1. **Let X be the set of all persons living in Delhi. The persons a and b in X are said to be related if the difference in their ages is at most 5 years. The relation is**
   (a) an equivalence relation
   (b) reflexive and transitive but not symmetric
   (c) symmetric and transitive but not reflexive
   (d) reflexive and symmetric but not transitive

2. **The matrix** $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & x-1 & 1 \\ 2 & 7 & x-3 \end{bmatrix}$ **will have inverse for every real number x except for**
   (a) $x = \frac{11\pm\sqrt{5}}{2}$ (b) $x = \frac{9\pm\sqrt{5}}{2}$
   (c) $x = \frac{11\pm\sqrt{3}}{2}$ (d) $x = \frac{9\pm\sqrt{3}}{2}$

3. **If the value of the determinant** $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix}$ **is positive, where** $a \neq b \neq c$, **then the value of abc**
   (a) cannot be less than 1
   (b) is greater than -8
   (c) is less than -8
   (d) must be greater than 8

4. **Consider the following statements in respect of the determinant**

$$\begin{vmatrix} \cos^2\frac{\alpha}{2} & \sin^2\frac{\alpha}{2} \\ \sin^2\frac{\beta}{2} & \cos^2\frac{\beta}{2} \end{vmatrix}$$

where α, β **are complementary angles**

(1) The value of the determinant is $\frac{1}{\sqrt{2}}\cos\left(\frac{\alpha-\beta}{2}\right)$.

(2) The maximum value of the determinant is $\frac{1}{\sqrt{2}}$.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

5. **What is** $(1000000001)_2 - (0.0101)_2$ **equal to?**
   (a) $(512.6775)_{10}$ (b) $(512 \cdot 6875)_{10}$
   (c) $(512 \cdot 6975)_{10}$ (d) $(512.0909)_{10}$

6. **If** $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & -3 & 4 \end{bmatrix}$, **then the matrix X for which** $2X + 3\vec{A} = 0$ **holds true is**
   (a) $\begin{bmatrix} -\frac{3}{2} & 0 & -3 \\ -3 & -\frac{9}{2} & -6 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{3}{2} & 0 & -3 \\ 3 & -\frac{9}{2} & -6 \end{bmatrix}$
   (c) $\begin{bmatrix} \frac{3}{2} & 0 & 3 \\ 3 & \frac{9}{2} & 6 \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{3}{2} & 0 & 3 \\ -3 & \frac{9}{2} & -6 \end{bmatrix}$

7. **If** z_1 **and** z_2 **are complex numbers with** $|z_1| = |z_2|$, **then which of the following is/are correct?**

(1) $z_1 = z_2$

(2) Real part of z_1 **= Real part of** z_2

(3) Imaginary part of z_1 **= Imaginary part of** z_2

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) 3 only (d) None

8. If $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ **and** $B = \begin{bmatrix} -1 & -2 & -1 \\ 6 & 12 & 6 \\ 5 & 10 & 5 \end{bmatrix}$**, then which of the following is/are correct?**

(1) A and B commute.

(2) AB is a null matrix.

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

9. The number of real roots of the equation $x^2 - 3|x| + 2 = 0$ **is**
(a) 4 (b) 3
(c) 2 (d) 1

10. If the sum of the roots of the equation $ax^2 + bx + c = 0$ **is equal to the sum of their squares, then**
(a) $a^2 + b^2 = c^2$ (b) $a^2 + b^2 = a + b$
(c) $ab + b^2 = 2ac$ (d) $ab - b^2 = 2ac$

11. If $A = \{x \in \mathbb{R}: x^2 + 6x - 7 < 0\}$ **and** $B = \{x \in \mathbb{R}: x^2 + 9x + 14 > 0\}$**, then which of the following is/are correct?**

(1) $(A \cap B) = (-2, 1)$

(2) $(A \setminus B) = (-7, -2)$

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

12. A, B, C and D are four sets such that $A \cap B = C \cap D = \phi$**. Consider the following:**

(1) $A \cup C$ **and** $B \cup D$ **are always disjoint.**

(2) $A \cap C$ **and** $B \cap D$ **are always disjoint.**

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both I and 2 (d) Neither 1 nor 2

