PHYSICS

1. **(c)**For an ideal gas, mean kinetic energy per molecule

$=\frac{f}{2}k_BT$

For diatomic gas, degree of freedom,

$f=5$

$\therefore$ K.E per molecule $=\frac{5k_BT}{2}$

2. **(d)**The displacement of a particle executing SHM is $x=A\sin\omega t$

$\therefore$ Velocity, $v=\frac{dx}{dt}=A\omega\cos\omega t$

$\therefore v=A\omega\sin\left(\omega t+\frac{\pi}{2}\right)$

$\therefore$ Phase difference, $\Delta\phi=\left(\omega t+\frac{\pi}{2}\right)-\omega t=\frac{\pi}{2}$rad.

3. **(a)** As, the mass density of a nucleus is constant

$\therefore$ Mass density, $=\frac{3m}{4\pi R_0^3}$

4. **(b)** When two capacitors are connected then their common potential difference is

$V=\frac{C_1V_1+C_2V_2}{C_1+C_2}$

Given, $C_1=2\mu\,F, V_1=14\,V, C_2=5\mu\,F, V_2=0\,V$

$\therefore\ V=\frac{2\times14+5\times0}{2+5}=4V$

5. **(a)** For an electromagnetic wave, Speed of electromagnetic wave, $c=\frac{E_0}{B_0}$

$\therefore\frac{B_0}{E_0}=\frac{1}{c}=$ reciprocal of speed

6. **(b)**Initial kinetic energy is

$K=\frac{1}{2}mv^2\quad\ldots\ldots(i)$

Final kinetic energy is

$K'=\frac{1}{2}mv^2=\frac{1}{2}m(u\cos30°)^2\qquad[\because v=u\cos30°]$

$=\frac{3}{4}\left(\frac{1}{2}mu^2\right)=\frac{3}{4}K\qquad$ [From Eq. (i)]

7. **(c)**The refractive index of water,

$\mu=\frac{4}{3}$

The real depth of image of bubble from the surface of water,

$d=24+8=32$ cm

$\therefore$ Apparent depth of the image of bubble is

$d'=\frac{d}{\mu}=\frac{32}{4/3}=24$ cm

8. **(b)**Current gain factor, $\alpha=0.96$

$R=1000\Omega$, Voltage drop across collector resistor $=0.5$ V

$\therefore\ \beta=\frac{\alpha}{1-\alpha}=\frac{0.96}{1-0.96}=24$

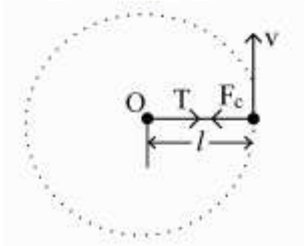
$\therefore$ Collector current,

$i_c=\frac{\text{Voltage drop across collector resistor}}{R}$

$=\frac{0.5}{1000}=0.5\times10^{-3}$ A

The base current, $i_b=\frac{i_c}{\beta}=\frac{0.5\times10^{-3}}{24}=20.8\mu$ A

9. **(a)** Consider the string of length l connected to a particle as shown in the figure



Speed of the particle is v. As, the particle is in uniform circular motion, the net force on the particle must be equal to centripetal force which is provided by the tension (T).

Net force $=$ Centripetal force

$=$ Tension in string

$\Rightarrow \frac{mv^2}{l} = T$

10. **(c)** According to conservation of angular momentum,

$I\omega =$ constant

i.e. we can write,

$I_1\omega_1 = I_2\omega_2$

or $\mathrm{MR^2}\omega = (\mathrm{M}+4\mathrm{m})\mathrm{R}^2\omega_2 \qquad (\because \omega_1 = \omega)$

or $\omega_2 = \left(\frac{\mathrm{M}}{\mathrm{M}+4\mathrm{m}}\right)\omega$

11. **(d)** Given, ratio of radius of two wires,

$\frac{r_1}{r_2} = \frac{2}{1}$

and ratio in their lengths,

$\frac{l_1}{l_2} = \frac{1}{2}$

When same force is applied on them, then ratio change in their lengths,

$\frac{\Delta l_1}{\Delta l_2} = ?$

We known that, Young's modulus,

$\mathrm{Y} = \frac{\mathrm{Fl}}{\mathrm{A}\Delta l} \Rightarrow \Delta \mathrm{l} = \frac{\mathrm{F}l}{\mathrm{AY}}$

$\Delta \mathrm{l} \propto \frac{\mathrm{l}}{\mathrm{A}}$

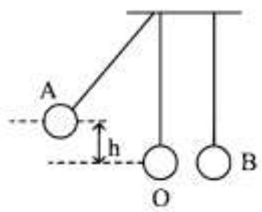
$\therefore \frac{\Delta l}{\Delta l_1}\frac{l}{\mathrm{A}} = \frac{l_1/\mathrm{A}_1}{l_2/\mathrm{A}_2} = \frac{l_1\mathrm{A}_2}{l_2\mathrm{A}_2}$

$\Rightarrow \frac{\Delta l_1}{\Delta l_2} = \frac{l_1/\mathrm{A}_1}{r_2^2/r_1^2} = \frac{1}{2}\cdot\left(\frac{1}{2}\right)^2 = \frac{1}{8}$

12. **(d)** When bobA strikes to the bob B, then

$\mathrm{mu} = (\mathrm{m}+\mathrm{m})\mathrm{v}'$

$\Rightarrow \mathrm{v}' = \frac{\mathrm{u}}{2}$......(i)



The potential energy of A at height h gets converted into kinetic energy of this mass, at point O, i.e.

$\mathrm{Mgh} = \frac{1}{2}\mathrm{mu}^2$

$\Rightarrow \mathrm{u} = \sqrt{2\mathrm{gh}} \qquad$ [from Eq.(i)]

$\therefore\ v' = \frac{\sqrt{2gh}}{2} = \sqrt{\frac{gh}{2}}$

Let the combined mass moves to a height h', then total mass $= 2m$

Then, $2mgh' = \frac{1}{2}(2m)v'^2$

$\Rightarrow\ gh' = \frac{gh}{4} \Rightarrow h' = \frac{h}{4}$

13. **(b)** Net work done by the gas = Area enclosed in $p - V$ curve, i.e. area of $\triangle$ ABC.

$W_{net} = \frac{1}{2} \times 5 \times 10^{-3} \times 4 \times 10^5\ J = 10^3\ J = 1000\ J$

14. **(a)** The gases carbon monoxide (CO) and nitrogen (N_2) are diatomic, so both have equal kinetic energy $\frac{5}{2}KT$, i.e. $E_1 = E_2$

15. **(c)** According to the question,

For wire 1 : Area of cross-section $= A_1$,

Force applied $= F_1$

and increase in length $= \Delta l$.

From the relation of Young's modulus of elasticity,

$Y = \frac{Fl}{A\Delta l}$

Substituting the values for wire 1 in the above relation, we get

$\Rightarrow\ Y_1 = \frac{F_1 l_1}{A_1 \Delta l}$ ....(i)

For wire 2: Area of cross-section $= A_2$

Force applied $= F_2$

increase in length $= \Delta l$

Similarly, $Y_2 = \frac{F_2 l_2}{A_2 \Delta l}$.....(ii)

$\because$ Volume, $V = Al$ or $l = \frac{V}{A}$

Substituting the value of l in Eqs. (i) and (ii), we get

$Y_1 = \frac{F_1 V}{A_1^2 \Delta l}$ and $Y_2 = \frac{F_2 V}{A_2^2 \Delta l}$

As it is given that the wires are made up of same material,

i.e. $Y_1 = Y_2$

$\Rightarrow\ \frac{F_1 V}{A_1^2 \Delta l} = \frac{F_2 V}{A_2^2 \Delta l} \Rightarrow \frac{F_1}{F_2} = \frac{A_1^2}{A_2^2} = \frac{A^2}{9A^2} = \frac{1}{9}$ ($\because\ A_1 = A$ and $A_2 = 3A$)

or $F_2 = 9F_1 = 9F$ (given, $F_1 = F$)

16. **(b)** Acceleration due to gravity at a depth d below the surface of the earth is given by

$g_{depth} = g_{surface}\left(1 - \frac{d}{R}\right)$

$= g_{surface}\left[\frac{R-d}{R}\right] = d_{surface}\left(\frac{r}{R}\right)$

Also, for a point at height h above surface,

$g_{height} = g_{surface}\left[\frac{R^2}{(R+h)^2}\right]$

Therefore, we can say that value of g increases from centre to maximum at the surface and then decreases as depicted in graph (b).

17. **(c)** Potential energy of the spring,

$U = \frac{1}{2}kx^2$ .....(i)

and $U' = \frac{1}{2}k(nx)^2 \Rightarrow U' = n^2\frac{1}{2}kx^2$

$\Rightarrow\ U' = n^2 U$ [From Eq. (i)]

18. **(d)** By using $v' = v\left[\frac{v - v_0}{v - v_s}\right]$

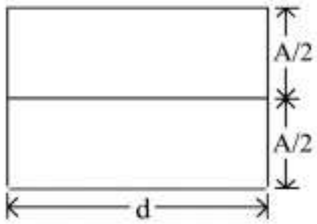
$\Rightarrow\ 2v = v\left[\frac{v - v_0}{v - 0}\right]$

$\Rightarrow v_0 = -v$

Negative sign indicates that observer is moving opposite to the direction of velocity of sound.

19. **(c)** The dielectric is introduced such that half of its area is occupied by it.

In the given case, the two capacitors are in parallel.

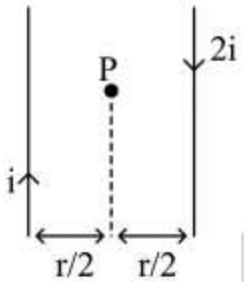


$\therefore\ C' = C_1 + C_2$

But $C_1 = \frac{A\varepsilon_0}{2d}$ and $C_2 = \frac{KA\varepsilon_0}{2d}$

Thus, $C' = \frac{A\varepsilon_0}{2d} + \frac{KA\varepsilon_0}{2d} \Rightarrow C' = \frac{C}{2}(1+K)$

20. **(a)** The magnetic field induction at P due to currents through both the wires is



$B = \frac{\mu_0}{4\pi}\frac{2i}{(r/2)} + \frac{\mu_0}{4\pi}\frac{2(2i)}{(r/2)} = \frac{\mu_0}{4\pi}\cdot\frac{12i}{r}$

acting perpendicular to plane of wire inwards. Now, B and v are acting in the same direction, i.e. $\theta = 0°$ Force on charged particle is $F = qvB\sin\theta = qvB \times 0 = 0$.

21. **(a)** Given, $\phi = (4t^2 + 6t + 9)$Wb and $t = 2$ s

We know that, $|\varepsilon| = \left|\frac{d\phi}{dt}\right|$

Here, $\left|\frac{d\phi}{dt}\right| = |8t + 6| \Rightarrow \left(\frac{d\phi}{dt}\right)_{t=2} = 8 \times 2 + 6$

Induced emf in the coil $= 22$ V

22. **(d)** Given,

$i = \frac{1}{\sqrt{2}}\sin(100\pi t)$amp

and $e = \frac{1}{\sqrt{2}}\sin(100\pi t + \pi/3)$ volt

$\Rightarrow\ i_0 = \frac{1}{\sqrt{2}}$ and $e_0 = \frac{1}{\sqrt{2}}$

We know that, average power,

$P_{av} = V_{rms} \times i_{rms}\cos\phi = \frac{1}{2} \times \frac{1}{2} \times \cos 60°$

$(\because i_{rms} = i_0/\sqrt{2}$ and $V_{rms} = V_0/\sqrt{2})$

$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ W

23. **(b)** When the sound is reflected from the cliff, it approaches the driver of the car. Therefore, the driver, acts as an observer and both the source (car) and observer are moving.

