1. **(d)** Given $|A|=\frac{1}{2\sqrt{2}}, |B|=\frac{1}{729}$

$|2 B(\operatorname{adj}(3 A)|=|2 B||\operatorname{adj}(3 A)|$

We know that $|\operatorname{adj} A|=|A|^{n-1}$ and $|kA|=k^{n}|A|$

$=2^{3}|B||3 A|^{3-1}=8|B|(3^{3}|A|)^{2}$

$$=8|B|3^{6}|A|^{2}=8 \cdot \frac{1}{729} \times 3^{6}\left(\frac{1}{2\sqrt{2}}\right)^{2}=8 \times \frac{1}{8}=1$$

2. **(b)** Since that $iz^{3}+z^{2}-z+i=0$ put $z=i$

$i^{4}+i^{2}-i+i=0 \Rightarrow 1-1-i+i=0$

Satisfied then, $(|z|+1)^{2}=[|i|+1]^{2}=4$

3. **(d)** Number of four digit numbers begins with 1 .

$=3 \times 2 \times 1=6$ and repeatation of digit $=\frac{6}{3}=2$

Digit total for each place $=(0+4+5) \times 2=18$ Total of all four-digit numbers beginning with 1

$=1 \times 6 \times 1000+18 \times 100+18 \times 10+18=7998$

Number of four-digit numbers begins with 4 .

$=3 \times 2 \times 1=6$ and repeatation of digit $=\frac{6}{3}=2$

Digit total for each place $=(0+1+5) \times 2=12$

Total of all four-digit numbers beginning with 4 .

$=4 \times 6 \times 1000+12 \times 100+12 \times 10+12=25332$

Number of four-digit numbers begins with 5

$=3 \times 2 \times 1=6$ and repetition of digit $=\frac{6}{3}=2$

Digit total for each place $=(0+1+4) \times 2=10$ Total of all four-digit numbers beginning with 5

$=6 \times 5 \times 1000+10 \times 100+10 \times 10+10=31110$

Required sum $=7998+25332+31110=64440$.

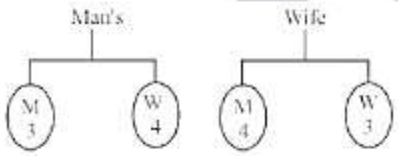
4. **(a)** Considering that $-x, y$ and z are the cube roots of unity

Assume $x=1, y=\omega$ and $z=\omega^{2}$

So, $xy+yz+zx=\omega+\omega^{3}+\omega^{2}$

$=\omega+1+\omega^{2}=0 (\because \omega^{3}=1)$

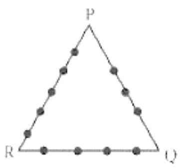
5. **(c)** Given,



According to question,

Case-1	0	3	3	0	$\rightarrow {}^{3}C_{0} \times {}^{4}C_{3} \times {}^{4}C_{3} \times {}^{3}C_{0}=16$
Case-2	3	0	0	3	$\rightarrow {}^{3}C_{3} \times {}^{4}C_{0} \times {}^{4}C_{0} \times {}^{3}C_{3}=1$
Case-3	1	2	2	1	$\rightarrow {}^{3}C_{1} \times {}^{4}C_{2} \times {}^{4}C_{2} \times {}^{3}C_{1}=324$
Case-4	2	1	1	2	$\rightarrow {}^{3}C_{2} \times {}^{4}C_{1} \times {}^{4}C_{1} \times {}^{3}C_{2}=\frac{114}{485}$

6. **(a)**



Required number of triangles

$={}^{12}C_{3}-({}^{3}C_{3}+{}^{4}C_{3}+{}^{5}C_{3})=220-(1+4+10)=205$

7. **(a)** Given, $\log_{b} a=p \Rightarrow a=b^{p}$

also, $\log_{b} c=2p \Rightarrow c=d^{2p}$

also, $\log_{f} e=3p \Rightarrow e=f^{3p}$

Now, $(\text{ace})^{1/p}=(b^{p} d^{2p} f^{3p})^{1/p}=bd^{2} f^{3}$

8. (c) As irrational roots occurs in pair and $-\sqrt{2}$ and $\sqrt{3}$ are roots of the given equation. So, $\sqrt{2}$ and $-\sqrt{3}$ are also roots of the given equation.

Thus, $x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

$= (x + \sqrt{2})(x - \sqrt{2})(x - \sqrt{3})(x + \sqrt{3})$

$= (x^2 - 2)(x^2 - 3) = x^4 - 5x^2 + 6$

On comparing $a_3 = 0, a_2 = -5, a_1 = 0$ and $a_0 = 6$

9. (c)

Consider $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

Now, $\left|\frac{z_1 + z_2}{z_1 - z_2}\right| = 1 \Rightarrow |z_1 + z_2| = |z_1 - z_2|$

$\Rightarrow |(x_1 + x_2) + i(y_1 + y_2)| = |(x_1 - x_2) + i(y_1 - y_2)|$

$\Rightarrow (x_1 + x_2)^2 + (y_1 + y_2)^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$\Rightarrow x_1^2 + x_2^2 + 2x_1x_2 + {y_1}^2 + y_2^2 + 2y_1y_2 = x_1^2 + x_2^2 -$

$2x_1x_2 + {y_1}^2 + y_2^2 - 2y_1y_2$

$\Rightarrow x_1x_2 + y_1y_2 = 0 \ldots\ldots$ (i)

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} \times \frac{x_2 - iy_2}{x_2 - iy_2} = \frac{(x_1x_2 + y_1y_2) + i(y_1x_2 - x_1y_2)}{x_2^2 + {y_2}^2}$$

$= 0 + \frac{i(y_1x_2 - x_1y_2)}{x_2^2 + y_2^2}$ $\therefore$ Re $\left(\frac{z_1}{z_2}\right) = 0$

Required answer Re $\left(\frac{z_1}{z_2}\right) + 1 = 1$

10. (b) Given, $26! = \mathrm{n}8^{\mathrm{k}} = \mathrm{n}2^{3\mathrm{k}}$ maximum power of 2

$= \left[\frac{26}{2}\right] + \left[\frac{26}{4}\right] + \left[\frac{26}{8}\right] + \left[\frac{26}{16}\right] = 13 + 6 + 3 + 1 = 23$

As power of 2 is multiple of 3

So, maximum value of $3\mathrm{k} = 21$

$\mathrm{k} = 7$

11. (b)

(A) We know that $\mathrm{adj}(AB) = (\mathrm{adj}B) \cdot (\mathrm{adj}A)$ Hence, statement A is not correct.

(B) We know that $\mathrm{AB} \neq \mathrm{BA}$

$\Rightarrow \mathrm{adj(AB)} \neq \mathrm{adj(BA)}$

Hence, statement B is not correct.

(C) We know that $\mathrm{A} \cdot \mathrm{adjA} = |\mathrm{A}|\mathrm{I}$

$\mathrm{(AB)\ adj\ (AB)} - |\mathrm{AB}|\mathrm{I_n}$

$= |AB|I_n - |AB|I_n =$ Null matrix

Hence, statement C is correct.

12. (d) As, $\mathrm{adj\ A^T} = (\mathrm{AdjA})^{\mathrm{T}} \Rightarrow \mathrm{A}(\mathrm{adjA^T}) = \mathrm{A}(\mathrm{AdjA})^{\mathrm{T}}$

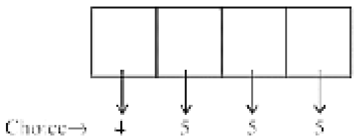
Statement A is correct

We know that, If $\mathrm{A^n} = \mathrm{A}$ then A is identify matrix.

$\therefore$ Statements B and C are correct

Hence, all statement are correct.

13. (b) All even digit are 0,2,4,6,8



$\therefore$ Total required numbers $= 4 \times 5 \times 5 \times 5 = 500$

14. (b) Given, $(z-100)^3+1000=0 \Rightarrow (z-1)^3=(-10)^3$

$\therefore\ z-100=-10(\omega,\omega^2,1) \therefore z-100=-10 \Rightarrow z=90$

$z-100=-10w \Rightarrow z=100-10\omega$

$z-100=-10w^2 \Rightarrow z=100-10\omega^2$

15. (d) $(1+i)^4+(1-i)^4=[(1+i)^2]^2+[(1-i)^2]^2$

$=(1-1+2i)^2+(1-1-2i)^2=(2i)^2+(-2i)^2$

$=4i^2+4i^2=-4-4=-8$

16. (a) All the diagonal elements of skew-symmetric matrix are zero.

Hence, statements 1 and 2 are correct we know that if $AA^T=I$ then A is called orthogonal matrix.

But $AA^T=A(-A)=-A^2[\ A^T=-A]$

Hence, Statement C is not correct.

17. (d) We know that a number is divisible by 4 if its last 2 digits is divisible by 4 .

Total number divisible by 4

$=0\ 0\ 0\ 21\times 2\times 3\times 1=1\times 2\times 3\times 1=6$

18. (a) $2^{120}=(2^3)^{40}=8^{40}=(1+7)^{40}$

$=1+{}^{40}C_1 7+{}^{70}C_2 7^2+\cdots\ldots {}^{40}C_{40}7^{40}$

$=1+7[{}^{40}C_1+{}^{40}C_2 7+\cdots\ldots\ldots {}^{40}C_{40}7^{39}]$

$\Rightarrow$ Remainder $=1$ [using division algorithm]

19. (c) Given,

$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,n-2) \\ C(11,6) & C(11,5) & C(12,n) \\ C(m,7) & C(m,6) & C(m+1,n+1) \end{vmatrix}=0$$

Applying $C_3 \rightarrow C_1+C_2-C_3$

$$\begin{vmatrix} C(9,4) & C(9,3) & C(10,4)-C(10,n-2) \\ C(11,6) & C(11,5) & C(12,6)-(12,n) \\ C(m,7) & C(m,6) & C(m+1,7)-C(m+1,n+1) \end{vmatrix}=0$$

Since determinant value is zero.

