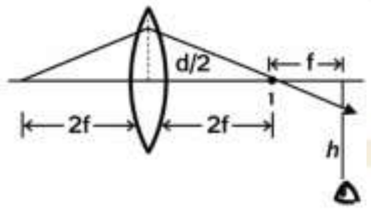
# PHYSICS

1. **(a)** During the phenomenon of resonance, the amplitude of oscillation becomes large.
2. **(c)** The variation of g with angular velocity ω is given by
$g' = g - R\omega^2$
If earth were to spin faster, that is angular velocity increases, then except at poles, the weight of bodies will decrease at all places.
3. **(b)** Given,
$A_m = 20$ cm
$A_c = 40$ cm
∴ Modulation index,
$\mu = \frac{A_m}{A_c} = \frac{20}{40} = 0.5$
4. **(a)**Given,
$T_2 = 27^\circ C = 300$ K
$T_1 = 677^\circ C = 950$ K
$\eta = \frac{W}{Q_1} = \frac{Q_1 - O_2}{Q_1}$
$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$
$\therefore \eta = 1 - \frac{300}{950} = \frac{13}{19}$
$\therefore \eta W = \eta Q_1 = 100 \times 10^3 \times \frac{13}{19}$ cal
Also, 4.2 J = 1cal
$\therefore W = 100 \times 10^3 \times \frac{13}{19} \times 4.2$
$\Rightarrow W = 2.87 \times 10^5 = 0.28 \times 10^6$ J
5. **(c)** During alpha-decay, atomic mass of parent nuclei is decreased by 4 units while atomic number is decreased by 2 units.
6. **(d)** From two similar triangles,
$\frac{h}{\frac{d}{2}} = \frac{f}{2}f$
$h = \frac{d}{4}$




7. **(b)** Given, collector resistance $= R_{out} = 4k\Omega$
$\beta = 100$
$R_{in} = 2k\Omega$
$V_{out} = 5$ V
∴ Voltage amplification,
$A_v = \beta \frac{R_{out}}{R_{in}} = \frac{V_{out}}{V_{in}}$
$V_{in} = \frac{V_{out} \times R_{in}}{\beta \times R_{out}}$
$= \frac{5 \times 2 \times 10^3}{100 \times 4 \times 10^3}$
$= \frac{1}{40} = 25$mV

**8.** **(a, b)** As work done = area under p -V curve, work done is maximum in case I. Change in internal energy is independent of the path from A to B. Therefore, in all cases change in internal energy is same.

**9.** **(a, d)** As Young's modulus Y for two wires is different, hence strain is different but stress is same as equal force F on each wire having same diameter and area of cross-section act on both wires.

**10.** **(a)** As, $K=\frac{1}{2}mv^2$

$\Rightarrow K=\frac{1}{2}mp^2$

$\Rightarrow p^2=2mK$

$\Rightarrow p=\sqrt{2mK}$

Suppose, $p_1$ is the momentum of the body and $K_1$ = kinetic energy of the body initially. As the kinetic energy of the body is increased by 300%, new kinetic energy of the body $=K_1+3K_1$

Hence, percentage change in momentum

$=\frac{p_2-p_1}{p_1}\times 100$

$=\frac{\sqrt{2mK_2}-\sqrt{2mK_1}}{\sqrt{2mK_1}}\times 100$

$=\frac{\sqrt{2m}\sqrt{K_2}-\sqrt{K_1}}{\sqrt{2m}\sqrt{K_1}}\times 100$

$=\frac{\sqrt{K_1+3K_1}-\sqrt{K_1}}{\sqrt{K_1}}\times 100$

$\frac{\sqrt{4K_1}-\sqrt{K_1}}{\sqrt{K_1}}\times 100=100\%$

**11.** **(c)** The electric field intensity at a point lying outside the sphere (non-conducting) is

$E=\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$

$\Rightarrow E\propto\frac{1}{r^2}\cdots\cdots(i)$

The electric field intensity at the sphere ( $r=R$ ),

$E=\frac{1}{4\Pi\varepsilon_0}\frac{q}{R^2}$

$\Rightarrow E=\propto\frac{1}{R^2}$

where, R being the radius of sphere.

The electric field intensity inside the sphere is

$E=\frac{1}{4\Pi\varepsilon_0}\cdot\frac{q}{R^3}$

$\Rightarrow E\propto r$

At the centre of sphere, $E=0$

**12.** **(a)** Resultant current is superposition of two currents,

i.e. I (instantaneous total current) $=6+I_0\sin\omega t$

DC ammeter will read average value

$=6+I_0\sin\omega t$

$=6\text{ A}(\because I_0\sin\omega t=0)$

AC ammeter will read average value

$=\sqrt{(6+I_0\sin^2\omega t)^2}$

$=\sqrt{36+12I_0\sin\omega t+I_0^2\sin^2\omega t(\because I_0\sin\omega t=0)}$

Since, $\sin^2\omega t = \frac{1}{2}$ and $I_{rms} = 8 = \frac{I_0}{\sqrt{2}}$

$\therefore$ AC reading $= \sqrt{36 + \frac{{I_0}^2}{2}} = \sqrt{36 + 64}$

$= 10$ A

**13.** **(c)**Let the velocity of the bullet before collision be v , then according to law of conservation of linear momentum,

$mv = (m + M)v' \ldots\ldots(i)$

As the mass m of the bullet gets embedded in the wall, hence velocity of bullet + target just after collision will be same,

$$\frac{1}{2}(M + m)v'^2 = (m + M)gn$$

$$\Rightarrow v' = \sqrt{2gh}$$

Putting the value of $v'$ in Eq.(i), we get

$$\Rightarrow v = \sqrt{2gh}\frac{(m + M)}{m} = \sqrt{2gh}\left(1 + \frac{M}{m}\right)$$

**14.** **(a)** The potential energy is given by

$$U = \int F \cdot dr = \int \frac{K}{r^2}dr = K\int r^{-2}dr$$

$$= K\left[\frac{r^{-1}}{-1}\right] = \frac{-K}{r}$$

The kinetic energy is given by

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{K}{r}\right)\left[\because \frac{mv^2}{4} = \frac{K}{r^2}\right]$$

Total energy, $E = U + K$

$$= -\frac{K}{r} + \frac{K}{2r} = \frac{-K}{2r}$$

**15.** **(b)** As, acceleration due to gravity at height h,

$$g' = \frac{GM}{(R + h)^2}$$

$$\Rightarrow \frac{g}{16} = \frac{GM}{R^2}\left(\frac{R^2}{(R + h)^2}\right) = g\frac{R^2}{(R + h)^2}\left(\because g' = \frac{g}{16}\right)$$

$$\Rightarrow \frac{1}{4} = \frac{R}{R + h} \Rightarrow h = 3R$$

**16.** **(d)** Lyman series lies in the ultraviolet region whereas Paschen, Brackett and Pfund series lie in the infrared region.

**17.** **(b)** Speed of EM wave in medium is given as

$$v = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

**18.** **(b)** Wavelength in Balmer series for hydrogen are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$= R_H\left(\frac{1}{4} - \frac{1}{n^2}\right), n = 3,4,5\ldots.$$

The second line in Balmer series corresponds to $n = 4$

Hence, $\frac{1}{\lambda_2} = R_H\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R_H}{16}$

or, $\lambda_2 = \frac{16}{3R_H}$

The wavelength of the first line ( $n = 2$ ) in Lyman series is

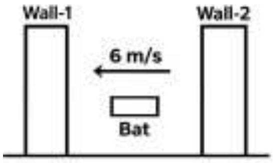
$$\frac{1}{\lambda_1} = R_H\left(1 - \frac{1}{2^2}\right) = R_H\left(1 - \frac{1}{4}\right) = \frac{3R_H}{4}$$

or, $\lambda_1 = \frac{4}{3R_H}$

$$\therefore \frac{\lambda_1}{\lambda_2} = \frac{4}{3R_H} \times \frac{3R_H}{16} = \frac{1}{4}$$

$\Rightarrow \lambda_1 = \frac{\lambda_2}{4} = \frac{486.4}{4} = 121.6$ nm

**19. (a)**




Frequency received by bat after reflection from wall - 1 ,

$f_1 = f\left[\frac{v+u}{v-u}\right] = 4.5 \times 10^4 \left[\frac{330+6}{330-6}\right]$

Given, $f = 4.5 \times 10^4 HZ$

$\Rightarrow f_1 = 4.67 \times 10^4 HZ$

Frequency received by bat after reflection from wall – 2

$f_2 = f\left[\frac{v-u}{v+u}\right]$

$= 4.5 \times 10^4 \left[\frac{330-6}{330+6}\right]$

$= 4.34 \times 10^4 HZ$

**20. (c)** Given that, moment of inertia of disc, $I_1 = 4 \text{ kg} - m^2$

$I_2 = 8 \text{ kg} - m^2$

Angular velocities of discs,

$\omega_1 = 16 rad/s$

$\omega_2 = 4 rad/s$

From angular momentuim conservation principle,

$I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega$

$\Rightarrow 4 \times 16 + 8 \times 4 = (4+8)\omega$

$\Rightarrow 64 + 32 = 12\omega$

$\Rightarrow \omega = \frac{96}{12} = 8 rad/s$

**21. (d)** From on a moving charged p[article in uniform magnetic field,

$F = Bqv\sin\theta$ ... .... (i)

Since, charge particle moves along the axis of circular current carrying loop, therefore, $\theta = 0°$ or $180°$.

When $\theta = 0°, F = Bqv\sin 0°$ [From Eq. (i)]

$F = 0$

When $\theta = 180°, F = Bqv\sin 180°$

$F = 0$

**22. (c)**As we know that, emf induced $e = \frac{d\phi_B}{dt}$

ore $= \frac{dB}{dt}$ (When area is constant $\phi_B \propto B$ )

Now for television wave, $e_1 = \frac{d}{dt}(B)$

$= \frac{d}{dt}\left(B_0\sin(2\pi f_1 t + \phi)\right)$

$e_1 = B_{02}\pi f_1 \cos(2\pi f_1 t + \phi)$

For radiowave, $e_2 = \frac{d}{dt}(B)$

$= \frac{d}{dt}(B_0\sin(2\pi f_2 \cos(2\pi f_2 + \phi)$

The ratio of emf induced

$= \frac{e_1}{e_2} = \frac{B_{02}\pi f_1 \cos(2\pi f_1 t + \phi)}{B_{02}\pi f_2 \cos(2\pi f_2 t + \phi)}$

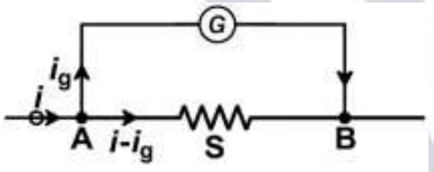
Since, $f_1 = 100MHZ, f_2 = 1MHZ$
and $\cos(2\pi f_1 t + \phi) = \cos(2\pi f_2 t + \phi) = \cos\phi$ (For complete cycle)
We get, $\frac{e_1}{e_2} = \frac{100}{1} = 100$

**23.** **(d)** According to the question,
$x = 5\sin\left(4t - \frac{\pi}{6}\right)$
When $x = 3$
$\Rightarrow \frac{3}{5} = \sin\left(4t - \frac{\pi}{6}\right)$
$\Rightarrow 4t - \frac{\pi}{6} = 37^\circ$
Hence, velocity $= \frac{dx}{dt} = 5 \times 4\cos\left(4t - \frac{\pi}{6}\right)$
$= 5 \times 4\cos\left(4t - \frac{\pi}{6}\right)$
$= 5 \times 4 \times \cos 37^\circ$
$= 16\ ms^{-1}\ \left(\because \cos 37^\circ = \frac{4}{5}\right)$

**24.** **(d)** Volume of wood $= \frac{\text{Mass}}{\text{Density}}$
$= \frac{120}{600} = 0.2\ m^3$
Weight of displaced water,
$vdg = 0.2 \times 1000 \times 10 = 2000\ N$
$Mg = vdg$
$\Rightarrow (120 + m)g = 2000$
$\Rightarrow m = 80\ kg$

**25.** **(b)** If we differenciate the equation of motion, $x = 1 - t - t^2$w.x.t. time, we get velocity.
$v = \frac{dx}{dt} = -1 - 2t$
Comparing with $v = u + at$, we have $u = -1\ ms^{-1}$ and $a = -2\ ms^{-2}$
At $t = 0, x = 1$ m. Then, u and a both are negative. Hence, x-coordinate of particle will go on decreasing.

