1. (b) We are given that the sum of the first k terms of a series S is:
$S_k = 3k^2 + 5k$
To determine the nature of the sequence, we first find the general term T_k (i.e., the k-th term) using:
$T_k = S_k - S_{k-1}$
Let's compute T_k :
$T_k = S_k - S_{k-1} = (3k^2 + 5k) - [3(k-1)^2 + 5(k-1)]$
First compute $(k-1)^2 = k^2 - 2k + 1$, so:
$S_{k-1} = 3(k^2 - 2k + 1) + 5(k-1) = 3k^2 - 6k + 3 + 5k - 5 = 3k^2 - k - 2$
Now subtract:
$T_k = (3k^2 + 5k) - (3k^2 - k - 2) = 3k^2 + 5k - 3k^2 + k + 2 = 6k + 2$
So the general term:
$T_k = 6k + 2$
This is clearly a linear expression in k, which means the sequence is an arithmetic progression.
First term $T_1 = 6(1) + 2 = 8$
Second term $T_2 = 6(2) + 2 = 14$
Third term $T_3 = 6(3) + 2 = 20$
So, common difference $= T_2 - T_1 = 14 - 8 = 6$

2. (b) Given,
The sum of the first 8 terms of a GP is 5 times the sum of its first 4 terms.
The formula for the sum of the first n terms of a GP is:
$$S_n = a\frac{1-r^n}{1-r}$$
Substitute for S_8 and S_4 into the equation:
$$a\frac{1-r^8}{1-r} = 5 \times a\frac{1-r^4}{1-r}$$
$$\frac{1-r^8}{1-r^4} = 5$$
$$r^8 - 5r^4 + 4 = 0$$
Let $x = r^4$, giving the quadratic equation:
$x^2 - 5x + 4 = 0$
Solving for x , we get:
$x = 4$ or $x = 1$
Since $x = r^4$, this gives:
$r^4 = 4 \Rightarrow r = \pm\sqrt{2}$
The number of possible real values of r is two: $\pm\sqrt{2}$.

3. (b) Given:
Equation is $x^2 - kx + k = 0$
One root is greater than the other by $2\sqrt{3}$.
Let roots be β and $\beta + 2\sqrt{3}$
Step 1: Use sum and product of roots
Sum $= 2\beta + 2\sqrt{3} = k$
Product $= \beta(\beta + 2\sqrt{3}) = k$
From sum:
$$\beta = \frac{k - 2\sqrt{3}}{2}$$
Substitute into product:
$$\left(\frac{k-2\sqrt{3}}{2}\right)\left(\frac{k+2\sqrt{3}}{2}\right) = k \Rightarrow \frac{k^2 - (2\sqrt{3})^2}{4}$$
$$= k \Rightarrow \frac{k^2 - 12}{4} = k \Rightarrow k^2 - 4k - 12 = 0$$
Solve:
$$k = \frac{4 \pm \sqrt{64}}{2} = \frac{4 \pm 8}{2} \Rightarrow k = 6 \text{ or } -2$$

4. (a) Given:

$x+\frac{5}{y}=4$

$y+\frac{5}{x}=-4$

Step 1: Add both equations

$x+y+5\left(\frac{1}{x}+\frac{1}{y}\right)=0$

Step 2: Multiply both equations

$\left(x+\frac{5}{y}\right)\left(y+\frac{5}{x}\right)=-16 \Rightarrow xy+10+\frac{25}{xy}=-16 \Rightarrow xy+\frac{25}{xy}=-26$

Let $xy=z$:

$z+\frac{25}{z}=-26 \Rightarrow z^2+26z+25=0 \Rightarrow z=-1 \text{ or } -25$

Try $xy=-1$ in (i):

$x+y+5\left(\frac{x+y}{-1}\right)=0 \Rightarrow (x+y)(1-5)=0 \Rightarrow (x+y)(-4)=0 \Rightarrow x+y=0$

5. **(a)** Given: 5th, 7th, and 13th terms of an AP are in GP.

Let first term $=a$, common difference $=d$

Then:

5 th term $=a+4d$

7 th term $=a+6d$

13 th term $=a+12d$

Since they are in GP:$(a+6d)^2=(a+4d)(a+12d)$

Expand:

LHS: $a^2+12ad+36d^2$

RHS: $a^2+16ad+48d^2$

Subtract:

$a^2+12ad+36d^2=a^2+16ad+48d^2 \Rightarrow -4ad-12d^2=0 \Rightarrow a/d=-3$

6. **(c)** Given:

$p,1,q$ in AP $\rightarrow p+q=2$

$p,2,q$ in GP $\rightarrow pq=4$

Statement I: $p,4,q$ in HP

$\Rightarrow$ Check if $\frac{1}{p},\frac{1}{4},\frac{1}{q}$ in AP

$\frac{1}{p}+\frac{1}{q}=\frac{p+q}{pq}=\frac{2}{4}=\frac{1}{2} \Rightarrow$ Average $=\frac{1}{4}$

True

Statement II: $\frac{1}{p},\frac{1}{4},\frac{1}{q}$ in AP

Already shown above.

7. **(d)** Given:

$x=(1111)_2=15$,

$y=(1001)_2=9$,

$z=(110)_2=6$

We need:

$x^3-y^3-z^3-3xyz$

Use identity:

$$x^3-y^3-z^3-3xyz=(x-y-z)^3$$

$x-y-z=15-9-6=0 \Rightarrow (0)^3=0=(0)_2$

8. **(a)** Given:

$\Delta=\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$

Cofactors:

B,C for b,c,

D for d,

G for g
Expression:
$bB + cC - dD - gG$
Key Idea:
Cofactor expansion of a row with cofactors of a different row/column gives 0 .
So this expression is not a full expansion of any row or column →
Hence:
$bB + cC - dD - gG = 0$

9. **(c)** Given determinant:

$$\Delta = \begin{vmatrix} k(k+2) & 2k+1 & 1 \\ 2k+1 & k+2 & 1 \\ 3 & 3 & 1 \end{vmatrix}$$

Step: Apply $R_1 \to R_1 - R_2$, and $R_2 \to R_2 - R_3$
After operations:

$$\Delta = \begin{vmatrix} k(k+2)-(2k+1) & (2k+1)-(k+2) & 0 \\ (2k+1)-3 & (k+2)-3 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} k^2 & k-1 & 0 \\ 2k-2 & k-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$$

Now expand along 3rd column (has two zeros):

$$\Delta = 1 \cdot \begin{vmatrix} k^2 & k-1 \\ 2k-2 & k-1 \end{vmatrix} = k^2(k-1) - (2k-2)(k-1)$$

Factor ($k-1$):
$= (k-1)[k^2 - (2k-2)] = (k-1)(k^2 - 2k + 2)$
Now analyze:
If $k = 0: \Delta = (-1)(0 - 0 + 2) = -2 \to$ Not zero
So Statement III is false
Try:
$k > 0 \Rightarrow k - 1$ and $k^2 - 2k + 2$ both positive for $k > 1 \to \Delta > 0$
$k < 0 \Rightarrow k - 1 < 0$, and quadratic part is always positive $\Rightarrow \Delta < 0$

10. **(b)** -6

11. **(b)**

12. **(a)** Given determinant:

$$\Delta = \begin{vmatrix} x+1 & \omega & \omega^2 \\ \omega & x+\omega^2 & 1 \\ \omega^2 & 1 & x+\omega \end{vmatrix} = 0$$

Try $x = \omega$:
Then:
$x + \omega^2 = -1$
$x + \omega = 2\omega$
So determinant becomes:

$$\begin{vmatrix} \omega+1 & \omega & \omega^2 \\ \omega & -1 & 1 \\ \omega^2 & 1 & 2\omega \end{vmatrix} = 0$$

13. **(a)** Let's simplify the base
Multiply numerator and denominator by the conjugate of the denominator:

$$\frac{\sqrt{3}+i}{\sqrt{3}-i} \cdot \frac{\sqrt{3}+i}{\sqrt{3}+i} = \frac{(\sqrt{3}+i)^2}{(\sqrt{3})^2 - i^2} = \frac{(\sqrt{3}+i)^2}{3+1} = \frac{(\sqrt{3}+i)^2}{4}$$

Now expand numerator:
$(\sqrt{3}+i)^2 = 3 + 2\sqrt{3}i + i^2 = 3 + 2\sqrt{3}i - 1 = 2 + 2\sqrt{3}i$
So the expression becomes:

$$\frac{2+2\sqrt{3}i}{4} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

Step 2: Convert to polar form

$\frac{1}{2}+\frac{\sqrt{3}}{2}i=\text{cis}\left(\frac{\pi}{3}\right)$

So:

$\left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^3=\left(\text{cis}\frac{\pi}{3}\right)^3=\text{cis}(\pi)=-1$

14. (c) We are given:

$x^2-x+1=0\Rightarrow x+\frac{1}{x}=1$ (since multiplying both sides by x)

Let:

$y=x-\frac{1}{x}$

We know:

$\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2\Rightarrow 1^2=x^2+\frac{1}{x^2}+2\Rightarrow x^2+\frac{1}{x^2}=-1$

Now use identity:

$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2=-1-2=-3\Rightarrow y^2=-3$

We are asked to find:

$y^2+y^4+y^8$

We know:

$y^2=-3$

$y^4=(-3)^2=9$

$y^8=9^2=81$

Now add:

$y^2+y^4+y^8=-3+9+81=87$

15. (c) Word: CAPITAL → 7 letters, with A repeated twice

Consonants: $C,P,T,L(4)$

Vowels: A, A, I (3)

Treat 4 consonants as 1 block, so arranging: [CPTL], A, A, I

⇒ Total = 4 items (1 block + 3 vowels), with 2 A's

Ways to arrange:

$\frac{4!}{2!}=12$

Ways to arrange consonants in the block:

$4!=24$

Total $=12\times 24=288$

16. (a) We are given a non-zero complex number z, and asked to find:

$\arg(z)+\arg(\bar{z})$

Key Concept:

For any non-zero complex number $z=re^{i\theta}$, its conjugate is $\bar{z}=re^{-i\theta}$

So:

$\arg(z)=\theta,\arg(\bar{z})=-\theta\Rightarrow\arg(z)+\arg(\bar{z})=\theta+(-\theta)=0$

17. (c) To find the number of sides n in a polygon with 20 diagonals, use the formula:

$$\text{Number of diagonals }=\frac{n(n-3)}{2}$$

Set it equal to 20:

$\frac{n(n-3)}{2}=20\Rightarrow n(n-3)=40\Rightarrow n^2-3n-40=0$

Solve the quadratic:

$n=\frac{3\pm\sqrt{(-3)^2+4\cdot 40}}{2}=\frac{3\pm\sqrt{9+160}}{2}=\frac{3\pm\sqrt{169}}{2}=\frac{3\pm 13}{2}$

So $n=\frac{16}{2}=8$ (reject negative root)

18. (c) Word: DELHI (5 letters)