13. If A is an invertible matrix of order n and k is any positive real number, then the value of $[\det(kA)]^{-1} \det A$ **is**
(a) k^{-n} (b) k^{-1}
(c) k^n (d) nk

14. The value of the infinite product $6^{\frac{1}{2}} \times 6^{\frac{1}{2}} \times 6^{\frac{3}{8}} \times 6^{\frac{1}{4}} \times \ldots$ **is**
(a) 6 (b) 36
(c) 216 (d) ∞

15. If the roots of the equation $x^2 - nx + m = 0$ **differ by 1 , then**
(a) $n^2 - 4m - 1 = 0$ (b) $n^2 + 4m - 1 = 0$
(c) $m^2 + 4n + 1 = 0$ (d) $m^2 - 4n - 1 = 0$

16. If different words are formed with all the letters of the word 'AGAIN' and are arranged alphabetically among themselves as in a dictionary, the word at the 50th place will be

(a) NAAGI (b) NAAIG
(c) IAAGN (d) IAANG

17. The number of ways in which a cricket team of 11 players be chosen out of a batch of 15 players so that the captain of the team is always included, is

(a) 165 (b) 364
(c) 1001 (d) 1365

18. In the expansion of $\left(\sqrt{x}+\frac{1}{3x^2}\right)^{10}$ the value of constant term (independent of x) is

(a) 5 (b) 8
(c) 45 (d) 90

19. The value of

$$\sin^2 5^\circ + \sin^2 10^\circ + \sin^2 15^\circ + \sin^2 20^\circ + \cdots .. + \sin^2 90^\circ$$

is

(a) 7 (b) 8
(c) 9 (d) $\frac{19}{2}$

20. On simplifying

$$\frac{\sin^3 A + \sin 3A}{\sin A} + \frac{\cos^3 A - \cos 3A}{\cos A},$$

we get

(a) $\sin 3A$ (b) $\cos 3A$
(c) $\sin A + \cos A$ (d) 3

21. The value of $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$ is

(a) $-\frac{7}{17}$ (b) $\frac{5}{16}$
(c) $\frac{5}{4}$ (d) $\frac{7}{17}$

22. Two poles are 10 m and 20 m high. The line joining their tops makes an angle of 15° with the horizontal. The distance between the poles is approximately equal to

(a) 36.3 m (b) 37.3 m
(c) 38.3 m (d) 39.3 m

23. If $g(x)=\frac{1}{f(x)}$ and $f(x)=x, x\neq 0$, then which one of the following is correct?

(a) $f(f(f(g(g(f(x)))))) = g(g(f(g(f(x)))))$

(b) $f(f(g(g(g(f(x)))))) = g(g(f(g(f(x)))))$

(c) $f(g(f(g(g(f(g(x))))))) = g(g(f(g(f(x)))))$

(d) $f(f(f(g(g(f(x)))))) = f(f(f(g(f(x)))))$

24. Consider the following:

(1) $\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{3}{5}=\frac{\pi}{2}$

(2) $\tan^{-1}\sqrt{3}+\tan^{-1}1=-\tan^{-1}(2+\sqrt{3})$

Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

25. If A is an orthogonal matrix of order 3 and $B=\begin{bmatrix}1 & 2 & 3\\ -3 & 0 & 2\\ 2 & 5 & 0\end{bmatrix}$**, then which of the following is/are correct?**

(1) $|AB|=\pm 47$

(2) $AB=BA$

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

26. If a, b, c are the sides of a triangle ABC, then $a^{\frac{1}{p}}+b^{\frac{1}{p}}-c^{\frac{1}{p}}$ **where** $p>1$**, is**
(a) always negative
(b) always positive
(c) always zero
(d) positive if $1<p<2$ and negative if $p>2$

27. If a, b, c are real numbers, then the value of the determinant $\begin{vmatrix}1-a & a-b-c & b+c\\ 1-b & b-c-a & c+a\\ 1-c & c-a-b & a+b\end{vmatrix}$ **is**
(a) 0 (b) $(a-b)(b-c)(c-a)$
(c) $(a+b+c)^2$ (d) $(a+b+c)^3$

28. If the point $z_1=1+i$ **where** $i=\sqrt{-1}$ **is the reflection of a point** $z_2=x+iy$ **in the line** $i\bar{z}-iz=5$**, then the point** z_2 **is**
(a) $1+4i$ (b) $4+i$
(c) $1-i$ (d) $-1-i$