$\therefore C_3=0$

So, $n-2=4 \Rightarrow n=6$

20. (b) Consider

$$\begin{vmatrix} \cos C & \sin B & 0 \\ \tan A & 0 & \sin B \\ 0 & \tan(B+C) & \cos C \end{vmatrix}$$

Expanding along C_1

$=\cos C(-\sin B\cdot\tan(B+C))-\tan A(\sin B\cos C)$

$=-\sin B\cdot\cos C\left[\dfrac{\sin(B+C)}{\cos(B+C)}+\dfrac{\sin A}{\cos A}\right]$

$=-\sin B\cdot\cos C\left[\dfrac{\sin(B+C)\cdot\cos A+\sin A\cos(B+C)}{\cos A\cdot\cos(B+C)}\right]$

$=-\sin B\cos C\dfrac{\sin(A+B+C)}{\cos A\cdot\cos(B+C)}$

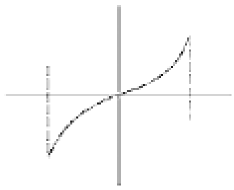
$=\dfrac{-\sin B\cos C\sin(\pi)}{\cos A\cdot\cos(B+C)}=0$

21. (d) Possible order of matrices with 4 entries

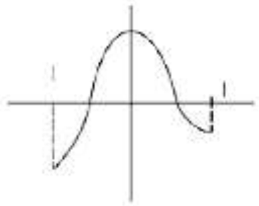
$1\times 4, 2\times 2, 4\times 1$

$\therefore$ Total number of matrices $=4\times 4\times 4\times 4\times 3=768$

22. (a)



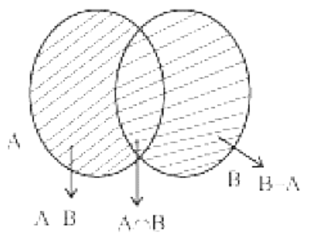
$f(x) = x|x|$
is one-one and onto



g(x) = cos(πx) is not one-one as any horizontal line passes g(x) twice.

23. (d)
Given $xRy \Rightarrow |x + y| < 2$ in $(-1,1)$
for reflexive
$|x + x| < 2 \Rightarrow |x| < 1$ true $\Rightarrow R$ is reflexive
for symmetric
Let $xRy \Rightarrow |x + y| < 2$
$\Rightarrow |y + x| < 2 \Rightarrow yRx \Rightarrow R$ is symmetric
for transitive $\because -1 < x < 1$
Let $|x + y| < 2$ and $|y + 2| < 2$
then $|x + z| < 2$
$\Rightarrow R$ is transitive.

24. (a) $(A \cup B) - \{(A - B) \cup (B - A) \cup (A \cap B\}$
From the venn diagram
$(A - B) \cup (B - A) \cup (A \cap B) = A \cup B$



$\Rightarrow (A \cup B) - (A \cup B) = \phi($ Null set $)$

25. (a)

$$\begin{vmatrix} a^2 & b\sin A & c\sin A \\ b\sin A & 1 & \cos A \\ c\sin A & \cos A & 1 \end{vmatrix}$$

Expanding along R_1
$= a^2(1 - \cos^2 A) - b\sin A(b\sin A - c\cos A\sin A) + c\sin A$
$(b\sin A \cdot \cos A - c\sin A)$
$= a^2\sin^2 A - b^2\sin^2 A + bc\sin^2 A \cdot \cos A + bc\sin^2 A \cdot \cos A$
$-c^2\sin^2 A$
$= a^2\sin^2 A - b^2\sin^2 A - c^2\sin^2 A + 2bc\sin^2 A \cdot \cos A$
$= \sin^2 A(a^2 - b^2 - c^2 + 2bc\cos A) = \sin^2 A \times 0 = 0$

26. (a) Given, a, b, c are in AP $\Rightarrow$ 2 b = a + c ….. (i)
also, b, c, d are in $G.P \Rightarrow c^2 = bd$…..(ii)
also, c, d, e are in H.P $\Rightarrow \frac{2}{d} = \frac{1}{c} + \frac{1}{e}$ …..(iii)
from (i) and (ii)

$2c^2 = (a+c)d \Rightarrow \frac{2}{d} = \frac{a+c}{c^2} \Rightarrow \frac{1}{c} + \frac{1}{e} = \frac{a}{c^2} + \frac{1}{c}$ from (iii)

$\frac{1}{e} = \frac{a}{c^2} \Rightarrow c^2 = ae$

So, a, c, e are in G.P.

27. (b)

Given, $\log_4(x-1) = \log_2(x-3)$

$\Rightarrow \frac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow \log_2(x-1) = \log_2(x-3)^2$

$\Rightarrow x - 1 = x^2 - 6x + 9 \Rightarrow x^2 - 7x + 10 = 0$

$\Rightarrow x^2 - 5x - 2x + 10 = 0 \Rightarrow (x-5)(x-2) = 0$

$x = 2,5$, but $x - 3 > 0 \Rightarrow x > 3$

$\Rightarrow x = 5$ only solution

28. (d)

Given, $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right) = k$

$\Rightarrow \log_x x - \log_x y + \log_y y - \log_y x = k$

$= 1 - \log_x y + 1 - \frac{1}{\log_x y} = k [\because \log_a a = 1]$

Let $\log_x y = t$

$\therefore\ 2 - t - \frac{1}{t} = k \Rightarrow 2t - t^2 - 1 = kt \Rightarrow t^2 + (k-2)t + 1 = 0$

For solution $D = b^2 - 4ac \geq 0$

$(k-2)^2 - 4 \geq 0 \Rightarrow (k-2)^2 \geq 4 \Rightarrow k - 2 \geq 2$ or $k - 2 \leq -2$

$k \geq 4$ or $k \leq 0 \Rightarrow k \neq 1$

29. (c) Given

$$A = \begin{vmatrix} \sin 2\theta & -\cos 2\theta & 0 \\ \cos 2\theta & \sin 2\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

We know that if $|A| = \pm 1$ then A is orthogonal matrix So $A^{-1} = A^T$ and $A^{-1} = \text{adj}A$

30. (a)

$$(1-x^2)^{20}\left(-\left(\frac{1}{x^2} + x^2 - 2\right)\right)^{-5}$$

$$= -(1-x^2)^{20}\left(x - \frac{1}{x}\right)^{-10}$$

$$= -(1-x^2)^{20}(x^2-1)^{-10} \cdot x^{10} = -(1-x^2)^{10} \cdot x^{10}$$

General term $T_{r+1} = -\,{}^{10}c_r(1)^{10-r}(-x^2)^{10-r} \cdot x^{10}$

$= (-1)^{11-r} \cdot {}^{10}c_r x^{30-2r}$

$\because\ 30 - 2r = 10 \Rightarrow r = 10$

So, coefficient $x^{10} = (-1)^{10}C_{10} = -1$.

31. (b) Given,

$$T_4 = T_{3+1} = {}^nC_3(mx)^{n-3}\left(\frac{1}{x}\right)^3 = \frac{5}{2}$$

$${}^nC_3(\,m)^{n-3}x^{n-6} = \frac{5}{2}x^0$$

On comparing $n - 6 = 0 \Rightarrow n = 6$

and ${}^6C_3\ m^3 = \frac{5}{2}$

$\Rightarrow \frac{6.5.4}{3.2}\ m^3 = \frac{5}{2} \Rightarrow\ m^3 = \frac{1}{8} \Rightarrow\ m = \frac{1}{2}$

$\therefore\ mn = 6 \times \frac{1}{2} = 3$

32. (d) Since a, b, c are in G.P $\Rightarrow b^2 = ac$

Now, for equal $ax^2 + bx + c = 0$

$D = b^2 - 4ac = ac - 4ac = -3ac < 0$

$\therefore$ Roots are imaginary

Let $a = 2, b = 4, c = 8$

$\therefore\ x^2 + 2x + 4 = 0 \Rightarrow x = \frac{-2 \pm \sqrt{4-16}}{2} = -1 \pm \sqrt{3}$

Ratio of roots $= \frac{-1+\sqrt{3}}{-1-\sqrt{3}} = \frac{\omega}{\omega^2} = \frac{1}{\omega}$

Product of roots $= \frac{(-1+\sqrt{3})}{2}\frac{(-1-\sqrt{3})}{2} \times 4 = \omega.\omega^2.4$

$= 4 = \frac{b^2}{a^2}$

33. (c)

$\therefore f(x) = x^2 + mx + n \in I, \forall x \in I$

$f(0) = 0 + 0 + n \in I \Rightarrow x \in I$

$f(1) = I + m + n \in I \Rightarrow\ m + n \in I$

$\Rightarrow\ m \in I\ (\because n \in I)$

So, both m and n are integer.

34. (a) Given,

$(x+y)^{2n+1} \cdot (x-y)^{2n+1} = (x^2 - y^2)^{2x+1}$

Middle term $\frac{2n+2}{2}, \frac{2n+4}{2} = (x+1)^{\text{th}}$ term, $(n+2)^{\text{th}}$ term

$T_{n+1} = 2n + 1_{C_n}(x^2)^{n+1}(y^2)^n$

$T_{n+2} = 2n + 1_{C_{n+1}}(x^2)^n(y^2)^{x+1}$

${}^{2n+1}C_n(x^2)^{n+1}(y^2)^n = {}^{2n+1}C_{n+1}(x^2)^n(y^2)^{n+1}$

$\frac{{}^{2n+1}C_n}{{}^{2n+1}C_{n+1}} = \frac{x^{2n}y^{2n+2}}{x^{2n+2}y^{2n}} \Rightarrow \frac{(2n+1)!}{(n+1)!\,n!} \cdot \frac{n!\,(n+1)!}{(2n+1)!} = \frac{y^2}{x^2}$

$\Rightarrow \frac{y^2}{x^2} = 1$

35. (c) Given, $n(A) = 5$ and $n(B) = 2$

Number of onto function from A to B $= 2^5 - {}^2C_1(2-1)^5 = 32 - 2 = 30$.