Vowels: E, I → at positions 2 and 5
Consonants: D, L, H → at positions 1, 3, and 4
We are to keep vowel and consonant positions fixed, only rearrange within their groups.
Step 1: Rearranging vowels (E,I) in their 2 fixed positions:
$2! = 2$ ways
Step 2: Rearranging consonants (D, L, H) in their 3 fixed positions:
$3! = 6$ ways
Total arrangements:
$2! \times 3! = 2 \times 6 = 12$
19. (c) We are given the equation:
$x + y + z = 5$
where x, y, z are positive integers.
Formula:
The number of positive integer solutions of
$x_1 + x_2 + \cdots + x_r = n$
is:
$\binom{n-1}{r-1}$
Here:
$n = 5$
$r = 3$ (variables: x, y, z)
Solutions $= \binom{5-1}{3-1} = \binom{4}{2} = 6$
20. (c) We are given the expression:
$(\sqrt{3} + \sqrt[4]{5})^{12} = \left(3^{1/2} + 5^{1/4}\right)^{12}$
We need to find how many rational terms are in the binomial expansion.
General Term in Expansion:
The general term is:
$T_k = \binom{12}{k} \cdot \left(3^{1/2}\right)^{12-k} \cdot \left(5^{1/4}\right)^k = \binom{12}{k} \cdot 3^{(12-k)/2} \cdot 5^{k/4}$
This term is rational only if both exponents are integers:
So:
$\frac{12-k}{2} \in \mathbb{Z} \Rightarrow 12 - k$ is even $\Rightarrow k$ is even
$\frac{k}{4} \in \mathbb{Z} \Rightarrow k$ is divisible by 4
So: k must be divisible by 4 and even $\Rightarrow k = 0,4,8,12$
That gives 4 values of $k \Rightarrow$ 4 rational terms.
21. (c) We are given:
Sum of binomial coefficients in $(x + y)^n = 256$
Step 1: Use the identity
$\sum_{k=0}^{n} \binom{n}{k} = 2^n \Rightarrow 2^n = 256 \Rightarrow n = 8$
Step 2: Find term with greatest binomial coefficient in expansion of $(x + y)^8$
The maximum binomial coefficient occurs at:
Middle term(s):
If n is even, max is at $k = n/2$
So for $n = 8$, max at $\binom{8}{4}$
That corresponds to the fifth term (since term index $= k + 1$)
22. (b) Given,
The inequality is: $k < (\sqrt{2} + 1)^3 < k + 2$, where k is a natural numb
We need to compute $(\sqrt{2} + 1)^3$
$(\sqrt{2} + 1)^3 = 2\sqrt{2} + 6 + 3\sqrt{2} + 1 = 5\sqrt{2} + 7$
Approximating $\sqrt{2} \approx 1.414$, we find:
$5\sqrt{2} + 7 \approx 5 \times 1.414 + 7 = 7.07 + 7 = 14.07$

The inequality becomes:
$k < 14.07 < k + 2$
This implies that $k > 12.07$ V, so the smallest integer value of k is:
$k = 13$
∴ The value of k is 13 .
23. (b) -1
24. (b) 1
25. (c)
26. (d)
27. (d) Given:
$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$
Find $A^2 - 4A$
Step-by-step:
1. $A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$
2. $4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}$
3. Subtract:
$$A^2 - 4A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = 5I_3$$
28. (b) We are given:
Number of selections of r as well as $n + r$ items from $5n$ different items are equal.
That means:
$$\binom{5n}{r} = \binom{5n}{n+r}$$
Step 1: Use binomial identity
We know:
$$\binom{n}{k} = \binom{n}{n-k}$$
So:
$$\binom{5n}{r} = \binom{5n}{n+r} = \binom{5n}{5n-(n+r)} = \binom{5n}{4n-r}$$
So we now have:
$$\binom{5n}{r} = \binom{5n}{4n-r} \Rightarrow r = 4n - r \Rightarrow 2r = 4n \Rightarrow r = 2n$$
29. (d) We are asked:
Number of selections of at most 3 things from 6 different things.
Step: "At most 3" means selecting:
0 things
1 thing
2 things
3 things
So total number of selections:
$$\binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} = 1 + 6 + 15 + 20 = 42$$
30. (b) We are given that:
$A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ is an ** orthogonal matrix**
By definition, a matrix A is orthogonal if:
$A^T A = I \Rightarrow A^{-1} = A^T$
We are asked: What is A^2 ?

Step:
If a matrix is orthogonal and symmetric (i.e., $A = A^T$), then:
$A^T = A \Rightarrow A^{-1} = A \Rightarrow AA = I \Rightarrow A^2 = I$
Let's check if A is symmetric:
$A^T = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix} = A \Rightarrow A$ is symmetric
And given that A is orthogonal →
So:
$A^2 = I$
31. (b) We are given:
$p - \sin 35°$
$q - \sin 25°$
$r = \sin(-95°)$
Step 1: Use identity
$\sin(-\theta) = -\sin(\theta) \rightarrow r - \sin(-95°) - -\sin 95°$
So:
$p + q + r - \sin 35° + \sin 25° - \sin 95°$
Step 2: Use identity for sum:
$\sin A + \sin B - 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$
Apply to $\sin 35° + \sin 25°$:
$-2\sin(30°)\cos(5°) - 2 \cdot \frac{1}{2} \cdot \cos(5°) - \cos(5°)$
So now:
$p + q + r - \cos(5°) - \sin(95°)$
But:
$\sin(95°) - \cos(5°) \rightarrow p + q + r - \cos(5°) - \cos(5°) - 0$
32. (a) We are continuing from the previous definitions:
$p - \sin 35°$
$q - \sin 25°$
$r - \sin(-95°) - -\sin 95° = -\cos 5°$
Now we need to find:
$pq + qr + rp$
Step 1: Approximate values
Use calculator or known values:
$\sin 35° \approx 0.574$
$\sin 25° \approx 0.423$
$\cos 5° \approx 0.996 \rightarrow r = -0.996$
Now compute:
$pq - (0.574)(0.423) \approx 0.243$
$qr - (0.423)(-0.996) \approx -0.421$
$rp - (-0.996)(0.574) \approx -0.572$
$pq + qr + rp \approx 0.243 - 0.421 - 0.572 - -0.750 = -\frac{3}{4}$
33. (c) We are given:
$p - \sin 35° \approx 0.574$
$q - \sin 25° \approx 0.423$
$r - \sin(-95°) - -\sin 95° - -\cos 5° \approx -0.996$
We are asked to find:
$p^2 + q^2 + r^2$
Step-by-step:
$p^2 \approx (0.574)^2 \approx 0.329$
$q^2 \approx (0.423)^2 \approx 0.179$
$r^2 \approx (-0.996)^2 \approx 0.992$
Add them:

$p^2+q^2+r^2 \approx 0.329+0.179+0.992-1.5=\frac{3}{2}$

34. (a) We are given:
$p-|\sin\alpha-\sin(\alpha-90°)|$
Step 1: Use identity
$\sin(\alpha-90°)--\cos\alpha$
So:
$p-|\sin\alpha-(-\cos\alpha)|-|\sin\alpha+\cos\alpha|$
Step 2: Minimize $p=|\sin\alpha+\cos\alpha|$
We know:
$|\sin\alpha+\cos\alpha| \in [0,\sqrt{2}]$
So minimum value is:
0 (when $\sin\alpha--\cos\alpha$, e.g., $\alpha-225°$)

35. (b) We are given:
$p-|\sin\alpha-\sin(\alpha-90°)|$
Step 1: Use identity
$\sin(\alpha-90°)--\cos\alpha \rightarrow p-|\sin\alpha+\cos\alpha|$
Step 2: Find maximum of $|\sin\alpha+\cos\alpha|$
We know:
$|\sin\alpha+\cos\alpha| \leq \sqrt{2}$ (maximum when $\sin\alpha-\cos\alpha-\frac{1}{\sqrt{2}}$)

36. (c) Given triangle sides: $\mathrm{AB}=3$ cm, $\mathrm{BC}=5$ cm, $\mathrm{CA}=7$ cm.
Let $\mathrm{a}=5, \mathrm{b}=7, \mathrm{c}=3$. Use cosine rule for angle B :
$\cos B=\frac{a^2+c^2-b^2}{2ac}$
$=\frac{25+9-49}{30}$
$=\frac{-15}{30}--0.5$
⇨ $\angle B=120°$ (obtuse)
So Statement I is correct.
Now total angle $=180°$, and one angle is $120°$, so other two add up to $60°$ (which is acute).
So Statement II is also correct.

37. (b) $105°$

38. (a) Given sides of triangle ABC :
$\mathrm{AB}=3$ cm, $\mathrm{BC}=5$ cm, $\mathrm{CA}=7$ cm
Let:
$a=BC=5$
$b=CA=7$
$c=AB=3$
Use Heron's formula:
Step 1: Semi-perimeter
$s=\frac{a+b+c}{2}=\frac{5+7+3}{2}=\frac{15}{2}$
Step 2: Area
Area $=\sqrt{s(s-a)(s-b)(s-c)}$
$=\sqrt{\frac{15}{2}\left(\frac{15}{2}-5\right)\left(\frac{15}{2}-7\right)\left(\frac{15}{2}-3\right)}$
$=\sqrt{\frac{15}{2}\cdot\frac{5}{2}\cdot\frac{1}{2}\cdot\frac{9}{2}}=\sqrt{\frac{15\cdot5\cdot1\cdot9}{16}}=\sqrt{\frac{675}{16}}=\frac{\sqrt{675}}{4}$
$\sqrt{675}=\sqrt{25\cdot27}=5\sqrt{27}$

$= 5 \cdot 3\sqrt{3} = 15\sqrt{3} \Rightarrow$ Area $= \dfrac{15\sqrt{3}}{4}$

39. (b)

40. (a) $\left(\frac{3+\sqrt{3}}{2}\right) b$

41. (c) We are given:

$p = \tan 2\alpha - \tan \alpha, q = \cot \alpha - \cot 2\alpha$

We are to find $\frac{p}{q}$.

Step 1: Use identity

$\cot \theta = \dfrac{1}{\tan \theta} \Rightarrow q = \dfrac{1}{\tan \alpha} - \dfrac{1}{\tan 2\alpha} = \dfrac{\tan 2\alpha - \tan \alpha}{\tan \alpha \cdot \tan 2\alpha}$

Step 2: Now,

$\dfrac{p}{q} = \dfrac{\tan 2\alpha - \tan \alpha}{\dfrac{\tan 2\alpha - \tan \alpha}{\tan \alpha \cdot \tan 2\alpha}} = \tan \alpha \cdot \tan 2\alpha$

42.(d) Given:

$p = \tan 2\alpha - \tan \alpha, q = \cot \alpha - \cot 2\alpha = \dfrac{\tan 2\alpha - \tan \alpha}{\tan \alpha \cdot \tan 2\alpha}$

So,

$p + q = (\tan 2\alpha - \tan \alpha)\left(1 + \dfrac{1}{\tan \alpha \cdot \tan 2\alpha}\right)$

$= (\tan 2\alpha - \tan \alpha) \cdot \dfrac{\tan \alpha \cdot \tan 2\alpha + 1}{\tan \alpha \cdot \tan 2\alpha}$

Try $\alpha = 15^\circ$:

Then $p + q \approx 2.309$, and $2\csc 4\alpha = 2\csc 60^\circ = 2 \times \frac{2}{\sqrt{3}} \approx 2.309$

$= 2\text{cosec}4\alpha$

43. (c) We are given:

$p = \tan 2\alpha - \tan \alpha, q = \cot \alpha - \cot 2\alpha$

We are to find $\tan^2 \alpha$ in terms of p and q.

