PHYSICS

1. **(a)** Lyman series of a hydrogen atom belongs to the ultraviolet region of electromagnetic spectrum.
2. **(c)** Insulator can be charged by rubbing or friction with another body. For example, or rubbing a glass rod with the silk gets negatively charged while the glass rod gets positively charged.
3. **(a)** In an adiabatic process,
$pV^{\gamma} =$ constant (k)
where p is pressure, V is volume and γ is the ratio of the specific heats.
Given, $\frac{C_p}{C_V} = \gamma = \frac{3}{2}$
Since, for adiabatic process $pV^{\gamma} = k$
$$\Rightarrow pV^{3/2} = k \Rightarrow \log p + \frac{3}{2}\log V = \log k$$
$$\therefore \frac{\Delta p}{p} + \frac{3}{2}\frac{\Delta V}{V} = 0$$
$$\Rightarrow \frac{\Delta V}{V} \times 100 = -\frac{2}{3}\left(\frac{\Delta p}{p} \times 100\right) = -\frac{2}{3} \times \frac{2}{3} = -\frac{4}{9}\%$$
Negative sign implies that volume decreases by $\frac{4}{9}\%$.
4. **(d)** Given, $f_1 = 20$ cm, $f_2 = 40$ cm
Since, power $P = \frac{1}{f} \Rightarrow P_1 = \frac{1}{f_1} = \frac{100}{20} = 5D$
and $P_2 = \frac{1}{f_2} = \frac{100}{40} = 25D$
Effective power, $P = P_1 + P_2 = 5 + 25 = 75D$
5. **(d)** The reactance of a capacitor of capacitance C connected to a AC source of frequency f is given by
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$
6. **(a)** For a freely falling body, $v^2 - u^2 = 2gx$ ($\therefore u = 0$)
Therefore, $v^2 = 2gx$ or $x = \frac{v^2}{2g}$
Thus, graph will be a parabola, symmetric along + ve X-axis
7. **(a)** Given, $x = ae^{-pt} + be^{qt}$
$$\Rightarrow v = \frac{dx}{dt} = -pae^{-pt} + qbe^{qt}$$
and $a = \frac{dv}{dt} = p^2ae^{-pt} + q^2be^{qt}$
Acceleration is positive, so velocity goes on increasing with time.
8. **(a)** As we know that, Impulse = Force × Time
$= [MLT^{-2}] \times [T]$
$= [ML^1 T^{-1}]$
Momentum = Mass × Velocity
$p = mv$
$[p] = [m] \times [v] = [M][LT^{-1}] = [ML^1 T^{-1}]$
Thus, dimension of impulse represent dimensions of momentum.
9. **(c)** As we know that, maximum height,
$H = \frac{u^2\sin^2\theta}{2g}$ ....(i)
Horizontal range, $R = \frac{u^2\sin 2\theta}{g}$....(ii)
Dividing Eq. (i) by Eq. (ii), we get
$$\frac{H}{R} = \frac{\tan\theta}{4} \Rightarrow \theta = \tan^{-1}\left(\frac{4H}{R}\right)$$
10. **(b)** The resonant frequency of a LC circuit is given by
$$f = \frac{1}{2\pi\sqrt{LC}}$$
when $L' = \frac{L}{3}$, as per question,

$$f' = f \Rightarrow \frac{1}{2\pi\sqrt{L'C'}} = \frac{1}{2\pi\sqrt{LC}}$$

$$\Rightarrow LC = L'C' \Rightarrow C' = \frac{LC}{L'} = \frac{LC}{L/3} = 3C$$

11. **(d)** The de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{mv_e}}{\frac{h}{mv_p}} = \frac{v_p}{v_e} = \frac{\frac{v_e}{4}}{v_e} = \frac{1}{4}\left(\because v_p = \frac{1}{4} \times v_e\right)$$

12. **(d)**For an ideal gas coefficient of volume expansion can be deduced as

According to ideal gas equation,

$pV = nRT$ ....(i)

at constant pressure, $p\Delta V = nR\Delta T$ ....(ii)

Dividing Eq (ii) by Eq. (i), we get

$\frac{\Delta V}{V} = \frac{\Delta T}{T}$ ....(iii)

But $\frac{\Delta V}{V} = \alpha \Delta T$ ....(iv)

∴ From Eqs. (iii) and (iv), we get

For ideal gas $\alpha = \frac{1}{T}$

So, for an ideal gas, coefficient of volume expansion is given by $\frac{1}{T}$.

13. **(d)** Green house gases are $CO_2, CH_4, N_2O, CF_xCl_x$ and O_3. Thus, H_2O is not a greenhouse gas.

14. **(a)** Force on a charge particle moving in a uniform magnetic field at angle θ is given by

$F = qvB\sin\theta \Rightarrow F \propto q$

Hence, $q_1 = q$ and $q_2 = 2q$

$$\therefore \frac{F_1}{F_2} = \frac{q_1}{q_2} = \frac{q}{2q} = \frac{1}{2}$$

15. **(c)** Given, $W = 50$ J and $q = 25C$

$$\text{Potential difference, } V = \frac{\text{Work done (W)}}{\text{Charge (q)}}$$

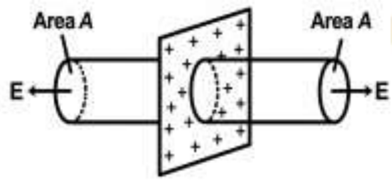
$$\Rightarrow V = \frac{50}{25} = 2\text{ V}$$

16. **(c)** The increasing order of frequency is Microwave < UV Rays < X-rays

The frequency is the inverse of wavelength. So, the order of increasing wavelength is

X-rays < UV rays < Microwave

17. **(a)** For an infinite plane sheet as shown below



As, the lines of force are parallel to the currect surface of cylinder, the flux passing through the curved surface is zero. The flux through the plane-end faces of the cylinder is $\phi_E = EA + EA = 2EA$

By Gauss's law,

$$\phi_E = \int E \cdot ds = \frac{q}{\varepsilon_0} \Rightarrow 2EA = \frac{\sigma A}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

18. **(b)** Infrared rays are used for the treatment of muscles ache. Due to their heating properties they are used in infrared therapy.

19. **(c)** The output of gate is given by

$Y = \overline{A \cdot B}$

It is a NAND gate output and its truth table is given by

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

20. (c) LED is a light emitting diode Which emits light in the spectral range of visible light is the range of wavelengths 0.4μ m to 0.7μ m. The energy corressponding to these- wavelength is from 1.8 eV to 3 eV. Hence, for the fabrication of LED in the visible range, the minimum band gap energy should be 1.8 eV. It convert electrical energy into light energy.

21. (b) Given, $y = 20\cos(\omega t + 4z)$

On comparing if with general equation,

$y = A\cos(kx + \omega t)$

we get, $A = 20$

Therefore, amplitude of the wave is 20 .

22. (a) Given, frequencies, $v_1 = v - 1$ and $v_2 = v + 1$

Beats $=$ Difference in frequencies

$= (v + 1) - (v - 1) = 2$

23. (b) $y = \log\omega t$ is a monotonically increasing function. It cannot repeat its value, therefore it is a non-periodic function.

24. (b) The equation of motion of this system about mean position is given by

$F = k_1x + k_2x$

$\Rightarrow m\frac{d^2x}{dt^2} = k_1x + k_2x \Rightarrow \frac{d^2x}{dt^2} = x\left(\frac{k_1 + k_2}{m}\right)$

$\because a = \frac{d^2x}{dt^2} = \omega^2x$

$\because \omega^2 = \frac{k_1 + k_2}{m}$ or $\omega = \sqrt{\frac{k_1 + k_2}{m}}$

25. (c) Volume of material remain same after stretching.

As volume remains same,

$A_1l_1 = A_2l_2$

Here, $l_2 = nl_1$

$\Rightarrow A_2 = \frac{A_1l_1}{l_2} = \frac{A_1}{n}$

Resistance of wire after stretching,

$R_2 = \frac{\rho l_2}{A_2} = \rho\frac{nl_1}{A_1/n} = n^2\rho\frac{l_1}{R_1} = n^2R\left(\because R = \rho\frac{l_1}{A_1}\right)$

26. (d) Since, the incidence light beam is unpolarised, the intensity of light on being transmitted through first nicol polaroid, will become

$I = \frac{I_0}{2}$

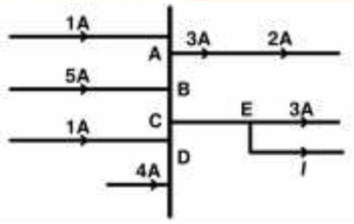
So, according to law of Malus, intensity on passing through second nicol will be

$I' = I\cos^2\theta = \frac{I_0}{2}\cos^2 60° = \frac{I_0}{2} \times \left(\frac{1}{2}\right)^2 = \frac{I_0}{8}$

27. (a) The collision of the molecules of an ideal gas is taken as elastic.