Hence, apparent frequency heard by the observer (driver) is given by

$f' = f\left(\frac{v+v_o}{v-v_s}\right)$ ....(i)

where, v_s = velocity of sound

and v_o = velocity of the car

Thus, Eq. (i) becomes

$2f = f\left(\frac{v + v_o}{v - v_o}\right)$

$\Rightarrow 2v - 2v_o = v + v_o \Rightarrow 3v_o = v \Rightarrow v_o = \frac{v}{3}$

24. (a) Given, Escape velocity on the surface of the earth is given by i.e:

$v_e = \sqrt{2gR_e}$

$v_e = \sqrt{\frac{2GM_e}{R_e}}$ ......(i)

Mass of the moon, $M_m = \frac{M_e}{81}$

Radius of the moon, $R_m = \frac{R_e}{4}$

∴ Escape velocity on the surface of the moon

$v_m = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{\frac{2G\left(\frac{Me}{81}\right)}{\frac{R_e}{4}}} = \frac{2\sqrt{2}}{9}\sqrt{\frac{GM_e}{R_e}}$

From Eq. (i),

$= \frac{2}{9}\sqrt{\frac{2GM_e}{R_e}} = \frac{2}{9}v_e$

Escape velocity $v_e = 11.1$ km/h

$= \frac{2}{9} \times 11.1 = 2.46 \text{ km h}^{-1}$

25. (a) For all processes 1,2 and 3

Change in internal energy, i.e.

$\Delta U = U_B - U_A$

$\therefore \Delta U_1 = \Delta U_2 = \Delta U_3 \Rightarrow Q = \Delta U + \Delta W$

Now, $\Delta W =$ work done by the gas, i.e. area under $p - V$ curve.

As, area $(1) > \text{area}(2) > \text{area}(3)$

$\therefore \ \Delta W_1 > \Delta W_2 > \Delta W_3$

$\therefore Q_1 > Q_2 > Q_3$

26. (d) Here, $L = 1H, C = 20\mu\ F, R = 300\Omega$

$X_L = 2\pi f L = 2\pi\left(\frac{50}{\pi}\right) \times 1 = 100\Omega$

$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\left(\frac{50}{\pi}\right) 20 \times 10^{-6}} = 500\Omega$

Impedance, $Z = \sqrt{R^2 + (X_C - X_L)^2}$

$= \sqrt{(300)^2 + (400)^2} = 500\Omega$

27. (a) Here, $\frac{I_{bright}}{I_{dark}} = \frac{I_{max}}{I_{min}} = 3$ (given)

$\Rightarrow \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{3}{1} \Rightarrow \frac{a_1 + a_2}{a_1 - a_2} = \sqrt{3}$

$\Rightarrow \ a_1 + a_2 = \sqrt{3}(a_1 - a_2) \Rightarrow \frac{a_1}{a_2} = \left(\frac{\sqrt{3} + 1}{\sqrt{3} - 1}\right)$

28. (c) Lyman series of H -atom, we can write

$\frac{hc}{\lambda} = Rhc\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

where, symbols have their usual meaning and for second line of Balmer series of H -like ion

$\frac{hc}{\lambda} = Z^2 Rhc\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

Therefore, $\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = Z^2\left(\frac{1}{4} - \frac{1}{16}\right)$

$\left(1-\frac{1}{4}\right)=Z^2\left(\frac{1}{4}-\frac{1}{16}\right)$

$\Rightarrow Z = 2.19 \simeq 2$

The approximately value of Z will be 2 .

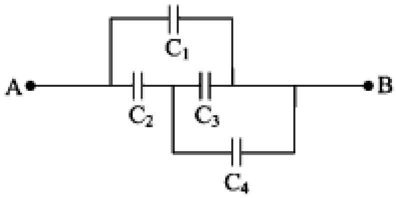
29. **(a)** From the first law of thermodynamics, if an amount of heat Q is given to a system, a part of it is used in increasing the internal energy ΔU of the system and the rest in doing work W by the system.

$\therefore Q = \Delta U + W$

$\Rightarrow \Delta U = Q - W \Rightarrow \Delta U = 150 - 110$

$= 40$ J

30. **(d)** From figure,

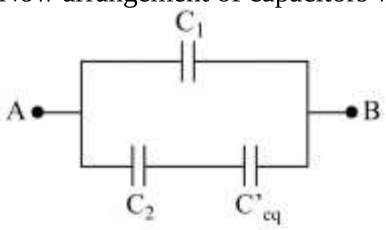


$C_1 = C_2 = C_3 = C_4 = 3\mu$ F (given)

$\because C_4$ and C_3 are in parallel, i.e.

$C_{eq} = C_4 + C_3 = (3+3)\mu F = 6\mu$ F

Now arrangement of capacitors will be as follows



$\because C_1$ is in parallel with the series combination of C_2 and C'_{eq}.

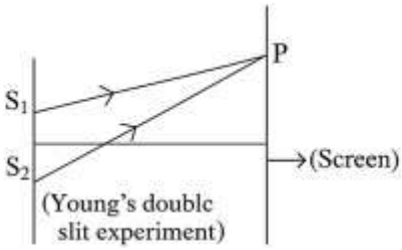
$\therefore C''_{eq}$ between A and B

$=\frac{(C_2 \times C'_{eq})}{C_2 + C'_{eq}} + C_1 \Rightarrow \left(\frac{3\times 6}{3+6}\right) + 3 = 5\mu$ F

31. **(b)**We know that, $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow \frac{1}{\lambda} = R\left(\frac{1}{1_1^2} - \frac{1}{2^2}\right) = \frac{3R}{4} \Rightarrow \lambda = \frac{4}{3R}$ cm

32. **(c)** Assume, Young's double slit experiment, amplitudes corresponding to the sources S_1 and S_2 be a_1 and a_2 and intensities be I_1 and I_2 respectively.



So, $I_{max} \propto a_{max}^2$ and $I_{min} \propto a_{min}^2$

$\Rightarrow \frac{I_{max}}{I_{min}} = \left(\frac{a_{man}}{a_{min}}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$

According to given question,

$\frac{I_{max}}{I_{min}} = \frac{9}{1}$

$\Rightarrow 9 = \left(\frac{a_1/a_2 + 1}{a_1/a_2 - 1}\right)^2$

$\Rightarrow (3)^2 = \left(\frac{a_1/a_2+1}{a_1/a_2-1}\right)^2$ (given)

$$\Rightarrow\ 3=\frac{a_1/a_2+1}{a_1/a_2-1}\Rightarrow 3=\frac{\left(\frac{a_1+a_2}{a_2}\right)}{\left(\frac{a_1-a_2}{a_2}\right)}\Rightarrow 3=\frac{a_1+a_2}{a_1-a_2}$$

$$\Rightarrow\ 3(a_1-a_2)=a_1+a_2\Rightarrow 3a_1-3a_2=a_1+a_2$$

$$\Rightarrow\ 2a_1=4a_2\Rightarrow\frac{a_1}{a_2}=\frac{4}{2}\Rightarrow a_1:a_2=2:1$$

33. **(b)** In case of an inductor, voltage leads the current by phase difference of $\frac{\pi}{2}$.

34. **(b)** Acceleration due to gravity at a height h from the earth's surface,

$$g'=\frac{GM}{(R+h)^2}$$

Given, $g'=1\%$ of $g=\frac{g}{100}$

$$\Rightarrow\ \frac{g}{100}=\frac{GM}{(R+h)^2}$$

$\frac{(R+h)^2}{100}=\frac{GM}{g}$ or $\frac{(R+h)^2}{100}=R^2\left(\because g=\frac{GM}{R^2}\right)$

or $R+h=10R$

$h=9R$

where, R is the radius of the earth.

35. **(a)** We know that energy, $E=\frac{1}{2}CV^2$

Dimensions of CV^2 = Dimensions of energy, $E=[ML^2\,T^{-2}\,A^0]$

36. **(c)** The output,

$Y=\overline{\bar{A}+\bar{B}}=\bar{\bar{A}}\cdot\bar{\bar{B}}=A\cdot B$

This is the boolean expression for AND gate.

37. **(b)** The energy of light of wavelength λ is given by

$$E=h\nu=\frac{hc}{\lambda}$$

$\Rightarrow\ \lambda=\frac{hc}{E}\quad\ldots(i)$

Here, h = Planck's constant $=6.63\times10^{-34}\ \mathrm{J-s}$

c= speed of light $=3\times10^8\ \mathrm{m/s}$

E= energy gap $=1.9\mathrm{eV}$

$=1.9\times1.6\times10^{-19}\ \mathrm{J}$

Substituting the given values in Eq. (i), we get

$$\Rightarrow\lambda=\frac{6.63\times10^{-34}\times3\times10^8}{1.9\times1.6\times10^{-19}}=6.54\times10^{-7}\ \mathrm{m}$$

$=654\ \mathrm{nm}$

38. **(b)**Let v be initial velocity of vertical projection and t be the time taken by the body to reach a height h from ground.

Here, $u=u, a=-g, s=h, t=t$

Using, $s=ut+\frac{1}{2}at^2$, we have

Or $h=ut+\frac{1}{2}(-g)t^2$

$gt^2-2ut+2h=0$

$$\therefore t=\frac{2u\pm\sqrt{4u^2-4g\times2h}}{2g}=\frac{u\pm\sqrt{u^2-2gh}}{g}$$

It means t has two values, i.e.

$$t_1=\frac{u+\sqrt{u^2-2gh}}{g}\Rightarrow t_2=\frac{u-\sqrt{u^2-2gh}}{g}$$

$$t_1+t_2=\frac{2u}{g}\text{ or }u=\frac{g(t_1+t_2)}{2}$$

39. **(b)** We know that, $eV_0=h\nu-\phi_0$ where, V_0 = stopping potential and ν = frequency of light.

Case (I) $eV_0 = h\nu - \phi_0$ ....(i)

Case (II) $\frac{eV_0}{4} = \frac{h\nu}{2} - \phi_0$

$\Rightarrow eV_0 = 2h\nu - 4\phi_0$ ....(ii)

From Eqs. (i) an i (ii), we get

$h\nu - \phi_0 = 2h\nu - 4\phi_0$

$\Rightarrow h\nu = 3\phi_0 \Rightarrow h\nu = 3h\nu_0$ $(\because \phi_0 = h\nu_0)$

$\Rightarrow \nu_0 = \frac{h\nu}{3h} \Rightarrow \nu_0 = \frac{\nu}{3}$

40. **(b)** When object is in rarer medium and observer is in denser medium, then normal shift,

$d = (n-1)h$

where, $h =$ real depth $= y$

Now, apparent depth or the apparent height of the bird from the surface of the water $= y + (n-1)y = n.y$. The total distance of the bird as estimated by fish is $x + ny$.

41. **(b)** Speed of car, $v = 180\,kmh^{-1} = 50\,ms^{-1}$ and time, $t = 10$ s

$\therefore$ Acceleration of the car,

$$a = \frac{v-u}{t} = \frac{50-0}{10} = 5\,ms^{-2}$$

Thus, the distance covered by the car,

$$s = \frac{1}{2}at^2 = \frac{1}{2} \times 5 \times (10)^2 = 250\text{ m}$$

42. **(a)** Magnetic field,

$B = \frac{E}{c}$, where $c = 3 \times \frac{10^8\text{ m}}{\text{s}}$.

$$B = \frac{6}{3 \times 10^8} = 2 \times 10^{-8}\text{ T}$$

43. **(a)** Area of a parallelogram $= |\vec{a} \times \vec{b}|$ where, a and b are sides of parallelogram.

Given, $\vec{a} = \vec{p} = 5\hat{\imath} - 4\hat{\jmath} + 3\hat{k}$

and $\vec{b} = \vec{q} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 5 & -4 & 3 \\ 3 & 2 & -1 \end{vmatrix}$$

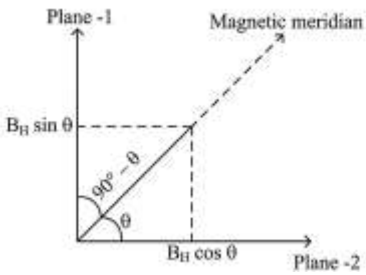
$\Rightarrow \vec{a} \times \vec{b} = \hat{\imath}(4-6) - \hat{\jmath}(-5-9) + \hat{k}(10+12)$

$\Rightarrow \vec{a} \times \vec{b} = -2\hat{\imath} + 14\hat{\jmath} + 22\hat{k}$

Thus, $|\vec{a} \times \vec{b}| = \sqrt{(2)^2 + (14)^2 + (22)^2} = \sqrt{684}$ sq units

44. **(a)** Let the B_H and B_V be the horizontal and vertical component of the earth's magnetic field B .