36. (d)

$$\frac{\sqrt{3}\cos 10^\circ - \sin 10^\circ}{\sin 25^\circ \cdot \cos 25^\circ} = \frac{2\left(\frac{\sqrt{3}}{2} \cdot \cos 10^\circ - \frac{1}{2}\sin 10^\circ\right)}{\frac{1}{2}(2\sin 25^\circ \cdot \cos 25^\circ)}$$

$$= \frac{4 \cdot \sin(60^\circ - 10^\circ)}{\sin 50^\circ} = \frac{4\sin 50^\circ}{\sin 50^\circ} = 4$$

37. (c)

$$\sin 9^\circ - \cos 9^\circ = \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 9^\circ - \frac{1}{\sqrt{2}}\cos 9^\circ\right)$$
$$= \sqrt{2}\sin(45^\circ - 9^\circ) = \sqrt{2}\sin 36^\circ$$
$$= \sqrt{2}\sqrt{1-\cos^2 36^\circ}\ \left[\because \cos 36^\circ = \frac{\sqrt{5}+1}{4}\right]$$
$$= \sqrt{2}\sqrt{1-\left(\frac{\sqrt{5}+1}{4}\right)2}$$
$$= \sqrt{2}\cdot\frac{\sqrt{10-2\sqrt{5}}}{4} = \frac{\sqrt{5-\sqrt{5}}}{2}$$

38. (d) Given

$\sin^3 A + \sin^3 B + \sin^3 C = 3\sin A\cdot\sin B\cdot\sin C$

$\Rightarrow\ \sin A + \sin B + \sin C = 0$

$\Rightarrow\ ak + kb + kc = 0 \Rightarrow a + b + c = 0$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} a+b+c & b & c \\ b+c+a & c & a \\ c+a+b & a & b \end{vmatrix} \text{(Applying } C_1 \to C_1 + C_2 + C_3)$$

$$= \begin{vmatrix} 0 & b & c \\ 0 & c & a \\ 0 & a & b \end{vmatrix} = 0$$

39. (c)

Given $\cos^{-1}x = \sin^{-1}x \Rightarrow \frac{\pi}{2} - \sin^{-1}x = \sin^{-1}x$

$\Rightarrow 2\sin^{-1}x = \frac{\pi}{2} \Rightarrow \sin^{-1}x = \frac{\pi}{4}$

$\Rightarrow x = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

40. (b)

Given, $(\sin\theta - \cos\theta)^2 = 2$

$\Rightarrow \sin^2 + \cos^2\theta - 2\sin\theta\cdot\cos\theta = 2$

$\Rightarrow \sin 2\theta = -1$

$\Rightarrow$ Two solution possible in $(-\pi, \pi)$

41. (c)

$$\frac{\cos A + \cos B}{\cos\left(\frac{A-B}{2}\right)} = \frac{2\cos\frac{A+B}{2}\cdot\cos\left(\frac{A-B}{2}\right)}{\cos\left(\frac{A-B}{2}\right)}$$
$$= 2\cos\left(\frac{\pi - C}{2}\right) = 2\cos\left(\frac{\pi}{2} - \frac{C}{2}\right) = 2\sin\frac{C}{2}$$
$$= 2\sin 30^\circ = 2\times\frac{1}{2} = 1$$

42. (c) $\sqrt{15 + \cot^2\left(\frac{\pi}{4} - 2\cot^{-1}3\right)}$

Consider, $2\cot^{-1}3 = 2\tan^{-1}\frac{1}{3} = \tan\frac{\frac{2}{3}}{1-\frac{1}{9}}$

$= \tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right) = \tan^{-1}\frac{3}{4}$

So, $\sqrt{15+\cot^2\left(\tan^{-1}1-\tan^{-1}\frac{3}{4}\right)}$

$= \sqrt{15+\cot^2\left(\tan^{-1}\left(\frac{1-\frac{3}{4}}{1+\frac{3}{4}}\right)\right)}$

$= \sqrt{15+\cot^2\left(\tan^{-1}\frac{1}{7}\right)} = \sqrt{15+\cot^2(\cot^{-1}7)}$

$= \sqrt{15+49} = 8$

43. (b) $\sin 10^\circ \cdot \sin 50^\circ + \sin 50^\circ \cdot \sin 250^\circ + \sin 250 \cdot \sin 10^\circ$

$= \frac{1}{2}(\cos 40^\circ - \cos 60^\circ + \cos 200^\circ - \cos 300^\circ + \cos 240^\circ$
$-\cos 260^\circ)$

$= \frac{1}{2}\Big[\cos 40^\circ - \frac{1}{2} + \cos(180+20)^\circ - \cos(360-60)^\circ +$
$\cos(180+60)^\circ - \cos(180+80)^\circ\Big]$

$= \frac{1}{2}\left[\cos 40^\circ - \frac{1}{2} - \cos 20^\circ - \cos 60^\circ - \cos 60^\circ + \cos 80^\circ\right]$

$= \frac{1}{2}\left[\cos 40^\circ - \cos 20^\circ + \cos 80^\circ - \frac{1}{2} - \frac{1}{2} - \frac{1}{2}\right]$

$= \frac{1}{2}\left[-2\sin 30^\circ \cdot \sin 10^\circ + \cos(90^\circ - 10^\circ) - \frac{3}{2}\right]$

$= \frac{1}{2}\left[-\sin 10^\circ + \sin 10^\circ - \frac{3}{2}\right] = -\frac{3}{4}$

44. (b)

$\tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{a-b}{a+b}\right)$

$= \tan^{-1}\left(\frac{a}{b}\right) - \tan^{-1}\left(\frac{\frac{a}{b}-1}{1+\frac{a}{b}\cdot 1}\right)$

$= \tan^{-1}\frac{a}{b} - \tan^{-1}\frac{a}{b} + \tan^{-1}1 = \tan^{-1}(1) = \frac{\pi}{4}$

45. (d) For real roots, discriminant ≥ 0

$\Rightarrow\ (\cos B)^2 - 4\sin B(\cos B - 1) \geq 0$

$(\cos B)^2 + 4\sin B(1-\cos B) \geq 0$

As $-1 \leq \cos B \leq 1$ and $\sin B \geq 0$ for $B[-0, \pi]$

So, it is only possible when $1 - \cos B \geq 0$

46. (c)

$\cos 2A + \cos 2B + \cos 2C$ [$\because \cos 2x = 1 - 2\sin^2 x$]

$= 1 - 2\sin^2 A + 1 - 2\sin^2 B + 1 - 2\sin^2 C$

$= 3 - 2(\sin^2 A + \sin^2 B + \sin^2 C)$

$= 3 - 2\left(\left(\frac{16}{65}\right)^2 + \left(\frac{63}{65}\right)^2 + 0\right)$ [since, $16^2 + 63^2 = 65^2$

$\Rightarrow [\angle B = 90^\circ] = 3 - 2 = 1.$

47. (b)

Given $\alpha+\beta=\frac{5\pi}{4}$ $\tan(\alpha+\beta)=\tan\frac{5\pi}{4}$

$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}=\tan\left(\pi+\frac{\pi}{4}\right)=\tan\frac{\pi}{4}$

$\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}=1$

$\tan\alpha+\tan\beta+\tan\alpha\cdot\tan\alpha=1$

Also, $\mathrm{f}(\theta)=\frac{1}{1+\tan\theta}$

so, $\mathrm{f}(\alpha)\cdot\mathrm{f}(\beta)=\frac{1}{1+\tan\alpha}\times\frac{1}{1+\tan\beta}$

$=\frac{1}{1+\tan\alpha+\tan\beta+\tan\alpha\cdot\tan\beta}=\frac{1}{1+1}=\frac{1}{2}$

48. (b)

$\tan\alpha+\tan\beta=6$

$\tan\alpha,\tan\beta=8$

$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\cdot\tan\beta}=\frac{6}{1-8}=\frac{6}{-7}$

$\cos(2\alpha+2\beta)=\cos2(\alpha+\beta)$

$=\frac{1-\tan^2(\alpha+\beta)}{1+\tan^2(\alpha+\beta)}=\frac{1-\frac{36}{49}}{1+\frac{36}{49}}=\frac{13}{85}$

49. (c)

$\tan(90-65)^\circ+2-2\tan40^\circ-\tan25^\circ$

$=\cot25^\circ-\tan25^\circ-2\tan40^\circ+2$

$=\frac{\cos^2 25-\sin^2 25}{\sin25\cdot\cos25}-2\tan40^\circ+2$

$=\frac{2\cos50^\circ}{\sin50^\circ}-2\tan40^\circ+2$

$=2\cot50^\circ-2\tan40^\circ+2=2\tan40^\circ-2\tan40^\circ+2=2$

50. (a)

1. $\cot\ A\cot\ B\cot C>0$

$\therefore\ \cot A>0,\cot\ B>0,\cot C>0$

$\therefore 0<A<\frac{\pi}{2},0<B<\frac{\pi}{2},0<C<\frac{\pi}{2}$

$\therefore\triangle$ ABC is an acute angled triangle.

Hence, statement A is correct.

2. $\tan A\tan B\tan C>0$

If $\triangle ABC$ is obtuse angled triangle then two of $\tan A$, tan B, $\tan C<0$

$\Rightarrow$ Two of angles are obtuse but it in not possible

Hence, statement B is not correct.

51. (a) $x^2+y^2+2x+6y+1=0$

To obtain the center (a, b) and the radius c of a circle equation, rewrite it as $(x-h)^2+(y-k)^2=r^2$, where (h,k) is the circle's center and r is its radius.

Consider, $x^2+y^2+2x+6y+1=0$

To obtain the equation in the desired form use completing the square

$(x^2+2x+1)-1+(y^2+6y+9)-9+1=0$

$\Rightarrow(x+1)^2+(y+3)^2=9$

$\Rightarrow(x-(-1))^2+(y-(-3))^2=(3)^2$

Center of the circle is $(-1,-3)$ and radius is 3 .

$\therefore\ a=-1,b=-3$ and $c=3$

$\therefore\ a^2+b^2+c^2=(-1)^2+(-3)^2+(3)^2=1+9+9=19$

52. (a) Given: Equation of sphere $x^2+y^2+z^2+2ux+2vy+2wz-1=0$ with AB as its diameter and $A\equiv(1,-1,2)$ and $B\equiv(2,1,-1)$ So, C will be midpoint of AB .

$C = \left(\frac{1+2}{2}, \frac{1-1}{2}, \frac{-1+2}{2}\right) = \left(\frac{3}{2}, 0, \frac{1}{2}\right)$

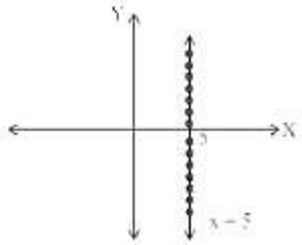
Since centre of the sphere is $(-u, -v, -w)$

On comparing we get

$u = \frac{-3}{2}, v = 0, w = \frac{-1}{2}$

Hence, $u + v + w = \frac{-3}{2} + 0 - \frac{1}{2} = \frac{-4}{2} = -2$

53. (d)



$x = 5$ represents a line parallel to y-axis so there are infinite points on x y - plane

54. (b) Equation of plane is $ax + by + cz = d$

Given the plane equation is $2x - 3y + 6z + 4 = 0$

On comparing we get $a = 2, b = -3, c = 6, d = -4$

Here we need to find the direction cosine of a normal to the plane i.e $\langle l, m, n\rangle$

Direction cosines of normal to plane

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{2}{\sqrt{49}} = \frac{2}{7}$$

$$m = \frac{b}{\sqrt{a^2 + b^2 + c^2}} = \frac{-3}{\sqrt{49}} = \frac{-3}{7}$$

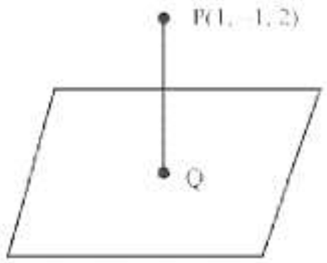
$$n = \frac{c}{\sqrt{a^2 + b^2 + c^2}} = \frac{6}{\sqrt{49}} = \frac{6}{7}$$