29. If $\sin x+\sin y=a$ **and** $\cos x+\cos y=b$**, then** $\tan^2\left(\frac{x+y}{2}\right)+\tan^2\left(\frac{x-y}{2}\right)$ **is equal to**
(a) $\frac{a^4+b^4+4b^2}{a^2b^2+b^4}$ (b) $\frac{a^4-b^4+4b^2}{a^2b^2+b^4}$
(c) $\frac{a^4-b^4+4a^2}{a^2b^2+a^4}$ (d) None of the above

30. A vertical tower standing on a levelled field is mounted with a vertical flag staff of length 3 m. From a point on the field, the angles of elevation of the bottom and tip of the flag staff are 30° and 45° respectively. Which one of the following gives the best approximation to the height of the tower?
(a) 3.90 m (b) 4.00 m
(c) 4.10 m (d) 4.25 m

For the next (31 to 33) items that follow:
Consider the expansion of $(1+x)^{2n+1}$

31. If the coefficients of x^r **and** x^{r+1} **are equal in the expansion, then r is equal to**
(a) n (b) $\frac{2n-1}{2}$
(c) $\frac{2n+1}{2}$ (d) $n+1$

32. The average of the coefficients of the two middle terms in the expansion is

(a) ${}^{2n+1}C_{n+2}$ (b) ${}^{2n+1}C_{n}$

(c) ${}^{2n+1}C_{n-1}$ (d) ${}^{2n}C_{n+1}$

33. The sum of the coefficients of all the terms in the expansion is

(a) 2^{2n-1} (b) 4^{n-1}

(c) 2×4^{n} (d) None of the above

34. The nth term of an A.P. is $\frac{3+n}{4}$, then the sum of first 105 terms is

(a) 270 (b) 735

(c) 1409 (d) 1470

35. A polygon has 44 diagonals. The number of its sides is

(a) 11 (b) 10

(c) 8 (d) 7

36. If p, q, r are in one geometric progression and a, b, c are in another geometric progression, then ap, bq, cr are in

(a) Arithmetic progression

(b) Geometric progression

(c) Harmonic progression

(d) None of the above

For the next (37 to 38) items that follow:
Consider a triangle ABC satisfying

$$2a\sin^2\left(\frac{C}{2}\right) + 2c\sin^2\left(\frac{A}{2}\right) = 2a + 2c - 3b$$

37. The sides of the triangle are in

(a) G.P.

(b) A.P.

(c) H.P.

(d) Neither in G.P. nor in A.P. nor in H.P.

38. sin A, sin B, sin C are in

(a) GP.

(b) A.P.

(c) H.P.

(d) Neither in G.P. nor in A.P. nor in H.P.

39. If $p = \tan\left(-\frac{11\pi}{6}\right)$, $q = \tan\left(\frac{21\pi}{4}\right)$ and $r = \cot\left(\frac{283\pi}{6}\right)$, then which of the following is/are correct?

(1) The value of $p \times r$ is 2 .

(2) p, q and r are in G.P.

Select the correct answer using the code given below:

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

40. The number of ways in which 3 holiday tickets can be given to 20 employees of an organization if each employee is eligible for any one or more of the tickets, is

(a) 1140 (b) 3420

(c) 6840 (d) 8000

41. What is the sum of n terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\cdots$?

(a) $\frac{n(n-1)}{\sqrt{2}}$ (b) $\sqrt{2}n(n+1)$

(c) $\frac{n(n+1)}{\sqrt{2}}$ (d) $\frac{n(n-1)}{2}$

42. The coefficient of x^{99} in the expansion of $(x-1)(x-2)(x-3)\ldots\ldots(x-100)$ is

(a) 5050 (b) 5000

(c) -5050 (d) -5000

43. $z\bar{z}+(3-i)z+(3+i)\bar{z}+1=0$ represents a circle with

(a) centre $(-3,-1)$ and radius 3

(b) centre $(-3,1)$ and radius 3

(c) centre $(-3,-1)$ and radius 4

(d) centre $(-3,1)$ and radius 4

44. The number of 3-digit even numbers that can be formed from the digits 0, 1, 2, 3, 4 and 5 , repetition of digits being not allowed, is