28. (c) The average energy associated with a particle of a monoatomic gas is $\frac{3}{2}K_BT$.

29. (d) According to Kirchhoff's first law, redrawing the circuit, we get



Current at A $= 3 - 2 = 1$ A
Current at B $= 1 + 5 = 6$ A
Current at D $= 1 + 4 = 5$ A
Current at C $= 6 + 5 = 11$ A
$\therefore\ I = 11 - 3 = 8$ A

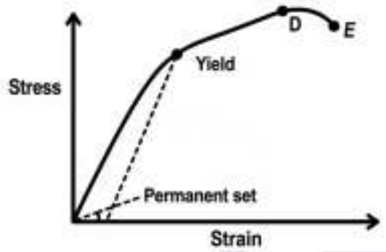
30. **(d)** When an electron jumps from orbit n_i to n_f, then the frequency of emitted photon is

$$\nu = cR\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Here, $n_i = 3, n_f = 2$

$$\Rightarrow\ \nu = cR\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] = \frac{5Rc}{36}$$

31. **(d)** The graph of stress versus strain is given below



From the graph it is clear that within the elastic limit, the corresponding stress refers to yield point.

32. **(a)** If a wire is stretched to double its length, then

$$\text{Strain} = \frac{\text{Change in length}}{\text{Actual length}} = \frac{2L - L}{L} = 1$$

33. **(b)** From Kepler's second law of planetary motion,

$$\frac{dL}{dt} = \frac{L}{2m}$$

Thus, angular momentum remains conserved.

34. **(b)** As we know that, PE of satellite revolving at height h from the surface of the earth.

$$= -\frac{GM_e m}{R_e + h}$$

KE of satellite, $= \frac{1}{2}\frac{GM_e m}{2(R_e + h)}$

Therefore, PE $= -2$KE

$\therefore\ r = -2$

35. **(d)** Given, $n = 20\ \text{cm}^{-1} = 2000\ \text{m}^{-1}$

$B = 20\text{mT} = 20 \times 10^{-3}$ T

Using, $B = \mu_0 a I$

$20 \times 10^{-3} = 4\pi \times 10^{-7} \times 2000 \times I$

$$\Rightarrow\ I = \frac{20 \times 10^{-3}}{4\pi \times 10^{-7} \times 2000} = 8\ \text{A}$$

36. **(d)** The magnetic field through the circle is constant as current through the straight wire is constant. So,

$$\frac{d\phi_B}{dt} = 0$$

As, $E = \frac{d\phi_B}{dt} = 0\ \therefore\ I = \frac{E}{R} = 0$

Here, no current is flowing in circle.

37. **(b)** Pressure at a point inside a liquid is given by

$p = \frac{F}{A} = \rho hg$

Therefore, at same height, pressure in a fluid at rest will be the same.

38. **(a)** The interatomic forces decreases or becomes very weak at the boiling point of a liquid. Therefore, surface tension becomes zero.

39. **(b)** If all the masses will remain same, then centre of mass will lie at the centre'O. But as the mass at B is 4m, so the centre of mass of the system will shift towards B. So centre of mass will be on line OB.

40. **(c)** The external torque changes the angular momentum, therefore the second law in rotational motion is given by,

$\tau_{ext} = \frac{dL}{dt}$

41. **(d)** Given, $y = ut + gt^2$

Comparing with equation of motion $y = ut + \frac{1}{2}at^2$ we get, $a = 2\text{ g}$

$\therefore$ Force $= m \times a = m \times 2g = 2mg$

42. **(a)** The centripetal force is balanced by the normal reaction exerted i.e. $mv^2/r \le \mu_s N$ where, μ_s is the coefficient of friction.

$\therefore \frac{mv^2}{r} = f \le \mu_s N$

43. **(a)** Given, $n = 2\text{ min to }10\text{ min} = 5$ half-lives and

$N_0 = 80\text{ g}$

We know, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

$\Rightarrow N = \frac{N_0}{2^n} = \left(\frac{80}{2^5}\right) = \frac{80}{32} = 25\text{ g}$

44. **(c)** Energy released in the nuclear process is

$E = mc^2 - mc^2\left(\frac{2}{3}\right)^2 = \left(1 - \frac{4}{9}\right)mc^2 = \frac{5}{9}mc^2$

Hence, energy released decrease by a factor of $\frac{5}{9}$.

45. **(d)** Given $|e| = 220\text{ V}, di = 0.9\text{ A}, v = 50\text{ Hz}$

As $v = \frac{1}{t} \Rightarrow t = \frac{1}{v} = \frac{1}{50}$

Since, $|e| = L\frac{di}{dt} \Rightarrow 220 = L \times \frac{0.9}{1/50}$

$\Rightarrow L = \frac{220}{0.9 \times 50} = 4.88\text{H} \simeq 5\text{H}$

46. **(c)** Given $M = \frac{f_o}{f_e} = 9 \Rightarrow f_o = 9f_e$

Also, $f_o + f_e = 20$

$\Rightarrow 9f_e + f_e = 20 \Rightarrow f_e = 2\text{ cm}$

$\therefore f_o = 9 \times 2 = 18\text{ cm}$

47. **(b)** Given that, $\frac{1}{2}m_1v_1^2 = \frac{1}{2}m_2v_2^2$

$\Rightarrow \frac{1}{2} \times 1 \times v_1^2 = \frac{1}{2} \times 9 \times v_2^2 \Rightarrow v_1^2 = 9v_2^2 \Rightarrow v_1 = 3v_2$

Now, momentum $p = mv$

Therefore, $\frac{m_1v_1}{m_2v_2} = \frac{1 \times v_1}{9 \times v_2} = \frac{3v_2}{9v_2} = \frac{1}{3}$

48. **(a)** When kinetic energy is conserved in a collision, then the collision is said to be elastic.

49. **(a)** Given, $R = 10\text{k}\Omega = 10^4\Omega$

$C = 100\text{pF} = 100 \times 10^{-12}\text{ F}$

Time constant of RC circuit,

$\tau = RC = 10^4 \times 100 \times 10^{-12} = 10^{-6}\text{ s}$

For demodulation $\frac{1}{f_C} \ll RC \Rightarrow f_C \gg \frac{1}{RC}$

or $f_C \gg \frac{1}{10^{-6}} = 10^6\text{ Hz}$

$\therefore f_C \gg 1\text{MHz}$

50. **(b)** For motion under central forces, the total mechanical energy i.e. the sum of kinetic energy and potential energy of a body is constant. The familiar example is the free fall of an object under gravity.

51. **(c)** The statement given in the option (c) is incorrect, it can be corrected as
A hypothesis is a supposition without assuming that it is true.

52. **(c)** Using relation, $\tan\theta = \frac{X_L - X_C}{R}$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{Z}{R}\right)$$

Here, $Z = \sqrt{3}R$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{3}R}{R}\right) = \tan^{-1}(\sqrt{3}) = 60^\circ$$

53. **(d)** Time period of simple harmonic motion,

$$T = 2\pi\sqrt{\left|\frac{\theta}{\alpha}\right|} = 2\pi\sqrt{\frac{I}{mH}}$$

$$\Rightarrow T \propto \sqrt{I} \Rightarrow T \propto \sqrt{m} \qquad \left[\because I = \frac{ml^2}{12}\right]$$

$$\frac{T_1}{T_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m}{4m}} = \frac{1}{2}$$

$$\Rightarrow T_2 = 2T_1 = 2T \quad [\because T_1 = T]$$

So, motion remains simple harmonic with time period 2T.

54. **(a)** Given, $\mu_r = 6000$

As we know, $\mu_r = 1 + X_m$

$\Rightarrow X_m = \mu_r - 1 = 6000 - 1 = 5999$

55. **(d)** Since, frequency of spring oscillator is smallest among all the given options, hence small time interval cannot be measured with spring oscillator.

56. **(a)** The error in measurement of length 1.02 cm is ± 0.01 cm and error in measurement of length 9.89 cm is ± 0.01 cm.

$\therefore$ The relative error is 1.02 cm

$$= \left[\pm\frac{0.01}{1.02} \times 100\%\right] = \pm 0.98\% = \pm 1\%$$

Similarly, the relative error in 9.89 cm

$$= \left[\pm\frac{0.01}{9.89} \times 100\%\right] = \pm 0.1\%$$

Therefore, relative errors in measurement of two masses are $\pm 1\%$ and $\pm 0.1\%$.

57. **(c)** Given, $\lambda = 589 \text{ nm} = 589 \times 10^{-9}$ m

$d = 0.589 \text{ mm} = 0.589 \times 10^{-3}$ m

In Young's double slit experiment, half angular width of central maxima is

$$\sin\theta = \frac{\lambda}{d} = \frac{589 \times 10^{-9}}{0.589 \times 10^{-3}} = 10^{-3} \Rightarrow \theta = \sin^{-1}(0.001)$$

58. **(b)** The graph between stopping potential and frequency is a straight line, so stopping potential and hence maximum kinetic energy of the photoelectrons depends linearly on the frequency.

