2}-COOH$$

94. (b) For a stable molecule the value of bond order must be positive.

$\because$ Stability is directly proportional to bond order.

95. (a) The second order reaction is

$$\underset{\text{Methyl ethanoate}}{CH_3COOCH_3}+NaOH\longrightarrow\underset{\substack{\text{Sodium salt of}\\\text{carboxylic acid}}}{CH_3COONa}+H_2O$$

96. (c) According to Baeyer's strain theory, cyclopentane is highly stable as, it has lowest angle strain. The desired angle for sp^3 hybridised carbon in cyclopentane should be $109^{\circ}28'$ and the actual angle is 108°. Thus the angle strain is minimum.

97. **(c)** Configuration of $O_2^{2-} = \sigma 1s^2, \sigma^{*}1s^2, \sigma 2s^2, \sigma^{*}2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^{*}2p_x^2 = \pi^{*}2p_y^2$.

$\therefore$ Number of antibonding electron pairs is four.

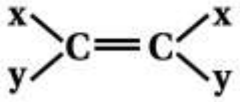
98. **(a)** For 1 M HCl ,

$[H^+] = 1 = 10^0$

$[H^+][OH^-] = 10^{-14}$

$[OH^-] = 10^{-14}$ moldm^{-3}.

99. **(c)** Geometrical isomerism is shown by alkene with different group attached to carbon i.e.



where, $x \neq y$

100. **(a)** Oxidation state of iron (x) in $K_4[Fe(CN)_6]$ is

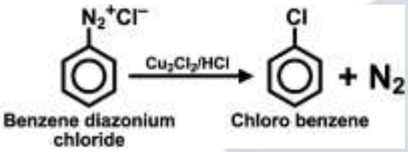
$4 + x + 6 \times (-1) = 0$

$x = 2$

101. **(c)** During sublimation of naphthalene, phase changes from solid to gaseous state. Hence, the randomness of the particles increases drastically.

$\therefore$ It has maximum increase in entropy.

102. **(c)** Decomposition of benzene diazonium chloride by using Cu_2Cl_2/HCl to form chlorobenzene is called Sandmeyer reaction.

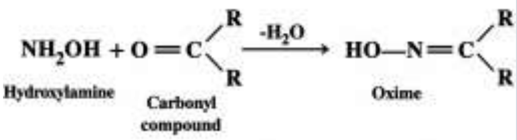


103. **(d)** The complex $[CoCl_3(NH_3)_3]$ cannot ionise in the solution as there is no primary valency in this compound and all the ions are within the coordination entity.

104. **(a)** The reaction $C(s) + O_2(g) \longrightarrow CO_2(g) + 3935$ kJ is a combustion reaction.

For combustion reaction. ΔH and ΔG are negative but ΔS is positive.

105. **(c)** The product formed, when hydroxylamine condenses with a carbonyl compound is called oxime.



106. **(b)** The electronic configuration of Sc is

$1s^2 2s^2 2p^6 3s^2 3p^6 3d^1 4s^2$

Electronic configuration of Sc^{3+} is $1s^2 2s^2 2p^6 3s^2 3p^6$.

So, Sc^{3+} is colourless due to the absence of unpaired electrons in d-orbital.

107. **(b)** When a sulphur sol is evaporated, sulphur is obtained.

On mixing with water, sulphur sol is not formed.

The sol is hydrophobic. Hydrophobic sols are irreversible in nature.

There is a little affinity between the dispersed phase and the dispersion medium.v

108. **(d)** When an alkyl halide reacts with alcoholic ammonia in a sealed tube, the product formed will be mixture of primary, secondary and tertiary amine.

109. **(c)** When conc. H_2SO_4 is heated with P_2O_5, the acid is converted into sulphur trioxide.

$H_2SO_4 + P_2O_5 \longrightarrow SO_3 + 2HPO_3$

110. **(a)** According to second law of thermodynamics, entropy of the universe is continuously increasing.

111. **(d)** KCN is a salt of strong base (KOH) and weak acid (HCN).

$KCN + H_2O \rightleftharpoons \underset{\text{Strong base}}{KOH} + \underset{\text{Weak Acid}}{HCN}$

$\therefore$ pH of the solution is greater than 7 at $25^\circ C$.

112. **(c)** The reagent used in Clemmenson's reduction is $Zn - Hg/$ conc. HCl. It is used for reduction of carbonyl group to alkanes.

113. **(d)** When KBr is dissolved in water, K^+ions are hydrated due to ionic nature. KBr is soluble in water due to hydration of K^+ion by H_2O molecule.

114. **(a)** When the mixture of noble gas is cooled in coconut bulb at 173 K , the gases that are not absorbed are He and Ne. As they have low critical temperature.