(a) 60 (b) 56

(c) 52 (d) 48

45. If $\log_8 m+\log_8\frac{1}{6}=\frac{2}{3}$, then m is equal to

(a) 24 (b) 18

(c) 12 (d) 4

46. The area of the figure formed by the lines $ax+by+c=0$, $ax-by+c=0$, $ax+by-c=0$ and $ax-by-c=0$ is

(a) $\frac{c^2}{ab}$ (b) $\frac{2c^2}{ab}$

(c) $\frac{c^2}{2ab}$ (d) $\frac{c^2}{4ab}$

47. If a line is perpendicular to the line $5x-y=0$ and forms a triangle of area 5 square units with co-ordinate axes, then its equation is

(a) $x+5y\pm5\sqrt{2}=0$ (b) $x-5y\pm5\sqrt{2}=0$

(c) $5x+y\pm5\sqrt{2}=0$ (d) $5x-y\pm5\sqrt{2}=0$

48. Consider any point P on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ in the first quadrant. Let r and s represent its distances from $(4,0)$ and $(-4,0)$ respectively, then $(r+s)$ is equal to

(a) 10 unit (b) 9 unit

(c) 8 unit (d) 6 unit

49. A straight line $x=y+2$ touches the circle $4(x^2+y^2)=r^2$. The value of r is

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) 2 (d) 1

50. The three lines $4x+4y=1$, $8x-3y=2$, $y=0$ are

(a) the sides of an isosceles triangle

(b) concurrent

(c) mutually perpendicular

(d) the sides of an equilateral triangle

51. The line $3x+4y-24=0$ intersects the x-axis at A and y-axis at B. Then the circumcentre of the triangle OAB where O is the origin is

(a) (2,3) (b) (3,3)

(c) (4,3) (d) None of the above

52. The eccentricity of the hyperbola $16x^2 - 9y^2 = 1$ is

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$

(c) $\frac{4}{5}$ (d) $\frac{5}{4}$

53. The product of the perpendiculars from the two points $(\pm 4, 0)$ to the line $3x\cos\phi + 5y\sin\phi = 15$ is

(a) 25 (b) 16

(c) 9 (d) 8

54. If the centre of the circle passing through the origin is $(3, 4)$, then the intercepts cut off by the circle on x-axis and y-axis respectively are

(a) 3 unit and 4 unit (b) 6 unit and 4 unit

(c) 3 unit and 8 unit (d) 6 unit and 8 unit

55. The lines $2x = 3y = -z$ and $6x = -y = -4z$

(a) are perpendicular

(b) are parallel

(c) intersect at an angle 45°

(d) intersect at an angle 60°

56. Two straight lines passing through the point $A(3, 2)$ cut the line $2y = x + 3$ and x-axis perpendicularly at P and Q respectively. The equation of the line PQ is

(a) $7x + y - 21 = 0$ (b) $x + 7y + 21 = 0$

(c) $2x + y - 8 = 0$ (d) $x + 2y + 8 = 0$

57. The radius of the sphere

$$3x^2 + 3y^2 + 3z^2 - 8x + 4y + 8z - 15 = 0$$

is

(a) 2 (b) 3

(c) 4 (d) 5

58. The direction ratios of the line perpendicular to the lines with direction ratios $\langle 1, -2, -2\rangle$ and $\langle 0, 2, 1\rangle$ are

(a) $\langle 2,-1,2\rangle$ (b) $\langle -2,1,2\rangle$

(c) $\langle 2,1,-2\rangle$ (d) $\langle -2,-1,-2\rangle$

59. What are the co-ordinates of the foot of the perpendicular drawn from the point $(3, 5, 4)$ on the plane $z = 0$?

(a) (0,5,4) (b) (3,5,0)

(c) (3,0,4) (d) (0,0,4)

60. The lengths of the intercepts on the co-ordinate axes made by the plane $5x + 2y + z - 13 = 0$ are

(a) 5, 2, 1 unit (b) $\frac{13}{5}, \frac{13}{2}, 13$ unit

(c) $\frac{5}{13}, \frac{2}{13}, \frac{1}{13}$ unit (d) 1, 2, 5 unit

61. The area of the square, one of whose diagonals is $3\hat{i} + 4\hat{j}$ is

(a) 12 square unit

(b) 12.5 square unit

(c) 25 square unit

(d) $156 \cdot 25$ square unit

62. ABCD is a parallelogram and P is the point of intersection of the diagonals. If O is the origin, then $\overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} + \overrightarrow{OD}$ is equal to