59. **(b)** Total work done by the gas

$W_{Total} = W_{A\to B} + W_{B\to C} + W_{C\to D} + W_{D\to A}$

Work done in isothermal process by ideal gas

$$W_{A\to B} = \mu RT, \log\frac{V_2}{V_1} \qquad \ldots\text{(i)}$$

Work done in adiabatic process by ideal gas

$$W_{B\to C} = \frac{\mu R(T_1 - T_2)}{r-1} \qquad \ldots\text{(ii)}$$

Work done in isothermal process by ideal gas,

$$W_{C\to D} = -\mu RT_2 \log\frac{V_3}{V_4} \qquad \ldots\text{(iii)}$$

Work done in adiabatic process, by ideal gas

$W_{D\to A} = \frac{-\mu R(T_1 - T_2)}{r-1}$ ....(iv)

Adding Eqs. (i), (ii), (iii) and (iv) we get

$W_{Total} = \mu RT_1 \log\frac{V_2}{V_1} - \mu RT_2 \log\frac{V_3}{V_4}$

Adding Eqs. (i), (ii), (iii) and (iv) we get

$W_{Total} = \mu RT_1 \log\frac{V_2}{V_1} - \mu RT_2 \log\frac{V_3}{V_4}$

60. (b) Given, $\lambda = 1.3\mu\text{ m} = 1.3 \times 10^{-6}$ m

Bandwidth of channel, $BW = 20\text{kHz} = 20 \times 10^3$ Hz

Optical source frequency,

$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{1.3 \times 10^{-6}}$

$= 2.3 \times 10^{14}$ Hz

$\therefore$ Number of channels, $n = \frac{f}{BW} = \frac{2.3 \times 10^{14}}{20 \times 10^3}$

$= 1.15 \times 10^{10}$

CHEMISTRY

61. (a) As we know that,

$\Delta U = q - p_{ext}\,\Delta V$

Now, if a process of expansion of a gas occurs in vaccum and at constant volume, then

$\Delta U = qV \quad (\because \Delta V = 0, p_{ext} = 0)$

62. (a) Carbon monoxide binds with haemoglobin to form carboxy haemoglobin, which is about 300 times more stable than the oxygen haemoglobin complex.

63. (b) Sucrose is formed by the condensation of α-D-glucopyranose and β-D-fructofuranose.

64. (c) Henry's law does not hold true when the gases are placed under extremely high pressure. It is one of the limitations of Henry's law.

65. (a) Both chromate and dichromate radicals on reaction with acidified H_2O_2 give blue solution which turns green on standing. In the presence of organic solvents such as diethyl ether, amyl alcohol or pyridine, permanent blue colour obtained.

$$K_2Cr_2O_4 + H_2SO_4 + H_2O_2 \xrightarrow{\text{Ether}} CrO_5 + K_2SO_4 + 3H_2O$$

$$K_2Cr_2O_7 + 4H_2SO_4 + 4H_2O_2 \xrightarrow{\text{Ether}} \underset{\text{(Blue colour)}}{2CrO_5} + K_2SO_4 + 5H_2O$$

66. (b) We know,

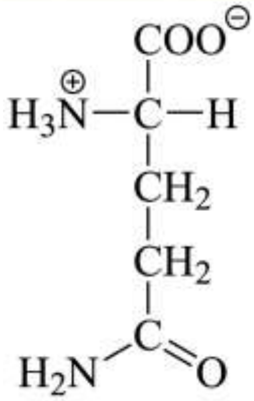
specific rate constant, $k = \frac{0.693}{t_{1/2}}$

$\therefore\ k = \frac{0.693}{1402} \Rightarrow k = 0.49 \times 10^{-3}\ s^{-1}$

67. (a) Given, $a = 4.29$Å

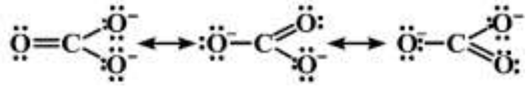
$r = \frac{\sqrt{3}}{4} \times a = \frac{\sqrt{3}}{4} \times 4.29\text{Å} = 1.86\text{Å}$

68. (d) Glutamine contains polar R group as amide group is present in the side chain of glutamine.

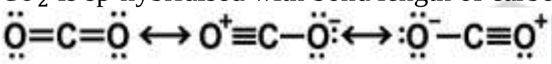


Glutamine

69. (d) A multiple bond is always shorter bond the corresponding single bond. The C -atom in CO_3^{2-} is sp^2-hybridised.



CO_2 is sp-hybridised with bond length of carbon oxygen is 122 pm .



The C -atom in CO is sp-hybridised with $C-O$ bond length 110 pm .

$^{+}C \equiv O^{-}$

So, the correct order of bond length is $CO < CO_2 < CO_3^{2-}$.

70. (c) The total number of nodes are given by $(n-1)$, i.e. sum of angular nodes l and radial nodes ($n-l-1$).

71. (a) $AB \rightleftharpoons A + B$

or $\quad K = \frac{[A][B]}{[AB]}$

if concentration of A is doubled, the equilibrium concentration of B becomes half to maintain K constant.

72. (b) Antipyretics are used to reduce fever. Antibiotics are used to treat bacterial infection. Analgesic are pain reliever and tranquilisers are used to treat mental illness.

73. (b) The given reaction is Gattermann reaction. In this reaction, chlorine or bromine can be introduced in the benzene ring by treating the diazonium salt solution with corresponding halogen acid in the presence of copper powder.

74. (b) The melting point of d-block elements is due to the presence of unpaired electrons in the last d-orbital or s-orbital. So, the element which has all the electrons in pairs will have the least melting point.

•Vanadium(V) with atomic number 23. [Ar] $3d^3 4s^2$ has 3 unpaired electrons.

•Zinc (Zn) with atomic number (30). [Ar] $3d^{10} 4s^2$ has zero unpaired electron.

•Manganese (Mn) with atomic number 25. $[Ar]3d^5 4s^2$ has five unpaired electrons.

•Copper (Cu) with atomic number 29. $[Ar]3d^{10} 4s^1$ has one unpaired electron. Therefore zinc has least melting point.

75. (c) The zymase enzyme is involved in the catalytic reaction of glucose into ethyl alcohol and carbon dioxide.

$$\underset{\text{Glucose}}{C_6H_{12}O_6(aq)} \xrightarrow{\text{Zymase}} \underset{\text{Ethyl alcohol}}{2C_2H_5OH(g)} + \underset{\text{Carbon dioxide}}{2CO_2(g)}$$

76. (d) Among the given mixture, only CCl_4 and $CHCl_3$ form non-ideal solution, thus they form azeotropic mixture as only non-ideal solutions form azeotropic mixture.

77. (d) k varies with temperature as given by Arrhenius equation.

$$k = Ae^{-E_a/RT} \Rightarrow k \propto T \text{ and } k \propto \frac{1}{E_a}$$

$\therefore$ k has least value at low T and high E_a.

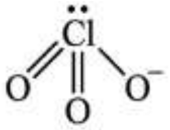
78. (a) The correct decreasing order of the stabilities of given carbocations is

(iii) $>$ (ii) $>$ (i) $>$ (iv)

Because electron releasing effect stabilises the carbocation. Electron releasing order is

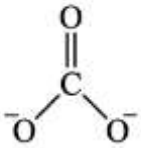
$NH_2 > OH > OCH_3 > Cl$.

79. (a) In ClO_3^-, total number of electrons $= 42$



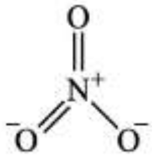
Pyramidal shape

In CO_3^{2-}, total number of electrons $= 32$



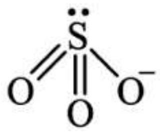
Trigonal planar shape

In NO_3^-, total number of electrons $= 32$



Trigonal planar shape

In SO_3^{2-}, total number of electrons $= 42$



Pyramidal shape

So, the option (a) is correct.

80. (c) Only $\frac{1}{8}$ th of an atom or molecule or ion actually belongs to a particular unit cell.

81. (d) We know that, $K_p = K_C(RT)^{\Delta n_g}$

for the given reaction,

$\Delta n_g = (2+1) - 2 = 1$

$K_p = 3.75 \times 10^{-6}(0.0831 \times 1069) = 0.00033$

82. (c)

Artificial Sweetener	**Sweetness value in comparison to cane sugar**
Aspartame	100
Saccharin	550
Sucralose	600
Alitame	2000

83. (c) Coupling reaction is an example of electrophilic substitution reaction.

84. (d) In the structure of CrO_5, it has O -atoms as peroxide which have oxidation number $= -1$ and one O as oxide (double bonded O) atom with oxidation number $= -2$

Let the oxidation number of Cr be x.

Then, $x + 4 \times (-1) + 1 \times (-2) = 0$

or $\quad x = +6$

$\therefore$ The oxidation number of Cr in CrO_5 is +6 .