$\therefore \langle l, m, n\rangle = \left\langle \frac{2}{7}, \frac{-3}{7}, \frac{6}{7} \right\rangle$

$$\therefore 49(7l^2 + m^2 - n^2) = 49\left(7\left(\frac{2}{7}\right)^2 + \left(\frac{-3}{7}\right)^2 - \left(\frac{6}{7}\right)^2\right)$$

$$= 49\left(\frac{28}{49} + \frac{9}{49} - \frac{36}{49}\right) = 1$$

55. (c)



Assume the line from $(1, -1, 2)$ meet the plane at Q Direction ratios of the line from the point $(1, -1, 2)$ to the given plane is $\langle 3, 2, 2\rangle$

So the equation of the line passing through P and with direction ratios will be:

$$\frac{x-1}{3} = \frac{y+1}{2} = \frac{z-2}{2} = \lambda$$

$x = 3\lambda + 1, y = 2\lambda - 1, z = 2\lambda + 2$

Now, since Q lies on the plane so it must satisfy the equation of the plane.

i.e $x+2y+3z=18 \therefore 3\lambda+1+4\lambda-2+6\lambda+6=18$

$13\lambda+5=18 \Rightarrow \lambda=1$

Therefore, coordinates of Q are

$(3+1,2-1,2+2)=(4,1,4)$

56. (d) Let Plane $ax+by+cz+d=0$ is passing through $(1,0,0)(0,1,0)\&(0,0,1)$

We will find the equation of plane

Hence $a+b=0, b+d=0, c+d=0$

Solving above three equations, we get $a=b=c=-d$

Now Let $a=b=c=-d=\lambda$

So, equation of plane will be $x+y+z-1=0$

Perpendicular distance from origin to the plane

$(p)=\frac{|-1|}{\sqrt{(1)^2+(1)^2+(1)^2}}=\frac{1}{\sqrt{3}}$ unit

Hence, $3p^2=3\times\frac{1}{3}=1$

57. (b) Given $l+2m+n=0$.....(i)

and $2l-2m+3n=0$.....(ii)

From (i)

$l=-2m-n \ldots.(iii)$

Substitute (iii) in (ii)

$2(-2m-n)-2m+3n=0$

$-4m-2n-2m+3n=0 \Rightarrow -6m+n=0 \Rightarrow n=6m$

From (iii) we get, $l=-2m-n=-2m-6m=-8m$

$$\frac{l}{-8m}=\frac{m}{m}=\frac{n}{6m}$$

$$\therefore \frac{l}{-8}=\frac{m}{1}=\frac{n}{6}=\frac{\sqrt{l^2+m^2+n^2}}{\sqrt{(-8)^2+(1)^2+(6)^2}}=\frac{1}{\sqrt{101}}$$

$$\therefore l=-\frac{8}{\sqrt{101}}, n=\frac{1}{\sqrt{101}}, n=\frac{6}{\sqrt{101}}$$

Hence, $l^2+m^2-n^2=\frac{64}{101}+\frac{1}{101}-\frac{36}{101}=\frac{29}{101}$

58. (b)

59. (b) Rewrite the equation as follows:

$y\sin\theta=9-x\cos\theta$

$$\Rightarrow y=\frac{9}{\sin\theta}-x\frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow y=-x\frac{\cos\theta}{\sin\theta}+\frac{9}{\sin\theta}\ldots.(i)$$

$\because$ The general equation of line is.

$y=mx+c \ldots.(ii)$

On comparing (i) and (ii), we get

$$m=-\frac{\cos\theta}{\sin\theta}$$

Since, the slope of perpendicular line are negative inverse of each other.

So, the slope m_1 of the required line can be

$$m_1=-\left(\frac{1}{-\frac{\cos\theta}{\sin\theta}}\right)\Rightarrow m_1=\frac{\sin\theta}{\cos\theta}$$

Also, the line passes through the point $(-\sin\theta,\cos\theta)$ So, the equation of the required line is

$$y-\cos\theta=\frac{\sin\theta}{\cos\theta}(x+\sin\theta)$$

$\Rightarrow \cos\theta y-\cos^2\theta=x\sin\theta+\sin^2\theta)$

$\Rightarrow x\sin\theta-y\cos\theta+\sin^2\theta+\cos^2\theta=0$

$\Rightarrow x\sin\theta-y\cos\theta+1=0$

60. (a) Given that P and Q lie on line $y = 2x + 3$
For P put $x = a$ then $y = 2a + 3$
For Q put $x = b$ then $y = 2b + 3$
Coordinates $\mathrm{P} = (\mathrm{a}, 2\mathrm{a} + 3)$ and $\mathrm{Q} = (\mathrm{b}, 2\,\mathrm{b} + 3)$
From $\mathrm{R} = (1,5)$ using distance formula, we have

$$\mathrm{PR} = \sqrt{(\mathrm{a}-1)^2 + (2\mathrm{a}+3-5)^2} = 2 \ldots\ldots (\mathrm{i})$$

$$\mathrm{QR} = \sqrt{(\mathrm{b}-1)^2 + (2\,\mathrm{b}+3-5)^2} = 2 \ldots. (\mathrm{ii})$$

From (i) we get

$$(a-1)^2 + (2a-2)^2 = 4$$

$$a^2 - 2a + 1 + 4a^2 + 4 - 8a = 4$$

$$5a^2 - 10a + 1 = 0$$

From (ii), we get

$$5b^2 - 10b + 1 = 0$$

By using quadratic formula

$$(a, b) = \frac{10 \pm \sqrt{100-80}}{10} \Rightarrow \mathrm{a} = 1 \pm \frac{2}{\sqrt{5}}$$

If $\mathrm{a} = 1 + \frac{2}{\sqrt{5}}$ then

Coordinate of $\mathrm{P} = (\mathrm{a}, 2\mathrm{a}+3) = \left(1 + \frac{2}{\sqrt{5}}, 2 + \frac{4}{\sqrt{5}} + 3\right)$

$$= \left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right)$$

If $\mathrm{a} = 1 - \frac{2}{\sqrt{5}}$ then Coordinate of $\mathrm{P} = (\mathrm{a}, 2\mathrm{a}+3) = \left(1 - \frac{2}{\sqrt{5}}, 2 - \frac{4}{\sqrt{5}} + 3\right)$

$$= \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$$

Coordinate of $\mathrm{Q} = \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$ or

$$\left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right)$$

Required coordinates of the point P and Q are

$$\left(1 + \frac{2}{\sqrt{5}}, 5 + \frac{4}{\sqrt{5}}\right), \left(1 - \frac{2}{\sqrt{5}}, 5 - \frac{4}{\sqrt{5}}\right)$$

61. (a) Given that two side of a square lie on the lines

$$2x + y - 3 = 0 \ldots\ldots (\mathrm{i})$$

$$4x + 2y + 5 = 0 \ldots\ldots (\mathrm{ii})$$

Divide (ii) by 2 we get, $2x + y + \frac{5}{2} = 0$ ... (iii)

Here $2x + y - 3 = 0$ and $2x + y + \frac{5}{2} = 0$ are parallel lines

So, length of the side of the square = Distance between parallel side $= \frac{|\mathrm{C}_1 - \mathrm{C}_2|}{\sqrt{\mathrm{a}_1^2 + \mathrm{b}_1^2}}$

where $C_1 = -3$ and $C_2 = \frac{5}{2}$

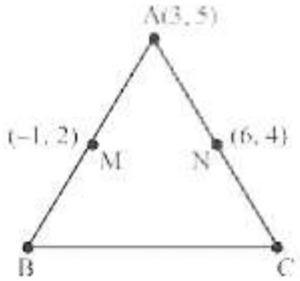
$$(\mathrm{a}_1, \mathrm{b}_1) = (2,1)$$

$$\Rightarrow \frac{\left|-3 - \frac{5}{2}\right|}{\sqrt{(2)^2 + (1)^2}} = \frac{\left|-\frac{11}{2}\right|}{\sqrt{5}} = \frac{11}{2\sqrt{5}}$$

So, length of the side of square $= \frac{11}{2\sqrt{5}}$ units

Required area of the square $= (\text{ side })^2 = \left(\frac{11}{2\sqrt{5}}\right)^2 = 6.05$ square units.

62. (b) Given, $A \equiv (3,5)$ and mid-points of sides AB and AC are $(-1,2)$ and $(6,4)$ respectively



Consider $B = (x_1, y_1)$ and $C = (x_2, y_2)$

By using mid point formula

$$M = (-1,2) = \left(\frac{x_1+3}{2}, \frac{y_1+5}{2}\right)$$

$$N = (6,4) = \left(\frac{x_2+3}{2}, \frac{y_2+5}{2}\right)$$

By comparing we get

$$\frac{x_1+3}{2} = -1, \frac{y_1+5}{2} = 2 \Rightarrow x_1 = -5, y_1 = -1$$

$$\text{and } \frac{x_2+3}{2} = 6, \frac{y_2+5}{2} = 4$$

$$\Rightarrow x_2 = 9, y_2 = 3$$

So, $B = (-5,-1)$ and $C(9,3)$

$$\text{Centroid of } \triangle ABC = \left(\frac{3-5+9}{3}, \frac{5-1+3}{3}\right)$$

$$= \left(\frac{7}{3}, \frac{7}{3}\right)$$

63. (b)

64. (c) Equation of parabola is $y^2 = 8x$

$\therefore\ a = 2$

Since focal distance of point $P_{(x_1,y_1)}$ is $x_1 + a$

So, focal distance from point $(6, 4\sqrt{3})$ is $6 + 2 = 8$

Hence, Statement A is correct.

Now, distance of point $P(6, 4\sqrt{3})$ from direction

$$PF = \sqrt{(6-2)^2 + (4\sqrt{3}-0)^2} = \sqrt{16+48} = 8$$

Hence, statement B is also correct.

65. (d)

66. (d) Given,

$\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

Using vector triple product

$$\vec{a} \times (\vec{b} \times \vec{a}) = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$$

$$= ((\hat{\imath} - \hat{\jmath} + \hat{k}) \cdot (\hat{\imath} - \hat{\jmath} + \hat{k}))\vec{b} - ((\hat{\imath} - \hat{\jmath} + \hat{k}) \cdot (\hat{\imath} + 2\hat{\jmath} - \hat{k}))\vec{a}$$

$$= (1+1+1)\vec{b} - (1-2-1)\vec{a}$$

$$= 3(\hat{\imath} + 2\hat{\jmath} - \hat{k}) + 2(\hat{\imath} - \hat{\jmath} + \hat{k})$$

$$= 5\hat{\imath} + 4\hat{\jmath} - \hat{k} \ldots\ldots (i)$$

Given $\vec{a} \times (\vec{b} \times \vec{a}) = \alpha\hat{\imath} - \beta\hat{\jmath} + \gamma\hat{k}$

Compare with (i), we get

$$\alpha = 5, \beta = -4, \gamma = -1$$

$$\Rightarrow \alpha + \beta + \gamma = 5 - 4 - 1 = 0$$

67. (a)

68. (b) Statement 1 is not correct

We know that $t = \vec{r} \times \vec{F}$

The moment of force about a point is dependent of application of force.