$\tan\theta = \frac{B_v}{B_H} \Rightarrow \cot\theta = \frac{B_H}{B_v}$....(i)



Let plane - 1 and 2 be mutually perpendicular planes makings angle θ and ($90° - \theta$) with magnetic meridian. The vertical component of the earth is magnetic field remain same in two plane but effective horizontal components in the two planes is given by

$B_1 = B_H\cos\theta$ ....(ii)

and $B_2 = B_H\sin\theta$ ....(iii)

Then, $\tan\theta_1 = \frac{B_V}{B_1} = \frac{B_V}{B_H\cos\theta}$

$\cot\theta_1 = \frac{B_H\cos\theta}{B_V}$ .....(iv)

Similarly, $\Rightarrow \tan\theta_2 = \frac{B_V}{B_2} = \frac{B_V}{B_H \sin\theta}$

$\Rightarrow \cot\theta_2 = \frac{B_H \sin\theta}{B_V}$ .....(v)

From Eqs. (iv) and (v), we get

$\Rightarrow \cot^2\theta_1 + \cot^2\theta_2$

$= \frac{B_H^2 \cos^2\theta}{B_V^2} + \frac{B_H^2 \sin^2\theta}{B_V^2}$

$\Rightarrow \cot^2\theta_1 + \cot^2\theta_2 = \frac{B_H^2}{B_V^2}(\cos^2\theta + \sin^2\theta)$

$\Rightarrow \cot^2\theta_1 + \cot^2\theta_2 = \cot^2\theta$

45. (c) Here, $K_1 = \frac{hc}{\lambda_1} - W$ .....(i)

and $K_2 = \frac{hc}{\lambda_2} - W$ .....(ii)

Substituting $\lambda_1 = 2\lambda_2$ in Eq. (i), we get

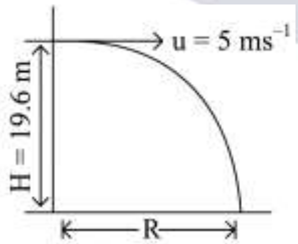
$K_1 = \frac{hc}{2\lambda_2} - W$

$\Rightarrow K_1 = \frac{1}{2}\left(\frac{hc}{\lambda_2}\right) - W = \frac{1}{2}(K_2 + W) - W$

$K_1 = \frac{K_2}{2} - \frac{W}{2}$

$\Rightarrow K_1 < \frac{K_2}{2}$

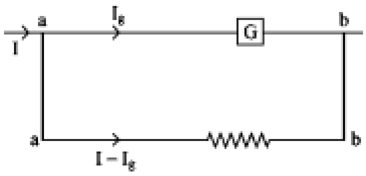
46. (b) The time taken to hit the ground is given by



$T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 19.6}{9.8}} = 2s$

47. (b) Let total current through the 1 parallel combination be I, the current through the galv anometer be I_g and the current through the shunt be $I - I_g$.

The shunted galvanometer is shown in the figure.



The potential difference, $V_{ab} = (V_a - V_b)$ is the same for both paths, so $I_g G = (I - I_g)S$

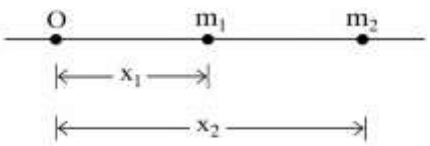
$\Rightarrow I_g(G + S) = IS \Rightarrow \frac{I_g}{T} = \frac{S}{S + G}$

The fraction of current passing through shunt

$= \frac{I - I_g}{I} = 1 - \frac{I_g}{I} = 1 - \frac{S}{S + G} = \frac{G}{S + G}$

$= \frac{8}{2 + 8} = \frac{8}{10} = 0.8$ A

48. (c) The centre of mass of the system is given by



$$x=\frac{m_1x_1+m_2x_2}{m_1+m_2}$$

49. (d) Applying Archimedes' principle, submerged part = replaced water

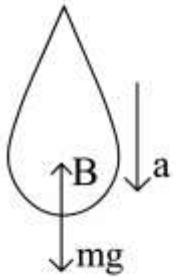
$\Rightarrow \frac{4}{5}h\rho_{wo}g = h\times\rho_l\times g$

where, ρ_w = density of water and ρ_l = density of the liquid.

$\therefore \rho_l=\frac{4}{5}\times\rho_w=\frac{4}{5}\times 1000$

$\Rightarrow \rho_l = 800\mathrm{kgm}^{-3}$

50. (a)



When the balloon is descending down with an acceleration a,

$mg - B = ma \quad \ldots(i)$

where, B = buoyant force.

Here, we should assume that while removing some mass the volume of balloon and hence buoyant force will not change.

Let the new mass of the balloon is m', so

$B - m'g = m'a \quad \ldots.(ii)$

On solving Eqs. (i) and (ii), we get

$mg - m'g = ma + m'a$

$\Rightarrow m(g-a)=m'(g+a)\Rightarrow m'=\frac{m(g-a)}{g+a}$

So, mass removed, $\Delta m = m - m'$

$=m\left[1-\frac{(g-a)}{(g+a)}\right]=m\left[\frac{g+a-g+a}{g+a}\right]=\frac{2ma}{g+a}$

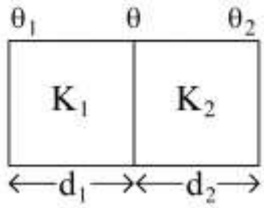
51. (d) Given,

$i = 10\ A, B = 0.15\ T$

$\theta = 45^\circ$ and $l = 2\ m$

Here, $F = ilB\sin\theta = 10\times 2\times 0.15\sin 45^\circ = \frac{3}{\sqrt{2}}\ N$

52. (c) For first slab,



Heat current, $H_1=\frac{K_1(\theta_1-\theta)A}{d_1}$

For second slab,

Heat current, $H_2=\frac{K_2(\theta-\theta_2)A}{d_2}$

As slabs are in series, $H_1 = H_2$

$\therefore \dfrac{K_1(\theta_1-\theta)A}{d_1}=\dfrac{K_2(\theta-\theta_2)A}{d_2}$

$\Rightarrow \theta=\dfrac{K_1\theta_1 d_2+K_2\theta_2 d_1}{K_2 d_1+K_1 d_2}$

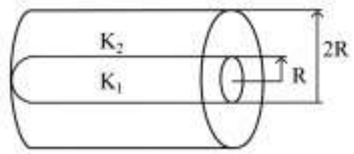
53. **(b)** $B=4\left[\frac{\mu_0}{4\pi}\cdot\frac{I}{a}(\sin\phi_1+\sin\phi_2)\right]$

Here, $a=\frac{l}{2}$ and $\phi_1=\phi_2=45^\circ$

$B=4\times\dfrac{\mu_0}{4\pi}\dfrac{I}{(l/2)}\left[\dfrac{1}{\sqrt{2}}+\dfrac{I}{\sqrt{2}}\right]$

$\therefore B=\dfrac{16}{\sqrt{2}}\left[\dfrac{\mu_0}{4\pi}\dfrac{I}{l}\right]=8\sqrt{2}\left(\dfrac{\mu_0}{4\pi}\cdot\dfrac{I}{l}\right)$

54. **(d)** Both the cylinders are in parallel, for the heat flow from one end as shown



So, $K_{eq}=\frac{K_1A_1+K_2A_2}{A_1+A_2}$

where, A_1 = area of cross-section of inner cylinder $=\pi R^2$

and A_2 = area of cross-section of cylindrical shell

$=\pi\{(2R)^2-(R)^2\}=3\pi R^2$

$\Rightarrow K_{eq}=\dfrac{K_1(\pi R^2)+K_2(3\pi R^2)}{\pi R^2+3\pi R^2}=\dfrac{1}{4}(K_1+3K_2)$

55. **(c)** Due to the specific shape of wings, when the aeroplane runs, air passes at higher speed over it as compared to its lower surface. This difference of air speeds above and below the wings, in accordance with Bernoulli's principle, creates a pressure difference, due to which an upward force called 'dynamic lift' acts on the plate.

$\therefore$ Upward lift = pressure difference × area of the wings [$\because F=p\times A$]

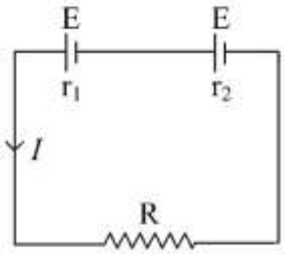
From Bernoulll's equation,

$p_1+\dfrac{1}{2}\rho v_1^2=p_2+\dfrac{1}{2}\rho v_2^2$

$\because p_2-p_1=\dfrac{1}{2}\rho(v_1^2-v_2^2)$

Hence, upward lift $=\frac{1}{2}\rho A(v_1^2-v_2^2)$

56. **(c)** Here are two batteries with emf E each and the internal resistances r_1 and r_2, respectively



We have $I(R+r_1+r_2)=2E$

Current in the circuit, $I=\frac{2E}{R+r1+r2}$ ....(i)

Now, the potential difference across the first cell would be equal to $V=E-Ir_1$. From the question, $V=0$, there

$E=Ir_1=\frac{2Er_1}{R+r_1+r_2}$ [from Eq.(i)]

where, $R+r_1+r_2=2r_1$

Hence, $R=r_1-r_2$.

57. **(a)** The frequency of a vibrating string can be expressed by the formula:

$f=\dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

Where:

• f is the frequency of the string,

•L is the length of the string,

•T is the tension in the string, and

• μ is the linear mass density of the string.

•Initially, the frequency of the string is 200 Hz when the length is L and the tension is T_1. The formula becomes:

$$200 = \frac{1}{2L}\sqrt{\frac{T_1}{\mu}}$$

When the length of the string is doubled to 2L and the tension is altered to T_2, the frequency becomes 300 Hz . The formula then changes to:

$$300 = \frac{1}{4L}\sqrt{\frac{T_2}{\mu}}$$

We can rearrange each equation to express the square root of tension over linear mass density:

$$\sqrt{\frac{T_1}{\mu}} = 400L \quad \sqrt{\frac{T_2}{\mu}} = 1200L$$

Squaring both sides of each equation gives:

$$\frac{T_1}{\mu} = (400L)^2 \quad \frac{T_2}{\mu} = (1200L)^2$$

Take the ratio $\frac{T_2}{T_1}$:

$$\frac{T_2}{T_1} = \frac{(1200L)^2}{(400L)^2}\ \frac{T_2}{T_1} = \left(\frac{1200}{400}\right)^2 \frac{T_2}{T_1} = (3)^2 \frac{T_2}{T_1} = 9$$

Thus, the ratio of the new tension T_2 to the original tension T_1 is $9:1$, indicating that the new tension is 9 times greater than the original tension.

58. **(b)** Charge, $q = ne$

$\Rightarrow\ q = 10^{19} \times 1.6 \times 10^{-19} = 1.6C$

59. **(c)** Here, phase difference

$$\tan\phi = \frac{X_L - X_C}{R} \Rightarrow \tan\frac{\pi}{3} = \frac{X_L - X_C}{R}$$

When L is removed, $\tan\frac{\pi}{3} = \frac{X_C}{R} = \sqrt{3}$

$\therefore\ X_C = \sqrt{3}R$ .....(i)

Similarly, when C is removed

$$\tan\frac{\pi}{3} = \frac{X_L}{R} = \sqrt{3}$$

$\Rightarrow X_L = \sqrt{3}R$

$\Rightarrow X_C = X_L$....(ii)[from Eq.(i)]

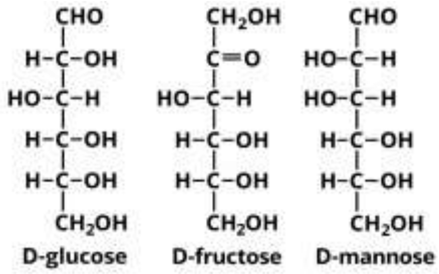
Now, $\tan\phi = 0$

$\Rightarrow\ \phi = 0°$

$\therefore$ Power factor, $\cos\phi = \cos 0° = 1$

CHEMISTRY

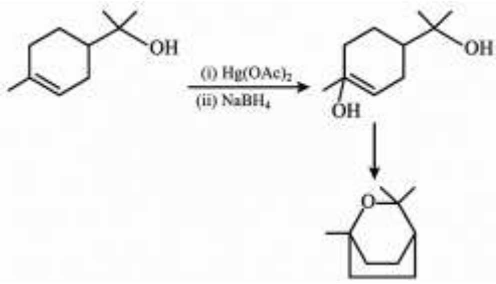
60. **(b)** D-glucose, D-fructose and D-mannose form the same osazone treated with excess phenyl hydrazine because they differ only Ist and 2nd carbon atoms which are transformed to the same form.