**26.** **(b)**From figure,




Shunt current $= i - i_g$
galvanometer current $= i_g$
$i =$ total current
$\therefore i = 1.5$ A
By KVL,
$i_g - G = (i - i_g)S$
Putting $G = 4\Omega$ and $S = 2\Omega$ and $i = 1.5$ A
$\therefore i_g \times 4 = (1.5 - i_g) \times 2$
$4i_g = 3 - 2i_g$
$6i_g = 3 \Rightarrow i_g = 0.5$ A
$\therefore$ Current passing through shunt $= i - i_g$
$= 1.5 - 0.5 = 1$ A

**27.** **(d)** Given, $d = 0.2\ mm = 2 \times 10^{-4}$ m
$D = 1\ m, \lambda = 500\ nm = 5 \times 10^{-7}$ m
The distance between 6th maxima and 10th minima is given as
$\Delta x = (x_{10})_{dark} - (x_6)_{bright}$

$= \frac{(2\times10-1)D\lambda}{2\ d} - \frac{6D\lambda}{d}$

$= \frac{D\lambda}{d}\left[\frac{19}{2}-6\right]$

$= \frac{1\times5\times10^{-7}}{2\times10^{-4}}\left[\frac{7}{2}\right] = \frac{35}{4}\times10^{-3}\ \mathrm{m}$

$= 8.75\times10^{-3}\ \mathrm{m}$

$= 8.75\ \mathrm{nm}$

Hence, 8 mm is closest to 8 nm .

**28.** **(b)** As $V = I_r msR$

$I_{rns} = \frac{V_R}{R} = \frac{65}{100} = 0.65\ A$

$V_L = I_{rns}\times X_L$

$X_L = \frac{V_L}{I_{rns}} = \frac{204}{0.65} = 313.85\Omega$

$X_L = \omega L = 2nfL$

or $L = \frac{X_L}{2\pi f}$

$L = \frac{313.85}{2\times\pi\times50} = 1.0H$

$X_C = \frac{V_C}{I_{rns}} = \frac{415}{0.65} = 638.46\Omega$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

$C = \frac{1}{2\times\pi\times50\times638.46}$

$= 5\times10^{-6} = 5\mu\ F$

**29.** **(a)** Let number of turns of the regular hexagon $= n$

Now, $n\times6\ m = 24\ m$

$\therefore n = 4$

Magnetic moment, $M = nIA = 4IA$

$\therefore$ Area of hexagon

$= \frac{1}{2}\ m^2\sin^2 60^\circ + m\cdot 2\ m\sin60^\circ + \frac{1}{2}\ m^2\sin120^\circ$

$= \frac{\sqrt{3}}{4}\ m^2 + \sqrt{3}\ m^2 + \frac{\sqrt{3}\ m^2}{4}$

$= \frac{6\sqrt{3}\ m^2}{4} = \frac{3\sqrt{3}\ m^2}{2}$

Hence, magnetic moment $= 4I\left(3\sqrt{3}\frac{m^2}{2}\right)$

$= 6\sqrt{3}I\ m^2$

**30.** **(a)** From the given circuit diagram, output $\gamma$ is given as

$\gamma = (A+B)AB$

$= (A\cdot B)AB$[By De morgan' slaw, $A+B = A\cdot B$]

$= (AB)(AB)$ $[\because A\cdot A = A]$

$= AB$

$=$ output of AND gate.

**31.** **(c)** According to fluid dynamics, for maximum level of water in tank,

$A_1v_1 = A_2v_2$

Given, $A_1 = A$,

velocity, $v_1 = 4\ ms^{-1}$

area of hole $A_2 = \frac{A}{2}$

$\Rightarrow A \times 4 = \frac{A}{2} \times \sqrt{2gh}$

$\Rightarrow 16 = \frac{2gh}{4}$

$\Rightarrow h = \frac{16 \times 4}{2 \times 10} = 3.2 \text{ m}$

**32.** **(b)** The capacitance of a conductor is defined as the ratio of the charge given to the rise in the potential of the conductor.

$C = \frac{q}{V} = \frac{q^2}{W}\left(V = \frac{W}{q}\right)$

$C = \frac{\text{ampere }^2 - \text{ second }^2}{\text{kg}^2 - \text{ metre }^2 \text{ second }^{-2}}$

Hence, dimensionally $[M^{-1} L^{-2} T^4 A^2]$

From Ohm's law, $V = IR$,

Hence, $R = \frac{V}{I} = \frac{\text{Volt}}{\text{Ampere}} = \frac{W}{q} \times \frac{t}{q}$

$R = \frac{F \times s \times t}{q^2} = kg\frac{m}{s^2} \times \frac{m \times s}{A^2 \times s^2}$

$= kgmm^2\ s^{-3}\ A^{-2}$

Dimesionally, $R = [ML^2\ T^{-3}\ A^{-2}]$

Hence, $CR = [M^{-1}L^{-2}T^4A^2][M^2T^{-3}A^{-2}]$

$= [M^0\ L^0\ T]$

**33.** **(a)** Net work done by a gas,

$\Delta W_{AB} = p \cdot \Delta V = 10 \times (2-1) = 10 \text{ J}$

$\Delta W_{BC} = 0(\text{ as} V = \text{ constant })$

From first law of thermodynamics,

we have, $\Delta Q = \Delta W + \Delta U$

$U = 0$

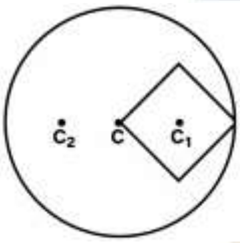
(process ABCA is cyclic)

Therefore, $\Delta Q = \Delta W_{AB} + \Delta W_{BC} + \Delta W_{CA}$

$\therefore \Delta W_{CA} = (\Delta Q - W_{AB} - W_{BC})$

$= (5 - 10 - 0)J = -5J$

**34.** **(a)**




Radius of the circular disc $= R$

Suppose, centre of mass of remaining portion is at a distance $C_2$ from the C .

Now, $A_1(CC_1) = A_2(CC_2)$

For square, $(\text{side})^2 + (\text{side})^2 = (\text{radius})^2$

$2(\text{side})^2 = R^2$

$(\text{side})^2 = \frac{R^2}{2}$

$\text{side} = \frac{R}{\sqrt{2}}$

Now, $A_1(CC_1) = A_2(CC_2)$

$\therefore CC_2 = \frac{A_1}{A_2}(CC_1)$

$A_1 = \text{ area of the square } = \left(\frac{R}{\sqrt{2}}\right)^2$

$A_2 =$ area of the remaining part of circle

$$= nR^2 - \left(\frac{R}{\sqrt{2}}\right)^2$$

$$CC_2 = \frac{\left(\frac{R}{\sqrt{2}}\right)^2}{nR^2 - \left(\frac{R^2}{\sqrt{2}}\right)^2} \cdot \left(\frac{R}{2}\right)$$

$$= \frac{R^2}{2nR^2 - R^2} \cdot \frac{R}{2}$$

$$= \frac{R \cdot R \cdot R}{R^2(2n-1)2} = \frac{R}{4\pi - 2}$$

**35. (b)** At neutral point

$B = B_H$

$$\Rightarrow \frac{\mu_0}{4\pi} \times \frac{M}{(r^2+1^2)^{\frac{3}{2}}} = B_H$$

$$\Rightarrow 10^{-7} \times \frac{m \times 21}{(r^2+1^2)^{\frac{3}{2}}} = 1.2 \times 10^{-5}$$

$$\Rightarrow 10^{-7} \times \frac{m \times 2 \times 3 \times 10^{-2}}{[(4 \times 10^{-2})^2 + (3 \times 10^{-2})^2]^{\frac{3}{2}}} = 1.2 \times 10^{-5}$$

$$\Rightarrow \frac{6\ m \times 10^{-4}}{[(5 \times 10^{-2})^2]^{\frac{3}{2}}} = 1.2$$

$$\Rightarrow \frac{6\ m \times 10^{-4}}{125 \times 10^{-6}} = 1.2$$

$$\Rightarrow\ m = \frac{1.2 \times 125 \times 10^{-2}}{6}$$

$$= 0.25\ A - m^2$$

**36. (a)** As per formula, the Young's modulus, $\gamma = \frac{\text{stress}}{\text{strain}}$

$$= \frac{\frac{F}{A}}{\frac{\Delta l}{1}}$$

$$\Rightarrow\ F = \frac{\gamma A \Delta l}{1}$$

In first case, $\gamma = \frac{\frac{F}{A_1}}{\frac{l_1}{L}}$

$\Rightarrow F = \frac{\gamma l_1 A_1}{L}$......(i)

Similarly, for the second case,

$F = \frac{\gamma l_2 A_2}{\Delta l}$......(ii)

From Eqs. (i) and (ii), we get

$$\frac{\gamma l_1 A_1}{L} = \frac{\gamma l_2 A_2}{4L}$$

As $\gamma$ is same, as the same material is taken, $l_1 A_1 = \frac{l_2 A_2}{4}$

$$l_1 \pi r^2 = \frac{l_2}{4} \pi (4r)^2$$

$$l_1 = \frac{l_2}{4} \cdot \frac{16r^2}{r^2} = 4l_2 \Rightarrow l_2 = \frac{l_1}{4}$$