Statement 2 is correct.
The moment of force about a line is a vector quantity. Because gross product of two vector is again a vector

$\vec{t} = \vec{r} \times \vec{F}$

$\downarrow \quad \downarrow \quad \downarrow$

Vec Vec Vec

69. (a) Let $\vec{I} = (\vec{r}.\hat{\imath})(\vec{r} \times \hat{\imath}) + (\vec{r} \cdot \hat{\jmath}) + (\vec{r} \times \hat{\jmath}) + (\vec{r} \cdot \hat{k})(\vec{r} \times \hat{k}) \ldots . (i)$

Let $\vec{r} = a\hat{\imath} + b\hat{\jmath} + c\hat{k}$

$(\vec{r}\hat{\imath}) = (a\hat{\imath} + bj + c\hat{k}).\hat{\imath} = a$

$(\vec{r} \cdot \hat{\jmath}) = b \Rightarrow (\vec{r} \cdot \hat{k}) = c$

and $\vec{r} \times \hat{I} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a & b & c \\ 1 & 0 & 0 \end{vmatrix} = -\hat{\jmath}(-c) - b\hat{k} = c\hat{\jmath} - b\hat{k}$

Similarly, $\vec{r} \times \hat{\jmath} = -c\hat{\imath} + a\hat{k}$ and $\vec{r} \times \hat{k} = b\hat{\imath} - a\hat{\jmath}$

Now substitute in equation (i), we get

$\vec{I} = a(c\hat{\jmath} - b\hat{k}) + b(-c\hat{\imath} + a\hat{k}) + c(b\hat{\imath} - a\hat{\jmath})$

$= ac\hat{\jmath} - ab\hat{k} - cb\hat{\imath} + ab\hat{k} + bc\hat{\imath} - ac\hat{\jmath} = \vec{0}$

70. (b)

71. (a)

72. (c) We have,

$y = e^x(a\cos x + b\sin x) \ldots \ldots (i)$

$\Rightarrow \dfrac{dy}{dx} = e^x(-a\sin x + b\cos x) + (a\cos x + b\sin x)e^x$

$\Rightarrow \dfrac{dy}{dx} = e^x(-a\sin x + b\cos x) + y(\text{ From (i) }) \ldots \text{ (ii)}$

Again differentiate w.r.t x.

$\dfrac{d^2y}{dx^2} = -e^x(a\cos x + b\sin x) + (-a\sin x + b\cos x)e^x + \dfrac{dy}{dx}$

Eliminating arbitrary constants

Eliminating arbitrary constants

$\Rightarrow \frac{d^2y}{dx^2} = -y + \frac{dy}{dx} - y + \frac{dy}{dx}$ [From (i) and (ii)]

$\Rightarrow \frac{d^2y}{dx^2} - \frac{2dy}{dx} + 2y = 0$ is the required differential equation

73. (d)

Given $f(x) = ax - b \ldots$ (i)

and $g(x) = cx + d \ldots .$ (ii)

Now $f(g(x)) = g(f(x))$

$\Rightarrow a(g(x)) - b = c(f(x)) + d$

$\Rightarrow a(cx + d) - b = c(ax - b) + d$[From (i) and (ii)]

$\Rightarrow\ acx + ad - b = acx - bc + d$

$ad - b = d - bc \Rightarrow ad - b + bc = d$

$\Rightarrow \therefore f(d) + g(b) = 2d$ [f(d) = ad − b& g(b) = bc + d)]

Let $I = \int_{-1}^{1}(3\sin x - \sin 3x)\cos^2 x dx$

74. (b) $I = \int_{-1}^{1}(3\sin x - 3\sin x + 4\sin^3 x)\cos^2 x dx^+$

$[\sin 3x = 3\sin x - 4\sin^3 x]$

$I = \displaystyle\int_{-1}^{1} 4\sin^3 x\cos^2 x dx = \int_{-1}^{1} f(x)dx$

Since, $f(x) = 4\sin^3 x\cos^2 x$ is an odd function as $f(x) = -f(x)$.

$\Rightarrow\ I = \displaystyle\int_{-1}^{1} 4\sin^3 x\cos^2 x dx = 0$

$[\int_{-a}^{a} f(x)dx = 0, \text{ if } f(x) = -f(x)]$

75. **(d)** Given : $\left\{2-\left(\frac{dy}{dx}\right)^2\right\}^{0.6}=\frac{d^2y}{dx^2}$

$$\left\{2-\left(\frac{dy}{dx}\right)^2\right\}^{\frac{3}{5}}=\frac{d^2y}{dx^2}$$

Raise power of 5 both sides

$$\left\{2-\left(\frac{dy}{dx}\right)^2\right\}^{3}=\left(\frac{d^2y}{dx^2}\right)^5$$

Now differential equation is free from decimal or fraction power.

Order $=2$, Degree $=5$.

76. **(a)** Given,

$$\frac{dy}{dx}=2e^xy^3\Rightarrow\frac{dy}{y^3}=2e^xdx$$

Integrating both sides wrt x

$$\frac{-1}{2y^2}=2e^x+C\ \ldots..(\text{i})$$

Using $y(0)=\frac{1}{2}$, in equation (i)

$$-2=2+c\Rightarrow c=-4$$

Now, $\frac{-1}{2y^2}=2e^x-4$

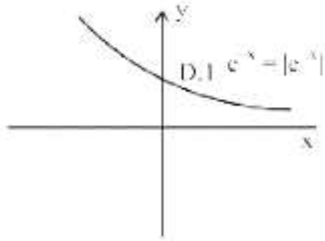
$$\Rightarrow 4y^2(e^x)-8y^2=-1\Rightarrow 4y^2(2-e^x)=1$$

77. **(d)** Given,

$$P=\int_a^b f(x)dx \text{ and } q=\int_a^b |f(x)|dx$$

As $f(x)=e^{-x}$ is always + ve $\forall x\in R$

$\Rightarrow |f(x)|=|e^{-x}|=e^{-x}=f(x)$



$$\Rightarrow \int_a^b f(x)dx=\int_a^b|f(x)|dx$$

Hence, p = q

78. **(a)**

Let $I=\int_0^{\frac{\pi}{2}}\frac{a+\sin x}{2a+\sin x+\cos x}dx\ \ldots..(i)$

$$\Rightarrow I=\int_0^{\frac{\pi}{2}}\frac{a+\cos x dx}{2a+\cos x+\sin x}\ \ldots..(ii)$$

$$\left[\int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

Adding equation (i) and (ii)

$$\Rightarrow 2I\int_0^{\frac{\pi}{2}}\frac{2a+\sin x+\cos x}{2a+\sin x+\cos x}dx=\int_0^{\frac{\pi}{2}}dx=[x]_0^{\frac{\pi}{2}}\Rightarrow I=\frac{\pi}{4}$$

79. **(d)** Given

$$f(x)=\frac{16x^3}{3}-4bx^2+x$$

$$f'(x)=16x^2-8bx+1$$

Since $f(x)$ is neither maximum nor minimum, then $f'(x)\neq 0$

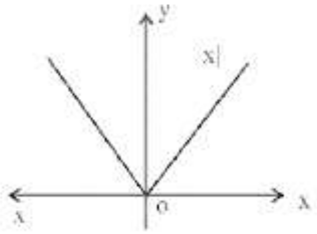
$\Rightarrow f'(x)>0$ or $f'(x)<0$ (but here $a>0$ for $f'(x)$ so not possible) for $f'(x)>0$

Discriminant of $f'(x)$ is $D<0$ and $a>0$

$$D=64b^2-64<0$$

$b^2 - 1 < 0$ and b is non-negative is $b \geq 0$
$b \in (-1,1)$ and $b \geq 0 \Rightarrow 0 \leq b < 1$.

80. (a)

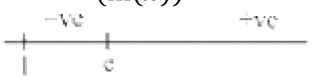


$f(x) = \frac{1}{\sqrt{|x|-x}}$ for domain of $f(x) = |x| - x > 0$
ie $|x| > x \Rightarrow x \in (-\infty, 0)$
$g(x) = \frac{1}{\sqrt{x-|x|}}$, for domain of $g(x), x - |x| > 0$
$\Rightarrow |x| < x$(Not possible $\because |x| \geq x$)
Let $I = \int \frac{3\cos x + 4\sin x}{2\cos x + 5\sin x} dx$
Let $3\cos x + 4\sin x = A(2\cos x + 5\sin x) +$
$B\left(\frac{d}{dx}(2\cos x + 5\sin x)\right)$
$3\cos x + 4\sin sx = A(2\cos x + 5\sin x) + B(-2\sin x + 5\cos x)$
Comparing coefficient of $\sin x$ and $\cos x$
$3 = 2A + 5B \ldots(i)$
$4 = 5A - 2B \ldots.(ii)$
Solving (i) and (ii) we get
$A = \frac{26}{29}, B = \frac{7}{29}$
Now $I = \int \frac{\frac{26}{29}(2\cos x + 5\sin x) + \frac{7}{29}(-2\sin x + 5\cos x)'}{2\cos x + 5\sin x} dx$
$\Rightarrow I = \int \frac{26}{29} dx + \frac{7}{29} \int \frac{(-2\sin x + 5\cos x)}{2\cos x + 5\sin x} dx$
$\Rightarrow I = \frac{26}{29} x + \frac{7}{29} \ln|2\cos x + 5\sin x| + c$
$\alpha = 26, \beta = 7$ (on comparing)

81. (d) $\alpha = 26$

82. (a) $\beta = 7$,

83. (d) Given
$f(x) = \frac{x}{\ln(x)} (x > 1)$
Differentiate w.r.t x
$f'(x) = \frac{\ln x - 1}{(\ln(x))^2}$



$f'(x)$ is the when $x \in (e, \infty)$ ie $f(x)$ is increasing in interval $(e, \infty) \Rightarrow$ (A) is correct.
$f'(x)$ is $-ve$, when $x \in (1, e)$ ie $f(x)$ is decreasing in the interval $(1, e) \Rightarrow (B)$ is correct.
Since $\ln x$ is an increasing function.
$7^9 > 9^7$
$\ln 7^9 > \ln 9^7$
$9\ln 7 > 7\ln 9 \Rightarrow$ statement (C) is correct
Hence, statement (A), (B), and (C) are correct.