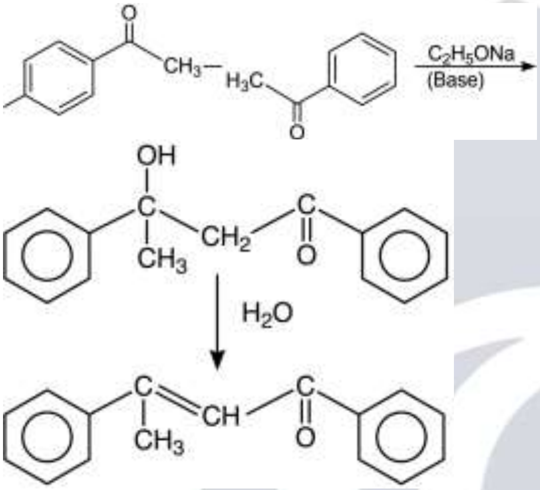
They form same osazone

$CH{=}NNHC_6H_5$
|
$C{=}NNHC_6H_5$
|
$(CHOH)_3$
|
CH_2OH

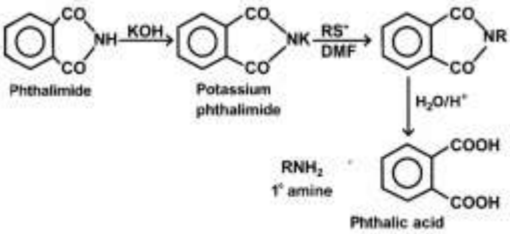
61. **(b)** Addition of OH at most substituted side of the ene and final product is formed by loss of H O resulting in the formation of ring.



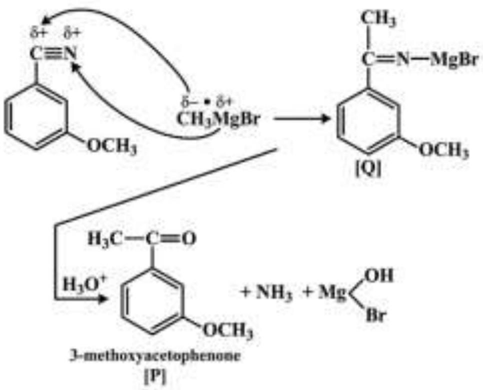
62. **(a)** Aldehydes or ketones with $\alpha - H$ atom, in presence of dilute base, undergoes aldol condensation to give β hydroxy aldehyde or ketone. On heating, aldol eliminate water molecule to form α,β-unsaturated compounds.



63. **(a)** Gabriel's synthesis is used for the preparation of 1° amines.

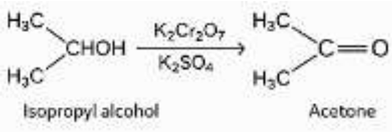


64. **(d)**



65. (d) α-D-glucose and β-glucose, differ in the orientation of -H and -OH groups are first carbon atom. Such isomers are called anomers. Starch is a mixture of amylase and amylopectin but amylase is a straight chain polymer while amylopectin is a branched chain polymer of α-D-glucose.

66. (b) Primary alcohols are easily oxidised to aldehydes and then to acid, both containing the same number of carbon atoms, while secondary alcohols are easily oxidised to ketones with same number of carbon atoms, but ketone oxidised to carboxylic acid containing lesser number of carbon atoms than original alcohol.



$\xrightarrow{[O]} \underset{\text{Acetic acid}}{CH_3COOH} + CO_2 + H_2O$

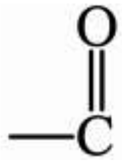
Thus all alcohols on oxidation finally give acids but acids obtained from 2° and 3° alcohols contain less C -atom.

67. (c) As, EDG on benzene increase the basicity and EWG decrease thus when (EWG) is directly attached to benzene nucleus it is, least basic strength.

(1)aromatic amine

(2)aliphatic $-CH_2NH_2$, thus maximum basic strength

(3)aromatic but $-NO_2$ group is (EWG) decreasing electron density at N -atom

(4) 

directly attached hence, least basic strength $4 < 3 < 1 < 2$

68. (a) In general, C_2H_2 and all 1 -alkynes given white precipitate with ammoniacal silver nitrate. Thus, in this question, propyne-1 $(CH_3C = CH)$ will give white precipitate with ammoniacal silver nitrate.

$CH_3C = \underset{\text{Propyne-1}}{CH} + NH_4OH + AgNO_3 \longrightarrow \underset{\text{White ppt.}}{CH_3C = CAg} + NH_4NO_3 + H_2O$

69. (a) For, $2NO(g) \rightleftharpoons N_2 + O_2$

$K_{C_1} = \frac{[N_2][O_2]}{[NO]^2} = 2.4 \times 10^{20}$

For, $NO(g) + \frac{1}{2}Br_2(g) \rightleftharpoons NOBr(g)$

$K_{C_2} = \frac{[NOBr]}{[NO][Br_2]^{1/2}} = 1.4$

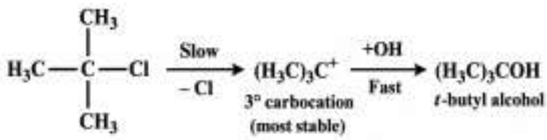
and $K_C = \frac{[NOBr]}{[N_2]^{1/2}[O_2]^{1/2}[Br_2]^{1/2}}$

or $K_C = \sqrt{\frac{1}{K_{C_1}}} \times K_{C_2} = \sqrt{\frac{1}{2.4 \times 10^{20}}} \times 1.4$

$K_C = 8.96 \times 10^{-11}$

70. (a) Most of the gases obey Henry's law when gases do not react with solvent chemically.

71. (a) Tertiary halide preferentially undergo S_N1 substitution as they can give stable carbocation.



72. (a) $He_2^+ : \sigma 1s^2, \sigma 1s^1$

$BO = \frac{1}{2}(N_b - N_a) = \frac{1}{2}(2-1) = \frac{1}{2} = 0.5$

$C_2^{2-} : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, (\pi 2p_x^2 \approx \pi 2p_y^2), \sigma 2p_z^2$

$BO = \frac{1}{2}(10-4) = \frac{6}{2} = 3.0$

$O_2^+ = \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2 \sigma 2p_z^2 (\pi 2p_x^2 \approx \pi 2p_y^2)$

$\pi^*2p_x^1$

$O_2^- = \frac{1}{2}(10-5) = \frac{5}{2} = 2.5$

$BO = \frac{1}{2}(10-7) = \frac{3}{2} = 1.5$

Thus, the correct order of decreasing bond order is

$C_2^{2-} > O_2^+ > O_2^- > He_2^+$

73. **(b)** Given, $\alpha = 6.9\%$

∴ Degree of dissociation,

$\alpha = \frac{6.9}{100}$

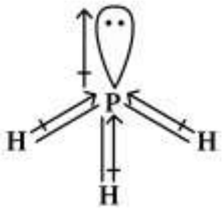
$\alpha = 0.069$

According to Ostwald's dilution law,

$K_a = \alpha^2 C = (0.069)^2 \times 0.2$

$K_a = 9.5 \times 10^{-4}$

74. **(a)** In ammoia,

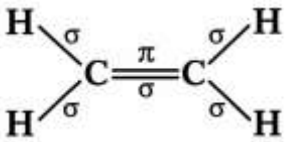


As, nitrogen is more electronegative than hydrogen, the resultant dipole moment adds up in the same directioin.

Dipole moment is a vector quantity.

O_2 and H_2 are homoatomic molecules thus, show no dipole.

75. **(b)** In ethene,



5σ-bond and 1π-bond.

76. **(a)** $pOH = pK_b + \log\frac{[\text{Salt}]}{[\text{Base}]}$

$= 5 + \log\frac{0.01}{0.01} = 5 + \log\frac{1}{10} = 5 + (-1) = 4$

$pH = 14 - pOH = 14 - 4 = 10$

77. **(a)** $[H^+] = 10^{-4}M$

No. of moles of NaOH in 100 mL solution

$= \frac{MV}{1000} = \frac{10^{-4} \times 100}{1000} = 10^{-5}$

Mass of NaOH in solution $= 10^{-5} \times 40 = 4 \times 10^{-4}$ g

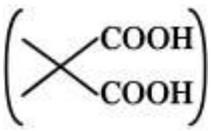
By Faraday's law,

$W = \frac{ItE}{96500}$

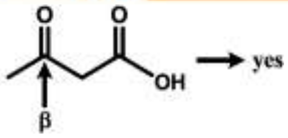
or $4 \times 10^{-4} = \frac{0.5 \times t \times 40}{96500}$

$t = \frac{4 \times 10^{-4} \times 96500}{0.5 \times 40} = 1.93$ s

78. **(c)**Gem dicarboxylic acid like

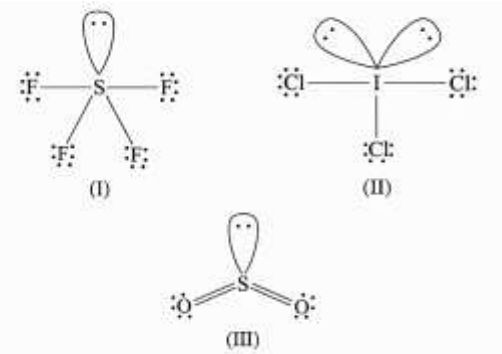


or an acid having a keto group at β-position is decarboxylated on heating.



(a)
(b)yes (c)no (d)yes

79. (d)



(I), (II) and (III) all of them have non-bonding electron (lone pair) on the central atom.

80. (a) For this given reaction, n = 2. At 25°C,

$$\therefore\ K = \text{antilog}\left[\frac{nE^\circ}{0.059}\right] = \text{antilog}\left[\frac{2\times(-0.295)}{0.059}\right]$$

$K = 1\times10^{10}$

81. (b) Negative deviation from Raoult's law are noticed when solvent-solvent and solute-solute interactions are weaker than solute-solvent interactions. In such solutions, the mixing of two components leads to the strengthening of intermolecular attractions and thus results in lesser values of vapour pressure of solution.

∴ Benzene-chloroform mixtures shows negative deviation.

82. (c) The elevation in boiling point is,

$$\Delta T_b = \frac{1000k_b\times w_2}{M_2\times w_1(\text{ in (g)})} \Rightarrow M_2 = \frac{1000\times k_b\times 5}{0.25\times 100}$$

$\Rightarrow M_2 = 20\text{ g}$

83. (a) Electron gain enthalpy (EGE) is the amount of energy released when an electron is added to a neutral atom in the gaseous state to form a negative ion. For non-metallic elements, EGE is typically more negative, reflecting the energy released when these elements gain electrons to attain a stable electronic configuration.

Fluorine (F) is the most electronegative element, which means it has a very high propensity to gain electrons, resulting in the most negative EGE. Oxygen (O), being close to fluorine in the periodic table and also very electronegative, has the next most negative EGE. Carbon (C), a non-metal located in the same period as oxygen, has a less negative EGE than oxygen but still more negative than the metals. Aluminium (Al) and Calcium (Ca) are metals and tend to lose electrons; thus, their EGE values are less negative than those of the non-metals listed. Thus, the correct decreasing order of negative EGE for the given elements is: $F > O > C > Al > Ca$.

84. (b) (I) and (II) are staggered and eclipsed conformers.

85. (a) $Ti(22) = [Ar]4s^2 3d^2$

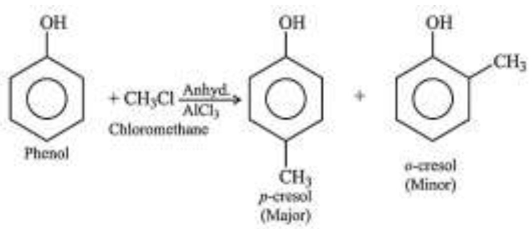
$Ti^{2+} = [Ar]3d^2$

$Ti^{4+} = [Ar]3d^0$

1	1			

Ti^{2+} has two unpaired electrons in 3d orbital and hence $d-d$ transition is possible due to absorption of light in visible region whereas Ti^{4+} is diamagnetic hence colourless.

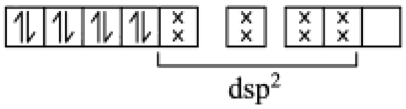
86. (c) The reaction is represented as



87. (a) In $[Ni(CN)_4]^{2-}$ CN^- is strong field ligand and causes pairing of electrons.