115. **(d)** Let the volume of 10 N HCl used is x litre.

$\therefore$ The volume of 4 N HCl used is $(1-x)$ litre.

$\therefore 7 = \dfrac{10 \times x + 4 \times (10 - x)}{1}$

$\therefore x = 0.5$ L

$\therefore$ Volume of 10 N HCl and 4 N HCl used is 0.5 L each.

116. **(c)** The metal present in insulin is zinc.

117. **(b)** The equivalent mass of oxygen in CO and CO_2 remains constant. While, that of carbon changes as it have different oxidation state in different compound.

118. **(d)** Maximum number of molecules of CH_3I, that can react with CH_3NH_2 is 3 . The reaction is as follows :

$$CH_3-NH_2 \xrightarrow{CH_3I} (CH_3)_2NH \xrightarrow{CH_3I} (CH_3)_3\,N \xrightarrow{CH_3I} \underset{\substack{\text{Tetramethyl}\\\text{ammonium}\\\text{iodide}}}{(CH_3)_4\overset{+-}{N}}$$

119. **(b)** Ellingham diagram represent a graph of ΔG°vsT. It shows the temperature dependence of the stability of compound.

120. **(b)** The ore not containing iron is carnallite.

It's formula is $KCl \cdot MgCl_2 \cdot 6H_2O$.

MATHEMATICS

121. **(d)** We have,

$7^{2\log_7 5}$

$= 7^{\log_8 5^2}$ $[\because m\log a = \log a^m]$

$= 5^2$ $[\because a^{\log_a b} = b]$

$= 25$

122. **(a)** The composition table is

$\otimes_{15}$	3	6	9	12
3	9	3	12	6
6	3	6	9	12
9	12	9	6	3
12	6	12	3	9

From the table 6 is the identity element.

123. **(a)** Since, identity element is its own inverse.

So, minimum number of element is 1 .

124. **(d)**We have.

$|a+b| = |a-b|$

$\Rightarrow |a+b|^2 = |a-b|^2$

$\Rightarrow (a+b)\cdot(a+b) = (a-b)\cdot(a-b)$

$\Rightarrow |a|^2 + |b|^2 + 2a\cdot b = |a|^2 + |b|^2 - 2a\cdot b$

$\Rightarrow 4a\cdot b = 0$

$\Rightarrow a\cdot b = 0$

$\Rightarrow a \perp b$

So, angle between a and b is 90°.

125. **(b)** We have,

$\dfrac{3x^2+1}{x^2-6x+8} = 3 + \dfrac{18x-23}{x^2-6x+8}$

$= 3 + \dfrac{18x-23}{(x-4)(x-2)}$

Again, let $\frac{18x-23}{(x-4)(x-2)}=\frac{A}{x-4}+\frac{B}{x-2}$

$\Rightarrow\ 18x-23=A(x-2)+B(x-4)$

Put $x=4$, we get

$72-23=2A\Rightarrow A=\frac{49}{2}$

Put $x=2$, we get

$36-23=-2B$

$\Rightarrow B=\frac{-13}{2}$

So, $\frac{3x^2+1}{x^2-6x+8}=3+\frac{49}{2(x-4)}-\frac{13}{2(x-2)}$

126. **(c)** We know that, any vector perpendicular to a and in the plane containing b and c is a × (b × c).

Now, $b\times c=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -5\\ 3 & 5 & -1\end{vmatrix}=23\hat{i}-14\hat{j}-\hat{k}$

Again, $a\times(b\times c)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & -1\\ 23 & -14 & -1\end{vmatrix}$

$=-17\hat{i}-21\hat{j}-97\hat{k}$

which is required vector.

127. **(a)** We have,

$|OA|=5,|OB|=6,\angle BOA=60^\circ$

Now, $OA\cdot OB=|OA||OB|\cos(\angle BOA)$

$=5\times 6\cos 60^\circ=5\times 6\times\frac{1}{2}=15$

128. **(a)** We have,

$A(1,-1,2), B(2,0,-1)$ and $C(0,2,1)$

$\therefore\ AB=(2-1)\hat{i}+(0-(-1))\hat{j}+(-1-2)\hat{k}$

$=\hat{i}+\hat{j}-3\hat{k}$

$AC=(0-1)\hat{i}+(2-(-1))\hat{j}+(1-2)\hat{k}$

and $=-\hat{i}+3\hat{j}-\hat{k}$

Now, required vector $=AB\times AC$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & \hat{1} & -3\\ -1 & 3 & -1\end{vmatrix}$