84. (d) Again differentiating w.r.t x

$$f''(x)=\frac{(\ln x)^2\times\frac{1}{x}-(\ln x-1)\times 2\frac{\ln x}{x}}{(\ln x)^4}$$

$$=\frac{(\ln x)^2-2(\ln x)^2+2\ln x}{x(\ln x)^4}$$

$$f''(e)=\frac{1-2+2}{e\times 1}=\frac{1}{e}(>0)\Rightarrow \text{ statement }(A)\text{ is correct}$$

As $f''(e)>0(+ve)$

So, f(x) attains local minima at x = e ⇒ statement (B) is correct

A local minimum value occurs at $x=e$, $f(x)$ at x = e, is f(e) $=\frac{e}{\ln e}=e$ ⇒ statement (C) is correct Hence, statement (A), (B) and (C) are correct.

Given :

$$g(x)=x-\frac{1}{x}\text{ and }\log(x)=x^3-\frac{1}{x^3}$$

$$f\left(x-\frac{1}{x}\right)=x^3-\frac{1}{x^3}\Rightarrow f\left(x-\frac{1}{x}\right)=\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right)$$

85. (a) $f(x)=x^3+3x\ \ldots.(i)$

So, $g[f(x)-3x]=f(x)-3x-\frac{1}{f(x)-3x}$

$$g[f(x)-3x]=x^3+3x-3x-\frac{1}{x^3+3x-3x}=x^3-\frac{1}{x^3}$$

$$g[f(x)-3x]=x^3-\frac{1}{x^3}$$

86. (d) $f'(x)=3x^2+3$

$f'(x)=6x$

87. (a)

(A) $\frac{\text{for } x<0}{f(x)=-x+1}$

so $f'(x)=-1$ then $f(x)$ is differentiable $\forall x<0$

⇒ statement (A) is correct

(B) at $x=.0001$

g(x) is continuous every where except for integer value as [x] is continuous $\forall x\in R$ except integers.

So $g(x)$ is continuous at $x=.0001$, ⇒ statement (B) is correct

(C) g(x) = [x] − 1 [x] ⇒ Always a integer

$$g'(x)=\frac{d}{dx}[x]-0$$

$g'(x)=0-0=0$ ⇒ statement (C) is not correct.

∴ (A), and (B) only correct.

88. (a)

Consider $\lim\limits_{x\to 0^-}h(x)+\lim\limits_{x\to 0^+}h(x)$

$$=\lim_{x\to 0^-}\frac{|x|+1}{[x]-1}+\lim_{x\to 0^+}\frac{|x|+1}{[x]-1}$$

$$=\frac{0+1}{-1-1}+\frac{0+1}{0-1}\begin{Bmatrix}[0-h]=-1\\ [0+h]=0\end{Bmatrix}$$

$$=-\frac{1}{2}-1=\frac{-3}{2}$$

89. (d) Given

$$\phi(a)=\int_a^{a+100\pi}|\sin x|dx$$

Since period of $|\sin x|=\pi$ and $|\sin x|=\sin x$ in the interval 0 to π.

$$\phi(a)=\int_0^{100\pi}|\sin x|dx=100\int_0^{\pi}|\sin x|dx=100\int_0^{\pi}\sin x dx$$

$$\phi(a)=100(-\cos x)_0^{\pi}=100\times 2=200$$

90. (a) $\phi(a)=200$

Differentiating w.r.t a

$\phi'(a)=0$
Given, $y=2f(x)+ax-b \ldots .(i)$
Differentiating w.r.t. ' x ' we get,
$\Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}}=2\mathrm{f}'(\mathrm{x})+\mathrm{a}(1) \ldots .(\mathrm{ii})$
Again differentiating w.r.t. x, we get,
$\Rightarrow \frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2}=2\mathrm{f}''(\mathrm{x}) \ldots .(\mathrm{iii})$

91. (c) Since $f(x)$ has a local maximum at $x=0$. Then
$f'(0)=0$ and $\mathrm{f}''(0)<0 \ldots (\mathrm{iv})$
So, option (c) is correct.

92. (b) Since y has a relative maxima at $\mathrm{x}=0$.
Then

$$\left(\frac{dy}{dx}\right)_{x=0}=0$$
$$\Rightarrow 2f'(0)+a=0 \ \{\text{from (ii)}\}$$
$$\Rightarrow 2(0)+a=0 \ \{\text{from (iv)}\}$$
$$\Rightarrow a=0$$
$$\text{also } \left(\frac{d^2y}{d^2}\right)_{x=0}<0 \Rightarrow 2f''(0)<0$$
$$\Rightarrow f''(0)<0$$

So, y has a relative maxima for $\mathrm{a}=0$ and all value of ' b '
We have given

$$f(x)=|x-1| \ldots .. \text{(i)}$$
$$g(x)=[x] \ldots \text{(ii)}$$
$$\text{and } h(x)=f(x)\cdot g(x)$$
$$\Rightarrow h(x)=|x-1|\cdot[x] \ldots \text{(iii)}$$

93. (a) Now

$$\int_{-1}^{0} h(x)dx=\int_{-1}^{0} f(x)\cdot g(x)\cdot dx=\int_{-1}^{0}|x-1|\cdot[x]dx$$
$$=\int_{-1}^{0}(1-x)\cdot(-1)dx=\int_{-1}^{0}(x-1)dx$$
$$=\left[\frac{x^2}{2}-x\right]_{-1}^{0}=\left[0-\left(\frac{1}{2}+1\right)\right]=-\frac{3}{2}$$

94. (d) Now

$$\int_{0}^{2} h(x)dx=\int_{0}^{1}|x-1|[x]dx+\int_{1}^{2}|x-1|[x]dx$$
$$=\int_{0}^{1}(1-x)\cdot(0)dx+\int_{1}^{2}(x-1)(1)dx$$
$$=\int_{1}^{2}(x-1)dx=\left[\frac{x^2}{2}-x\right]_{1}^{2}$$
$$=\left(\frac{4}{2}-2\right)-\left(\frac{1}{2}-1\right)=\frac{1}{2}$$

We have given,

$$\int \frac{dx}{\sqrt{x+1}-\sqrt{x-1}} = \alpha(x+1)^{\frac{3}{2}} + \beta(x-1)^{\frac{3}{2}} + C \ldots..(i)$$

Let $I = \int \frac{dx}{\sqrt{x+1}-\sqrt{x-1}} = \int \frac{(\sqrt{x+1}+\sqrt{x-1})dx}{(\sqrt{x+1}-\sqrt{x-1})(\sqrt{x+1}+\sqrt{x-1})}$

$$\Rightarrow I = \frac{1}{2}\int \sqrt{(x+1)}dx + \frac{1}{2}\int \sqrt{x-1}dx$$

$$\Rightarrow I = \frac{1}{2}\cdot\frac{(x+1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + \frac{1}{2}\frac{(x-1)^{\frac{3}{2}}}{\left(\frac{3}{2}\right)} + C$$

$$\Rightarrow I = \frac{1}{3}(x+1)^{\frac{3}{2}} + \frac{1}{3}(x-1)^{\frac{3}{2}} + C$$

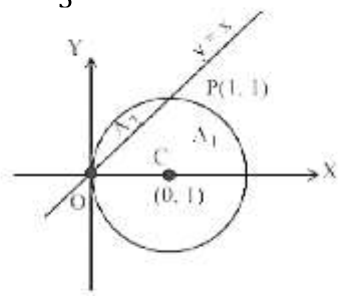
$$\Rightarrow \int \frac{dx}{\sqrt{x+1}-\sqrt{x-1}} = \frac{1}{3}(x+1)^{-} + \frac{1}{3}(x-1)^{-} + C \ldots.(ii)$$

95. (a) Comparing (i) and (ii) we get

$\alpha = \frac{1}{3}$

96. (c) Comparing (i) & (ii) we get

$\beta = \frac{1}{3}$



Equation of circle

$x^2 + y^2 - 2x = 0$

$\Rightarrow (x-1)^2 + y^2 = 1$

$\Rightarrow y^2 = 1 - (x-1)^2$

Area of minor segment $(A_2) = \int_0^1 \left(\sqrt{1-(x-1)^2} - x\right)dx$

$$\Rightarrow A_2 = \left[\frac{(x-1)}{2}\sqrt{1-(x-1)^2} + \frac{1}{2}\sin^{-1}(x-1) - \frac{x^2}{2}\right]_0^1$$

$$\Rightarrow A_2 = \left[\left(0 + \frac{1}{2}(0) - \frac{1}{2}\right) - \left(0 + \frac{1}{2}\sin^{-1}(-1) - 0\right)\right]$$

$$\Rightarrow A_2 = \frac{\pi - 2}{4}$$

Since Area of given circle $= \pi(1)^2 = \pi$

97. (d) $\Rightarrow A_1 + A_2 = \pi$

$\Rightarrow A_1 = \pi - \frac{\pi-2}{4} = \frac{3\pi+2}{4}$

98. (a) $\frac{2(A_1+A_2)}{A_1-3A_2} = \frac{2(\pi)}{\left(\frac{3\pi+2}{4}\right)-\frac{3(\pi-2)}{4}} = \pi$

99. (d) We have given, $3f(x) + f\left(\frac{1}{x}\right) = \frac{1}{x} + 1$…..(i)

Replacing (x) by $\left(\frac{1}{x}\right)$, we get,

$3f\left(\frac{1}{x}\right) + f(x) = x + 1$ ... (ii)