(a) $4\overrightarrow{OP}$ (b) $2\overrightarrow{OP}$

(c) $\overrightarrow{OP}$ (d) Null vector

63. **If $\vec{b}$ and $\vec{c}$ are the position vectors of the points B and C respectively, then the position vector of the point D such that $\overrightarrow{BD} = 4\overrightarrow{BC}$ is**

(a) $4(\vec{c} - \vec{b})$ (b) $-4(\vec{c} - \vec{b})$

(c) $4\vec{c} - 3\vec{b}$ (d) $4\vec{c} + 3\vec{b}$

64. **If the position vector $\vec{a}$ of the point $(5, n)$ is such that $|\vec{a}| = 13$, then the value/values of n can be**

(a) ± 8 (b) ± 12

(c) 8 only (d) 12 only

65. **If $|\vec{a}| = 2$ and $|\vec{b}| = 3$, then $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2$ is equal to**

(a) 72 (b) 64

(c) 48 (d) 36

66. **Consider the following inequalities in respect of vectors $\vec{a}$ and $\vec{b}$:**

(1) $|\vec{a} + \vec{b}| \leqslant |\vec{a}| + |\vec{b}|$

(2) $|\vec{a} - \vec{b}| \geqslant |\vec{a}| - |\vec{b}|$

Which of the above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

67. **If the magnitude of difference of two-unit vectors is $\sqrt{3}$, then the magnitude of sum of the two vectors is**

(a) $\frac{1}{2}$ unit (b) 1 unit

(c) 2 unit (d) 3 unit

68. **If the vectors $\alpha\hat{i} + \alpha\hat{j} + \gamma\hat{k}, \hat{i} + \hat{k}$ and $\gamma\hat{i} + \gamma\hat{j} + \beta\hat{k}$ lie on a plane, where α, β and γ are distinct non-negative numbers, then γ is**

(a) Arithmetic mean of α and β

(b) Geometric mean of α and β

(c) Harmonic mean of α and β

(d) None of the above

69. **The vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ are such that $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$. Which of the following is/are correct?**

(1) $(\vec{a} - \vec{d}) \times (\vec{b} - \vec{c}) = \vec{0}$

(2) $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$

Select the correct answer using the code given below:

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

70. **The value of $\int_a^b \frac{x^7 + \sin x}{\cos x} dx$ where $a + b = 0$ is**

(a) $2b - a\sin(b - a)$

(b) $a + 3b\cos(b - a)$

(c) $\sin a - (b - a)\cos b$

(d) 0

71. **If $f(x) = \sqrt{25 - x^2}$, then what is $\mathrm{Lim}_{x \to 1} \frac{f(x) - f(1)}{x - 1}$ equal to ?**

(a) $\frac{1}{5}$ (b) $\frac{1}{24}$

(c) $\sqrt{24}$ (d) $-\frac{1}{\sqrt{24}}$

72. **Consider the function**

$$f(x) = \begin{cases} ax - 2 & \text{for } -2 < x < -1 \\ -1 & \text{for } -1 \leqslant x \leqslant 1 \\ a + 2(x-1)^2 & \text{for } 1 < x < 2 \end{cases}$$

What is the value of a for which $f(x)$ is continuous at $x = -1$ and $x = 1$?

(a) -1 (b) 1

(c) 0 (d) 2

73. **The function $f(x) = \frac{1 - \sin x + \cos x}{1 + \sin x + \cos x}$ is not defined at $x = \pi$. The value of $f(\pi)$ so that $f(x)$ is continuous at $x = \pi$ is**

(a) $-\frac{1}{2}$ (b) $\frac{1}{2}$

(c) -1 (d) 1

74. **Consider the following functions:**

(1) $f(x) = \begin{cases} \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

(2) $f(x) = \begin{cases} 2x + 5 & \text{if } x > 0 \\ x^2 + 2x + 5 & \text{if } x \leqslant 0 \end{cases}$