85. (c) At constant pressure and temperature, the rate of diffusion or effusion of gas is inversely proportional to the square root of its density.

Rate of diffusion $\propto \frac{1}{\sqrt{d}}$

$$\frac{r_1}{r_2} = \sqrt{\frac{d_2}{d_1}} = \sqrt{\frac{M_2}{M_1}}$$

$r_1 = r_2$ only when $M_1 = M_2$

Hence, N_2O and CO_2 have equal rate of diffusion due to equal molecular weight.

86. (a) Carbon monoxide (CO) is the reducing agent which reduces iron oxide (Fe_2O_3) to iron in the blast furnace.

$Fe_2O_3 + 3CO \longrightarrow 2Fe + 3CO_2 \uparrow$

87. (b) $\mu^{\infty}_{Ba(OH)_2} = \lambda^{\infty}_{Ba^{2+}} + 2\lambda^{\infty}_{OH^-} = 523.28$ ….(i)

$\mu^{\infty}_{BaCl_2} = \lambda^{\infty}_{Ba^{2+}} + 2\lambda^{\infty}_{Cl^-} = 280.00$ ….(ii)

$\mu^{\infty}_{NH_4Cl} = \lambda^{\infty}_{NH_4^+} + \lambda^{\infty}_{Cl^-} = 129.80$ …..(iii)

$\mu^{\infty}_{NH_4OH} = \lambda^{\infty}_{NH_4^+} + \lambda^{\infty}_{OH^-}$

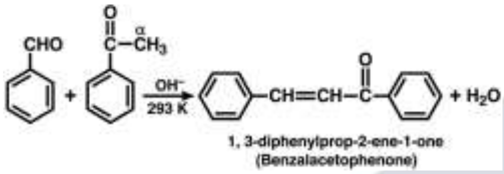
Eq. (iii) $+ \frac{\text{Eq.(i)}}{2} - \frac{\text{Eq. (ii)}}{2}$ will gives

$\Rightarrow \lambda^{\infty}_{NH_4^+} + \lambda^{\infty}_{OH^-} = \lambda^{\infty}_{NH_4OH} = \frac{502.88}{2}$

$= 251.44\Omega^{-1}\ cm^2\ mol^{-1}$

88. (a) Xerophthalmia disease is type of (hardening of cornea of eye) disease. It is caused by deficiency of vitamin-A.

89. (a) The final product from this reaction is



90. (d) Phase transformation involve the energy changes e.g., ice requires heat for melting. Normally this melting takes place at constant pressure and during phase change, temperature remains constant (at 273 K).

91. (a) Dettol is a mixture of chloroxylenol and terpineol.

92. (c) The chemical formula of Hinsberg's reagent is $C_6H_5SO_2Cl$.

93. (b) The increasing order of atomic radii of group-13 elements is

$Ga(135pm) < Al(143pm) < In(167pm) < Tl(170pm)$

On moving down the group the atomic radii increases. However due to poor shielding of nuclear charge by 3d-electron.

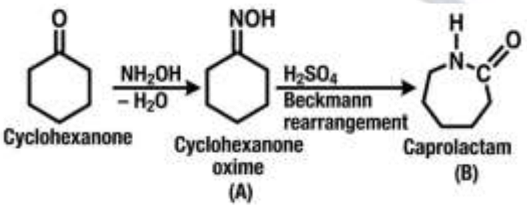
$\underset{(135pm)}{r_{Ca}} < \underset{(143pm)}{r_{Al}}$

The electrons in the outermost shell of Ga experience more force of attraction towards nucleus than Al which decreases atomic radius.

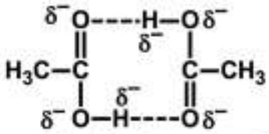
94. (a) Macromolecular colloids are quite stable and resemble to the true solutions in many respects.

95. (b) Wurtz reaction is used to prepare symmetrical alkanes having even number of carbon atoms. Option (b) contains an alkane with odd number of carbon atoms. Thus, it can be prepared by Wurtz reactions.

96. (c)



97. (a) Due to intermolecular hydrogen bonding, ketones and carboxylic acids have higher boiling point as compare to aldehyde.



Hydrogen bonding in carboxylic acid

98. (d) In case of bec lattice cell, packing efficiency = 68%

$\therefore$ Percent vacant space $= 100 - 68 = 32\%$

99. (d) Since, at equilibrium, rate of forward reaction = rate of backward reaction. Therefore, in the chemical reaction,$N_2 + 3H_2 \rightleftharpoons 2NH_3$ the same amount of ammonia is formed, as it is decomposed into N_2 and H_2.

100. **(b)** Heat absorbed,

$q = 500\text{cal}$

Work done by the system, $W = -350\text{cal}$

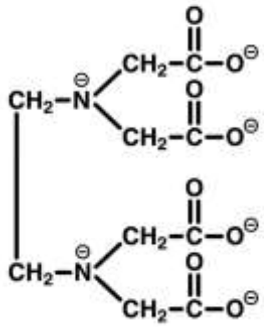
According to the first law of thermodynamics,

$\Delta E = q + W = 500 + (-350) = +150\text{cal}$.

101. **(c)** Phenelzine is used to treat panic attacks.

102. **(b)** $C_6H_5\ N_2^+X^-$ is most stable due to resonance.

103. **(b)** EDTA is a hexadentate ligand because it has six binding sites i.e. it has six donor atoms per ligand.



104. **(d)** $\frac{d}{p} = \frac{M}{RT}$

Let density of gas B be d.

∴ Density of gas A = 3d and let molecular weight of A be M.

∵ Molecular weight of B = 2M.

Since, R is a gas constant and T is same for both gases, so,

$p_A = \frac{d_A RT}{M_A}$ and $p_B = \frac{d_B RT}{M_B}$

$\frac{p_B}{p_A} = \frac{d_B}{d_A} \times \frac{M_A}{M_B} = \frac{d}{3d} \times \frac{M}{2M} = \frac{1}{6}$

105. **(b)** More the value of i, more will be the elevation in boiling point. Also, more dilute solution has low boiling point. Hence, the increasing order of boiling points is

$10^{-2}\text{MNaCl} > 10^{-3}\text{MMgCl}_2$

$(i = 2) \quad (i = 3)$

$> 10^{-4}\text{MNaCl} > 10^{-4}$ Murea

$(i = 2) \quad (i = 1)$

106. **(c)** Molarity of $BaCl_2 = \frac{1 \times 1000}{208 \times 200} = 0.024M$

Also, normality of $BaCl_2 = 0.024 \times 2 = 0.048$ N

$(\because N = M \times V \cdot f)$

$\lambda_m = \frac{\kappa \times 1000}{M} = \frac{0.0058 \times 1000}{0.024}$

$= 241.678\ \text{S cm}^2\ \text{mol}^{-1}$

Also, $\lambda_{eq.} = \frac{\kappa \times 1000}{N} = \frac{0.0058 \times 1000}{0.048}$

$= 120.83\ \text{S cm}^2 \text{eq}^{-1}$.

107. **(c)** $2\ N_2O_5 \longrightarrow 4NO_2 + O_2$

∵ The above reaction is first order.

so, $k = \frac{2.303}{t} \log \frac{P_0}{P_t}$

Put the values

$4.48 \times 10^{-5} = \frac{2303}{10 \times 60} \log \frac{600}{P_t}$

$\log \frac{600}{P_t} = 0.01167$

$\frac{600}{P_t} = \text{antilog}(0.01167)$

$\frac{600}{P_t} = 1.02$

$P_t = \frac{600}{1.02} \Rightarrow 588$ atm

108. **(a)** As we know that, $C_p - C_V = nR$

where, n = number of moles of gas

$R = 8314\ J/Kmol^{-1}$

$C_p - C_V = 10 \times 8.314 = 83.14\ J/K$

109. **(b)** Use of lead (Pb) in petrol causes emission of lead by vehicles.

110. **(d)** Trihydrides of group 15 elements are given on the basis of reducing character as follows :

$BiH_3 > SbH_3 > AsH_3 > PH_3 > NH_3$

The reducing character increases down the group because the bond strength of $E - H$ bond decreases. As the size of element E down the group increases and thus, the bond dissociation energy decreases. This facilitates the dissociation of hydrogen atoms and hence, down the group, the reducing character increase.