Hence, $l_1 = 1, l_2 = \frac{l_1}{4}$

**37. (a)** Given, power of laser beam,

$P = 30\ mW = 30 \times 10^{-3}\ W$

Area of cross section, $A = 15\ mm^2 = 15 \times 10^{-6}\ m^2$
Permittivity of free space, $\varepsilon_0 = 9 \times 10^{-12}$SI unit
Speed of light, $c = 3 \times 10^8\ m/s$
Intensity of electromagnetic wave is given by

$$I = \frac{1}{2} nc\varepsilon_0 E^2$$

where, n is refravctive index for air, $n = 1$

$$I = \frac{1}{2} c \cdot \varepsilon_0 E^2 \ldots\ldots (i)$$

$$I = \frac{P}{A} \ldots.. (ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2} C\varepsilon_0 E^2 = \frac{P}{A}$$

$$E^2 = \frac{2P}{Ac\varepsilon_0}$$

$$E = \sqrt{\frac{2 \times 30 \times 10^{-3}}{15 \times 10^{-6} \times 3 \times 10^8 \times 9 \times 10^{-12}}}$$

$$= 1.22 \times 10^3\ V/m = 1.22 kV/m$$

**38.** **(c)**The average molecular kinetic energy is expressed as

$$K = \frac{1}{2} mv^2 = \frac{3}{2} kT$$

Given, $T = 20^\circ C = 20 + 273 = 293\ K$
$k = 1.38 \times 10^{-23}\ J/K =$ Boltzmann's constant
Substituting all the values, we get

$$K = \frac{3}{2} \times 1.38 \times 10^{-23} \times 293$$

$$= 6.07 \times 10^{-21}\ J$$

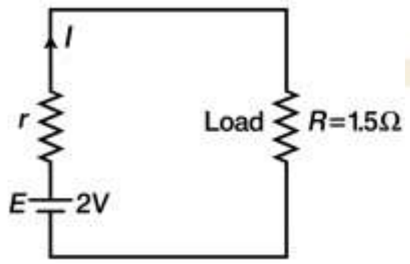
**39.** **(d)** As for the projectile motion, horizontal range,

$$R = \frac{u^2 \sin 2\theta}{g}$$

$\because$ Maximum range $= u^2/g$
$\because\ A = nR^2$
$\therefore\ A \propto R^2$
i.e., $A \propto u^2$

$$\therefore \frac{A_1}{A_2} = \frac{u_1^4}{u_2^4} = \left(\frac{2}{4}\right)^4 = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

**40.** **(a)** Given, emf of cell, $E = 2\ V$




Load resistance, $R_L = 1.5\Omega$
Current flowing the circuit is

$$I = \frac{E}{R + r}$$

Power transferered to the load, R ,

$$P = I^2 R = \left(\frac{E}{R + r}\right)^2 \cdot R = \frac{E^2 R}{(R + r)^2}$$

Power P will br maximum, if

$$\frac{dP}{dR} = 0$$
$$\Rightarrow \frac{d}{dR}\frac{E^2R}{(R+r)^2} = 0$$
$$\Rightarrow \frac{E^2(R+r)^2 - E^2R \cdot 2(R+r)}{(R+r)^3} = 0$$
$$\Rightarrow E^2(R+r)[R+r-2R] = 0$$
$$\Rightarrow E^2(R+r)(r-R) = 0$$
$$\Rightarrow r - R = 0$$
$$\Rightarrow r = R$$
$\therefore$ Maximum power, $P = \frac{E^2R}{(R+r)^2} = \frac{E^2R}{(R+R)^2} = \frac{E^2}{4R}$
$$\Rightarrow P = \frac{2^2}{4} \times 1.5 = \frac{1}{3} = 0.33W$$

**41.** **(d)** A thick glass mirror produces a number of images. There is an apparent shift of actual silvered surface towards the unsilvered face.

Effective distance of the reflecting surface from unsilvered face $= \frac{d}{\mu}$
$$= \frac{3}{\frac{3}{2}} \text{ cm} = 2 \text{ cm}$$
Distance of point object from effective reflecting surface
$= 9 \text{ cm} + 2 \text{ cm} = 11 \text{ cm}$
Distance of image from point objecrt
$= 11 \text{ cm} + 11 \text{ cm} = 2 \text{ cm}$
Distance of image from unsilvered face
$= (22-9)\text{cm} = 13 \text{ cm}$

**42.** **(d)** Given $K_{max} = hv - W_0$
$$= \frac{(6.6 \times 10^{-34}) \times (3 \times 10^8)}{99 \times 10^{-9}} - 1.0 \times 1.6 \times 10^{-19}$$
$$= 2 \times 10^{-18} - 1.6 \times 10^{-19}$$
$$= 1.84 \times 10^{-18} \text{ J}$$
Now, $\lambda = \frac{h}{\sqrt{2mK_{max}}}$
$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times (9.1 \times 10^{-31}) \times 1.84 \times 10^{-18}}}$$
$$= 0.36 \times 10^{-9} \text{ m} = 0.36 \text{ nm}$$

**43.** **(d)** If fringe widths of air and water are $\beta_{air}$ and $\beta_{water}$ repectively, then fringe width, $\beta_{water} = \frac{\beta_{air}}{\mu}$

Given, $\mu = \frac{4}{3}, \beta_{air} = 0.4$

Hence, $\beta_{water} = \frac{0.4}{\frac{4}{3}} = 0.3 \text{ nm}$

**44.** **(d)** Since, container are identical, so initial volume and pressure will be same for identical gases.For container $C_1$,
$p_1 V_1^{\,\gamma} = p_2 V_2^{\,\gamma}$ (adiabatic)
$$p_2 = \left(\frac{V_1}{V_2}\right)^\gamma p_1 = \left(\frac{V_1}{\frac{V_1}{2}}\right)^\gamma p_1 = 2^\gamma p_1 \ \ldots.(i)$$
For container $C_2$,
$p_1 V_1 = p_2 V_2$ (isothermal)
$$p_2' = \left(\frac{V_1}{V_2}\right) p_1 = \left(\frac{V_1}{\frac{V_1}{2}}\right) p_1 = 2p_1 \quad \ldots..(ii)$$
From Eqs. (i) and (ii), we get
$$\frac{p_2}{p_2'} = \frac{2^\gamma p_1}{2p_1} = 2^{\gamma-1} : 1$$

**45. (b)** Let $W_0$ be the work function of the metal, then kinetic energy of photoelectrons emitted when light of wavelength $\lambda_1$ is incident will be

$K_1 = \frac{hc}{\lambda_1} - W_0$

Similarly, $K_2 = \frac{hc}{\lambda_2} - W_0$

Now, $K_1 = K_2 = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = hc\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$

$= hc\left[\frac{1}{3\lambda_2} - \frac{1}{\lambda_2}\right]$

$K_1 - K_2 = \frac{-2hc}{3\lambda_2} = \frac{-2}{3}(K_2 + W_0)$

$\Rightarrow K_1 = K_2 - \frac{2}{3}K_2 - \frac{2}{3}W_0$

$= \frac{K_2}{3} - \frac{2}{3}W_0 \Rightarrow K_1 < \frac{K_2}{3}$

**46. (d)** Given, speed of car, $u = 54\ \text{km/h} = 54 \times \frac{5}{18}$

$= 15\ \text{m/s}$

Since, car stops in 3 s on applying brakes, so acceleration can be calculated from first equation of motion,

$v = u + at \Rightarrow 0 = 15 + a(3)$

$\Rightarrow a = -\frac{15}{3} = -5\ \text{ms}^{-2}$

Using second equation of motion,

$s = ut + \frac{1}{2}a^2$

$= 15 \times 3 + \frac{1}{2} \times (-5) \times (3)^2$

$= 45 - 22.5 = 22.5\ \text{m}$

**47. (c)** According to Doppler's effect, the apparent frequency of sound when both source and observer are moving towards each other

$\Rightarrow f = \frac{(v + v_0)f_0}{(v - v_0)}$

$\Rightarrow f = \left(\frac{v}{v - v_s}\right)f_0 + \left(\frac{v_0}{v - v_s}\right)f_0$

$\Rightarrow f = \left(\frac{f_0}{v - v_s}\right)v_0 + \left(\frac{f_0}{v - v_s}\right)v$

Equation of straight line, $y = mx + c$ which is equation of straight line making an intercept c on Y -axis,

Slope of the graph $= \frac{f_0}{v - v_s}$ and intercept $= \frac{vf_0}{v - v_s}$

This condition is represented in graph A plotted between f and $v_0$.

Hence, option (c) is correct.

**48. (a)** The energy emitted by a body, in the form of radiation on account of its temperature, is called thermal radiation. These radiations are heat radiations and travel along straight lines with speed of light.

**49. (b)** By the law of conservation of energy,

$(U + K)_{\text{surface}} = (U + K)_\infty$

$\Rightarrow \frac{-GM}{R} + \frac{1}{2}m(3v_c)^2 = 0 + \frac{1}{2}mv^2$

$\Rightarrow -\frac{GM}{R} + \frac{9v_c^2}{2} = \frac{1}{2}v^2$

Since, $v_c^2 = \frac{2GM}{R}$

$\therefore \frac{-v_c^2}{2} + \frac{9v_c^2}{2} = \frac{1}{2}v^2$

$\Rightarrow v^2 = 8v_c^2$

$v = 2\sqrt{2}v_c = 2\sqrt{2} \times 11.2 = 31.7\ \text{km/s}$

**50.** **(c)** Equivalent thermal conductivity of the compound slab,

$$K_{eq} = \frac{1_1 + 1_2}{\frac{1_1}{K_1} + \frac{1_2}{K_2}} = \frac{1+1}{\frac{1}{K} + \frac{1}{2\,K}} = \frac{21}{\frac{31}{2\,K}} = \frac{4\,K}{3}$$

**51.** **(c)** Magnetic Lorenz force acting on each electron,

$F = evB$..........(i)

As, current, $i = neAv$

$$ev = \frac{i}{nA}$$

Therefore, substituting value of ev into Eq.(i),

$$F = \frac{i}{nA}B$$

Given, $i = 5\text{ A}, n = 20\text{ turns}, B = 0.1\text{ T}$

$$F = evB = \frac{iB}{nA} = \frac{5 \times 0.1}{10^{29} \times 10^{-5}} = 5 \times 10^{-25}\text{ N}$$

**52.** **(c)** If a man slides down with some acceleration, then its apparent weight decreases. For critical condition, rope can bear only $\frac{2}{3}$ of his weight. If a is the minimum acceleration, then tension in the rope

$= m(g - a) =$ breaking strength

$$\Rightarrow m(g - a) = \frac{2}{3}mg$$

$$\Rightarrow a = g - \frac{2\,g}{3} = \frac{g}{3}$$

**53.** **(c)** In general, nth mode of a string fixed at ends has frequency,

$$v = \frac{nv}{2l}$$

where, $n = 1,2,3,\ldots\ldots$

where, v is the velocity of wave and I is the length of string.