$\therefore Ni^{2+} = 3d^8$

$[Ni(CN)_4]^{2-}$



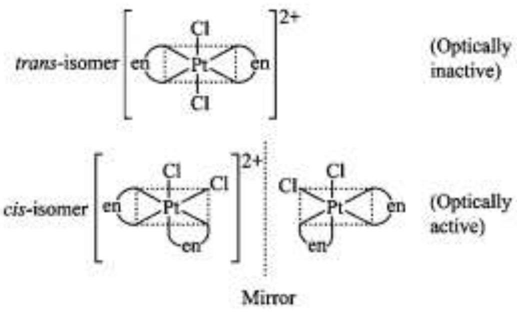
As, all the electrons are paired, hence, it is diamagnetic.

88. (b) Magnesium is an important structural component of chlorophyll molecules. It is present in the tetraphyrrole ring of chlorophyll in its centre.

89. (c) Nickel on heating in a steam of carbon monoxide forms a volatile complex nickel tetracarbonyl.

$Ni + 4CO \xrightarrow{330-350\ K} [Ni(CO)_4]$

90. (d) $[PtCl_2(en)_2]^{2+}$ shows both geometrical and optical isomerism as the complex forms cis and trans isomers. trans-isomer doesn't show optical isomerism since it is symmetrical but cis-isomer shows optical isomerism as is unsymmetrical.



(Non-superimposable mirror image)

91. (b) The balanced equation the reaction is

$\underset{100\ g}{CaCO_3} + \underset{73\ g}{2HCl} \longrightarrow CaCl_2 + H_2O + CO_2$

100 g of $CaCO_3$ completely reacts with 73 g of HCl

So, 15 g of $CaCO_3$ will react with $= \frac{73}{100} \times 15$ g of HCl

$= 10.95$ g of HCl

92. (d) Bohr radius for nth orbit $= 0.53\text{Å} \times \frac{n^2}{Z}$ where, Z = atomic number

$\therefore$ Bohr radius of 2 nd orbit of

$Be^{3+} = \frac{0.53 \times (2)^3}{4} = 0.53\text{Å}$

For option (d) Bohr radius of 1 st orbit of

$H = \frac{0.53 \times (1)^3}{1} = 0.53\text{Å}$

Hence, Bohr's radius of 2nd orbit of Be^{3+} is equal to that of first orbit of hydrogen.

93. (d) We known,

$2.303\log\frac{k_2}{k_1} = \frac{E_a}{R}\left[\frac{T_2 - T_1}{T_1 T_2}\right]$

$\therefore 2.303\log\frac{k_2}{k_1} = \frac{65 \times 10^3}{8.314}\left[\frac{25}{298 \times 273}\right]$

$\frac{k_2}{k} = 11.5$

$\therefore$ The reaction would proceed 11 times faster.

94. (a) The blue colouration is due to the formation of $Fe_4[Fe(CN)_6]_3$ (ferriferrocyanide).

$Fe(OH)_2 + 6NaCN \longrightarrow \underset{\text{Sodium ferrocyanide}}{Na_4[Fe(CN)_6]} + 2NaOH$

$3Na_4[Fe(CN)_6] + 2Fe_2(SO_4)_3 \longrightarrow \underset{\substack{\text{Ferric ferrocyanide} \\ \text{(Prussian bule)}}}{Fe_4[Fe(CN)_6]_3} + 6Na_2SO_4$

95. (b) CN^- is isoelectronic to CO as they have same number of electrons.

Number of electrons in the given species is as follows,

$CO = 6 + 8 = 14$

$O_2^- = 16 + 1 = 17$

$N_2^+ = 14 - 1 = 13$

$O_2^+ = 16 - 1 = 15$

$CN^- = 6 + 7 + 1 = 14$

96. (d) We know that

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{p_2}{p_1}\right)^{n-1}$$

where, n is the order of reaction and $t_{1/2}$ is half-life.

$$\frac{350}{175} = \left(\frac{20}{40}\right)^{n-1}$$

or $2 = \left(\frac{1}{2}\right)^{n-1}$

$\Rightarrow n - 1 = -1$

$n = 0$

∴ It is a zero order reaction.

97. (b) $2C_2H_6(g) + 7O_2(g) \longrightarrow 4CO_2(g) + 6H_2O(l)$;

$\Delta H = -4100$ kJ

Let, $\Delta H_f^\circ(C_2H_6) = x$,

$\Delta H = \Sigma H^\circ_{f(\text{ products })} - \Sigma H^\circ_{f(\text{reactants})}$

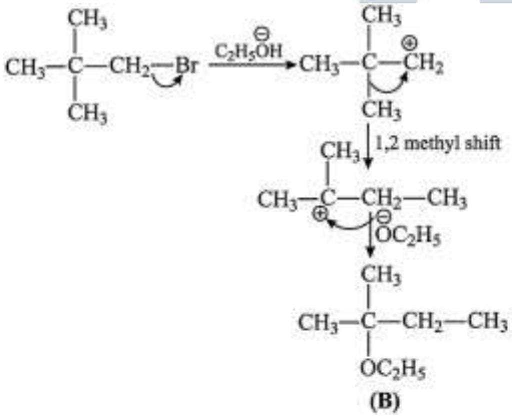
$-4100 = [4(-410) + 6(-285)] - [2x + 0]$

$x = -375$ kJ

98. (c) As the colligative properties depend only upon the number of particles of solute, so if the non-volatile solute dissociates or associates in the solution, the value of colligative properties deviates i.e., abnormal colligative properties are obtained.

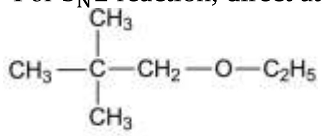
99. (c) As, H_2O is a weak field ligand and thus $[Cu(H_2O)_4]^{2+}$ absorbs red light in visible spectrum. But the colour appears blue because $[Cu(H_2O)_4]^{2+}$ is a labile complex and changes to $[Cu(NH_3)_4]^{2+}$, where NH_3 is strong field ligand thus absorbs yellow light of visible spectrum and impart blue colour.

100. (a) For S_N1 reaction, the stable carbocation formation leads to stable product thus, there is 1, 2-methyl shift.



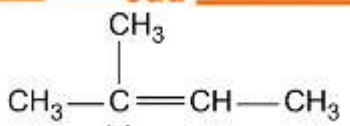
(B)

•For S_N2 reaction, direct attack occurs, thus no rearrangement occurs in the reaction



(A)

•For elimination, more stable alkene is fomred.



(c)

101. **(b)** The rate of reaction for $2A + B \longrightarrow$ products is rate $= k[A]^2[B]$

If concentration of A is doubled,

Rate $' = [2A]^2[B] = 4k[A]^2[B]$

Rate $' = 4$ Rate

102. **(a)** There is no heat exchange between the system and the surrounding for adiabatic change.

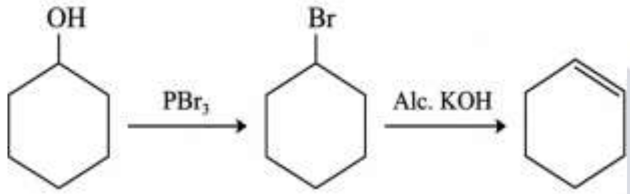
$\therefore\ q = 0$

103. **(b)** In strong base,

$4MnO_4^- + 4OH^- \longrightarrow 4MnO_4^{2-} + O_2 + 2H_2O$

104. **(a)** Octahedral complex having general formula $[M(AA)_2a_2]^{n\pm}$ or $[M(AA)_2ab]^{n\pm}$, shows optical isomersim. cis $-[Co(NH_3)_4Cl_2]^+$does not show optical isomerism due to symmetry, while other three complexes show optical isomerism.

105. **(a)**



106. **(b)** When $FeSO_4 \cdot 7H_2O$ reacts with $(NH_4)_2SO_4$, it forms a double salt known as ferrous ammonium sulphate $(FeSO_4(NH_4)_2SO_4 \cdot 6H_2O)$ or Mohr's salt.

MATHEMATICS

107. **(d)**Let$y = C^{\log_b a}$

$\Rightarrow\ \log_C y = \log_b a$

$\Rightarrow\ \dfrac{\log y}{\log c} = \dfrac{\log a}{\log b}\left[\because \log_a b = \dfrac{\log b}{\log a}\right]$

$\Rightarrow\ \dfrac{\log y}{\log a} = \dfrac{\log c}{\log b} \Rightarrow \log_a y = \log_b c$

$\Rightarrow\ y = a^{\log_b c}\ [\because \log_a x = y \Rightarrow a^y = x]$

$\therefore\ a^{\log_b c} - c^{\log_b a} = a^{\log_b c} - a^{\log_b c}$

$= 0$

108. **(c)** Given curve, $y = x^2 - xy$.

On differentiating the equation, $y = x^2 - xy$ w.r.t. x, we

get $\dfrac{dy}{dx} = 2x - \left(x\dfrac{dy}{dx} + y\right)$

$\Rightarrow\ (1 + x)\dfrac{dy}{dx} = 2x - y \Rightarrow \dfrac{dy}{dx} = \dfrac{2x - y}{1 + x}$

$\Rightarrow \left(\dfrac{dy}{dx}\right)_{(1,1/2)} = \dfrac{2(1) - \left(\frac{1}{2}\right)}{1 + (1)} = \dfrac{3/2}{2} = \dfrac{3}{4}$

109. **(b)** $\lim_{x\to 0} \frac{e^{ax} - e^{bx}}{2x}$

$$= \lim_{x\to 0} \frac{\left(1 + ax + \frac{(ax)^2}{2!} + \frac{(ax)^3}{3!} + \cdots\right) - \left(1 + bx + \frac{(bx)^2}{2!} + \frac{(bx)^3}{3!}\right)}{2x}$$

$$= \lim_{x\to 0} \frac{x\left[(a-b)+\left(\frac{a^2x}{2!}+\frac{a^3x^2}{3!}+\cdots\right)+\left(\frac{b^2x}{2!}+\frac{b^3x^2}{3!}+\cdots\right)\right]}{2x}$$

$$= \frac{1}{2}[(a-b)+(0+0+\cdots)+(0+0+\cdots)]$$

$$= \frac{a-b}{2}$$

110. **(b)** Given, $(x+1)^2+y^2=4 \ldots.(i)$

and $(x-1)^2+y^2=9 \ldots.(ii)$

On subtracting Eq. (ii) from Eq.(i),we get

$(x+1)^2-(x-1)^2=4-9$

$\Rightarrow (x^2+2x+1)-(x^2-2x+1)=-5$

$\Rightarrow 4x=-5$

$\Rightarrow x=-1.25$

On putting, $x=-1.25$ into Eq.(i),we get

$(-0.25)^2+y^2=4$

$\Rightarrow y^2=3.9375 \Rightarrow y=\pm\sqrt{3.9375}$

$\Rightarrow y=\pm\frac{3}{4}\sqrt{7}$

$\therefore a=-1.25$ and $b=\frac{3}{4}\sqrt{7}$

$\therefore (a,b)=\left(-1.25,\frac{3}{4}\sqrt{7}\right)$

111. **(a)** First, break the number 5.001 as $x=5$ and $\Delta x=0.001$ and use the relation

$f(x+\Delta x)\approx f(x)+\Delta x f'(x)$

consider $f(x)=x^3-7x^2+10 \Rightarrow f'(x)=3x^2-14x$

Therefore,

$f(x+\Delta x)\approx (x^3-7x^2+10)+\Delta x(3x^2-14x)$

$\Rightarrow f(5.001)\approx (5^3-7(5)^2+10)+(0.001)(3(5)^2-14(5))$

$=(125-175+10)+(0.001)(75-70)$

$=-40+(0.001)(5)=-40+0.005=-39.995$

112. **(d)** Given equation of circle is $x^2+y^2+3x-y+2=0$.

On comparing this equation with $x^2+y^2+2gx+2fy+c=0$, we get $g=\frac{3}{2}, f=-\frac{1}{2}$ and $c=2$

Now, the length of intercept on X -axis $=2\sqrt{g^2-c}$

$$=2\sqrt{\left(\frac{3}{2}\right)^2-2}=2\sqrt{\frac{9}{4}-2}=2\sqrt{\frac{1}{4}}=2\left(\frac{1}{2}\right)=1$$

113. **(b)** $\because f(x)=a+(x-4)^{4/9}$

$\therefore f'(x)=0+\frac{4}{9}(x-4)^{-5/9}$

Clearly, at $x=4, f'(x)$ is not defined

Hence, $x=4$ is the point of extremum.