111. **(a)** Given, mass of solute = 22.2 g

Volume of solution = 1 L

∴ Molarity

$(M) = \frac{\text{mass of solute (in g)}}{\text{mol.weight of solute } \times \text{ Volume of solution (in L)}}$

$= \frac{22.2\ g}{111\ g\ mol^{-1} \times 1\ L}$

[∵ Mol^2 weight of $CaCl_2 = 111\ g\ mol^{-1}$]

$= 0.2\ mol\ L^{-1} = 0.2M$

112. **(c)** The reaction for decolourisation of 1 mole of $KMnO_4$ by H_2O_2 are given as follow:

$2KMnO_4 + 3H_2SO_4 \longrightarrow K_2SO_4 + 2MnSO_4 + 3H_2O + 5[O]$

$5H_2O_2 + 5[O] \longrightarrow 5H_2O + 5O_2$

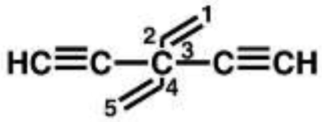
The overall reaction is

$2KMnO_4 + 3H_2SO_4 + 5H_2O_2 \longrightarrow K_2SO_4 + 2MnSO_4 + 8H_2O + 5O_2$

As 2 moles of $KMnO_4$ require $H_2O_2 = 5$ moles

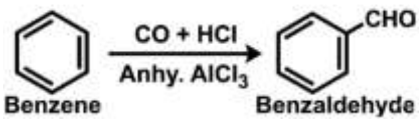
∴ 1 mole of $KMnO_4$ require $H_2O_2 = \frac{5}{2} = 2.5$ moles

113. **(a)** The correct IUPAC name of given molecule is 3, 3-diethynylpenta-1, 4-diene.



114. **(b)** In case of H_2O_2, $O - O$ bond is non-polar and $O - H$ bond is polar.

115. **(a)** This is a named reaction known as Gattermann-koch reaction.



116. **(a)**Balmer series of transitions in the spectrum of hydrogen atom fall in visible region. Lyman series fall in ultraviolet, while Paschen, Brackett and Pfund fall in infrared region.

117. **(d)** Given, $\Delta G° = 13.8\ kJ/mol = 13.8 \times 10^3\ J/mol$

∵ $\Delta G° = -RT\ln K_C$

$13800 = -8.314 \times 298 \ln K_C$

∴ $K_C = 3.81 \times 10^{-3}$

118. **(c)** Lyman series is a hydrogen spectral series of transitions and result in ultraviolet emission lines of the hydrogen atom as electron goes from $n \geq 2$ to $n = 1$ (where n is the principal quantum number) the lowest energy level of electron. Hence, it lies in ultraviolet region.

119. **(b)** $K_3[Fe(CN)_5NO]$

$(CN)_5$ = pentacyano

(NO) = nitro-N

So, IUPAC name of given compound is potassium pentacyanonitro-N-ferrate (III).

120. **(c)** Chemisorption occurs at very high temperature and high pressure as it involves chemical reaction between adsorbate and adsorbent molecules and involves high activation energy to start the reaction which requires high pressure and temperature.

MATHEMATICS

121. **(b)** Consider the given inequation as $2x + 3y = 6$.

Table for equation $2x + 3y = 6$ or $y = \frac{6-2x}{3}$ is

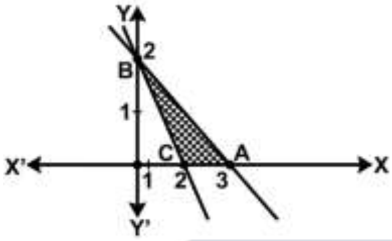
x	0	3
y	2	0

On putting (0,0) in the inequality $2x + 3y \leq 6$, we get Consider $x + y = 2$

x	0	2
y	2	0

$0 \leq 6$ which is true

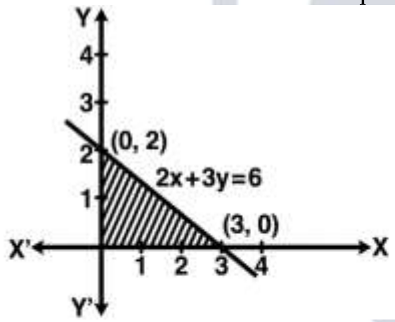
$\Rightarrow$ Feasible region will be towards origin



122. **(d)** Let $Z = x + y$

Given, constraints are $2x + 3y \leq 6, x \geq 0, y \geq 0$.

These constraints consider as equation $2x + 3y = 6$



This equation intersect X and Y-axes at $A(3,0)$ and $B(0,2)$. On putting $(0,0)$ in the inequality $2x + 3y \leq 6$, we get $2 \times 0 + 3 \times 0 \leq 6 \Rightarrow 0 \leq 6$ (which is true). So, the feasible region is towards the origin.

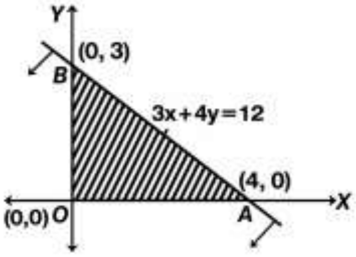
$\therefore$ The corner points of a feasible region are $O(0,0), A(3,0)$ and $B(0,2)$.

The value of Z on these corner points are given below

Corner points	**Value of $Z(Z = x + y)$**
$\alpha(0,0)$	$Z = 0 + 0 = 0$
$A(3,0)$	$Z = 3 + 0 = 3$ (maximum)
$B(0,2)$	$Z = 0 + 2 = 2$

Hence, maximum value of Z is 3 .

123. **(b)** The corner points of the feasible region are $O(0,0), A(4,0)$ and $B(0,3)$.



The value of Z on these corner points are given below

Corner points	**Value of Z($\mathbf{Z = x + y}$)**
O(0,0)	$Z = 0 + 0 = 0$
A(4,0)	$Z = 4 + 0 = 4$ (maximum)
B(0,3)	$Z = 0 + 3 = 3$

Hence, Z(max) is 4 at point $x = 4$ and $y = 0$.

124. **(c)** Let the vector be $x\hat{i} + y\hat{j} + z\hat{k}$

Since, unit vector along X-axis is $\hat{i}$

$\hat{i} = V_1 + V_2 + V_3$

$\hat{i} = \hat{i} - 3\hat{j} + 2\hat{k} + 3\hat{i} + 6\hat{j} - 7\hat{k} + x\hat{i} + y\hat{j} + z\hat{k}$

$\hat{i} + 0\hat{j} + 0\hat{k} = (4 + x)\hat{i} + (3 + y)\hat{j} + (z - 5)\hat{k}$

$\Rightarrow x + 4 = 1 \Rightarrow x = -3$

$\Rightarrow y + 3 = 0 \Rightarrow y = -3 \Rightarrow z - 5 = 0 \rightarrow z = 5$

The vector $-3\hat{i} - 3\hat{j} + 5\hat{k}$.

125. **(b)** Given, $|a| = 8, |b| = 3$ and $|a \times b| = 12$

We know that, $\sin\theta = \frac{|a\times b|}{|a||b|}$

$\Rightarrow \sin\theta = \dfrac{12}{8 \times 3} = \dfrac{1}{2} \Rightarrow \sin\theta = \sin\dfrac{\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}$ $\quad \left[\because \sin\frac{\pi}{6} = \frac{1}{2}\right]$

Hence, angle between a and b is $\frac{\pi}{6}$.

126. **(c)** Given, $a = 2\hat{i} + 3\hat{j} + \hat{k}, b = \hat{i} - 2\hat{j} + \hat{k}$

and $c = -3\hat{i} + \hat{j} + 2\hat{k}$

We know that, $[abc] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$\therefore [abc] = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ -3 & 1 & 2 \end{vmatrix}$

$= 2(-4 - 1) - 3(2 + 3) + 1(1 - 6)$

$= -10 - 15 - 5 = -30$

127. **(d)** Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now, $A(\text{Adj}A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad - bc & 0 \\ 0 & ad - bc \end{bmatrix}$

$\Rightarrow ad - bc = 8, -ab + ab = 0, dc - dc = 0$

$\Rightarrow -bc + ad = 8$

$\det(A) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc = 8$

128. **(c)** Given, $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4 + 4 & -4 - 4 \\ -4 - 4 & 4 + 4 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$

Given that, $A^2 = pA$

$\Rightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = p\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = \begin{bmatrix} 2p & -2p \\ -2p & 2p \end{bmatrix}$

Now, $2p = 8 \Rightarrow p = 4$

129. **(c)** $A(adjA) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} = -2I$

$A(adjA) = |A|I \Rightarrow -2I = |A|I$

$|A| = -2$

$\Rightarrow |adjA| = |A|^{n-1}, n = 3$

$\Rightarrow |adjA| = |A|^{3-1} = |A|^2$

$\Rightarrow |adjA| = (-2)^2 = 4$

130. **(b)**Given, $ax^2 + bx + c = 0$

For equal roots $D = 0 \Rightarrow b^2 = 4ac \ \Rightarrow b^2 = 4ac$

In Case I : $ac = 1$

$(a, b, c) = (1,2,1)$

Case II : $ac = 4$

$(a, b, c) = (1,4,4)$ or $(4,4,1)$ or $(2,4,2)$

Case III : $ac = 9$

$(a, b, c) = (3,6,3)$

Required probability $= \frac{5}{216}$

131. **(c)**$8^{3\log_8 5} \Rightarrow 8^{\log_8 5^3} = (5)^3 \qquad [a^{\log_a x} = x]$