Which of the above functions is/are derivable at $x = 0$?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

75. **The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is**

(a) $[0, \infty)$ (b) $(-\infty, 0)$

(c) $[1, \infty)$ (d) $(-\infty, 0]$

76. **Consider the following statements:**

(1) The function $f(x) = x^2 + 2\cos x$ is increasing in the interval $(0, \pi)$

(2) The function $f(x) = \ln\left(\sqrt{1 + x^2} - x\right)$ is decreasing in the interval $(-\infty, \infty)$

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

77. **The derivative of $\ln(x + \sin x)$ with respect to $(x + \cos x)$ is**

(a) $\frac{1 + \cos x}{(x + \sin x)(1 - \sin x)}$ (b) $\frac{1 - \cos x}{(x + \sin x)(1 + \sin x)}$

(c) $\frac{1 - \cos x}{(x - \sin x)(1 + \cos x)}$ (d) $\frac{1 + \cos x}{(x - \sin x)(1 - \cos x)}$

78. **If $y = \cot^{-1}\left[\frac{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}\right]$, where $0 < x < \frac{\pi}{2}$, then $\frac{dy}{dx}$ is equal to**

(a) $\frac{1}{2}$ (b) 2

(c) $\sin x + \cos x$ (d) $\sin x - \cos x$

79. **The function $f(x) = \frac{x^2}{e^x}$ is monotonically increasing if**

(a) $x < 0$ only (b) $x > 2$ only

(c) $0 < x < 2$ (d) $x \in (-\infty, 0) \cup (2, \infty)$

80. **If $x^a y^b = (x - y)^{a+b}$, then the value of $\frac{dy}{dx} - \frac{y}{x}$ is equal to**

(a) $\frac{a}{b}$ (b) $\frac{b}{a}$

(c) 1 (d) 0

81. If $f: R \rightarrow R, g: R \rightarrow R$ be two functions given by $f(x) = 2x - 3$ and $g(x) = x^3 + 5$, then $(f \circ g)^{-1}(x)$ is equal to

(a) $\left(\frac{x+7}{2}\right)^{\frac{1}{3}}$ (b) $\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

(c) $\left(x - \frac{7}{2}\right)^{\frac{1}{3}}$ (d) $\left(x + \frac{7}{2}\right)^{\frac{1}{3}}$

82. If $0 < a < b$, then $\int_a^b \frac{|x|}{x} dx$ is equal to

(a) $|b| - |a|$ (b) $|a| - |b|$

(c) $\frac{|b|}{|a|}$ (d) 0

83. $\int_0^{2\pi} \sin^5\left(\frac{x}{4}\right) dx$ is equal to

(a) $\frac{8}{15}$ (b) $\frac{16}{15}$

(c) $\frac{32}{15}$ (d) 0

84. If $f(x) = \frac{\sin(e^{x-2}-1)}{\ln(x-1)}$, then $\text{Lim}_{x \rightarrow 2} f(x)$ is equal to

(a) -2 (b) -1

(c) 0 (d) 1

85. Consider the following statements:

(1) $f(x) = \ln x$ is an increasing function on $(0, \infty)$.

(2) $f(x) = e^x - x(\ln x)$ is an increasing function on $(1, \infty)$.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

86. If $s = \sqrt{t^2 + 1}$, then $\frac{d^2s}{dt^2}$ is equal to

(a) $\frac{1}{s}$ (b) $\frac{1}{s^2}$

(c) $\frac{1}{s^3}$ (d) $\frac{1}{s^4}$

87. Consider the following statements:

Statement 1: The function $f: R \rightarrow R$ such that $f(x) = x^3$ for all $x \in \mathbb{R}$ is one-one.

Statement 2: $f(a) \approx f(b) \Rightarrow a = b$ for all $a, b \in R$ if the function f is one-one.

Which one of the following is correct in respect of the above statements?

(a) Both the statements are true and Statement 2 is the correct explanation of Statement 1.

(b) Both the statements are true and Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true but Statement 2 is false.

(d) Statement 1 is false but Statement 2 is true.