In fourth normal mode, $n = 4$

$$v = \frac{4v}{2l}$$

Given, $v = 500\text{HZ}, 1 = 2\text{ m}$

Hence, $500 = \frac{4v}{2\times2}$

$$\text{or, } v = \frac{500 \times 4}{4} = 500\text{ m/s}$$

**54.** **(b)** As we know, power factor $= \frac{R}{Z}$

$$= \frac{R}{\sqrt{R^2 + \omega L^2}}$$

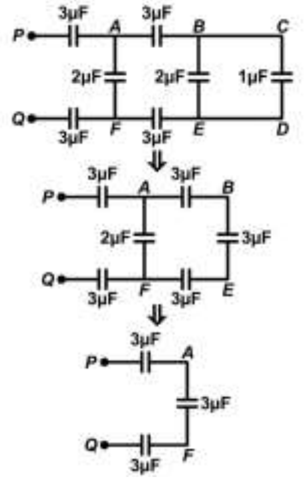
Given $\omega = 100, L = 400\text{mH}$

$$= \frac{30}{\sqrt{(30)^2 + (100 \times 400 \times 10^{-3})^2}}$$

$$= \frac{30}{\sqrt{900 + 160000 \times 10^{-3\times2}}}$$

$$= \frac{30}{59} = 0.6$$

**55.** **(a)**

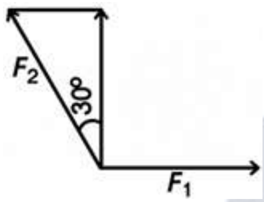



The equivalent capacitance between P and Q is given by

$$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1\mu F$$

$C_{eq} = 1\mu\ F$

**56.** **(a)**




From the diagram we get,

$$\tan 30^\circ = \frac{1}{\sqrt{3}} = \frac{x}{10}$$

$$\Rightarrow x = \frac{10}{\sqrt{3}}$$

**57.** **(a)** The most commonly used material in LEDs is gallium arsenide.

**58.** **(c)** Charge on electron is negative and its magnitude is $1.6 \times 10^{-19} C$

**59.** **(b)** Given, $q_1 = -20\mu C = -2 \times 10^{-5} C$

$q_2 = 60\mu C = 6 \times 10^{-5} C$

Force of attraction between the both charges, kept at a certain distance r

$$F_1 = \frac{Kq_1q_2}{r^2}$$

$$= \frac{K \times 2 \times 10^{-5} \times 6 \times 10^{-5}}{r^2}$$

$$= \frac{12 \times 10^{-10} K}{r^2}$$

When both charges are touched, then new charge on both spheres,

$$q_1{}' = q_2{}' = \frac{q_1 + q_2}{2} = \frac{-20 + 60}{2} = 20\mu C$$

$$= 2 \times 10^{-5} C$$

∴ Force between the two spheres,

$$F_2 = \frac{Kq_1' \cdot q_2'}{r^2}$$

$$= \frac{K.2 \times 10^{-5} \times 2 \times 10^{-5}}{r^2}$$

$$= \frac{4 \times 10^{-10}}{r^2} K$$

$\therefore \frac{F_1}{F_2} = \frac{\frac{12 \times 10^{-10} K}{r^2}}{\frac{4 \times 10^{-10}}{r^2}} = \frac{12}{4} = \frac{3}{1}$

$\therefore F_1 : F_2 = 3 : 1$

**60.** **(b)** An AC voltage across a resistance can be measured by using a moving coil galvanometer. It can be used as a voltmeter by inserting a resistor in series with it. It responds only to direct current. Hence, it requires a rectifier, so that coil deflects only in one direction.

## CHEMISTRY

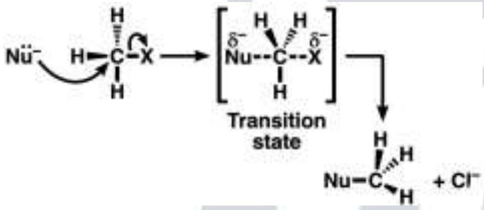
**61.** **(c)** Use of aspartame is limited to cold foods because it is unstable at cooking temperature.
Alltame is more stable than aspartame but the control of swentoness of food is difficult while using it.

**62.** **(a)** Lucess reapent: Anhyd. $\frac{ZnCl_2}{HCl}$ it is used to differentiate $1^\circ, 2^\circ$ and $3^\circ$ alcuhols.

**63.** **(b)** Acidic strength of oxb-acids containing the same halogen are in the order :
$HCl < HClO_2 < HClO_3 < HClO_4$
This is because $ClO_4{}^-$is the most stable due to dispersal of negative charge on four D-atoms (O being more electromegnetic than CI ).

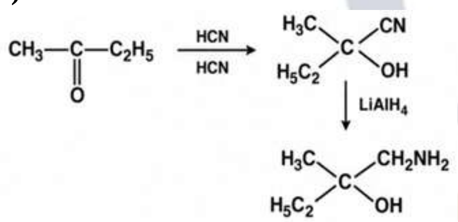
**64.** **(a)** Coagulation-The process of aggregation of colloidal particles into an insoluble precipitate by the addition of suitable electrolyte.
Diffusion-The process of movement of molecules from a region of higher concentration to a region of lower concentration.
Peptisation-The process of converting a precipitate into colloidal sol by shaking it with dispersion medium in the presence of small amount of electrolyte.
Electrolysis-The process by which electric current is passed through a substance to effect a chemical change.

**65.** **(d)**




Inversion of configuration takes place. There is no carbocation formation.

**66.** **(c)**




**67.** **(c)** Doping Si with In or B leads to the formation of p-type semiconductors.
In p-type semiconductors, trivalent impurities are used for doping, whereas, in n-type semiconductors, pentavalent impurities are used for doping.

**68.** **(d)** According to Le-Chatelier's principle when concentration of reactant increases, the equilibrium shifts in favour of forward reaction.

**69.** **(d)** Thermodynamic parameters which depend only on the initial and final states of system are called state functions, such as enthalpy ( $H = q + W$ ), Gibb's free energy ( $G = H - TS$).Whereas, thermodynamic parameters which depend on the path by which the process is performed and not on initial and final states, are called path functions, such as work done (W ), heat (q) etc.

**70.** **(a)** When the non-ideal binary solution shows the negative deviation, it is known as a maximum boiling azeotrope e.g. acetone-chloroform. Rest all three options show positive deviation from Raoult's law.

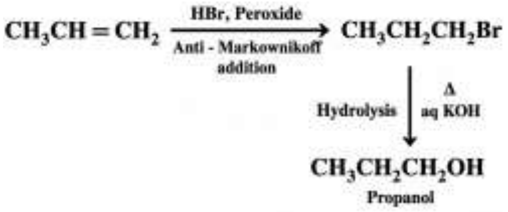
**71.** **(a)** $MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O] \times 2$
$C_2O_4^{2-} \rightarrow CO_2 + 2e^-] \times 5$

$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

5 moles of $C_2O_4^{2-}$ need 2 moles of $KMnO_4$.

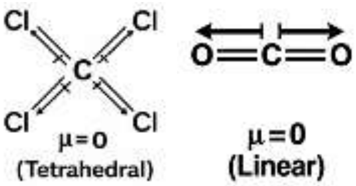
1 mole of $C_2O_4^{2-}$ would need $= \frac{2}{5}$ mole of $KMnO_4$.

**72.** **(c)**

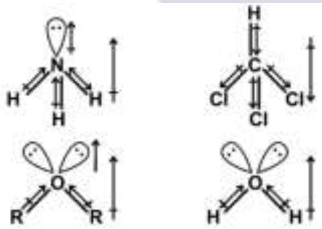



**73.** **(a)** Ethanol undergoes intermolecular H -bonding due to the presence of a hydrogen attached to a electronegative oxygen atom. Hence, ethanol exists as associated molecules and a large amount of energy is required to break these H -bonds. Therefore, the boiling point of ethanol is higher than that of methoxy methane which does not form H -bonds.

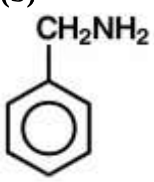
**74.** **(c)**



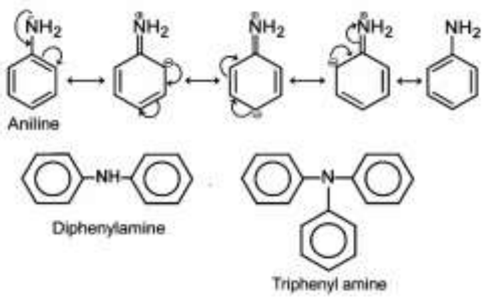

Dipole moments cancel out reach other in $CCl_4$ and $CO_2$ resulting in net dipole moment as zero because these are symmetrical structures.



**75.** **(b)**




i.e. benzylamine is the most basic among other given options because lone pairs of N are available for donation. whereas lone pair of N in aniline, diphenylamine and triphenyl amine are delocalised over the benzene ring, hence are not available for donation, making them weaker bases.




**76.** **(b)** All natural processes are generally irreversible and net work done in an irreversible process is some what less than in reversible process.

**77.** **(d)** Platelets are pieces of very large cells in bone marrow. They help in forming blood clots to stop bleeding and wounds heal.

**78.** **(a)** Hydrolysis of maltose yields two moles of $\alpha -$ D-glucose in which $C - 1$ of one glucose linked to $C - 4$ of another glucose.

**79. (c)** The solubility of any gas in a particular liquid is the volume of gas that can be dissolved in unit volume of liquid. It only depend upon pressure, temperature and nature of gas and liquid.

**80. (b)** $H_2O_2$ has open book structure with $O-O$ spins. Its dihedral angle is $111^\circ$.

**81. (a)** Radioactive nuclide follow first order kinetics

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{6.93} = 0.1 \text{ s}$$

**82. (b)** ZnS and AgCl show Frenkel defect. NaCl show Schottky defect but AgBr show both defect.

**83. (d)** The essential amino acids are histidine, isoleucine, leucine, lysine, methionine, phenylalanine, threonine, tryptophan and valine.

**84. (d)** According to MOT, bond order is given by the formula, $BO = \frac{N_b - N_a}{2}$ where $N_a, N_b$ are the number of electrons in antibonding and bonding molecular orbital respectively.

Bond order for $O_2 = \frac{10-6}{2} = 2$

$$O_2^- = \frac{10-7}{2} = 1.5$$

$$O_2^+ = \frac{10-5}{2} = 2.5$$

$$O_2^{2-} = \frac{10-8}{2} = 1$$

**85. (a)** Total number of nodes is given by $(n-1)$

$2s = (2-1) = 1$

$3s = (3-1) = 2$

$4s = (4-1) = 3$

$5s = (5-1) = 4$

**86. (c)** As KOH is a base.

$pOH = -\log[OH^-]$

$pOH + -\log[10^{-3}] = 3$

$pH + pOH = 14$

$p_H = 14 - 3 = 11$

**87. (d)** Barbiturates are the depressants drug used to treat depression.

**88. (a)** Wurtz reaction is kind of reaction in which there is a formation of simple higher alkane from alkyl halides. Minimum two carbon chains should be present to form alkane in this reaction. So, methane cannot be formed.