$=8\hat{i}+4\hat{j}+4\hat{k}$

129. **(d)** Let $S_n=\frac{1}{2\times5}+\frac{1}{5\times8}+\frac{1}{8\times11}+\cdots+\frac{1}{(3n-1)(3n+2)}$

$=\frac{1}{3}\left[\frac{3}{2\times5}+\frac{3}{5\times8}+\frac{3}{8\times11}+\cdots+\frac{3}{(3n-1)(3n+2)}\right.$

$=\frac{1}{3}\left[\frac{5-2}{2\times5}+\frac{8-5}{5\times8}+\frac{11-8}{8\times11}+\cdots+\frac{(3n+2)-(3n-1)}{(3n-1)(3n+2)}\right]$

$=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{5}+\frac{1}{5}-\frac{1}{8}+\frac{1}{8}-\frac{1}{11}+\cdots+\frac{1}{3n-1}-\frac{1}{3n+2}\right]$

$=\frac{1}{3}\left[\frac{1}{2}-\frac{1}{3n+2}\right]=\frac{1}{3}\left[\frac{3n+2-2}{2(3n+2)}\right]$

$=\frac{n}{2(3n+2)}=\frac{n}{6n+4}$

130. **(c)** $T_9=T_{8+1}$

$={}^8C_8(3x)^{8-8}\left(\frac{-1}{2x}\right)^8$

$={}^8C_8(3x)^0\left(\frac{-1}{2x}\right)^8$

$= 1 \times 1 \times \frac{1}{2^8x^8} = \frac{1}{256x^8}$

131. **(d)** We have,

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix} \text{ and } B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$$

Since, B is inverse of A, then

$AB = 1$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{10}\begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & 0 & 5-\alpha \\ 0 & 10 & \alpha-5 \\ 0 & 0 & 5+\alpha \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$\therefore 5 - \alpha = 0 \text{ or } \alpha - 5 = 0 \text{ or } 5 + \alpha = 10$

$\Rightarrow \alpha = 5$

132. **(b)** Since, A is singular matrix, then

$|A| = 0$

$$\begin{vmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{vmatrix} = 0$$

$\Rightarrow -x[x^2 - 144] = 0 \Rightarrow x = 0, \pm 12$

133. **(d)** We have,

$$A = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

$$\therefore |A| = \begin{bmatrix} 1 & -2 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$$

$= 1[8 - 6] + 3[6 - 4] = 2 + 6 = 8$

Now, we know that

$$A \cdot (\text{adj}A) = 8I = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

134. **(d)** Since, $f(x) = |x|$ is neither one-one nor onto, so $f(x)$ is not invertible.

$\therefore f^{-1}(x)$ does not exist.

135. **(a)** We have,

$$\begin{bmatrix} x & p & q \\ p & x & q \\ p & q & x \end{bmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get

$$\begin{vmatrix} x+p+q & p & q \\ x+p+q & x & q \\ x+p+q & q & x \end{vmatrix}$$

Taking common $(x + p + q)$ from C_1, we get

$$(x+p+q)\begin{vmatrix} 1 & p & q \\ 1 & x & q \\ 1 & q & x \end{vmatrix}$$

Applying $R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$, we get

$$(x+p+q)\begin{vmatrix} 0 & p-x & 0 \\ 0 & x-q & q-x \\ 1 & q & x \end{vmatrix}$$

$= (x + p + q) \cdot 1[(p - x)(q - x) - 0]$

$= (x - p)(x - q)(x + p + q)$

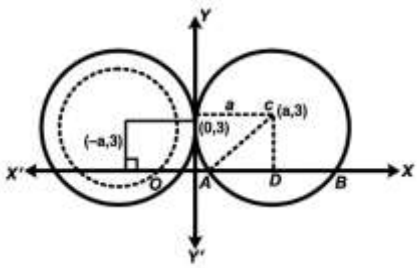
136. **(c)**We have,

Equation of circle is $x^2+y^2=4$. Its centre is $C_1(0,0)$ and radius (r_1) is 2 .
Equation of another circle is
$x^2+y^2-6x-8y-24=0$
i.e. $(x-3)^2+(y-4)^2=(7)^2$. Its centre is $C_2(3,4)$ and radius (r_2) is 7 .
Now, $C_1C_2=\sqrt{(3-0)^2+(4-0)^2}=5$
and $|r_1-r_2|=|2-7|=5$
$\therefore\ C_1C_2=|r_1-r_2|$
So, both circles touch each other internally.
Hence, number of common tangents is 1 .