Now, on $3 \times$ [Eq. (i)] $-$ Eq. (ii), we get

$$8f(x) = \frac{3}{x} + 3 - x - 1$$

$$\Rightarrow f(x) = \frac{3}{8x} - \frac{x}{8} + \frac{1}{4} \dots (iii)$$

100. (a)

$$8\int_1^2 f(x)dx = 8\int_1^2 \left(\frac{3}{8x} - \frac{x}{8} + \frac{1}{4}\right) dx$$

$$= \int_1^2 \left(\frac{3}{x} - x + 2\right) dx = \left[3\ln x - \frac{x^2}{2} + 2x\right]_1^2$$

$$= \left(3\ln 2 - \frac{4}{2} + 4\right) - \left(3\ln 1 - \frac{1}{2} + 2\right)$$

$$= \frac{1}{2} + 3\ln 2 = \frac{1}{2}\ln e + \ln 2^3$$

$$= \ln\sqrt{e} + \ln 8 = \ln(8\sqrt{e})$$

101. (c) No. of ways to choose two black from the bag $= {}^5C_2$ No. of ways to choose two white ball from the bag $= {}^4C_2$ Number of ways to choose both of the ball of same colour $= {}^5C_2 + {}^4C_2$ So, the required probability is

$$P(E) = \frac{{}^5C_2 + {}^4C_2}{{}^9C_2} = \frac{10 + 6}{36}$$

$$P(E) = \frac{16}{36} = \frac{4}{9}$$

102. (c) Given, Mean $= 5$ and Variance $= 4$

$5^{23}P(X = 3) = \lambda 4^{\lambda}$

$\because$ Mean $= 5 \Rightarrow np = 5$

Variance $= 4 \Rightarrow npq = 4$

$$\Rightarrow 5q = 4 \Rightarrow q = \frac{4}{5}$$

Now, $p = 1 - q = 1 - \frac{4}{5} = \frac{1}{5}$

$\because np = 5 \Rightarrow n = 25$

Now, $5^{23}P(X = 3) = 5^{23}[{}^nC_3 p^3 q^{n-3}]$

$$= 5^{23}\left[{}^{25}C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^{22}\right]$$

$$= 5^{23}\left[\frac{25 \times 24 \times 23}{6} \times \frac{1}{5^3} \times \frac{4^{22}}{5^{22}}\right] = 4 \times 23 \times 4^{22}$$

$$\Rightarrow 5^{23}P(X = 3) = 23 \times 4^{23} \dots (ii)$$

From equation (i) & (ii) :-

$\lambda = 23$

103. (b) Given data

x	y	xy	x^2
-4	1	-4	16
-1	2	-2	1
2	7	14	4
3	1	3	9

$\Sigma x = -4 - 1 + 2 + 3 = 0$
$\Sigma y = 1 + 2 + 7 + 1 = 11$
$\Sigma xy = -4 - 2 + 14 + 3 = 11$
$\Sigma x^2 = 16 + 1 + 4 + 9 = 30$
$\bar{x} = \frac{\Sigma x}{4} = \frac{0}{4} = 0$
$\bar{y} = \frac{\Sigma y}{4} = \frac{11}{4}$

$$b_{yx} = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{4}}{\Sigma x^2 - \frac{(\Sigma x)^2}{4}} = \frac{11-0}{30-0} = \frac{11}{30}$$

Equation of regression line of y on x will be
$y - \bar{y} = b_{yx}(x - \bar{x})$
$\Rightarrow y - \frac{11}{4} = \frac{11}{30}(x-0) \Rightarrow y = \frac{11}{30}x + \frac{11}{4}$
$\therefore\ a = \frac{11}{4}$ and $b = \frac{11}{30}$
$2a + 15b = 2 \times \frac{11}{4} + 15 \times \frac{11}{30} = 11$

104. **(a)** Slope of line $x + 2y + 1 = 0 \Rightarrow m_1 = -1/2$
Slope of line $2x + 3y + 4 = 0 \Rightarrow m_2 = -2/3$
Then, $\tan\theta = \left|\frac{-\frac{1}{2}+\frac{2}{3}}{1+\frac{1}{2}\times\frac{2}{3}}\right| = \frac{1}{8}$
And, $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

$$= \frac{\frac{3}{8} - \frac{1}{8^3}}{1 - \frac{3}{64}} = \frac{192-1}{8^3} \times \frac{8^2}{61} = \frac{191}{8\times 61}$$

Therefore,

$$488\tan 3\theta = 488 \times \frac{191}{8\times 61} = 191$$

105. **Step 1:** Solve for Y in terms of X
Given the equation $\frac{2X-3Y}{5X+4Y} = 4$, multiply both sides by (5X + 4Y): $2X - 3Y = 20X + 16Y$ Rearrange the equation to solve for Y: $19Y = -18XY = -\frac{18}{19}X$
Step 2: Determine the variance of X
Given X follows a binomial distribution with $n = 10$ and $p = 1/2$, the variance of X is:
$\text{Var}(X) = np(1-p) = 10^*(1/2)^*(1/2) = 2.5$
Step 3: Calculate the variance of Y
Since $Y = -\frac{18}{19}X$, the variance of Y is:

$$\text{Var}(Y) = \left(-\frac{18}{19}\right)^2 \text{Var}(X) = \left(\frac{18}{19}\right)^2 * 2.5 = \frac{324}{361} * 2.5 = \frac{810}{361}$$

Final Answer $\frac{810}{361}$

106. **(b)** If a, b, and c are in harmonic progression (HP), then their reciprocals $\frac{1}{a}, \frac{1}{b}$, and $\frac{1}{c}$ are in arithmetic progression (AP). This implies that:

$$2 \cdot \frac{1}{b} = \frac{1}{a} + \frac{1}{c}$$

Rearranging, we get:

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

Therefore, the expression $\frac{1}{b-a} + \frac{1}{b-c}$ simplifies to $\frac{2}{b}$, which corresponds to option (A).

Additionally, since $\frac{2}{b}=\frac{1}{a}+\frac{1}{c}$, option (B) is also correct.
Brainly
Option (C) states that $\frac{1}{b-a}+\frac{1}{b-c}=\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$. Given that $\frac{1}{a}+\frac{1}{c}=\frac{2}{b}$, we can express:
$$\frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=\frac{1}{2}\left(\frac{2}{b}+\frac{1}{b}\right)=\frac{3}{2b}$$
Since $\frac{1}{b-a}+\frac{1}{b-c}=\frac{2}{b}\neq\frac{3}{2b}$, option (C) is incorrect.
Therefore, the correct answer is (b) A and B only.

107. **(a)** To find the average price of oil per litre over the four years, we need to consider that an equal amount of money was spent each year. Let's assume the amount spent each year is X rupees.

Given: The prices were $150, 200, 250$, and 300 rupees per litre in four consecutive years.
Let q_1, q_2, q_3, and q_4 be the quantities of oil bought in each year respectively.
Then, we have the following equations based on spending an equal amount each year:
$$150q_1=X$$
$$200q_2=X$$
$$250q_3=X$$
$$300q_4=X$$
From these equations, the quantities can be expressed as:
$$q_1=\frac{X}{150},$$
$$q_2=\frac{X}{200},$$
$$q_3=\frac{X}{250},$$
$$q_4=\frac{X}{300}.$$
The total quantity of oil purchased over the four years is:
$$q_1+q_2+q_3+q_4=\frac{X}{150}+\frac{X}{200}+\frac{X}{250}+\frac{X}{300}.$$
The average price per litre is given by the total expenditure over the total quantity,
Average Price $=\frac{4X}{q_1+q_2+q_3+q_4}$.
Substituting the expressions for q_1, q_2, q_3, and q_4, we get:
Average Price $=\frac{4X}{\frac{X}{150}+\frac{X}{200}+\frac{X}{250}+\frac{X}{300}}$.
The X cancels out:
$$\text{Average Price }=\frac{4x}{\frac{x}{150}+\frac{x}{200}+\frac{x}{250}+\frac{x}{300}}$$
The X cancels out:
Average Price $=\frac{4}{\frac{1}{150}+\frac{1}{200}+\frac{1}{250}+\frac{1}{300}}$.
Calculate the reciprocal of each price:
$$\frac{1}{150}=0.00667, \frac{1}{200}=0.005, \frac{1}{250}=0.004, \frac{1}{300}=0.00333$$
Their sum is:
$$0.00667+0.005+0.004+0.00333=0.019$$
Thus, the average price is:
Average Price $=\frac{4}{0.019}\approx 210.53$.
Therefore, the approximate average price of oil per litre is $\mathbf{210}$ rupees.

108. **(b)** To find the probability that both Ts in the word "TIRUPATI" are always consecutive,
we need to consider both the total number of possible arrangements of the letters and the number of favorable arrangements where the Ts are consecutive.

First, let's calculate the total number of possible arrangements of the letters in "TIRUPATI". The word "TIRUPATI" is composed of 8 letters where 'T' repeats twice. Therefore, the total number of arrangements can be calculated by the formula for permutations of multiset:

$$\frac{n!}{n_1!\, n_2! \ldots n_k!}$$

Where n is the total number of elements, and $n_1, n_2, \ldots, n_k$ are the frequencies of each distinct element.
For "TIRUPATI":
Total number of arrangements $= \frac{8!}{2!} = \frac{40320}{2} = 20160$
Next, we need to calculate the number of favorable arrangements where both Ts are consecutive. To do this, treat both Ts as a single entity or block. Hence, we have 7 entities to arrange: TT, I, R, U, P, A, I.
The effective number of letters now is 7 where 'I' repeats twice. The number of such arrangements is given by:

$$\frac{7!}{2!} = \frac{5040}{2} = 2520$$

Each of these arrangements can occur in two ways (TT or TT), so we multiply by 2 :
$2520 \times 2 = 5040$
Therefore, the number of favorable arrangements is 5040 . The probability that both Ts are consecutive then the ratio of the favorable arrangements to the total number of arrangements:

$$\text{Probability } = \frac{\text{Number of favorable arrangements}}{\text{Total number of arrangements}} = \frac{5040}{20160} = \frac{1}{4}$$

Hence, the probability that both Ts are consecutive is:
Option b: $\frac{1}{4}$

109. **(c)** To determine the units digit of a number raised to a power, we only need to look at the units digit of the base number. The units digit of 77 is 7 . Let's see what happens when we raise 7 to different powers:

$7^1 = 7$
$7^2 = 49$
$7^3 = 343$
$7^4 = 2401$

Notice a pattern in the units digits: 7,9,3,1. This pattern repeats. The units digit of any power of 7 will be one of these four digits.
Since the units digit cycles through four values, the probability that the units digit of 77^n is 1 is 1 out of 4 . Therefore, the correct answer is Option c : 1/4.