Step-by-step (short form):

Write q in terms of tangent:

$q = \dfrac{1}{\tan \alpha} - \dfrac{1}{\tan 2\alpha}$

$= \dfrac{p}{\tan \alpha \cdot \tan 2\alpha}$

$\Rightarrow \tan^2 \alpha$

$= \dfrac{p}{p + 2q}$

44. (c) We are given:

$2\sin \alpha + \cos \alpha = 2$, where $0^\circ < \alpha < 90^\circ$

We are to find $\tan \alpha$.

Step 1: Let's square both sides:

$(2\sin \alpha + \cos \alpha)^2 = 2^2 = 4$

$4\sin^2 \alpha + 4\sin \alpha\cos \alpha + \cos^2 \alpha = 4$

Now use identity:

$\sin^2 \alpha + \cos^2 \alpha = 1 \Rightarrow 4(1 - \cos^2 \alpha) + 4\sin \alpha\cos \alpha + \cos^2 \alpha = 4$

Simplify:

$4 - 4\cos^2 \alpha + 4\sin \alpha\cos \alpha + \cos^2 \alpha = 4 \Rightarrow -3\cos^2 \alpha + 4\sin \alpha\cos \alpha = 0$

Factor:

$\cos \alpha(-3 \cos \alpha + 4 \sin \alpha)$

$= 0 \Rightarrow -3 \cos \alpha + 4 \sin \alpha$

$= 0 \Rightarrow \dfrac{\sin \alpha}{\cos \alpha} = \dfrac{3}{4}$

$\Rightarrow \tan \alpha = \dfrac{3}{4}$

45. (b) We are given:
$2\sin\alpha + \cos\alpha = 2$ with $0° < \alpha < 90°$
We are to find:
$2\sin 2\alpha + \cos 2\alpha$
Step 1: From previous solution, we found:
$\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}, \cos\alpha = \frac{4}{5}$ (Using Pythagorean triple)
Step 2: Use double angle identities:
$\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$
$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16-9}{25} = \frac{7}{25}$
Now compute:
$2\sin 2\alpha + \cos 2\alpha = 2\cdot\frac{24}{25} + \frac{7}{25}$
$= \frac{48+7}{25} = \frac{55}{25} = \frac{11}{5}$
46. (b)
47. (c) We are given the triangle with angles:
$\angle B = x = 30°$
$\angle A = 3x = 90°$
$\angle C = 180° - 120° = 60°$
Step 1: Check if the sum of the angles is $180°$:
$\angle A + \angle B + \angle C = 90° + 30° + 60° = 180°$
This confirms that the angles satisfy the angle sum property of a triangle.
Statement I. The triangle is right-angled.
Since $\angle A = 90°$, the triangle is right-angled.
Statement III: III. The angles A, C and B of the triangle are in AP.
The angles $30°60°$, and90 $°$ are in Arithmetic Progression because:
$60° - 30° = 30°$ and $90° - 60° = 30°$
This confirms that the angles are in AP.
Statement II is not correct because there is no mention of a side being 3 times the other.
∴ The correct answer is Option (I) and (III) are correct.
48. (b) We are given:
A man is standing at point M , which is 100 m away from the base P of a chimney.
The height of the chimney (PR) is 50 m .
The highest point of the smoke is Q , and the angle of elevation from M to Q is $45°$.
Points P, R, Q lie on a vertical straight line, perpendicular to PM .
You are to find the angle ∠RMQ.
Step 1: Coordinates Setup
Let's place the figure in a coordinate plane:
Let point $P = (0,0)$ (base of the chimney)
Then $\mathrm{R} = (0,50)$ (top of chimney)
Since smoke continues vertically above R_r let $\mathrm{Q} = (0, \mathrm{h})$ for some $h > 50$
Point $M = (100,0)$
We're given:
$\angle QMP = 45°$
So angle of elevation from M to Q is $45°$, thus:
$\tan(45°) = \frac{h}{100} = 1 \Rightarrow h = 100$
So the height of point Q is 100 m .
Step 2: Now Find ∠RMQ
We need to find angle RMQ , i.e., the angle between vectors MR and MQ .
Let's find vectors:
Vector $\mathrm{MR} = \mathrm{R} - \mathrm{M} = (0,50) - (100,0) = (-100,50)$
Vector $\mathrm{MQ} = \mathrm{Q} - \mathrm{M} = (0,100) - (100,0) = (-100,100)$

Use dot product formula:

$$\cos(\theta) = \frac{\overrightarrow{MR} \cdot \overrightarrow{MQ}}{|\overrightarrow{MR}||\overrightarrow{MQ}|}$$

Dot product:

$$\overrightarrow{MR} \cdot \overrightarrow{MQ} = (-100)(-100) + (50)(100) = 10000 + 5000 = 15000$$

Magnitudes:

$$|\overrightarrow{MR}| = \sqrt{(-100)^2 + 50^2} = \sqrt{10000 + 2500} = \sqrt{12500}$$

$$|\overrightarrow{MQ}| = \sqrt{(-100)^2 + 100^2} = \sqrt{10000 + 10000} = \sqrt{20000}$$

Now:$\cos(\theta) = \frac{15000}{\sqrt{12500 \cdot 20000}} = \frac{15000}{\sqrt{2.5 \times 10^4 \times 2 \times 10^4}} = \frac{15000}{\sqrt{5 \times 10^8}} = \frac{15000}{\sqrt{5} \cdot 10^4}$

Now calculate:

$$\cos(\theta) = \frac{15000}{10^4 \cdot \sqrt{5}} = \frac{3}{2\sqrt{5}}$$

So:

$$\theta = \cos^{-1}\left(\frac{3}{2\sqrt{5}}\right)$$

Instead of evaluating that, we can use tangent for better match with options.

Let's compute tan(∠RMQ) using vector formula:

$$\tan(\theta) = \frac{|MR \times MQ|}{MR \cdot MQ}$$

Compute cross product magnitude in 2D:

$$|\overrightarrow{MR} \times \overrightarrow{MQ}| = |(-100)(100) - (-100)(50)| = |-10000 + 5000| = 5000$$

So:

$$\tan(\theta) = \frac{5000}{15000} = \frac{1}{3} \Rightarrow \theta = \tan^{-1}\left(\frac{1}{3}\right)$$

49. (d) Given k is a root of $x^2 - 4x + 1 = 0$, solving the quadratic gives:

$$k = \frac{4 \pm \sqrt{16 - 4}}{2} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

So $k = 2 + \sqrt{3}$ or $2 - \sqrt{3}$, both are positive.

We are asked to find $\tan^{-1}(k) + \tan^{-1}(1/k)$.

Using the identity:

$$\tan^{-1}(a) + \tan^{-1}\left(\frac{1}{a}\right) = \frac{\pi}{2} \quad \text{for } a > 0, a \neq 1$$

Since $k > 0$, we apply the identity:

$$\tan^{-1}(k) + \tan^{-1}(1/k) = \frac{\pi}{2}$$

50. (c) We are given:

$$\tan^{-1}(k) + \tan^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{4}$$

Step 1: Use identity

$$\tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}\left(\frac{a + b}{1 - ab}\right), \text{ if } ab < 1$$

So,

$$\tan^{-1}(k) + \tan^{-1}\left(\frac{1}{2}\right) = \tan^{-1}\left(\frac{k + \frac{1}{2}}{1 - \frac{k}{2}}\right)$$

Set equal to $\frac{\pi}{4} \Rightarrow \tan^{-1}$ (something) $= \tan^{-1}(1) \Rightarrow$ something $= 1$

So:

$$\frac{k + \frac{1}{2}}{1 - \frac{k}{2}} = 1$$

Step 2: Solve the equation

Multiply both sides by denominator:

$k+\frac{1}{2}=1-\frac{k}{2}$

Multiply entire equation by 2 to eliminate fractions:

$2k+1=2-k \Rightarrow 3k=1 \Rightarrow k=\frac{1}{3}$

51. (a) We are given two lines:

$m^2x+ny-1=0$

$n^2x-my+2=0$

We need to find the condition under which these two lines are perpendicular.

Step 1: Find slopes

Convert each line into slope-intercept form $y=mx+c$:

Line 1:

$m^2x+ny=1 \Rightarrow y=\frac{-m^2}{n}x+\frac{1}{n} \Rightarrow$ slope $=-\frac{m^2}{n}$

Line 2:

$n^2x-my=-2 \Rightarrow y=\frac{n^2}{m}x+\frac{2}{m} \Rightarrow$ slope $=\frac{n^2}{m}$

Step 2: Use condition for perpendicular lines

Slopes of perpendicular lines satisfy:

$m_1 \cdot m_2=-1$

So:

$\left(-\frac{m^2}{n}\right)\cdot\left(\frac{n^2}{m}\right)=-1 \Rightarrow -\frac{m^2\cdot n^2}{n\cdot m}$

$=-1 \Rightarrow -\frac{mn}{1}=-1 \Rightarrow mn=1$

=mn−1=0

52. (d) Given points $A(p,1), B(1,q), C(0,0)$ form an equilateral triangle with $p,q \in (0,1)$.

Find distances:

$AB=\sqrt{(1-p)^2+(q-1)^2}$

$BC=\sqrt{1+q^2}$

$CA=\sqrt{p^2+1}$

Using $AB=BC$ and $AB=CA$, and solving the two equations:$(1-p)^2+(q-1)^2=1+q^2$ and $(1-p)^2+(q-1)^2=p^2+1$

Solving gives $p=q$, substitute into one equation:

$p^2-4p+1=0 \Rightarrow p=2-\sqrt{3}$

So $p+q=4-2\sqrt{3}$

53. (a) Given triangle with vertices:

$A(1,1), B(0,0), C(2,0)$

We are to find point P , the intersection of angular bisectors - i.e., the incenter of the triangle.