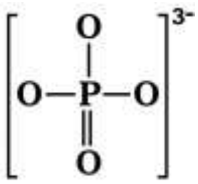
**89. (a)** The $CrO_4{}^{2-}$ (yellow) exist in acidic medium with $pH = 6$ and changes to orange in basic medium as $Cr_2O_7{}^{2-}$ with $pH = 8$

**90. (b)** Bromine acts as mild oxidising agent and oxidation of glucose with bromine water form gluconic acid. This show presence of aldehyde group in glucose.

**91. (c)** Those azeotropes which boils at a temperature lower than boiling point of each component forms the minimum boiling azeotrope. e.g. Heptane + octane, ethanol + water, acetone + carbon dioxide

**92. (c)** The incorrect statement about activation energy is option (c) because the SI unit of activation energy is J/mol or kJ/mol or kcal /mol.

**93. (d)** The structure of $PO_4{}^{3-}$ is



Bond order $= \frac{\text{Number of bonds}}{\text{Number of resonating structure}}$

$= \frac{5}{4} = 1.25$

**94. (a)** Those species which have same number of electrons are called isoelectronic species.
$O_2^{\ -}, F^-, Mg^{2+}, Al^{3+}$ have 10 electrons.
Hence, 4 isoelectronic species are present.

**95. (d)** The hcp (hexagonal close packing) has a coordination number of 12 and contains 6 atoms per unit cell.

**96. (b)** $3C_2H_2(g) \rightleftharpoons C_6H_6(g)$

$$K_C = \frac{[C_6H_6]}{[C_2H_2]^3} = \frac{0.5}{[C_2H_2]^3}$$

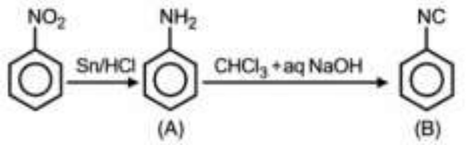
$$4 = \frac{0.5}{[C_2H_2]^3}$$

$$[C_2H_2] = \sqrt[3]{\frac{0.5}{4}} = \frac{1}{2}$$

$[C_2H_2] = 0.5 \text{ mol L}^{-1}$
Therefore, the concentration of ethylene is $0.5 \text{ mol/L}$.

**97. (b)** Aspartame is used in soft drinks. It is an artificial sweetening agent which is unstable at higher temperatures. It is a compound formed from aspartic acid and phenyl alanine. It is 100 times as sweet as cane sugar.

**98. (b)** Complete reaction of the given reaction is as follows.




**99. (b)** Sulphur dioxide gas readily decolourises the purple colour of acidifide $KMnO_4$ solution. In this case, $KMnO_4$ acts as an oxidising agent and sulphur dioxide gas acts as a reducing agent.

**100. (b)** Let, the rate of diffusion of gas X, $r_1 = b$. Therefore, rate of diffusion of methane, $r_2 = 2b$ According to Graham's law of diffusion,

$$\left(\frac{r_1}{r_2}\right) = \left(\sqrt{\frac{M_2}{M_1}}\right)$$

$M_1 =$ molecular mass of gas X
$M_2 =$ molecular mass of methane $= 16$ g

$$\therefore \frac{b}{2\,b} = \sqrt{\frac{16}{M_2}}$$

$$\Rightarrow \left(\frac{1}{4}\right) = \frac{16}{M_2}$$

$\Rightarrow M_2 = 64$ g
Therefore, molecular mass of X is 64 g .

**101. (c)** Fire clay bricks is used to prepare the inner lining of a blast furnance. It is highly refractory in nature and does not melt, even at huge temperature. Its main components are silica and alumina.

**102. (c)** For the given cell representation, the cell reaction will be,
$Fe(s) + 2H^+(aq) \rightarrow Fe^{2+}(aq) + H_2(g)$
The standard emf of the cell will be,

$$E^\circ_{\text{cell 1}} = E^\circ \frac{H^+}{H} - E^\circ_F \frac{e^{2+e}}{F}$$

$$= 0 - (-0.44)^2 = 0.44 \text{ V}$$

The Nernst equation for the cell reaction at 25°C,

$$E_{\text{cell}} = E^\circ_{\text{cell}} - \frac{0.0591}{n} \log \frac{[Fe^{2+}]}{[H^+]}$$

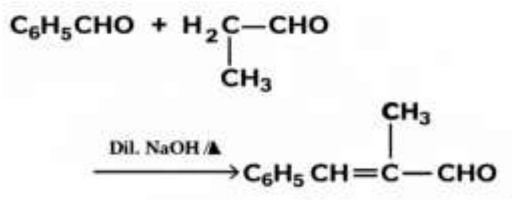
$$= 0.44 - \frac{0.0591}{2} \log \frac{[0.001]}{[0.01]^2}$$

$$= 0.44 - 0.02955(\log 10)$$

$= 0.44 - 0.02955(1)$
$= 0.41045\ \mathrm{V} = 0.41\ \mathrm{V}$

**103.** **(b)** Beri-beri disease is caused by the deficiency of vitamin $B_1$.

**104.** **(d)**



2-methyl-3-phenylprop-2-en-1-al

**105.** **(d)** Among the given options, entropy decreases during the crystallisation of sucrose from solution. It is because entropy is a measure of randomness and during the process of crystallisation liquid state changes into solid state. Hence, entropy decreases.

**106.** **(c)** Bithional is an antiseptic which is widely used in medicinal soaps, so that soaps can impart antiseptic properties.

**107.** **(a)** Only aliphatic primary amines can react with nitrous acid to produce alcohol, water and nitrogen gas.

$$\underset{1-\text{amine}}{R - NH_2} + HONO \xrightarrow{273-278\ K} \underset{\text{Al chohal}}{R - Omine} + N_2 + H_2O$$

So, dimethylamine which is a secondary amine cannot liberate $N_2$ with HONO .

**108.** **(d)** Octahedral complex of general formula $[M(AA)_2a_2]^{n\pm}$ shows optical isomerism. Cis - $[Co(NH_3)_4Cl_2]^{+}C$ does not show optical isomerism due to symmetry, while other three complexes show optical isomerism.

**109.** **(b)** We know, pV = nRT

or $pV = \frac{W}{M}RT\left[Here,\ n = \frac{\text{Weight of gas taken(W)}}{\text{Molar mass of gas(M)}}\right]$

or $p = \frac{W}{VM}RT$

or $p = \frac{dRT}{M}\left[\text{Here, density (d)} = \frac{\text{Mass}}{\text{Volume}}\right]$

$$\therefore d = \frac{pM}{RT} = \frac{3 \times 28}{0.082 \times 503} = \frac{84}{41.246} = 2.03\ g/mL$$

**110.** **(a)** Higher the value of van't Hoff factor (i), higher will be depression in freezing point and lower will be the freezing point of solution.

So, 'i' value, $0.2MK_2SO_4 = 3$

$0.2MKCl = 2$

$0.2MNaNO_3 = 2$

$0.2MMgSO_4 = 2$

Hence, $0.2MK_2SO_4$ will have lowest freezing point.

**111.** **(d)** Molar conductance $= \lambda_m$

$$\lambda_m = \frac{K \times 10^{-3}}{M}\ mol^{-1}\ m^3$$

$$= \frac{(12.04 \times 10^{-2} Sm^{-1}) \times 10^{-3}(\ mol^{-1}\ m^3)}{0.025}$$

$$= 481.6 \times 10^{-5} Sm^2\ mol^{-1}$$

**112.** **(a)** $2\ N_2O_5 \rightarrow 4NO_2 + O_2$

$$\begin{bmatrix} \text{I nitial} & p_0 & 0 & 0 \\ \text{F inal} & p_0 - 2x & 4x & x \end{bmatrix}$$

$$k = \frac{2.303}{t}\log\frac{p_0}{p_t}$$

$$3.38 \times 10^{-5} = \frac{2.303}{20 \times 60}\log\frac{100}{p_t}$$

$0.0176=\log\frac{100}{p_t}$

$p_t=96$ atm

**113.** **(a)** Heat capacity at constant.

Pressure $=C_p$

Heat capacity at constant volume $=C_v$

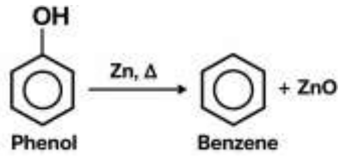
As we all know,

$C_P-C_V=R$

**114.** **(d)** Among the given options, sulphur dioxide and nitrogen oxides are responsible for acid rain. It is because sulphur dioxide and nitrogen oxide reacts with water to give sulphuric acid and nitric acid respectively.

**115.** **(b)** N belongs to second period as it has no d -orbital. So, five pairs of bonding electrons around nitrogen is not possible. As a result $NCl_5$ does not exist.

**116.** **(c)**Phenol is converted to benzene on heating with zine dust.




Hence, benzene is the major product.

**117.** **(a)** AsH $e^+$ and H contain same number of electron. So, they show same type of spectra.

**118.** **(b)** Paschen series of atomic spectrum of hydrogen gas lies in infrared region.

**119.** **(d)** [ $Ni(CO_4)$ ]

$x+4(0)=0$

$x=0$

Therefore, oxidation state of Ni is zero.

**120.** **(c)** Low temperature and high pressure favours physical adsorption because van der Waals' forces of attraction will increase.

## MATHEMATICS

**121.** **(a)** We have,

$3^{\log_4 5}-5^{\log_4 3}=3^{\log_4 5}-3^{\log_4 5}=0$

**122.** **(b)** Given curve, $xy+ax+by=0$.....(i)

(1,1) lie on curve (i)

$\therefore 1+a+b=0$........(ii)

From Eq.(i)

$x\frac{dy}{dx}+y\cdot 1+a+b\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{-(a+y)}{(b+x)}$

$\therefore \left(\frac{dy}{dx}\right)_{(1,1)}=-\frac{(a+1)}{b+1}=2(\because m=\tan\theta)$

$\therefore -\frac{(-b)}{b+1}=2$ [from Eq.(ii)]

$\Rightarrow b=2b+2$

$\therefore\ b=-2$

From Eq.(ii), $a=1$

Hence, $a=1, b=-2$

**123.** **(b)** Divide and multiply by $e^{\frac{1}{x}}$, then

$\lim_{x\to 0}\frac{(1+a^3)+8e^{1/x}}{1+(1-b^3)e^{1/x}}=2$

$\Rightarrow \frac{0+8}{0+1-b^3}=2\Rightarrow 1-b^3=4\Rightarrow b^3=-3\backslash$

$\Rightarrow b=-3^{\frac{1}{3}}$

Then, $a \in R$.