137. **(d)** We have, $3x+y+k=0$ is tangent to the circle $x^2+y^2=10$.
So, distance of line $3x+y+k=0$ from centre of the circle will be equal to the radius of circle.
$\therefore \left|\frac{3(0)+(0)+K}{\sqrt{(3)^2+(1)^2}}\right|=\sqrt{10}$
$\Rightarrow \frac{K}{\sqrt{10}}=\pm\sqrt{10}\Rightarrow K=\pm 10$

138. **(a)** Negation of $\sim(p\rightarrow q)=p\wedge\sim q$
So, negation of given statement is If 2 is prime and 3 is not odd.

139. **(b)**



Equation of circle will be
$(x\pm a)^2+(y-3)^2=a^2$ ….(i)
Now, in $\triangle$ ACD,
$AC^2=AD^2+CD^2$
$\Rightarrow a^2=(4)^2+(3)^2$
$\left[\because AC=a, AD=\frac{1}{2}AB=4, CD=3\right]$
$\Rightarrow a^2=25\Rightarrow a=5$
So, equation of circles will be
$(x\pm 5)^2+(y-3)^2=5^2$
$\Rightarrow x^2\pm 10x+25+y^2-6y+9=25$
$\Rightarrow\ x^2+y^2\pm 10x-6y+9=0$

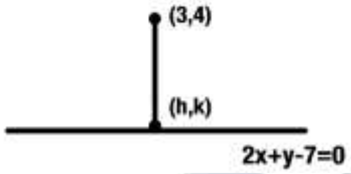
140. **(a)** The triangle formed with vertices $A(0,0), B\left(0,\frac{3}{2}\right)$ and $C(-5,0)$ is a right-angled triangle at A. So, BC is hypotenuse.
So, orthocentre of $\triangle$ ABC is mid-point of BC.
$\therefore$ Orthocentre $=\left(\frac{0-5}{2},\frac{\frac{3}{2}+0}{2}\right)=\left(\frac{-5}{2},\frac{3}{4}\right)$

141. **(d)** Let
$S_1: x^2+y^2-6x-6y+4=0$
$S_2: x^2+y^2-2x-4y+3=0$
$S_3: x^2+y^2+2kx+2y+1=0$
Now, radical cnetre of S_1 and S_2 is given by
$S_1-S_2=0$
$\Rightarrow -4x-2y+1=0$
$\Rightarrow 4x+2y-1=0$ ….(i)
Radical centre of S_2 and S_3 is given by

$S_2 - S_3 = 0$
$-2(1+k)x - 6y + 2 = 0$
$(1+k)x + 3y - 1 = 0 \quad ....(ii)$
Now, Eq. (ii) becomes, if
$k = 1: 2x + 3y - 1 = 0$
$k = 2: 3x + 3y - 1 = 0$
$k = 4: 5x + 3y - 1 = 0$
$k = 5: 6x + 3y - 1 = 0$
Now, only $6x + 3y - 1 = 0$ is parallel to $4x + 2y - 1 = 0$ So, for this radical centre does not exists.
So, $k \neq 5$

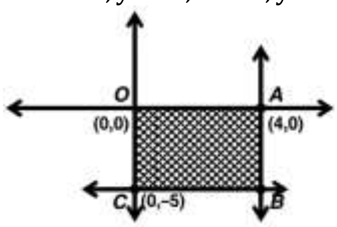
142. **(b)** We have,
$S_1: x^2 + y^2 - 2x - 2y - 7 = 0$
$S_2: x^2 + y^2 + 4x + 2y + k = 0$
Since, S_1 and S_2 cut orthogonally, then
$2g_1g_2 + 2f_1f_2 = c_1 + c_2$
$\Rightarrow 2(-1)(2) + 2(-1)(1) = -7 + k$
$\Rightarrow -4 - 2 = -7 + k$
$\Rightarrow k' = 1$
Now, let r_1 and r_2 be the radius of S_1 and S_2, respectively. Then,
$r_1 = \sqrt{(-1)^2 + (-1)^2 - (-7)} = 3$
$r_2 = \sqrt{(2)^2 + (1)^2 - 1} = 2$
Now, length of common chord
$= \frac{2r_1r_2}{\sqrt{r_1^2 + r_2^2}}$
$= \frac{2 \times 3 \times 2}{\sqrt{(3)^2 + (2)^2}} = \frac{12}{\sqrt{13}}$

143. **(b)**



Let (h, k) be the coordinate of foot of perpendicular. Now, slope of line $2x + y - 7 = 0$ is $\left(\frac{-2}{1}\right)$ i.e. -2 .
So, slope of perpendicular line.
$= \frac{-1}{(-2)} = \frac{1}{2}$
$\therefore$ Equation of perpendicular line is given by
$y - 4 = \frac{1}{2}(x - 3)$
$\Rightarrow 2y - 8 = x - 3$
$\Rightarrow x - 2y + 5 = 0$
On solving $2x + y - 7 = 0$ and $x - 2y + 5 = 0$, we get $x = \frac{9}{5}$ and $y = \frac{17}{5}$
So, $(h, k) = \left(\frac{9}{5}, \frac{17}{5}\right)$