88. $\int \frac{dx}{1+e^{-x}}$ is equal to

(a) $1 + e^x + c$ (b) $\ln(1 + e^{-x}) + c$

(c) $\ln(1 + e^x) + c$ (d) $2\ln(1 + e^{-x}) + c$

where c is the constant of integration

89. $\int_{-1}^{1} x|x| dx$ is equal to

(a) 0 (b) $\frac{2}{3}$

(c) 2 (d) -2

90. The area bounded by the coordinate axes and the curve $\sqrt{x}+\sqrt{y}=1$, is

(a) 1 square unit (b) $\frac{1}{2}$ square unit

(c) $\frac{1}{3}$ square unit (d) $\frac{1}{6}$ square unit

For the next (91 to 92) items that follow:
Consider the function

$$f(x)=\left(\frac{1}{x}\right)^{2x^2}, \text{where } x>0$$

91. At what value of x does the function attain maximum value?

(a) e (b) $\sqrt{e}$

(c) $\frac{1}{\sqrt{e}}$ (d) $\frac{1}{e}$

92. The maximum value of the function is

(a) e (b) $e^{\frac{2}{e}}$

(c) $e^{\frac{1}{e}}$ (d) $\frac{1}{e}$

For the next (93 to 94) items that follow:
Consider $f'(x)=\frac{x^2}{2}-kx+1$ such that $f(0)=0$ and $f(3)=15$

93. The value of k is

(a) $\frac{5}{3}$ (b) $\frac{3}{5}$

(c) $-\frac{5}{3}$ (d) $-\frac{3}{5}$

94. $f''\left(-\frac{2}{3}\right)$ is equal to

(a) -1 (b) $\frac{1}{3}$

(c) $\frac{1}{2}$ (d) 1

For the next (95 to 96) items that follow:
Consider the function

$$f(x)=-2x^3-9x^2-12x+1$$

95. The function $f(x)$ is an increasing function in the interval

(a) $(-2,-1)$ (b) $(-\infty,-2)$

(c) $(-1,2)$ (d) $(-1,\infty)$

96. The function $f(x)$ is a decreasing function in the interval

(a) $(-2,-1)$ (b) $(-\infty,-2)$ only

(c) $(-1,\infty)$ only (d) $(-\infty,-2)\cup(-1,\infty)$

For the next (97 to 98) items that follow:
Consider the integrals

$$A=\int_0^{\pi}\frac{\sin x dx}{\sin x+\cos x} \text{ and } B=\int_0^{\pi}\frac{\sin x dx}{\sin x-\cos x}$$

97. Which one of the following is correct?

(a) $A=2B$ (b) $B=2A$

(c) $A=B$ (d) $A=3B$

98. What is the value of B ?

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$

(c) $\frac{3\pi}{4}$ (d) π

For the next (99 to 100) items that follow:
Consider the function

$$f(x) = \begin{cases} -2\sin x & \text{if } x \leqslant -\frac{\pi}{2} \\ A\sin x + B & \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2} \\ \cos x & \text{if } x \geqslant \frac{\pi}{2} \end{cases}$$

which is continuous everywhere.

99. The value of A is

(a) 1 (b) 0

(c) -1 (d) -2

100. The value of B is

(a) 1 (b) 0

(c) -1 (d) -2

101. The degree of the differential equation $\frac{dy}{dx} - x = \left(y - x\frac{dy}{dx}\right)^{-4}$ is

(a) 2 (b) 3

(c) 4 (d) 5

102. The solution of $\frac{dy}{dx} = \sqrt{1 - x^2 - y^2 + x^2y^2}$ is

(a) $\sin^{-1} y = \sin^{-1} x + c$

(b) $2\sin^{-1} y = \sqrt{1 - x^2} + \sin^{-1} x + c$

(c) $2\sin^{-1} y = x\sqrt{1 - x^2} + \sin^{-1} x + c$

(d) $2\sin^{-1} y = x\sqrt{1 - x^2} + \cos^{-1} x + c$

where c is an arbitrary constant

103. The differential equation of the family of circles passing through the origin and having centres on the x-axis is

(a) $2xy\frac{dy}{dx} = x^2 - y^2$ (b) $2xy\frac{dy}{dx} = y^2 - x^2$

(c) $2xy\frac{dy}{dx} = x^2 + y^2$ (d) $2xy\frac{dy}{dx} + x^2 + y^2 = 0$

104. The order and degree of the differential equation of parabolas having vertex at the origin and focus at $(a, 0)$ where $a > 0$, are respectively

(a) 1,1 (b) 2,1

(c) 1,2 (d) 2,2

105. $f(xy) = f(x) + f(y)$ is true for all

(a) Polynomial functions f

(b) Trigonometric functions f

(c) Exponential functions f

(d) Logarithmic functions f

106. Three digits are chosen at random from $1, 2, 3, 4, 5, 6, 7, 8$ and 9 without repeating any digit. What is the probability that the product is odd?