Step: Use incenter formula

For triangle with vertices $A(x_1,y_1), B(x_2,y_2), C(x_3,y_3)$, the incenter is:

$P=\left(\frac{ax_1+bx_2+cx_3}{a+b+c},\frac{ay_1+by_2+cy_3}{a+b+c}\right)$

where $a=BC, b=AC, c=AB$

Find side lengths:

$a=BC=\sqrt{(2-0)^2+(0-0)^2}=2$

$b=AC=\sqrt{(2-1)^2+(0-1)^2}=\sqrt{2}$

$c=AB=\sqrt{(1-0)^2+(1-0)^2}=\sqrt{2}$

Now plug into incenter formula:

$x=\frac{2(1)+\sqrt{2}(0)+\sqrt{2}(2)}{2+\sqrt{2}+\sqrt{2}}=\frac{2+2\sqrt{2}}{2+2\sqrt{2}}=1$

$y = \frac{2(1) + \sqrt{2}(0) + \sqrt{2}(0)}{2 + 2\sqrt{2}}$

$= \frac{2}{2 + 2\sqrt{2}} = \frac{1}{1 + \sqrt{2}} = \sqrt{2} - 1$

54. (b) Given $A(3, -1), B(1,1)$,
Midpoint $P = (2,0)$
Slope of AB $= -1 \Longrightarrow$ Perpendicular bisector slope = 1
Line: $y = x - 2$
Let $Q = (x, x - 2)$, and distance $PQ = \sqrt{2}$:
$\sqrt{(x-2)^2 + (x-2)^2} = \sqrt{2} \Rightarrow 2(x-2)^2 = 2 \Rightarrow (x-2)^2 = 1 \Rightarrow x = 1$ or 3
So points: $(1, -1), (3,1)$

55. (d) We are given:
Triangle ABC is equilateral.
AD is altitude from vertex $A(1,2)$ to side BC, meeting at point $D(-2,6)$.
We are to find the equation of line BC , which is perpendicular to AD .
Step 1: Find slope of AD

slope of $AD = \frac{6-2}{-2-1} = \frac{4}{-3} = -\frac{4}{3}$

Since $AD \perp BC$, the slope of BC is the negative reciprocal:

slope of $BC = \frac{3}{4}$

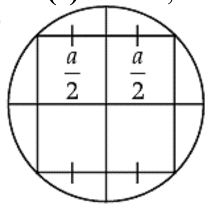
Step 2: Use point-slope form with point $D(-2,6)$

$y - 6 = \frac{3}{4}(x + 2) \Rightarrow 4(y - 6) = 3(x + 2)$

$\Rightarrow 4y - 24 = 3x + 6 \Rightarrow 3x - 4y + 30 = 0$

56. (b) We are given:
Diameter of circle $= 10$ cm $\Rightarrow$ radius $= 5$ cm
Two diameters are along lines: $x + y = 0$ and $x - y = 0$
These lines are perpendicular and intersect at the center of the circle
Step 1: Find center of the circle
The intersection point of $x + y = 0$ and $x - y = 0$
Solve:
From $x - y = 0 \Rightarrow x = y$
Substitute into $x + y = 0: x + x = 0 \Rightarrow x = 0 \Rightarrow y = 0$
So, center $= (0,0)$
Step 2: Write equation of circle
Standard form:
$(x - 0)^2 + (y - 0)^2 = r^2 \Rightarrow x^2 + y^2 = 25$

57. (c) Given,



The circle's equation is
$x^2 + y^2 + 2x + 2y + 1 = 0$
Rewrite by completing squares:
$(x + 1)^2 + (y + 1)^2 = 1$
$\therefore$ center $= (-1, -1)$, radius r $= 1$
For a square inscribed in this circle with sides parallel to the axes, its diagonal equals the circle's diameter $= 2$. If the square has side length a, then
$a\sqrt{2} = 2 \Longrightarrow a = \sqrt{2}$

Each vertex lies a half-side $= \frac{a}{2} = \frac{1}{\sqrt{2}}$ from the center along both axes. Since the center is $(-1,-1)$, the four vertices are

$$\left(-1 \pm \frac{1}{\sqrt{2}}, -1 \pm \frac{1}{\sqrt{2}}\right)$$

One of these vertices is

$$\left(-1 + \frac{1}{\sqrt{2}}, -1 - \frac{1}{\sqrt{2}}\right)$$

58. (d) We are given:

Parabola: $y^2 = 4x$

A tangent is inclined at 45° to the x-axis $\Rightarrow$ slope $m = \tan 45^\circ = 1$

We are to find the point of contact of this tangent with the parabola.

Step 1: General form of tangent to $y^2 = 4x$ with slope m

For parabola $y^2 = 4x$, the tangent with slope m has equation:

$$y = mx + \frac{1}{m}$$

For $m = 1$, the tangent becomes:

$$y = x + 1$$

Step 2: Find point of contact by solving with the parabola

Substitute into $y^2 = 4x$:

$$(x+1)^2 = 4x \Rightarrow x^2 + 2x + 1 = 4x$$

$$\Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2$$

$$= 0 \Rightarrow x = 1 \Rightarrow y = 1 + 1 = 2$$

So, point of contact is $(1,2)$

59. (b) We are given the hyperbola:

$$25x^2 - 75y^2 = 225$$

Convert to standard form

Divide entire equation by 225 :

$$\frac{25x^2}{225} - \frac{75y^2}{225} = 1 \Rightarrow \frac{x^2}{9} - \frac{y^2}{3} = 1$$

This is a standard hyperbola of the form:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

where:

$a^2 = 9 \Rightarrow a = 3$

$b^2 = 3$

Use formula for distance between foci

For hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

distance between foci $= 2c$, where $c = \sqrt{a^2 + b^2}$

$c = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3} \Rightarrow$ Distance $= 2c = 4\sqrt{3}$

60. (b) We are given a parametric point on an ellipse:

$$(x, y) = (3\sin\alpha, 5\cos\alpha)$$

This matches the standard parametric form of an ellipse:

$$x = a\sin\theta, y = b\cos\theta \Rightarrow a = 3, b = 5$$

Step 1: Use eccentricity formula

For ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a < b$,

eccentricity $e = \sqrt{1 - \frac{a^2}{b^2}}$

$$e = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

61. (b) We are given that a line in 3D makes angles α, β, γ with the positive $\mathbf{x}-, \mathbf{y}-, \mathbf{z}$-axes, respectively. In 3D geometry, if a line makes angles α, β, γ with the coordinate axes, then:

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ (Direction cosine identity)
We are to find the value of:
$\cos(\alpha+\beta)\cos(\alpha-\beta)$
Step 1: Use trigonometric identity
We use the identity:
$$\cos(\alpha+\beta)\cos(\alpha-\beta) = \frac{1}{2}[\cos(2\beta) + \cos(2\alpha)]$$
Step 2: Express in terms of $\cos\alpha$ and $\cos\beta$
Use identities:
$\cos(2\alpha) = 2\cos^2\alpha - 1, \cos(2\beta) = 2\cos^2\beta - 1$
So:
$\cos(\alpha+\beta)\cos(\alpha-\beta)$
$= \frac{1}{2}[(2\cos^2\alpha - 1) + (2\cos^2\beta - 1)]$
$= \frac{1}{2}[2(\cos^2\alpha + \cos^2\beta) - 2]$
$= \cos^2\alpha + \cos^2\beta - 1$
Now use the identity:
$\cos^2\alpha + \cos^2\beta = 1 - \cos^2\gamma$
$\Rightarrow \cos^2\alpha + \cos^2\beta - 1 = -\cos^2\gamma$

62. (c) We are given three vertices of a rectangle in 3D:
$A(1,2,-1), B(2,5,-2), C(4,4,-3)$
We are to find the area of the rectangle.
Compute vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$
$\overrightarrow{AB} = B - A = (2-1, 5-2, -2+1) = (1,3,-1)\overrightarrow{AC}$
$= C - A = (4-1, 4-2, -3+1) = (3,2,-2)$
Area of a rectangle = product of two adjacent side lengths
So we check if $\overrightarrow{AB} \perp \overrightarrow{AC}$ using dot product:
$\overrightarrow{AB} \cdot \overrightarrow{AC} = (1)(3) + (3)(2) + (-1)(-2) = 3 + 6 + 2 = 11 \neq 0 \Rightarrow \overrightarrow{AB} \perp \overrightarrow{AC}$ is false
Try $\overrightarrow{BC} = C - B = (4-2, 4-5, -3+2) = (2,-1,-1)$
Now check $\overrightarrow{AB} \cdot \overrightarrow{BC}$:
$(1)(2) + (3)(-1) + (-1)(-1) = 2 - 3 + 1 = 0 \Rightarrow \overrightarrow{AB} \perp \overrightarrow{BC}$
So adjacent sides: $\overrightarrow{AB}$ and $\overrightarrow{BC}$
Find lengths
$|\overrightarrow{AB}| = \sqrt{1^2 + 3^2 + (-1)^2} = \sqrt{1+9+1} = \sqrt{11}$
$|\overrightarrow{BC}| = \sqrt{2^2 + (-1)^2 + (-1)^2} = \sqrt{4+1+1} = \sqrt{6}$
Area $= |\overrightarrow{AB}| \cdot |\overrightarrow{BC}| = \sqrt{11} \cdot \sqrt{6} = \sqrt{66}$

63. (d)

64. (a) Given line:
$\frac{x+1}{p} = \frac{y-1}{q} = \frac{z-2}{r}$, where $p = 2q = 3r$
Let $r = t \Rightarrow p = 3t, q = \frac{3t}{2}$
Direction ratios $= \left(3t, \frac{3t}{2}, t\right)$
Angle with **y**-axis $\vec{j} = (0,1,0)$:
$$\cos\theta = \frac{\frac{3t}{2}}{\sqrt{9t^2 + \frac{9t^2}{4} + t^2}} = \frac{\frac{3t}{2}}{\frac{7t}{2}} = \frac{3}{7}$$
Now,
$$\cos(2\theta) = 2\cos^2\theta - 1 = 2\left(\frac{3}{7}\right)^2 - 1 = \frac{18}{49} - 1 = -\frac{31}{49}$$

65. (b) We are given:

The plane passes through point (1,1,1)
It is perpendicular to a line with direction ratios $\langle 3,2,1\rangle$
Step 1: Use point-normal form of plane
If a plane passes through point (x_0, y_0, z_0) and has normal vector $\langle a, b, c\rangle$, its equation is:
$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$
Here:
Point: (1,1,1)
Normal vector = direction ratios of the line = $\langle 3,2,1\rangle$
Substitute:
$3(x-1)+2(y-1)+1(z-1)$
$=0 \Rightarrow 3x-3+2y-2+z-1$
$=0 \Rightarrow 3x+2y+z=6$
66. (d) We are given:
A line makes angles α, β, γ with the coordinate axes
So, by direction cosine identity:
$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$
Also,
$\vec{a}=\sin^2\alpha\hat{\imath}+\sin^2\beta\hat{\jmath}+\sin^2\gamma\hat{k}$ and $\vec{b}=\hat{\imath}+\hat{\jmath}+\hat{k}$
Step 1: Use identity
Since $\sin^2\theta=1-\cos^2\theta$,
$\vec{a}=(1-\cos^2\alpha)\hat{\imath}+(1-\cos^2\beta)\hat{\jmath}+(1-\cos^2\gamma)\hat{k}$
Step 2: Take dot product $\vec{a}\cdot\vec{b}$
$\vec{a}\cdot\vec{b}=(1-\cos^2\alpha)+(1-\cos^2\beta)+(1-\cos^2\gamma)$
$=3-(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)=3-1=2$
67. (a) We are given a vector:
$\vec{d}=(\vec{a}\times\vec{b})\times\vec{c}$
We are to analyze two statements:
⋄ Statement I:
" $\vec{d}$ is coplanar with $\vec{a}$ and $\vec{b}$ "
Recall the vector triple product identity:
$(\vec{a}\times\vec{b})\times\vec{c}=[\vec{c}\cdot\vec{a}]\vec{b}-[\vec{c}\cdot\vec{b}]\vec{a}$
This shows that $\vec{d}$ is a linear combination of $\vec{a}$ and $\vec{b}$
$\Longrightarrow$ So, $\vec{d}$ lies in the plane of $\vec{a}$ and $\vec{b}$
Statement I is correct
68. (b) We are given:
$3\vec{a}-4\vec{b}+\vec{c}=\vec{0} \Rightarrow \vec{c}=4\vec{b}-3\vec{a}$
Let's find vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$:
$\overrightarrow{AB}=\vec{b}-\vec{a}$
$\overrightarrow{BC}=\vec{c}-\vec{b}=(4\vec{b}-3\vec{a})-\vec{b}=3\vec{b}-3\vec{a}=3(\vec{b}-\vec{a})$
So,
$\overrightarrow{BC}=3\cdot\overrightarrow{AB} \Rightarrow$ Magnitude ratio: $AB:BC=1:3$
69. (a) Use identity and substitute $\vec{c}$
We already know:
$$\vec{b}\times\vec{c}=\vec{b}\times\left(\cos^2\theta\vec{a}+\sin^2\theta\vec{b}\right)=\cos^2\theta(\vec{b}\times\vec{a})+\sin^2\theta(\vec{b}\times\vec{b})=-\cos^2\theta(\vec{a}\times\vec{b})$$
(since $\vec{b}\times\vec{a}=-\vec{a}\times\vec{b}$, and $\vec{b}\times\vec{b}=\vec{0}$)
Similarly,
$\vec{c}\times\vec{a}=\left(\cos^2\theta\vec{a}+\sin^2\theta\vec{b}\right)\times\vec{a}=\cos^2\theta(\vec{a}\times\vec{a})+\sin^2\theta(\vec{b}\times\vec{a})=-\sin^2\theta(\vec{a}\times\vec{b})$
(again using $\vec{a}\times\vec{a}=\vec{0}$, and $\vec{b}\times\vec{a}=-\vec{a}\times\vec{b}$)
Add all terms
$\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}=\vec{a}\times\vec{b}-\cos^2\theta\vec{a}\times\vec{b}-\sin^2\theta\vec{a}\times\vec{b}$
Factor:
$=(1-\cos^2\theta-\sin^2\theta)\cdot(\vec{a}\times\vec{b})=(1-1)\cdot(\vec{a}\times\vec{b})=\vec{0}$

70. (a) We are given:
Vectors $\vec{a}, \vec{b}, \vec{a} \times \vec{b}$ are all unit vectors
Step 1: Use property of cross product
For any two vectors $\vec{a}$ and $\vec{b}$:
$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$
Given:
$|\vec{a}| = |\vec{b}| = 1$
Also, $|\vec{a} \times \vec{b}| = 1$ (it's a unit vector)
So,
$|\vec{a} \times \vec{b}| = \sin\theta = 1 \Rightarrow \theta = 90^\circ$
$\Rightarrow \vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = \cos\theta$
$= \cos 90^\circ = 0$
71. (d) Given:
$x = \sec\theta - \cos\theta, y = \sec^4\theta - \cos^4\theta$
Differentiate both:
$$\frac{dx}{d\theta} = \sec\theta\tan\theta + \sin\theta$$
$$\frac{dy}{d\theta} = 4\sec^4\theta\tan\theta + 4\cos^3\theta\sin\theta$$
So,
$$\left(\frac{dy}{dx}\right)^2 = \left(\frac{dy/d\theta}{dx/d\theta}\right)^2 = \frac{16(y^2-4)}{x^2-4}$$
72. (c) $-16x$
73. (c) Given: Right-angled triangle at B , with $AB + AC = 3$.
Let $AB = x, BC = y$, then $AC = \sqrt{x^2+y^2}$
From $AB + AC = 3$:
$x + \sqrt{x^2+y^2} = 3 \Rightarrow y^2 = 9 - 6x$
Area $A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{9-6x}$
Differentiate and maximize:
$$A'(x) = \frac{1}{2}\left[\sqrt{9-6x} - \frac{6x}{2\sqrt{9-6x}}\right] = 0$$
$\Rightarrow x = 1, y = \sqrt{3}$
$\Rightarrow \tan\angle A = \frac{y}{x} = \sqrt{3} \Rightarrow \angle A = \frac{\pi}{3}$
74. (a) Given:
Triangle ABC is right-angled at B
$AB + AC = 3$ units
We need to find the maximum area
Step 1: Place triangle on coordinate plane
Let:
$B = (0,0)$
$A = (x, 0) \Rightarrow AB = x$
$C = (0, y) \Rightarrow BC = y$
Then $AC = \sqrt{x^2+y^2}$
Given:
$AB + AC = 3 \Rightarrow x + \sqrt{x^2+y^2} = 3$
$\Rightarrow \sqrt{x^2+y^2} = 3 - x \Rightarrow x^2 + y^2$
$= (3-x)^2$
$= 9 - 6x + x^2$
$\Rightarrow y^2 = 9 - 6x$
Area of triangle

$$\text{Area } = \frac{1}{2} \cdot AB \cdot BC = \frac{1}{2} x \cdot y = \frac{1}{2} x \cdot \sqrt{9-6x}$$

Let $A(x) = \frac{1}{2} x \sqrt{9-6x}$

Maximize area

Differentiate:

$$A'(x) = \frac{1}{2}\left[\sqrt{9-6x} - \frac{6x}{2\sqrt{9-6x}}\right] = 0$$

$$\Rightarrow \sqrt{9-6x} - \frac{6x}{2\sqrt{9-6x}} = 0$$

$\Rightarrow 2(9-6x) = 6x$

$\Rightarrow 18 - 12x = 6x \Rightarrow x = 1$

Then $y = \sqrt{9-6x} = \sqrt{3}$

So area:

$$\frac{1}{2} \cdot x \cdot y = \frac{1}{2} \cdot 1 \cdot \sqrt{3} = \frac{\sqrt{3}}{2}$$

75. (d) is independent of both p and q

76. (a) $\frac{y}{x}$

77. (a) Integrate the derivative

$f'(x) = 4 \Rightarrow f(x) = 4x + C$

Using $f(0) = 0$:

$0 = 4 \cdot 0 + C \Rightarrow C = 0 \Rightarrow f(x) = 4x$

So the curve is:

$y = 4x$

Which is a straight line passing through the origin.

Check which option this matches.

Check which point lies on the line

For $x = 1, y = 4 \Rightarrow$ Point $(1,4)$ lies on the line.

78. (c) We are given:

$f'(x) = 4 \Rightarrow f(x) = 4x$ (since the slope is constant and the curve passes through origin)

We are to find the area under the curve from $x = 0$ to $x = 4$

Step 1: Area under the curve

$$\text{Area } = \int_0^4 f(x)dx = \int_0^4 4x dx = 4\int_0^4 x dx$$

$$= 4 \cdot \left[\frac{x^2}{2}\right]_0^4 = 4 \cdot \left(\frac{16}{2}\right) = 4 \cdot 8 = 32$$

79. (c) We are given a piecewise function:

$$f(x) = \begin{cases} x^3, & x^2 < 1 \\ x^2, & x^2 \geq 1 \end{cases}$$

We are to find:

$\lim_{x \to 0} f'(x)$

Step 1: Determine which branch applies near $x = 0$

Since $x^2 < 1$ is true for all x near 0 , we only consider:

$f(x) = x^3$ near $x = 0 \Rightarrow f'(x) = 3x^2$

Now:

$\lim_{x \to 0} f'(x) = \lim_{x \to 0} 3x^2 = 0$

80. (d) We are given a piecewise function:

$$f(x) = \begin{cases} x^3, & x^2 < 1 \\ x^2, & x^2 \geq 1 \end{cases}$$

That is:

For $|x| < 1: f(x) = x^3$

For $|x| \geq 1: f(x) = x^2$

We need to check:

Statement I: Function is continuous at $x = -1$
Statement II: Function is differentiable at $x = 1$
Check Statement I: Continuity at $x = -1$
Left of -1 : use x^3
Right of -1 : use x^2
Check:
$\lim_{x\to-1^-} f(x) = (-1)^3 = -1$
$\lim_{x\to-1^+} f(x) = (-1)^2 = 1$
Since limits from left and right are not equal → Not continuous
Statement I is false
Check Statement II: Differentiability at $x = 1$
Left of 1 : use x^3, derivative is $3x^2$, so left derivative at $1 = 3(1)^2 = 3$
Right of 1 : use x^2, derivative is $2x$, so right derivative at $1 = 2(1) = 2$
Since left and right derivatives are not equal, function is not differentiable at $x = 1$
Statement II is false

81. (b) Understand the domain restriction
Since $\cos x = 1$ at $x = 2n\pi$, we exclude these points because they make the denominator zero.
So, $1 - \cos x \neq 0 \Rightarrow \cos x \neq 1$
Behavior of the function
We analyze:
$$y = \frac{1}{1 - \cos x}$$
$\cos x \in [-1,1)$ (since 1 is excluded)
Therefore, $1 - \cos x \in (0,2]$
So $y = \frac{1}{1-\cos x} \in [0.5, \infty)$
Minimum value:
When $\cos x = -1$:
$$y = \frac{1}{1-(-1)} = \frac{1}{2} = 0.5$$
As $\cos x \to 1^-$:
$1 - \cos x \to 0^+$, so $y \to \infty$

82. (b) We are given:
$y = \frac{1}{1 - \cos x}$ and we are to evaluate: $\int \frac{1}{1-\cos x} dx$
Step 1: Use trigonometric identity
$$1 - \cos x = 2\sin^2\left(\frac{x}{2}\right)$$
So:
$$\frac{1}{1-\cos x} = \frac{1}{2\sin^2\left(\frac{x}{2}\right)} = \frac{1}{2}\csc^2\left(\frac{x}{2}\right)$$
Step 2: Use substitution
Let $u = \frac{x}{2} \Rightarrow dx = 2du$
$$\int \frac{1}{1-\cos x} dx = \int \frac{1}{2}\csc^2\left(\frac{x}{2}\right) dx$$
$$= \int \csc^2(u)\, du = -\cot u + C$$
$$= -\cot\left(\frac{x}{2}\right) + C$$

83. (b) We are given:
$f(x) = \sin[x]$, where $[x]$ is the greatest integer ≤ x
$g(x) = |x|$
We need to evaluate:
$$\lim_{x\to0} f(x)\cdot g(x) = \lim_{x\to0} \sin[x]\cdot|x|$$
Step 1: Understand behavior of $[x]$ near 0