144. **(b)** We have,
$xy = 0, x - 4 = 0, y + 5 = 0$
$\Rightarrow x = 0, y = 0, x = 4, y = -5$



So. area enclosed = Area of rectangle OABC

$= AB \times BC$

$= 5 \times 4$

$= 20$sq units

145. **(a)** Equation of ellipse is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Then, equation of auxillary circle will be

$x^2+y^2=a^2\ (a>b)$

Now, it is given that

Area of auxillary circle $= 2 \times$ Area of ellipse

$\Rightarrow\ \pi a^2 = 2\times \pi ab$

$\Rightarrow\ a = 2b$

Now, eccentricity of the ellipse,

$$e=\sqrt{1-\frac{b^2}{a^2}}=\sqrt{1-\frac{b^2}{(2b)^2}}$$

$$=\sqrt{1-\frac{b^2}{4b^2}}=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$$

146. **(a)** Since, the number of vertices of odd degree is always even and number of even degree is always even.

So, $m+n$ is an even number.

147. **(c)** We have, equation of ellipse is

$$\frac{x^2}{36}+\frac{y^2}{16}=1 \Rightarrow \frac{x^2}{(6)^2}+\frac{y^2}{(4)^2}=1$$

$\therefore a=6, b=4$

Now, $PS+PS'=2a=2\times 6=12$

148. **(d)** We have,

$$\sin\left(2\cos^{-1}\frac{\sqrt{5}}{3}\right)=\sin\left\{\cos^{-1}\left(2\times\left(\frac{\sqrt{5}}{3}\right)^2-1\right)\right\}$$

$$=\sin\left\{\cos^{-1}\left(\frac{10}{9}-1\right)\right\}$$

$$=\sin\left(\cos^{-1}\frac{1}{9}\right)$$

$$=\sin\left\{\sin^{-1}\sqrt{1-\left(\frac{1}{9}\right)^2}\right\}$$

$$=\sin\left\{\sin^{-1}\left(\frac{4\sqrt{5}}{9}\right)\right\}=\frac{4\sqrt{5}}{9}$$

149. **(d)** We have,

$$\frac{x^2}{36}-\frac{y^2}{k^2}=1$$

Option (a) (3,1)

$$\frac{9}{36}-\frac{1}{k^2}=1$$

$$\Rightarrow \frac{1}{k^2}=\frac{-27}{36}\Rightarrow k^2=\frac{-36}{27}$$

which is not possible.

Option (b) (-3,1)

$$\frac{9}{36}-\frac{1}{k^2}=1\Rightarrow k^2=\frac{-36}{27}$$

which is not possible.

Option (c) (5,2)

$\frac{25}{36} - \frac{4}{k^2} = 1$

$\frac{4}{k^2} = \frac{-11}{36}$

$k^2 = \frac{-144}{11}$

which is not possible.

Option (d) (10,4)

$\frac{100}{36} - \frac{16}{k^2} = 1 \Rightarrow \frac{16}{k^2} = \frac{64}{36}$

$\Rightarrow k^2 = 9$, which is possible.

150. **(a)**If your equation is $3x^2 - 2x + 4y + 5 = 0$:

Vertex (h, k): (1/3, -7/6)

Focal length (a): $1/3$ (opens downward)

Focus: $(1/3, -7/6 - 1/3) = (1/3, -3/2)$

151. **(c)** We have,

$$\tan^{-1}x + 2\cot^{-1}x = \frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}x + \cot^{-1}x + \cot^{-1}x = \frac{2\pi}{3}$$

$$\Rightarrow \frac{\pi}{2} + \cot^{-1}x = \frac{2\pi}{3}$$

$$\Rightarrow \cot^{-1}x = \frac{\pi}{6}$$

$$\Rightarrow x = \cot\frac{\pi}{6} = \sqrt{3}$$

152. **(a)** We have,

$\sin^2 17.5° + \sin^2 72.5° = \sin^2 17.5° + \cos^2(90° - 72.5°)$

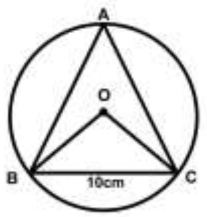
$= \sin^2 175° + \cos^2 17.5° = 1 = \tan^2 45°$

153. **(c)** Let $Z = \frac{(1+i)^2}{1-i} = \frac{1+i^2+2i}{1-i} = \frac{2i}{1-i} \times \frac{1+i}{1+i}$

$$= \frac{2i + 2i^2}{1 - i^2} = \frac{2i - 2}{1 + 1} = \frac{2i - 2}{2} = -1 + i$$

$\therefore \bar{Z} = -1 - i$

154. **(b)**



We know that,

$\angle BOC = 2\angle BAC = 2 \times 30° = 60°$

Since, $OB = OC$

$\therefore \angle OBC = \angle OCB = \frac{1}{2}(180° - \angle BOC) = 60°$

So, $\triangle$ BOC is an equilateral triangle.