$= 5 \times 5 \times 5 = 125$

132. **(a)** Given curve is $y = (1 + x)^y + \sin^{-1}(\sin^2 x)$

On differentiating both sides w.r.t. x,

$\frac{dy}{dx} = (1 + x)^y \left[\frac{y}{1+x} + \log(1 + x)\frac{dy}{dx}\right] + \frac{2\sin x\cos x}{\sqrt{1-\sin^4 x}}$

$\left(\frac{dy}{dx}\right)_{at(0,1)} = +1$ [at $x = 0, y = 1$]

$\Rightarrow$ Slope of normal at $(x = 0) = -1$

Hence, equation of the normal at $(0,1)$ is $x + y = 1$

133. **(a)** Given, $L = \lim_{x\to 0} \frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}$ $\left[\frac{0}{0}\text{ form }\right]$

$= \lim_{x\to 0} \frac{0-\frac{(0-2x)}{2\sqrt{a^2-x^2}}-\frac{2x}{4}}{4x^3}$ [by using L′ hospital rule]

$= \lim_{x\to 0} \frac{x\left(\frac{1}{\sqrt{a^2-x^2}} - \frac{1}{2}\right)}{4x^3} = \lim_{x\to 0} \frac{\frac{1}{\sqrt{a^2-x^2}} - \frac{1}{2}}{4x^2}$

$\left(\frac{\frac{1}{a} - \frac{1}{2}}{0}\text{ form}\right)$

For limit to be exist $\Rightarrow \frac{1}{a} - \frac{1}{2} = 0 \Rightarrow a = 2.$

134. **(a)** Given, centre $(h, k) = (1,2)$

and circle touches on X axis.

$\therefore$ Radius(r) = y coordinate of centre = 2

So, equation of circle is

$(x - 1)^2 + (y - 2)^2 = 2^2 \quad [\because (x - h)^2 + (y - k)^2 = r^2]$

$\Rightarrow x^2 - 2x + 1 + y^2 - 4y + 4 = 4$

$[\because (a - b)^2 = a^2 + b^2 - 2ab]$

$\Rightarrow \ x^2 + y^2 - 2x - 4y + 1 = 0$

135. **(a)** First, break the number 5.001 as $x = 5$ and $\Delta x = 0.001$ and use the relation

$f(x + \Delta x) = f(x) + \Delta x f'(x)$

Consider,

$f(x) = x^3 - 7x^2 + 15$

$\Rightarrow\ f'(x) = 3x^2 - 14x$
Let $x = 5$ and $\Delta x = 0.001$
Also, $f(x + \Delta x) = f(x) + \Delta x f'(x)$
Therefore,
$f(x + \Delta x) - (x^3 - 7x^2 + 15) + \Delta x(3x^2 - 14x)$
$\Rightarrow\ f(5.001) = (5^3 - 7 \times 5^2 + 15) +$
$(3 \times 5^2 - 14 \times 5)(0.001)$
as $x = 5, \Delta x = 0.001]$
$= (125 - 175 + 15) + (75 - 70)(0.001)$
$= -35 + (5)(0.001) = -35 + 0.005 = -34.995$

136. **(a)** Given equation is

$$2x^2 + 2y^2 + 3x + 4y + \frac{9}{8} = 0.$$

On dividing both sides by 2 , we get

$x^2 + y^2 + \frac{3}{2}x + 2y + \frac{9}{16} = 0$ .....(i)

From Eq. (i), we have coefficient of $x = \frac{3}{2}$ and coefficient of $y = 2$

$$\therefore\ \alpha = \frac{-1}{2}\left(\frac{3}{2}\right) = -\frac{3}{4} \text{ and } \beta = \frac{-1}{2}(2) = -1$$

Hence, centre $= \left(-\frac{3}{4}, -1\right)$

From Eq. (i), we have constant term $= \frac{9}{16}$

$$\therefore\ \text{Radius} = \sqrt{\left(\frac{-3}{4}\right)^2 + (-1)^2 - \frac{9}{16}} = \sqrt{\frac{9}{16} + 1 - \frac{9}{16}} = 1$$

137. **(b)** For maximum value of $f(x) = \frac{1}{4x^2+2x+1}$

$4x^2 + 2x + 1$ must be minimum.
For minimum values of $4x^2 + 2x + 1$.

$$4x^2 + 2x + 1 = 4x^2 + 2x + \frac{1}{4} + \frac{3}{4} = \left(2x + \frac{1}{2}\right)^2 + \frac{3}{4}$$

So, $4x^2 + 2x + 1$ becomes minimum when $2x + \frac{1}{2} = 0$

Then, minimum value of $4x^2 + 2x + 1$ becomes $\frac{3}{4}$.

Hence, the maximum value of $f(x) = \frac{1}{3/4} = \frac{4}{3}$

138. **(c)** Given, $f(x) = \begin{cases} ax + 3, & x \le 2 \\ a^2x - 1, & x > 2 \end{cases}$

Continuity at $x = 2$,
$\text{LHL} = \lim_{x\to 2^-} f(x) = \lim_{x\to 2}(ax + 3) = 2a + 3$
$\text{RHL} = \lim_{x\to 2^+} f(x) = \lim_{x\to 2}(a^2x - 1) = 2a^2 - 1$
Since, $f(x)$ is continuous for all values of x.
$\therefore$ LHL = RHL
$\Rightarrow 2a + 3 = 2a^2 - 1 \Rightarrow 2a^2 - 2a - 4 = 0$
$\Rightarrow a^2 - a - 2 = 0 \Rightarrow a^2 - 2a + a - 2 = 0$
$\Rightarrow a(a - 2) + 1(a - 2) = 0 \Rightarrow (a + 1)(a - 2) = 0$
$\therefore\ a = -1, 2$

139. **(d)** Let $y = \lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{2/x}$

$$\Rightarrow \log y = \lim_{x\to 0}\frac{2}{x}\log\left(\frac{a^x + b^x + c^x}{3}\right)$$

$$\Rightarrow \log y = \lim_{x\to 0}\frac{2\log(a^x + b^x + c^x) - \log 3}{x}\left(\frac{0}{0}\text{ form}\right)$$

$\Rightarrow\ \log y = \log(abc)^{2/3}$
[using L' Hospital's rule]

$\therefore\ y = (abc)^{2/3}$

140. **(a)** Coordinates of centre of given circle is $(1,2)$ and it passes through the point $(4,6)$ the radius of the required circle is equal to the distance from the centre to a point on a circle.

$\therefore$ Radius of circle $= \sqrt{(1-4)^2 + (2-6)^2} = \sqrt{9+16} = 5$

Hence, the required equation of the circle is

$(x-1)^2 + (y-2)^2 = (5)^2$

$x^2 + y^2 - 2x - 4y - 20 = 0$

141. **(d)** We know that, any line is parallel to the plane, then normal to the plane is perpendicular to the line.

Given, line is $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$

So, DR's of line are $\langle 3,4,5 \rangle$.

Let us consider the equation of plane be

$2x + y - 2z = 0$ [from the given options]

Hence, DR's of a plane are $\langle 2,1,-2 \rangle$.

Now, $3 \times 2 + 4 \times 1 + 5 \times (-2) = 6 + 4 - 10$

$= 10 - 10 = 0$

142. **(a)** Equation of plane passing through intersection of two planes

$x + 2y + 3z = 2$ and $x - y + z = 3$

$\Rightarrow\ (x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

$\Rightarrow\ (1+\lambda)x + (2-\lambda)y + (3+\lambda)z - (2+3\lambda) = 0$

whose distance from $(3,1,-1)$ is $\frac{2}{\sqrt{3}}$.

$$\therefore \frac{|3(1+\lambda) + 1 \cdot (2-\lambda) - 1(3+\lambda) - (2+3\lambda)|}{\sqrt{(1+\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2}} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \lambda = -\frac{7}{2}$$

Hence, equation of plane is $5x - 11y + z - 17 = 0$.

143. **(c)** The given lines can be rewritten as $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$

$\therefore$ Angle between the lines is

$$\theta = \cos^{-1}\left(\frac{3 \times 2 + 2(-12) - 6(-3)}{\sqrt{3^2 + 2^2 + (-6)^2}\sqrt{(2)^2 + (-12)^2 + (-3)^2}}\right) = 0$$

$= \ \theta\ = 90°$

144. **(c)** Given lines are

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = r$ [say]….(i)

and $\frac{x-4}{5} = \frac{y-1}{2} = z$ …..(ii)

Any point on the line (i) is $(2r+1, 3r+2, 4r+3)$.

If they intersect, then this point satisfies the second line, we get

$$\frac{2r+1-4}{5} = \frac{3r+2-1}{2} = 4r+3$$

$$\Rightarrow \frac{2r-3}{5} = \frac{3r+1}{2} \Rightarrow r = -1$$

Hence, the required point is $(-1,-1,-1)$.