**124.** **(a)** Centre and radii of the given circles are $C_1(1,3)$, $r_1 = r$ and $C_2(4,-1), r_2 = 3$ respectively. Since, circles intersect in two distinct points, then

$|r_1 - r_2| < C_1C_2 < r_1 + r_2$

$\Rightarrow |r-3| < 5 < r+3 \ldots\ldots$(i)

From last two relations, $r > 2$

From first two relations,

$|r-3| < 5$

$\Rightarrow -5 < r-3 < 5$

$\Rightarrow -2 < r < 8 \ldots\ldots\ldots$(ii)

$\therefore$ From Eq.(i) and Eq.(ii), we get

$2 < r < 8$

**125.** **(b)** Let $\delta(x) = x^{\frac{1}{3}}$

Now, $f(x+\delta x) - f(x) = f'(x)\delta(x) = \frac{\delta x}{3x^{\frac{2}{3}}}$

We may write $0.007 = 0.008 - 0.001$

Taking $x = 0.008$ and $\delta x = -0.001$

We have,

$$f(0.007) - f(0.008) = \frac{-0.001}{3(0.008)^{\frac{2}{3}}}$$

or $f(0.007) - f(0.008) = \frac{-1}{120}$

$$\therefore f(0.007) = f(0.008) - \frac{1}{120} = 0.2 - \frac{1}{120}$$

$$\Rightarrow (0.007)^{\frac{1}{3}} = \frac{23}{120}$$

**126.** **(d)** Intercept on Y-axis $= 2\sqrt{f^2 - c}$

$$= 2\sqrt{\frac{49}{4} - 12} = 1$$

**127.** **(c)** $\because f(x) = a - (x-3)^{\frac{8}{9}}$

$$\therefore f'(x) = 0 - \frac{8}{9}(x-3)^{-\frac{1}{9}}$$

At $x = 3, f'(x)$ is not defined.

Hence, $x = 3$ is the point of extremum.

Hence, maximum value of $f(x) = a at x = 3$

**128.** **(a)** We have.

$$f(x) = \begin{cases} ax^2 + b & x < -1 \\ bx^2 + ax + 4 & x \geq -1 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 2ax & x < -1 \\ 2bx + a & x \geq -1 \end{cases}$$

$\because f(x)$ is differentiable at $x = -1$

$\therefore$ It is continuous at $x = -1$ and hence

$\lim_{(x\to -1^-)} f(x) = \lim_{x\to -1^+} f(x)$

$\Rightarrow a + b = b - a + 4$

$\Rightarrow a = 2$

and also, $\lim_{(x\to -1^-)} f'(x) = \lim_{x\to -1^+} f'(x)$

$\Rightarrow -2a = -2b + a$

$\Rightarrow 3a = 2b = b = 3(\because a = 2)$

Hence, $a = 2, b = 3$

**129.** **(c)** $\lim_{x\to\infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$

$\Rightarrow \lim_{x\to\infty}\left(1+\frac{(ax+b)}{x^2}\right)^2 x = e^2$..........(i)

$\lim_{x\to\infty}\frac{ax+b}{x^2}$ must be equal tp zero.

$\Rightarrow \lim_{x\to\infty}\frac{ax+b}{x^2} = 0$

$\Rightarrow \lim_{x\to\infty}\left(\frac{a}{x}+\frac{b}{x^2}\right) = 0$

$\therefore$ a and b $\in$ R

From Eq. (i), $\lim_{e^{x\to\infty}}\left(1+\frac{ax+b}{x^2}-1\right)2x = e^2$

$\Rightarrow \lim_{e^{x\to\infty}}\frac{2(ax+b)}{x} = e^2$

$\therefore e^{2a} = e^2$

$\Rightarrow a = 1$ and $b \in R$

**130.** **(a)** $S(-3,-2) = 9+4-6-6+1 = 2 > 0$

$\therefore (-3,-2)$ outside of S and $S'(-3,-2) = 9+4-12-6+2 = -3 < 0$

$\therefore (-3,-2)$ inside of $S'$.

**131.** **(d)** $x^2+4x+c = 0$

For real roots, $D = b^2-4ac \geq 0$

$16-4c \geq 0$

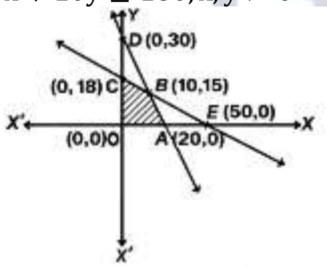
So, $c = 1,2,3,4$ will satisfy the above inequality.

$\therefore$ Required probability $= \frac{4}{9}$

**132.** **(b)**Given constraints are

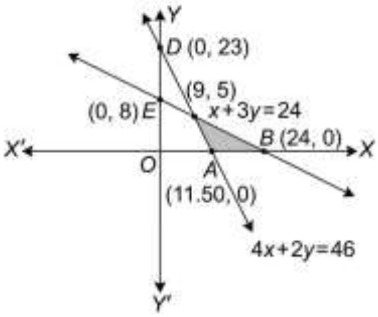
$6x+4y \leq 120$

$3x+10y \leq 180, x, y > 0$




Here, OABCO is the required feasible region whose corner points are O, A, B and C .

**133.** **(b)**Feasible region ABCD and $Z = 4x+2y$




At point $C(9,5), Z = 4\times 9+2\times 5 = 46$

At point $A(11.5,0) Z = 4\times 11.5+2\times 0 = 46$

At point $B(24,0), Z = 4\times 24+0 = 26$

Hence, maximum value of Z is 96.

**134.** **(c)** Given, $A(\text{adj}A) = \begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}$

By using property $A(\text{adj}A) = |A|1n$

$\Rightarrow |A|In = \begin{bmatrix}8 & 0\\0 & 8\end{bmatrix}$

$\Rightarrow |A|1n = 8\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} \Rightarrow |A| = 8$

**135.** **(a)**Given, $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

Now, $|A| = a^{\wedge}3$

$\therefore |A||\text{adj}A| = |A||A|^{n-1} = (a^3)^n = a^{3n}$

**136.** **(d)** Given, $A = \begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix}$

Since, the Matrix A is singular

$\therefore |A| = 0$

$\Rightarrow A = \begin{bmatrix} 2-k & 2 \\ 1 & 3-k \end{bmatrix} = 0$

$\Rightarrow (2-k)(3-k) - 2 = 0$

$\Rightarrow 6 - 5k + k^2 - 2 = 0$

$\Rightarrow 4 - 5k + k^2 = 0$

$\Rightarrow 5k - k^2 = 4$

**137.** **(a)** $\cos\theta = \frac{a\cdot b}{|a||b|}$

$$= \frac{(2\hat{\imath} + 2\hat{\jmath} - \hat{k})\cdot(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k})}{\sqrt{2^2 + 2^2 + (-3)^2}\sqrt{6^2 + (-3)^2 + 2^2}}$$

$$= \frac{12 - 6 - 2}{\sqrt{4 + 4 + 1}\sqrt{36 + 9 + 4}} = \frac{4}{21}$$

**138.** **(c)** Given; $a = x\hat{\imath} + 2\hat{\jmath}, b = u\hat{\jmath} + 3\hat{k}$ and $c = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

Now, $a \times b = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ x & 2 & 0 \\ 0 & y & 3 \end{vmatrix}$

$= \hat{\imath}(6-0) - \hat{\jmath}(3x-0) + \hat{k}(xy-0)$

$= 6\hat{\imath} - 3x\hat{\jmath} + xy\hat{k}$

$= 6\hat{\imath} = 3x\hat{\jmath} + xy\hat{k} = z\hat{\imath} - 3\hat{\jmath} + \hat{k}$

On equating the coefficients of $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$, we get

$z = 6, x = 1$ and $xy = 1$

$\because xy = 1y = 1$

$\Rightarrow a = \hat{\imath} + 2\hat{\jmath}, b = \hat{\jmath} + 3\hat{k}$

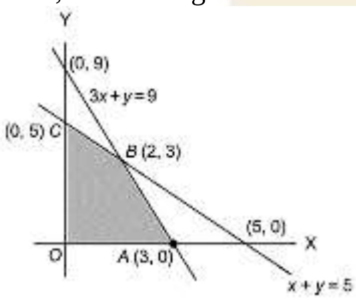
and $c = \hat{\imath} + \hat{\jmath} + 6\hat{k}$

$\therefore [abc] = \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 1 & 1 & 6 \end{vmatrix}$

$[abc] = 1(6-3) - 2(0-3) + 0 = 3 + 6 = 9$

**139.** **(b)** Given, constraints are $x \geq 0, y \geq 0, x + y \leq 5$ and $3x + y \leq 9$ and $z = 12x + 3y$

Here, feasible region is OABCO.




At point $O(0,0), z = 12(0) + 3(0) = 0$

At point $A(3,0), z = 12(3) + 3(0) = 36$

At point $B(2,3), z = 12(2) + 3(3) = 33$

At point $C(0,5), z = 12(0) + 3(5) = 15$

Hence, maximum value of z is 36 .

**140.** **(a)**$3p + q - 2r$

$= 2(\hat{i}+\hat{j}) + (4\hat{k}-\hat{j}) - 2(\hat{i}+\hat{k})$

$= \hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore$ Unit vector in the director of

$3p + q - 2r = \frac{1}{3}(\hat{i} + 2\hat{j} + 2\hat{k})$

**141.** **(c)** Given equation of line is

$\frac{x-3}{4} = \frac{y-4}{5} = \frac{z-5}{6}$

So, DR's of line are (4,5,6)

Since, line is parallel to the plane, therefore the normal to plane is perpendicular to the line.

$\therefore a_1a_2 + b_1b_2 + c_1c_2 = 0$

Consider the plane is

$x - 2y + z = 0$

Here, $a_2 = 1,\ b_2 = -2, c_1$

So, $a_1a_{2\,b1}\ b_2 + c_1c_2$

$= (4 \times 1) + (5 \times -2) + (6 \times 1) = 4 - 10 + 6$

$= 10 - 10 = 0$

Hence, required plane is

$x - 2y + z = 0$

**142.** **(c)** Let $I = \int \frac{1}{x\sqrt{ax-x^2}}dx$

Putting $x = \frac{a}{t}$

$\Rightarrow dx = \frac{-a}{t^2}dt$

$\therefore I = \int \frac{1}{\frac{a}{t}\sqrt{a\cdot\frac{a}{t} - \frac{a^2}{t^2}}} \times -\frac{a}{t^2}dt$

$= \int \frac{-t}{t^2\sqrt{\frac{a^2}{t} - \frac{a^2}{t^2}}}dt$

$= \int \frac{-1}{at\sqrt{\frac{1}{t} = \frac{1}{t^2}}}dt$

$= \frac{-1}{a}\int \frac{1}{\sqrt{\frac{t^2}{t} - \frac{t^2}{t^2}}}dt$

$= \frac{-1}{a}\int \frac{1}{\sqrt{t-1}}dt = \frac{-1}{a}\int (t-i)^{-\frac{1}{2}}dt$

$= \frac{-1}{a}\frac{(t-1)^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{-2}{a}\left(\frac{a}{x} - 1\right)^{\frac{1}{2}} + C$

$I = \frac{-2}{a}\sqrt{\frac{a-x}{x}} + C$

**143.** **(c)** We have, $I = \int \frac{3^x}{\sqrt{1-9^x}}dx = \int \frac{3^x}{\sqrt{1-(3^x)^2}}dx$

Putting $3^x = t$

$\Rightarrow 3^x \log 3\ dx = dt$

$\Rightarrow 3^x dx = \frac{1}{\log 3}dt$

$\therefore I = \frac{1}{\log 3}\int \frac{1}{\sqrt{1-t^2}}dt$

$= \frac{1}{\log 3}\sin^{-1}t + C$

$= \frac{1}{\log 3} \sin^{-1}(3^x) + C$

**144.** **(a)** Here $f(x) = \cos x$

$f(-x) = \cos(-x) = \cos x = f(x)$

So, f is even function.