$\therefore\ OB = BC = 10$ cm

Therefore, area of circumcircle $= \pi(OB)^2 = \pi(10)^2$

$= 100\pi$sq cm

155. **(a)** We have,

$\Rightarrow \sin 3\theta = \sin\theta$

$\Rightarrow 3\sin\theta - 4\sin^3\theta = \sin\theta$

$\Rightarrow 4\sin^3\theta - 2\sin\theta = 0$

$\Rightarrow 2\sin\theta(2\sin^2\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$ or $\sin\theta = \pm\dfrac{1}{\sqrt{2}}$

If $\sin\theta = 0$ and if $\sin\theta = \pm\frac{1}{\sqrt{2}}$

$\theta = 0, \pi, -\pi$

$\Rightarrow \theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{-5\pi}{4}, \dfrac{-7\pi}{4}, \dfrac{-\pi}{4}, \dfrac{-3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$

$\therefore$ There are nine solutions.

156. **(b)** We have,

$e^{ix} = \cos x + i\sin x$

Put $x = \frac{\pi}{2}$

$\Rightarrow e^{i\frac{\pi}{2}} = \cos\dfrac{\pi}{2} + i\sin\dfrac{\pi}{2}$

$\Rightarrow e^{i\frac{\pi}{2}} = i \Rightarrow e^{i^2\frac{\pi}{2}} = (i)^i \Rightarrow i^i = e^{-\frac{\pi}{2}}$

$\therefore$ Imaginary part of i^i is zero.

157. **(a)** Let $Z = (1+i)^5$

$= [(1+i)^2]^2[1+i]$

$= [1+i^2+2i]^2[1+i] = (2i)^2(1+i)$

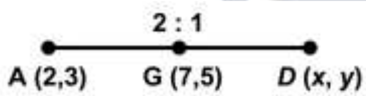
$= -4(1+i) = -4-4i$

Let θ be amplitude of z. Then,

$\theta = \tan^{-1}\left|\dfrac{-4}{-4}\right| - \pi = \tan^{-1}(1) - \pi$

$= \dfrac{\pi}{4} - \pi = \dfrac{-3\pi}{4}$

158. **(a)** We know that, centroid divides median in the ratio 2:1.



$(7,5) = \left(\dfrac{2x+2}{2+1}, \dfrac{2y+3}{2+1}\right)$

$(7,5) = \left(\dfrac{2x+2}{3}, \dfrac{2y+3}{3}\right)$

$\therefore \dfrac{2x+2}{3} = 7$ and $\dfrac{2y+3}{3} = 5$

$\Rightarrow x = \dfrac{19}{2}$ and $y = 6$

$\therefore$ D is $\left(\dfrac{19}{2}, 6\right)$.

159. **(b)** We have,

$\lim\limits_{x\to1}\dfrac{\tan(x^2-1)}{x-1}$

$= \lim\limits_{x\to1}\frac{\sec^2(x^2-1)\cdot 2x}{1}$ [using L' Hospital Rule]

$= 2\sec^2 0 = 2$

160. **(c)** We have,

$y = 2^{\log x}$

$\therefore \dfrac{dy}{dx} = 2^{\log x}\cdot\log 2\cdot\dfrac{d}{dx}(\log x) = \dfrac{2^{\log x}\log 2}{x}$

161. **(b)** We have,

$\sec^{-1}\left(\dfrac{1+x}{1-y}\right) = a \Rightarrow \dfrac{1+x}{1-y} = \sec a$

$\Rightarrow \dfrac{1(1-y)-(1+x)\left(-\frac{dy}{dx}\right)}{(1-y)^2} = 0$

$\Rightarrow\ 1-y+(1+x)\dfrac{dy}{dx}=0 \Rightarrow \dfrac{dy}{dx}=\dfrac{y-1}{x+1}$