$\because f(4)=a+(4-4)^{4/9}=a$

$\therefore$ The minimum value of $f(x)$ is a.

114. **(d)** At, $x=\frac{\pi}{2}$

LHL $=\lim_{x\to\pi/2^-}(a\sin x+b)=a+b$

RHL $=\lim_{x\to\pi/2^+}(\cos x)=0$

Since, $f(x)$ is continuous at $x=\pi/2$

$\therefore a+b=0 \qquad \ldots.(i)$

At, $x=-\frac{\pi}{2}$

LHL $= \lim_{x\to-\pi/2^-}(2\sin x) = -2$

RHL $= \lim_{x\to-\pi/2^+}(a\sin x + b) = -a + b$

Since, $f(x)$ is continuous at $x = -\pi/2$

$\therefore\ -a + b = -2 \ldots$ (ii)

On solving Eqs. (i) and (ii), we get $a = 1$ and $b = -1$

115. **(b)** Let $L = \lim_{x\to\infty}\left(\frac{x^2-2x+1}{x^2-4x+2}\right)^{2x}$

$$\Rightarrow \ln L = \lim_{x\to\infty}\left[2x\ln\left(\frac{x^2-2x+1}{x^2-4x+2}\right)\right]$$

$$\Rightarrow \ln L = \lim_{x\to\infty}\left[2x\ln\left(1+\frac{2x-1}{x^2-4x+2}\right)\right]$$

$$\Rightarrow \ln L = \lim_{x\to\infty}\left[2x\left(\frac{2x-1}{x^2-4x+2}-\frac{\left(\frac{2x-1}{x^2-4x+2}\right)^2}{2}+\frac{\left(\frac{2x-1}{x^2-4x+2}\right)^3}{3}\right)\cdots\right]$$

$$\Rightarrow \ln L = \left(\lim_{x\to\infty}\frac{2x(2x-1)}{x^2-4x+2}\right) - 0 + 0\cdots$$

$$\Rightarrow \ln L = \lim_{x\to\infty}\frac{4x^2\left(1-\frac{1}{2x}\right)}{x^2\left(1-\frac{4}{x}+\frac{2}{x}\right)}$$

$$\ln L = \frac{4(1-0)}{(1-0+0)}$$

$\Rightarrow\ \ln L = 4$

$\Rightarrow\ L = e^4$

116. **(a)** $S(-2,-1) = (-2)^2 + (-1)^2 - 2(-2) - 4(-1) - 4$

$= 4 + 1 + 4 + 4 - 4 = 9 > 0$

$\therefore (-2,-1)$ lies outside of S

$S(-2,-1) = (-2)^2 + (-1)^2 - 4(-2) - 2(-1) - 16$

$= 4 + 1 + 8 + 2 - 16 = -1 < 0$

$\therefore (-2,-1)$ lies inside of S'

Thus, $(-2,-1)$ lies inside S' only.

117. **(b)** Given, $S = \{1,2,3 \ldots,50\}$

$A + \left\{n \in S: n + \frac{50}{n} > 27\right\}$

$= \{n \in S: n^2 - 27n + 50 > 0\}$

$= \{n \in S: (n-25)(n-2) > 0\}$

$= \{n \in S: n < 2 \text{ or } n > 25\}$

$= \{1,26,27,28,\ldots,50\}$

$\Rightarrow n(A) = 26$

$B = \{n \in S: n \text{ is prime }\}$

$= \{2,3,5,7,11,13,17,19,23,29,31,37,41,43,47\}$

$\Rightarrow n(B) = 15$

$C = \{n \in S: n \text{ is a square }\} = \{1,4,9,16,25,36,49\}$

$\Rightarrow n(C) = 7$

$\therefore p(A) = \frac{n(A)}{n(S)} = \frac{26}{50}$

$\Rightarrow p(B) = \frac{n(B)}{n(S)} = \frac{15}{50}$

$p(C) = \frac{n(C)}{n(S)} = \frac{7}{50}$

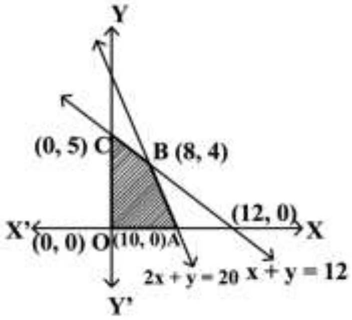
$\therefore p(A) > p(B) > p(C)$

118. **(a)** The given inequations are $x+2y \geq 4, 2x+y \leq 6, x, y \leq 0$.

According to the inequtions $x, y \geq 0$, the feasible region be the first quadrant (including positive X and positive Y-axis). According to the inequation $x+2y \geq 4$, the feasible region be the region above or on the line $x+2y=4$. According to the inequation $2x+y \leq 6$, the feasible region be the region below or on the line $2x+y=6$. Now, he common feasible region will be the required feasible region.

119. **(b)** Given, constraints are $x \geq 0, y \geq 0, x+y \leq 12, 2x+y \leq 20$

The feasible region is OABCO.



$\because Z = 10x + 16y$

At, $O(0,0), Z = 10(0) + 16(0) = 0$

At, $A(10,0), Z = 10(10) + 16(0) = 100$

At, $B(8,4) Z = 10(8) + 16(4) = 144$

At, $C(0,12), Z = 10(0) + 16(12) = 192$

Hence, the maximum value of Z is 192 .

120. **(b)** Given, $A = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{bmatrix}$

$\therefore |A| = \begin{bmatrix} 2 & 2 \\ 3 & 4 \end{vmatrix} = 2 \times 4 - 3 \times 2 = 8 - 6 = 2$

Now, $A_{11} = 4, A_{12} = -3, A_{21} = -2$ and $A_{22} = 2$

$\therefore \operatorname{adj}_A = \begin{bmatrix} 4 & -3 \\ -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \operatorname{adj} A = \frac{1}{2} \begin{bmatrix} 4 & -2 \\ -3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 \\ -3/2 & 1 \end{bmatrix}$

121. **(c)** Given, $A(\operatorname{adj} A) = 10I$

We know that $A(\operatorname{adj} A) = |A|I$

$\therefore 10I = |A|I$

$\Rightarrow |A| = 10$

We know that $|\operatorname{adj} A| = |A|^{n-1}$, where n is order of A

$\therefore |\operatorname{adj} A| = |A|^{4-1} = 10^3 = 1000$

122. **(c)** Given, $A = \begin{bmatrix} k+1 & 2 \\ 4 & k-1 \end{bmatrix}$ is a singular matrix.

$\therefore |A| = 0$

$\Rightarrow \begin{vmatrix} k+1 & 2 \\ 4 & k-1 \end{vmatrix} = 0$

$\Rightarrow (k+1)(k-1) - 4 \times 2 = 0$

$\Rightarrow k^2 - 1 - 8 = 0$

$\Rightarrow k^2 - 9 = 0$

$\Rightarrow k^2 = 9$

$\Rightarrow k = \pm 3$

123. **(d)** We have, $a = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and $b = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

Clearly, $|a| = \sqrt{1+4+4} = \sqrt{9} = 3$

and $|b| = \sqrt{1+4+4} = \sqrt{9} = 3$

$\because a \cdot b = |a||b| \cos\theta$

$\Rightarrow\ \cos\theta = \dfrac{1-1+2\cdot 2+2\cdot(-2)}{3\times 3}$

$\Rightarrow\ \cos\theta = \dfrac{1+4-4}{9} \Rightarrow \cos\theta = \dfrac{1}{9}$

$\therefore\ \theta = \cos^{-1}\left(\dfrac{1}{9}\right)$

124. **(b)** Since, vectors a, b and c are coplanar

$\therefore [abc] = 0 \Rightarrow a\cdot(b\times c) = 0$

Now,

$$b\times c = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ m & -1 & 2 \end{vmatrix}$$

$= \hat{i}(4-1) - \hat{j}(2+m) + \hat{k}(-1-2m)$

$= 3\hat{i} - (2+m)\hat{j} - (1+2m)\hat{k}$

and

$a\cdot(b\times c) = (2\hat{i} - 3\hat{j} + 4\hat{k})\cdot(3\hat{i} - (2+m)\hat{j} - (1+2m)\hat{k})$

$= 2(3) + 3(2+m) - 4(1+2m)$

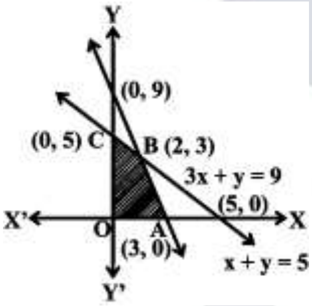
$= 6 + 6 + 3m - 4 - 8m = 8 - 5m$

$\therefore\ a\cdot(b\times c) = 0$

$\therefore\ 8 - 5m = 0 \Rightarrow m = \dfrac{8}{5}$

125. **(b)** Given constraints are $x \geq 0, y \geq 0, x + y \leq 5$ and $3x + y \leq 9$ and $z = 12x + 13y$

The feasible region is OABCO.



$\because Z = 12x + 13y$

At, $O(0,0), Z = 12(0) + 13(0) = 0$

At, $A(3,0), Z = 12(3) + 13(0) = 36$

At, $B(2,3), Z = 12(2) + 13(3) = 63$

At, $C(0,5), Z = 12(0) + 13(5) = 65$

Here, maximum value of Z is 65 .

126. **(a)** Given, $a = 2\hat{i} + \hat{j} - \hat{k}, b = \hat{i} - \hat{j}$ and $c = 5\hat{i} - \hat{j} + \hat{k}$

$\therefore a + b - c = (2 + 1 - 5)\hat{i} + (1 - 1 + 1)\hat{j} + (-1 + 0 - 1)\hat{k}$

$= -2\hat{i} + \hat{j} - 2\hat{k} = -(2\hat{i} - \hat{j} + 2\hat{k})$

Now, the unit vector in the direction of $a + b - c$ be

$\dfrac{-(2\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{2^2 + (-1)^2 + 2^2}} = -\dfrac{1}{3}(2\hat{i} - \hat{j} + 2\hat{k})$

$\therefore$ The required unit vector be $\frac{1}{3}(2\hat{i} - \hat{j} + 2\hat{k})$.

127. **(a)** Consider the equation of line given in option (a). The DR's of this line or (4,5,6).

We know that if the line $\frac{x-x_0}{a_1} = \frac{y-y_0}{b_1} = \frac{z-z_0}{c_1}$ is parallel to the plane $a_2x + b_2y + c_2z + d = 0$, then $a_1a_2 + b_1b_2 + c_1c_2 = 0$, that is the normal to the plane is perpendicular to the line.

Here, the vector $\hat{i} - 2\hat{j} + \hat{k}$ is normal to the plane $x - 2y + z = 0$ and $4(1) + 5(-2) + 6(1) = 4 - 10 + 6 = 10 - 10 = 0$

128. **(a)** Let $I = \int \frac{xdx}{2(1+x)^{3/2}}$

On putting, $1 + x = t$, we get $dx = dt$

$\therefore\ I = \int \frac{(t-1)dt}{2t^{3/2}} = \frac{1}{2}\left[\int t^{-1/2}dt - \int t^{-3/2}dt\right]$

$= \frac{1}{2}\left[\frac{t^{\frac{1}{2}}}{\frac{1}{2}} - \frac{t^{-\frac{1}{2}}}{-\frac{1}{2}}\right] + C$

$= \frac{1}{2} \times 2\left[\sqrt{t} + \frac{1}{\sqrt{t}}\right] + C$

$= \frac{t+1}{\sqrt{t}} + C = \frac{x+2}{\sqrt{1+x}} + C$

129. **(c)** Let $I = \int \frac{4^x}{\sqrt{1-16^x}}dx$

$= \int \frac{4^x}{\sqrt{1-(4^x)^2}}dx$

On putting, $4^x = t$, we get $4^x \log 4 dx = dt$

$\Rightarrow\ 4^x dx = \frac{dt}{\log 4}$

$\therefore\ I = \frac{1}{\log 4}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{\log 4}\sin^{-1}t + C$

$= \frac{1}{\log 4}\sin^{-1}4^x + C$

130. **(c)** Let $f(x) = \sin^2 x$

Now,$f(-x) = \sin^2(-x) = (\sin(-x))^2$

$= (-\sin x)^2 = \sin^2 x = f(x)$

So, f is an even function

$\therefore \int_{-\pi/2}^{\pi/2} \sin^2 x = 2\int_0^{\pi/2} \sin^2 x dx$

$\left[\because \int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx, \text{ if f is even}\right]$

$2\int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2}dx = \left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$

$=\left(\frac{\pi}{2} - \frac{\sin 2\times\pi/2}{2}\right) - \left(0 - \frac{\sin(2\times 0)}{2}\right)$

$= \left(\frac{\pi}{2} - 0\right) - (0-0) = \frac{\pi}{2}$

131. **(b)** Let $L_1: \frac{x-1}{2} = \frac{y-4}{4} = \frac{z-2}{3}$

and $L_2 = \frac{1-x}{1} = \frac{y-2}{5} = \frac{3-z}{a}$

the line L_2 can be written as $\frac{x-1}{-1} = \frac{y-2}{5} = \frac{z-3}{-a}$

Now, the DR's of lines L_1 and L_2 are $(2,4,3)$ and $(-1,5,-a)$ respectively.