$\therefore \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = 2 \int_0^{\frac{\pi}{2}} \cos x dx$

$\left[\because \int_{-a}^{a} f(x)dx = 2 \int_0^a f(x)dx, \text{ if f is even}\right]$

$= 2[\sin x]_0^{\frac{\pi}{2}}$

$= 2\left[\sin\frac{\pi}{2} - \sin 0\right] = 2[1 - 0] = 2$

**145.** **(c)** The angle between two lines

$\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$ and $\frac{x - x_2}{a_2} = \frac{y - y_2}{b_2} = \frac{z - z_2}{c_2}$

is given by

$\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$

$= \frac{3 \times 1 + 5 \times 1 + 4 \times 2}{\sqrt{3^2 + 5^2 + 4^2}\sqrt{1^2 + 1^1 + 2^2}}$

$= \frac{16}{\sqrt{50}\sqrt{6}} = \frac{16}{\sqrt{300}}$

$\Rightarrow \cos\theta = \frac{16}{10\sqrt{3}}$

$\Rightarrow \cos\theta = \frac{8}{5\sqrt{3}}$

$\Rightarrow \theta = \cos^{-1}\left(\frac{8}{5\sqrt{3}}\right)$

**146.** **(d)** Given lines are

$\frac{x - 1}{1} = \frac{y - 1}{2} = \frac{z - 2}{3} = \lambda(\text{ let })$

$\Rightarrow x = \lambda + 1, y = 2\lambda + 1, z = 3\lambda + 2$

and $\frac{x - 5}{2} = \frac{y - 2}{1} = z = \mu(\text{ let })$

$\Rightarrow x = 2\mu + 5, y = \mu + 2, z = \mu$

For point of intersection, we have

$\lambda + 1 = 2\mu + 5$ and $2\lambda + 1 = \mu + 2$ and $3\lambda + 2 = \mu$

$\lambda - 2\mu = 4$.........(i)

$2\lambda - \mu = 1$.........(ii)

$3\lambda - \mu = -2$........(iii)

On solving Eqs. (ii) and (iii), we get

$2\lambda - \mu = 1$

$3\lambda - \mu = -2$

$- \ + \ +$ So, $\mu = -7$

$-\lambda = 3$

$\lambda = -3$

Now, put $\lambda = -3$ and $\mu = -7$ in Eq. (i), we get

$-3 + 14 = 11 \neq 4$

Hence, the lines do not intersect.

**147.** **(a)**Given, 5 persons are in a queue of a shop. A and B itself can arrange in two ways thereforenumber of arrangement for A and B $= 2! = 2$ So, probability that A and B are always together

$= \frac{2! \times 4!}{5!} = \frac{2 \times 4!}{5 \times 4!} = \frac{2}{5}$

**148.** **(b)** $P(A) = 0.2, P(B) = 0.3$

$\therefore P(A) = 0.8, P(B) = 0.7$

$\therefore$ Required probability

$= P(A)P(B) + P(A)P(B) + P(A)P(B)$

$= 0.2 \times 0.7 + 0.8 \times 0.3 + 0.2 \times 0.3$

$= 0.44$

**149.** **(d)**Number of triangles formed $= {}^7C_3$

Number of isosceles triangles $= 7 \times 3 = 21$

So, probability $= \frac{21}{{}^7C_3} = \frac{21}{\frac{7!}{3!(7-3)!}}$

$= \frac{21 \times 3! \times 4!}{7 \times 6 \times 5 \times 4!} = \frac{3}{5}$

**150.** **(b)** Given that A, B and C are three mutually exclusive events

$\Rightarrow P(A) = 2P(B) = 3P(C)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= 2P(B) + P(B) + \frac{2}{3}P(B)$

$= \frac{11}{3}P(B)$

$\because P(A \cup B \cup C) = 1$

$\Rightarrow \frac{11}{3}P(B) = 1 \Rightarrow P(B) = \frac{3}{11}$

$\Rightarrow P(B) = \frac{6}{22}$

**151.** **(b)** We know that $U_{n+1} = 3U_n - 2U_{n-1} \ldots\ldots$(i)

Step-(I) Given, $U_1 = 3 = 2 + 1 = 2^1 + 1$, which is true for
$n = 1$, put $n = 1$ in Eq.(i),

Then $U_{1+1} = 3U_1 - 2U_{1-1}$

$\Rightarrow U_2 = 2U_1 - 2U_0$

$= 3 \times 3 - 2 \times 2 = 5 = 2^2 + 1$

.Which is true for $n = 2$

$\therefore$ The result are true for $n = 1$ and $n = 2$

Step-(II) Assume it is true for $n = k$, then it is also true for
$n = k - 1$

Then, $U_k = 2^k + 1 \ldots\ldots$(ii)

and $U_{k-1} = 2^{k-1} + 1 \ldots\ldots$(iii)

Step-(III) On putting $n = k$ in Eq.(i), we get

$U_{k+1} = 3U_k - 2U_{k-1}$

$= 3(2^k + 1) - 2(2^{k-1} + 1)$[From Eqs. (ii) and (iii)

$= 3.2^k + 3 - 2.2^{k-1} - 2$

$= 3.2 + 3 - 2^k - 2$

$= (3 - 1)2^k + 1$

$= 2.2^k + 1 = 2^{k+1} + 1$

This shows that the result is true for $n = k + 1$. Hence by the principle of mathematical induction the result is true for all $n \in N$.

**152.** **(a)** We have, $P(n) = 4^n + 15n + P$

For $n = 1, P(1) = 4 + 15 + p = 19 + p$ is divisible by 9 .

Thus, P should be -1 .

Since, $19 - 1 = 18$ is divisible by 9 .

**153.** **(b)** For $n=1, 2^{3n}-1$ has value $2^3-1=7$

For $n=2, 2^{3n}-1$ has value $2^6-1=63=7\times 9$

which is divisible by 7 not by 6 or 8 or 9.

**154.** **(a)** We know, $T_n=\frac{(n+1)^2-n}{n(n+1)}$

$$=1+\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

$$\therefore S_{10}=10+\left(1-\frac{1}{11}\right)=\frac{120}{11}$$

**155.** **(c)** We have, n . n!

$$=(n+1-1)\cdot n!$$

$$=(n+1)!-n!$$

$$=V(n)-V(n-1)$$

$$\Rightarrow \sum_{n=1}^{m} n(n)!=V(m)-V(0)=(m+1)!-1$$

**156.** **(c)** We have,

$a=-14$ and $a+4d=2$

$\Rightarrow -14+4d=2 \Rightarrow 4d=14+2$

$\Rightarrow 4d=16$

$\therefore d=4$

Now, $S_n=\frac{n}{2}[2a+(n-1)d]$

$$\Rightarrow 40=\frac{n}{2}[-28+(n-1)4]$$

$\Rightarrow n^2-8n-20=0$

$\Rightarrow (n-10)(n+2)=0$

$\Rightarrow n=10$

**157.** **(d)** Given, differential equation is $\frac{d^2y}{dx^2}=0$

On integrating, we get

$\frac{dy}{dx}=c$, where c is a constant

$\Rightarrow dy=cdx$

Again integrating,

$\Rightarrow \int dy=c\int dx$

$\Rightarrow y=cx+d$, where d is constant.

$\therefore$ The solution of differential equation represents all straight lines in a plane.

**158.** **(b)** Given, $\frac{dy}{dx}+\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}=0$

$$\Rightarrow \frac{dy}{\sqrt{1-y^2}}+\frac{dx}{\sqrt{1-x^2}}=0$$

On integrating, we get

$$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}}+\int \frac{dx}{\sqrt{1-x^2}}=0$$

$\Rightarrow \sin^{-1}y+\sin^{-1}x=c$

**159.** **(b)** Given, $x\frac{dy}{dx}=\cot y$

$$\Rightarrow \tan y\,dy=\frac{dx}{x}$$

On integrating, we get

$$\Rightarrow \int \tan y\,dy=\int \frac{dx}{x}$$

$\Rightarrow \log\sec y=\log x+\log c_1$

$\Rightarrow \log\sec y=\log x\cdot c_1$

$\Rightarrow \sec y=x\cdot c_1$

$\Rightarrow x\cos y = c,$

Where $c = \frac{1}{c_1}$

**160.** **(b)** We have, ${}^{n}C_3 = 220$

$\Rightarrow \frac{n(n-1)(n-2)}{6} = 220$

$\Rightarrow n(n-1)(n-2) = 1320$

$\Rightarrow n = 12[\because 12 \times 11 \times 10 = 1320]$

**161.** **(b)** Total number of questions, $n = 5$

Probability of correct answer, $p = \frac{1}{3}$

Probability of incorrect answer, $q = \frac{2}{3}$

Required probability $= P(x \geq 4)$

$= p(x = 4) + p(x = 5)$

$= {}^5C_4 \times \left(\frac{1}{3}\right)^4 \times \left(\frac{2}{3}\right)^1 + {}^5C_5\left(\frac{1}{3}\right)^5 \times \left(\frac{2}{3}\right)^0$

$= \frac{10}{3^5} + \frac{1}{3^5} = \frac{11}{3^5}$

**162.** **(c)** Given, $\sin A + \sin B = a \ldots\ldots(i)$

$\cos A + \cos B = b \ldots\ldots.(ii)$

Dividing Eq.(i) by Eq. (ii), we get

$\frac{\sin A + \sin B}{\cos A + \cos B} = \frac{a}{b}$

$\Rightarrow \frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)} = \frac{a}{b}$

$\Rightarrow \tan\frac{A+B}{2} = \frac{a}{b} \ldots\ldots.(iii)$

Now, $\cos(A+B) = \frac{1-\tan^2\left(\frac{A+B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)}\left(\because \text{ using } \cos\theta = \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right)$

$\Rightarrow \cos(A+B) = \frac{1-\frac{a^2}{b^2}}{1+\frac{a^2}{b^2}}$ [ using Eq.(iii)]

$\therefore \cos(A+B) = \frac{b^2-a^2}{b^2+a^2}$

**163.** **(c)** $\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \frac{\cos\theta}{1-\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta}{1-\frac{\cos\theta}{\sin\theta}}$

$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} + \frac{\sin^2\theta}{\sin\theta-\cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta-\sin\theta} - \frac{\sin^2\theta}{\cos\theta-\sin\theta}$

$= \frac{\cos^2\theta-\sin^2\theta}{\cos\theta-\sin\theta}$

$= \frac{(\cos\theta+\sin\theta)(\cos\theta-\sin\theta)}{(\cos\theta-\sin\theta)}$

[Using identitya $a^2 - b^2 = (a+b)(a-b)$ ]