162. **(d)** We have,

$y=\cos^2\dfrac{3x}{2}-\sin^2\dfrac{3x}{2} \Rightarrow y=\cos3x$

$\Rightarrow \dfrac{dy}{dx}=-\sin3x\cdot3=-3\sin3x$

and $\dfrac{d^2y}{dx^2}=-3\cos3x\cdot3=-9\cos3x=-9y$

163. **(d)** We have,

$f(x)=\begin{cases}\dfrac{1-\cos x}{x^2}, & x\neq0\\ k, & x=0\end{cases}$

Since, f(x) is continuous at $x=0$, then

$f(0)=\lim\limits_{x\to0}f(x) \Rightarrow k=\lim\limits_{x\to0}\dfrac{1-\cos x}{x^2}$

$\Rightarrow k=\lim\limits_{x\to0}\dfrac{\sin x}{2x}$ [using L' Hospital Rule]

$\Rightarrow k=\dfrac{1}{2}$

164. **(b)** We have,

$(1+\omega)(1+\omega^2)(1+\omega^4)(1+\omega^8)$

$=(1+\omega)(1+\omega^2)(1+\omega)(1+\omega^2)$

$[\because \omega^4=\omega^3\cdot\omega=\omega \text{ and } \omega^8=(\omega^3)^2\cdot\omega^2=\omega^2]$

$=\left((1+\omega)(1+\omega^2)\right)^2=(1+\omega+\omega^2+\omega^3)^2$

$=(\omega^3)^2\quad [\because 1+\omega+\omega^2=0]$

$=(1)^2=1\ [\because \omega^3=1]$

165. **(c)** We have,

$x^x=y^y \Rightarrow x\log x=y\log y$

$\Rightarrow 1\cdot\log x+\dfrac{x}{x}=y'\log y+\dfrac{y}{y}y'$

$\Rightarrow \log x+1=y'(\log y+1)$

$\Rightarrow \dfrac{dy}{dx}=\dfrac{1+\log x}{1+\log y}$

166. **(b)** We have,

$y^2=x$

$\Rightarrow\ 2y\dfrac{dy}{dx}=1 \Rightarrow \dfrac{dy}{dx}=\dfrac{1}{2y}$

Let the required point be (x_1, y_1). So, slope of tangent

$=\left.\dfrac{dy}{dx}\right|_{(x_1,y_1)}$

$\Rightarrow\ m=\dfrac{1}{2y_1}$ ....(i)

Since, tangent makes 45° with X-axis, so

$m=\tan45°=1$ ....(ii)

From Eqs. (i) and (ii), we have

$\dfrac{1}{2y_1}=1 \Rightarrow y_1=\dfrac{1}{2}$

Again, (x_1,y_1) lies on th curve $y^2=x$. So,

$y_1^2=x_1 \Rightarrow x_1=\left(\dfrac{1}{2}\right)^2=\dfrac{1}{4}$

So, required point is $\left(\frac{1}{4},\frac{1}{2}\right)$.

167. **(d)** We have,

$x^2y^2=a^4 \Rightarrow 2xy^2+x^2(2yy')=0$

$\Rightarrow y' = \frac{-2xy^2}{2x^2y} = \frac{-y}{x}$

$\therefore \left.\frac{dy}{dx}\right|_{(-a,a)} = \frac{-a}{-a} = 1$

Now, length of subtangent $= \left|\frac{y}{dy/dx}\right| = \left|\frac{a}{1}\right| = a$

168. **(d)** We have,

$252 = 2^2 \times 3^2 \times 7^1$

$\therefore$ Number of positive divisors of

$252 = (2+1)(2+1)(1+1) = 3 \times 3 \times 2 = 18$

169. **(b)** We have,

$5^{124} = (5^3)^{41} \cdot 5$

Now, $5^3 = 1 \cdot (\text{mod}124)$

$(5^3)^{41} = 1(\text{mod}124)$

$(5^3)^{41} \cdot 5 = 1 \times 5(\text{mod}124) = 5(\text{mod}124)$

So, the remainder obtained when 5^{124} is divided by 124 is 5 .

170. **(a)** Option (a) Since, sum of two odd integers is an even integer. Closure property does not hold.

So, it is not a group.

Option (b) Since, sum of two even integers is an even integer.

So, it is a group.

Option (c) $1 \times (-1) = -1$, which belongs to given set.

So, it is a group.

Option (d) $0 + 0 = 0$, which belongs to given set.

So, it is a group.

171. **(b)** We have,

$y = -x^2 + 6x - 3 \Rightarrow \frac{dy}{dx} = -2x + 6$

Now, for increasing.

$\frac{dy}{dx} > 0 \Rightarrow -2x + 6 > 0$

$\Rightarrow 2x < 6 \Rightarrow x < 3$

So, for $x < 3, f(x)$ is increasing.