Since, L_1 and L_2 are perpendicular to each other.

$\therefore 2(-1) + 4)(5) + 3(-a) = 0$

$\Rightarrow\ -2 + 20 - 3a = 0$

$\Rightarrow\ -3a = -18 \Rightarrow a = 6$

132. **(a)** Let $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda$

Now, any point P that lies on the lines L_1 has the form $(1+2\lambda, -1+3\lambda, 1+4\lambda)$.

Now, on putting $x = 1+2\lambda, y = -1+3\lambda$ and $z = 1+4\lambda$ into the equation of lines L_2, we get

$\frac{1+2\lambda-3}{1} = \frac{-1+3\lambda-k}{2} = 1+4\lambda$

$\Rightarrow \dfrac{1+2\lambda-3}{1}=1+4\lambda$

$\Rightarrow -2\lambda=3 \Rightarrow \lambda=\dfrac{-3}{2}$

and $\dfrac{-1+3\lambda-k}{2}=1+4\lambda$

$\Rightarrow -1+3\lambda-k=2+8\lambda$

$\Rightarrow -5\lambda=3+k$

$\Rightarrow -5\left(-\dfrac{3}{2}\right)=3+k\ [\because \lambda=-3/2]$

$\Rightarrow k=\dfrac{15}{2}-3$

$\Rightarrow k=\dfrac{9}{2}$

$\Rightarrow 2k=9$

133. **(a)** The total number of possible five-digit numbers $=5!$

The total number of possible five-digit numbers in which 1 and 5 are always together $=2\times 4!$

$\therefore$ Required probability $=\frac{2\times 4!}{5!}=\frac{2\times 4!}{5\times 4!}=\frac{2}{5}$

134. **(c)** Here, number which are divisible by either 3 or 5 are $12,15,18,20,21,24,27,30$.

$\therefore$ Total numbers $=9$

P (number either divisible by 3 or 5) $=\frac{9}{20}$

P (number neither divisible by 3 nor 5)

$=1-$ P (number either divisible by 3 or 5)

$=1-\dfrac{9}{20}=\dfrac{11}{20}$

135. **(b)** Number of triangles formed $= {}^9C_3$

Number of isosceles triangles $=9\times\left(\frac{9-1}{2}\right)$

$=9\times 4=36$

So, required probability

$=\dfrac{36}{{}^9C_3}=\dfrac{36}{\frac{9!}{3!\,(9-3)!}}=\dfrac{36\times 3\times 2\times 6!}{9\times 8\times 7\times 6!}=\dfrac{3}{7}$

136. **(d)** Given, $P(B)=\frac{3}{2}P(A)$ and $P(C)=\frac{1}{2}P(B)$

Since, A, B and C are mutually exclusive and exhaustive events.

$\therefore\ P(A)+P(B)+P(C)=1$

$\Rightarrow\ P(A)+\dfrac{3}{2}P(A)+\dfrac{1}{2}\times\dfrac{3}{2}P(A)=1$

$\Rightarrow\ P(A)\left(1+\dfrac{3}{2}+\dfrac{3}{4}\right)=1$

$\Rightarrow\ P(A)\left(\dfrac{13}{4}\right)=1 \Rightarrow P(A)=\dfrac{4}{13}$

$\therefore\ P(C)=\dfrac{1}{2}\times\dfrac{3}{2}P(A)=\dfrac{3}{4}\times\dfrac{4}{13}=\dfrac{3}{13}$

Also, A, B and C are mutually exclusive.

$\therefore P(A\cap B)=P(B\cap C)=P(C\cap A)=0$

Now, $P(A\cup C)=P(A)+P(C)-P(A\cap C)$

$=\dfrac{4}{13}+\dfrac{3}{13}-0=\dfrac{7}{13}$

137. **(b)** Given, $a_0=1, a_{n+1}=3n^2+n+a_n$

$\Rightarrow\ a_1=3(0)^2+(0)+a_0=0+0+1=1$

$\Rightarrow\ a_2=3(1)^2+(1)a_1=3+1+1=5$

From option (b),

Let $P(n)=n^3-n^2+1$
$P(0)=(0)^3-(0)^2+1=1=a_0$
$P(1)=(1)^3-(1)^2+1=1-1+1=a_1$
$P(2)=(2)^3-(2)^2+1=8-4+1=5=a_2$
Thus, $a_n=n^3-n^2+1$

138. **(a)** Let $P(n)=49^n+16^n+k$
For $n=1$, we get
$P(1)=49^{(1)}+16^{(1)}+k=65+k$
As $P(1)$ is divisible by 64 , we take
$k=-1$
$\therefore$ $P(1)=65-1=64$, which is divisible by 64.
Thus, the least negative integral value of k be -1 .

139. **(c)** Let $P(n)=2^{3n}-7n-1$
$\Rightarrow$ $P(1)=2^{3(1)}-7(1)-1=8-8=0$
$\Rightarrow$ $P(2)=2^{3(2)}-7(2)-1=64-15=49$
$P(1)$ and $P(2)$ are divisible by 49 .
Let $P(k)=2^{3k}-7k-1=49t$, where t is an integer
Now,
$P(k+1)=2^{3(k+1)}-7(k+1)-1=2^{3k}\cdot 2^3-7k-7-1$
$=8\left(2^{3k}-7k-1\right)+49k$
$=8(49t)+49k$
$=49(8t+k)$, where $8t+k$ is an integer
Thus, $2^{3n}-7n-1$ is divisible by 49 .

140. **(b)** Given series is
$\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+\cdots$
The sum of the given series upto n-terms
$\frac{4}{3}+\frac{10}{9}+\frac{28}{27}+\cdots$ upto n-terms
$=\left(1+\frac{1}{3}\right)+\left(1+\frac{1}{9}\right)+\left(1+\frac{1}{27}\right)+\cdots$ upto n-terms
$=(1+1+1+\cdots$ upto n-terms $)$
$+\left(\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\cdots \text{ upto n-terms }\right)$
$=n+\frac{1}{3}\left(\frac{1-\frac{1}{3^n}}{1-\frac{1}{3}}\right)=n+\frac{(3^n-1)}{2(3^n)}=\frac{3^n(2n+1)-1}{2(3^n)}$

141. **(a)** Given, $\frac{1}{2!}+\frac{2}{3!}+\cdots+\frac{99}{100!}$
$=\frac{2-1}{2!}+\frac{3-1}{3!}+\frac{4-1}{4!}+\cdots+\frac{100-1}{100!}$
$=\left(\frac{1}{1!}-\frac{1}{2!}\right)+\left(\frac{1}{2!}-\frac{1}{3!}\right)+\left(\frac{1}{3!}-\frac{1}{4!}\right)+\cdots+\left(\frac{1}{99!}-\frac{1}{100!}\right)$
$=1-\frac{1}{100!}=\frac{100!-1}{100!}$

142. **(b)** Here, $T_{12}=a+11d$ and $T_{22}=a+21d$
Since, $100=T_{12}+T_{22}$
$\therefore$ $100=a+11d+a+21d$
$\Rightarrow a+16d=50$ ... (i)
Now, $S_{33}=\frac{33}{2}[2a+(33-1)d]$
$=33(a+16d)=33\times 50$
$=1650$ [From Eq. (i)]
Thus, required sum be 1650.

143. **(a)** The general equation of all non-vertical lines in a plane is $ax+by=1$, where $b\neq 0$.
On differentiating both sides w.r.t. x, we get,$a+b\frac{dy}{dx}=0$

Again, differentiating w.r.t x, we get,$b\frac{d^2y}{dx^2}$

$\Rightarrow \frac{d^2y}{dx^2} = 0\ [\because b \neq 0]$.

144. **(a)** Given, $\left(\frac{dy}{dx}\right)^2 = 1 - x^2 - y^2 + x^2y^2$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = (1-y^2) - x^2(1-y^2) = (1-x^2)(1-y^2)$

$\therefore \frac{dy}{dx} = \sqrt{(1-x^2)(1-y^2)}$

$\Rightarrow \frac{dy}{\sqrt{1-y^2}} = \sqrt{1-x^2}dx$

On integrating both sides, we get

$\int \frac{dy}{\sqrt{1-y^2}} = \int \sqrt{1-x^2}dx$

$\Rightarrow \sin^{-1}y = \frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x + \frac{C}{2}$

$\Rightarrow 2\sin^{-1}y = x\sqrt{1-x^2} + \sin^{-1}x + C$

145. **(c)** Given, differential equation is

$\left(\frac{dy}{dx}\right)\tan y = \sin(x+y) + \sin(x-y)$

$\Rightarrow \left(\frac{dy}{dx}\right)\tan y = 2\sin\left(\frac{x+y+x-y}{2}\right)$

$\Rightarrow \left(\frac{dy}{dx}\right)\frac{\sin y}{\cos y} = 2\sin x\cos y$

$\Rightarrow \frac{\sin y}{\cos^2 y}dy = 2\sin x dx$

On integration both sides, we get

$\int \frac{\sin y}{\cos^2 y}dy = \int 2\sin x dx$

$\Rightarrow -\frac{(\cos y)^{-2+1}}{(-2+1)} = -2\cos x + C$

$\Rightarrow \frac{1}{\cos y} = -2\cos x + C \Rightarrow \sec y = -2\cos x + C$

146. **(c)** We know that if ${}^nC_x = {}^nC_y$, then either $x = y$

or $x + y = n$

Since, ${}^nC_{10} = {}^nC_{12}$

$\therefore 10 + 12 = n$

$\Rightarrow n = 22$

Now, ${}^nC_{21} = {}^{22}C_{21}$

$= \frac{22!}{(22-21)!\,21!} = \frac{22!}{1!\,21!} = \frac{22 \times 21!}{1 \times 21!} = 22$

147. **(d)** Let the probability of failure and success be p and q, respectively.