(a) $\frac{2}{3}$ (b) $\frac{7}{48}$
(c) $\frac{5}{42}$ (d) $\frac{5}{108}$

107. Two events A and B are such that $P(\text{ not } B)=0.8, P(A \cup B)=0.5$ and $P(A \mid B)=0.4$. Then $P(A)$ is equal to

(a) 0.28 (b) 0.32
(c) 0.38 (d) None of the above

108. If mean and variance of a Binomial variate X are 2 and 1 respectively, then the probability that X takes a value greater than 1 is

(a) $\frac{2}{3}$ (b) $\frac{4}{5}$
(c) $\frac{7}{8}$ (d) $\frac{11}{16}$

109. Seven unbiased coins are tossed 128 times. In how many throws would you find at least three heads?

(a) 99 (b) 102
(c) 103 (d) 104

110. A coin is tossed five times. What is the probability that heads are observed more than three times?

(a) $\frac{3}{16}$ (b) $\frac{5}{16}$
(c) $\frac{1}{2}$ (d) $\frac{3}{32}$

111. The geometric mean of the observations $x_1, x_2, x_3, \ldots x_n$ is G_1. The geometric mean of the observations $y_1, y_2, y_3, \ldots . y_n$ is G_2. The geometric mean of observations $\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, \ldots \frac{x_n}{y_n}$ is

(a) $G_1 G_2$ (b) $\ln(G_1 G_2)$
(c) $\frac{G_1}{G_2}$ (d) $\ln\left(\frac{G_1}{G_2}\right)$

112. The arithmetic mean of $1, 8, 27, 64, \ldots$.. up to n terms is given by

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n(n+1)^2}{2}$

(c) $\frac{n(n+1)^2}{4}$ (d) $\frac{n^2(n+1)^2}{4}$

113. An unbiased coin is tossed until the first head appears or until four tosses are completed, whichever happens earlier. Which of the following statements is/are correct?

(1) The probability that no head is observed is $\frac{1}{16}$.

(2) The probability that the experiment ends with three tosses is $\frac{1}{8}$.

Select the correct answer using the code given below:

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

114. If $x \in [0,5]$, then what is the probability that $x^2-3x+2 \geqslant 0$?

(a) $\frac{4}{5}$ (b) $\frac{1}{5}$
(c) $\frac{2}{5}$ (d) $\frac{3}{5}$

115. A bag contains 4 white and 2 black balls and another bag contains 3 white and 5 black balls. If one ball is drawn from each bag, then the probability that one ball is white and one ball is black is

(a) $\frac{5}{24}$ (b) $\frac{13}{24}$
(c) $\frac{1}{4}$ (d) $\frac{2}{3}$

116. A problem in statistics is given to three students A, B and C whose chances of solving it independently are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that the problem will be solved is

(a) $\frac{1}{12}$ (b) $\frac{11}{12}$

(c) $\frac{1}{2}$ (d) $\frac{3}{4}$

117. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probabilities of an accident involving a scooter driver, car driver and a truck driver are 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. The probability that the person is a scooter driver is

(a) $\frac{1}{52}$ (b) $\frac{3}{52}$

(c) $\frac{15}{52}$ (d) $\frac{19}{52}$

118. A coin is tossed 5 times. The probability that tail appears an odd number of times, is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{5}$ (d) $\frac{1}{5}$

119. The regression coefficients of a bivariate distribution are -0.64 and -0.36 . Then the correlation coefficient of the distribution is

(a) 0.48 (b) -0.48

(c) 0.50 (d) -0.50

120. What is the probability that the sum of any two different single digit natural numbers is a prime number?

(a) $\frac{5}{27}$ (b) $\frac{7}{18}$

(c) $\frac{1}{3}$ (d) None of the above