145. **(c)** Let $I = \int \frac{2^x}{\sqrt{1-4^x}} dx = \int \frac{2^x}{\sqrt{1-(2^x)^2}} dx$

Putting $2^x = t \Rightarrow 2^x \cdot \log 2 dx = dt$

$$\Rightarrow\ 2^x dx = \frac{dt}{\log 2}$$

$$\therefore\ I = \frac{1}{\log 2}\int \frac{dt}{\sqrt{1-t^2}} = \frac{1}{\log 2}\sin^{-1}(t) + C$$

$$= \frac{1}{\log 2}\sin^{-1}(2^x) + C$$

146. **(c)** $\int_{-\pi/2}^{\pi/2} \sin x dx = \int_{-\pi/2}^{0} \sin x dx + \int_{0}^{\pi/2} \sin x dx$

$= -[\cos x]_{-\pi/2}^{0} - [\cos x]_0^{\pi/2}$

$= (-1-0)-(0-1) = 0$

147. **(d)** Let $I = \int \frac{dx}{x^2[1+x^4]^{\frac{3}{4}}} = \int \frac{dx}{x^2\left(x^4\left(\frac{1}{x^4}+1\right)\right)^{3/4}}$

[taking x^4 common from denominator]

$$\Rightarrow \int \frac{dx}{x^5\left(\frac{1}{x^4}+1\right)^{3/4}}$$

Let $\left(\frac{1}{x^4}+1\right) = t^4 \Rightarrow \frac{-4dx}{x^5} = 4t^3dt \Rightarrow \frac{dx}{x^5} = -t^3dt$

$\therefore I = \int \frac{-t^3dt}{t^3} = -t + C = -\left(\frac{1}{x^4}+1\right)^{1/4} + C$

148. **(c)** In a regular hexagon, there are two equilateral triangles are possible.

$\therefore$ Required probability $= \frac{2}{{}^6C_3} = \frac{2}{\frac{6\times5\times4}{3\times2}} = \frac{2}{20} = \frac{1}{10}$

149. **(c)** Total number of ways $= 5!$

and favourable number of ways $= 2\cdot 4!$

$\therefore$ Required probability $= \frac{2\cdot 4!}{5!} = \frac{2}{5}$

150. **(d)** Given, $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$

Since, A, B and C are exclusive events.

$\therefore\ P(A)+P(B)+P(C) = 1$

$\Rightarrow\ P(A) + \frac{3}{2}P(A) + \frac{1}{2}\times\frac{3}{2}P(A) = 1$

$\Rightarrow\ P(A)\left(1+\frac{3}{2}+\frac{3}{4}\right) = 1$

$\Rightarrow \frac{13}{4}P(A) = 1 \Rightarrow P(A) = \frac{4}{13}$

$\Rightarrow\ P(C) = \frac{1}{2}\times\frac{3}{2}P(A) = \frac{3}{4}\times\frac{4}{13} = \frac{3}{13}$

Also, A, B and C are mutually exclusive.

$\therefore P(A\cap B) = P(B\cap C) = P(C\cap A) = 0$

$\Rightarrow P(A\cup C) = P(A)+P(C)-0 = \frac{4}{13}+\frac{3}{13} = \frac{7}{13}$

151. **(c)** Let E_1 = Student does not know the answer

E_2 = Student knows the answer

and E = Student answer correctly

$\therefore\ P(E_1) = 1-p \Rightarrow P(E_2) = p$

$\Rightarrow\ P\left(\frac{E}{E_2}\right) = 1$ and $P\left(\frac{E}{E_1}\right) = \frac{1}{5}$

Note that, the probability that student did not know the answer randomly = The probability that student know the answer.

$$\therefore\ P\left(\frac{E_2}{E}\right) = \frac{P(E_2)P\left(\frac{E}{E_2}\right)}{P(E_1)P\left(\frac{E}{E_1}\right)+P(E_2)P\left(\frac{E}{E_2}\right)}$$

$$= \frac{p(1)}{(1-p)\frac{1}{5}+p(1)} = \frac{p}{\frac{1-p+5p}{5}} = \frac{5p}{1+4p}$$

152. **(d)** Given, $\cos x + \cos y = \frac{3}{2}$

$\Rightarrow\ 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$ ....(i)

and $\sin x + \sin y = \frac{3}{4}$

$\Rightarrow 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{4}$ ....(ii)

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{3/4}{3/2} \Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{1}{2}$$

$$\therefore \sin(x+y) = \frac{2\tan\left(\frac{x+y}{2}\right)}{1+\tan^2\left(\frac{x+y}{2}\right)} = \frac{2\times\frac{1}{2}}{1+\left(\frac{1}{2}\right)^2}$$

$$= \frac{4}{4+1} = \frac{4}{5}$$

153. **(b)** $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A} = \frac{\sin A}{\cos A}\times\frac{\sin A}{\sin A-\cos A} + \frac{\cos A}{\sin A}\times\frac{\cos A}{\cos A-\sin A}$

$= \frac{1}{\sin A-\cos A}\left\{\frac{\sin^3 A-\cos^3 A}{\cos A\sin A}\right\}$

$= \frac{\sin^2 A+\cos^2 A+\sin A\cos A}{\cos A\sin A} = \sec A\operatorname{cosec} A + 1$

154. **(d)** We have, $\sin 2x = 4\cos x \Rightarrow 2\sin x\cos x = 4\cos x$

$\Rightarrow \cos x(\sin x - 2) = 0 \Rightarrow \cos x = 0$ $[\because \sin x \neq 2]$

$\Rightarrow \cos x = 0 = \cos\pi/2 \Rightarrow x = 2n\pi \pm \frac{\pi}{2}, n \in Z$

155. **(b)** Given, $2f(x) + f(1-x) = x^2$ ....(i)

Replacing x by $(1-x)$, we get

$2f(1-x) + f(x) = (1-x)^2$

$\Rightarrow 2f(1-x) + f(x) = 1 + x^2 - 2x$ ....(ii)

Multiplying Eq. (i) by 2 and subtracting Eq. (ii) from Eq. (i), we get

$3f(x) = x^2 + 2x - 1 \Rightarrow f(x) = \frac{x^2+2x-1}{3}$

156. **(b)** Given, $A = \{1,2,5,6\}$ and $B = \{1,2,3\}$

$A\times B = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),$
$(5,1),(5,2),(5,3),(6,1),(6,2),(6,3)\}$

$B\times A = \{(1,1),(1,2),(1,5),(1,6),(2,1),(2,2),$
$(2,5),(2,6),(3,1),(3,2),(3,5),(3,6)\}$

$\therefore (A\times B)\cap(B\times A) = \{(1,1),(1,2),(2,1),(2,2)\}$

157. **(a)** If a set A has n elements, then its power set will contain 2^n elements.

$\therefore$ Total number of elements is in power set of $A = 2^{15}$

158. **(d)** Let $z = \frac{(1+i)(2+i)}{(3-i)} = \frac{2+3i+i^2}{3-i} = \frac{2+3i-1}{3-i}$

$[\because i^2 = -1]$

$= \frac{1+3i}{3-i}\times\frac{3+i}{3+i} = \frac{3+10i+3i^2}{9-i^2}$

$= \frac{3+10i-3}{9+1} = \frac{10i}{10} = i$ $[\because i^2 = -1]$

$\therefore z = 0 + i\cdot 1$

So, $\arg(z) = \tan^{-1}\left(\frac{1}{0}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$

159. **(a)** We have,

$\left[i^{18} + \left(\frac{1}{i}\right)^{25}\right]^3$ $\{i^{18} = i^{16+2} = 1\cdot i^2 = -1\}$

$= [-1 + (-i)^{25}]^3 = [-1-i]^3 = -[1+i]^3$

$= -[1+3i-3-i] = -[-2+2i] = 2(1-i)$

160. **(b)** $(\sqrt{3}+i)^{100} = 2^{99}(a+ib) \Rightarrow (-2i\omega)^{100} = 2^{99}(a+ib)$

$\left[\because \omega = -\frac{1+\sqrt{3}i}{2}, i\omega = -\frac{i-\sqrt{3}}{2},\right.$

$\left.-2i\omega = i+\sqrt{3} \text{ and } \omega^{100} = \omega^{99}\omega = 1\cdot\omega = \omega\right]$

$\Rightarrow 2^{100}\omega = 2^{99}(a+ib)$

$\Rightarrow 2\left\{\frac{-1-\sqrt{3}i}{2}\right\} = a+ib \Rightarrow a=-1 \text{ and } b=\sqrt{3}$

$\therefore a^2+b^2 = (-1)^2+(\sqrt{3})^2 = 4$

161. **(b)** Given, $a_0 = 1, a_{n+1} = 3n^2+n+a_n$

Put $n=0 \Rightarrow a_1 = 3(0)+0+a_0 = 1$

$\Rightarrow a_2 = 3(1)^2+1+a_1 = 3+1+1 = 5$

From option (b),

Let $P(n) = n^3-n^2+1$

$\therefore P(0) = 0-0+1 = 1 = a_0$

$P(1) = 1^3-1^2+1 = 1 = a_1$

and $P(2) = (2)^3-(2)^2+1 = 5 = a_2$

162. **(d)** Put $n=1$ in $49^n+16n+P$, then

$(49)^1+16(1)+P = 49+16+P = 65+P$

So, here if $P=-1$

Then, the given expression is divisible by 64 .