As $x \to 0^-, [x] = -1$, so $\sin[x] = \sin(-1)$
As $x \to 0^+, [x] = 0$, so $\sin[x] = \sin(0) = 0$
Step 2: Left-hand limit
$\lim_{x\to 0} f(x)g(x) = \sin(-1) \cdot |x| = -\sin(1) \cdot (-x) = \sin(1)x \to 0$
(because $x \to 0^- \Rightarrow x \to 0$, so product tends to 0)
84. (d) We are given:
$f(x) - \sin[x]$, where $[x]$ is the greatest integer $\leq$ x
$g(x) - |x|$
We are to compute:
$$\lim_{x\to 0} \frac{f(x)}{g(x)} - \lim_{x\to 0} \frac{\sin[x]}{|x|}$$
Behavior near $x = 0$
As $x \to 0^-$:
$[x] = -1 \to \sin[x] - \sin(-1) - -\sin 1$
$|x| - -x$
So:
$$\lim_{x\to 0^-} \frac{\sin[x]}{|x|} - \frac{-\sin 1}{-x} - \frac{\sin 1}{x} \to -\infty$$
As $x \to 0^+$:
$[x] - 0 \to \sin[x] - 0$
$|x| - x$
So:
$$\lim_{x\to 0^+} \frac{0}{x} - 0$$
Since LHL $\neq$ RHL $\to$ limit does not exist
85. (c) We are given the function:
$f(x) = |x - 3|$
Step 1: Understand absolute value function
The absolute value function $|x - 3|$ is defined for all real numbers.
It simply gives:
$$f(x) = \begin{cases} x - 3 & \text{if } x \geq 3 \\ -(x - 3) & \text{if } x < 3 \end{cases}$$
So, no restrictions on x.
$(-\infty, \infty)$
86. (d) We are given:
$f(x) = |x - 3|$ and $y = 3$
We are to find the area bounded between the curve $f(x)$ and the line $y = 3$.
Understand the geometry
$f(x) = |x - 3|$ is a V-shaped graph with vertex at (3,0)
Line $y = 3$ is horizontal
We want the area between the curve and the line, i.e., where:
$$\text{Area } = \int_a^b (3 - |x - 3|)dx$$
This is only meaningful where $f(x) \leq 3$, i.e., between the points where $|x - 3| = 3$
Solve:
$|x - 3| = 3 \Rightarrow x - 3 = \pm 3 \Rightarrow x = 0$ or $x = 6$
So, area bounded between the curves from $x = 0$ to $x = 6$:
Compute area
$$\text{Area } = \int_0^6 (3 - |x - 3|)dx$$
Break the integral at $x = 3$:
For $x \in [0,3]$: $|x - 3| = 3 - x$
For $x \in [3,6]$: $|x - 3| = x - 3$
So:

Area $= \int_0^3 (3-(3-x))dx + \int_3^6 (3-(x-3))dx$

$= \int_0^3 x dx + \int_3^6 (6-x)dx$

Evaluate:

$\int_0^3 x dx = \frac{3^2}{2} = \frac{9}{2} = 4.5, \int_3^6 (6-x)dx$

$= \frac{(6-3)^2}{2} = \frac{9}{2} = 4.5$

Total area $= 4.5 + 4.5 = 9$

87. (c) We are given:

A function f={(1,1),(2,4),(3,7),(4,10)}

f(x)=px+q (a linear function), and we are to find p+q

Use any two points to form equations

Using $f(1) = 1$ and $f(2) = 4$:

From $f(1) = 1$:

$p(1) + q = 1 \Rightarrow p + q = 1$

From $f(2) = 4$:

$2p + q = 4$

Subtract (1) from (2):

$(2p + q) - (p + q) = 4 - 1 \Rightarrow p = 3$

Put back into (1):

$3 + q = 1 \Rightarrow q = -2$

Step 2: Find $p + q$

$p + q = 3 + (-2) = 1$

88. (a) We are given the function:

$f = \{(1,1), (2,4), (3,7), (4,10)\}$

Let's analyze the statements:

Statement I: **f** is one-one

A function is one-one (injective) if every input maps to a unique output - i.e., no two inputs have the same output.

Here:

$f(1) = 1$

$f(2) = 4$

$f(3) = 7$

$f(4) = 10$

All outputs are distinct $\Rightarrow f$ is one-one

Statement II: **f** is onto if codomain is natural numbers

The range of the function $= \{1,4,7,10\}$

Codomain $= \mathbb{N} = \{1,2,3,4,5,6,7,\ldots\}$

Since the function only maps to a few natural numbers and not every natural number is hit (e.g., 2,3,5, etc. are missing),

$\Rightarrow f$ is NOT onto when codomain $= \mathbb{N}$

89. (a) We are given:

$f(x) = x^2 - 1,$ and we need to compute $\lim_{x\to 1} (f \circ f)(x)$

Step 1: Understand composition

$(f \circ f)(x) = f(f(x)) = f(x^2 - 1)$

So,

$f(f(x)) = f(x^2 - 1) = (x^2 - 1)^2 - 1$

Step 2: Evaluate the limit

$\lim_{x\to 1} f(f(x)) = \lim_{x\to 1} [(x^2 - 1)^2 - 1]$

At $x = 1$:

$x^2 - 1 = 0$

So, $(x^2 - 1)^2 - 1 = 0^2 - 1 = -1$

90. (c)

91. (d) We are given:

$$y = \sin^{-1}\left(x - \frac{4x^3}{27}\right)$$

We are to express this in simpler form, and the options suggest that this is a standard triple-angle identity.

Step 1: Recall triple-angle identity for sine:

$\sin(3\theta) = 3\sin\theta - 4\sin^3\theta$

Let:

$$x = \sin\theta \Rightarrow x - \frac{4x^3}{27} = 3\sin\left(\frac{\theta}{3}\right) - 4\sin^3\left(\frac{\theta}{3}\right)$$

But we can reverse the identity:

Let $x = \frac{1}{3}\sin\theta$

Then:

$$\Rightarrow x - \frac{4x^3}{27} = \sin\theta \Rightarrow \sin^{-1}\left(x - \frac{4x^3}{27}\right) = \theta = 3\sin^{-1}(x/3)$$

92. (c)

93. (c) We are given:

$$f(x) = x^2 + 9 \Rightarrow \lim_{x\to 0}\frac{\sqrt{f(x)} - 3}{\sqrt{f(x) + 7} - 4} = \lim_{x\to 0}\frac{\sqrt{x^2+9} - 3}{\sqrt{x^2+16} - 4}$$

Step 1: Use binomial approximation or rationalization

Let's rationalize numerator and denominator separately.

Step 2: Use identity:

$$\sqrt{a+h} \approx \sqrt{a} + \frac{h}{2\sqrt{a}} \text{ when } h \to 0$$

Apply this:

$$\sqrt{x^2+9} \approx 3 + \frac{x^2}{6}$$

$$\sqrt{x^2+16} \approx 4 + \frac{x^2}{8}$$

So:

$$\frac{\sqrt{x^2+9} - 3}{\sqrt{x^2+16} - 4} \approx \frac{\frac{x^2}{6}}{\frac{x^2}{8}} = \frac{1/6}{1/8} = \frac{8}{6} = \frac{4}{3}$$

94. (d) Neither I nor II

95.(d) We are given:

Functional equation:

$$f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)} \text{ for all positive real } x, y$$

And $f(2) = 3$

We are to find:

$f(16) = ?$

Step 1: Try to express 16 using powers of 2

Since $f(2) = 3$, we try to find a pattern:

Let's find $f(4)$, then $f(8)$, then $f(16)$

Step 2: Use the identity cleverly

Let $x = 4, y = 2$

$$f\left(\frac{4}{2}\right) = \frac{f(4)}{f(2)} \Rightarrow f(2)$$

$$= \frac{f(4)}{f(2)} \Rightarrow 3 = \frac{f(4)}{3} \Rightarrow f(4) = 9$$

Now find $f(8)$:

$f\left(\frac{8}{2}\right)=\frac{f(8)}{f(2)} \Rightarrow f(4)$

$=\frac{f(8)}{3} \Rightarrow 9=\frac{f(8)}{3} \Rightarrow f(8)=27$

Now find $f(16)$:

$f\left(\frac{16}{2}\right)=\frac{f(16)}{f(2)} \Rightarrow f(8)$

$=\frac{f(16)}{3} \Rightarrow 27=\frac{f(16)}{3} \Rightarrow f(16)=81$

96. (c) Given:

$f\left(\frac{x}{y}\right)=\frac{f(x)}{f(y)}$, and $f(2)=3$

Find: $f(1) \cdot f(4)$

Step 1:

Let $x=2, y=2$:

$f(1)=\frac{f(2)}{f(2)}=\frac{3}{3}=1$

Step 2:

Let $x=4, y=2$:

$f(2)=\frac{f(4)}{f(2)} \Rightarrow 3=\frac{f(4)}{3} \Rightarrow f(4)=9$

Result:

$f(1) \cdot f(4)=1 \cdot 9=9$

97. (d) We are given:

Functional equation: $f(xy)=f(x+y)$ for all real x, y

$f(5)=10$

We need to find: $f(0)$

Step 1: Use the functional equation

Let's take $x=0$, so:

$f(0 \cdot y)=f(0+y) \Rightarrow f(0)=f(y)$

So, $f(y)=f(0)$ for all y

This means f is a constant function, i.e.,

$f(x)=f(0)$ for all x

But we are also told:

$f(5)=10 \Rightarrow f(0)=10$

98. (c) Given:

$f(xy)=f(x+y)$, and $f(5)=10$

Let $x=0$:

$f(0)=f(y) \Rightarrow f$ is constant

Since $f(5)=10$,

$f(x)=10$ for all x

So,

$f(20)+f(-20)=10+10=20$

99. (b) We are given:

$f(x)=[x^2]$, where $[\cdot]$ denotes the greatest integer function

and asked to compute the integral:

$$\int_{\sqrt{2}}^{\sqrt{3}} f(x) dx=\int_{\sqrt{2}}^{\sqrt{3}}[x^2] dx$$

Step 1: Understand the behavior of $f(x)=[x^2]$

We analyze how x^2 changes between $x=\sqrt{2} \approx 1.414$ and $x=\sqrt{3} \approx 1.732$:

- At $x=\sqrt{2}, x^2=2$
- At $x=\sqrt{3}, x^2=3$

So, $x^2 \in [2,3)$ as $x \in [\sqrt{2}, \sqrt{3})$
Thus, for all $x \in [\sqrt{2}, \sqrt{3})$,
$[x^2] = 2$
But we must be careful at the point where $x^2 = 3$, i.e., at $x = \sqrt{3}$, but since $\sqrt{3} \notin$ interior of interval, it doesn't affect value of the integral.
Step 2: Evaluate the integral
Since $f(x) = 2$ throughout the interval $[\sqrt{2}, \sqrt{3})$, we get:

$$\int_{\sqrt{2}}^{\sqrt{3}} f(x)dx = \int_{\sqrt{2}}^{\sqrt{3}} 2dx = 2(\sqrt{3} - \sqrt{2})$$