$= \sin\theta + \cos\theta$

**164.** **(c)** $\sin 2x + \cos x = 0$

$\Rightarrow 2\sin x\cos x + \cos x = 0$

$\Rightarrow \cos x(2\sin x + 1) = 0$

$\Rightarrow \cos x = 0$ or $2\sin x + 1 = 0$

$\Rightarrow \cos x = \cos\frac{\pi}{2} \Rightarrow x = n\pi \pm \frac{\pi}{2}, n \in z$

or $\sin x=-\frac{1}{2}=-\sin\left(\frac{\pi}{6}\right)$

$=\sin\left(\pi+\frac{\pi}{6}\right)$

$\Rightarrow \sin x=\sin\left(\frac{7\pi}{6}\right)$

$\therefore x=n\pi+(-1)^n\frac{7\pi}{6}, n\in Z$

**165.** **(c)** Given, $f(x)+2f(1-x)=x^2+5\ldots\ldots(i)$

Replace $x\rightarrow 1-x$

$\therefore f(1-x)+2f(x)=(1-x)^2+5\ldots\ldots(ii)$

Multiplying Eq. (ii) by 2 and subtracting Eq. (i) from it

$\Rightarrow 2f(1-x)+4f(x)-f(x)-2f(1-x)$

$=2(10x)^2+5\times 2)-x^2-5$

$\Rightarrow 3f(x)=2(x^2-2x+1)-x^2+5=x^2-4x+7$

$\therefore f(x)=\frac{(x-2)^2+3}{3}$

**166.** **(b)** $A\times B=\{(3,1),(3,2),(3,3),(3,5),(5,1),(52),$
$(5,3),(5,5),(7,1),(7,2),(7,3),(7,5)\}$

$B\times A=\{(1,3),(1,5),(1,7),(3,3),(2,5),(2,7),(3,3),$
$(3,5),(3,7),(5,3),(5,5),(5,7)\}$

$\therefore A\times B\cap B\times A=\{(3,3),(3,5),(5,3),(5,5)\}$

**167.** **(d)** For a given set A with n elements, number of elements in power set P(A) is $2^n$.

Here, $n=17$

$\therefore$ Power set constant $2^{17}$ elements.

**168.** **(d)** $\frac{1-i\sqrt{3}}{1+i\sqrt{3}}=\frac{1-i\sqrt{3}}{1+i\sqrt{3}}\times\frac{1-i\sqrt{3}}{1-i\sqrt{3}}=\frac{(1-i\sqrt{3})^2}{4}$

[ Using identity $(a+b)(a-b)=a^2-b^2$]

$=\frac{1-2\sqrt{3}i-3}{4}=\frac{-2-2\sqrt{3}i}{4}$

$=\frac{-1}{2}-\frac{\sqrt{3}}{2}i$

$\therefore \arg\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)=-(\pi-\tan^{-1}\sqrt{3})=\frac{-2\pi}{3}$

**169.** **(d)** $\left[i^{22}+\left(\frac{1}{i}\right)^{25}\right]^3=\left[(i^2)^{11}+\frac{1}{(i^2)^{12}i}\right]^3$

$=\left[(-1)^{11}+\frac{1}{i}\right]^3=\left(-1+\frac{1}{i}\right)^3\ [\because i^2=-1]$

$=(-1)^3+\left(\frac{1}{i}\right)^3+3(-1)\left(\frac{1}{i}\right)\left(-1+\frac{1}{i}\right)$

[Using $(a+b)^3=a^3+b^3+3ab(a+b)$]

$=-1+\frac{1}{i^2 i}-\frac{3}{i}\left(-1+\frac{1}{i}\right)$

$=-1-\frac{1}{i}+\frac{3}{i}-\frac{3}{i^2}$

$=-1+\frac{2}{i}+3(\because i^2=-1)$

$=2+\frac{2i}{1\times 1}=2-2i=2(1-i)$

**170.** **(a)** $(1+\sqrt{3})^{100}=1\times(i+\sqrt{3})^{100}=i^{100}(i+\sqrt{3})^{100}$

( Asi $1^{4\,m}=1$, where $=25\therefore 1^{100}=1$)

$=\left(i^2+i\sqrt{3}\right)^{100}=(-1+i\sqrt{3})^{100}=2\omega\Big)^{100}$

Similarly, $(i-\sqrt{3})^{100} = (-1+i\sqrt{3})^{100} = (2\omega^2)^{100}$

$\therefore (i+\sqrt{3})^{100} + (i-\sqrt{3})^{100} + 2^{100}$

$= (2\omega)^{100} + (2\omega^2)^{100} + 2^{100}$

$= 2^{100}(\omega^{100} + \omega^{200} + 1)$

$= 2^{100}(0) = 0$

$\therefore (i+\sqrt{3})^{100} + (i-\sqrt{3})^{100} + 2^{100} = 0$

**171.** **(b)** From 12 given points ${}^{12}C_2$ straight lines can be drawn.

But 3 points are collinear, using 3 points ${}^{3}C_2$ straight lines can be drawn.

So, total straight lines without the straight line using three points $= {}^{12}C_2 - {}^{3}C_2$

From 3 collinear points 1 straight line can be drawn

So, total number of straight lines $= {}^{12}C_2 - {}^{3}C_2 + 1$

$= \dfrac{12!}{2! \times 10!} - \dfrac{3!}{2! \times 1!} + 1$

$= \dfrac{12 \times 11 \times 10!}{2 \times 1 \times 10!} - \dfrac{3 \times 2!}{2! \times 1} + 1$

$= 66 - 3 + 1 = 64$

**172.** **(b)** Total number formed by using 5 digits $= \frac{5!}{2!}$

$= \dfrac{5 \times 4 \times 3 \times 2!}{2!} = 60$

For number greater than 50,000, digit 2 cannot come at first place.

Hence, number formed in which 2 is at the first place

$= \dfrac{4!}{2!} = \dfrac{4 \times 3 \times 2!}{2!} = 12$

Hence, total number formed greater than 50000

$= 60 - 12 = 48$

**173.** **(d)** A polygon of n sides has number of diagonals $= 170$

$\Rightarrow n\dfrac{(n-3)}{2} = 170$

$\Rightarrow n^2 - 3n = 340$

$\Rightarrow n^2 - 3n - 340 = 0$

$\Rightarrow n^2 - 20n + 17n - 340 = 0$

$\Rightarrow n(n-20) + 17(n-20) = 0$

$\Rightarrow (n-20)(n+17) = 0$

$\Rightarrow n - 20$ or $n + 17 = 0$

$n = 20$ or $n = -17$ [it is not possible]

So, $n = 20$

**174.** **(c)** Equation of angle of equation of pair of straight line $ax^2 + 2hxy + by^2$ is

$\dfrac{x^2 - y^2}{a - b} = \dfrac{xy}{h}$

For $x^2 - 4xy - 5y^2 = 0$

$a = 1, h = -2, b = -5$

So, equation of angle bisector is

$\dfrac{x^2 - y^2}{1 - (-5)} = \dfrac{xy}{-2}$

$\Rightarrow x^2 - y^2 = -3xy$

$\Rightarrow x^2 + 3xy - y^2 = 0$

**175.** **(c)** We know to find, PQ

IMAGE

The slope of the line $2x - y = 5$

$\Rightarrow y = 2x - 5$

On comparing with $y = mx + c$, we get

$m = 2$

Also, slope of PQ, $m_{PQ} = 2$
Eq. of PQ, $(y - 2) = 2(x - 1)$
$\Rightarrow y - 2 = 2x - 2$
$\Rightarrow y = 2x$
So, from $x + y + 2 = 0 \Rightarrow x + 2x + 2 = 0$
$\Rightarrow 3x + 2 = 0 \Rightarrow x = -\frac{2}{3}$
$y = 2 \times -\frac{2}{3} = -\frac{4}{3}$
$\therefore$ The point Q is $\left(\frac{-2}{3}, \frac{-4}{3}\right)$
and $P = (1,2)$
Distance, $PQ = \sqrt{\left(\frac{-2}{3} - 1\right)^2 + \left(\frac{-4}{3} - 2\right)^2}$
$= \sqrt{\left(\frac{-5}{3}\right)^2 + \left(\frac{-10}{3}\right)^2}$
$= \sqrt{\frac{25}{9} + \frac{100}{9}} = \sqrt{\frac{125}{9}} = \frac{\sqrt{125}}{3} = \frac{5\sqrt{5}}{3}$

**176.** **(a)** We have, $\theta = \frac{\Pi}{4}$
Given, line is $2x - y = -7$
$y = 2x + 7$
Here, slope $m_1 = 2$
The required line makes an angle $45^\circ$ with this line
So, $\tan 45^\circ = \left|\frac{2 - m_2}{1 + 2m_2}\right|$
$\Rightarrow 1 + 2m_2 = 2 - m_2$ or $1 + 2m_2 = -(2 - m_2)$
$2m_2 + m_2 = 2 - 1$ or $1 + 2m_2 = -2 + m_2$
$\Rightarrow 3m_2 = 1 \Rightarrow 1m_2 - m_2 = -2 - 1$
$\therefore m_2 = \frac{1}{3}, m_2 = -3$

**177.** **(c)** If 2,3 are intercepts of a line $L = 0$, then
x - intercept $= 2 = a$
y - intercept $= 3 = b$
Equation of line is $\frac{x}{a} + \frac{y}{b} = 1$
$\Rightarrow \frac{x}{2} + \frac{y}{3} = 1$
$\Rightarrow 3x + 2y - 6 = 0$
$\therefore$ Required distance from origin is given by
$= \left|\frac{3(0) + 2(0) - 6}{\sqrt{3^2 + 2^2}}\right| = \frac{|-6|}{\sqrt{13}} = \frac{6}{\sqrt{13}}$
$\therefore$ Distance $= \frac{6}{\sqrt{13}}$ units

**178.** **(c)** The expansion $(x + y)^{100} + (x - y)^{100}$ has
$\left(\frac{100}{2} + 1\right) = 51$ terms

**179.** **(a)** Given expansion is $(1 + 3x + 2x^2 + x^3)^{20}$
$= ((1 + x)^3)^{20}$
$= (1 + x)^{60}$.........(i)
In the expansion of $(a + b)$^n, $(r + 1)$ th term given by
$t_{r+1} = {}^nC_r a^r \cdot b^{n-r}$

or, $t_{r+1} - {}^{n}C_r a^{n-r} \cdot b^r$

$\therefore$ 21st term in Eq.(i) will contain the coefficient of $x^{20}$ and it is given by

$t_{21} = {}^{60}C_{20} x^{20}, 1^{40} = {}^{60}C_{20} x^{20}$

$\therefore$ Coefficient of $x^{20}$ in $(1+x)^{60}$ is ${}^{60}C_{20}$ or ${}^{60}C_{40}$.

**180.** **(c)** Given expansion is $(1 - 3x + 3x^2 - x^2)^{2n}$

$= [(1-x)^3]^{2n}$

$= (1-x)^{6n}$

$\therefore$ Middle term $= \frac{6n+2}{2}$ th term [ here, 6n is even]

$= (3n+1)$ th term