110. **(c)** To determine the probability that the product of two of the numbers is equal to the third number when three different numbers are selected at random from the first 15 natural numbers, we first need to consider the possible ways to choose these numbers and then find how many of those ways meet the given condition.
Step 1: Calculate Total Number of Ways to Choose 3 Numbers from 15.
The total number of ways to choose 3 different numbers out of 15 is given by the combination formula:

$$\binom{15}{3} = \frac{15!}{3!\,(15-3)!} = \frac{15 \times 14 \times 13}{3 \times 2 \times 1} = 455$$

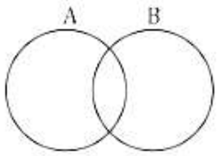
Step 2: Identify the Sets Where the Product of Two Numbers Equals the Third.
Let's identify possible sets of numbers $(\mathrm{a}, \mathrm{b}, \mathrm{c})$ where $a \times b = c$. The numbers must be distinct and come from the first 15 natural numbers 1 through 15.
We'll go through each number and find suitable pairs:
For $c = 6$: $1 \times 6 = 6$ or $2 \times 3 = 6$. So, the set can be (1,6,6) or (2,3,6). Only (2,3,6) satisfies our condition with distinct numbers.
For $c = 8$: $1 \times 8 = 8$ or $2 \times 4 = 8$. So, the set can be (1,8,8) or (2,4,8). Only (2,4,8) satisfies our condition with distinct numbers.
For $c = 9$: $1 \times 9 = 9$ or $3 \times 3 = 9$. So, the set can be (1,9,9) or (3,3,9). None of these satisfy our condition with distinct numbers.
For $c = 10$: $1 \times 10 = 10$ or $2 \times 5 = 10$. So, the set can be (1,10,10) or (2,5,10). Only (2,5,10) satisfies our condition with distinct numbers.
For $c = 12$: $1 \times 12 = 12$ or $2 \times 6 = 12$ or $3 \times 4 = 12$. The sets can be (1,12,12), (2,6,12), or (3,4,12). Both (2,6,12) and (3,4,12) satisfy our condition with distinct numbers.
For $c = 14$: $1 \times 14 = 14$ or $2 \times 7 = 14$. So, the set can be (1,14,14) or (2,7,14). Only (2,7,14) satisfies our condition with distinct numbers.

For $c = 15: 1 \times 15 = 15$ or $3 \times 5 = 15$. The set can be (1,15,15) or (3,5,15). Only (3,5,15) satisfies our condition with distinct numbers.
The valid sets are (2,3,6), (2,4,8), (2,5,10), (2,6,12), (3,4,12), (2,7,14), (3,5,15). These are 7 sets in total.
Step 3: Calculate the Probability
The probability P is the number of favorable outcomes over the total number of outcomes:
$$P = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{7}{455} = \frac{1}{65}$$
Hence, the correct answer is Option c : $\frac{1}{65}$

111. (d)
$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$
$$\geq 0.75 + 0.125$$
$$\geq 0.875$$

112. (c)
Step 1: Express $P(A) + P(B)$ in terms of $P(A \cup B)$ and $P(A \cap B)$
Using the formula for the probability of the union of two events: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Therefore, $P(A) + P(B) = P(A \cup B) + P(A \cap B)$.
Step 2: Determine the minimum value of $P(A) + P(B)$
Given $P(A \cup B) \geq 0.75$ and $P(A \cap B) \geq 0.125$, the minimum value of $P(A) + P(B)$ occurs when $P(A \cup B) = 0.75$ and $P(A \cap B) = 0.125$. Therefore, the minimum value is $0.75 + 0.125 = 0.875$.
Step 3: Determine the maximum value of $P(A) + P(B)$
Since probabilities cannot exceed 1 , the maximum value of $P(A \cup B)$ is 1 . The maximum value of $P(A \cap B)$ is 0.375. Therefore, the maximum value of $P(A) + P(B)$ is $1 + 0.375 = 1.375$. However, $P(A) + P(B)$ cannot exceed 2. Given $P(A \cup B) \geq 0.75$, and $P(A \cap B) \leq 0.375$, the maximum value of $P(A) + P(B)$ will be constrained by $P(A \cup B) \leq 1$. Let's examine this. If $P(A \cup B) = 1$, then $1 = P(A) + P(B) - P(A \cap B)$, so $P(A) + P(B) = 1 + P(A \cap B)$. The maximum value will be $1 + 0.375 = 1.375$
Final Answer
Minimum value of $P(A) + P(B) = 0.875$; Maximum value of $P(A) + P(B) = 1.375$

113. (b)



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.8 = 0.6 + 0.4 - P(A \cap B)$$
$$P(A \cap B) = 0.2$$

114. (c) We are given the following probabilities:
$$P(A) = 0.6$$
$$P(B) = 0.4$$
$$P(C) = 0.5$$
$$P(A \cup B) = 0.8$$
$$P(A \cap C) = 0.3$$
$$P(A \cap B \cap C) = 0.2$$
$$P(A \cup B \cup C) \geq 0.85$$
First, use the given information to find $P(A \cap B)$:
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.8 = 0.6 + 0.4 - P(A \cap B)$$
$$0.8 = 1.0 - P(A \cap B) \Longrightarrow P(A \cap B) = 1.0 - 0.8 = 0.2.$$
We now know:
$$P(A \cap B) = 0.2$$
$$P(A \cap C) = 0.3$$
$$P(A \cap B \cap C) = 0.2 \text{ (given)}$$
Next, consider the formula for the union of three events:
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap$$

Substitute the known values:
$P(A \cup B \cup C)=0.6+0.4+0.5-0.2-0.3-P(B \cap C)+0.2$
Combine like terms:

$$
\begin{aligned}
& P(A \cup B \cup C)=(0.6+0.4+0.5)-(0.2+0.3)-P(B \cap C)+0.2 \\
& =1.5-0.5-P(B \cap C)+0.2 \\
& =1.5-0.5+0.2-P(B \cap C) \\
& =1.2-P(B \cap C)
\end{aligned}
$$

We have the constraint:
$P(A \cup B \cup C) \geq 0.85$
Substitute $P(A \cup B \cup C)=1.2-P(B \cap C)$:
$1.2-P(B \cap C) \geq 0.85$
Solve for $P(B \cap C)$:

$$
\begin{aligned}
& -P(B \cap C) \geq 0.85-1.2 \\
& -P(B \cap C) \geq-0.35 \\
& P(B \cap C) \leq 0.35
\end{aligned}
$$

Now we have an upper bound: $P(B \cap C) \leq 0.35$.
Check if this is feasible given all other constraints. The triple intersection is 0.2 , and there are no direct contradictions preventing $P(B \cap C)$ from reaching 0.35 . A larger $P(B \cap C)$ (e.g., 0.45) would reduce the union to $1.2-0.45=0.75$, which is less than 0.85 , violating the given constraint. Therefore, the maximum value of $P(B \cap C)$ that satisfies all conditions is 0.35 .
Final Answer: 0.35

115. (b)

Step 1: Expressing p and q in terms of n

The probability of getting at least one tail is: $p=1-\left(\frac{1}{2}\right)^n$

The probability of getting exactly one tail is given by the binomial probability formula: $\binom{n}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1}=n\left(\frac{1}{2}\right)^n$

The probability of getting at least two tails is: $q=1-\left(\frac{1}{2}\right)^n-n\left(\frac{1}{2}\right)^n=1-(1+n)\left(\frac{1}{2}\right)^n$

Step 2: Solving for **nusing** the given equation

We are given that $p-q=\frac{5}{32}$. Substituting the expressions for p and q, we get:

$$
\left[1-\left(\frac{1}{2}\right)^n\right]-\left[1-(1+n)\left(\frac{1}{2}\right)^n\right]=\frac{5}{32}
$$

This simplifies to: $n\left(\frac{1}{2}\right)^n=\frac{5}{32}$

We can test integer values of n :

If $\mathrm{n}=1,\left(\frac{1}{2}\right)^1=\frac{1}{2} \neq \frac{5}{32}$

If $\mathrm{n}=2,2\left(\frac{1}{2}\right)^2=\frac{1}{2} \neq \frac{5}{32}$

If $\mathrm{n}=3,3\left(\frac{1}{2}\right)^3=\frac{3}{8} \neq \frac{5}{32}$

If $\mathrm{n}=4,4\left(\frac{1}{2}\right)^4=\frac{4}{16}=\frac{1}{4}=\frac{8}{32} \neq \frac{5}{32}$

If $\mathrm{n}=5,5\left(\frac{1}{2}\right)^5=\frac{5}{32}$

Therefore, $\mathrm{n}=5$.

116. (a)

Step 1: Calculating p and q for $n=5$

$$
p=1-\left(\frac{1}{2}\right)^5=1-\frac{1}{32}=\frac{31}{32} q=1-(1+5)\left(\frac{1}{2}\right)^5=1-\frac{6}{32}=\frac{26}{32}
$$

Step 2: Calculating $\mathbf{p}+\mathbf{q}$

$$
p+q=\frac{31}{32}+\frac{26}{32}=\frac{57}{32}
$$

Final Answer

The value of $p+q$ is $\frac{57}{32}$.

117. **(b)** Given

$$\sum x_i f_i = 1.1 + \frac{2}{2} + \frac{3}{2^2} + \cdots . + \frac{n}{2^{(n-1)}} \ldots . . (i)$$

and $\frac{1}{2}\Sigma x_i f_i = \frac{1}{2} + \frac{2}{2^2} + \cdots + \frac{n-1}{2^{n-1}} + \frac{n}{2^n} \ldots . . \text{(ii)}$

Subtracting (ii) from (i), we get

$$\Rightarrow \left(1 - \frac{1}{2}\right)\Sigma x_i f_i = \left[1.1 + \frac{2-1}{2} + \frac{3-2}{2^3} + \frac{4-3}{2^4} + \cdots + \frac{1}{2^{n-1}}\right] - \frac{n}{2^n}$$

$$\Rightarrow \frac{1}{2}\Sigma x_i f_i = \left[1 + \frac{1}{2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots + \frac{1}{2^{n-1}}\right] - \frac{n}{2^n}$$

$$\Rightarrow \frac{1}{2}\Sigma x_i f_i = \frac{1\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} - \frac{1}{2^n} = \frac{2(2^n - 1)}{2^n} - \frac{n}{2^n}$$

$$\Rightarrow \Sigma x_i f_i = \frac{2^{n+1} - n - 2}{2^{n-1}}$$

118. **(c)**

$$\Sigma f_i = 1 + \frac{1}{2} + \frac{1}{2^2} + \cdots . + \frac{1}{2^{(n-1)}}$$

$$\Sigma f_i = \frac{1\left(1 - \frac{1}{2^n}\right)}{1 - \frac{1}{2}} = \frac{2^n - 1}{2^{n-1}}$$

Calculating mean $= \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2^{n+1} - n - 2}{2^{n-1}} \cdot \frac{2^{n-1}}{2^n - 1}$

$$= \frac{2^{n+1} - n - 2}{2^n - 1}$$

Ascending data is, 15,18,19,21,24,32,35,41,47,50 Largest 5 observation is, 32,35,41,47,50

Mean $\mu = \frac{32+35+41+47+50}{5} = \frac{205}{5} \Rightarrow \mu = 41$

119. **(c)**

Mean deviation $= \frac{\Sigma|x_i - \mu|}{n}$

$$= \frac{|32-41|+|35-41|+|141-41|+|47-41|+|50-41|}{5}$$

$$= \frac{9+6+0+6+9}{5} = 6$$

120. **(d)**

$$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{5} = \frac{9^2+6^2+0^2+6^2+9^2}{5}$$

$$= \frac{234}{5} = 46.8$$