100. **(a)** We are given the function:
$f(x) = [x^2]$
where $[\cdot]$ denotes the greatest integer function (also called floor function), and we need to compute:

$$\int_{\sqrt{2}}^{2} [x^2]dx$$

•Step 1: Understand the behavior of $f(x) = [x^2]$
We examine the values of x^2 in the interval $[\sqrt{2}, 2]$:
• When $x = \sqrt{2}, x^2 = 2$
•When $x = 2, x^2 = 4$
We now break the interval $[\sqrt{2}, 2]$ into subintervals where $[x^2]$ is constant:
• $x^2 \in [2,3) \Rightarrow x \in [\sqrt{2}, \sqrt{3})$
• $x^2 \in [3,4) \Rightarrow x \in [\sqrt{3}, 2)$
• At $x = 2, x^2 = 4 \Rightarrow [x^2] = 4$, but this is just a point so it doesn't affect the integral.
So the integral becomes:

$$\int_{\sqrt{2}}^{\sqrt{3}} 2dx + \int_{\sqrt{3}}^{2} 3dx$$

• Step 2: Evaluate each integral
First part:

$$\int_{\sqrt{2}}^{\sqrt{3}} 2dx = 2(\sqrt{3} - \sqrt{2})$$

Second part:

$$\int_{\sqrt{3}}^{2} 3dx = 3(2 - \sqrt{3})$$

• Step 3: Add the results
$2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3}) = 2\sqrt{3} - 2\sqrt{2} + 6 - 3\sqrt{3}$
Combine like terms:
$(2\sqrt{3} - 3\sqrt{3}) + (-2\sqrt{2}) + 6 = -\sqrt{3} - 2\sqrt{2} + 6$

101. **(c)** Class intervals and corresponding number of students are given.
• Height (in cm): 160-162, Number of Students: 12
• Height (in cm): 162-164, Number of Students: 15
• Height (in cm): 164-166, Number of Students: 24
• Height (in cm): 166-168, Number of Students: 13
How to solve
Sum the number of students in the height ranges less than or equal to 165 cm. (8)
1.Step 1: Calculate the total number of students with height less than or equal to 165 cm.
• Add the number of students in the height ranges 160-162, 162-164, and 164-166.
• Total students $= 12 + 15 + 24$
2.Step 2: Calculate the sum.
•$12 + 15 + 24 = 51$

102. **(c)** To find the median height from a grouped frequency distribution, we use the median formula:

$$\text{Median } = L + \left(\frac{\frac{N}{2} - F}{f} \right) \times h$$

Where:
• L = lower boundary of the median class
• N = total number of students
• F = cumulative frequency before the median class
• f = frequency of the median class
• h = class width

Step 1: Prepare the cumulative frequency table

Height (cm)	Frequency (f)	Cumulative Frequency (CF)
160-162	12	12
162-164	15	27
164-166	24	51
166-168	13	64

Step 2: Find N and $\frac{N}{2}$

$N = 12 + 15 + 24 + 13 = 64, \frac{N}{2} = 32$

Step 3: Find the median class

Since $\frac{N}{2} = 32$, look for the class where the cumulative frequency just exceeds or contains 32.

From the table:
• 164-166 has cumulative frequency 51 → This is the median class.

Step 4: Use the formula
• $L = 164$ (lower boundary of 164-166)
• $F = 27$ (cumulative frequency before 164-166)
• $f = 24$ (frequency of median class)
• $h = 2$ (class width)

$$\text{Median } = 164 + \left(\frac{32 - 27}{24}\right) \times 2 = 164 + \left(\frac{5}{24}\right) \times 2 = 164 + \frac{10}{24} = 164 + 0.4167 \approx 164.41 \text{ cm}$$

103. **(d)** To find the height which occurs most frequently, we need to determine the mode of the frequency distribution. The mode lies in the modal class, which is the class interval with the highest frequency.

Step 1: Identify the modal class

Given frequencies:
• 160-162: 12
• 162-164:15
• 164-166: 24 - Highest frequency → Modal class
• 166-168: 13

So, modal class $= 164 - 166$

Step 2: Use the mode formula

$$\text{Mode } = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Where:
• l = lower boundary of modal class = 164
• f_1 = frequency of modal class = 24
• f_0 = frequency of class before modal class = 15
• f_2 = frequency of class after modal class = 13
• h = class width = 2

Step 3: Plug in values

$$\text{Mode } = 164 + \left(\frac{24 - 15}{2 \times 24 - 15 - 13}\right) \times 2 = 164 + \left(\frac{9}{48 - 28}\right) \times 2 = 164 + \left(\frac{9}{20}\right) \times 2$$

$= 164 + 0.9 = 164.9$ cm

104. **(c)** The most appropriate graphical representation of a continuous frequency distribution (like the one given, with height intervals and frequencies) is:
(c) histogram

Explanation:
• A histogram is used for continuous data and shows the distribution of data over intervals.
• Unlike bar charts, in histograms the bars are adjacent (no gaps) to reflect the continuity of the data.

105. **(b)** We are given:
• Number of observations: $n = 50$
• $\sum X = 200, \sum Y = 250$
• $\sum X^2 = 900, \sum Y^2 = 1400$

We are to compare Variance (X) and Variance (Y).
Step 1: Compute Mean

$$\bar{X} = \frac{\sum X}{n} = \frac{200}{50} = 4$$

$$\bar{Y} = \frac{\sum Y}{n} = \frac{250}{50} = 5$$

Step 2: Use formula for variance:

$$\operatorname{Var}(X) = \frac{1}{n}\sum X^2 - \bar{X}^2 = \frac{900}{50} - 4^2 = 18 - 16 = 2$$

$$\operatorname{Var}(Y) = \frac{1}{n}\sum Y^2 - \bar{Y}^2 = \frac{1400}{50} - 5^2 = 28 - 25 = 3$$

Conclusion:
$\operatorname{Variance}(X) = 2 < \operatorname{Variance}(Y) = 3$

106. **(a)** We are given:
• Number of observations, $n = 50$
• $\sum X = 200, \sum X^2 = 900$
• $\sum Y = 250, \sum Y^2 = 1400$

We need to compare coefficient of variation (CV) for X and Y .
Step 1: Compute Mean
For X :

$$\bar{X} = \frac{\sum X}{n} = \frac{200}{50} = 4$$

For Y :

$$\bar{Y} = \frac{\sum Y}{n} = \frac{250}{50} = 5$$

Step 2: Compute Variance
The formula for variance is:

$$\operatorname{Var}(X) = \frac{\sum X^2}{n} - (\bar{X})^2$$

For X :

$$\operatorname{Var}(X) = \frac{900}{50} - (4)^2 = 18 - 16 = 2$$

For Y :

$$\operatorname{Var}(Y) = \frac{1400}{50} - (5)^2 = 28 - 25 = 3$$

Step 3: Compute Standard Deviation
$\sigma_X = \sqrt{2} \approx 1.414, \sigma_Y = \sqrt{3} \approx 1.732$

Step 4: Compute Coefficient of Variation (CV)

$$\mathrm{CV} = \frac{\text{Standard Deviation}}{\text{Mean}} \times 100$$

For X :

$$\mathrm{CV}_X = \frac{1.414}{4} \times 100 \approx 35.35\%$$

For Y:

$$\mathrm{CV}_Y = \frac{1.732}{5} \times 100 \approx 34.64\%$$

Conclusion:
$\mathrm{CV}_X > \mathrm{CV}_Y$

107. **(c)** Given, 9P $(X = 4) = P(X = 2)$

$\therefore 9 \cdot {}^6C_4 p^4 (1-p)^2 = {}^6C_2 p^2 (1-p)^4$

$\Rightarrow \frac{(1-p)^2}{p^2} = 9$

$\Rightarrow \frac{1-p}{p} = 3$

$\Rightarrow p = 1/4$

108. **(a)** Given, $9P(X = 4) = P(X = 2)$

$9 \cdot {}^6C_4 p^4 (1-p)^2 = {}^6C_2 p^2 (1-p)^4$

$\Rightarrow \frac{(1-p)^2}{p^2} = 9$

$\Rightarrow \frac{1-q}{p} = 3$

$\Rightarrow p = 1/4$

109. **(a)** We are given:
- 7 gentlemen
- 4 ladies
- A committee of 6 members is to be formed
- We want the probability that the committee has exactly 3 gentlemen

Step 1: Total number of ways to form a committee of 6 people from $11(7\mathrm{G} + 4\mathrm{~L})$

$$\text{Total ways } = \binom{11}{6}$$

Step 2: Number of favorable ways to choose exactly 3 gentlemen and 3 ladies
- Choose 3 gentlemen out of 7 :

$\binom{7}{3}$
- Choose 3 ladies out of 4:

$\binom{4}{3}$
- Total favorable ways:

$\binom{7}{3} \cdot \binom{4}{3}$

Step 3: Calculate the values
- $\binom{11}{6} = 462$
- $\binom{7}{3} = 35$
- $\binom{4}{3} = 4$

So,

$$\text{Favorable ways } = 35 \cdot 4 = 140$$

Step 4: Probability

$$P = \frac{140}{462} = \frac{70}{231} = \frac{10}{33}$$

110. **(d)** We are given:
- Total people = 7 gentlemen + 4 ladies = 11 people
- Committee size $=$ 6 members
- We need to find the probability that the committee includes at least 2 ladies.