Let X represents the number of failure

According to the question, $q = 2p$

$\because p + q = 1$ and $q = 2p$

$\because p = \frac{1}{3}$ and $q = \frac{2}{3}$

Now, required probability $= P(X \leq 2)$

$= P(X=0) + P(X=1) + P(X=2)$

$= {}^6C_0p^0q^6 + {}^6C_1p^1q^5 + {}^6C_2p^2q^4$

$= \left(\frac{2}{3}\right)^6 + 6\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^5 + 15\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4$

$$= \frac{1}{729}(64 + 192 + 240) = \frac{496}{729}$$

148. **(c)** Given, $\cos A = m\cos B \Rightarrow \frac{\cos A}{\cos B} = m$

On applying componendo and dividendo rule, we get

$$\frac{\cos A + \cos B}{\cos A - \cos B} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{B-A}{2}\right)} = \frac{m+1}{m-1}$$

$$\Rightarrow \frac{\cot\left(\frac{A+B}{2}\right)}{\tan\left(\frac{B-A}{2}\right)} = \frac{m+1}{m-1}$$

$$\Rightarrow \cot\left(\frac{A+B}{2}\right) = \left(\frac{m+1}{m-1}\right)\tan\left(\frac{B-A}{2}\right)$$

$$\therefore \lambda = \frac{m+1}{m-1}$$

149. **(b)** Given, $\frac{2\tan A}{1-\cot A} + \frac{2\cot A}{1-\tan A}$

$$= \frac{\frac{2\sin A}{\cos A}}{1 - \frac{\cos A}{\sin A}} + \frac{\frac{2\cos A}{\sin A}}{1 - \frac{\sin A}{\cos A}}$$

$$= \left(\frac{\sin A}{\cos A}\right)\left(\frac{2\sin A}{\sin A - \cos A}\right) + \left(\frac{\cos A}{\sin A}\right)\left(\frac{2\cos A}{\cos A - \sin A}\right)$$

$$= \frac{2}{\sin A - \cos A}\left[\frac{\sin^2 A}{\cos A} - \frac{\cos^2 A}{\sin A}\right]$$

$$= \frac{2}{\sin A - \cos A}\left[\frac{\sin^3 A - \cos^3 A}{\sin A\cos A}\right]$$

$$= \frac{2(\sin A - \cos A)(\sin^2 A + \cos^2 A + \sin A\cos A)}{(\sin A - \cos A)(\sin A\cos A)}$$

$$= 2\left(\frac{1}{\sin A\cos A} + 1\right)$$

$$= 2(\sec A\operatorname{cosec} A + 1) = 2\sec A\operatorname{cosec} A + 2$$

150. **(b)** Given, $2\cos 4x + \sin^2 2x = 0$

$$\Rightarrow 2\cos 4x + \left(\frac{1 - \cos 4x}{2}\right) = 0$$

$$\Rightarrow 3\cos 4x + 1 = 0$$

$$\Rightarrow \cos 4x = -\frac{1}{3}$$

$$\Rightarrow \sin 4x = \pm\frac{2\sqrt{2}}{3}$$

$$\Rightarrow 4x = n\pi + (-1)^n \sin^{-1}\left(\pm\frac{2\sqrt{2}}{3}\right)$$

$$\Rightarrow x = \frac{n\pi}{4} + \frac{(-1)^n}{4}\sin^{-1}\left(\pm\frac{2\sqrt{2}}{3}\right)$$

151. **(a)** Given, $2f(x^2) + 3f\left(\frac{1}{x^2}\right) = x^2 - 1$.....(i)

Replacing x by $\frac{1}{x}$, we get

$2f\left(\frac{1}{x^2}\right) + 3f(x^2) = \frac{1}{x^2} - 1$......(ii)

On multiplying Eq. (i) by 2, Eq. (ii) by 3 and then subtracting Eq. (i) from Eq. (ii), we get

$$5f(x^2) = 3\left(\frac{1}{x^2} - 1\right) - 2(x^2 - 1)$$

$\Rightarrow 5f(x^2) = \frac{3}{x^2} - 2x^2 - 1$

$\Rightarrow f(x^2) = \frac{1}{5}\left(\frac{3}{x^2} - 2x^2 - 1\right)$

$\Rightarrow f(x^2) = \frac{1}{5x^2}(3 - 2x^4 - x^2)$

$\Rightarrow f(x^2) = \frac{(2x^2+3)(1-x^2)}{5x^2}$

$\therefore f(x^8) = \frac{(1-x^8)(2x^8+3)}{5x^8}$

152. **(a)** Given, $A = \{a, bc\}, B = \{b, c, d\}$ and $C = \{a, d, c\}$

Now, $A - B = \{a, b, c\} - \{b, c, d\} = \{a\}$

and $B \cap C = \{b, c, d\} \cap \{a, d, c\} = \{c, d\}$

$\therefore (A - B) \times (B \cap C) = \{a\} \times \{c, d\} = \{(a, c), (a, d)\}$

153. **(b)** Given; $n(A) = p$ and $n(B) = q$

$\therefore n(A \times B) = pq$

The number of relations from a set A to a set B is same as the total number of subset of the set $A \times B$.

We know that if $n(A) = k$, then $n(P(A)) = 2^k$

Now, the total number of subset of $A \times B$ be 2^{pq}

$\therefore$ Then number of relations from the set A to the set B is 2^{pq}.

154. **(b)** Given, $Z = \sqrt{3} + i$

$\therefore \arg(z^2 e^{z-i}) = \arg\left[(\sqrt{3}+i)^2 e^{\sqrt{3}+i)-i}\right]$

$= \arg\left[(2 + 2\sqrt{3}i)e^{\sqrt{3}}\right]$

$= \arg\left[2e^{\sqrt{3}}(1 + \sqrt{3}i)\right]$

$= \arg[(1 + \sqrt{3}i)]$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \tan^{-1}\left(\tan\frac{\pi}{3}\right) = \frac{\pi}{3}$

155. **(d)** Given, $i^n + i^{n+1} + i^{n+2} + i^{n+3} = i^n(1 + i + i^2 + i^3)$

$= i^n(1 + i + (-1) + (i^2)i)$

$= i^n(1 + i + (-1) + (-1)i)$

$= i^n[(1 + i) - (1 + i)] = i^n(0) = 0$

156. **(d)** We have, $\left(\frac{3}{2} + i - \frac{\sqrt{3}}{2}\right)^{50} = 3^{25}(x + iy)$

$\Rightarrow (\sqrt{3})^{50}\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)^{50} = 3^{25}(x + iy)$

$\Rightarrow 3^{25}\left[-i\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\right]^{50} = 3^{25}(x + iy)$

$\Rightarrow (-i)^{50}\omega^{50} = x +. iy$

$\Rightarrow (i^4)^{12} \cdot i^2 \cdot (\omega^3)^{16} \cdot \omega^2 = x + iy$

$\Rightarrow (1)^{12} \cdot (-1) \cdot (1)^{16} \cdot \left(-\frac{1}{2} - i\frac{\sqrt{3}}{2}\right) = x + iy$

$\Rightarrow \frac{1}{2} + i\frac{\sqrt{3}}{2} = x + iy$

$\Rightarrow 1 + i\sqrt{3} = 2x + i(2y)$

$\therefore (2x, 2y) = (1, \sqrt{3})$

157. **(b)** From 10 given points, ${}^{10}C_2$ straight lines can be drawn.

But 4 points are collinear, using 4 points, ${}^4C_2$ straight lines can be drawn.

From 4 col linear points, 1 straight line can be drawn. So, total number of straight lines $= {}^{10}C_2 - {}^4C_2 + 1$

$= \frac{10!}{8!\,2!} - \frac{4!}{2!\,2!} + 1$

$= 45 - 6 + 1 = 40$

158. **(c)** Since, numbers should be greater than 1000 but less than 4000 .

$\therefore$ The first digit: must be either 2 or 3 .

It is clear that required numbers must be 4 digit numbers.

Now, there are four choices (0,2,3,4), for each unit, ten and hundred place digit.

Thus, total number $= {}^2C_1 \times 4 \times 4 \times 4$

$= 2 \times 4 \times 4 \times 4 = 128$

159. **(c)** The total number of lines joining any two points of the polygon is given by ${}^nC_2$

So, ${}^nC_2 = 105$

$\Rightarrow \dfrac{n(n-1)}{2} = 105$

$\Rightarrow n^2 - n = 210$

$\Rightarrow n^2 - n - 210 = 0$

$\Rightarrow n^2 - 15n + 14n - 210 = 0$

$\Rightarrow n(n-15) + 14(n-15) = 0$

$\Rightarrow (n-15)(n+14) = 0$

either $n - 15 = 0$ or $n + 14 = 0$

$\Rightarrow n = 15$ or -14

$\because$ Number of sides cannot be negative

$\therefore n = 15$

160. **(d)** Equation of angles of bisector of pair of straight line, $ax^2 + 2bxy + by^2$ is $\frac{x^2-y^2}{a-b} = \frac{xy}{h}$

$\therefore$ For, $3y^2 - 5xy - 2x^2 = 0$

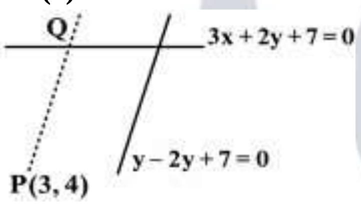
$a = 3, b = -2, h = -5$

So, equation of angle bisector is

$\dfrac{x^2 - y^2}{3-(-2)} = \dfrac{xy}{-5}$

$\Rightarrow \dfrac{x^2 - y^2}{5} = \dfrac{xy}{-5} \Rightarrow x^2 - y^2 + xy = 0$

161. **(a)**



The slope of the line, $y - 2x + 7 = 0$

$\Rightarrow y = 2x - 7$

Slope $(m) = 2$

$\therefore$ Slope of PQ $= m_{PQ} = 2$

Equation of PQ,

$(y-4) = 2(x-3)$

$\Rightarrow y = 2x - 2$.....(i)

and $3x + 2y + 7 = 0$.....(ii)

On putting, the value of y in Eq. (ii), we get

$3x + 2(2x-2) + 7 = 0$

$\Rightarrow 7x = -3 \Rightarrow x = -\dfrac{3}{7}$

Then, $y = 2 \times \left(-\frac{3}{7}\right) - 2 = -\frac{20}{7}$

So, coordinates of $Q = \left(-\frac{3}{7}, \frac{-20}{7}\right)$

Thus, distance

$$PQ = \sqrt{\left(3 - \left(\frac{-3}{7}\right)^2\right) + \left(4 - \left(-\frac{20}{7}\right)\right)^2} = \frac{24\sqrt{5}}{7}$$

162. **(a)** Slope of the line,

$y - 3x + 18 = 0$

$\Rightarrow y = 3x - 18 \Rightarrow$ Slope $(m_1) = 3$

and angle $(\theta) = 60°$

so, $\tan 60° = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\Rightarrow \sqrt{3} = \left|\frac{3 - m_2}{1 + 3m_2}\right|$

$\Rightarrow \frac{3 - m_2}{1 + 3m_2} = \pm\sqrt{3}$

Either, $\frac{3-m_2}{1+3m_2} = \sqrt{3}$ or $\frac{3-m_2}{1+3m_2} = -\sqrt{3}$

$\Rightarrow 3 - m_2 = \sqrt{3} + 3\sqrt{3}m_2$

or $3 - m_2 = -\sqrt{3} - 3\sqrt{3}m_2$

$\Rightarrow m_2(1 + 3\sqrt{3}) = 3 - \sqrt{3}$

or $m_2(1 - 3\sqrt{3}) = 3 + \sqrt{3}$

$\Rightarrow m_2 = \frac{3 - \sqrt{3}}{1 + 3\sqrt{3}}$ or $m_2 = \frac{3 + \sqrt{3}}{1 - 3\sqrt{3}}$

$\therefore m_2 = \frac{3 - \sqrt{3}}{1 + 3\sqrt{3}}, \frac{3 + \sqrt{3}}{1 - 3\sqrt{3}}$

163. **(c)** If 3 and 5 are intercepts of a line $L = 0$, then

x-intercept $= a = 3$

y-intercept $= b = 5$

Equation of line is

$\frac{x}{3} + \frac{y}{5} = 1 \Rightarrow 5x + 3y - 15 = 0$

$\therefore$ Required distance

$= \left|\frac{5(3) + 3(7) - 15}{\sqrt{5^2 + 3^2}}\right| = \frac{21}{\sqrt{34}}$

164. **(d)** $(x + y)^{60} = {}^{60}C_0 x^{60} - {}^{60}C_1 x^{59} y + \cdots + {}^{60}C_0 x^{60}$ ….(i)

$(x - y)^{60} = {}^{60}C_0 x^{60} + {}^{60}C_1 x^{59} y + \cdots + {}^{60}C_{60} y.$ …..(ii)

By adding Eq. (i) and Eq. (ii)

$(x + y)^{60} + (x - y)^{60}$

$= \underbrace{2\ {}^{60}C_0 x^6 + {}^{60}C_2 x^{58} y^2 + \cdots + {}^{60}C_{60} y^{60})}_{31 \text{ terms}}$

Hence, the expansion of $(x + y)^{60} + (x - y)^{60}$ has 31 terms.

165. **(c)** $(1 - 3x + 3x^2 - x^3)^{15} = [(1 - x)^3]^{15}$

$= (1 - x)^{45}$

So, $T_{r+1} = {}^{45}C_r (1)^{45-r} (-x)^r$

For coefficient of x^{29}, put $r = 29$

Then, $T_{30} = {}^{45}C_{29} (1)^{45-29} (-x)^{29} = -{}^{45}C_{29} x^{29}$

Hence, coefficient of $x^{29} = -{}^{45}C_{29}$ and $-{}^{45}C_{29} = -{}^{45}C_{16}$

166. **(b)** $\because$ Middle term has greatest binomial coefficient. In the expansion of $(1 + 3x + 3x^2 + x^3)^{2n}$

$= ((1 + x)^3)^{2n} = (1 + x)^{6n}$

$\because$ 6n is even

So, middle term of $(1 + x)^{6n} = T_{\left(\frac{6n}{2}+1\right)}$

$= T_{(3n+1)} = (3n + 1)$ th term.