172. **(c)** Let $I = \int_0^{\pi/2} (\sin^{100}x - \cos^{100}x)dx$ ....(i)

$$\Rightarrow I = \int_0^{\pi/2} \left(\sin^{100}\left(\frac{\pi}{2} - x\right) - \cos^{100}\left(\frac{\pi}{2} - x\right)\right) dx$$

$$\left[\because \int_0^a f(x)dx = \int_0^a f(a-x)dx\right]$$

$\Rightarrow I = \int_0^{\pi/2} (\cos^{100}x - \sin^{100}x)dx$ ....(ii)

On adding Eqs. (i) and (iii), we get

$2I = 0 \Rightarrow I = 0$

173. **(b)** After time t hour distance covered by X and Y are 4t and 3t respectively.

Now, shortest distance between X and Y after time t hour is given by

$AB^2 = OA^2 + OB^2 - 2OA \cdot OB\cos\theta$

$= (4t)^2 + (3t)^2 - 2 \times 4t \times 3t\cos 120°$

$= 16t^2 + 9t^2 - 24t^2 \times \left(\frac{-1}{2}\right)$

$= 16t^2 + 9t^2 + 12t^2 = 37t^2$

$\Rightarrow AB = \sqrt{37}t$

Now, $\frac{d(AB)}{dt} = \sqrt{37}$

$\therefore$ Rate at which the shortest distance

between x and y is increasing is $\sqrt{37}$ km/h.

174. **(b)** Let $I = \int_0^3 tf(t)$

Put $t = 3x \Rightarrow dt = 3dx$

When $t = 0 \Rightarrow x = 0$

and $t = 3 \Rightarrow x = 1$

$$\therefore\ I = \int_0^1 3xf(3x)(3dx)$$

$$= 9\int_0^1 xf(3x)dx$$

$$= k\int_0^1 xf(3x)dx$$

$\therefore\ k = 9$

175. **(c)** Let $I = \int \frac{1}{1+\cos 8x}dx$

$$= \int \frac{1}{2\cos^2 4x}dx = \frac{1}{2}\int \sec^2 4xdx$$

$$= \frac{1}{2}\left(\frac{\tan 4x}{4}\right) + C = \frac{1}{8}\tan 4x + C$$

176. **(a)** Let $I = \int e^x(x^5 + 5x^4 + 1)dx$

$$= \int e^x_{II}(x^5_I + 1)dx + \int e^x(5x^4)dx$$

$$= (x^5 + 1)\int e^x dx - \int \left[\left(\int e^x dx\right)\frac{d}{dx}(x^5 + 1)\right]dx$$

$$+ \int e^x(5x^4)dx$$

$$= (x^5 + 1)e^x - \int e^x(5x^4)dx + \int e^x(5x^4)dx$$

$$= (x^5 + 1)e^x + C = e^x \cdot x^5 + e^x + C$$

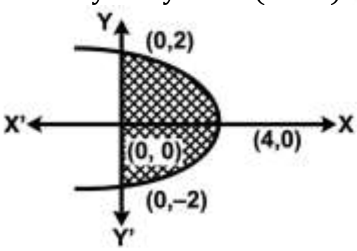
177. **(d)** Let $I = \int \frac{x^2+1}{x^2-1}dx = \int \frac{x^2-1+2}{x^2-1}dx$

$$= \int \left(1 + \frac{2}{x^2 - 1}\right)dx = x + 2 \times \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$$

$$= x + \log\left|\frac{x-1}{x+1}\right| + C$$

178. **(d)** We have,

$x = 4 - y^2 \Rightarrow y^2 = -(x - 4)$



$$\text{Required area} = 2\int_0^4 ydx = 2\int_0^4 \sqrt{4-x}dx$$

$$= 2\left[\frac{-(4-x)^{3/2}}{3/2}\right]_0^4$$

$$= \frac{-4}{3}\left[(4-4)^{3/2} - (4-0)^{3/2}\right]$$

$$= \frac{-4}{3}(-8) = \frac{32}{3}\ \text{sq units}$$

179. **(b)** We have, equation of line is

$y = mx + C$

But $m = C$ (given)

$\therefore\ y = Cx + C$

Now, $\frac{dy}{dx} = C$

$$\therefore\ y = \frac{dy}{dx}(x+1) \Rightarrow (x+1)\frac{dy}{dx} - y = 0$$

which is required differential equation.

180. **(c)** We have,

$$\left[1 + \left(\frac{dy}{dx}\right)^5\right]^{1/3} = \frac{d^2y}{dx^2} \Rightarrow \left(\frac{d^2y}{dx^2}\right)^3 = 1 + \left(\frac{dy}{dx}\right)^5$$

$\therefore$ Order $= 2$ and degree $= 3$