Step 1: Total number of ways to form a committee of 6 members from 11 people:

$$\text{Total ways } = \binom{11}{6}$$

$$\binom{11}{6} = 462$$

Step 2: Calculate number of favorable cases (committees with at least 2 ladies)
This means committees having:

• 2 ladies + 4 gentlemen
• 3 ladies +3 gentlemen
• 4 ladies + 2 gentlemen
• Case 1: 2 ladies + 4 gentlemen
$\binom{4}{2} \cdot \binom{7}{4} = 6 \cdot 35 = 210$
• Case 2: 3 ladies + 3 gentlemen
$\binom{4}{3} \cdot \binom{7}{3} = 4 \cdot 35 = 140$
• Case 3: 4 ladies + 2 gentlemen
$\binom{4}{4} \cdot \binom{7}{2} = 1 \cdot 21 = 21$
Total favorable cases:
$210 + 140 + 21 = 371$
Probability = Favorable / Total
$P(\text{ at least 2 ladies }) = \frac{371}{462}$
Reduce the fraction:
$\frac{371}{462} = \frac{53}{66}$

111. **(c)** We are given:
•Probabilities that A, B, and C become managers:
$P(A) = \frac{3}{10}, P(B) = \frac{1}{2}, P(C) = \frac{4}{5}$
• Probabilities that bonus scheme is introduced given they become managers:
$P(Bonus \mid A) = \frac{4}{9}, P(Bonus \mid B) = \frac{2}{9}, P(Bonus \mid C) = \frac{1}{3}$
To find the total probability that the bonus scheme is introduced, we use the law of total probability:
$P(\text{ Bonus }) = P(A) \cdot P(\text{ Bonus } \mid A) + P(B) \cdot P(\text{ Bonus } \mid B) + P(C) \cdot P(\text{ Bonus } \mid C)$
Now, substitute values:
$P(\text{ Bonus }) = \left(\frac{3}{10} \cdot \frac{4}{9}\right) + \left(\frac{1}{2} \cdot \frac{2}{9}\right) + \left(\frac{4}{5} \cdot \frac{1}{3}\right)$
Compute each term:
• $\frac{3}{10} \cdot \frac{4}{9} = \frac{12}{90} = \frac{2}{15}$
• $\frac{1}{2} \cdot \frac{2}{9} = \frac{2}{18} = \frac{1}{9}$
• $\frac{4}{5} \cdot \frac{1}{3} = \frac{4}{15}$
Now sum:
$\frac{2}{15} + \frac{1}{9} + \frac{4}{15}$
Convert all to a common denominator (LCM of 15 and 9 is 45):
• $\frac{2}{15} = \frac{6}{45}$
• $\frac{1}{9} = \frac{5}{45}$
• $\frac{4}{15} = \frac{12}{45}$
So total probability:
$P(\text{ Bonus }) = \frac{6+5+12}{45} = \frac{23}{45}$

112. **(a)** We are given:
• $P(A \text{ becomes manager }) = 3/10$, and $P(\text{ Bonus} \mid A) = 4/9$
•$P(B \text{ becomes manager }) = 1/2$, and $P(\text{ Bonus} \mid B) = 2/9$
•$P(C \text{ becomes manager }) = 4/5$, and $P(\text{ Bonus} \mid C) = 1/3$
Let:
•M_A, M_B, M_C : events that A, B, or C becomes manager respectively.
• B : event that bonus scheme is introduced.

We are to find:

$$P(M_B \mid B)=\frac{P(M_B\cap B)}{P(B)}$$

Step 1: Find $P(B)$

Using total probability:

$$P(B)=P(M_A)\cdot P(B\mid M_A)+P(M_B)\cdot P(B\mid M_B)+P(M_C)\cdot P(B\mid M_C)$$

$$P(B)=\frac{3}{10}\cdot\frac{4}{9}+\frac{1}{2}\cdot\frac{2}{9}+\frac{4}{5}\cdot\frac{1}{3}$$

$$=\frac{12}{90}+\frac{2}{18}+\frac{4}{15}$$

Let's simplify:

- $\frac{12}{90}=\frac{2}{15}$
- $\frac{2}{18}=\frac{1}{9}$
- $\frac{4}{15}$ remains

Now get common denominator:

$$P(B)=\frac{2}{15}+\frac{1}{9}+\frac{4}{15}$$

Find LCM of 15 and 9 $\rightarrow$ LCM $=45$

$$P(B)=\frac{6}{45}+\frac{5}{45}+\frac{12}{45}=\frac{23}{45}$$

Step 2: Find $P(M_B\cap B)$

$$P(M_B\cap B)=P(M_B)\cdot P(B\mid M_B)=\frac{1}{2}\cdot\frac{2}{9}=\frac{2}{18}=\frac{1}{9}$$

Step 3: Use Bayes' Theorem

$$P(M_B\mid B)=\frac{P(M_B\cap B)}{P(B)}=\frac{\frac{1}{9}}{\frac{23}{45}}=\frac{1}{9}\cdot\frac{45}{23}=\frac{5}{23}$$

113. **(a)** We are given:

- The original arithmetic mean of 100 observations is 50 .
- A transformation is applied to each observation:

$$\text{New value }=\frac{\text{Original value }-5}{20}$$

Step 1: Mean of original observations = 50

Let the original observations be $x_1, x_2, \ldots, x_{100}$.

Then:

$$\frac{x_1+x_2+\cdots+x_{100}}{100}=50 \Rightarrow \sum x_i=5000$$

Step 2: Apply the transformation to each observation

Each new observation is:

$$y_i=\frac{x_i-5}{20}$$

The new mean is:

$$\frac{1}{100}\sum_{i=1}^{100} y_i=\frac{1}{100}\sum_{i=1}^{100}\frac{x_i-5}{20}$$

Factor out the constants:

$$=\frac{1}{100}\cdot\frac{1}{20}\sum_{i=1}^{100}(x_i-5)=\frac{1}{2000}\left(\sum x_i-5\cdot 100\right)$$

$= \frac{1}{2000}(5000 - 500) = \frac{4500}{2000} = 2.25$

114. **(b)** We are given:
• Original standard deviation $\sigma = 10$
• Number of observations = 100 (but standard deviation is unaffected by the number of observations unless we are computing standard error - not the case here)
• Each observation is transformed by:
$x' = \frac{x+5}{20}$
Step-by-step:
Let the original data be $x_1, x_2, \ldots, x_{100}$
We apply the transformation:
$x_i' = \frac{x_i+5}{20}$
We need to understand how standard deviation changes under this transformation.
Important Properties:
• Adding a constant (like +5) does not change the standard deviation.
• Multiplying/dividing all observations by a constant scales the standard deviation by the absolute value of that constant.
So:
$\sigma_{\text{new}} = \left|\frac{1}{20}\right| \cdot \sigma = \frac{10}{20} = 0.5$

115. **(b)** We are given:
• $P(A) = \frac{1}{3}$
• $P(B) = \frac{1}{2}$
• $P(A \cap B) = \frac{1}{4}$
We are asked to find:
$P(B \mid A^c)$
Step 1: Use the conditional probability formula:
$P(B \mid A^c) = \frac{P(B \cap A^c)}{P(A^c)}$
We already know:
• $P(A^c) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$
Now find $P(B \cap A^c)$:
$P(B \cap A^c) = P(B) - P(A \cap B) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$
Now plug in:
$P(B \mid A^c) = \frac{\frac{1}{4}}{\frac{2}{3}} = \frac{1}{4} \cdot \frac{3}{2} = \frac{3}{8}$

116. **(b)** We are given:
• $P(A) = \frac{1}{3}$
• $P(B) = \frac{1}{2}$
• $P(A \cap B) = \frac{1}{4}$
We are asked to find:
$P(A^c \cap B^c)$, which is the probability that neither A nor B occurs.
This can be written using the formula for the union:
$P(A^c \cap B^c) = 1 - P(A \cup B)$
We use the formula:
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Substitute values:

$P(A \cup B)=\frac{1}{3}+\frac{1}{2}-\frac{1}{4}$

To calculate this, use a common denominator (LCM of 3,2,4 is 12):

$P(A \cup B)=\frac{4}{12}+\frac{6}{12}-\frac{3}{12}=\frac{7}{12}$

So,

$P(A^{c} \cap B^{c})=1-\frac{7}{12}=\frac{5}{12}$

117. **(b)** We are given that two fair dice are tossed, and we are to find the probability that the sum of the numbers on the dice is strictly greater than 7 .

Step 1: Total number of outcomes

When two dice are tossed, the total number of possible outcomes is:

$6 \times 6=36$

Step 2: Favorable outcomes (sum > 7)

We need to count how many outcomes have a sum greater than 7 , i.e., sum $=8,9,10,11$, or 12 .

- Sum $=8:(2,6),(3,5),(4,4),(5,3),(6,2) \rightarrow 5$ outcomes
- Sum $=9:(3,6),(4,5),(5,4),(6,3) \rightarrow 4$ outcomes
- Sum $=10:(4,6),(5,5),(6,4) \rightarrow 3$ outcomes
- Sum $=11:(5,6),(6,5) \rightarrow 2$ outcomes
- Sum $=12:(6,6) \rightarrow 1$ outcome

Total favorable outcomes $=5+4+3+2+1=15$

Step 3: Probability

$P(\text{ sum }>7)=\frac{15}{36}=\frac{5}{12}$

118. **(c)** We are given:

•Probability of hitting the target (success) $=p=\frac{1}{5}$

•Probability of missing the target $=q=1-p=\frac{4}{5}$

•Number of trials $=n=7$

• We need the probability that the man hits the target at least 2 times, i.e., $P(X \geq 2)$

This is best found by the complement method:

$P(X \geq 2)=1-P(X=0)-P(X=1)$

Let $X \sim \operatorname{Binomial}(n=7, p=1/5)$

Step 1: Compute $P(X=0)$:

$P(X=0)=\binom{7}{0}\left(\frac{1}{5}\right)^{0}\left(\frac{4}{5}\right)^{7}=\left(\frac{4}{5}\right)^{7}$

Step 2: Compute $P(X=1)$:

$P(X=1)=\binom{7}{1}\left(\frac{1}{5}\right)^{1}\left(\frac{4}{5}\right)^{6}=7 \cdot \frac{1}{5} \cdot\left(\frac{4}{5}\right)^{6}$

$$P(X=1)=\frac{7}{5} \cdot\left(\frac{4}{5}\right)^{6}$$

Now sum $P(X=0)+P(X=1)$:

$P(X<2)=\left(\frac{4}{5}\right)^{7}+\frac{7}{5} \cdot\left(\frac{4}{5}\right)^{6}=\left(\frac{4}{5}\right)^{6}\left[\frac{4}{5}+\frac{7}{5}\right]=\left(\frac{4}{5}\right)^{6} \cdot \frac{11}{5}$

119. **(b)** We are given that a random variable X follows a binomial distribution, i.e.,

$X \sim B(n, p)$

Properties of Binomial Distribution:

- Mean $\mu=n p=200$
- Variance $\sigma^{2}=n p(1-p)=160$

Step 1: Use the mean to find p in terms of n :

$n p=200 \Rightarrow p=\frac{200}{n}$

Step 2: Plug into variance formula:

$n p(1-p)=160$

Substitute $p=\frac{200}{n}$:

$$n \cdot \frac{200}{n}\left(1-\frac{200}{n}\right)=160 \Rightarrow 200\left(1-\frac{200}{n}\right)=160$$

$$200-\frac{200 \cdot 200}{n}=160 \Rightarrow \frac{40000}{n}=40 \Rightarrow n=\frac{40000}{40}=1000$$

120. **(c)** We are given the numbers:

$8^2, 9^2, 10^2, 11^2, 12^2, 13^2, 14^2, 15^2$

That is, the squares of the integers from 8 to 15 (inclusive). First, compute their squares:

$8^2=64$
$9^2=81$
$10^2=100$
$11^2=121$
$12^2=144$
$13^2=169$
$14^2=196$
$15^2=225$

Now add them up:

$$\begin{aligned}\text{Sum } &= 64+81+100+121+144+169+196+225\\ &=(64+81)+(100+121)+(144+169)+(196+225)\\ &=145+221+313+421=1100\end{aligned}$$

There are 8 numbers.

$$\text{Mean } =\frac{1100}{8}=137.5$$