163. **(c)** Let $P(n) = 2^{3n}-7n-1 \Rightarrow P(1) = 0, P(2) = 49P$

(1) and P(2) are divisible by 49.

Let $P(k) = 2^{3k}-7k-1 = 491$

$P(k+1) = 2^{3k+3}-7k-8 = 8(49I+7k+1)-7k-8$

$= 49(8I)+49k = 49\lambda$

where, $\lambda = 8I+k$, which is an integer.

164. **(a)** Given, $\frac{\sec^2x}{\tan x}dx = -\frac{\sec^2y}{\tan y}dy$

$\Rightarrow \int \frac{\sec^2x}{\tan x}dx = -\int \frac{\sec^2y}{\tan y}dy$

Put $\tan x = u \Rightarrow \sec^2x dx = du$

and $\tan y = v \Rightarrow \sec^2y dy = dv$

$\therefore \int \frac{du}{u} = -\int \frac{dv}{v}$

$\Rightarrow \log u = -\log v + \log C \Rightarrow uv = C \Rightarrow \tan x \cdot \tan y = C$

165. **(c)** Given differential equation can be rewritten as

$$\frac{dy}{dx} = \frac{y}{x} + \frac{x\phi\left(\frac{y^2}{x^2}\right)}{y\phi'\left(\frac{y^2}{x^2}\right)}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Given equation becomes,

$$v + x\frac{dv}{dx} = \frac{vx}{x} + \frac{x\phi\left(\frac{v^2x^2}{x^2}\right)}{vx\phi'\left(\frac{v^2x^2}{x^2}\right)}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\phi(v^2)}{v\phi'(v^2)} \Rightarrow \frac{v\phi'(v^2)}{\phi(v^2)}dv = \frac{dx}{x}$$

On integrating both sides, we get

$$\frac{1}{2}\log\phi(v^2) = \log x + \log C_1 \Rightarrow \log\phi(v^2) = 2\log xC_1$$

$$\Rightarrow \phi(v^2) = (xC_1)^2 \Rightarrow \phi\left(\frac{y^2}{x^2}\right) = x^2C_1^2 \Rightarrow \phi\left(\frac{y^2}{x^2}\right) = x^2C$$

[put $C_1^2 = C$]

166. **(a)** Given differential equation is

$$(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0$$

$\Rightarrow (1+y^2)\frac{dx}{dy} = -x + e^{\tan^{-1}y}$

$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1}y}}{1+y^2}$

which is a linear differential equation.

Here, $P = \frac{1}{1+y^2}, Q = \frac{e^{\tan^{-1}y}}{1+y^2}$ and

$\therefore\ IF = e^{\int Pdy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$

Hence, required solution is $x \cdot IF = \int\ Q \cdot IFdy + C$

$\Rightarrow\ xe^{\tan^{-1}y} = \int\ \left(\frac{e^{\tan^{-1}y}}{1+y^2} \cdot e^{\tan^{-1}y}\right) dy + \frac{C}{2}$

$\Rightarrow xe^{\tan^{-1}y} = \frac{e^{2\tan^{-1}y}}{2} + \frac{C}{2}$

$\Rightarrow 2xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + C$

167. **(c)** We have, $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + \cdots + n \cdot n!$

$= \sum_{r=1}^{n} r \cdot (r!) = \sum_{r=1}^{n} [(r+1)r! - r!] = \sum_{r=1}^{n} [(r+1)! - r!]$

$= (2! - 1!) + (3! - 2!) + \cdots + [(n+1)! - n!]$

$= (n+1)! - 1! = (n+1)! - 1$

168. **(a)** Here, $T_n = 1 + 2 + 2^2 + 2^3 + \cdots + 2^n$

$= \frac{1(2^{n+1} - 1)}{2-1} = 2^{n+1} - 1$

$\therefore \Sigma T_n = \Sigma(2^{n+1} - 1) = \Sigma 2^{n+1} - \Sigma 1$

$= (2^2 + 2^3 + \cdots + 2^{n+1}) - n = 2(2 + 2^2 + \cdots + 2^n) - n$

$= \frac{4(2^n-1)}{2-1} - n = 2^{n+2} - n - 4$

169. **(a)** Given, $2b = a + c$ and $(c-b)^2 = (b-a)a$

$\therefore\ (b-a)^2 = (b-a)a$

$\Rightarrow\ b = 2a \Rightarrow c = 3a \Rightarrow a:b:c = 1:2:3$

170. **(b)**Number of triangles $= {}^{n+3}C_3 = 220$

$\Rightarrow \frac{(n+3)!}{3!\,n!}$

$\Rightarrow (n+1)(n+2)(n+3)\ = 1320 = 12 \times 10 \times 11$

$= (9+1)(9+2)(9+3)$

$\therefore\ n = 9$

171. **(a)** Required number of straight lines

$=\ {}^8C_2 -\ {}^3C_2 + 1 = 26$

172. **(d)** Total numbers formed by using given 5 digits $= \frac{5!}{2!}$

For number greater than 40000 , digit 2 cannot come at first place.

Hence, numbers formed in which 2 is at the first place

$= \frac{4!}{2!}$

$\therefore$ Total numbers formed greater than 40000 $= \frac{5!}{2!} - \frac{4!}{2!}$

$= 60 - 12 = 48$

173. **(a)** A polygon of n sides has number of diagonals

$= \frac{n(n-3)}{2} = 275$ [given]

$\Rightarrow\ n^2 - 3n - 550 = 0 \Rightarrow (n-25)(n+22) = 0$

$\therefore\ n = 25\ \ [\because n \neq -22]$

174. **(c)** Bisector of the angles between the lines

$x^2 - 2mxy - y^2 = 0$ is

$\frac{x^2-y^2}{xy}=\frac{[1-(-1)]}{-m}=\frac{2}{-m}\Rightarrow mx^2+2xy-my^2=0$

But it is also represented by $x^2-2nxy-y^2=0$.

$\therefore\ \frac{m}{1}=\frac{2}{-2n}\Rightarrow mn=-1$

175. **(c)** Let equation of line parallel to $3x-y=7$ be

$3x-y=\lambda$

Since, it passes through (1,2).

$\therefore\ 3-2=\lambda\Rightarrow\lambda=1$

Now, line is $3x-y=1$.

Since, the point of intersection of $x+y+5=0$ and $3x-y=1$ is $(-1,-4)$.

So, distance between (1,2) and ($-1,-4$)

$=\sqrt{\{1-(-1)\}^2+\{2-(-4)\}^2}$

$=\sqrt{(2)^2+(6)^2}=\sqrt{4+36}=\sqrt{40}$

176. **(d)** Let m be the required slope.

$\left|\frac{m-3}{1+3m}\right|=1\Rightarrow\frac{m-3}{1+3m}=\pm1$

$\Rightarrow m-3=1+3m$ and $m-3=-1-3m$

$\Rightarrow m=-2$ and $m=\frac{1}{2}$

177. **(d)** Equation of line is $\frac{x}{3}+\frac{y}{4}=1\Rightarrow 4x+3y-12=0$

Now, distance from origin $=\left|\frac{4\times0+3\times0-12}{\sqrt{3^2+4^2}}\right|=\frac{12}{5}$ units

178. **(c)** Let $f(x)=(x+a)^{53}+(x-a)^{53}$

Here $n=53$, which is odd.

$\therefore$ Total number of terms $=\frac{n+1}{2}=\frac{53+1}{2}=\frac{54}{2}=27$

179. **(c)** The given series is in GP. Hence, its sum

$=S\frac{1\{(1+x)^{20+1}-1\}}{(1+x)-1}\qquad[\because a=1,r=1+x]$

$=\frac{(1+x)^{21}-1}{x}$

Therefore, the required coefficient of x^{10} in the expansion of $\frac{(1+x)^{21}-1}{x}$

$=$ Coefficient of x^{11} in the expansion of $(1+x)^{21}-1={}^{21}C_{11}$

180. **(b)** The given expression is

$\left(x^2+\frac{1}{x^2}+2\right)^n=\left[\left(x+\frac{1}{x}\right)^2\right]^n=\left(x+\frac{1}{x}\right)^{2n}$

The binomial contains ($2n+1$) terms in its expansion.

$\therefore$ The middle terms is T_{n+1}.

$T_{n+1}={}^{2n}C_n(x)^{2n-n}\left(\frac{1}{x}\right)^n={}^{2n}C_n=\frac{(2n)!}{(2n-n)!\,n!}=\frac{(2n)!}{(n!)^2}$